Quality Engineering, 6(1), 39-56 (1993-94)

IMPROVING PARAFFIN WAX YIELD THROUGH PROCESS OPTIMIZATION USING TAGUCHI'S METHOD OF EXPERIMENTATION

K. N. ANAND
SQC Unit
Indian Statistical Institute
Bangalore 560 059, India

Key Words

Yield; Slack wax; Paraffin wax; Oil extraction; Orthogonal arrays; Process optimization; Linear graphs.

Experimentation in Industry

Experiments in industry are carried out to improve quality and yield, or to reduce the cost. In the conventional approach, one factor is studied while the others are kept constant. When the influencing factors are few, it may be possible to arrive at an optimum condition using such an approach. However, when the factors are numerous, it is difficult to find the solution through such a conventional approach. It becomes complex when the interaction between two or more factors is also present, which is quite common in process industries.

Here, statistical designs, like factorial experiments and response surface designs are available. However, the former requires a large number of experimental runs. Most of the information can be obtained at less cost by resorting to fractional factorial experiments which involve confounding of higher-order interactions. Fractional factorial experiments can be constructed by the use of orthogonal arrays. It has been investigated by Rao (1), Kempthorne (2), Plackette and Burman (3), Addelman (4), and Taguchi (5). The process of assigning an orthogonal array to a specific experiment has been made easy by a graphical tool, called a linear graph (6), developed by Taguchi to represent interactions between pairs of columns in an orthogonal array. The use of linear graphs enables a scientist or an engineer to design (7) and analyze complicated experiments without requiring a basic knowledge of the construction of designs using a Galois field.

Manufacturing Process

Slack wax, a by-product in an oil refinery, is used in the manufacture of "Paraffin Wax." Slack wax contains 20 to 25% oil, whereas paraffin wax is permitted to have a maximum of 3.5% oil (8). A hydraulic press process is used for removing excess oil from the slack wax which is melted in a tank at 80 to 90°C. The melted wax is allowed to settle down for about half an hour at room temperature. The sediment is tapped off and the molten material is then slabbed using galvanized iron trays. The slabs, which are about 2 in. in height, are wrapped in a filter canvas cloth and pressed at high pressures in a hydraulic press provided with a hot-water circulation arrangement. All of the low melting fractions and the oil in the slack wax are collected separately. Further impurities of the deoiled wax are removed by acid treatment, where the material passes through decolorization, neutralization, and filtration. The resulting wax is slabbed again using galvanized iron trays. The flow diagram of the manufacturing process is given in Figure 1.

A composite sample drawn at random from the melted wax emerging from the filtration tank is tested for the requirements laid down in the relevant Indian Standard (8), of which the oil content is important. Its maximum permissible content is 3.5% for Type 3 paraffin wax, and, if it is exceeded, the material has to be recycled for extraction for excess oil.

Background

The yield of paraffin wax was 35 to 40% versus an expected yield of 60 to 65% in the chemical plant. Ten to 15% of the finished product was recycled

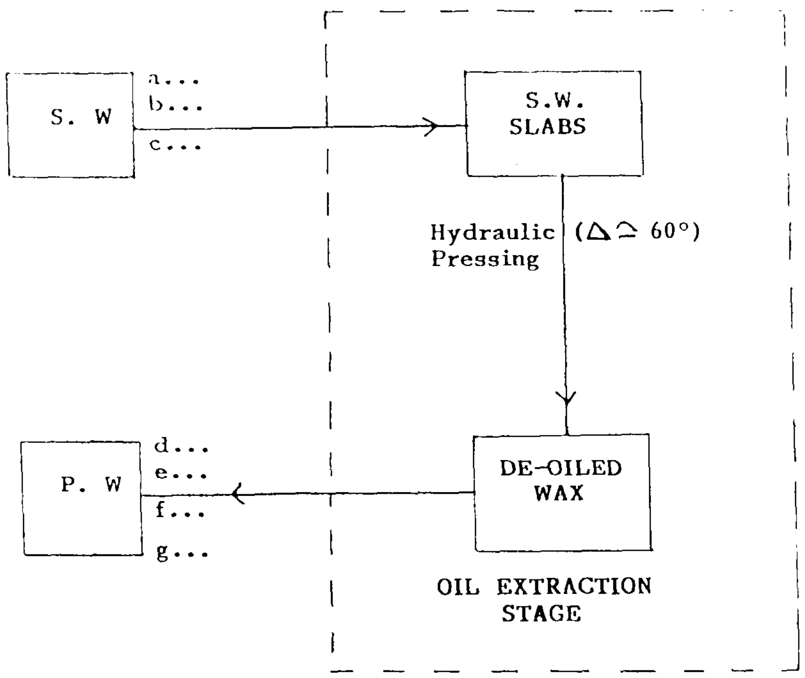

Figure 1. Process flow chart. a—Δ (80–90°C); b—removal of sediment; c—slab casting; d acid treatment; S.W.—slack wax; e—decolorization and neutralization; f—filtration; g slab casting; P.W.–paraffin wax.

because of higher oil content. The actual and expected recovery at different stages are given in Table 1.

