Indian Statistical Institute

Backpaper Examinations, 2009-2010, Semester II

M. Tech. in Computer Science I

Automata, Languages and Computations

31/08/10

Date : ~~September 3, 2010~~ Maximum Marks : 75 Time : 2 Hours

***Attempt all questions**. The paper carries a total of marks 82. Maximum you can score is 75. Figures in the right margin indicate the marks on the different parts of a question.*

1. State and prove the pumping lemma for regular languages.
   Use the above lemma to check if the language L = {x ∈ {a,b,c}* | number of a's in x is not equal to number of b's in x and number of c's in x is even} is regular. [(2+5)+13=20]

2. When do you say that a context-free grammar is in Chomsky Normal Form? Prove that any context-free language is generated by a context-free grammar in Chomsky Normal Form. [3+12=15]

3. Define a Push-down Automaton. Let C be a context-free language and R be a regular language. Prove that C ∩ R is context-free. [4+10=14]

4. Define **formally** a finite automaton that accept strings of zeros and ones which contains an even number of zeros or an odd number of ones - not both! [10]

5. When do you say that a language is decidable? Prove that every context-free language is decidable. [2+8=10]

6. Prove that if L_1 and L_2 are two regular languages, then so is $L_1 \cup L_2$.
   Let $\mathbf{C}$ be any family of regular languages. $\mathbf{C_U}$ = {s | s ∈ L for some L ∈ $\mathbf{C}$}. Is $\mathbf{C_U}$ regular? Justify your answer. [5+8=13]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : 2010-11(First Semester)

M.Tech (CS) I year

Elements of Algebraic Structures

Date :13.09.10 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. The maximun you can score is 100. Notation is as used in the class.

1. (a) Define a cyclic group. Show that a subgroup of a cyclic group is cyclic.

   (b) Condider the additive group of integers $\mathbb{Z}$. Let H be a non-trivial subgroup of $\mathbb{Z}$. Show that there exits a positive integer d such tha $H = d\mathbb{Z} = \{dn : n \in \mathbb{Z}\}$.

   (c) Let G_1, G_2 be two multiplicative groups. Consider the cartesian product $G_1 \times G_2$. For $(x_1, y_1), (x_2, y_2) \in G_1 \times G_2$ define their product to be (x_1x_2, y_1y_2). Show that $G_1 \times G_2$ is a group. This is called the **direct product** of G_1 and G_2.

   Show that the direct product of two cyclic groups need not be cyclic.

   [8+6+(5+6)]

2. When is a subgroup H of G said to be normal. Let H be a normal subgroup of G. Define the quotient group G/H

   Show that if G is finite, then $|G/H| = |G|/|H|$

   Let $f : G \to G'$ be a homomorphism of the group G *onto* G'. Show that the kernel $ker(f)$ is normal. Prove that $G/ker(f)$ is isomorphic to G'.

   Condider the unit circle C of the complex plane as a multiplicative group. Apply the above result to show that the additive group $\mathbb{R}/\mathbb{Z}$ is isomorphic to C [8+5+10+7]

3. Show that every permutation can be expressed as a product of transpositions.

Express the following permutation as a product of disjoint cycles.

$$\pi(1) = 3, \pi(2) = 5, \pi(3) = 1, \pi(4) = 2, \pi(5) = 4.$$

Find the order of each cycle.

If π is an r-cycle, then what is its sign? Justify. [8+10+7]

4. (a) For finite groups, write down the *class equation* or *class formula.* Explain all notation.

   (b) Use the class equation to show that if a group G has order p^2, where p is prime, the G is commutative.

   (c) Define an ideal in a ring R. When is it said to be maximal. Show that if M is a maximal ideal in a commutative ring R then the quotient ring R/M is a field.

   Hence, or otherwise, show that $\mathbb{Z}_p$, p prime, is a field.

   (d) Show that $p\mathbb{Z}$ is a prime ideal in the ring of integers $\mathbb{Z}$, where p is prime.

[7+6+(8+5)+6]

INDIAN STATISTICAL INSTITUTE

Mid-Semester-Examination: 2010-2011 M.Tech. (CS) First Year

Computer Organization

Date: 14.9.2010 **Maximum marks = 60** Credit: 30%

Time: 3 hours

Name:_____ **Roll No.:**_____

Instructions (Read carefully)

A. This is an **OPEN BOOK/OPEN NOTES** exam. Answer all questions; partial credit may be given for incomplete/incorrect answers.

B. Total points = 70; **maximum score = 60.**

C. You may write your answer on the test booklet.

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1. **(2 points)** Convert the following sign-magnitude number into 16-bit 2's complement binary:

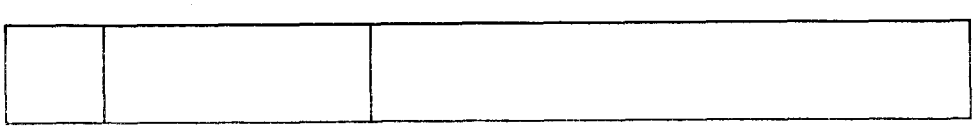
   10011001

2. **(2 points)** Convert the following 2's complement number into 16-bit sign magnitude form:

   1111 1001

3. **(3 points)** Show the IEEE 754 floating point representation of the following decimal number using single precision: -250×10^{-3}

4. **(3 points)** Write the smallest positive denomalized number in the IEEE 754 single precision format:



5. **(3 points)** Consider the following two floating point numbers A and B in IEEE 754 single precision format:

A: 0 0111 1111 1000 0000 0000 0000 0000 000

B: 1 1000 0000 1000 0000 0000 0000 0000 000

The result of floating point multiplication (A * B) in IEEE 754 single precision format is

6. **(3 points)** Consider the following two floating point numbers A and B in IEEE 754 singl precision format:

A: 0 0111 1111 0000 0000 0000 0000 0000 000

B: 0 0111 1111 0100 0000 0000 0000 0000 000

The result of floating point addition (A + B) is:

(i) 0 1000 0000 0100 0000 0000 0000 0000 000

(ii) 0 1111 1110 0100 0000 0000 0000 0000 000

(iii) 0 1000 0000 0010 0000 0000 0000 0000 000

(iv) 0 0111 1111 0100 0000 0000 0000 0000 000

(v) none of the above.

7. **(2 points)** Two integers $A = a_{n-1}\, a_{n-2} \ldots a_0$ and $B = b_{n-1}\, b_{n-2} \ldots b_0$ are being added using complement adder to produce the sum $S =. s_{n-1}\, s_{n-2} \ldots s_0$. Let C_i denote the carry bit generate the $(i-1)^{th}$ bit. Then an overflow (incorrect addition result) will occur if and only if (choose

(i) $C_n = 1$ and $C_{n-1} = 0$, (ii) $C_n = 1$ and $C_{n-1} = 1$, (iii) $C_n \oplus C_{n-1} = 0$,

(iv) $a_{n-1} b_{n-1} \bar{s}_{n-1} \oplus \bar{a}_{n-1} \bar{b}_{n-1} s_{n-1} = 1$, (v) none of the above.

8. **(4 points)** Two (nk)-bit numbers $A = a_{nk}\, a_{nk-1} \ldots a_1$ and $B = b_{nk}\, b_{nk-1} \ldots b_1$ are being added the following scheme.: the bits are partitioned into n groups, each group consisting of k bits. each group of k bits, a ripple carry adder is employed to compute the sum. The carry input of these (n of them) adders is being generated by a carry-lookahead-tree circuit (CLT)

implemented by a prefix network. The asymptotic cost and delay values of the proposed scheme in order notations are (choose one):

(i) cost = O(nk log (nk)); delay = O(log (nk))

(ii) cost = O(nk log k); delay = O(n + log k)

(iii) cost = O(nk log k); delay = O(n log k + k)

(iv) cost = $O(\sqrt{(nk)}$ log k); delay = $O(\sqrt{(nk)}$ log k)

(v) none of the above.

9a. **(3 points)** In a machine, the multiply instruction takes 12 clock cycles for execution, and in a typical program, multiply instructions account for 10% of the total number of instructions. Each of the remaining 90% instructions requires 4 clock cycles on the average. **What** percentage of time does the CPU spend on multiplication?

9b. **(7 points)** A designer suggests that the hardware of the above multiplier can be modified so that the number of clock cycles required for the multiply instruction reduces to 6, but at the cost of increasing the delay of the circuit resulting in a 20% increase in the clock cycle time. **Would** you accept this modification as far as performance is concerned? **Justify** your argument.

10. **(5 points)** Two enhancements (namely 1 and 2) on an old machine are proposed for a new architecture with the following speed-ups: Speed_up_1 = 30; Speed_up_2 = 20. Only one enhancement is usable at a time. If enhancements 1 and 2 are usable for 20% and 30% of the time respectively, what will be the overall_Speed_up? Use Amdahl's law to justify your argument.

11. **(4 points)** Write MIPS code for the following statement: X[4] = X[5] + C, where C is in register $t0, and the base address of the array, i.e. address of X[0] is
1000 1111 1100 1110 1111 1111 1000 0000.

12. **(4 points)** Consider the following MIPS code:

lui $t1, 7FFF (in hex)
ori $t1, $t1, FFFF (in hex)
addu $t1, $t1, $t1
sll t1, t1, 2
addi $t1, $t1, 9

The content of the register $t1 after execution of the above code is (choose one):
(i) 0, (ii) 1, (iii) –1, (iv) a number causing overflow, (v) none of these

13 **(5 points)** The total number of bytes transferred from the main memory to CPU when the following code is executed is (choose one):

(i) 18 (ii) 28 (iii) 36 (iv) 52 (v) 56 (vi) none of these.

```
lw $t2, 7($t5)
lw $t3, 10($t2)
add $ t1, $t2, $t3
addi $t1, $t1, 1
sll t1, t1, 1
lw $t2, 12($t1)
lw $t3, -20($t2)
add $ t1, $t2, $t3
sw $t2, -36($t1)
```

14. **(13 points: (6+7))** In the single-cycle data path implementation of a MIPS processor as sho in Figure 1, let us assume that the operation time for the major functional units are as follows:

Memory units: 2 ns; register file: 1 ns; ALU: 2 ns; adder for PC+4: 2 ns; adder fo branch address computation: 2 ns; MUX blocks: 1 ns; sign-extend/shift-left: 1 ns other units: negligible.

(a) Assuming that the machine implements only a small subset (*lw, sw, add, beq, j*) of instructions, calculate the minimum clock cycle time (CCT).

(b) Consider the following MIPS code. Based on the above operation time of the units, calculate the total amount of time required to execute the code.

```
lw $t1, 1000($t2)
lw $t2, 1000($t2)
add $ t2, $t2, $t2
Loop:  add $ t1, $t1, $t1
beq $t1, $t2, Loop
sw $t2, 1000($t3)
```

15. (7 points) Consider a multicycle implementation of datapaths for a MIPS subset as show in Figure 2. What will be status of the following control lines ALUSrcA, ALUSrcB, and ALUOp during the cycle of memory address computation for the instruction: lw $t2, 1000($t3)? Further, if the ALUSrcA line is permanently stuck-at logic 1 under a fault, which instructions will malfunction?

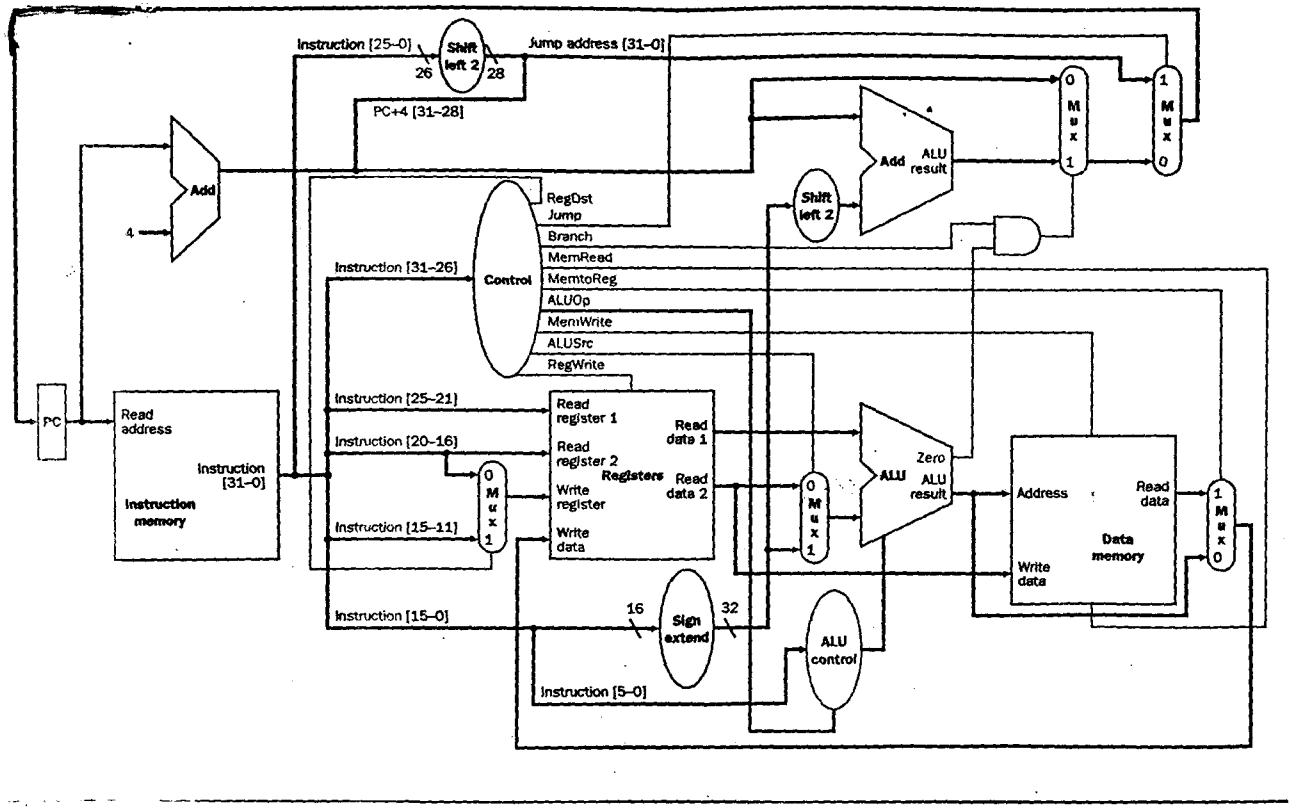

Instruction [25–0]
Shift left 2
26
28
Jump address [31–0]
PC+4 [31–28]
Add
4
M u x
0
1
Add
ALU result
Shift left 2
RegDst
Jump
Branch
MemRead
MemtoReg
ALUOp
MemWrite
ALUSrc
RegWrite
Control
Instruction [31–26]
PC
Read address
Instruction [31–0]
Instruction memory
Instruction [25–21]
Instruction [20–16]
Instruction [15–11]
Instruction [15–0]
Instruction [5–0]
Read register 1
Read register 2
Registers
Write register
Write data
Read data 1
Read data 2
16
Sign extend
32
ALU
Zero
ALU result
ALU control
Address
Read data
Write data
Data memory

Figure – 1 .

