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ASYMPTOTIC DESIGN-CUM-MODEL APPROACH FOR CONVEX WEIGHTING OF DIRECT AND INDIRECT COMPONENTS OF SMALL DOMAIN PREDICTORS

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ABSTRACT

We consider predicting domain totals in survey sampling by 'composites' of 'synthetic' and 'non-synthetic' versions of generalized regression predictors. Neither with 'traditional' nor 'alternative' design-based variance and covariance estimators of the predictors one can be sure that 'the linear combinations' may be 'convex'. But with an 'asymptotic design-cum-model' based approach for a specific model a truly 'convex weighting' procedure is developed. A simulation-based numerical illustration is presented to check how the various procedures may work.
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1. INTRODUCTION

We consider sampling with unequal probabilities from a survey population and using the sample to estimate the totals of a real variable for a number of its non-overlapping domains. An auxiliary variable with known population values is supposed to be available motivating the use of generalized regression (greg) predictors. For domains of relatively small sizes sample representation becomes too inadequate leading to inefficient prediction, if one restricts to the use of 'direct' predictors utilizing domain-specific sampled values alone for the variable of interest. For an improvement one may employ 'indirect' predictors using, in addition, sampled values outside the specific domains assuming similarities of domains. A linear compound of the two may be preferred with appropriate weighting. Optimal weights for the composite predictors usually involve unknown parameters as was noted in particular by Schaible (1978, 1992). If appropriate statistics are substituted for the latter the weights need not remain positive proper fractions. To get over this difficulty we apply Brewer's (1979) asymptotic design-based analytic approach. Postulating a simplistic linear regression model we work out the optimal weight that minimizes the limiting design-cum-model expectation of the square error of the composite estimator and find the resulting composite a truly 'convex' combination of the 'direct' and 'indirect' versions of the greg predictors. The theory is briefly presented in section 2 and a simulation-based illustration of a numerical exercise to check how the procedure works is reported in section 3. We close with a few concluding remarks in section 4. Of course with an empirical Bayesian or 'mixed linear modelling' approach methods for convex weighting are well known but they do not relate to design-based procedures.

2. CONVEX WEIGHTING OF 'DIRECT' AND 'INDIRECT' GREG PREDICTORS

We consider a survey population $U = (1, \cdots, i, \cdots, N)$ consisting of D non-overlapping domains of sizes N_d, $d = 1, \cdots, D$. On it is defined a real variable y with unknown values y_i with a total Y_d for U_d, $d = 1, \cdots, D$. An auxiliary variable x with known values x_i, $i \in U$ with domain totals X_d is also available. The problem is to estimate Y_d, $d = 1, \cdots, D$, on drawing a sample s of size n from U with a probability $p(s)$, adopting a suitable design p. We assume the inclusion-probabilities π_i for i and π_{ij} for i, j to be positive. We assume y to be so related to x that a super-population model may be plausibly postulated permitting us to write

$$y_i = \beta_d x_i + \epsilon_i, \; i \in U_d, \; d = 1, \cdots, D \, . \tag{1}$$

Here β_d's are unknown constants and ϵ_i's are 'independently' distributed random variables with means and variances

$$E_m(\epsilon_i) = 0, \text{ and } V_m(\epsilon_i) = \sigma_i^2 \, .$$

If we are justified further to 'suppose the domains to be alike' then we may take

$$\beta_d = \beta, \; \forall d = 1, \cdots, D \, . \tag{2}$$

The model (1) will be denoted by $\underline{M_d}$ and that under (2) by $\underline{M}$. Choosing suitable constants $Q_i (> 0)$ we may employ Särndal's (1980) generalized regression predictors for Y_d recognizing their two versions - 'non-synthetic' 'direct' motivated by $\underline{M}_d$ and 'synthetic' 'indirect' motivated by $\underline{M}$ - applicable in the present situation described below. We shall throughout write $\sum, \sum\sum$ to denote sums over i in U and $i,j\,(i<j)$ in U, $\sum'$, $\sum'\sum'$, those respectively in s and $I_{di} = 1$ if $i \in U_d$ and 0, else. Let

