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A Note on a Unified Approach to the Frontier Production Function Models With Correlated Non-Normal Error Components: The Case of Cross Section Data

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ABSTRACT

This paper reviews the Stochastic Frontier Production Function (SFPF) Models with error components, termed as technical inefficiency error and noise variable in a cross section framework. It tries to unify all such models incorporating (i) truncation or non-normality of the technical inefficiency error term and (ii) correlation between the two error components. Allocative inefficiency error terms are also introduced and are assumed to be correlated among themselves.

JEL Classification: C10

Keywords : Frontier Production Function; Non - Normal Errors; Truncated Errors; Allocative Inefficiency; Technical Inefficiency

1. INTRODUCTION

The Frontier Production Function (FPF) can be defined as the locus of maximum possible output levels for different levels of input combinations. The output of each firm is thus bounded above by a frontier. The frontier function becomes stochastic if a noise variable is introduced besides the inefficiency error. The error is then composed of two additive terms - (i) noise variable which is assumed to be normally distributed and (ii) inefficiency error which takes only negative values. More specifically, the model can be written as:

$$Y = g(X_1, X_2,\ldots, X_n) \exp(\tau + v), \tag{1}$$

where v is the general statistical noise variable and τ is the inefficiency error term giving rise to the technical inefficiency of the firm. Y is the level of output and X_i's are inputs. Since $\tau \leq 0$, the firm is said to be maximum efficient if $\tau = 0$ and the firm is on the frontier. If we ignore the noise variable, the firms are either efficient ($\tau = 0$) or have various degrees of inefficiency depending on the specific values of τ. Thus the scatter points lie on or below the frontier function. It is the noise variable which introduces flexibility to the frontier function. In that case some of the points of the scatter diagram may lie above the frontier function. However, the calculation of the efficiency of a firm is done through the value of τ. Besides, there is another type of inefficiency of firm which is known as allocative inefficiency. Allocative inefficiency of a firm can not be introduced in a single equation model. It is necessary to take a system approach.

A firm either maximizes it's profit or output function subject to a given level of costs of inputs or minimizes it's cost function subject to a given level of output. The solution is the optimum combination of input levels.[1] If a firm is able to use the optimum levels of inputs then the firm is called allocative efficient. Otherwise the firm is allocative inefficient. Allocative inefficiency thus comes from the first order conditions:

$$w_i = p\,(\frac{\partial g}{\partial x_i})\exp(\tau) \text{ for all}^2 \text{ i},$$

where w_i is the ith input price and p is the output price. It is here the distortion term from the optimum level is introduced for each input to capture the allocative inefficiencies of the firm (Kumbhakar (1987)). The first order conditions are rewritten as·

$$w_i = p\,(\frac{\partial g}{\partial x_i})\exp(\tau)\exp(u_i) \text{ for all i,} \qquad (2)$$

where u_i is the error term of the ith input giving rise to allocative inefficiency. Thus $u_i = 0$ means the firm is allocative efficient.

Rewriting equations (1) and (2) we get,

$$\tau + v = f(y, \underline{x}) \qquad \text{... (3a)}$$

$$\tau + u_i = f_i(y, \underline{x}), \text{ for all i.} \qquad \text{... (3b)}$$

The observations are y and x_i values of the firms and the prices of inputs and the output. Once the joint distribution of τ, v and u_i's are defined, it is possible to get the maximum likelihood estimates of the parameters of the model.

2. THE DISTRIBUTIONAL ASSUMPTIONS OF THE ERROR COMPONENTS

In the standard literature it is assumed that τ, v and u_i's are independent. In this case the joint pdf is $f_\tau.f_v.f_u$, the product of the densities of τ, v and u. E.g., v and u_i's can be assumed to follow a normal distribution and the technical inefficiency term is assumed

1 We shall only discuss maximization of profit/output function. A Frontier Cost Function may analogously be defined and technical and allocative inefficiency found (See for example: Schmidt and Lovell (1979, 1980), Kopp and Diewert (1982), Bauer (1990), Ferrier and Lovell (1990)).

