The evolution of a layer of electron vortices produced by the interaction of an intense laser pulse and plasma

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The instability of the large-scale motion of a thin layer of electron vortices produced by the interaction of intense laser pulse and a uniform plasma is studied here. The standard point vortex models and vortex sheet models of the flow field involve unphysical singularities in flow field, leading to ill-posed problems. To make the problem well posed a description in terms of vortex blobs is considered. If the vortex sheet has a smooth localized core then the linear growth rates rapidly decrease to zero at high wave numbers, thus providing a natural ultraviolet cutoff. Such a model can then be further used to study the development of small-scale inhomogeneity in the magnetic field produced by the ultraintense laser beam.

I. INTRODUCTION

When a short powerful pulse of laser radiation is made to propagate through plasma, a large magnetic field is generated directly as a consequence of laser plasma interaction.[1–3] Very recently, Gorbunov *et al.*[4] found that a quasistatic magnetic field was generated at the fourth order with respect to the parameter q_E/c (where q_E is the electron quiver velocity and c is the light velocity), if one applies perturbation theory to the relativistic cold electron fluid equations and Maxwell equations. Their result is valid for high-frequency electromagnetic waves ($\omega \gg \omega_{pe}$). It seems from this study that even in a uniform plasma, an azimuthal magnetic field can be generated by density perturbation produced by the plasma wave. In the wake region the magnetic field has a homogeneous component and a component oscillating along the longitudinal direction.[4] The magnetic diffusion skin-layer thickness $\Delta s=(c/\omega_{pe})\delta$, where $\delta=(\nu_{ei}/\omega)^{1/2}$ and so at these high wave numbers, the magnetic diffusion is active and so the electron flow remains laminar[4] and the small-scale inhomegeneity does not develop. However, at larger laser amplitudes and narrow widths, this magnetic field will become unstable and soon turbulence will set in, introducing small-scale inhomogeneity in the magnetic field. In this paper we will study the development of inhomogeneity in the magnetic field, produced by an intense narrow laser pulse.

In the study of Inertial Confinement Fusion (ICF) by a laser, the interaction of an ultrashort, relativistically strong pulse of a laser plasma may give rise to self-focusing.[5–7] An ultraintense laser beam may be self-focused to a narrow bullet leaving behind a trail of the magnetic field.[8,9] As $\nabla \times H$ is proportional to current density and hence the electron fluid velocity, and the isomagnetic curves correspond to the streamlines of the electron motion. The vortex structure and magnetic field correspond to a narrow central sheet carried by relativistic electrons produced by the wave break of the plasma wakefield.[8] Charge neutrality is ensured by two electron current sheets that run at the periphery due to the opposite current repulsion. In this paper we show how inhomogeneity can develop even in a initially homogeneous magnetic field due to Kelvin–Helmholtz-type instability. We consider electron motions, which are slow compared to the Langmuir time and at speeds much smaller than the speed of light. So the relativistic effects are neglected. Here the electron fluid is regarded as a homogeneous, incompressible fluid. The description of transport in an inhomogeneous magnetic field is complicated by the fact that the drift of the charged particles due to the inhomogeneity leads to charge separation in the plasma. The resulting electric field adversely affects the transport of particles and energy across the magnetic field. However, in this preliminary study we neglect the drift currents and assume that ions just form a stationary neutralizing background. The electron fluid equations, then in two dimensions, reduces to the Hasegawa–Mima equation. This same equation also describes the large-scale motion in the atmosphere and it differs from the standard two-dimensional Euler's equation due to the presence of an additional term that in the atmospheric flows arises due to the coriolis force. Bulanov *et al.*[8] replaced the thin layer of vorticity field by a number of point vortices. It has been shown for two-dimensional Euler flows[10] that for the point vortex scheme, neither a finer time step nor a large number of point vortices lead to a smooth rollup. So it is better to formulate the problem in terms of a continuous vortex sheet. Basu *et al.*[11] have studied numerically the motion of a continuous vortex sheet using a panel method to model a mixing layer problem. They find that the vortex sheet develops curvature singularities, although the problem still remains numerically tractable. On the other hand, it is well known that the motion of a vortex sheet can be mathematically described in terms of the Birkoff–Rota equations.[12] It is found that if we expand this sheet in terms of analytical functions, then it leads to a finite

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time curvature singularity,[13] and one does not know how to continue the solution beyond certain critical time t_c. However, in real physical flows, the vortex sheet diffuses instantaneously to become a smooth layer of vorticity, with an exponentially decaying core structure. In the flow of electron fluid considered here, the magnetic diffusivity gives rise to such a core. In fact, at low intensities, for a wide laser pulse the electron flow is laminar.[4] So here we will study the instability of the vortex sheet, which has a smooth localized core structure.