$$\begin{aligned} \widehat{\beta}_1 &= \frac{\sum' y_i x_i Q_i I_{di}}{\sum' x_i^2 Q_i I_{di}}, \quad B_1 = \frac{\sum y_i x_i Q_i I_{di}\pi_i}{\sum x_i^2 Q_i I_{di}\pi_i}, \\ e_{1i} &= y_i - \widehat{\beta}_1 x_i, \quad E_{1i} = y_i - B_1 x_i, \; i \in U; \\ \widehat{\beta}_2 &= \frac{\sum' y_i x_i Q_i}{\sum' x_i^2 Q_i}, \quad B_2 = \frac{\sum y_i x_i Q_i \pi_i}{\sum x_i^2 Q_i \pi_i}, \\ e_{2i} &= y_i - \widehat{\beta}_2 x_i, \quad E_{2i} = y_i - B_2 x_i, \; i \in U. \end{aligned}$$

Then, the 'direct' greg predictor for Y_d is

$$\begin{aligned} t_1 &= \sum{}' \frac{y_i}{\pi_i} g_{1i} I_{di}, \text{ where} \\ g_{1i} &= 1 + \left(X_d - \sum{}' \frac{x_i}{\pi_i} I_{di}\right) \frac{x_i Q_i \pi_i}{\sum' x_i^2 Q_i I_{di}} \end{aligned}$$

and the 'indirect' greg predictor for Y_d is

$$\begin{aligned} t_2 &= \sum{}' \frac{y_i}{\pi_i} g_{2i}, \text{ where} \\ g_{2i} &= I_{di} + \left(X_d - \sum{}' \frac{x_i}{\pi_i} I_{di}\right) \frac{x_i Q_i \pi_i}{\sum' x_i^2 Q_i}. \end{aligned}$$

By E_p, V_p, C_p we shall mean the operators for design-based expectation, variance and covariance. We follow Sărndal (1982) to approximate $V_p(t_j)$, $j = 1, 2$ by the respective formulae

$$\begin{aligned} V_1 &= \sum\sum \Delta_{ij}\pi_{ij} \left(\frac{E_{1i} I_{di}}{\pi_i} - \frac{E_{1j} I_{dj}}{\pi_j}\right)^2, \\ V_2 &= \sum\sum \Delta_{ij}\pi_{ij} \left(\frac{E_{2i} I_{di}}{\pi_i} - \frac{E_{2j} I_{dj}}{\pi_j}\right)^2, \end{aligned}$$

writing

$$\Delta_{ij} = \frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}, \; i,j \in U.$$

Two variance estimators each for t_j, $j = 1,2$, also following Särndal (1982) respectively are

$$v_k(1) = \sum{}' \sum{}' \Delta_{ij} \left(\tfrac{e_{1i} I_{di}}{\pi_i} a_{ki} - \tfrac{e_{1j} I_{dj}}{\pi_j} a_{kj} \right)^2 ,$$
$$k = 1,2 \; ; \; a_{1i} = 1, \; a_{2i} = g_{1i}, \; i \in s$$
$$v_k(2) = \sum{}' \sum{}' \Delta_{ij} \left(\tfrac{e_{2i} I_{di}}{\pi_i} b_{ki} - \tfrac{e_{2j} I_{dj}}{\pi_j} b_{kj} \right)^2 ,$$
$$k = 1,2 \; ; \; b_{1i} = 1, \; b_{2i} = g_{2i}, \; i \in s \; .$$

Doubting the appropriateness of $\underline{M}$ against $\underline{M}_d$ especially, if D is large, one may prefer the composite predictor which is a 'convex' combination of t_1 and t_2, namely,

$$t_c = \alpha t_1 + (1-\alpha) t_2 \; , \tag{3}$$

with α suitably chosen in $[0,1]$. Noting that

$$C_p(t_1, t_2) = \sum \sum \Delta_{ij} \pi_{ij} \left(\frac{E_{1i} I_{di}}{\pi_i} - \frac{E_{1j} I_{dj}}{\pi_j} \right) \left(\frac{E_{2i} I_{di}}{\pi_i} - \frac{E_{2j} I_{dj}}{\pi_j} \right)$$
$$= C, \text{ say} ,$$

one may estimate it by four alternatives

$$c_{kr} = \sum{}' \sum{}' \Delta_{ij} \left(\frac{e_{1i} I_{di}}{\pi_i} a_{ki} - \frac{e_{1j} I_{dj}}{\pi_j} a_{kj} \right) \left(\frac{e_{2i} I_{di}}{\pi_i} b_{ri} - \frac{e_{2j} I_{dj}}{\pi_j} b_{rj} \right) ,$$
$$k = 1,2 \text{ and } r = 1,2 \; .$$