2 For maximization, the noise variable v is ignored. The existence of v makes the model flexible and the estimates robust against extreme observations (Pal, Ghosh and Neogi (2003)).

to follow a half-normal distribution (Aigner, Lovel and Schmidt (1977), Pal, Neogi and Ghosh (1998)). If moreover u_i's are also assumed to be independent of each other then the joint distribution of τ, v and u ($u = (u_1, u_2, \ldots, u_n)^T$) is then

$$f_\tau . f_v . f_u = \{(2/\sqrt{2\pi}\,\sigma_\tau)\exp(-\tau^2/(2\sigma_\tau^2))\}.\{(1/\sqrt{2\pi}\,\sigma_v)\exp(-v^2/(2\sigma_v^2))\}.$$

$$\{(1/(\sqrt{2\pi})^{n/2}\,\sigma_1.\,\sigma_2 \ldots \sigma_n)\exp(-\Sigma u_i^2/2\sigma_i^2)\}. \quad -\infty<\tau<0,\ -\infty<v<\infty \text{ and } -\infty<u_i<\infty,$$

where σ_τ, σ_V and $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ are the standard deviations of τ, v and u's respectively.

An immediate generalization of the above formulation is to the multivariate normal distribution of u_i's. The product of the individual pdf's of the u_i's are replaced by the multivariate normal density as given below.

$$f_\tau . f_v . f_u = \{(2/\sqrt{2\pi}\,\sigma_\tau)\exp(-\tau^2/(2\sigma_\tau^2))\}.\{(1/\sqrt{2\pi}\,\sigma_v)\exp(-v^2/(2\sigma_v^2))\}.$$

$$\{(1/(\sqrt{2\pi})^{n/2}\,|\Sigma^{-1/2}|)\exp(-\underline{u}^T\Sigma^{-1}\underline{u}/2)\}. \quad -\infty<\tau<0,\ -\infty<v<\infty \text{ and } -\infty<u_i<\infty,$$

where Σ is the covariance matrix of the random vector u.

The u_i's in this case are assumed to be correlated among themselves.

The assumption of half-normal distribution of τ has been criticized in the literature on the ground that it gives the maximum frequency at $\tau = 0$, i.e., there are maximum number of firms near the frontier line. This is unlikely. Instead we can take a truncated normal distribution for τ (Stevenson (1980), Kumbhakar (1987)). This necessitates the following change in the pdf of τ.

$$f_\tau = (1/\sqrt{2\pi}\,\sigma_\tau)\exp(-(\tau-\mu_\tau)^2/(2\sigma_\tau^2)) / \int_{-\infty}^{0} (1/\sqrt{2\pi}\,\sigma_\tau)\exp(-(\tau-\mu_\tau)^2/(2\sigma_\tau^2))d\tau, \quad -\infty<\tau<0,$$

where μ_τ is the mean value of the untruncated τ.

This is in fact a generalization of the half-normal distribution. $\mu_\tau = 0$ leads the distribution to the half-normal. Even here $\mu_\tau \geq 0$ implies that there is a maximum frequency near the frontier function (Diagram 1). Thus it captures all the possibilities.

We often face situations where there are only a very few observations near the frontier curve. The assumption of truncated normal distribution may not be tenable in these situations. Ideally one should seek a unimodal density function which has zero value at zero like negative gamma (Greene (1980,1990)), negative exponential or negative lognormal[3].

