*Aligarh Journal of Statistics*
*Vol. 2 (1982) 38-40*

# A NOTE ON THE ESTIMATION OF MEAN SQUARE ERROR

PULAKESH MAITI & T. P. TRIPATHI

In an earlier paper, Maiti and Tripathi (1981) obtained a biased Subclass in $T_2$ class of linear estimators, where the Well-known Horvitz-Thompson estimator fails to be better, better in the sense of having smaller mean square error than one belonging to the biased sub-class, of course, under some moderate conditions. In this note, we present the conditions under which the estimates of mean square error will be non-negative.

**Introduction and result**—Maiti and Tripathi (1981) revisited $T_2$-class of liner estimators, $T_2 = \sum_{i \epsilon s} \beta_i y_i$ for population total y in search of the biased estimators better than the well-known Horvitz-Thompson estimator, $\hat{y}_{H\text{-}T} = \sum_{i \epsilon s} y_i/\pi_i$ and observed that there does not exist UMMSE estimator for y in $T_2$. However some biased estimators were detected to be better than Horvitz-Thompson estimator for Y in case of a family of sampling designs and in case some apriori information on co-efficient of variation $C_y$, as $C_{(1)} \leqslant C_y$ is available. In this note, we consider the problem of estimation of mean square errors of $T'_2$ and $T_2^*$ defined in (2.5) and (3.1) on pages 54 and 56 of Maiti and Tripathi (1981). From (1.2) of Maiti and Tripathi (1981), it may be shown that

$$M(T_2) = \sum_{i=1}^{N} \sum_{j=1}^{N} y_i y_j (\beta_i \beta_j \pi_{ij} + 1 - 2 \beta_j \pi_j) \tag{1.1}$$

An unbiased estimator of (M $(T_2)$ is given by

$$\hat{M}(T_2) = \sum_{i \epsilon s} \sum_{i \epsilon s} y_i y_j (\beta_i \beta_j \pi_{ij} + 1 - 2 \beta_j \pi_j)/\pi_{ij}) \tag{1.2}$$

and hence unbiased estimates for M $(T'_2)$ and M $(T_2^*)$ can be obrained from (1.2) by replacing $\beta_i$ by $\lambda/\pi_i$ and $\lambda^*/p_i$ respectively. (1.2) Let $y_i>0$ for $i = 1, 2, ..., N$. A set of sufficient conditions for non-negativity of (1.2) would be

$$\beta_i \beta_j + (1-2\beta_j \pi_j)/\pi_{ij} \geqslant 0 \quad \text{for } i, j = 1, 2, \ldots, N \tag{1.3}$$

In fact, the condition (1.3) should hold only for $i \neq j$, because for $i=j$, the condition (1.3) reduces to

$$\beta_i^2 - 2\beta_i + 1/\pi_i \geqslant 0 \tag{1.3}$$

i.e, $(\beta_i - 1)^2 + \{(1-\pi_i)/\pi_i\} \geqslant 0$

Which always holds true

Thus from (1.3), it is found that $\hat{M}(T'_2)$ will be non-negative if for all $i \neq j$,

$$\lambda^2/\pi_i \pi_j + (1-2\lambda)/\pi_{ij} \geqslant 0 \tag{1.4}$$

The condition (1.4) will always be true, irrespective of the sampling design in case $\lambda \leqslant \frac{1}{2}$. Thus for all those populations with $C^2(\hat{y}_{H\text{-}T}) \geqslant 1$, $\hat{M}(T'_2)$ will be non-negative in case the optimum estimator $T'_{02}$($T'_2$ with $\lambda = \lambda_0$) is used. Further in case,

$1 \leqslant C^2_{(1)}(\hat{y}_{H\text{-}T}) \leqslant C^2(\hat{y}_{H\text{-}T})$, $\lambda$ in $T'_2$ is such that

$$1/(1 + C^2_{(1)}(\hat{y}_{H\text{-}T})) \leqslant \lambda \leqslant \tfrac{1}{2}$$

$\hat{M}(T'_2)$ will be non-negative.

Further, the condition (1.4) may be expressed as

$$\frac{1}{\pi_i \pi_j}[\lambda^2 - \{(2\lambda-1)\pi_i \pi_j/\pi_{ij}\} \geqslant 0$$

$$\text{or} \quad (\lambda-1)^2 + \{(2\lambda-1)(\pi_{ij} - \pi_i\pi_j)/\pi_{ij}\} \geqslant 0 \tag{1.5}$$

i. e. the same condition $\pi_{ij}/\pi_i\pi_j \geqslant 1$ for $i \neq j$ under which $\hat{V}(\hat{y}_{H\text{-}T})$ is non-negative [ (Horvitz and Thonpson, (1952)] will make $\hat{M}(T'_2)$ also non-negative.

Similarly from (1.3) $\hat{M}(T_2^*)$ will be non-negative

$$\text{if } \left\{ \lambda^{*2}/p_i p_j \right\} + \left\{ \left(1 - \frac{2\lambda^* \pi_j}{p_j}\right) / \pi_{ij} \right\} \geqslant 0 \quad \text{for } i \neq j \qquad (1.6)$$

## REFERENCES


Horvitz, D. G. and Thompson D.J. (1952) : A generalisation of sampling without replacement from a finite universe. *J. Ame:. Statist. Assoc.* 47, 663-685

Maiti, P. and Tripathi T.P. (1981) : Some T2 class of estimators betrer than H-T estimator. The *Aligarh J. Statist.* 52-58


*Recevied:* Feb. 24,


P. Maiti
Opratious Reseesch Group
Dr Vikram Sarabliai Marg,
Baroda-390007

T. P. Tripathi
University of Ilorin Nigeria
Indian Statisics Institute,
Calcutta