We shall illustrate uses only of c_{11} and c_{22}. For simplicity, we shall write v_1, v_2 for $v_k(1), v_k(2)$ respectively and $\widehat{c}$ for c_{11}, c_{22}. As $V_p(t_c)$ we shall take

$$V = \alpha^2 V_1 + (1-\alpha)^2 V_2 + 2\alpha(1-\alpha) C \; .$$

Then, the obvious optimal choice of α is

$$\alpha_0 = \frac{V_2 - C}{V_1 + V_2 - 2C} \; .$$

This, in application should be replaced by

$$\widehat{\alpha}_0 = \frac{v_2 - \widehat{c}}{v_1 + v_2 - 2\widehat{c}} \; . \tag{4}$$

But this may go outside $[0,1]$ and

$$\hat{t}_c = \hat{\alpha}_0 t_1 + (1-\hat{\alpha}_0)\, t_2 \;,$$

need not be a 'convex' combination of t_1 and t_2.

To get over this difficulty we adopt the "measure of error in t_c as a predictor of Y_d" as

$$\lim E_p E_m \,(t_c - Y_d)^2 = M, \text{ say } . \tag{5}$$

The meaning of the operator $\lim E_p$ is given below following Brewer (1979).

According to Brewer (1979), U along with $\underline{Y} = (y_1, \cdots, y_i, \cdots, y_N)$, $\underline{X} = (x_1, \cdots, x_i, \cdots, x_N)$, $\underline{Q} = (Q_1, \cdots, Q_i, \cdots, Q_N)$ etc., is supposed to conceptually re-appear $T(>1)$ times. On each re-appearance an 'independent' sample of the type s adopting the design p is drawn. The samples so drawn are amalgamated into a pooled sample, denoted by s_T. The resulting design giving the selection probability for s_T is denoted by p_T. If $e = e(s)$ based on s be a predictor for Y_d then $e\,(s_T)$ should predict TY_d and so

$$\lim_{T\to\infty} E_{p_T}\left(\frac{1}{T}e\,(s_T)\right), \text{ abbreviated as ``}\lim E_p\,(e(s))\text{''}$$

should be close to Y_d. Introducing this asymptotic approach one may apply Slutzky's (vide Cramér (1946)) limit theorems on sequences of functions to conveniently derive useful asymptotic results. In our present case, for the models $\underline{M}_d$ and $\underline{M}$ we further assume that

$$\sigma_i^2 = \sigma^2 f_i, \text{ with } \sigma(>0, \text{ unknown}) \text{ and} f_i(>0, \text{ known}),\ i \in U\ , \tag{6}$$

and denote the respective models by $\underline{M}_d(f)$ and $\underline{M}(f)$. Then the choice of α that minimizes M in (5) is

$$\alpha_m = \frac{\lim E_p V_m\,(t_2) - \lim E_p C_m\,(t_1, t_2)}{\lim E_p V_m\,(t_1) + \lim E_p V_m\,(t_2) - 2\lim E_p C_m\,(t_1, t_2)}$$

$$= \frac{\dfrac{\sum f_i x_i^2 Q_i^2 \pi_i}{E_p\left(\sum' x_i^2 Q_i\right)^2}}{\dfrac{\sum f_i x_i^2 Q_i^2 \pi_i}{E_p\left(\sum' x_i^2 Q_i\right)^2} + \dfrac{\sum f_i x_i^2 Q_i^2 I_{di} \pi_i}{E_p\left(\sum' x_i^2 Q_i I_{di}\right)^2}} \,.$$