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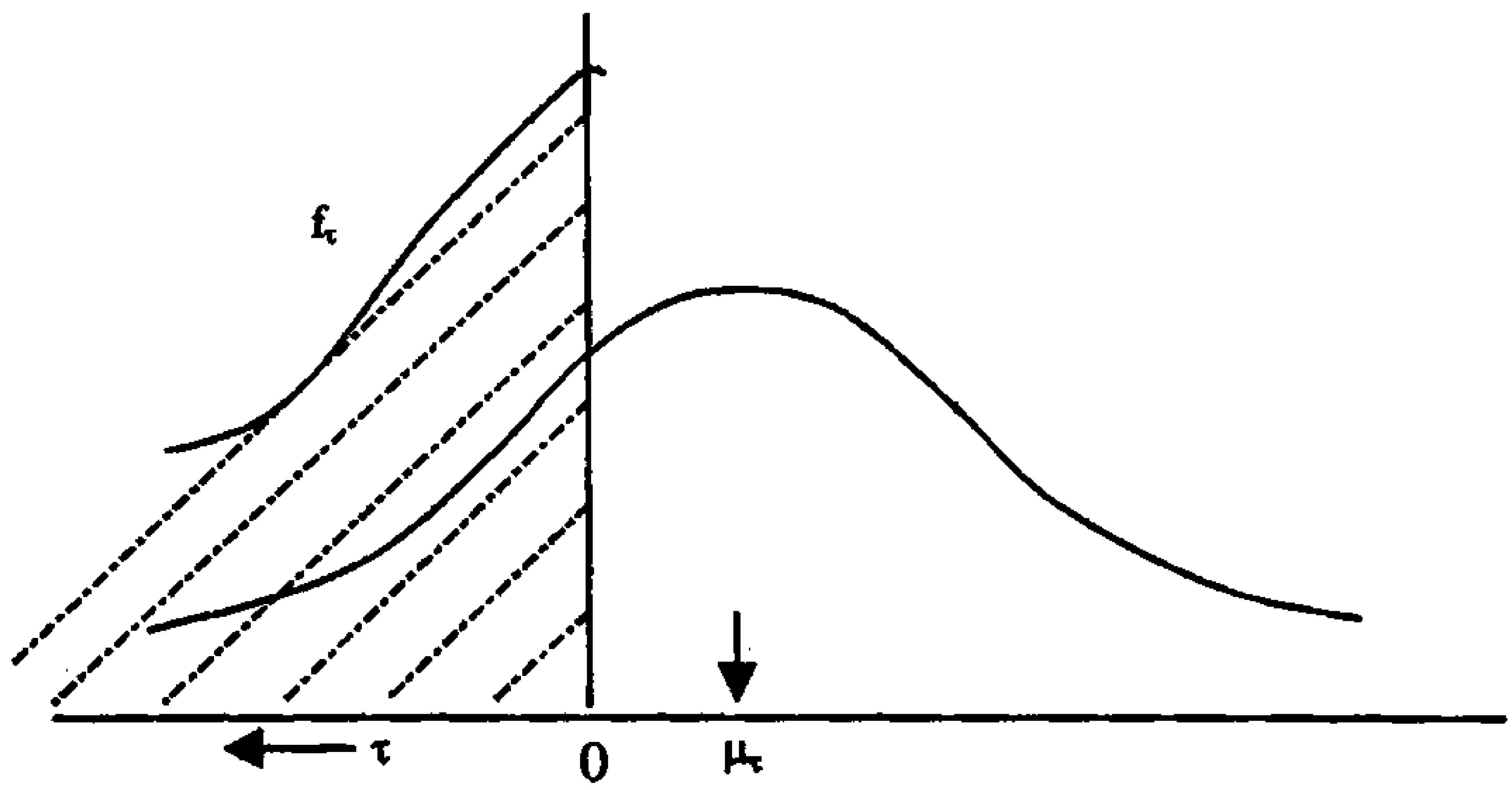
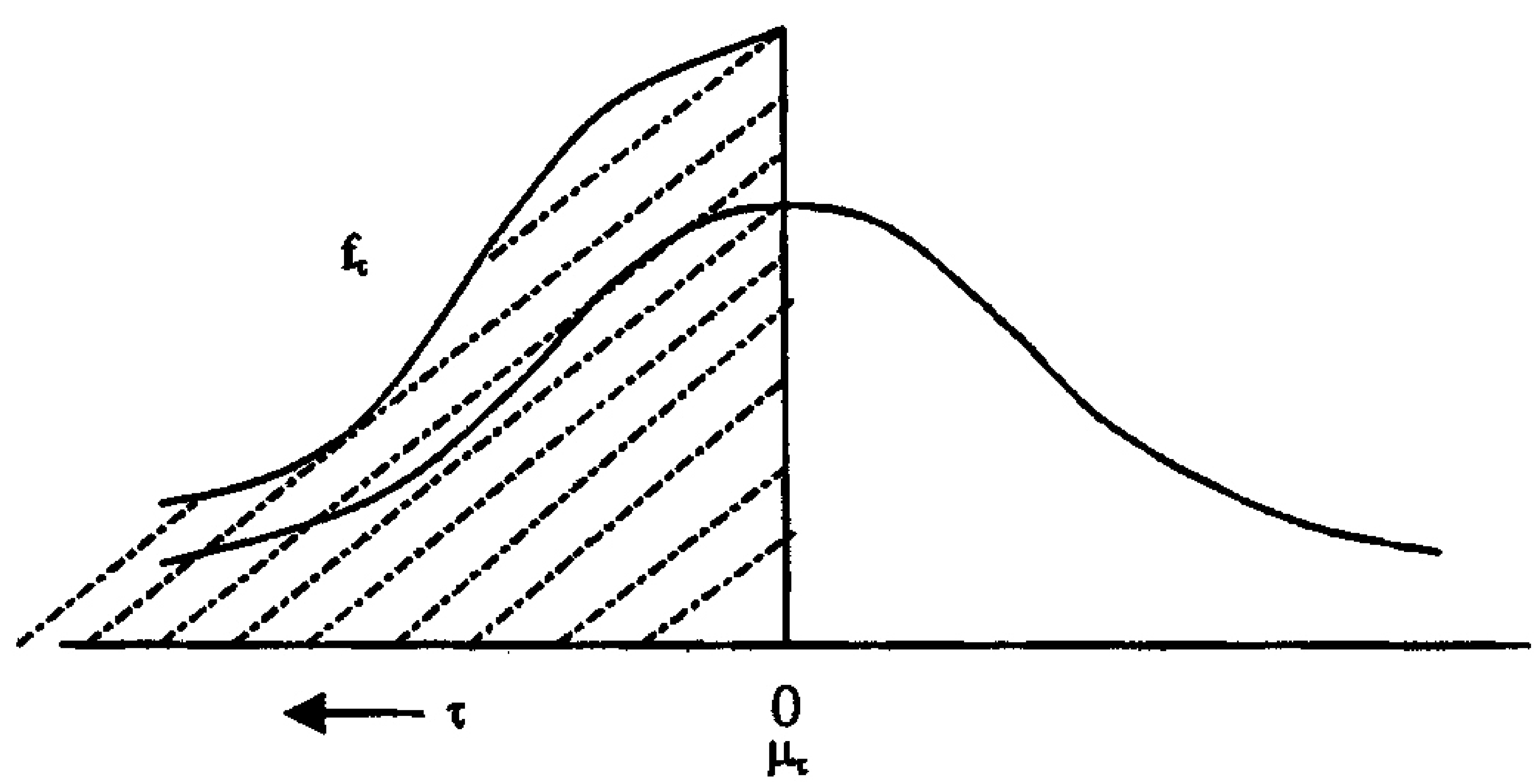
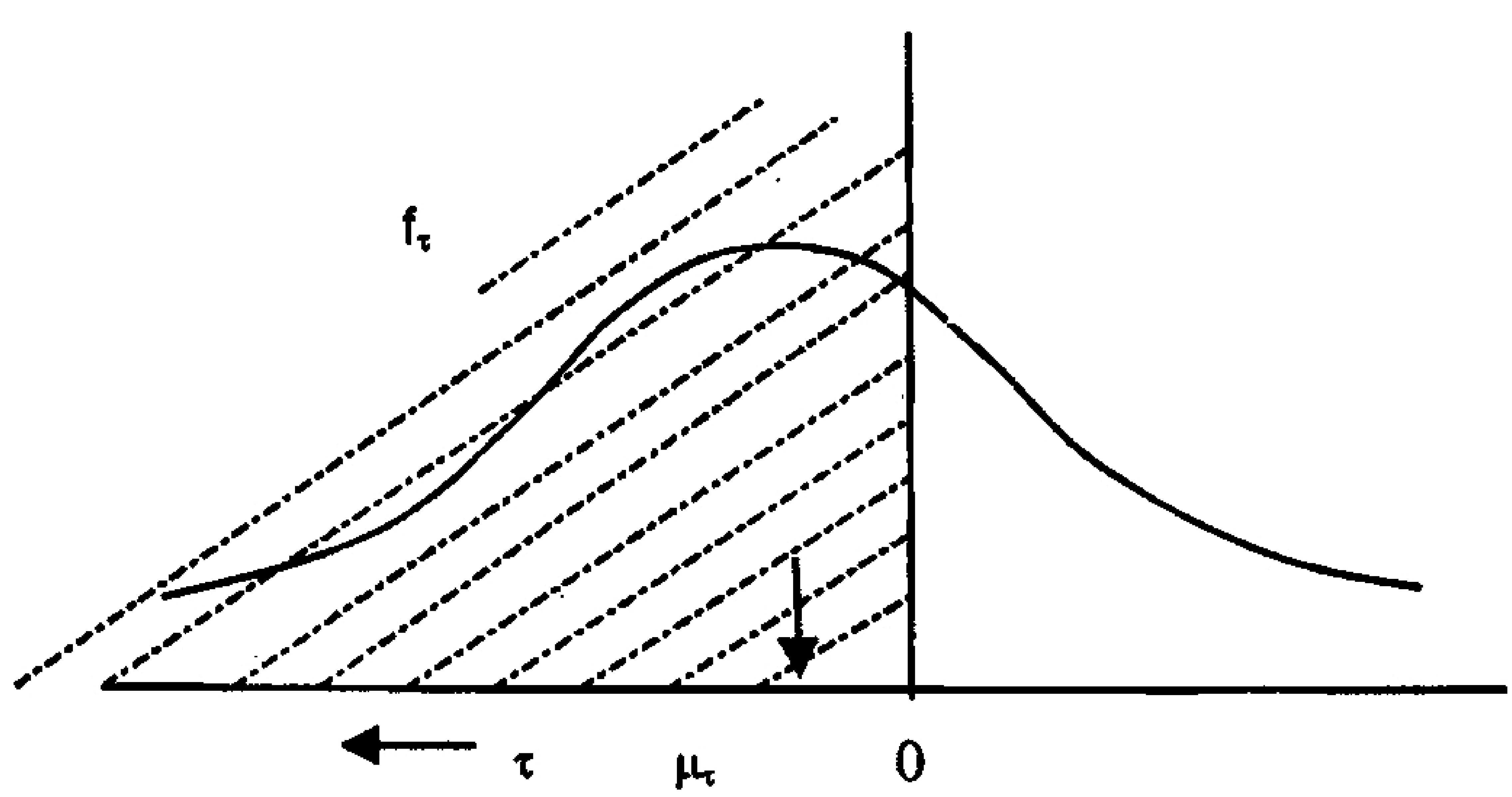
3 The readers are referred to the following papers: Afriat (1972), Richmond (1974), Schmidt (1976), Aigner et al. (1977), Meeusen and Broeck (1977), Broeck et al. (1980) and Stevenson (1980).