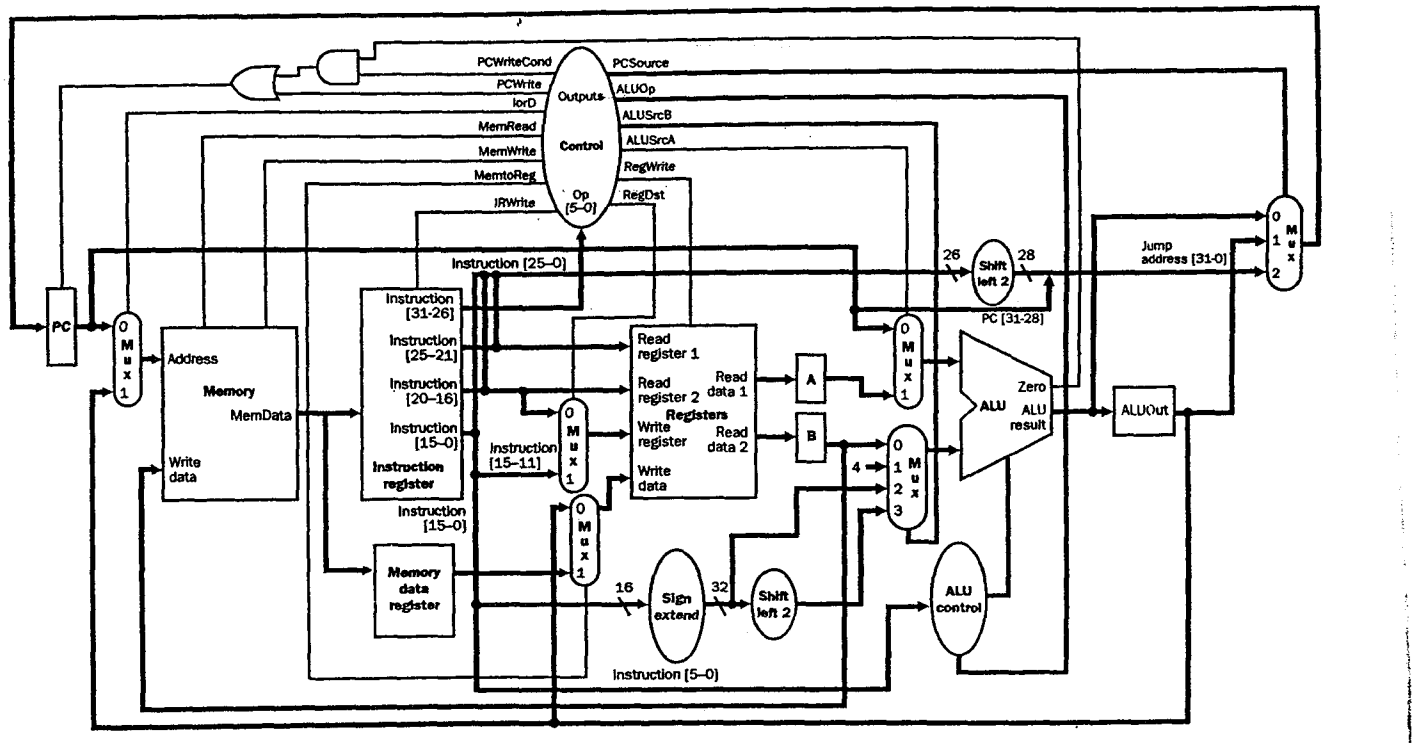

PCWriteCond
PCWrite
IorD
MemRead
MemWrite
MemtoReg
IRWrite
Outputs
Control
Op [5–0]
PCSource
ALUOp
ALUSrcB
ALUSrcA
RegWrite
RegDst
PC
0 M u x 1
Address
Memory
MemData
Write data
Instruction [31-26]
Instruction [25–21]
Instruction [20–16]
Instruction [15–0]
Instruction register
Instruction [15–0]
Memory data register
Instruction [25–0]
Instruction [15–11]
Read register 1
Read register 2
Registers
Write register
Write data
Read data 1
Read data 2
A
B
4
ALU
Zero
ALU result
ALUOut
26
Shift left 2
28
PC [31-28]
Jump address [31-0]
0 1 2 3 M u x
16
Sign extend
32
Shift left 2
ALU control
Instruction [5–0]

Figure-2

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination (2010-2011)

M.Tech. (CS) – First year

Switching Circuit and Logic Design

Maximum marks: 80

Date:16.9.10 **Time : 2 hours 30 mins**

Answer all the questions

1a) What is an Unate function? What is the condition for a function to be Unate? [5]

b) Determine which of the following functions are Unate?

i) **$F=\Sigma$ (1,2,3,8.9,10,11,12,14)**

ii) **$G=\Sigma$ (1,4,5,7,9,12,13,15,29,31)**

[8]

c) A three variable function **F (x_1 ,x_2 ,x_3)** realized by a two-level **NOR-NOR** network remains unaltered when all the **NOR** gates of the network are replaced by **NAND** gates.

i) Find a suitable function F (x_1 ,x_2 ,x_3) satisfying the above criterion.

ii) Is the above function unique? If not, find at least one more function that could have the same property. [12+5]

2a) Use map method to find a minimal set of tests for detecting all multiple faults for the two-level AND-OR realization of the function given below.

$F(w,x,y,z)= wz' + xy' + wx'y + w'x$ [10]

3. A PLA has the following SSR (Set of subsuming rows) specifications:

A = {1,2,6}, B = {1,5}, C = {2,3,5,6}, D = {3,4,7,9}

E = {2,3,4,8,9}, F = {7,8,9},

i) Find the compatibility matrix of the PLA.

ii) Find the foldable compatibility matrix(FCM) of the above problem

iii)Find the ordered foldable pairs for maximum folding of the above PLA by FCM .

[5 + 10 + 5]

P.T.O

4 i)Determine a minimum-row state table for a clock mode sequence detector with one input and one output line, which produces an output **1** when either of the sequences **011** or **100** is detected. Once a sequence is detected, no part of it can be taken to form part of another sequence: that is overlapping sequence is not allowed.

ii) Design the circuit using **D** flip-flops.

[15 + 5]

INDIAN STATISTICAL INSTITUTE
Periodical Examination
M. Tech. I year: 2010–2011
Optimization Techniques

Date: 17. 9. 2010 Marks: 60 Time: 2.5 Hours

1. (a) Define a convex set and its corner points.
   (b) Prove that in a convex set, every point is a convex combination of the corner points.
   (c) Prove that the optimal solution of a convex programming problem lies on one of the corner points.

   [(2+2)+6+6 = 16]

2. Prove that if a feasible solution corresponds to two distinct bases then it must be degenerate. Is the converse true? Justify your answer. [5+5 = 10]

3. Solve the following LPP.

$$\begin{array}{lrcl} \text{minimize} & x+y & & \\ \text{subject to} & 2x+3y & \geq & 5 \\ & x+2y & \leq & 6 \\ & -x+y & \geq & 1 \\ & x,y & \geq & 0 \end{array}$$

   [14]

4. (a) Define dual of a linear program.
   (b) Prove that if an LP has an optimal solution then so does its dual and at optimality their costs are equal.
   (c) Write down the dual of the LPP of Question 3. Solve the dual.

   [3+5+2 = 10]

5. (a) Suppose that, while searching for an augmenting path from a node u of a graph G with respect to a matching M, we discover a blossom b. Prove that there is an augmenting path from u in G with respect to M iff there is one from u in G/b with respect to M/b.
   (b) Formulate an LP for finding the maximum weight matching with exactly m edges, for a fixed m, in a non-bipartite graph.

   [8+2 = 10]

INDIAN STATISTICAL INSTITUTE

PERIODICAL EXAMINATION
M.TECH.(CS) I YEAR

PROBABILITY AND STOCHASTIC PROCESSES

Date: 20.9.2010 Maximum marks: 80 Duration: 2 hrs and 30 mins

The paper contains 100 marks. Answer as much as you can, the maximum you can score is 80.

Group – A

1. Among the digits 1,2,3,4,5 first one is chosen, and then a second selection is made among the remaining four digits. Assume that all twenty possible results have the same probability. Find the probability that an odd digit will be selected
   (a) the first time,
   (b) the second time,
   (c) both times.

   (10)

2. (a) Two dice are thrown r times. Find the probability p_r that each of the six combinations $(1,1),\ldots,(6,6)$ appears at least once.
   (b) A die is thrown as long as necessary for an ace to turn up. Assuming that the ace does not turn up at the first throw, what is the probability that more than three throws will be necessary?
   (c) If the probability of hitting a target is 1/5 and ten shots are fired independently, what is the probability of the target being hit at least twice?
   (d) Let X, Y and Z be independent random variables with the same geometric distribution $\{q^k p\}$. Find $\Pr[X+Y \leq Z]$.

   $(5+5+5+5=20)$

Group – B

1. Let X and Y be random variables having joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{x} & 0 < y \leq x \leq 1; \\ 0 & \text{otherwise.} \end{cases}$$

   Compute $\Pr[X^2 + Y^2 \leq 1]$.

2. Define the σ-algebra $\mathcal{B}$ of Borel sets of $\mathbb{R}$. Let $\pi(\mathbb{R}) = \{(-\infty, x] : x \in \mathbb{R}\}$. Show $\mathcal{B} \subseteq \sigma(\pi(\mathbb{R}))$ where $\sigma(\pi(\mathbb{R})$ denotes the smallest σ-algebra generated by the set $\pi(\mathbb{R})$.

3. Let $X_1, X_2, \ldots, X_n$ be independent Bernoulli trials with $\Pr[X_i = 1] = p_i$ and let $X = X_1 + X_2 + \cdots + X_n$ and $\mu = p_1 + p_2 + \cdots + p_n$. Show that for $0 < \delta \leq 1$,

$$\Pr[X < \mu - \mu\delta] \leq \exp\left(\frac{-\mu\delta^2}{2}\right).$$

4. Suppose that a deck of n numbered cards are put in a random order so that all $n!$ permutations are equally likely. Let S_n be the number of cards in its proper place. Compute $\mathrm{Var}(S_n)$.

5. Suppose there are two urns each containing n balls. An urn is chosen at random and a ball is taken out from the urn (and not replaced). This experiment is continued until the chosen urn is found to be empty. When this event occurs, what is the probability that the other urn has exactly r balls?

$(10 + 10 + 10 + 10 + 10 = 50)$

Group – C

1. Urn R contains n red balls and urn B contains n blue balls. At each stage, a ball is selected at random from each urn and they are swapped. Show that the expected number of balls in urn R after stage k is $\frac{1}{2}(1 + (1 - 2/n)^k)$.

2. Let $X_1, \ldots, X_n$ be a sequence of independent random variables where each X_i takes values from the set $\{0, 1, 2\}$ with probabilities p_0, p_1, p_2 respectively. Show how to extract independent and uniformly distributed random *bits* from such a sequence.

$(10 + 10 = 20)$

INDIAN STATISTICAL INSTITUTE
Mid-semestral Examination
M.Tech (CS) –I year (semester – I)
Data and File Structures

Date: September 21, 2010 Maximum Marks: 80 Duration: 3:30 Hours

This exam is open-book and open-notes. But you are not allowed to consult, or pass any documents to, or collect documents from others.