Obviously,

$$t_m = \alpha_m t_1 + (1-\alpha_m)\, t_2 \tag{7}$$

is a 'convex' combination of (t_1, t_2). For $V_p\,(t_m)$ we shall employ the estimator

$$v_k(t_m) = \alpha_m^2 v_k(1) + (1-\alpha_m)^2 v_k(2) + 2\alpha_m(1-\alpha_m) c_{kk},\ k = 1,2.$$

Alternatively, since $v_k(j)$, $k = 1,2$; $j = 1,2$ are not known to have any specific properties we prefer to employ their two sets of modifications, under $\underline{M}_d(f)$ and $\underline{M}(f)$, namely,

$$v'_k(j) = \frac{E_m V_p(t_j)\, v_k(j)}{\lim E_p E_m v_k(j)} \text{ and}$$

$$v''_k(j) = \frac{\left(\lim E_p E_m (t_j - Y)^2\right) v_k(j)}{\lim E_p E_m v_k(j)},$$

$$k = 1,2;\ j = 1,2.$$

The genesis of these variance estimators may be found in Chaudhuri and Maiti (1992) and in the chapter one of this thesis. Numerical illustrations appear in table 2 of section 3. Also we may replace c_{kk}, $k = 1,2$ by

$$c'_{kk} = c_{kk} \frac{E_m C_p(t_1, t_2)}{\lim E_p E_m c_{kk}},\ k = 1,2.$$

However replacing $v_k(j)$ by $v'_k(j), v''_k(j)$ and c_{kk} by c'_{kk} in (4) one need not necessarily get a 'convex' combination of t_1 and t_2. So, our recommendation is in favour of (7).

If x_i- values are not available for i outside s, though X_d is known, an appropriate alternative may be to estimate α_m by

$$\widehat{\alpha}_m = \frac{\dfrac{\sum' f_i x_i^2 Q_i^2}{\left(\sum' x_i^2 Q_i\right)^2}}{\dfrac{\sum' f_i x_i^2 Q_i^2}{\left(\sum' x_i^2 Q_i\right)^2} + \dfrac{\sum' f_i x_i^2 Q_i^2 I_{di}}{\left(\sum' x_i^2 Q_i I_{di}\right)^2}}$$

which satisfies

$$\lim E_p \widehat{\alpha}_m = \alpha_m \tag{8}$$

and proceed to estimate M treating it as a measure of error of t_c. For this, writing

$$M = \alpha^2 \lim E_p V_m(t_1) + (1-\alpha)^2 \lim E_p V_m(t_2) + 2\alpha(1-\alpha) \lim E_p C_m(t_1, t_2) - V_m(Y_d), \tag{9}$$

an estimator for it may be taken as

$$\widehat{M} = \widehat{\sigma}^2 \left[\widehat{\alpha}_m^2 A + (1-\widehat{\alpha}_m)^2 B + 2\widehat{\alpha}_m(1-\widehat{\alpha}_m) D - \sum' \frac{f_i I_{di}}{\pi_i}\right] \tag{10}$$

where,

$$A = \sum{}' \frac{f_i}{\pi_i^2} I_{di} + \frac{\sum{}' x_i^2 Q_i^2 f_i I_{di}}{\left(\sum{}' x_i^2 Q_i I_{di}\right)^2} \sum{}'\sum{}' \Delta_{ij} \left(\frac{x_i}{\pi_i} I_{di} - \frac{x_j}{\pi_j} I_{dj}\right)^2$$

$$B = \sum{}' \frac{f_i}{\pi_i^2} I_{di} + \frac{\sum{}' x_i^2 Q_i^2 f_i}{\left(\sum{}' x_i^2 Q_i\right)^2} \sum{}'\sum{}' \Delta_{ij} \left(\frac{x_i}{\pi_i} I_{di} - \frac{x_j}{\pi_j} I_{dj}\right)^2$$

$$D = \sum{}' \frac{f_i}{\pi_i^2} I_{di}, \quad \widehat{\sigma}^2 = \frac{\sum{}' \frac{e_{2i}^2}{f_i}}{(n-2) + \frac{\sum{}' \frac{x_i^2}{f_i} \sum{}' x_i^2 Q_i^2 f_i}{\left(\sum{}' x_i^2 Q_i\right)^2}} .$$