The probability density function (f_τ) of τ for $\mu_\tau>0$



The probability density function (f_τ) of τ for $\mu_\tau=0$



The probability density function (f_τ) of τ for $\mu_\tau<0$

Diagram 1: The probability density functions of τ truncated at 0 for different values of μ_τ.

A negative gamma distribution for t can be defined as

$$f(\tau) = (\theta^p/\Gamma(p))\,(-\tau)^{p-1}\exp(-\theta\,(-\tau))$$
$$\text{i.e., } -\tau = \Gamma(\theta,p), \text{ with}$$
$$\mu_\tau = E(\tau) = -p/\theta,\ \sigma_\tau^2 = p/\theta^2 \text{ and } E(-\tau)^r = \Gamma(p+r)/\{\theta^r\,\Gamma(p)\}.$$

There is no problem of finding the joint distribution if the technical and allocative inefficiencies are assumed to be independent. But they are likely to be dependent (Greene (1980)). Schmidt and Lovell (1980) put forward a model of stochastic frontiers where technical and allocative inefficiencies are assumed to be correlated. Earlier Farrell (1957), Johansen (1972), Forsund and Hjalmarsson (1974) noted that the relation between technical and allocative inefficiency is relevant in time series data. There is no reason why these two errors should not be independent in a cross section data also. Formulation of the models assuming dependencies between these inefficiency error terms were done by Schmidt and Lin (1984), Forsund, Lovel and Schmidt (1980), Kumbhakar (1987), Schmidt and Lovel (1980) and many others.

The question of relation between the noise variable and the inefficiency error can also be raised. The random shock leading to higher output is likely to influence management to become more efficient. One may argue that the problem here is also a dynamic one (Pal and Sengupta (1999)). Nevertheless, this model enables us in testing whether there is any correlation between the two error components. So far as τ, v and u are independent we do not have any problem in finding the joint pdf as this is nothing but the product of individual pdfs. But once dependency between τ and v are introduced, it is necessary to define their joint pdf. The problem arises because τ is not normal and is not defined over the whole real line. Here we introduce a novel idea. We first define the conditional distribution of v given t and then multiply the conditional pdf of τ to get the joint pdf. The conditional pdf can be taken as normal with mean

$$\rho(\sigma_v/\sigma_\tau)(\tau-\mu_\tau)$$

and variance $\sigma_v^2(1-\rho^2)$ as we get from a bivariate normal distribution with ρ being the correlation coefficient between v and τ. Symbolically,

$$v|\tau \sim N\{\rho(\sigma_v/\sigma_t)\,(\tau-\mu_\tau),\ \sigma_v^2(1-\rho^2)\}$$

leading to the joint pdf as

$$f_{v,\tau} = f_{v|\tau}\cdot f_\tau,\quad -\infty<\tau<0,\ -\infty<v<\infty.$$

In a similar manner we can extend this idea to u and τ also (Pal and Sengupta (2003)). E.g., let us assume truncated normal distribution for the technical inefficiency ~~component~~ τ instead of taking half-normal. Symbolically, we can write