1. A lower triangular matrix is a matrix $A=(a_{ij})$ in which $a_{ij} = 0$ for $i < j$ and thus it is written

$$A = \begin{pmatrix} a_{11} & & & & & \\ a_{21} & a_{22} & & & & \\ a_{31} & a_{32} & a_{33} & & & \\ . & . & . & . & & \\ . & . & . & . . & & \\ a_{n1} & a_{n2} & a_{n3} & . . & . & a_{nn} \end{pmatrix}$$

Design a technique to allocate this matrix in an array of size $n(n+1)/2$ such that the location of a_{ij} should be a simple function of i, j and the location of a_{11}. [5]

2. Suppose we change the definition of height balanced tree so as to allow a maximal difference of 2 instead of 1 between the heights of the two subtrees of each node. Design rebalancing algorithms so as to ensure logarithmic search times. What are the most skewed height balanced trees in this case. [20]

3. Given a set A of n sorted integers, the position of some value x is the number of elements smaller than or equal to x in set A (including x itself if it is in the set). let us define the position of y as the kth smallest entry greater than some value x whenever the position of y is the $i+k$ th position where the position of x is i. It may not exist if $i+k$ is greater than n.

Suggest a modification to balanced binary search tree data structure such that given A, x and k, the kth smallest entry greater than some value x can be obtained in $O(\log n)$ time and also present the algorithm to locate the element. [20]

4. Find the minimum height AVL tree that is not in WB[1/10]. [8]

P.T.O

5. Let T_1, T_2 be 2 arbitrary binary trees each having n unlabelled nodes. Show that it is possible to transform from tree T_1 to T_2 by applying at most $2(n-1)$ single rotations. [10]

6. Let $A[1, \ldots, n]$ be an array of numbers. Design a data structure and an algorithm to perform an sequence of the following operations:
   - *Add(i, y)* - add y to $A[i]$.
   - *PartialSum[i]* - return the sum of the first i numbers in the array.

   Both the operations should run in time $O(\log(n))$. You may use as a workspace arrays of size n. First specify the data structure and then give your algorithm. [15]

7. A linked list has exactly n nodes. The elements in these nodes are selected from the set $\{0,1,\ldots,n\}$. There are no duplicates in the list. Design an O(n) worst case time algorithm to find which one of the elements from the above set is missing in the given linked list using only a constant amount of additional storage. [10]

8. There are people standing in a circle waiting to be executed. After the first person is executed, certain number of people are skipped and one person is executed. Then again, people are skipped and a person is executed. The elimination proceeds around the circle (which is becoming smaller and smaller as the executed people are removed), until only the last person remains, who is given freedom.

   There are n persons, numbered 1 to n, around a circle. We eliminate second of every two remaining persons until one person remains. Given n and x, write a 'C' program to determine the xth person to be eliminated using circular queue.
   For example, if n=10, x=3, answer will be 5. [15]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination

M. Tech (CS) - I Year, 2010-2011 (Semester - I)

Discrete Mathematics

Date : 23.09.2010 Maximum Marks : 60 Duration : 3.0 Hours

Note: This is a **cheat sheet based** examination. You can carry with yourself **two A4 sized sheets** with your name and roll number written neatly on top of both the sheets. You have to submit the cheat sheets after the examination is over. Cheat sheets cannot be shared.

Answer as much as you can, but the maximum you can score is 60.

This is a 2 page question paper with 8 questions.

(Q1) Prove or disprove the following statement. Given two non-negative functions $f(n)$ and $g(n)$, either $f(n) = O(g(n))$ or $g(n) = O(f(n))$. [6]

(Q2) Let P be a set of n points lying on the circumference of a circle such that if lines are drawn connecting every point to every other point, then no three of these lines intersect in a single point inside the circle. Let A be the set of all points of intersection of the lines in the interior of the circle. Note that, the points of P are not included in A. Derive an expression for the cardinality of the set A? [12]

[Hint: Can you define functions and count?]

(Q3) A binary relation R on a set X, denoted as (X, R) is a subset of the Cartesian product $X \times X$. We use $\sim aRb$ to denote that (a, b) is not in R.

A binary relation is irreflexive if $\sim aRa, \forall a \in X$.

A binary relation is asymmetric if $aRb \Rightarrow \sim bRa, \forall a, b \in X$.

A binary relation is antisymmetric if aRb and bRa implies $a = b, \forall a, b \in X$.

Now, prove the following statement. *A binary relation is irreflexive, transitive and antisymmetric if and only if it is transitive and asymmetric.* [8]

(Q4) (i) Let us consider our disrete mathematics class. If Shaondip is late, then Kalyan is late, and if both Shaondip and Kalyan are late, then the class is boring. Suppose that the class is not boring! What can you conclude about Shaondip?

[Hint: Formulate the problem in terms of logic.]

(ii) Write the following statement in predicate logic and then negate it. Clearly mention what is your domain and predicates.

Let x and y be real numbers. If x is rational and y is irrational, then $x + y$ is irrational.

[4+(3+3)=10]

[PTO]

(Q5) (i) Find out the number of ways in which you can distribute n distinguishable balls into k distinguishable cells where no cell remains empty. Give justifications for your answer.

(ii) Find out the number of ways in which you can distribute n distinguishable balls into k indistinguishable cells where cells can be empty. Give justifications for your answer.

[4+6=10]

(Q6) A box contains 10 blue balls, 20 red balls, 8 green balls, 15 yellow balls and 25 white balls. How many balls must we choose to ensure that we have 12 balls of the same colour? [5]

(Q7) Prove that if $n^2 + 1$ points are placed in an equilateral triangle (the region inside as well as the perimeter) of side length 1, then there are two points whose distance is at most $\frac{1}{n}$. [8]

[Hint: Start with small values of n and then try to generalize.]

(Q8) Consider a building having a staircase with n stairs. In how many ways can a person climb the staircase, if she can climb by 1 or by 2 stairs in each step? Find out a closed form expression in terms of n. [8]

INDIAN STATISTICAL INSTITUTE
M. Tech. (CS) I: 2010–2011
Programming Languages and Methodology
Mid Semester Examination

Date: 24. 09. 2010 Marks: 50 Time: 2 Hours

Answer any part of any question. The question is of 55 marks. The maximum marks you can get is 50. Please write all the part answers of a question at the same place.

1. (a) Write a C program, that will accept a character string and inform you if there is any palindrome of length at least 5 characters.

   (b) What is the prototype of the "strtok" library function in C? How can you implement it? $8 + (2 + 5) = 15$

2. (a) Write a function in C that finds the GCD as well as the LCM of two unsigned integers and show how the function executes when the two integers are 2310 and 96900.

   (b) Will your functions work properly for all pairs of unsigned integers? Explain. $6 + 4 = 10$

3. (a) Write a piece of code in C to dynamically allocate a two dimensional array of integers.

   (b) Consider the following C program.

```
#include <stdio.h>
main(){
        int arr[10][10], i, j;
        for (i = 0; i < 10; i++)
        for (j = 0; j < 10; j++)
                arr[i][j] = i*j + 1;
        printf("%x\n", arr); printf("%x\n", arr+1);
        printf("%x\n", *arr); printf("%x\n", *arr+1);
        printf("%x\n", **arr);
}
```

   The first line of output of this program is fef56970. What are the rest of the outputs? Give proper justification to your answer. $3 + 7 = 10$

4. (a) Write C functions using iteration as well as recursion to calculate the n-th Fibonacci number.

   (b) Explain with proper examples and figures of activation records which one is more efficient. $5 + 5 = 10$

5. Explain the parameter passing strategy (to functions) in C programming language. More credit will be given for clear examples. 10

INDIAN STATISTICAL INSTITUTE

Semester Examination : 2010-11(First Semester)

M.Tech (CS) I year

Elements of Algebraic Structures

Date :06.12.10 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. The maximun you can score is 100. Notation is as used in the class.

1. (a) For a positive integer n, let

$$\mathbb{Z}_n^* = \{a \in \mathbb{Z} : gcd(a, n) = 1\}.$$

Show that $\mathbb{Z}_n^*$ is a multiplicative group where multiplication is defined modulo n.

Hence, or otherwise, show that for any integer a with $gcd(a, n) = 1$,

$$a^{\phi(n)} \equiv 1 \bmod n.$$

[10]

2. Show that every ideal I in the ring $\mathbf{F}[x]$, where $\mathbf{F}$ is a field is generated by a single polynomial i.e. $I =< p(x) >$ for some $p(x) \in \mathbf{F}[x]$.

Show that the ideal generated by $1 + x + x^2$ is maximal in $\mathbb{R}[x]$. (Hint: Imitate the proofs for $\mathbb{Z}$.)

Prove that $1 + x^2$ is irreducible but $1 + x + x^2$ is reducible over $\mathbb{Z}_7$. Hence find the factors in the latter case. [7+8+10]

3. (a) Define a vector space over a field $\mathbf{F}$.

(b) Show that any two vector spaces of the same dimension are isomorphic.

(c) Find a basis of the following subspace of $\mathbb{R}^3$

$$\mathcal{S} = \{(x, y, z) \in \mathbb{R}^3 : 2x - 3y + z = 0\}.$$

[4+6+6]

P.T.O.

4. Suppose U, W are subspaces of a finite-dimensional vector space V. Define the space $U + W$. Prove that

$$dim(U + W) = dim(U) + dim(W) - dim(U \bigcap W).$$

[10]

5. (a) Given a linear map $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$, obtain an $m \times n$ matrix A such that

$$f(\mathbf{x}) = A.\mathbf{x}; \text{ for all } \mathbf{x} \in \mathbb{R}^n,$$

where $\mathbf{x}$ is regarded as a column vector.

(b) Define the *rank* and *nullity* of a matrix.
Show that for an $m \times n$ matrix A over a field $\mathbf{F}$,

$$\rho(A) + \nu(A) = n,$$

where ρ, ν denote rank and nullity respectively.

[6+10]

6. (a) Consider the following system of linear equations

$$A\mathbf{x} = \mathbf{b},$$

where A is an $m \times n$ matrix $\mathbf{x} = (x_1, \ldots, x_n)^T$ and $\mathbf{b}^T \in \mathbb{R}^m$. Show that the system is consistent iff $\rho(A) = \rho([A : \mathbf{b}])$.

(b) Show that the system

$$\begin{aligned} 2x + 4y &= 1 \\ 4x + 3y &= 2 \end{aligned}$$

has a unique solution over $\mathbb{R}$. Does it have a unique solution over $\mathbb{Z}_5$? Justify and find all the solution(s).

[8+10]

7. Construct a finite field $\mathbf{F}$ of order 8. Explain the field multiplication and addition. Show that with respect to these operations it is indeed a field. Can the field elements be repsented by bit strings? [10]

8. Show that every element of a finite field of order p^n is a root of the polynomial $x^{p^n} - x$. [6]

INDIAN STATISTICAL INSTITUTE

Semestral Examination (2010-2011)

M.Tech. (CS) – First year

Switching Circuit and Logic Design

Maximum marks: 100

Date: 6.12.10 **Time : 3 hours**

Answer any four questions

1(a).A logic circuit has five inputs x_1, x_2, x_3, x_4 and x_5. Output Z_0 is **1**, when majority of the inputs are **0**. Output Z_1 is **1**, when fewer than four inputs are **1**, provided that at least one input is **1**. Output Z_2 is **1**, when two ,three or four of the inputs are **1**. Find a minimum cost realization of the above circuit using Karnaugh maps. [20]

(b).Using minimum number of Nor gates realize EXCLUSIVE-OR function. [5]

2. i) Determine a minimum-row state table for a clock mode sequence detector with one input and one output line, which produces an output **1** for every fourth **1** input (not necessarily consecutive).

ii) Design the above circuit using **D** flip-flops. [15 + 10]

3.A fundamental -mode sequential circuit has two inputs x_1 and x_2, and one output **Z**. The output becomes **1** only when x_1 changes its value from **1 to 0** while x_2=1. Once **1**,the output remains **1**, and returns to **0** only when x_2 changes from **1 to 0** while x_1 =**1.**

(a) Derive a minimum row state table for the above circuit?
(b) Determine the output equation. [15+10]

4.(a) For the machine shown in table below, find a preset experiment which will identify the initial state. The machine is initially provided with an input sequence **001** to which it responds by producing an output sequence **101** .

[P.T.O]

Present state	Next state,Output X=0	x=1
A	B, 0	E,0
B	E,1	C,0
C	D,1	A,1
D	A,1	B,0
E	C,0	D,1

(b).It is next provided with the sequence **011011011011011.** Show that this sequence determines the number of states of the machine.

(c). Find the correct output sequence for this input sequence.

[10+10+5]

5. For an incompletely specified machine, whose state table is shown below, determine a minimum state reduced machine for the given state table. Draw the merger table. State if this minimization is unique.

Present state	Next state,Output X=0	x=1
A	E,1	F,-
B	D,0	-,-
C	A,-	E,1
D	F,1	E,0
E	C,-	A,1
F	B,0	D,-

[12 + 8 + 5]

INDIAN STATISTICAL INSTITUTE

First-Semestral-Examination: 2010-2011 M.Tech. (CS) First Year

Computer Organization

Date: December 07, 2010 **Maximum marks = 100** Credit: 50%

Time: 3 hours

Name:_____ **Roll No.:**_____

Instructions (Read carefully)

A. Answer all questions; partial credit may be given for incomplete/incorrect answers.
B. This is an **OPEN-BOOK/OPEN-NOTES** exam.
C. Total points = 110; **maximum score =100.**

---

1. (15 points) All appearances of the following code sequence

add R1, R1, R2
lw Rd, 0 (R1), in machine X,
are being replaced by: *lw* Rd, 0 (R1 + R2) in a new machine Y.

