COMMUN. STATIST.—THEORY METH., 22(9), 2619-2622 (1993)

COEFFICIENT OF VARIATION FOR THE $\mathcal{L}$-CLASS OF LIFE DISTRIBUTIONS

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Key Words : Coefficient of Variation

ABSTRACT

It is shown that the coefficient of variation for the $\mathcal{L}$-class of life distributions is less than or equal to 1.We prove that a d.f. F, belonging to the $\mathcal{L}$-class of life distributions , is exponential if and only if its coefficient of variation is equal to 1.

1. INTRODUCTION AND SUMMARY

Klefsjö (1983) has defined and extensively studied the $\mathcal{L}$-class of life distributions. A distribution function F with support on $[0,\infty)$ and finite mean μ is said to belong to $\mathcal{L}$-class if for $s \geq 0$

$$\int_0^\infty e^{-st}\,\overline{F}(t)\,dt \geq \frac{\mu}{1+s\mu}, \tag{1.1}$$

where $\overline{F}=1-F$. The right hand side of (1.1) may be seen to be the Laplace transform of an exponential distribution with mean μ. It is easy to show that (see Klefsjö (1983)) the $\mathcal{L}$-class is strictly larger than the harmonically new better than used in expectation (HNBUE) class of life distributions.Hence the $\mathcal{L}$-class also contains the smaller NBUE, NBU, IFRA and IFR classes of life distributions.

Klefsjö (1982) has shown that the coefficient of variation for the HNBUE class of life distributions is less than or equal to 1.Klefsjö's work was supplemented by Basu and Bhattacharjee (1984) who prove that F,belonging to the HNBUE class of life distributions ,is exponential if and only if its coefficient of variation is equal to 1.We obtain the similar results for the $\mathcal{L}$-class of life distributions.

2. RESULTS

We first prove a few lemmas.

Lemma 2.1: If F belongs to the $\mathcal{L}$-class of life distributions with finite mean μ, then

$$\int_0^\infty e^{-st}\, dF(t) \leq (1+s\mu)^{-1}, \quad s \geq 0 \tag{2.1}$$

Proof: We have

$$\int_0^\infty e^{-st}\, \bar{F}(t)dt = \int_0^\infty e^{-st}(1-F(t))dt = \Big(1 - \int_0^\infty e^{-st}\, dF(t)\Big)/s.$$

Now by applying (1.1),we get the result. □

Lemma 2.2: If F belongs to the $\mathcal{L}$-class of life distributions with mean μ,then

$$\int_0^\infty t^2\, dF(t) \leq 2\mu^2 \tag{2.2}$$

Proof: Define

$$g(s) = \int_0^\infty e^{-st} dF(t) - (1+s\mu)^{-1}, \quad s \geq 0$$

Then $g(s) \leq 0$ for all $s \geq 0$, by Lemma 2.1.

Note that $g(0)=0=g'(0)$ and $g''(0) = \int_0^\infty t^2 dF(t) - 2\mu^2$.

Using the Taylor's expansion (Lagrange's form),we can write :

$$g(s)=g(0) + g'(0)s + g''(\xi)(s^2/2) \text{ for some } \xi, \ 0<\xi<s.$$

Thus, $$g''(\xi)=\Big\{ 2g(s)/s^2 \Big\} \leq 0.$$

Now making $s \to 0+$, we get $g''(0) \leq 0$. This proves the lemma. □

Now we state the key result in the following theorem.

Theorem 2.3 : The coefficient of variation for the $\mathcal{L}$-class of life distributions is less than or equal to one.

Proof: Let X be distributed as $F\epsilon\mathcal{L}$ with mean μ and variance σ^2.

Thus, C.V. $\leq 1 \Leftrightarrow (\sigma/\mu)^2 \leq 1 \Leftrightarrow \sigma^2 \leq \mu^2 \Leftrightarrow E\,X^2 \leq 2\mu^2$.

Now the theorem follows by Lemma 2.2. □

Theorem 2.4: Suppose F belongs to the $\mathcal{L}$-class of life distributions with finite mean μ.Then F is exponential if and only if its coefficient of variation is equal to one.

Proof: That the coefficient of variation for exponential distribution is equal to 1 is trivial.

Conversely, suppose that the C.V.=1. We have to show that F is exponential. If possible, suppose F is not exponential.Since the Laplace transform uniquely determines a d.f., for $F\epsilon\ \mathcal{L}$,but not exponential,we have

$$\int_0^\infty e^{-st}\, dF(t) < (1 + \mu s)^{-1},\ s>0.$$

Thus, we have $g''(\xi) < 0$ for some ξ, $0<\xi<s$.Here $g(\cdot)$ is as in Lemma 2.2.Now choose s to be sufficiently small.Hence, due to the continuity of $g''(x)$ at $x=\xi$, we have $g''(0)<0$.Consequently, C.V.<1,which is a contradiction.This completes the proof of the theorem. □

ACKNOWLEDGEMENT

The author is grateful to the referee for many helpful suggestions which have subtantially improved on an earlier version of this paper.

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Received November 1992; Revised March 1993