II. BASIC EQUATIONS

The motion of electron fluid is described by the equation

$$m_e\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)=-e\left(\mathbf{E}+\frac{1}{c}(\mathbf{v}\times\mathbf{B})\right)-\frac{1}{n}\nabla p. \tag{1}$$

Here we assume that the ions form a stationary neutralizing background. Neglecting the drift current in the Maxwell's equation, we obtain the velocity of the electron fluid as

$$\mathbf{v}=-\frac{c}{4\pi en}\nabla\times\mathbf{B}. \tag{2}$$

Taking the *curl* of Eq. (1) and using the Maxwell's equation,

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{3}$$

for two-dimensional flow, we obtain,

$$\frac{\partial\mathbf{\Omega}}{\partial t}+u\frac{\partial\mathbf{\Omega}}{\partial x}+v\frac{\partial\mathbf{\Omega}}{\partial y}=0, \tag{4}$$

where $\mathbf{\Omega}=\nabla\times\mathbf{v}-(e/m_ec)\mathbf{B}$ and $\mathbf{v}=(u,v,0)$.

Here, since the electron fluid is assumed to be homogeneous, we neglect the baroclinic terms that produce the magnetic field.

Using Eq. (2) and the fact that $\mathbf{B}$ is divergence-free, we can write

$$\mathbf{\Omega}=\frac{c}{4\pi en}\nabla^2\mathbf{B}-\frac{e}{m_ec}\mathbf{B}. \tag{5}$$

Since we restrict ourselves to two-dimensional flow in the (x,y) plane, $\mathbf{\Omega}=\Omega\hat{z}$, $\mathbf{B}=B\hat{z}$. Now we obtain the governing equations of motion, i.e., the Hasegawa–Mima equation,

$$\frac{\partial\Omega}{\partial t}+\left\|\frac{\partial(B,\Omega)}{\partial(x,y)}\right\|=0, \tag{6}$$

where $\Omega=\nabla^2B-B$.

Here the space coordinates are normalized to c/ω_{pe} and the time coordinate is normalized to $1/\omega_{pe}$, where c is the velocity of light and ω_{pe} is the plasma frequency. The magnetic field unit is $m_e\omega_{pe}c/e$.

The Hasegawa–Mima equation coincides exactly with two-dimensional equations on the tangent plane decribing the large-scale motion in the atmosphere.[14] For small-scale spatial modes this reduces to the two-dimensional Euler's equations,

$$\frac{\partial\Omega}{\partial t}+\left\|\frac{\partial(\psi,\Omega)}{\partial(x,y)}\right\|=0, \tag{7}$$

and the corresponding fluid vorticity is $\Omega=\nabla^2\psi$, where $\psi(x,y,t)$ is the streamfunction.

In fact, as can be seen from Eqs. (6) and (7), the steady-state solutions of the Euler's equations and the Hasegawa–Mima equation is identical. The dynamics, however, could be different.

Let $K(x,y)$ be the Green's function that satisfies the identity

$$\nabla^2K-K=\delta(x)\delta(y), \tag{8}$$

where $\delta(\cdot)$ is the Dirac's delta distribution.

Then we have

$$K(x,y)=-\frac{1}{2\pi}K_0(r), \tag{9}$$

where $r=(x^2+y^2)^{1/2}$ and $K_0(r)$ is the modified Bessel function of zeroth order.