It is seen that the maximum loss in recovery is at the oil extraction stage. Therefore, the study was confined to the deoiling stage.

Pressing of slack wax in the hydraulic press is done in four stages. The process specifications at different stages are given in Table 2.

Table 1. Actual and Expected Stagewise Recovery

SERIAL NO.	STAGE	PROCESS	EXPECTED RECOVERY (% BY MASS)	ACTUAL RECOVERY (% BY MASS)
1	Melting of slack wax	Removal of 2–3% impurities	97–98	97–98
2	Deoiling	Removal of excess oil (20–25%)	70–75	40–50
3	Acid treatment, neutralization, and filtration	Decolorization	60–65	35–40

Table 2. Process Parameters and Specification

PROCESS PARAMETERS	SPECIFICATION
a. Temperature of inlet water at the time of pressing of slack wax	65°C
b. Stage I pressing (1100 lb/in.^{2}) time in minutes	28–30
c. Stage II pressing (1550 lb/in.^{2}) time in minutes	10–12
d. Stage III pressing (1800 lb/in.^{2}) time in minutes	5–6
e. Stage IV pressing (2100 lb/in.^{2}) time in minutes	1–2

Investigation

Pressing the wax longer or pressing it at a higher temperature results in low recovery. Similarly, pressing the wax for less time or at a lower temperature results in recycling of the product due to the high oil content. Preliminary observations revealed that the plant was not adhering to the given specifications fully. A batch of material was processed as per the specifications to examine whether the low recovery was due to

- lack of proper control
- inadequacy of process specifications, or
- both

Results of the trial runs were 58.5% recovery at the deoiling stage and an oil content of 2.9%. Though there was an increase of about 10% in the yield, it was still much below the expected level of 70–75%. Thus, there was a need to evolve optimum process conditions which would maximize recovery of the deoiled wax of desired quality. This could be achieved by conducting experiments using factors suspected to improve the yield and oil content. These were temperature and time of pressing at different pressure levels. Factors and levels determined after detailed technical discussion for the experiment are given in Table 3.

Another factor likely to influence the yield was the height of wax slab. Throughout the experiments, this was maintained at a constant level of 2 in.

There are five factors, of which four are at two levels and one at three levels. A full factorial will require $2^4 \times 3 = 48$ trials. An orthogonal array approach was adopted to reduce the number of experimental runs. Five main effects, A, B, C, D, E, and the interactions $A \times B$, $A \times C$, $A \times D$, and $A \times E$ were included in the investigation.

The linear graph technique (5) invented by Taguchi is used to design the present experiment.

Table 3. Factors and Levels

FACTORS	LEVELS 1	2	3
A. Temperature of inlet water (°C)	65	55	
Time in minutes of pressing at:			
B. 1100 lb/in.^{2}	20	28	
C. 1550 lb/in.^{2}	10	7	
D. 1800 lb/in.^{2}	6	3	
E. 2100 lb/in.^{2}	0	1	2

Linear Graph

Linear graphs represent the interaction information graphically and make it easy to assign factors and interactions to the various columns of an orthogonal array with the help of an interaction table (6). In the linear graph, the columns of an orthogonal array are represented by the nodes and lines. When two nodes are connected by a line, it means that the interaction of the two columns represented by the nodes is confounded with the column represented by the line. In a linear graph, each node and each line has a distinct column number associated with it. Further, every column of the array is represented in its linear graph once and only once. The principal utility of linear graphs is for creating a variety of different orthogonal arrays from the standard ones to fit real problem situations. The linear graphs are useful for creating 4-level and 3-level columns in 2-level orthogonal arrays. A 4-level factor in a 2-level orthogonal array is represented by two nodes and the line joining them.

The assignment of a 3-level factor in a 2-level orthogonal array is done by first generating a 4-level column by the multilevel technique (5) and then one of the levels is made a dummy level. Multilevel and dummy-level techniques and interactions between 2- and 4-level factors are explained in the Appendix.

Selection of Design Layout Using Linear Graphs

The steps followed in the selection of the layout are as follows:

1. Express the information required in an experiment by means of linear graphs. In a graph, a main effect is represented by a node, and an interaction between two factors is represented by the line joining the nodes. This is termed the required linear graph.

2. Compute the total degrees of freedom required to estimate all the factorial effects that are of interest. The minimum number of experimental runs will be the total degrees of freedom computed to estimate the effects plus one. Choose an orthogonal array closest to the size of the experiment thus determined.
3. Compare the standard linear graph (6) of the chosen array with the required linear graph as obtained in Step 1.
4. Modify the selected standard linear graph by deletion of edges joining a pair of nodes or by joining unconnected nodes as required so as to make the standard graph correspond to the required linear graph. Thus, each factorial effect on the required linear graph is made to correspond with each column number on the standard or modified linear graph, respectively.
5. Assign each factor to the respective column of the standard orthogonal table.