The frequency of *lw* (load) is 20% in a program P running on X, and the above combination of addressing mode is used for 10% of all loads. Finally, the machine Y is upgraded to a third machine Z, in which the hardware for executing the *lw* instruction is improved by a factor of 10 compared to that in machine Y. Assume CPI(X) = CPI(Y) = CPI(Z), and CCT(X) = CCT(Y) = CCT(Z). Compute the overall speed-up of machine Z compared to machine X for the program P.

2. (10 points) Consider the following MIPS code:

```
Loop: add $t1, $t2, $t3
      add $ t1 $t1, $t1
      slt $t1, $t2,  $t3
      addi $t2, $t2, -1000
      sw $t2, 1000($t3)
      beq $t2, $t3, Loop
```

Write in machine code, the last instruction of the above MIPS code.

3. (10 points) Write down the following numbers in IEEE 754 single precision floating point format:

(i) the largest denormal number;
(ii) the smallest positive number.

P.T.O.

4. (10 points) Consider a (32 × 32) array A[i, j] stored as row major in memory which consist 16 banks interleaved on word basis. The banks share the same address and data lines, and hav memory controller. The following code is to be executed:

```
int A [32] [32]
        for (j = 0; j < 32;  j = j+1)
                for (i = 0; i < 32; i = i +1)

                        A[i][j] = 2 * A[i] [j];
```

What problem of data access might arise in utilizing the full bandwidth of the interleaved mem system while executing the above code? How should this code be re-written to improve memo performance?

5. (15 points) Consider a memory system with 4-way set associative, write-back cache consist of 64 1-word blocks. Assume 1 word = 32 bits. The main memory consists of 1 K words. The cache hit ratio is 0.85, average cache access time per word is 100 ns, and average main memor access time per word is 500 ns.

(i) Compute the total size of the cache in terms of the number of bits.

(ii) Compute the average overall memory access time (considering both cache and main memory) as seen by the CPU.

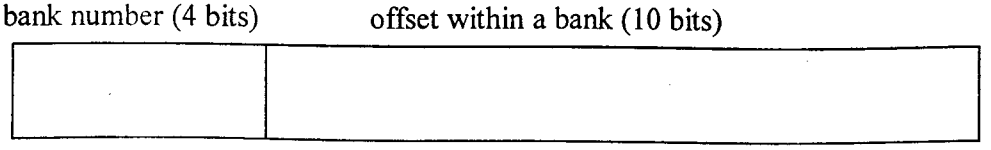
6. (5 points) Compulsory misses in a cache can be reduced (choose one):

(i)	by increasing associativity;	(ii)	by increasing block size;
(iii)	by using a victim cache;	(iv)	by enhancing compiler techniques;
(v)	by using both (i) and (iii);	(vi)	none of the above.

7. (5 points) Consider a cache in which each block consists of multiple words. Sending the criti words to cache from main memory with higher priority, on a request, will reduce (choose one):

(i) capacity miss rate;
(ii) conflict miss rate;
(iii) miss penalty;
(iv) hit time;
(v) none of the above.

8. (10 points) Consider a machine with 16 K main memory words (address ranging from 0 through 16383). The memory is organized as 16 banks (bank-0, bank-1, .. , bank-15) interleaved sequentially on word basis. Each bank consists of 1 K memory locations. What would the binary address of memory location 5000 in the following format?



9. The memory hierarchy of a computer system consists of virtual memory, main memory and cache and is organized as follows. Assume 1 word = 32 bits.

(i) Virtual memory: 8 virtual pages; page size = 2K words;
page replacement policy is FIFO.

(ii) Main memory (MM):
4 physical page frames; page size = 2K words; MM has 2-way interleaving;
Each line per bank of MM = 1 word;
time to send the line address from CPU to MM = 5 clock cycles;
time to access a line in MM = 15 clock cycles;
time to deliver a word from MM to cache = 4 clock cycles;
width of the memory data bus = 32 bits.

(iii) Cache:
The cache is physically addressed, write-through, direct-mapped, has a capacity of 4K words, cache block size = 2 words;

Currently, virtual pages 0, 1, 4, 5 are in MM, and they were loaded in the MM in the same order in time (i.e., page 0 first, page 1 next, so on). The current page table is given by:

Virtual page	Page frame
0	3
1	1
2	not in MM
3	not in MM
4	2
5	0
6	not in MM
7	not in MM

The CPU requests access to an item X whose virtual address is 0100 1001 0010 10. Assume that currently cache is empty.

(i) (5 points) Write the sequence of events that would take place while processing on this request (e.g, cache hit/miss, address translation, page fault, etc.).

(ii) (5 points) What will be the physical address of X after being uploaded to MM (in binary)?

(iii) (5 points) Where should X be located in the cache (cache index in binary)?

(iv) (5 points) What would be the miss penalty of the cache for a read miss in clock cycles?

10. (10 points) A computer has 3 levels of cache denoted as L1, L2, and L3, where L1 lies closest to the CPU. Let H_i and M_i denote the hit time and miss rate of the i-th level cache, $1 \leq i \leq 3$, and let P_3 be the miss penalty of L3. Write the expression for the average memory access time in terms of the above parameters.

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech (CS) - I Year (Semester - I)

Data and File Structures

Date : 09.12.10 Maximum Marks : 100 Duration : 4:00 Hours

Note : You may answer any part of any question, but maximum you can score is 100.

1. You are given a min-heap that is stored in an array A. You can assume that all elements are distinct. Describe an algorithm ThirdSmallest(A) that returns the third smallest element in A or Nil if A contains less than 3 elements. Your algorithm needs to be strictly faster than linear time, that is, simply scanning through the whole array is not an acceptable answer. Full points will be awarded only for a constant time solution. You should analyse the running time of your algorithm and argue its correctness. **15**

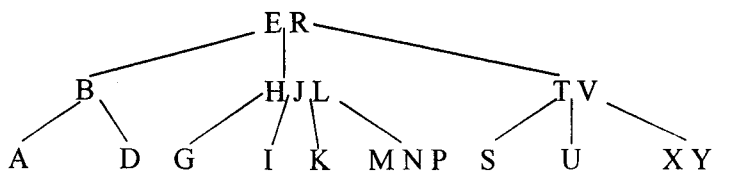
2.(a) Using double hashing with the hash functions $h(k; i) = (h_1(k)+i.h_2(k))$ mod 11, $h_1(k) = k$ mod 13, and $h_2(k) = 1+(k \bmod 7)$, insert the numbers 20, 14, and 30 into this hash table:

0	1	2	3	4	5	6	7	8	9	10
	1			17				8		36

(b) Which problem occurs if we choose a hash table of size 12 instead of 11 and replace $h(k; i)$ by the function $h_0(k; i) = (h_1(k) + i \,.\, h_2(k))$ mod 12? Explain your answer.

(c) We want to store a set S. 10000 numbers in the range $0; \ldots; 100000000$ and we want to be able to test efficiently if a particular number is in S. We decide to use a hash table $T[0::10000]$ of size 10001 and to resolve collisions with chaining. As a hash function we choose $h(x) = \sqrt{x}$. Explain why this is not a good hash function. **5+5+5=15**

3. Consider the following B-tree with a minimum branching factor of $t = 2$.



(a) Show the B-tree that results from inserting Q into the above B-tree.

(b) Show the B-tree that results from deleting E in what you obtained AFTER inserting Q.

5+5=10

4. Give an $O(n \log k)$ algorithm to merge k sorted lists $L_1, \ldots, L_k$ (sorted in increasing order) into one sorted list, where n is the total number of items in all the lists combined. **10**

5. Consider the function defined by $f(1)=1, f(2)=2, f(n)=f(n-1)+f(n-2)$ for all $n>2$. Show that this function grows exponentially. **5**

6. Suppose you are given a collection S of n intervals $[x_{lo}, x_{hi}]$. Derive an efficient data structure and algorithm to report all the intervals of S that have a nonempty intersection with a query interval $Q = [q_0, q_1]$. Your query algorithm should run in $O(k + \log n)$ time where k is the number of intervals reported. **15**

7. (a) Prove that in any red-black tree, no path from any node v, to a leaf is more than twice as long as any other path from v to any other leaf.
   (b) Prove that if a black node has just one child, that child must be red.
   (c) Show the tree that results from inserting the values 2, 1, 4, 5, 9, 3, 6, 7 sequentially into an initially empty red-black tree. Show the tree after each insertion.

4+3+8=15

8. Suppose that a search for key k in a binary search tree ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Provide a small counterexample that disproves the following claim: *Any three keys $a \in A$; $b \in B$; $c \in C$ must satisfy $a \leq b \leq c$.*

12

9. An alternative method of performing an in order tree walk of an n-node binary search tree finds the minimum element in the tree and then finding its n - 1 successors, one at a time. In other words, if minimum is a, then first find a, then find b =successor of a, then c=successor of b, etc. Prove that this algorithm runs in $O(n)$ time. [Note that one cannot claim that "successor" operation always takes $O(1)$ time, even for an approximately balanced binary search tree.] **13**

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2010-11
M. Tech. (CS) I Year
Optimization Techniques

Date: 10.12.10 Maximum Marks: 100 Duration: 3 Hours

Answer all questions

1.(a) Define a convex set; give example of a convex set as well as of a non-convex set.

(b) Define a convex function; give example of a convex function as well as of a non-convex function.

(c) Define an extreme vertex of a convex set and prove that an optimal solution of a convex programming problem lies on an extreme point.

[5+5+(2+8)=20]

2.(a) Can two distinct basis of an LPP correspond to same bfs ? Give an example.

(b) Give example of an infeasible LPP and also of an unbounded LPP.

[6+4+4=14]

3. Solve the following LPP using two-phase method.

maximize $x_1 + 2x_2 + x_3$

subject to

$$2x_1 - x_2 + 2x_3 = 8$$
$$x_1 + 2x_2 - x_3 = 6$$
$$x_1, x_2, x_3 \geq 3$$

[15]

4.(a) Find the rank of the following matrix

$$\begin{matrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{matrix}$$

P.T.O.

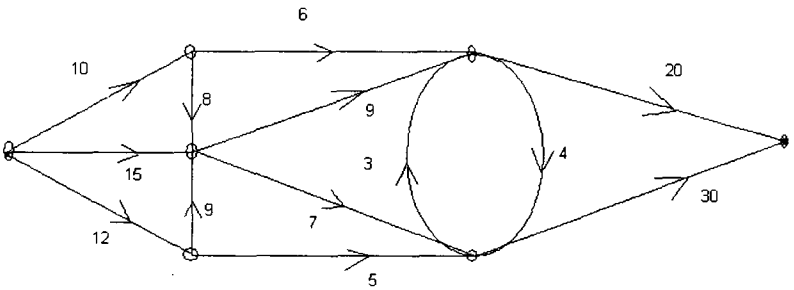
(b) Write an algorithm to find a bfs in the transportation problem.

[6+10=16]

5.(a) Define the max-flow problem.

(b) Write down the Ford and Fulkersan Labelling Algorithm.

(c) Find a maximal flow in the following network. Where the numbers associated with the edges are the capacities in the direction shown.



[4+6+10=20]

6. Solve the following assignment problem.

marks / jobs	A	B	C	D	E
1	20	30	10	50	10
2	10	15	10	15	12
3	70	60	40	30	60
4	20	30	10	10	20
5	40	40	40	50	50

[15]

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INDIAN STATISTICAL INSTITUTE
M. Tech. (CS) I: 2010–2011
Programming Languages and Methodology
Semestral Examination

Date: 16. 12. 2010 Marks: 100 Time: 3 Hours

Answer any 5 questions.

1. (a) Write a C program to implement a Tic-Tac-Toe game between the computer and the user.

   (b) How can you upgrade your program so that the computer will never be defeated?

   $10 + 10 = 20$

2. Describe, with suitable examples, the differences between parameter passing strategy (to the functions) in C, C++ and Java. 20

3. (a) Can a member function of a class be called independently without an object or pointer? Explain.

   (b) Explain briefly the role of the "this" pointer in determining the object on which a member function gets called?

   (c) We want to create a static C library that Ms. Sen will use in order to keep track of the memory allocations that are being done in her program. We have already defined the functions named "UserMalloc" and "UserFree" in the library. These functions will not only allocate memory and free it if legitimate - it will also do so by book-keeping of the allocated memory and prompt for appropriate errors. What code should we write in our header file "UserMemoryLeakReporter.h" so that Ms. Sen can include the header file in her program (also link our library with her program during compilation) and then keep using the function names "malloc" and "free" as she would normally do, but the actual call will happen for the functions provided in our library? By including the header file, the book-keeping of her memory allocations will automatically be done and her irregularities in handling memory will also be pointed out appropriately. Just writing the content of the header file will be sufficient. [Note: The preprocessor replaces the statement "#include <header.h>" with the content of the file "header.h"]

   $(5 + 5) + 10 = 20$

P. T. O.

4. (a) Explain briefly one significant use of Copy Constructors using an example.