For small values of r this corresponds to the logarithmic Green's function of the Euler's case. For a smooth initial generalized vorticity, the corresponding magnetic field is given by

$$B(x,y,t)=\int_{\mathrm{R}^2}K(x-x',y-y')\Omega(x',y',t)dx'\,dy'. \tag{10}$$

Substituting this in (2), one obtains,

$$\mathbf{v}=-\int_{\mathrm{R}^2}\nabla\times\hat{z}K(x-x',y-y')\Omega(x',y',t)dx'\,dy'. \tag{11}$$

We can replace the integral by the discrete sum

$$\mathbf{v}=-\sum_k\nabla\times\hat{z}K(x-x_k,y-y_k)\gamma_k. \tag{12}$$

This is equivalent to considering the motion of point vortices. Bulanov *et al.*[8] considered the motion of many such point vortices as solutions to Eq. (6). However, the point vortex solutions are not regular solutions of the governing equations. To obtain regular solutions of (6), one has to use vortex blobs, instead of points. Instead of considering the Green's function we can consider the smooth solutions of the equation,

$$\nabla^2K_\delta-K_\delta=\phi(r/\delta), \tag{13}$$

where $\phi(r/\delta)$ is a smooth (C^∞) localized function with a rapid decay, such that as $\delta\to 0$, $K_\delta(x,y)\to K(x,y)$ pointwise. For two-dimensional Euler flows it has been shown[15,16] that smooth solutions can be obtained using vortex blobs. instead of point vortices. In fact, if these governing equations are considered in the limit when the effect of magnetic diffusion is small, then we will have to consider only such smooth solutions. Here, following Refs. 15 and 16, we replace the above equation by

$$\mathbf{v}=-\sum_k\nabla\times\hat{z}K_\delta(x-x_k,y-y_k)\gamma_k, \tag{14}$$

where

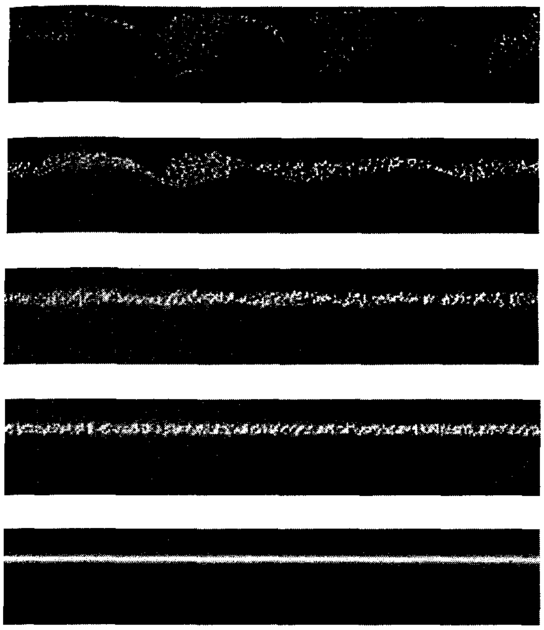

FIG. 1. The vorticity field Ω of a current sheet in a channel of size 1×5 at different times $t=0,5,10,15,20$ bottom to top.

$$K_\delta(x,y)=\int_{\mathbb{R}^2} K(x-x',y-y')\phi_\delta(x',y')dx'\,dy', \quad (15)$$

$\phi(x,y)$ being a well-localized smooth function that is usually normalized and has some vanishing moments.[16] For the Euler's equation using a multilevel vortex method,[17] the evolution of a thin layer of vorticity was simulated in a flat two-dimensional channel using smooth vortex bloblets. The results of the simulation are displayed in Fig. 1. The initial vorticity field was taken to be of the form $\Omega=\exp[-(y-r_L/2)^2/\delta^2]-\exp[-(y+r_L/2)^2/\delta^2]$, where $r_L=0.2$ and $\delta=0.1$. Here r_L is the width of the laser pulse and δ is the skin-layer thickness. The channel has fixed boundaries at $y=\pm 0.5$ and in the longitudinal direction it extends from $x=0$ to $x=5$. This corresponds to a thin current sheet in the plasma. We first note the qualitative similarities between the structure of the vorticity field in Fig. 1 and the structure of the vorticity field obtained in Ref. 8 using particle-in-cell simulations. In Fig. 1, at a later stage, the vorticity field is seen to develop the so-called cat's eye structure.