The required linear graph for the present experiment is given in Figure 2.

The degrees of freedom (d.f.) required to estimate all main effects and the interactions *AB*, *AC*, *AD*, and *AE* is 11. The minimum number of experimental runs is 11 + 1 = 12 and the nearest orthogonal array is $L_{16}(2^{15})$.

Therefore, the experiment is designed as an L_{16} orthogonal array, the layout of which is given in Table 4.

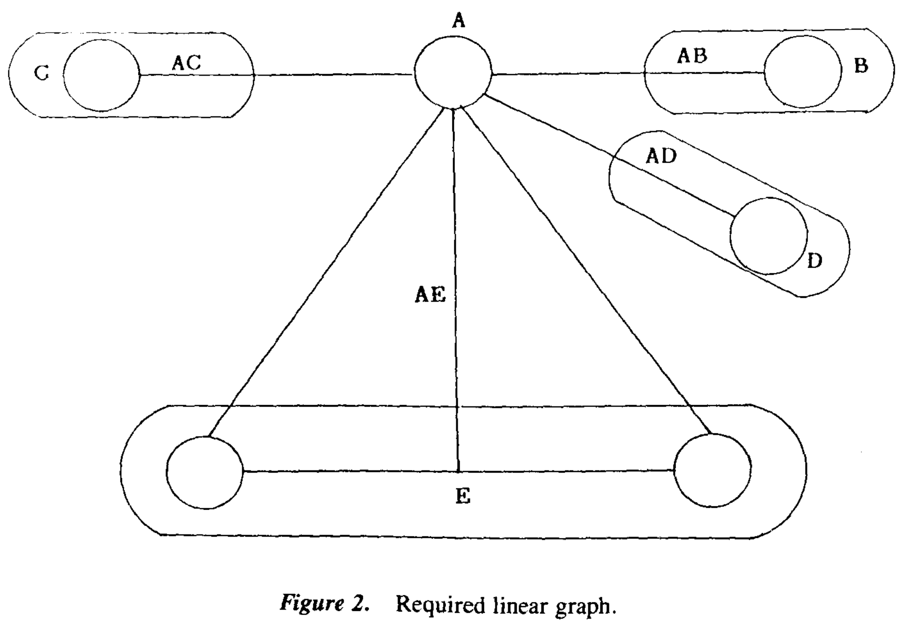

Figure 2. Required linear graph.

Table 4. $L_{16}(2^{15})$ Orthogonal Array Layout

NO.	COLUMN 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
	Group 1	Group 2		Group 3				Group 4							

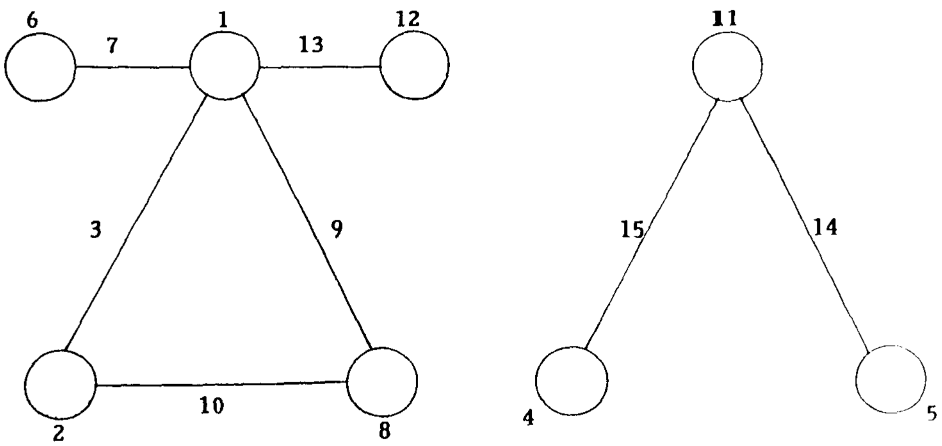

Figure 3. Standard linear graph.