   (b) What will be the output of the following code?

```
#include <iostream>
using namespace std;
class Base
{ /* private members defined */
  public:
    Base() { cout<<"Constructor of class Base\n";}
    ~Base() { cout<<"Destructor of class Base\n";}
};
class Derived: public Base
{ /* private members defined */
  public:
    Derived(){
/* some dynamic memory allocation */
                    cout<<"Constructor of class Derived\n";}
    ~Derived(){
/*freeing of the dynamically allocated memory*/
                     cout<<"Destructor of class Derived\n";}
};
int main()
{
    Base *pBase;
    pBase = new Base();
    delete pBase;
    pBase = new Derived();
    delete pBase;
}
```

   (c) What modification(s) would you suggest to make sure the above code frees all memory allocated in the constructor of the class "Derived"? What would be the output of the program after you have made the changes.

$$7 + 6 + (3 + 4) = 20$$

5. In the following pieces of code, what are the errors that we will get while compilation? You may not stick to the language used by the compiler for error prompting.

   (a)

```
class Base
{ public: int n;
};
class Derived
{ int m;
};
int main ()
{ Base b1; Derived d1;
  b1.n = 1; d1.n=2; d1.m = 3;
```

```
}
```

Correct the above code without deleting any existing code, so that it compiles successfully. Explain very briefly the impact that your change will have on the size of two objects.

(b)
```
class Base
{ int n;
  virtual int f()=0;
}
class Derived: public Base
{ int m;
  int f()
  { m = n = 0;
  }
}
int main ()
{ Base b1;
  Derived d1;
}
```

Correct the above code to make it compile successfully. You may add or delete words here.

(c)
```
int main ()
 { const int n;
   int * pi;
   const int * pci;
   int * const cpi;

   pi = &n; pci = &n;
   cpi = (int * const) &n;
 }
```

Correct the above code to make it compile successfully. You may add or delete words here.

$8 + 7 + 5 = 20$

6. (a) Explain briefly (use diagrams and examples if necessary) the working of virtual functions. Give examples.

   (b) Write a template class for the generic data structure "stack". Define the constructor, destructor, copy constructor, operator= and the member functions "push" and "pop".

$8 + 12 = 20$

7. (a) Write a C++ program that will use the ofstream class to write the first ten Fibonacci numbers to a file named "fibo.txt".

   (b) Write the overloaded "operator<<" function (definition and body only) so that it can work as a friend function to the following class "ThreeD-Point" such that the point (2.3, 4.5, 6.7) is output as: "(2.3, 4.5, 6.7)".

P. T. O.

Also indicate the change that needs to be incorporated within the class definition.

```
class ThreeDPoint
{ private: double x,y,z;
  public:
  ThreeDPoint(double a, double b, double c):
              x(a), y(b), z(c){}
};
```

(c) What will be the output of the following program?

```
#include <iostream>
using namespace std;
class A
{
      int n;
   public:
      A (int m=0):n(m) {}
      friend ostream& operator<< (ostream &, A &);
};
ostream& operator<< (ostream &out, A &a)
{
   out << a.n << " Hello !!!" << endl;
}
int main ()
{
   A a1, a2(1), a3(2), a4(3);
   Q1: cout << a1 << a2 << a3 << a4;
}
```

Explain very briefly the working of the line labelled with "Q1" causing the output.

(d) What will be the output of the following program?

```
#include <iostream>
using namespace std;
int main ()
{
   cout << "The cow has ";
   try
   {
      throw 2;
      cout << "The horn is for its defence." << endl;
   }
   catch (int n)
   {
      cout << n << " horns. " << endl;
   }
   cout << "The goat has two horns too." << endl;
}
```

Explain very briefly the flow of the program.

$5 + (3 + 2) + (2 + 3) + (2 + 3) = 20$

8. Write short notes (any two).

   (a) Variable argument functions in C.

   (b) Function pointers in C.

   (c) Functional Programming Language and Lambda Calculus.

   (d) Java Virtual Machine.

$10 + 10 = 20$

Midsemester Examination: Semester II(2010-11)
M.Tech(CS) I Year
Theory of Automata, Languages and Computation

Date: 21/02/11 Maximum Marks:80 Duration:3hours

Note: Answer as many Questions as you can. Maximum score is 100
Unless otherwise stated, all notations used are as defined in the class.

1. (a) When is a finite automaton said to be non-deterministic? Consruct a DFA accepting all strings $a_1a_2..a_n$ over the alphabet $\{a, b\}$ such that $a_{n-1} = b$ [8]

2. Given two DFA's $\mathcal{M}_1$ and $\mathcal{M}_2$ accepting $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, construct a **single** DFA $\mathcal{M}$ that accepts $\mathcal{L}_1 \Delta \mathcal{L}_2 \stackrel{\text{def}}{=} (\mathcal{L}_1 - \mathcal{L}_2) \bigcup (\mathcal{L}_2 - \mathcal{L}_1)$. [10]

3. Define regular expressions over a given alphabet Σ.

   Let $\mathcal{M} =< \Sigma, Q, q_1, \delta, F >$ be a DFA accepting a regular language L. Suppose $Q = \{q_1, q_2, ..., q_n\}$. Define for $i, j > 0, k \geq 0$,

   $$R^k_{i,j} = \{x \in \Sigma^* : \delta^*(q_i, x) = q_j \text{ and } \mathcal{M}$$

   $$\text{passes through no state } q_l \text{ with } l > k \text{ as it reads } x\}.$$

   Express L in terms of the sets $R^k_{i,j}$. Assuming that each $R^k_{i,j}$ is regular, suppose the regular expression $r^k_{i,j}$ represents $R^k_{i,j}$ for each i, j, k. Find a regular expression for L. [5+5]

4. (a) Find an algorithm to test a given DFA $\mathcal{M}$ whether the language it accepts is non-empty. Hence, or otherwise, obtain an algorithm to test, given DFA's $\mathcal{M}_1$ and $\mathcal{M}_2$, whether $\mathrm{L}(\mathcal{M}_1) = \mathrm{L}(\mathcal{M}_2)$ or not.

   (b) Show that the following language is not regular.

   $$\mathcal{L} = \{a^n b^m : 0 < n \leq m\}.$$

Ram Bam

(c) Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two languages. The quotient $\mathcal{L}_1/\mathcal{L}_2$ is defined as follows:

$$\mathcal{L}_1/\mathcal{L}_2 = \{x : \text{ for some } y \in \mathcal{L}_2, x.y \in \mathcal{L}_1\}.$$

Show that if $\mathcal{L}_1$ is regular, then so is $\mathcal{L}_1/\mathcal{L}_2$. [8+7+7]

5. Consider the grammar G given by

$$S \rightarrow aS|aSbS|\lambda.$$

Show that if $x \in \mathcal{L}(G)$, then each prefix of x has as many a's as b's. [8]

6. Show that if $\mathcal{L}_1$ and $\mathcal{L}_2$ are context-free languages, then $\mathcal{L}_1.\mathcal{L}_2$ is also context-free. [7]

7. When is a context-free grammar said to be in Chomsky Normal Form(CNF)? Find CNF grammar equivalent to the following

$$S \rightarrow aB|bA$$

$$A \rightarrow a|aS|bAA$$

$$B \rightarrow b|bS|aBB.$$

[2+8]

8.
- State the Bar-Hillel's Pumpinf lemma for context-free languages.
- Use the above lemma to show that $\mathcal{L} = \{0^{i^2} : i > 0\}$ is not context-free. [5+7]

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INDIAN STATISTICAL INSTITUTE

Periodical Examination

M.Tech. (Computer Science)

First Year, Semester II, 2010—2011

Computer Architecture

Date : 22.02.2011

Time : 2 hours Maximum Marks : 60

Note: *Answer **three** questions. Marks on different parts of a question are shown in the right margin within parentheses. Precise answers will fetch more credit.*

1) a) Suppose a computer spends 90% of its time handling a particular type of computation when running a given program. Its manufacturer makes a change that improves the performance on that type of computation by a factor of 10.

   i) If the program originally was taking 100 seconds to execute, what will its execution time be after the change? (3)

   ii) What is the speedup from the old system to the new system? (3)

   iii) What fraction of its execution time does the new system spend for the type of computation that was improved? (3)

   b) Describe the Von Neumann's architecture and its limitation. (6)

   c) Distinguish between RISC and CISC. (5)

2) a) Describe SISD, SIMD, MISD and MIMD. (4X3=12)

   b) Given an expression:

   C=A+B

   Write the sequence of instructions to implement it by

   i) Stack architecture

   (ii) Accumulator architecture. (2X2=4)

P.T.O.

c) Distinguish the following instructions according to their addressing mode.

i) MOV A, B

ii) MVI A, 05

iii) LXI H, 2500H

MOV A, M

iv) STA 2400H (4X1=4)

3) a) Describe different type of addressing modes. (10)

b) If a program has 400,000,000,000 instructions and takes 2 hours to run on a 50 MHz processor, what are the CPI and MIPS ratings for this processor on this program? (2x3=6)

c) Describe the different 'replacement algorithms' in the context of cache memory. (4)

4) a) Describe set association mapping. Explain how its advantageous over direct and fully associative mapping. (5+5)

b) A cache has 64KB capacity, 128 byte lines, and is 4-way set – associative. The system containing the cache uses 32 bit addresses.

i) How many lines and sets does the cache have? (2)

ii) How many entries are required in the tag array? (2)

iii) How many bits of tags are required in cache entry in the tag array? (2)

iv) If the cache is write-through, how many bits are required for each entry in the tag array. How much total storage is required for the tag array if an LRU replacement policy is used? (2+2)

Indian Statistical Institute

Mid-Semester Examination: 2010-2011

(first year second semester)

Course Name: M. Tech in Computer Science

Subject Name: Computer Networks

Date: 26-02-2011 Maximum Marks: 60 Duration: 2 hours

Instructions:

You **may** attempt **all** questions which cary a total of **72** marks. However, the maximum marks you can score is only **60**.

1. (a) Sketch the differential Manchester encoding for the bit stream: 0001110101. Assume the line is initially in the low state. [4]

   (b) What SNR is required to achieve a bandwidth efficiency of 1.2 for PSK for a bit error rate of 10^{-7}? Assume that the $\frac{E_b}{N_o}$ corresponding to the required bit error rate is 11.2 dB. [Assume $log_{10}(1.2) \approx 0.08$] [6]

   (c) Express $\frac{E_b}{N_o}$ in terms of spectral efficiency when data rate equal to the maximum channel capacity. [6]

   (d) Consider a channel with a 1 MHz capacity and an SNR of 24 dB. What is the upper limit to the date rate that the channel can carry? How many signal levels are needed to achieve this data rates? [Assume $log_2(252) \approx 8$] [8]

2. (a) Demonstrate how Hamming codes can be used to correct a single burst error of length k or less in the data stream. [4]

   (b) Seven-bit messages are transmitted using a Hamming code. How many check bits are needed to ensure that the receiver can detect and correct single bit errors? Show the bit pattern transmitted for the message 1101101. Assume that even parity is used in the Hamming code. [6]

   (c) A bit stream 10011101 is transmitted using the standard CRC method. The generator polynomial is x^3+1. Show the actual bit string transmitted. Suppose the third bit from the left is inverted during transmission. Show that this error is detected in the receiver's end. [4 + 4]

   (d) A group of stations share a 16 Kbps slotted Aloha channel. Each station outputs a 100-byte frame and buffers any outgoing frames until they can be sent. Assume that all the stations together attempt to send at the uniform rate of 20 frames (original plus retransmission of frames) per second. What is the throughput in frames/sec? [6]

3. (a) Consider the channel efficiency of basic bit-map protocol versus binary count-down protocol at low load. Which one is less? Explain your answer. [4]

   (b) What is piggybacking? What is its advantage? [4 + 2]

   (c) Frames of 1000 bits are sent over a 1 Mbps satellite channel whose propagation time from the earth to the satellite is 270 msec. Acknowledgements are always piggybacked on to data frames. The headers are very short. What is the maximum achievable channel utilization for 1) Stop-and-wait protocol and 2) Go-back-N protocol with window size of 7? [4 + 4]

   (d) What is the minimum frame size required for a CSMA/CD based computer network running at 1 Gbps on a 1 Km cable (with no repeaters) with a link speed of 200,000 Km/sec? [6]

Indian Statistical Institute

Semester-II 2010-2011

M.Tech.(CS) - First Year

Mid-term Examination (28 February, 2010)

Subject: Operating Systems

Total: 35 marks Maximum marks: 30 Duration: 2.5 hrs.

Please keep your answers brief and to the point.

1. (a) Using system calls and interrupts as examples, clearly explain the difference between synchronous and asynchronous events.

   (b) Describe the mechanism by which a user's program gets back a return value from a kernel function at the end of a system call.

   (c) What is the difference between the type of information stored by the OS in the *u area* and the *proc* structure of a process?

   (d) Using your answer above, explain in 1-2 lines each whether the following items of information should be stored in the *proc* structure or the *u* area. (i) process state (ii) open file descriptors (iii) disk quota and resource limits.

   (e) Consider the following sequence of steps used by the kernel to effect a context-switch from process P to P'.

1. Run the scheduling algorithm. Let P' be the selected process.
2. If $P == P'$, goto 5.
3. Save context of P.
4. Restore context of P'.
5. Return to user mode.

   (i) Now consider a context switch from process P'' to P. Clearly explain what problem arises if, in step 4. the context of P is restored faithfully using exactly the values saved in step 3 during the switch from P to P'.