(a) $v \sim N(\mu_v, \sigma_v^2)$,

(b) $(u, \tau) \sim$ Truncated $N_{n+1}((\mu, \Sigma)$,

where $-\infty < v < \infty$, $-\infty < u_i < \infty$ and $-\infty < \tau < h$, and

$$\mu = (\mu_{\tau,}\bar{0}), \qquad \Sigma = \begin{bmatrix} \sigma_\tau^2 & \sigma_{\tau 1} & \dots & \sigma_{\tau n} \\ \sigma_{\tau 1} & \sigma_1^2 & \dots & \sigma_{1n} \\ \dots & \dots & \dots & \dots \\ \sigma_{\tau n} & \sigma_{1n} & \dots & \sigma_n^2 \end{bmatrix}$$

More precisely, we are assuming t to be truncated normal and $u|\tau$ to be normal, and $f(u,\tau)$ is written as:

$$f(u,\tau) = f(u|\tau)\ f(\tau).$$

Thus the density function of (u,τ) is given by

$$f(u,\tau)=f(u|\tau)f(\tau)=\Phi^{-1}((h-\mu_\tau)/\sigma_\tau)(2\pi)^{-(n+1)/2}\sigma_\tau^{-1}|\Sigma_u-\sigma_{\tau u}\sigma_\tau^{-2}\sigma_{\tau u}'|^{-1/2}\exp(-Q/2), \qquad (5)$$

where $Q = ((\tau-\mu_\tau)/\sigma_\tau)^2 + (u-(\tau-\mu_\tau)\sigma_\tau^{-2}\sigma_{\tau u})'(\Sigma u-\sigma_{\tau u}\sigma_\tau^{-2}\sigma_{\tau u}')^{-1}(u-(\tau-\mu_\tau)\sigma_\tau^{-2}\sigma_{\tau u})$,

and $-\infty < u_i < \infty$ and $-\infty < \tau < h$,

and h is the point of truncation which is usually taken to be zero.

3. DERIVATION OF THE LIKELIHOOD FUNCTION AND ESTIMATION OF EFFICIENCIES

Once the specification of joint distribution function is complete for a firm, it is the routine work to find the likelihood function of all the observations by taking product of the joint pdfs of each firm and viewing it as a function of the parameters. One can then maximize it to estimate the underlying parameters.

The relations between v, τ and u with the observations y, $x_1, x_2, \dots, x_n$ are as follows

$$Y_f = g(X_{1f}, X_{2f}, \dots, X_{nf}) \exp(\tau_f + v_f),$$

And $w_{if} = p_f\,(\frac{\partial g}{\partial x_i}) \exp(\tau_f)$, for all $i = 1, 2, \dots, n$ and $f = 1, 2, \dots, F$,

Where F is the number of firms. We shall henceforth suppress the firm specific subscripts. Substituting $z_1=v+\tau$, $z_2=u+l\tau$ and $z_3=\tau$ where $l=(1\ 1\ 1 \dots\ 1)'$ and noting that Jacobian of transformation is one, we can easily get $f(z_1, z_2, z_3)$, the joint pdf of z_1, z_2 and z_3. The density function of (z_1, z_2) can be obtained by integrating $f(z_1, z_2, z_3)$ over z_3 in the appropriate range

$$f(z_1, z_2) = \int_{-\infty}^{h} f(z_1, z_2, z_3)\, dz_3.$$

To get the density function of the observed values namely, y_f, $x_{1f}, \ldots, x_{nf}$ from z_1, z_2, one has to multiply this by the relevant Jacobian of transformation again. The likelihood function is obtained by multiplying these density functions for all observations and writing it as a function of parameters.

The log-likelihood function can be maximized to estimate the parameters of the model. We can then estimate inefficiency errors namely the technical inefficiency error τ_f and the allocative inefficiency errors u_{if}'s for each firm f by taking conditional expectation of τ_f given $\tau_f + v_f$ in order to get an indirect estimate of τ_f for the technical efficiency. We can similarly find the conditional expectation of u_{if} given $(\tau_f + v_f, \tau_f + u_{1f}, \ldots, \tau_f + u_{nf})$ for the allocative inefficiency.