III. STABILITY ANALYSIS

In this paper we consider the Hasegawa–Mima equation (6). In the small wave number limit, Eq. (6) reduces to

$$\frac{\partial B}{\partial t}=0. \quad (16)$$

So it seems that in this case that the long-wave instabilities are damped. In this paper we investigate this aspect with regard to a thin layer of vorticity. To study the motion of a vortex sheet of zero thickness lying along the line $y=0$, we let

$$\Omega(x,y,0)=\delta(y)\,\gamma(x,0,0), \quad (17)$$

where $\gamma(x,0,0)$ is the strength of the vortex sheet at a point on the sheet.

We track the subsequent motion of this vortex sheet by using the parametric relations, $x=x(s,t), y=y(s,t)$, where $-\infty<s<\infty$. So from now onward we can refer to a point on the vortex sheet by using the parameter s, instead of two coordinates x,y. Then, using the differential relation $\gamma(s,t)ds=\Omega(x,y,t)dx\,dy$ in Eq. (11), we obtain the following equation (after switching to a complex variable form):

$$\frac{dz^*(s,t)}{dt}=⨍_{-\infty}^{\infty} K'[z(s)-z(\tau)]\gamma(\tau,t)d\tau, \quad (18)$$

where $\mathfrak{I}K(z)=K(x,y), z(s,t)=x(s,t)+\mathrm{i}y(s,t)$, * denotes a complex conjugate, ′ denotes a complex derivative, and $\mathfrak{I}$ denotes an imaginary part.

Here the above integral is to be considered as the Cauchy's principle part. Since by Eq. (6), $\Omega(s,t)$ is a flow invariant, we have $\gamma(s,t)=\gamma(s,0)$ for all $t>0$. Now making a change in the variables by letting $\gamma(s,0)ds=d\Gamma(s)$, $Z(\Gamma,t)=z(s,t)$, we arrive at the generalization of the Birkoff–Rota equation,[12]

$$\frac{\partial Z^*}{\partial t}(\Gamma,t)=⨍_{-\infty}^{\infty} K'[Z(\Gamma,t)-Z(\Gamma',t)]d\Gamma'. \quad (19)$$

Similarly, the motion of a vortex sheet with a smooth uniform core structure is described by

$$\frac{\partial Z^*}{\partial t}(\Gamma,t)=\int_{-\infty}^{\infty} K'_\delta[Z(\Gamma,t)-Z(\Gamma',t)]d\Gamma'. \quad (20)$$

Here we make the important assumption that the vortex core moves with the meridian of the vortex layer. This assumption is valid if we are concerned only with large-scale motion of the vortex sheet, and not with the secondary motions of the size of the core.

In this paper we consider the core structure function of the form

$$K'_\delta(z)=-\frac{i}{2\pi z}G(|z|^2). \quad (21)$$

Let us now perturb the vortex sheet and write

$$Z(\Gamma,t)=\Gamma+\epsilon\zeta(\Gamma,t), \quad (22)$$

where $\epsilon\ll 1$.

Using Eq. (20), we substitute $\theta=\Gamma'-\Gamma$ and collect terms of order ϵ to obtain

$$\frac{d\zeta^*}{dt}=-\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{G(\theta^2)[\zeta(\Gamma+\theta,t)-\zeta(\Gamma,t)]}{\theta^2}d\theta + \frac{i}{\pi}\int_{-\infty}^{\infty} G'(\theta^2)\mathfrak{R}[\zeta(\Gamma+\theta,t)-\zeta(\Gamma,t)]d\theta, \quad (23)$$

where $\mathfrak{R}$ stands for the real part.

If we now let

$$\zeta=\sum_{n=-\infty}^{\infty} a_n \exp(in\Gamma), \quad (24)$$

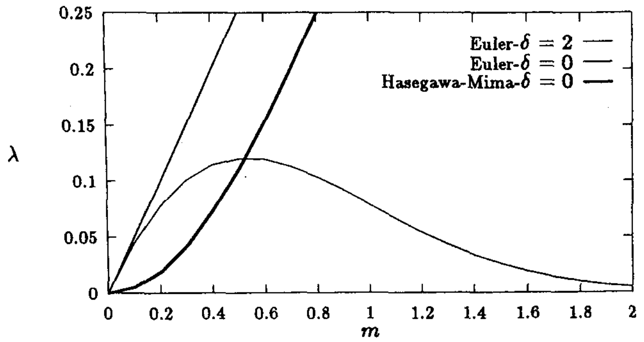

FIG. 2. The linear growth rate as a function of the wave number.