   (ii) How should the above sequence of steps be changed to circumvent this problem? (Your answer should clearly explain what happens in your modified scheme at the individual machine instruction level.)

[2+3+1+3+2+4=15]

2. (a) Recall that the Shortest Job First (SJF) scheduling algorithm cannot be used in the Operating Systems typically used in desktop computers. Describe an approximate version of this algorithm that *can* be used in such OSs.

   (b) Briefly describe an operating environment in which the *original* SJF scheduling algorithm can be used.

[2+2=4]

P.T.O.

3. (a) How is the static priority of a process determined under the Linux 2.6 kernel? What is its default value? For conventional processes, what possible values can the static priority take?

(b) In a system running the Linux 2.6 kernel, what happens to a process when it exhausts its allocated time quantum? Your answer should cover all possible scenarios.

(c) Draw a diagram of the main data structure(s) that is/are used by the Linux 2.6 scheduler to select the next process to run.

(d) Using the diagram, explain briefly why this scheduler is called a $\Theta(1)$ scheduler.

[2+2+3+1=8]

4. (a) Let $wait_b()$ and $signal_b()$ be the usual semaphore operations for *binary* semaphores. Show how you would implement a *counting* semaphore along with the usual *wait*() and *signal*() operations using the $wait_b()$ and $signal_b()$ functions.

(b) How many binary semaphores does your implementation need? Briefly justify (in 1-2 lines) the need for each binary semaphore.

[6+2=8]

INDIAN STATISTICAL INSTITUTE

Back-Paper Examination : 2010-11(First Semester)

M.Tech (CS) I year

Elements of Algebraic Structures

Date :2.2.2011 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. The maximun you can score is 100. Notation is as used in the class.

1. Show that every cyclic group is isomorphic to either $\mathbb{Z}$ or to $\mathbb{Z}_n$, for some integer n. [10]

2. Define a vector space.
   Give an example of each of the following:
   - A finite vector space.
   - A finite-dimensional vector space.
   - An infinite-dimensional vector space.

   [4+4+4+4=16]

3. Show that for subspaces U, W of a finite-dimensional vector space V,

$$dim(U \oplus W) = dim(U) + dim(W).$$

   [10]

4. Let $f : V \longrightarrow W$ be a linear map from a vector space V onto a vector space W.
   Define the kernel, $ker(f)$, of f. Show that it is a subspace of V. Prove that $V/(ker(f)$ is isomorphic to W. [15]

5. Define the column rank and the row rank of a matrix A. Show that they are equal. [10]

6. Show that the system of linear equations

$$Ax = b,$$

   has a unique solution iff A is of full column rank. [10]

P.T.O.

7. Obtain a basis of the system

$$\begin{aligned} 2x - 3y + 5z &= 0 \\ x - y + z &= 0 \end{aligned}$$

Hence find its general solution. [10]

8. Check whether $1 + 2x + x^3$ is irreducible over $\mathbb{Z}_3$, If not, then find its factorization. [10]

9. Show that every finite field has p^n elements for some prime p and for some positive integer n. [10]

INDIAN STATISTICAL INSTITUTE

Periodical Examination

M. Tech (CS) - I Year (Semester - II)

Design and Analysis of Algorithms

Date : 24.2.2011 Maximum Marks : 60 Duration : 3 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. Let L be a read-only array containing a sequence of n integers (you are not allowed to change the content of each cell of the array). Write an efficient algorithm for finding the median of the members in L. You may allocate $O(1)$ working storage in your algorithm. Analyze the worst case and expected time complexity of your algorithm.

   If the changing in the cells of L is allowed how will you modify the algorithm so that it can find the median of the elements of L using $O(1)$ extra storage in expected linear time.

   [4+4+7+5=20]

2. In the online version of convex hull problem, the number of points in the set P is not known in advance, and they come one by one. At every instance, we may need to report the convex hull of the present point set P. Show that, at every stage $O(n)$ time is enough to compute and report the convex hull. Is it possible to have an algorithm where each stage can be performed in $O(\log n)$ time? State the necessary data structure in each case.

   [8+7=15]

3. Consider the UNION-FIND problem, where each set is represented by a tree, and the set name is written at the root of that tree.

   The *union by rank* heuristic for the union operation of two sets works as follows: Let U and V be two sets represented by two trees rooted at u and v respectively. If $|U| < |V|$, then u is attached with v; otherwise, v is connected to u. The name of the new set $U \cup V$ is writen at the root (v or u) of the resulting tree, and its size is set to $|U| + |V|$.

   Show that if the *union by rank* heuristic is followed, then the height of the tree of a set containing k nodes can be at most $\lfloor \log k \rfloor$.

   State the method of path compression when a sequence of union and find operations are executed. What is the advantage of this method with respect to the execution time of the algorithm (you do not need to give any formal proof).

   You also state the data structures to be used for storing the sets in the implementation of the above two methods for the UNION-FIND problem.

   [5+5+5=15]

4 (a) Given a edge-weighted directed acyclic graph (DAG) and a pair of distinguished vertices s and t, give an algorithm to find the longest path from s to t in the graph. Will your algorithm work when the graph is (i) undirected, and (ii) directed but not acyclic?

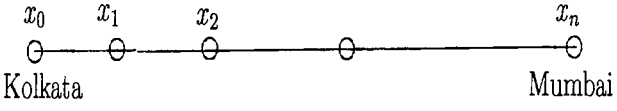
(b) An edge-weighted sparse graph $G = (V, E)$ with weight function $w : E \rightarrow R$ is given. It is known that the graph does not contain any negative weight cycle. But there may exist some edge(s) having negative weight(s). A student X wants to compute shortest paths for each pair of vertices in that graph. He reweights all the edges of the graph in the following way:

For each edge $(u, v) \in E$ compute $w^*(u, v) = w(u, v) + h(u) - h(v)$, where $h : V \rightarrow R$ is another function that ensures that new weight of the graph will make all the edge-weights non-negative. Now he executes Dijkstra for each vertex of the graph with new weight w^* to get all pair shortest paths. Is he correct? What is the time complexity of his algorithm?

[Hint: Prove or disprove that a path p connecting v_i and v_j is a shortest path with original weight function w if and only if p is a shortest path with new weight function w^*].

[7+8=15]

5. Suppose a person wants to go from Kolkata to Mumbai on road driving a car. He starts with a full tank and can go for 100 kilometers on a full tank. Let $x_1 < x_2 < ... < x_n$ denote the locations of the various petrol pumps along the way, measured in kilometers from Kolkata (as shown in the figure below). Present an algorithm that determines the fewest number of petrol pumps he needs to stop at to reach Mumbai without running out of fuel along the way. Give a short proof of the correctness of your algorithm.



[15]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination
M. Tech(CS) I year: 2010–2011
Software Engineering

Date: 01. 03. 2011 Marks : 50 Time : 2 Hours

Answer any part of any questions. The question if of 55 marks. The maximum marks you can get is 50. Please try to write all the part answers of a question at the same place.

1. List the differences between "writing a program for your own" and "writing a program that will be used by somebody else". Consider developing a "Stack" data structure in C programming language for detailed explanation. [20]

2. In our institute, students belonging to different streams can participate in two types of courses: a fixed number of credit courses and one or two audit courses. The marks obtained in the credit courses contribute to the overall grade of the student. The student must also score a minimum marks in each of the audit courses. The total marks in a particular subject consists of the marks obtained in the Assignments, Mid-Term examination and Semester examination. The weight-age of these three parts varies from one subject to another. After the completion of the examinations, the concerned teacher submits the marks before a given deadline. The students can view their marks after each of the examinations. The final score is calculated by the system, given the weight-age of the assignments, mid-term and semester examinations. The system then calculates the final marks and decides if the student has to take any supplementary examination. The system should be so designed that the students are allowed to view their marks by entering his/her name and a secret password.

   (a) Draw a DFD corresponding to the above problem.

   (b) Prepare an SRS for the same.

   [15 + 10 = 25]

3. With short pieces of codes, explain how can you handle the "CONTROL C" input from the keyboard in C and JAVA. [10]

Examination: Semester II(2010-11)
M.Tech I Year
Automata, Languages and Computations.

Date: 20.04.11 Maximum Marks:100 Duration:3hours

Note: Answer as many as you can. Maximum score is 100
Unless otherwise stated, notation used is as defined in the class.

1. (a) Explain in detail when a pushdown automata (PDA) accepts a string by empty stack.

   (b) Show that if $\mathcal{L}$ is a context-free language then there is a PDA that accepts $\mathcal{L}$.

   (c) When is a PDA said to be deterministic? Show that if $\mathcal{L}$ is a language accepted by a deterministic PDA by empty stack, then $\mathcal{L}$ has the prefix property *i.e.* no proper prefix of a string in $\mathcal{L}$ is in $\mathcal{L}$.

   (d) Show that if $\mathcal{L}$ is a context-free language and $\mathcal{R}$ is regular, then $\mathcal{L} \bigcap \mathcal{R}$ is context-free by constructing a suitable PDA.(Show only the major steps.)
   Hence, show that

$$\mathcal{L} = \{ww : w \in (0+1)^*\}$$

   is not context-free

[5+7+7+(6+7)]

2. State the Bar-Hillel's Pumping Lemma.
   Show that the language
   $\{w : w \in \{a, b, c\}^*$, w contains equal numbers of a's, b's and c's $\}$
   is not context-free.

   Construct a Turing Machine which accepts this language.

[(4+7+5)]

3. (a) Design (i) single-tape (ii) multitape Turing machines(TMs) that accept the set of all strings over $\{a, b\}$ containing an equal numbers of a's and b's. Compare the number of moves made by the respective TMs on accepting a string of length n.

Also, construct a TM that uses only $\lceil \log n \rceil$ cells (not counting cells on the input tape)

(b) A language $\mathcal{L}$ over Σ is said to be recursive if both $\mathcal{L}$ and $\Sigma^* - \mathcal{L}$ are recursively enumerable. Show that $\mathcal{L}$ is recursive iff there is a TM accepting $\mathcal{L}$ that halts on all inputs. [(5+5+2+5)+8]

4. (a) When is a function $f : \mathbb{N}^k \rightarrow \mathbb{N}$ said to be (Turing) computable? Show that the following functions are computable.

i. $f(n,m) = [n/m] = \begin{cases} 0 & \text{if } m = 0 \\ \text{the largest integer} \leq n/m & \text{if } m > 0 \end{cases}$

ii. $p(n) =$ the nth prime number.

iii. $lcm(n,m) = \begin{cases} 0 & \text{if } n = 0 \text{ or } m = 0 \\ \text{the least common multiple of } n, m & \text{otherwise} \end{cases}$

iv. $gcd(n,m)=$ the greatest common divisor of n and m.

v. $f(n) = \lceil \log n \rceil$

(b) Consider the following function on $\mathbb{N} \times \mathbb{N}$.

$$B(x,y) = \begin{cases} 0 & \text{if } x = 0 \text{ or } y = 0 \\ x \dot{-} (\mu z \leq x(\exists w \leq x(w.(x \dot{-} z) = x) & \\ \quad \text{and } \exists u \leq y(u.(x \dot{-} z) = y))) & \text{otherwise} \end{cases}$$

Here "'$\mu z \leq xP(x)$"' means "'the smallest $z \leq x$ such that $P(x)$ holds"'. Describe, in simple English, the function B.[(2+5×4) = 22+3]

5. (a) Define the classes $\mathcal{P}$ and $\mathcal{NP}$. When is a language said to be $\mathcal{NP}$-complete?

(b) Show that SAT is in $\mathcal{NP}$.

(c) Describe the COMPLETE SUBGRAPH problem. Assuming that SAT is $\mathcal{NP}$-complete, prove that COMPLETE SUBGRAPH is also $\mathcal{NP}$-complete.

(d) [7+5+8]

Indian Statistical Institute

Semester-II 2010-2011

M.Tech.(CS) - First Year

21·04·

Semestral Examination (April, 2011)

Subject: Operating Systems

Total: 110 marks Maximum marks: 100 Duration: 4 hrs.

Please keep your answers brief and to the point.

1. (a) List the most important constituents of a process' context that must be maintained on a per-thread basis.

   (b) Explain how asynchronous I/O can be useful for an implementation of a user-level threads library.

   (c) Using an example, explain why thread local storage is necessary.

   [4+6+6=16]

2. (a) Briefly describe a deadlock prevention approach which ensures that the hold-and-wait condition is never fulfilled in a system. Clearly explain the disadvantages of such an approach.

   (b) Define a *safe sequence* for a Resource Allocation State (RAS).

   (c) Consider the following Resource Allocation State involving 5 processes and 5 resources.

$$Total = [\ 5\ 4\ 5\ 3\ 6\ ]$$

$$Max = \begin{bmatrix} 5 & 1 & 1 & 1 & 5 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 2 & 3 & 4 \\ 4 & 0 & 4 & 0 & 4 \\ 2 & 0 & 1 & 2 & 3 \end{bmatrix} \quad Alloc = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad Req = \begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 1 & 0 & 0 & 0 & 3 \\ 2 & 0 & 1 & 1 & 1 \end{bmatrix}$$

   $Total[i]$ specifies the total number of instances of resource i that exist in the system (including both allocated and free instances). $Max[i,j]$, $Alloc[i,j]$, and $Req[i,j]$ denote, respectively:

   - the maximum number of instances that process i may request of resource j,
   - the number of instances of resource j currently allocated to process i, and
   - the number of instances of resource j that process i is currently requesting.