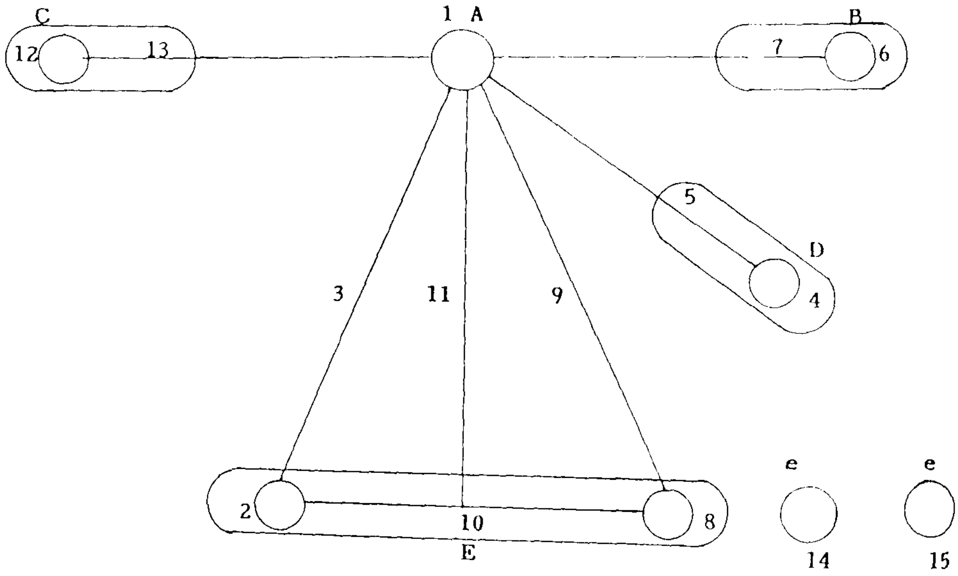

Figure 4. Linear graph for the experiment.

Table 5. Layout of the Experiment: $L_{16}(2^{15})$

EXPERIMENT	*A* (1)	*D* (4)	*B* (6)	*E* (2, 8, 10)	*C* (12)
	FACTOR COLUMN NO.				
1	1	1	1	1	1
2	1	1	1	2	2
3	1	2	2	1	2
4	1	2	2	2	1
5	1	1	2	3	1
6	1	1	2	2	2
7	1	2	1	3	2
8	1	2	1	2	1
9	2	1	1	1	1
10	2	1	1	2	2
11	2	2	2	1	2
12	2	2	2	2	1
13	2	1	2	3	1
14	2	1	2	2	2
15	2	2	1	3	2
16	2	2	1	2	1

Consider the standard linear graph [Fig. 3] (6). By erasing lines 14 and 15, five columns such as 4, 5, 11, 14, and 15 are made free. Column 11 is utilized for representing the interaction between columns 1 and 10 (6). Node 1 is joined to node 4 because column 5 (the interaction between 1 and 4) is free. Unutilized columns 14 and 15 are used for estimating the error. The linear graph for the present experiment is given in Figure 4.

Assignment of the factors to the columns was done from Figure 4. The experiment layout is given in Table 5.

Factor A (i.e., the temperature of inlet water), whose levels are difficult to change, is assigned to column 1 of the $L_{16}(2^{15})$ table (primary zone) (6).

Response

Responses considered during the experimentation were

(i) yield and
(ii) oil content of the deoiled wax

Conduct of the Experiment

Each trial required about 100 kg of slack wax for pressing in 11 daylight hydraulic presses. Two tons of material were melted to obtain a homogeneous melt with respect to oil content. Five samples were taken to determine the oil content in the slack wax, the average of which was found to be 20.2%. The melted wax was solidified in galvanized iron trays to obtain slabs of constant height (2 in.). A canvas cloth free from holes and other defects and capable of withstanding a pressure of up to 2500 lb/in.^{2} was used for wrapping the slab. Eleven slabs, each wrapped in the cloth, were pressed through hydraulic presses as per the conditions stipulated in the layout of the experiment. The weight was taken before and after pressing so as to arrive at the yield. The deoiled wax was then melted in a small tank. Two samples were then taken and tested for oil content as per the Indian Standard (8). The yield and the oil content for the 16 trials along with the actual physical layout are given in Table 6.

Analysis and Results

Analysis of variance (ANOVA) (9) was carried out on yield and oil content data. The results are given in Tables 7 and 8.

It is seen that factor A (the temperature of the inlet water), factor C (time of pressing at Stage II), factor E (time of pressing at Stage IV), and interactions between temperature and time of pressing at Stage I and Stage II (i.e., $A \times B$ and $A \times C$ interactions) are significant. The last column in the ANOVA table

Table 6. Physical Layout of the Experiment and Responses on Yield and Percent Oil Content at the O[illegible] Extraction Process

