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Bertrand–Edgeworth equilibrium large markets with non-manipulable residual demand

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Abstract

In a model of price competition among capacity-constrained firms with 'non-manipulable' residual demand, we demonstrate that if the number of firms is large, then the unique equilibrium yields the competitive price. The result goes through even when firms produce to order.

Keywords: Bertrand equilibrium; Non-manipulable residual demand

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1. Introduction

Consider a model of price competition where firms simultaneously decide on both price and quantity and are free to supply less than the quantity demanded. Edgeworth (1897) demonstrates that pure strategy equilibria may not exist in such models.[1] In a model with parallel residual demand, however, Vives (1986) demonstrates that the Edgeworth non-existence problem is resolved if there are many small firms. Moreover, the unique equilibrium price turns out to be the competitive one.

In this paper we consider two extensions of this result. We examine a class of 'non-manipulable' residual demand functions that is a generalization of the parallel residual demand function. We say that a residual demand function is non-manipulable if, by increasing its output level, a firm cannot increase the residual demand coming to it. Moreover, these functions satisfy some properties that are generalizations of those satisfied by the parallel residual demand. We demonstrate that if the number of firms is large enough, then a unique Nash equilibrium exists. Moreover, the Nash equilibrium price

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[1] See Dixon (1987), among others, for a formal statement of the problem.

is the competitive one. We demonstrate that the result goes through even when the firms produce to order.

Thus the results in Vives (1986) goes through under our extensions.

Other papers dealing with the limit properties of Bertrand competition include, among others, Allen and Hellwig (1986), Dixon (1987) and Borgers (1992). Allen and Hellwig (1986) examine a model with a proportional residual demand function. They demonstrate that as the market becomes large the equilibrium price converges in distribution to the competitive one. However, the monopoly price always belongs to the support of equilibrium strategies. Dixon (1987) introduces the notion of menu costs and demonstrates that in the presence of such costs an epsilon-Nash equilibrium exists if the economy is replicated. Moreover, if ϵ is small enough, and the industry large enough, then any ϵ-equilibrium will be approximately competitive. The replication procedure in Dixon (1987), however, is different in the sense that he replicates market demand as well. Another interesting paper is by Borgers (1992) where he shows that for the parallel rationing rule iterated elimination of dominated strategies yields prices close to the competitive price.[2]

2. The model

There are n identical firms, all producing the same homogeneous good. The market demand function $q = d(p)$ satisfies the following assumption.

Assumption 1. $d(p)$ is negatively sloped.

The cost function of all the firms, $c(q)$, is linear and capacity-constrained so that

$$c(q) = \begin{cases} cq & \text{if } q \leq k_n = \dfrac{K}{n} \\ \infty & \text{otherwise} \end{cases}$$

where $d(c) > K$.

The ith firm's strategy consists of simultaneously choosing *both* a price p_i and an output q_i. Firms are free to supply less than the quantity demanded. All firms move simultaneously. For n large we solve for the pure strategy Nash equilibrium of this game. (Papers that examine mixed strategy equilibria for Bertrand competition include Allen and Hellwig (1986) and Maskin (1986)). Note that with an increase in n firms become more numerous, as well as smaller (since k_n decreases).

We then impose some conditions on the residual (or the contingent) demand function.

Let $r_i(p_i, p, n)$ denote the residual demand facing the ith firm if firm i charges a price of $p_i \geq p$, and the other $(n-1)$ firms charge a price of p and produce $d(p)/n$. For simplicity we write $\partial_{r_i}(p_i, p, n)/\partial_{p_i} = r_i'(p_i, p, n)$.

[2] We refer the readers to Vives (1999) for a more detailed discussion of these papers.

Assumption 2.