   Show that the system is neither in a safe state nor in a deadlocked state.

   [(4+4)+2+(10+10)=30]

3. (a) In the context of address-binding schemes, what is a (i) logical address, (ii) physical address? Explain in 1-2 lines each whether the two addresses are identical or different for compile-time, load-time, and run-time address binding schemes.

   (b) Consider an operating system that uses a paging-based memory management scheme. Logical addresses are 24 bits long. Each page table entry occupies 4 bytes. Calculate the minimum and maximum page sizes for which exactly 2 levels of paging will be required.

   [NOTE: In a 2-level paging scheme, the second-level page table must fit in a single page.]

(c) Explain how page-fault frequency monitoring can be used to prevent thrashing. What (if any) hardware support is needed in this method?

(d) Consider a process running under Linux on the i386 architecture. How many entries do the process' global and local segment descriptor tables contain? Describe these entries.

[10+14+6+10=40]

4. (a) The directory hierarchy in an SVR2-like filesystem can be regarded as a graph, with nodes corresponding to files/directories, and each edge pointing from a directory to another directory (or file) contained within it. Is this graph always acyclic? Is it necessarily a tree? Clearly explain your answer (use an example if necessary).

(b) Suppose you have just created a directory called `os` in your home directory. You now create two files `hw1.c` and `hw2.c` in the directory. Next, you compile `hw1.c` to get `a.out`. After testing the programme, you delete `a.out`. Assuming that the directory is stored on an SVR2 filesystem, draw a diagram showing the data block of the `os` directory.

(c) Consider the following fragment from algorithm *iget*:

```
1. while (not done) {
2.    if (inode in cache) {
3.       if (inode locked) {
4.          sleep till inode becomes unlocked;
5.          continue;
6.       }
```

Carefully construct an example scenario to justify the use of the `continue` statement in line 5 (i.e., you should explain why a process must re-check condition 2 on waking up after line 4).

(d) Suppose that the block size in an SVR2 filesystem is 512 bytes. Calculate the maximum number of times that a process can go to sleep when reading 800 bytes from an open file. Justify your answer using appropriate extracts from the pseudo-code for the `read` system call.

[6+6+6+6=24]

Indian Statistical Institute

Semester Examination: 2010-2011

(first year second semester)

Course Name: M. Tech in Computer Science

Subject Name: Computer Networks

Date: 23-04-2011 Maximum Marks: 50 Duration: 3 hours

Instructions:

You **may** attempt **all** questions which carry a total of **60** marks. However, the maximum marks you can score is only **50**.

1. (a) A large number of consecutive IP addresses are available starting at 198.16.0.0. Suppose that four organizations, A, B, C and D, request 4000, 2000, 4000, and 8000 addresses, respectively, and in that order. For each of these, write the first IP address assigned. the last IP address assigned, and the mask in the $w.x.y.z/s$ notation. [4]

   (b) Write the purpose of the following two fields of $IPv6$ header:

      i. Flow label

      ii. Next header [2]

   (c) A router has the following $CIDR$ (Classless InterDomain Routing) entries in its routing table:

Address/mask	*Next hop*
194.24.0.0/21	*Interface* 0
194.24.8.0/22	*Interface* 1
194.24.16.0/20	*Interface* 2

   What does the router do if a packet with address 194.24.17.4 arrives? [2]

   (d) Assume that a router has received the following new IP addresses: 57.6.96.0/21, 57.6.104.0/21, 57.6.112.0/21, and 57.6.120.0/21. If all of them use the same outgoing line, can they be aggregated? If so, to what? If not, why not? [2]

2. (a) Write the purpose of the following two fields of TCP header:

      i. Window size

      ii. Urgent pointer [2]

   (b) How is congestion control effected by TCP? [4]

   (c) Suppose that the TCP congestion window is set to 18 Kilobyte and a timeout occurs. What should the size of the congestion window be if the next six transmission bursts are to be successful? Assume that the maximum segment size is 1 Kilobyte. [2]

(d) Two networks each of which provides reliable connection-oriented service. One of them offers a reliable byte stream and the other offers a reliable message stream. Are these identical? If so, why is the distinction made? If not, give an example of how do they differ. [2]

3. (a) What is Reverse Path Forwarding? [2]

(b) Consider Fig. 1 where (a) shows a subnet and (b) shows a sink tree for router I of that subnet. How many packets are generated by a broadcast from I, using reverse path forwarding (assume that the preferred path follows the sink tree)? Give a tree built by reverse path forwarding for a broadcast from I. [2+2]

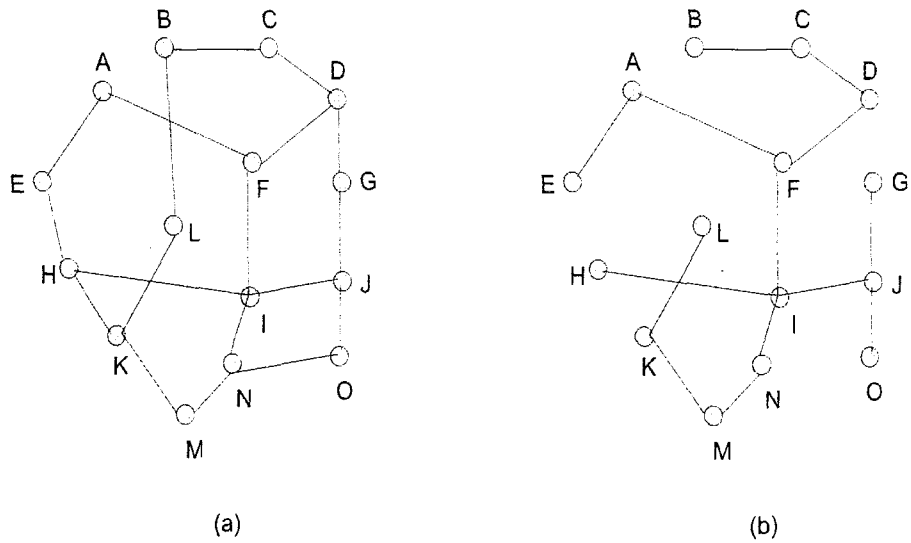


Figure 1: (a) A subnet (b) A sink tree

(c) Consider the subnet of Fig. 2 below. Distance vector routing is used, and the following vectors have just come into router C: from B: (5, 0, 8, 12, 6, 2); from D: (16, 12, 6, 0, 9, 10); and from E: (7, 6, 3, 9, 0, 4). The measured delays to B, D, and E, are 6, 3, and 5, respectively. What is C's new routing table? Give both the outgoing line to be used and the expected delay. [4]

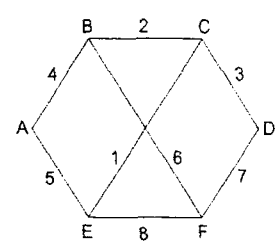


Figure 2: A subnet

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech (CS) - I Year (Semester - II)

Design and Analysis of Algorithms

Date : 25.4.2011 Maximum Marks : 100 Duration : 3.5 Hours

Note : You may answer any part of any question, but maximum you can score is 100.

1.(a) You are given k sorted arrays, where the i-th array contains n_i integer keys. State an efficient method for merging all these arrays in a single array. State the time and working space complexities of your algorithm.

(b) You are to organize a tournament involving n competitors. Each competitor must play exactly once against each of the $n-1$ possible opponents. Each competitor is to play at most one match per day.

Show that if n is a power of 2, then it is possible to design an optimal tournament that takes exactly $n-1$ days. Do this by giving an algorithm which takes n as input, and outputs the list of player pairings for each of the $n-1$ days. (Hint: Use divide-and-conquer approach.) [6+9=15]

2.(a) Design an efficient algorithm to assign distinct integers 1 through n to the n vertices of an acyclic digraph so that $number(u) < number(v)$ if $u \neq v$ and there is a directed path from u to v. Your algorithm should run in time $O(n+m)$, where n and m are respectively the number of vertices and edges of the digraph.

(b) Let $P1$ and $P2$ be two problems such that $P1$ is polynomial time reducible to $P2$, the time complexity of this reduction is $O(n^2)$, and the time complexity of solving $P2$ is $O(n^4)$. Here n is the input size of the problem P1. What can you say about the time complexity of the problem $P1$? Justify your answer. [8+7=15]

3.(a) Give the *fail* indices generated by the KMP-flowchart construction algorithm for the following patterns:

(a) AAAB,

(b) AABAACAABABA,

(c) ABRACADABRA,

(d) ASTRACASTRA.

(b) Write the KMP-scan algorithm.

(c) Show that KMP-scan algorithm does at most 2n character comparisons, where n is the size of the text string.

$[3 \times 4 + 5 + 5 = 22]$

4.(a) Show that if the Ford-Fulkersons' algorithm for computing maximum flow between a pair of vertices s and t in a flow network terminates, it produces the maximum flow value from s to t.

(b) Design an efficient algorithm for the maximum flow problem in an acyclic flow network with unequal edge capacities. Analyze the time and space complexities of your algorithm.

(c) Decide whether the following statement is *true* or *false*. If it is true, give proper justification, and if it is false, give a counterexample.

> Let $G = (V, E)$ be an arbitrary flow network, with a source s and a sink t. Each edge $e \in E$ is attached with a positive integer capacity c_e. Let (A, B) be the minimum $s - t$ cut in graph G with respect to the edge capacities $\{c_e, e \in E\}$. Now, if we add 1 to every edge capacity, then (A, B) still remains the minimum $s - t$ cut in the revised flow network.

[7+10+8 = 25]

5.(a) Let M and M' be two different matchings (may not be edge disjoint) in a bipartite graph G. Suppose that $|M'| > 2|M|$. Show that there is an edge $e' \in M'$ such that $M \cup \{e'\}$ is a matching in G.

(b) In the context of designing an algorithm for the non-bipartite matching problem, the following result is important: *If at some stage of augmenting the matching, there is no augmenting path from a node u, then there will never be an augmenting path from u in the subsequent stages.* Prove this statement, and state why it is important.

[10+10=20]

6. The 3-SAT problem and the subset-sum problem are defined as follows:

   3-SAT problem: *Given a CNF expression with n boolean variables such that each of its clauses contains at most 3 variables, does there exist any assignment of values to the boolean variables such that the value of the expression is 1?*

   Subset-sum problem: *Given a set of integers $S = \{s_1, s_2, \ldots, s_n\}$, and another integer K, does there exist a subset of S whose sum is exactly equal to K?*

   Prove that 3-SAT problem is polynomial time reducible to the subset-sum problem.

   [10]

7. Let $G = (V, E)$ be a weighted directed graph. Show that the problem of getting a traveling salesman tour in G of cost $\rho \times opt$ is NP-complete, where *opt* indicates the cost of the optimum traveling salesman tour in G, and ρ is a given constant.

   Also show that if the edge costs of G satisfy triangle inequality, then a traveling salesman tour of cost $2 \times opt$ can be found in polynomial time.

   [10+10=20]

INDIAN STATISTICAL INSTITUTE

Second-Semester Examination: 2010-11

Course Name: M. TECH. (QROR) I Yr.

Subject Name: Instrumentation and Computer Engineering

Date: 26.04.11 Maximum Marks: 100 Duration: 3 Hrs

Question no 1 is compulsory and answer any 6 questions from the rest.

1. Mention True or False [4]
   a) A ROM cannot be randomly accessed by CPU.
   b) Decreasing response time always improves the throughput of a computer.
   c) A photo multiplier tube can be used as an active transducer.
   d) Proportional control action is a discontinuous mode of operation.

2. a) Derive the expression of the output voltage for a 2 bit R-2R ladder digital to analog (D/A) convertor.

   b) Design a 3 bit synchronous counter such that, the binary equivalent of the series 5, 1, 4, 3, 0, 6, 2, 7 can be realized at the output. [5+11=16]

3. What is the difference between an active transducer and a passive transducer? How many types of active transducer are available? Discuss about the operation procedures of resistance strain gauges, photomultiplier tube and rotary plate capacitor. [3+2+3+4+4=16]

4. a) Explain the operation procedure of a proportional plus integral (PI) control action with related equations. How the output of the controller varies with a step change in error signal?

   b) What are the advantages and limitations of a PI controller?

   c) A PI(proportional+integral) controller is used for control of certain process. The settings are as follows. K_p (gain)=2%, P_o=40%, Reset rate=2%/min, E_p (error signal)=4t+6 where, t=time. What will be controller output in percentage after 2minutes? [(3+5)+3+5=16]

5. a) Draw the block diagram of an automatic control system and define manipulated variable, load variable, process lag, control lag and dead time.

b) What are the advantages of an automatic control system?

c) Derive the output of an PID (proportional plus integral plus derivative) control action for a step type error signal. Explain with a figure that how the output changes with error signal and time. [(2+5)+(2)+(4+3)=16]

6. Explain the operation of a RS flip flop and the contribution of it's individual gates for every possible input bits. [4+4+4+4=16]

7. a) An additional processor is added in a computer that uses multiple processors for separate tasks. Will it change the throughput and response time of the computer?

b) For a particular program find the relation between CPU execution time and clock rate.

c) A program runs in 10 seconds on a computer which has a 4GHz clock. A substantial increase in clock rate is possible but that will increase the clock cycles by 1.2 times for the same program to run. What clock rate will be required to run that program in 6 seconds.

d) A compiler designer is trying to decide between two code sequence 1 and 2 and both the code sequences use three different instruction classes A, B, and C. The required CPI (clock cycles per instruction) for A, B, and C are 1, 2 and 3 respectively. The number of instruction counts for A, B, and C are 2, 1 and 2 respectively, in code sequence 1. The number of instruction counts for A, B, and C are 4, 1 and 1 respectively, in code sequence 2. Which code sequence executes the most instructions? Which will be faster? What is the CPI for each sequence? [2+3+4+(2+3+2)=16]

8. What is the decimal value of this 32-bit two's complement (signed integer) number?
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1100_{two}$

How many bits are allocated for the exponent and fraction part of a double precision floating point number?