then (23) becomes

$$\sum_{n=-\infty}^{\infty} \frac{da_n^*}{dt} \exp(-in\Gamma)$$

$$= \frac{i}{2\pi} \sum_{n=-\infty}^{\infty} I(|n|) a_n \exp(in\Gamma), \tag{25}$$

where $I(m)$ is given by

$$I(m) = \int_{-\infty}^{\infty} \frac{G(\theta^2)[1-\cos(m\theta)]}{\theta^2} d\theta - 2\int_{-\infty}^{\infty} G'(\theta^2) \times [1-\cos(m\theta)] d\theta. \tag{26}$$

Let us now consider the logarithmic vortex with a smooth exponential core, viz.

$$G(\theta^2) = 1 - \exp(-\theta^2/\delta^2). \tag{27}$$

Substituting this into (26), we obtain

$$I(m) = \pi m - \pi m \ \mathrm{erf}(m\delta/2), \tag{28}$$

where erf(x) is the error function,

$$\mathrm{erf}(x) = \frac{2}{\sqrt{(\pi)}} \int_0^x \exp(-t^2) dt. \tag{29}$$

Equating the Fourier coefficients in Eq. (25) it can be shown that a_n grows like $\exp(\lambda t)$ where λ is given by

$$\lambda = \frac{I(m)}{2\pi}, \tag{30}$$

where $m = |n|$.

We note here that the growth rate λ is normalized by ω_{pe} and the wave number m is normalized by ω_{pe}/c. In Fig. 2, the growth rate λ is plotted against m. We see that the growth rate (the solid line) falls off to zero at higher wave numbers and the critical wave number (beyond which the growth rate falls to zero) is inversely proportional to the core size of the vortex sheet. Here we see that letting $\delta \to 0$, the growth rate for a vortex sheet of zero thickness is obtained as in Ref. 12.

For the (Bessel) vortex sheet of zero thickness, we have

$$G(\theta^2) = \theta K_1(\theta). \tag{31}$$

For this case, using (31) and the relation

$$\int_{-\infty}^{\infty} K_1(\theta)\sin(m\theta) d\theta = \frac{\pi m}{(1+m^2)^{1/2}}, \tag{32}$$

we obtain

$$I(m) = \frac{\pi m^2}{(1+m^2)^{1/2}}. \tag{33}$$

We see here that, as expected, the growth rate of instabilities in the magnetic field is reduced as we decrease m, and in the limit as $m \to 0$ the growth rate drops to zero, as is expected from Eq. (16). For the Bessel-vortex sheet, although the presence of a negative core around the vortex sheet does reduce the growth rate at small wave numbers, at large wave numbers the growth rate is essentially the same (cf., Fig. 2) as that of the standard logarithmic vortex sheet of zero thickness.

IV. DISCUSSION AND CONCLUSION

In this paper we have shown by the linear analysis that for the problem of the large-scale motion of a layer of electron vortices, if the vortex layer has a smooth localized vortex core then the disturbances grow exponentially at a rate that falls off rapidly to zero beyond a critical wave number. This critical wave number is inversely proportional to the size of the core. For the Bessel vortex sheet of zero thickness, the growth rate is less than that for the standard logarithmic vortex sheet, but the growth rate at high wave numbers is essentially the same as that of the standard vortex sheet of zero thickness. This indicates that just like in the Euler's case (i.e. the large wave number limit of the Hasegawa–Mima equation) it is necessary to consider the Bessel vortex sheet with a smooth localized structure to obtain physically meaningful solutions out of the Hasegawa-Mima equation.

Gorbunov *et al.*,[4] found that their analytical result matches quite well with simulation result for small amplitudes and wide laser pulses in which the electron flow remains laminar. For large amplitudes and narrow widths, the agreement failed possibly due the neglect of nonlinear effects. They found that in the wake region an anharmonic magnetic field component arises. This may be due to the onset of convective instabilities studied here. A further analytical study on the growth of secondary local instabilties of the vortex layer will reveal the development of spatial inhomogeneity in the magnetic field at the wake region. The study of the time evolution of the magnetic field may also provide some clue as to how the field affects the transport of laser energy from the critical density surface to the ablation surface in the case of laser-produced plasma.

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