Then we have the
Theorem:

$$\lim E_p \widehat{M} = M .$$

Proof: Follows applying Slutzky's limit theorem, on noting $E_m\left(\widehat{\sigma}^2\right) = \sigma^2$, $\lim E_p E_m\left(\widehat{\sigma}^2 A\right) = \lim E_p V_m\left(t_1\right)$, $\lim E_p E_m\left(\widehat{\sigma}^2 B\right) = \lim E_p V_m\left(t_2\right)$, $\lim E_p E_m\left(\widehat{\sigma}^2 D\right) = \lim E_p C_m\left(t_1, t_2\right)$, $\lim E_p E_m\left(\widehat{\sigma}^2 \sum{}' \frac{I_{di}}{\pi_i}\right) = V_m\left(Y_d\right)$. The resulting estimator t_c with α replaced by $\widehat{\alpha}_m$ will be denoted by $\widehat{t}_m$.

3. NUMERICAL STUDY OF PROCEDURES BY SIMULATION

In order to examine efficacy of a predictor e for Y_d paired with a variance estimator v we assume the distribution of the pivotal quantity

$$t = \frac{e - Y_d}{\sqrt{v}}$$

to be close to that of the standardized normal deviate τ with the $N(0,1)$ distribution. Then, with a choice of γ in $(0,1)$,

$$e \pm \tau_{\frac{\gamma}{2}} \sqrt{v}$$

provides a confidence interval (CI) for Y_d with a nominal confidence coefficient $100(1-\gamma)$, denoting by $\tau_{\frac{\gamma}{2}}$ the $100\frac{\gamma}{2}\%$ point on the right tail area of $N(0,1)$. In our numerical illustration we shall take $\gamma = .05$ and we shall illustrate only the choice $Q_i = \frac{1}{x_i}$, $i \in U$. We tried $Q_i = \frac{1}{\pi_i x_i}$, $Q_i = \frac{1-\pi_i}{\pi_i x_i}$ to get comparable results but got poor results with $Q_i = \frac{1}{x_i^2}$, $i \in U$.

For a simulation study we draw random samples of x_i from the exponential density

$$f(x/\lambda) = \frac{1}{\lambda} exp(-x/\lambda),\ \lambda > 0,\ x > 0 ,$$

taking $\lambda = 7.0$. Taking $\sigma_i^2 = \sigma^2 x_i^g$ with $\sigma = 1$, $g = 0.4$ and 1.6 and drawing random samples of ϵ_i from $N(0, \sigma_i^2)$ we generate y_i's subject to (1), choosing $\beta = 5.5$, taking $N = 767$. Generating z_i from $f(x/\lambda)$ with $\lambda = 15.0$, we take $w_i = 5 + z_i$ as size-measures of i to draw samples of size $n = 183$ following Lahiri's (1951) scheme of sampling. We divide $U = (1, \cdots, i, \cdots, N)$ into $D = 19$ disjoint domains, each consisting of consecutive units in succession of various sizes N_d, $d = 1, \cdots, D$. We take $R = 500$ replicates of samples and each time we identify the domains to which the sampled units respectively belong.

To examine the relative efficacies of various choices of (e, v) we evaluate the following criteria, denoting by $\sum_r$ the sum over the replicates: (I) *ACP* (Actual coverage percentage) $\equiv$ the percent of replicates for which the CI's cover Y_d - the closer it is to 95 the better.

(II) *ACV* (Average coefficient of variation) $\equiv \frac{1}{R}\sum_r \frac{\sqrt{v}}{e}$ - this reflects the length of the CI - the smaller it is the better.

(III) *ARE* (Absolute relative error) $\equiv \frac{1}{R}\sum_r \left| \frac{e - Y_d}{Y_d} \right|$.

(IV) *ARB* (Absolute relative bias) $\equiv \left| \frac{\bar{e} - Y_d}{Y_d} \right|$, where $\bar{e} = \frac{1}{R}\sum_r e$.