Show the IEEE 754 binary representation of the number -0.75 in 32 bit precision.

Add the numbers 0.5_{ten} and -0.4375_{ten} in binary using scientific notation and floating point addition with 4 bits of precision.

[3+2+5+6=16]

9. a) Explain the operating procedure of a CPU with 7 registers, MUX, ALU and decoder as major components.

b) The binary OPR word for adding contents in A BUS and B BUS in ALU is 00010. Find the 14 bit control word for performing the operation

Register1 Content ← Register2 Content+ Register3 Content

c) Explain the algorithm for performing a push (insertion) operation in a register stack.

d) Find the reverse polish notation for the expression
(A+B)*[C*(D+E)+F] [7+4+3+2=16]

10. Show and explain the connection procedure of 4 RAMS and 1 ROM (each having capacity of 128 words of eight bits per word) with CPU.

What is an Associative memory? Explain the operation of an associative memory with emphasis on match logic.
[(4+4)+(1+3+4)=16]

11. a) How can a shift register store a binary number 110?

b) Show the working procedure of a 4 bit asynchronous DOWN counter with circuit and timing diagram. [7+9=16]

INDIAN STATISTICAL INSTITUTE
Semestral Examination
M. Tech(CS) I year: 2010–2011
Software Engineering

Date: 28. 04. 2011 Marks: 100 Time : 3 Hours

Answer any part of any question. The maximum marks you can obtain is 100. The question paper is of 110 marks. Please try to write all the part answers of a question at the same place.

1. 10 lifts serve 100 floors (0 to 99). There are two constraints.

   (a) More than 6 lifts cannot move in the same direction. A lift can change its direction only when there is no more floor to serve moving in that direction.

   (b) At any point of time a lift can accommodate a maximum of 20 people.

   One needs to develop a software to control these lifts. Explain each of the phases of the software development process for this tiny software. Make any reasonable assumption that you may need and state the same with justification. [25]

2. Consider a Railway reservation System where the Passengers can reserve their tickets using Internet facility. The system should provide the particulars of the trains available on entering the source and destination stations and the date of journey. The number of available seats for each class should be displayed. The user then enters the number of passengers with their details and berth preferences after which the system calculates the total fare and displays the ticket. The passenger can buy the ticket online using credit cards. For this a card number is asked for. The system checks the validity of the credit card through a third party banking system. The printed ticket is considered to be a valid document. The system also prepares a list of passengers for a particular train with relevant details.

   Draw a DFD and prepare an SRS for this system. [10 + 15 = 25]

3. (a) Why cost estimation models are important for planning a software project?

   (b) Explain the COCOMO model in detail.

   [8+12 = 20]

4. (a) Explain the differences between Function-Oriented Design and Object-Oriented Design.

   (b) Briefly describe how a high level design can be executed using UML.

   [10 + 10 = 20]

5. (a) Describe the differences between White box and Black box testing.

   (b) Briefly explain (i) Unit Testing (ii) Integration Testing.

   (c) Explain the roles of (i) verification and (ii) validation in software testing.

   [8+6+6 = 20]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2010-11

M.Tech (Computer Science)

Computer Architecture

Date: 29·04·11 **Marks: 100** **Duration: 3hours**

NOTE: Answer any **five** questions. Each question carries 20 marks. Marks on different parts of a question are shown in the right margin within parentheses. Precise answers will fetch more credit.

(1)

a) Define pipelining. Explain the difference between sequential processing and pipelining. (2+2)

b) What do you mean by (i) structural hazards, (ii) data hazards, and (iii) control hazards in the context of instruction pipelining? Explain your answer with appropriate illustrative examples. (3X4=12)

c) Let us assume that there are five stages in a pipeline. Time required for the five stages are as follows: 10ns, 8ns, 10ns, 10ns and 7ns. Assume that pipeline adds 1ns of overhead. Find the speedup due to pipelining. (4)

(2)

a) What is instruction level parallelism (ILP)? Discuss different types of dependencies which can affect the performance of ILP. (2+8=10)

b) Consider a loop as follows:

```
for (i=1; i<=100; i=i+1) {
        A[i] = A[i] + B[i];     /*s1*/
        B [i+1] = C[i] + D[i]; /*s2*/
}
```

Find out the dependencies present between statements s1 and s2. Can s1 and s2 be run in parallel? If not, modify the loop to do so.

(4+6=10)

(P.T.O)

(3)

a) What is super pipelining? (2)

b) Explain different instruction issue policies in super scalar processors with examples. (10)

c) What is vector processing? (2)

d) Define Initiation rate, Convey and Chime. (6)

(4)

a) Discuss write policies that distinguish cache design. (5)

b) What are the techniques used for reducing cache miss rate? (5)

c) Classify instruction set architecture. (6)

d) Suppose that we are considering an enhancement that runs 10 times faster than the original machine but is usable only 40% of the time. What is the overall speedup gained by incorporating the enhancement? (4)

(5)

a) Distinguish between static and dynamic interconnection networks. (5)

b) Discuss (i) sequential consistency, (ii) processor consistency, (iii) weak consistency, (iv) release consistency, and (v) entry consistency (5x2=10)

c) A program repeatedly executes a loop that has 120 iterations. Each iteration takes 10,000 cycles. On multiprocessor systems 50,000 cycles are required to synchronize the processors once all iterations of the loop are completed.

(i) Find out the execution time of each loop on a uniprocessor system.

(ii) Find out the execution time of each loop on a 2-processor system, and the speedup over the uniprocessor system.

(P.T.O)

(iii) Find out the execution time of each loop on a 4-processor system, and the speedup over the uniprocessor system. (1+2+2=5)

)

a) Explain in detail superscalar processors, data flow processors and VLIW processor with examples. Discuss the drawbacks and advantages for each of them. (9+3=12)

b) Suppose it takes 2.5ns to access the tag array of a set-associative cache, 4ns to access data array, 1ns to perform the hit/miss comparison, and 1ns to return the selected data to the processor in case of hit.

(i) What is the time required to determine if a hit has occurred? Does the data-array access the critical path in case of a cache hit?

(ii) What is the cache hit latency of the system?

(iii) What would the cache hit latency of the system be if both the tag and data array access time were 3ns? (3+2+3=8)

Indian Statistical Institute

Semester-II 2010-2011

M.Tech.(CS) - First Year

Backpaper Examination (26 July, 2011)

Subject: Operating Systems

Total: 100 marks Maximum marks: 45 Duration: 3 hrs.

Please keep your answers brief and to the point.

1. (a) Write a C program that does the following:
   - creates a child process;
   - the parent process prints the child's process ID and exits;
   - the child executes `/bin/ls` and exits.

   (b) List the situations in which the kernel invokes the scheduler.

   (c) Using the above, explain the difference between preemptive and non-preemptive scheduling.

   [8+4+4=16]

2. Consider the following procedure to remove a node `p` from a doubly-linked list:

```
void remove(NODE *p)
{ NODE *prev, *next;
  prev = p->prev; next = p->next;
  if (next == NULL) tail = prev;
  else next->prev = prev;
  if (prev == NULL) head = next;
  else prev->next = next;
}
```

   Suppose two processes P_1 and P_2 share a doubly-linked list. Using an example, show how the list may get corrupted if the two processes execute `remove` concurrently.

   (You should show the initial state of the list, the calls to `remove` by the processes, the intended final state of the list, the interleaving of operations by the two processes, and the actual final state of the list.) [10]

3. (a) State and briefly explain the four necessary conditions for deadlock.

   (b) Consider the following variant of the dining philosophers problem. One of the philosophers is highly respected by everyone else. When he announces that he is hungry, his neighbours give him their forks. Can deadlock happen in the modified scenario? If yes, give an example of a deadlocked situation. If not, explain which of the necessary conditions is violated.

   [8+4=12]

4. (a) Define internal and external fragmentation.

   (b) For which of the following memory management schemes is internal / external fragmentation possible?

   (i) paging; (ii) segmentation.

   In each case, justify your answer in 2-3 lines.

   (c) What is Belady's anomaly? Name a page replacement algorithm that (i) exhibits Belady's anomaly; (ii) never exhibits Belady's anomaly.

   (d) Describe the format of logical and linear addresses generated by i386 processors. (Include the bit-wise breakup of the address field in your answer.)

[4+8+4+8=24]

5. (a) List and briefly describe the fields contained in a disk inode.

   (b) Briefly describe the data structures used to store the inode cache / inode table.

   (c) In the algorithm *iget*, if a process needs an inode that is locked, it sleeps till the inode becomes free. However, if it needs a free inode buffer, but the list of free inode buffers is empty, the kernel returns an error. Explain why the kernel returns an error in this case instead of sleeping until a buffer is available.

   (d) Sketch a figure showing the relationship between the user file descriptor table, the global file table, and the inode table. When can the reference count become greater than one for entries in the (i) inode cache and (ii) global file table?

   (e) When a file is "created" using the `creat` system call, explain what permissions are checked (i) if the file already exists (ii) if the file does not exist (i.e., it is *actually* created).

[14+6+6+8+4=38]

Examination: Semester II(2010-11)
M.Tech I Year
Automata, Languages and Computations.

Date:...04.11 Maximum Marks:100 Duration:3hours

Note: Answer as many as you can. Maximum score is 100
Unless otherwise stated, notation used is as defined in the class.

1. (a) Explain in detail when a pushdown automata (PDA) accepts a string by empty stack.

   (b) Show that if $\mathcal{L}$ is a context-free language then there is a PDA that that accepts

   (c) When is a PDA said to be deterministic? Show that if $\mathcal{L}$ is a language accepted by a deterministic PDA by empty stack, then $\mathcal{L}$ has the prefix property *i.e.* no proper prefix of a string in $\mathcal{L}$ is in $\mathcal{L}$.

   (d) Show that if $\mathcal{L}$ is a context-free language and $\mathcal{R}$ is regular, then $\mathcal{L} \bigcap \mathcal{R}$ is context-free by constructing a suitable PDA.(Show only the major steps.)
   Hence, show that

   $$\mathcal{L} = \{ww : w \in (0+1)^*\}$$

   is not context-free

   [5+7+7+(6+7)]

2. State the Bar-Hillel's Pumping Lemma.
   Show that
   $\{w : w \in \{a, b, c\}^*$ w contains equal numbers of a's, b's and c's $\}$
   is not context-free.

   Construct a Turing Machine which accepts this language.

   [(4+7+5)]

3. (a) Design (i)single-tape (ii) multitape Turing machines(TMs) that accept the set of all strings over $\{a, b\}$ containing an equal numbers of a's and b's. Compare the number of moves made by the respective TMs on accepting a string of length n.

Also, construct a TM that uses only $\lceil \log n \rceil$ cells (not counting cells on the input tape)

(b) A language $\mathcal{L}$ over Σ is said to be recursive if both $\mathcal{L}$ and $\Sigma^* - \mathcal{L}$ are recursively enumerable. Show that $\mathcal{L}$ is recursive iff there is a TM accepting $\mathcal{L}$ that halts on all inputs. [17+8]

4. (a) When is a function $f : \mathbb{N}^k \rightarrow \mathbb{N}$ said to be (Turing) computable? Show that the following functions are computable.

i. $f(n,m) = [n/m] = \begin{cases} 0 & \text{if } m = 0 \\ \text{the largest integer} \leq n/m & \text{if } m > 0 \end{cases}$

ii. $p(n) =$ the nth prime number.

iii. $lcm(n,m) = \begin{cases} 0 & \text{if } n = 0 \text{ or m} = 0 \\ \text{the least common multiple of } n, m & \text{otherwise} \end{cases}$

iv. $gcd(n,m)=$ the greatest common divisor of n and m.

v. $f(n) = \lceil \log n \rceil$

(b) Consider the following function.

$$B(x,y) = \begin{cases} 0 & \text{if } x = 0 \text{ or } y = 0 \\ x \dot{-} (\mu z \leq x(\exists w \leq x(w.(x \dot{-} z) = x) & \\ \quad \text{and } \exists u \leq y(u.(x \dot{-} z) = y))) & \text{otherwise} \end{cases}$$

Here "'$\mu z \leq x P(x)$"' means "'the smallest $z \leq x$ such that $P(x)$ holds"'. Describe in simple English the function B. [22+3]

5. (a) Define the classes $\mathcal{P}$ and $\mathcal{NP}$. When is a language said to be $\mathcal{NP}$-complete?

(b) Show that SAT is in $\mathcal{NP}$.

(c) Describe the COMPLETE SUBGRAPH problem. Assuming that SAT is $\mathcal{NP}$- complete, prove that COMPLETE SUBGRAPH is also $\mathcal{NP}$-complete.

(d) [7+5+8]