SERIAL NO.	EXPERIMENTAL CONDITIONS: A Temp. (°C) ±1°C	Time (in min) at pressing pressure: B 1100 ±100 lb/in.^{2}	C 1550 ±100 lb/in.^{2}	D 1800 ±100 lb/in.^{2}	E 2100 ±100 lb/in.^{2}	MASS (in kgs) OF: Slack Wax	Deoiled Wax	Yield of Deoiled Wax in Percent by Mass	Oil Content of Deoiled Wax for Sample: 1	2
1	65	20	10	6	0	96.5	60.8	63.00	2.80	3.05
2	65	20	7	6	1	96.0	62.2	64.79	2.85	3.19
3	65	28	7	3	0	96.0	59.3	61.77	2.70	3.10
4	65	28	10	3	1	96.0	58.5	60.93	2.70	2.90
5	65	28	10	6	2	95.0	57.0	60.00	2.55	2.80
6	65	28	7	6	1	96.5	59.5	61.65	2.90	3.10
7	65	20	7	3	2	98.0	62.0	63.27	2.95	3.19
8	65	20	10	3	1	95.5	60.0	62.82	2.76	3.15
9	55	20	10	6	0	95.0	64.5	67.89	3.53	3.73
10	55	20	7	6	1	96.5	69.5	72.02	3.28	3.45
11	55	28	7	3	0	96.5	72.5	75.13	3.45	3.20
12	55	28	10	3	1	97.0	68.0	70.10	3.28	3.06
13	55	28	10	6	2	100.5	68.0	67.67	3.12	2.95
14	55	28	7	6	1	96.0	69.5	72.40	3.12	3.25
15	55	20	7	3	2	96.5	68.0	70.47	3.19	3.32
16	55	20	10	3	1	96.0	64.0	66.67	3.40	3.25

gives the (ρ) percentage (degrees of contribution) for critical factors. The 97% of total variation is explained by the critical factors.

ANOVA for oil content is given in Table 8.

Here e_1, the error due to the experimental condition, is not significant. Therefore, a pooled estimate of error has been computed and the main effects and interactions have been tested against this pooled error.

Here the factor *A*, the temperature of the inlet water, and factor *B*, the time of pressing at 1100 lb/in.^{2}, are significant, 58.1% of the total variation being explained by the critical factors *A* and *B*.

The average responses for different levels of significant factors in the analyses for yield and oil contents as well as for different combinations of *AB* and *AC* (significant interaction for yield) are computed and given in Table 9.

The effect curves for critical factors are given in Figure 5.

The best levels of significant factors on yield and oil content based (from Table 9) on average responses are summarized in Table 10.

Table 7. ANOVA on Yield Data

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SUM OF SQUARES	F	ρ (%)
A	1	256.96	256.96	428.27[a]	77.8
B	1	0.10	0.10[b]		
C	1	31.42	31.42	52.37[a]	9.2
D	1	0.19	0.19[b]		
E	2	5.38	2.69	4.48[c]	1.3
$A \times B$	1	19.76	19.76	32.93[a]	5.8
$A \times C$	1	10.50	10.50	17.50[a]	3.0
$A \times D$	1	0.58	0.58[b]		
$A \times E$	2	1.71	0.86[b]		
Error	4	2.80	0.70		
Total	15	329.40			97.11
Pooled error	9	5.38	0.60		

[a] Significant at 1%.
[b] Pooled with error.
[c] Significant at 5%.

Table Values:

F_1, 6 at 0.05 = 5.99; at 0.01 = 13.74;

F_2, 6 at 0.05 = 5.14; at 0.01 = 10.92;

F_2, 9 at 0.05 = 4.26; at 0.01 = 8.02;

ρ Computation

$$\rho_A = \frac{S_A - S_e}{S_T} \times 100$$

$$= \frac{256.96 - 0.60}{329.4} \times 100$$

$$= 77.8$$

Table 8. ANOVA on Oil Content of the Deoiled Wax

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SUM OF SQUARES	F	ρ (%)
A	1	1.08413	1.08413	47.28[a]	47.8
B	1	0.26463	0.26463	11.54[a]	10.9
C	1	0.04575	0.04575	2.00	
D	1	0.00011	0.00011[b]		
E	2	0.13880	0.06940	3.03	
$A \times B$	1	0.00883	0.00883[b]		
$A \times C$	1	0.05533	0.05533	2.41	
$A \times D$	1	0.00756	0.00756[b]		
$A \times E$	2	0.10610	0.0530[b]	2.31	
e_1	4	0.03334	0.00833	N.S.[c]	
ST_1	15	1.74455			
e_2	16	0.47765	0.02985		
ST	31	2.22220			
Pooled error					
E′	23	0.52749	0.02293		

[a] Significant at 1%.
[b] Pooled with error.
[c] N.S. = not significant.