We can now illustrate the above formulation by taking specific cases to see how it can be done. Following Kumbhakar (1987) the relations between v, τ and u with the observations y, $x_1, x_2, \ldots, x_n$ can be taken as:

$$\ln y_f = \ln \beta_0 + \sum_{i=1}^{n} \alpha_i \ln x_{if} + \tau_f + v_f$$

$$\ln(w_{if}/p_f) - \ln\beta_0 - \ln\alpha_i - \ln x_i - \sum_{k=1}^{n} x_k \ln x_{kf} = \tau_f + u_{if} \quad (7)$$

Assuming t to follow a truncated normal distribution $f(z_1, z_2, z_3)$ in this case would be[4]

$$f(z_1, z_2, z_3) = \Phi^{-1}((h-\mu_\tau)/\sigma_\tau)(2\pi)^{-(n+2)/2}\sigma_\tau^{-1}\sigma_v^{-1}|\Sigma_u - \sigma_{\tau u}\sigma_\tau^{-2}\sigma_{\tau u}'|^{-1/2}\exp(-Q_1/2)$$

where $Q_1 = \{(z_3-\mu_0)/\sigma_0\}^2 + a_0$

$$\Sigma_{u\cdot\tau} = (\Sigma_u - \sigma_{\tau u}\sigma_\tau^{-2}\sigma_{\tau u}')$$

$$1/\sigma_0^2 = 1/\sigma_\tau^2 + 1/\sigma_v^2 + l^{*\prime}\Sigma_{u.\tau}^{-1} l^*$$

$$l^* = l + \sigma_{\tau u}\sigma_t^{-2}$$

$$\mu_0 = \sigma_0^2 [(z_1-\mu_v)/\sigma_v^2 + \mu_\tau/\sigma_\tau^2 + l^{*\prime}\Sigma_{u.\tau}^{-1} z_2^*]$$

$$z_2^* = z_2 + \sigma_{\tau u}\sigma_\tau^{-2}\mu_\tau$$

$$a_0 = [(\mu_\tau/\sigma_\tau)^2 + ((z_1-\mu_v)/\sigma_v)^2 + z_2^{*\prime}\Sigma_{u.\tau}^{-1} z_2^*] - (\mu_0/\sigma_0)^2$$

and $-\infty < z_1 < \infty$, $z_2 \in R^n_+$ and $-\infty < z_3 < h$.

The density function of (z_1, z_2) is

$$f(z_1, z_2) = \int_{-\infty}^{h} f(z_1, z_2, z_3)\, dz_3$$

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4 The derivation of the likelihood function and the subsequent efficiencies are given in Pal and Sengupta (2003).

$$= \Phi^{-1}((h-\mu_\tau)/\sigma_\tau)(2\pi)^{-(n+1)/2}\sigma_\tau^{-1}\sigma_v^{-1}|\Sigma_u-\sigma_{\tau u}\sigma_\tau^{-2}\sigma_{\tau u}'|^{-1/2}\ \sigma_0\ \exp(-a_0/2)\ \Phi\text{-}1((h-\mu_0)/\sigma_0)$$

To get the density function of the observed values namely, y_f, x_{1f},.....,x_{nf} one has to multiply this by the relevant Jacobian of transformation $(1-\sum\alpha_i)$. The log likelihood function is then:

$$L(\beta_0,\alpha,\sigma_\tau^2,\sigma_v^2,\Sigma,\mu_\tau) = -(F(n+1)/2)\ \ln(2\pi) - (F/2)\ \ln\sigma_v^2 - (F/2)\ \ln\sigma_\tau^2 - (F/2)\ \ln|\Sigma|$$

$$+(F/2)\ \ln\sigma_0^2 - (1/2)\ \Sigma_f\ a_{0f} - F\ \ln\Phi((h-\mu_\tau)/\sigma_\tau) + \Sigma_f \ln\Phi((h-\mu_{0f})/\sigma_0) + F\ \ln(1-\sum\alpha_i).$$

We can not get observation specific estimates of inefficiency of the firms, because the two errors namely the noise variable and inefficiency error are mingled together and there is no way of separating them. We can however get conditional distribution of τ given τ+v. The expectation or mode of this distribution for each firm would give an estimate of efficiency (Jondrow et al. (1982) and Kalirajan and Flinn (1983)). This is clearly not consistent.