(i) $r_i(p_i, p, n)$ is twice differentiable, decreasing and (weakly) concave in p_i. Moreover, $\lim_{n\to\infty} r_i'(p_i, p, n)|_{p_i=p} < 0$.
(ii) Consider a situation where m of the firms charge $\tilde{p}$ and all other firms charge prices that are strictly greater than $\tilde{p}$.
  (a) Then the residual demand facing all the firms charging $\tilde{p}$ is $d(\tilde{p})/m$.
  (b) Suppose that there is excess demand at the price $\tilde{p}$ and all the firms charging $\tilde{p}$ supply till capacity. If any one of the firms charging $\tilde{p}$ increases its price by a sufficiently small amount, then the residual demand facing it would be greater than its capacity.
  (c) If the demand $d(\tilde{p})$ is being met by the firms charging $\tilde{p}$, then all firms charging higher prices have no demand.

Assumption 2(ii)(a) formalizes the notion that the residual demand function is *non-manipulable*. Clearly, the firms charging $\tilde{p}$ cannot increase the residual demand coming to them by increasing their output level beyond $d(\tilde{p})/m$.[3]

Notice that for a downward sloping, concave market demand function, Assumption 2 is satisfied by the parallel (or the efficient) residual demand function. Suppose firm 1 charges p_1 and all other firms charge p, where $p_1 \geq p$. Moreover, let the other firms produce $d(p)/n$ each. Then the residual demand facing firm 1 is $\max\{d(p_1) - (n-1)d(p)/n, 0\}$. Clearly, if $d(p)$ is concave, then the residual demand function is decreasing and concave in p_1. Moreover, notice that $r_1(p_1, p, n)|_{p_1=p} = d(p)/n$ and $\lim_{n\to\infty} r_1'(p_1, p, n)|_{p_1=p} = d'(p) < 0$. Thus the non-manipulable residual demand can be thought of as a generalization of the parallel residual demand function

We then introduce a few notations.

Definition. Let p^* be $d^{-1}(K)$.[4]

Note that p^* is the competitive price.

Definition. Let $\hat{n}$ be the smallest possible integer such that $\forall N \geq \hat{n}$,

$$r_i'(p^*, p^*, N)[p^* - c] + \frac{d(p^*)}{N} < 0.$$ [5]

Proposition 1 below shows that for $n \geq \hat{n}$, p^* can be sustained as the unique Nash equilibrium of this game. Recall that p^* is the competitive price. Thus if the market comprises many small firms, the Bertrand price is the competitive one.

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[3] As an example of a residual demand function that is *manipulable*, one can mention the proportional residual demand function.

[4] Since $d(c) > K$, $p^* > c$.

[5] Notice that $\lim_{n\to\infty}[r_i'(p^*, p^*, n)(p^* - c) + d(p^*)/n] = \lim_{n\to\infty} r_i'(p^*, p^*, n)[p^* - c]$. Since $p^* > c$, and from Assumption 2(i), $\lim_{n\to\infty} r_i'(p^*, p^*, n) < 0$, this term is negative.

Proposition 1. *Let $n \geq \hat{n}$. Then the unique Nash equilibrium involves all the firms charging a price of p^* and producing $d(p^*)/n = k_n$.*

Proof. *Existence.* In equilibrium all the firms produce till capacity, hence undercutting p^* is not profitable.

We then argue that for all the ith firms, charging a higher price, p_i, is not profitable either. Clearly the deviant firm always supplies the whole of the residual demand coming to it. Hence the profit of a firm which charges a price $p_i(\geq p^*)$ is $\pi(p_i, r_i(p_i, p^*, n)) = [p_i - c]r_i(p_i, p^*, n)$. Clearly

$$\frac{\partial \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i} = r_i'(p_i, p^*, n)[p_i - c] + r_i(p_i, p^*, n). \tag{1}$$

Next, since $p_i > c$, from the concavity of $r_i(p_i, p^*, n)$ it follows that $\pi(p_i, r_i(p_i, p^*, n))$ is concave in p_i.[6] Moreover,

$$\left.\frac{\partial \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i}\right|_{p_i = p^*} = r_i'(p^*, p^*, n)(p^* - c) + \frac{d(p^*)}{n}. \tag{2}$$

Since $n \geq \hat{n}$, we have that $\partial \pi(p_i, r_i(p_i, p^*, n))/\partial p_i|_{p_i = p^*} < 0$. Next, from the concavity of $\pi(p_i, r_i(p_i, p^*, n))$ it follows that $\forall p_i \geq p^*$, the profit of any deviant firm is decreasing in p_i.