(V) *PCV* (Pseudo coefficient of variation) $\equiv \frac{1}{\bar{v}}\sqrt{\frac{1}{R}\sum_r (v - \bar{v})^2}$,

where $\bar{v} = \frac{1}{R}\sum_r v$.

For live data in table 3 we present all these criteria but in tables 1 and 2 which do not use the live data we present only the criteria I and II for brevity.

As a term of reference for relative performances we take the Horvitz-Thompson (1952) estimator for Y_d, namely,

$$t_H = \sum{}' \frac{y_i}{\pi_i} I_{di}$$

for which the variance estimator due to Yates and Grundy (1953) is

$$v_H = \sum{}'\sum{}' \Delta_{ij} \left(\frac{y_i}{\pi_i} I_{di} - \frac{y_j}{\pi_j} I_{dj} \right)^2 .$$

In tables 1 and 2 below we present the numerical evaluations for a few selected domains. In table 3 we illustrate performance of $\hat{t}_m$ paired with $\widehat{M}$ in (10) by referring to certain live data described below. For this we take $f_i = 1$ in (6) and use $\frac{1}{n-1}\sum{}' e_{2i}^2$ instead of $\hat{\sigma}^2$, noting by Cauchy inequality that it is a conservative estimator for σ^2.

Table 1

Relative performances of (e, v) for various alternative choices.
Values for $g = 1.6$ are separated by slashes following those for $g = 0.4$.

(e,v)	Domain size 5		Domain size 9		Domain size 161	
	ACP	$10^3 ACV$	ACP	$10^3 ACV$	ACP	$10^3 ACV$
(t_H, v_H)	100/100	673/681	91.4/87.7	629/631	93.0/92.6	202/211
$(t_1, v_1(1))$	59.6/59.6	17/74	54.3/45.7	10/36	91.8/88.4	5/20
$(t_1, v_2(1))$	47.8/58.4	9/35	50.9/39.9	8/27	94.2/91.0	5/20
$(t_2, v_1(2))$	97.2/91.0	28/121	88.7/90.2	15/59	94.8/91.8	5/19
$(t_2, v_2(2))$	97.2/91.0	28/120	88.3/90.2	15/58	94.8/92.0	5/19
$(t_m, v_1(t_m))$	96.6/91.0	28/120	88.7/90.2	15/58	93.8/91.0	5/19
$(t_m, v_2(t_m))$	96.6/91.0	28/119	88.3/89.6	15/58	95.0/91.8	5/19

Table 2

Relative efficacies of 'traditional' and 'alternative' procedures.
Values for $g = 1.6$ are separated by slashes following those for $g = 0.4$.

(e,v)	Domain size 8		Domain size 10		Domain size 125	
	ACP	$10^3 ACV$	ACP	$10^3 ACV$	ACP	$10^3 ACV$
(t_H, v_H)	71.8/70.8	719/723	76.4/71.8	688/695	91.2/91.0	230/239
$(t_2, v_1(2))$	93.6/77.2	32/118	95.8/84.0	18/67	95.4/92.2	5/22
$(t_2, v_1'(2))$	93.8/77.8	32/119	95.8/84.4	18/68	95.4/92.4	5/22
$(t_2, v_1''(2))$	93.8/78.4	32/120	96.2/85.0	18/68	95.6/92.6	5/22
$(t_2, v_2(2))$	93.6/77.4	31/117	96.0/84.2	18/67	95.6/92.6	5/22
$(t_2, v_2'(2))$	93.8/77.8	31/118	96.0/84.8	18/67	95.6/92.8	5/22
$(t_2, v_2''(2))$	93.8/78.8	32/119	96.2/85.2	18/68	95.6/92.8	5/22

The live data relate to $N = 1184$ workers of Indian Statistical Instituta, Calcutta, in April, 1992 divided into 39 disjoint 'units' taken as domains, treating y_i, x_i, z_i as their last month's dearness allowance (DA), gross pay and basic pay respectively. We take 500 replicates of samples of size $n = 200$ each by Lahiri's (1951) scheme and take $Q_i = \frac{1}{\pi_i x_i}$.