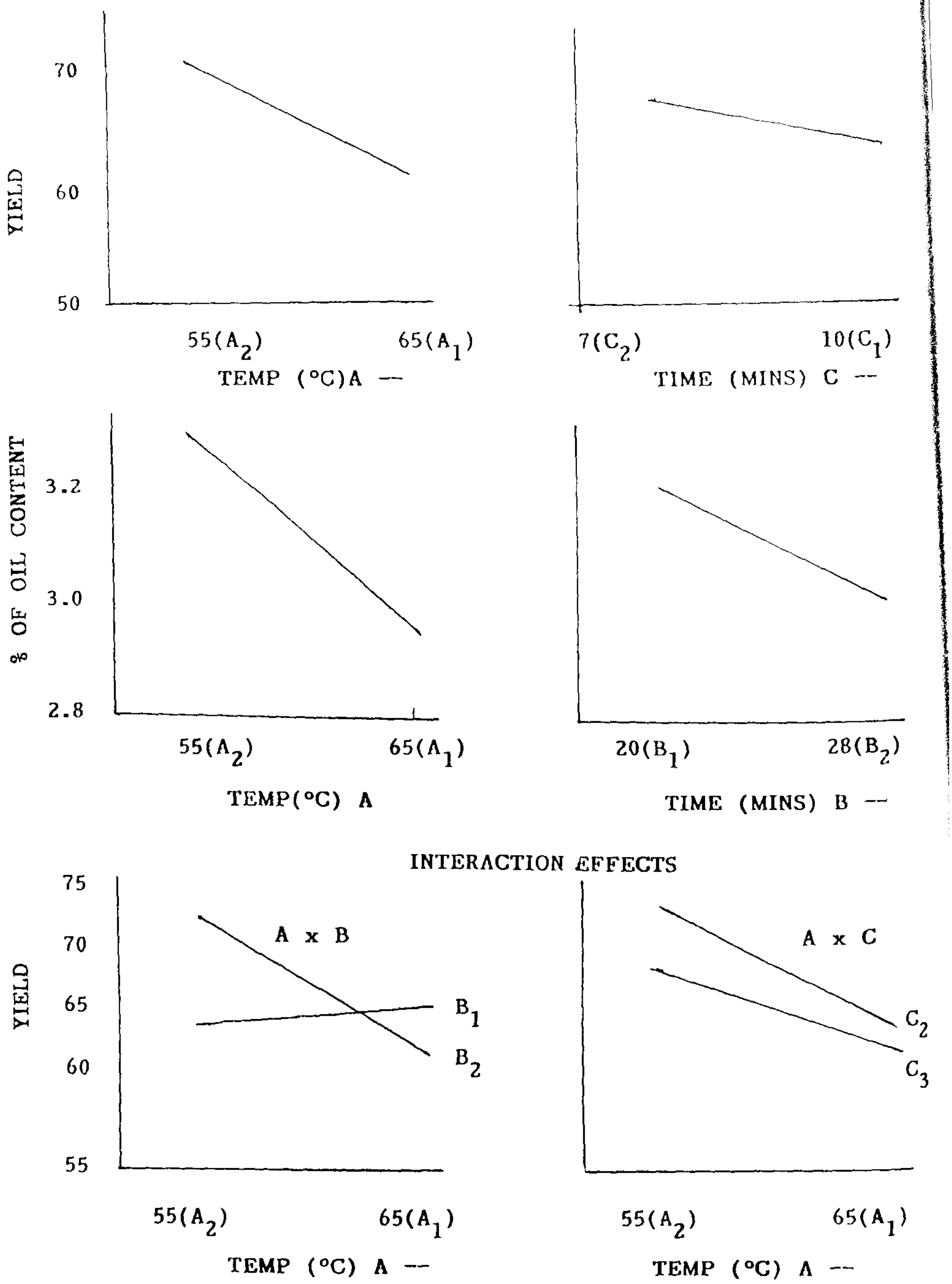

Figure 5. Effect curves on yield and oil content: Main effects and interactions.

Table 9. Average Responses of Significant Factors

FACTOR/ LEVEL	YIELD (%)	OIL CONTENT (%)
A_1	62.28	2.92
A_2	70.29	3.29
B_1		3.19
B_2		3.01
C_1	64.88	
C_2	67.69	
A_1B_1	63.47	
A_1B_2	61.09	
A_2B_1	69.26	
A_2B_2	71.32	
A_1C_1	61.69	
A_1C_2	62.87	
A_2C_1	68.08	
A_2C_2	72.50	
E_1	66.95	
E_2	66.72	
E_3	65.35	

Table 10. Best Levels of Critical Factors

RESPONSE	BEST LEVEL OF CRITICAL FACTORS
Yield	A_2, B_2, C_2, E_1
Oil content	A_1, B_2

Optimum Combination

An examination of the best level of significant factors in the above analysis reveals one area of conflict. The first level of factor A (temperature of the inlet water) is found to be better for oil content, whereas the second level of A is better for yield. Because the yield is more at A_2 and the oil content at A_2 is 3.29% which is well within the maximum limit specified (3.5%) in the standard, level A_2 is preferred. The interaction AB and AC was significant, and the maximum yield was obtained for the combination A_2B_2 and A_2C_2. Levels B_2 and C_2 become the right choice for factors B and C, respectively. The level for the noncritical factors (D) was chosen as D_2, the lower time for pressing. Thus, the optimum combination arrived at is $A_2B_2C_2D_2E_1$.