To get an estimate of the technical inefficiency of a firm we first get the conditional distribution of z_3 given z_1 and z_2 which is

$$f(z_3|z_1, z_2) = f(z_1, z_2, z_3)/f(z_1, z_2)$$

$$= \begin{cases} \dfrac{\exp\{-((z_3-\mu_0)/\sigma_0)^2/2\}}{\sqrt{2\pi}\sigma_0\Phi((h-\mu_0)/\sigma_0)} & \textit{for} \quad z_3 \le h \\ 0 & \textit{otherwise.} \end{cases}$$

And then get mode or expectation of this conditional distribution. The mode and the expectation are as given below:

$$M_0(z_3|z_1,z_2) = \begin{cases} \mu_0 & \textit{if} \quad \mu_0 \le h \\ h & \textit{if} \quad \mu_0 > h \end{cases}$$

$$E(z_3|z_1,z_2) = \mu_0 - \sigma_0\ \{\varphi((h-\mu_0)/\sigma_0)\}/\{\Phi((h-\mu_0)/\sigma_0)\}.$$

To get the allocative inefficiency we find the conditional distribution of u_j given z_1 and z_2 for all j and for each observation. For this we take the following transformations of v, τ and u.

$$z_1 = v + \tau, \quad z_2 = u + / \tau \quad z_3 = u_j.$$

The conditional distribution is:

$$f(z_3|z_1, z_2) = f(z_1, z_2, z_3)/f(z_1, z_2)$$

$$= \begin{cases} \dfrac{\exp\{-((z_3 - \mu_{00})/\sigma_{00})^2/2\}}{\sqrt{2\pi}\sigma_{00}\Phi((h-\mu_{00})/\sigma_{00})} & \textit{for} \quad z_{2j} - h < z_3 \\ 0 & \textit{otherwise.} \end{cases}$$

The mode and expectation of this distribution (the subscription f is suppressed) is similarly derived as

$$M_0(z_3 \mid z_1, z_2) = \begin{cases} \mu_{00} & \textit{if} \quad z_{2j} - h \le \mu_{00} \\ z_{2j} - h & \textit{if} \quad z_{2j} - h > \mu_{00} \end{cases}$$

$$E(z_3 \mid z_1, z_2) = \mu_{00} + \sigma_{00} \ \{\varphi((z_{2j}\text{-h-}\mu_{00})/\sigma_{00})\}/\{1\text{-}\Phi((\ z_{2j} \text{ - h-}\mu_{00})/\sigma_{00})\}.$$

It is a routine work to derive the likelihood function and the efficiencies by taking τ to follow a negative gamma distribution.

4. DISCUSSIONS AND CONCLUDING REMARKS

In this paper we have introduced a unified approach towards developing Stochastic Frontier Production Function Models. The unified approach has been illustrated by taking different distributional assumptions on the error terms. The novel idea is the derivation of joint distribution function defined through conditional distribution approach to introduce non-independence of the error components. This was necessary because of the non-normality assumption of the inefficiency error term. The form of the production function has also been kept open. Some of the specific forms were however taken for illustrations. One can also take translog or other forms of the production function (Kumbhakar (1991), Battese and Broca (1997), Kumbhakar and Lovell (2000), Fried, Lovell and Schmidt (1993), Greene (1997), etc.)

It should be mentioned here that there are several other estimation procedures than the Maximum Likelihood Method to estimate the parameters of the Frontier Production Function. E.g., Richmond (1974) suggested Correlated OLS estimates in which OLS intercept is shifted upwards appropriately. This estimate is consistent (Gabrielson (1975), Greene (1980)), but asymptotically less efficient than ML (Olsen et al. (1980)).

Though our discussion is confined to the case where only cross section data are available, it is worth noting that many avenues are opened up when one has panel data. In a panel data model consistent estimate of firm level efficiencies is possible. Hoch (1955, 1962) was first to introduce panel data model to estimate firm level efficiencies using average production function. Later similar techniques were employed for frontier production function and many generalizations were possible (Battese and Coelli (1988, 1991), Kumbhakar (1988,1990,1991), Seale (1990), Heshmati, Kumbhakar and Hjalmarsson (1995), Kumbhakar, Heshmati and Hjalmarsson (1997), Schmidt and Sickles (1984) etc.).

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