Finally, since $p^* > c$, it is optimal for all the firms to produce exactly $d(p^*)/n = k_n$.

Uniqueness. The proof is in several steps.

Step 1. We first claim that there cannot be an equilibrium where different firms charge different prices. Suppose to the contrary such an equilibrium exists and let $l < n$ firms charge the price p' and all other firms charge a higher price.

If $lK/n < d(p')$ then there is excess demand at the price p' and any one of the l firms can increase its price slightly and gain (Assumption 2(ii)(b)). If, however, $lK/n < d(p')$ then at the price p' all firms charging p' supply less than capacity. Thus any one of the l firms can increase its profits by undercutting slightly. So assume that $lK/n < d(p')$. Then none of the other $n - l$ firms charging a higher price has a positive demand (Assumption 2(ii)(c)). Since $p' > p^* > c$, any one of these firms can now match p' and have a strictly positive profit (Assumption 2(ii)(a)).

Step 2. Thus the only possible equilibria involve all the firms charging the same price p''. If $p'' > p^*$ then there is excess supply at the price p''. Hence all the firms can undercut slightly and increase their profit level. Whereas if $p'' < p^*$ then there is excess demand and all the firms can increase their price slightly and gain. □

The idea behind the existence result is quite simple. Consider a market price of p^*. If the number of firms is large then the residual demand coming to every firm is very small, so that it is residual demand rather than marginal cost which determines firm supply. In that case price would not equal marginal cost, and firms may no longer have an incentive to increase their price level. Assumption 2 specifies a set of conditions under which this is indeed true when all the firms charge p^* and produce $d(p^*)/n$.

[6] This follows since

$$\frac{\partial^2 \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i^2} = r_i''(p_i, p^*, n)[p_i - c'(r_i(p_i, p^*, n))] + 2r_i'(p_i, p^*, n).$$

We then examine if Proposition 1 extends to the case where the firms produce to order. Suppose the firms play a two stage game where, in Stage 1, the firms simultaneously announce their prices, and in Stage 2, they simultaneously decide on their output levels. We then solve for the subgame perfect Nash equilibrium of this game.

Proposition 2. *Consider the game where firms produce to order and assume that* $n \geq \hat{n}$. *Then the following strategies constitute the unique subgame perfect Nash equilibrium:*

Stage 1. *All firms charge a price of* p^*.

Stage 2.

Case (i) *Suppose that in Stage 1 all the firms charge* p^*. *Then, in Stage 2, all the firms produce* $d(p^*)/n = k_n$.

Case (ii) *Next suppose that in Stage 1,* $(n-1)$ *of the firms charge* p^*, *while one of the firms charges a price strictly greater than* p^*. *Then, in Stage 2, the firms charging* p^* *produce* $d(p^*)/n$, *while the output level of the other firms is* $r_i(p_i, p^*, n)$.

Proof. *Existence*. Its clear that the strategies in Stage 2 are optimal. Next consider the pricing strategies. If any one of the firms deviate by charging a price p_i greater than p^* then its residual demand is $r_i(p_i, p^*, n)$. We can now mimic the argument in Proposition 1 to claim that the profit of the deviant firm decreases.

Uniqueness. The argument mimics that of Proposition 1.

3. Conclusion

In this paper we consider two extensions of Vives (1986). Proposition 1 above extends Vives (1986) to the case of non-manipulable residual demand functions, while Proposition 2 shows that the argument goes through even when firms produce to order. We can thus argue that the essential feature of the residual demand function that drives the result in Vives (1986) is the fact that it is non-manipulable and satisfies Assumption 2.

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