The expected results with regard to yield and oil content for the optimum combination are given by

$$\begin{aligned}\text{Yield} &= \overline{A_2B_2C_2} + \bar{E}_1 - \bar{T}\\ &= 73.76 + 66.95 - 66.28\\ &= 74.3\%\end{aligned}$$

$$\begin{aligned}\text{Oil} &= \bar{A}_2 + \bar{B}_2 - \bar{T}\\ &= 3.2 + 3.01 - 3.10\\ &= 3.2\%\end{aligned}$$

Confirmatory Trials

The results of the two confirmatory trials carried out with the optimum combination ($A_2B_2C_2D_2E_1$) are given in Table 11.

Thus, the recovery of deoiled wax has increased from 58.5% (see section titled Investigation) to the average yield level of 72.7% and also the average oil content is below the maximum specified.

Implementation

The optimum combination thus achieved is implemented by the plant on a regular basis and it has increased the recovery of paraffin wax from the initial 35-40% to over 60% (final yield). Recycling of paraffin wax for high oil content is totally eliminated. Thus, it has been possible to realize an approximate saving of about Rs. one million ($40000) per annum.

Conclusion

It is been shown that a fractional factorial experiment using the orthogonal array layout developed by Taguchi has helped in identifying the critical process parameters and their best levels for improving the yield as well as quality (oil content). The yield of paraffin wax has improved to a level very close to the

Table 11. Results of Confirmatory Trials

TRIAL RESPONSE	YIELD IN % BY MASS	OIL CONTENT % BY MASS FOR SAMPLE	
		1	2
I	71.43	3.10	2.95
II	73.95	3.12	3.00
Average	72.69	3.04	

theoretically expected yield. The 10–15% recycling of the materials for excess oil is totally eliminated.

The experimentation has been quite economical because the results are achieved involving only 16 trials, whereas a full factorial experiment would have required 48 trials. Even some of the suspected first-order interactions, which turn out to be significant, could also be studied.

The experimentation has been highly successful for the yield improvement, as almost all the variation (97%) is explained by the significant main effect and interaction (Table 6). Such a high percentage of the explained variation resulted in highly reproducible results with respect to yield. This has gone a long way in securing a consistently high yield. A milestone achieved was an increase in profit by 6.5%.

Appendix

Multilevel and Dummy-Level Techniques and Interaction Between 2- and 4-Level Factors

Multilevel Technique

This technique is useful in designing fractional experiments when the levels of different factors are not the same. For such an experiment, a multilevel arrangement is applied; i.e., to arrange a 4- or 8-level column in 2-level series orthogonal tables, or to arrange a 9- or 27-level column in 3-level series orthogonal tables.

Let us consider the problem of accommodating a 4-level factor in the 2-level orthogonal array series. In the linear graph, the representation of a 4-level factor is made by the two nodes and the edge joining them. In other words, we use three columns of the array for a 4-level factor. The two columns corresponding to the two nodes give four possible level combinations: (1,1), (1,2), (2,1), and (2,2). We use the following one-to-one correspondence to obtain the corresponding levels of the 4-level factor.

(1,1)______1 (2,1)______3
(1,2)______2 (2,2)______4

The assignment using the multilevel technique is explained as follows:

Let us assume that A has four levels and B, C, D, and E have two levels each. The assignment using the linear graph is shown in Figure 6. Table 12 gives the assignment to an orthogonal array.

Dummy-Level Techique

The dummy-level technique is especially useful for accommodating 2-level factors in 3-level orthogonal array series or accommodating 3-level factors in 4-level orthogonal series.

In the above example, suppose factor A is at the 3-level. With the help of the multilevel technique, a 4-level column $(\overline{1,2,3})$ is first generated. Because fac-

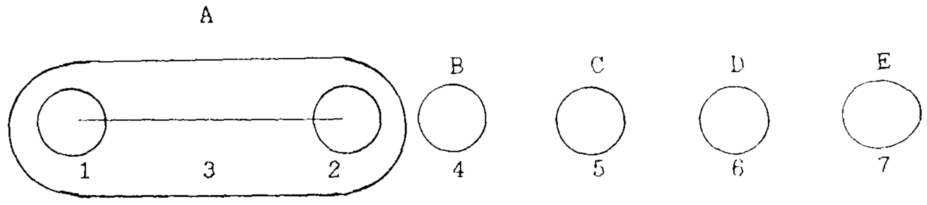

Figure 6. Linear graph for 4 × 2^4 design.

