LEFT-AND RIGHT-HANDEDNESS OF PLANT ORGANS IS IT DETERMINED BY A BERNOULLI PROCESS?

(Discussion of the Paper by Davis and Ramanujacharyulu)

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The statistical analysis of bilateral symmetry in plant organs by Davis and Ramanujacharyulu (1971)* (henceforth to be referred to as DR) is based on the assumption that the number of 'L's and 'R's in a set of n observations follows a binomial distribution. The object of this note is to point out that this assumption is very surely wrong, for there is unmistakable evidence to this effect in the various data reported in DR itself. An alternative model is proposed which could explain some of the peculiarities observed in the DR data, though the validity of this model could not be independently ascertained for lack of appropriate obervations.

The author's suspicion regarding the suitability of a binomial model was first aroused by the preponderence of very small values of chisquares recorded in the analysis of Tables 1 and 7 of DR. Infact no elaborate statistical testing is needed to establish the fact that these values could not have arisen in a random sample from the chisqure distribution with 1 d.f. Even though one would be inclined to accept the hypothesis of bilateral symmetry in all but two of the *Malvaceous species* (Table 1) and in the plants of *Euphorbia Nerü folia* (Table 7), it seems plausible that if one keeps a recorded of the individual plant organs (flowers in Table 1 and shoots in Table 7) in a chronological or some other meaningful order, the series may not conform to what one would expect in a Bernoulli sequence of trials. Several of such series relating to *Cordyline rubra* (Table 10) and *Scindapsus officinalis* (Table 11) show very definite signs of nonrandomness.

All these facts have lead the author to believe that as opposed to the assumption of a Bernoulli sequence, a simple two states (L, R) Markov Chain of the type illustrated in example (a) of Feller (*Introduction to Probability Theory and its Applications*, Vol. 1, (second edition) 1957, chapter XV, section 2) might provide a better description of the phenomenon studied in DR. The model could be described as follows, where X_i represents the i-th member of a sequence of random variables ($i = 1, 2, \ldots$)

(a) Each X_i is either L or R

(b) Prob $(X_1 = L) = $ Prob $(X_1 = R) = \frac{1}{2}$

(c) Prob $(X_{i+1} = L \mid X_i) = L =$ Prob $(X_{i+1} = R \mid X_i = R)$

Notice that here each X^i is distributed in the same way as X_1 implying thereby that in a sequence X_i such as this, the L's and R's would in the long run be recorded about

*See pp. 269–292.

equally often as observed in Tables 1 and 7 of DR. However, if S_n be the total number of L's recorded in the first n members of the sequence we have

$$\text{var}(S_n) = \frac{n}{4}+\frac{1}{2}\,[(n-1)(2p-i)+(n-2)(2p-1)^2+\ldots+(2p-1)^n]$$

which is $\quad < \frac{n}{4}$ if $p < \frac{1}{2}$.

Further if this Markov Chain model is true and this indeed is a verifiable proposition, $\left(S_n-\frac{n}{2}\right)^2\Big/$ var (S_n) is asymptotically distributed as chisquare on 1 d.f. If $p < \frac{1}{2}$, $(2S_n-n)^2/n$ is stochastically smaller than a chisquare on 1 d.f. and it provides a possible explanation of the small values of chisquares recorded in Tables 1 and 7 to which a reference is made in the second paragraph of this note unfortunately the data in Tables 1 and 7 were not maintained in the required form and for Tables 10 and 11 where they are so maintained the author is sceptical about the suitability of the proposed Markov Chain model since it does not take into account the positive association between the convolution of a lamina on the pliar spiral of the shoot on which the leaf is borne, very prominently brought about in Table 10.

For the proposed model, if the value of p is unknown, a very good consistent estimate can be obtained from the data using the $(n-1)$ consecutive pairs (X_i, X_{i+1}), $i = 1, 2, \ldots (n-1)$. Let C_{n-1} be the number of such pairs where X_i and X_{i+1} are of like types, i.e. : the pair is either (L, L) or (R, R). Then $p = C_{n-1}/(n-1)$ is such an estimate for p. Let the corresponding estimate for var (S_n) be denoted by var(S_n). When p is unknown, for large values of n, $\left(S_n-\frac{n}{2}\right)^2\Big/$var (S_n) can be used as chisquare on 1 d.f. for testing hypothesis of bilateral symmetry.

We shall conclude the note with the following remarks :

1. The values of p for all the shoots in Table 10 are well above 0.5 and 0.8 is a good representative figure.

2. One need not abandon the hypothesis of bilateral symmetry if the data of Table 10 is analysed under the Markov Chain model while the same data would be considered as very sure evidence of asymmetry if the binomial model were true.

3. Even if the proposed model turns out to be unsatisfactory, it is undisputable that the DR paper has thrown out many more problems than what it has solved and it is still a long way to understanding the laws operating in the expression of bilateral symmetry or asymmetry of plant organs which was the professed aim of Davis and Ramanujacharyulu.

Reference

Davis, T. A. and Ramanujacharyulu, C. (1971): Statistical analysis of bilateral symmetry in plant organs, *Sankhyā*, Series B, 33, 269–292.

Paper received : May, 1970.