4. CONCLUDING REMARKS AND RECOMENDATIONS

From table 1 we note that for small sizes 5 and 9 of domains (i) the 'direct' greg predictor is poor and as such the composite cannot improve upon the 'synthetic' greg predictor, the latter two being close performers and both quite good and far superior to the basic Horvitz-Thompson estimator. But when the domain size is large, the 'direct' does not really lag behind the 'synthetic' one though the model suits the latter and the composite fares well. Between $v_k(j)$ for $k = 1, 2$ with j fixed at 1,2 there is little to choose.

Table 3
Relative performances of $(t_1, v_2(1))$, $(t_2, v_2(2))$, $\left(\hat{t}, \widehat{M}\right)$ with values given successively downwards.

Domain Size	ACP	$10^3 ACV$	$10^5 ARE$	$10^5 ARB$	PCV
69	63	72.6	18	750	6.569
	92	56.8	15	174	0.905
	98	74.4	15	52	0.344
13	43	134.4	434	14648	2.022
	94	127.3	10	288	0.981
	94	137.4	7	247	0.537
6	36	244.4	27	46495	12.599
	51	118.7	64	3870	1.089
	88	745.3	63	3818	0.486
50	57	93.7	24	3378	13.159
	89	61.9	26	2	0.565
	94	56.8	26	113	0.368
10	68	164.0	21	568	1.853
	77	106.3	19	2093	0.879
	100	273.4	19	1768	0.547
21	61	110.2	59	4907	2.386
	87	59.8	12	110	1.025
	100	123.3	14	27	0.524
30	77	139.9	1	1879	1.698
	86	99.2	26	356	0.639
	98	126.3	25	201	0.451

From table 2 we see that the Horvitz-Thompson estimator is bad as it should be as it does not use x_i's at all. Our $v_k'(2)$ and $v_k''(2)$, $k = 1,2$ provide improved confidence intervals. The 'direct' greg predictor fares so badly with our simulation that we do not show its performances and it is not worth trying the composite t_m.

Our recommendations are therefore that (1) the composite t_m should be reckoned with in small domain estimation if one like Särndal (1992) is in favour of a 'design-based' approach and (2) the variance estimators $v_k'(j), v_k''(j)$ should be tried as possible improvements on $v_k(j)$, $k = 1, 2$, $j = 1, 2$.

With reference to table 3 we find that even though the direct estimator is worse compared to the synthetic predictor the copmosite is found better for varying domain sizes in respect of every criterion except ACV. So, we may recommned that $\left(\hat{t}_m, \widehat{M}\right)$ is a viable alternative.

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REFERENCES

Brewer, K.R.W. (1979). "A class of robust sampling designs for large-scale surveys." *Jour. Amer. Stat. Assoc.*, **74**, 911-915.

Chaudhuri, A. and Maiti, T. (1994). "Variance estimation in model assisted survey sampling." *Comm. Stat.: Theo. Meth.*, **23**, (4), 1203-1214.

Cramér, H. (1946). *Mathematical methods of statistics.* Princeton Univ. Press.

Horvitz, D.G. and Thompson, D.J. (1952). "A generalization of sampling without replacement from a finite universe." *Jour. Amer. Stat. Assoc.*, **47**, 663-685.

Lahiri, D.B. (1951). "A method of sample selection providing unbiased ratio estimation." *Bull. Int. Stat. Inst.*, **33:2**, 133-140.

Särndal, C.E. (1980). "On π- inverse weighting versus best linear weighting in probability sampling." *Biometrika*, **67**, 639-650.

— (1982). "Implication of survey design for generalized regression estimation of linear function." *Jour. Stat. Plan. Inf.*, **7**, 155-170.

— (1992). "Design based-approach in estimation for domains in 'small area statistics and survey designs." Int. Conf., Warsaw.

Schaible, W.L. (1978). "Choosing weights for composite estimators for small area statistics." *Proc. Sec., Survey, Research Methods, ASA*, 741-746.

— (1992). "Use of small area estimator in US Federal programs." Int. Conf., Warsaw.

Yates, F. and Grundy, P.M. (1953). "Selection without replacement from within strata with probability proportional to size." *Jour. Roy. Stat. Soc.*, **B**, **15**, 253-261.