Table 12. Assignment of 4 × 2^4 Design in $L_8(2^7)$ Using Multilevel Technique

EXPERIMENT NO.	1	2	3	*A* 1 2 3	*B* 4	*C* 5	*D* 6	*E* 7
1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2
3	1	2	2	2	1	1	2	2
4	1	2	2	2	2	2	1	1
5	2	1	2	3	1	2	1	2
6	2	1	2	3	2	1	2	1
7	2	2	1	4	1	2	2	1
8	2	2	1	4	2	1	1	2

Table 13. Assignment of 3 × 2^4 design in $L_8(2^7)$ Layout Using Dummy-Level Technique

EXPERIMENT NO.	*A* 1 2 3	*B* 4	*C* 5	*D* 6	*E* 7
1	1	1	1	1	1
2	1	2	2	2	2
3	2	1	1	2	2
4	2	2	2	1	1
5	3	1	2	1	2
6	3	2	1	2	1
7	1′	1	2	2	1
8	1′	2	1	1	2

Note: 1′—dummy level.

tor A consists of only three levels, the more important level of A is repeated whenever the symbol 4 appears in column (1,2,3). For example, if the first level of A is more important, then this level is replicated more often. For instance,

$$A_1 = A_1, \quad A_2 = A_2, \quad A_3 = A_3, \quad A_4 = A_1$$

Table 13 gives the assignment of 3×2^4 design in L_8 (2^7) layout using the dummy-level technique.

Interaction Between 2- and 4-Level Factors

Interaction between a 2-level factor and a 4-level factor is represented by three interaction columns obtained by joining the node representing the 2-level factor with two nodes and line joining the two nodes representing the 4-level factor.

Let column 1 represent a 2-level factor, say A, and columns 2, 4, and 6 represent a 4-level factor, say B. Consider the triangle whose vertices are 1, 2, and 4. The interaction between columns 1 and 2 is column 3; the interaction between columns 2 and 4 is column 6; the interaction between columns 1 and 6 is column 7 (6). This information can be pictorially represented as given in Figure 7 by drawing a perpendicular from node 1 to the base (2,4). The interaction $A \times B$ is then given by columns 3, 5, and 7.

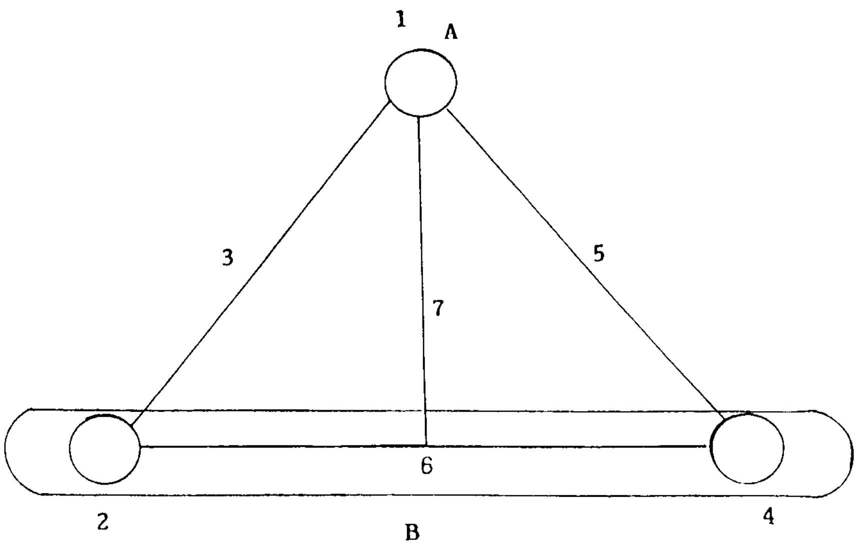

Figure 7. Linear graph representing interaction between 2- and 4-level factors.

Acknowledgment

The author thanks Dr. M. Jeyachandra of the Pennsylvania State University for his help and suggestion in finalizing this article.

References

1. Rao, C. R., Factorial Experiments Derivable from Combinatorial Arrangement of Arrays, *J. Roy. Stat. Soc., series B*, 9, 128–139 (1947).
2. Kempthorne, O., *The Design and Analysis of Experiments*, Robert E. Krieger Publishing Co., New York, 1979.
3. Plackette, R. L., and Burman, J. P., The Design of Optimal Multifactorial Experiments, *Biometrika*, 33, 305–325 (1943).
4. Addelman, S., Orthogonal Main Effect Plans for Asymmetrical Factorial Experiments, *Technometrics* 4, 21–46 (1962).
5. Taguchi, Genichi, *System of Experimental Design*, Volumes 1 and 2, UNIPUB, New York, and American Supplier Institute, Dearborn, MI.
6. Taguchi, Genichi, *Tables of Orthogonal Arrays and Linear Graphs*, Maruzen, Tokyo, 1962.
7. Anand, K. N., Increasing Market Share Through Improved Product and Process Design: An Experimental Approach, *Qual. Eng.*, 3(3) 361–370 (1991),
8. *I.S. 4654–1974 Specification for Paraffin Wax*, Bureau of Indian Standards, New Delhi, 1974.
9. Chakravarti, I. M., Laha, R. G., and Roy J., *Handbook of Methods of Applied Statistics*, Vol. 1, Prentice-Hall, Englewood Cliffs, NJ, 1985.

About the Author: K. N. Anand is Head of SQC & OR Unit at Indian Statistical Institute, Bangalore. The author has over 20 years of consultancy and teaching experience in the field of Quality Management and application of SQC techniques to a variety of Indian industries.