Sensitivity Analysis of Optical Waveguides

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Abstract—**The change in the characteristics of dielectric slab and fiber waveguides due to dimensional tolerances of their parameters has been analyzed. The analysis is helpful in fabrication and use of such guides in optical communication systems.**

DIELECTRIC and semiconductor optical waveguides are fabricated using a number of techniques [1], [2] such as sputtering, vapor deposition, solution deposition, oxidation, ion bombardment, and ion exchange. The dimensional tolerances of their parameters offered by different techniques of fabrication are different and a sensitivity analysis of their characteristics is useful in the design and use of an integrated optical circuit.

The sensitivity of a parameter x with respect to a parameter y is given by [3]

$$S_{x,y} = (y/x)(\partial x/\partial y). \tag{1}$$

If the characteristics C are a function of several independent parameters y_i, $i = 1, 2 \cdots n$, then the change in the characteristics ΔC is related to the tolerance Δy_i of y_i by the relation

$$\Delta C = C \sum_{i=1}^{n} (\Delta y_i/y_i) S_{c, y_i}. \tag{2}$$

The mode propagation constant β—a function of size, refractive indexes of different regions of the guide, and wavelength—is one of the most useful characteristics of the optical waveguides.

It is convenient to combine the three parameters into one as

$$V = (n_1^2 - n_2^2)^{1/2} Kd$$

where $K = 2\pi/\lambda$ is the free space propagation constant, n_1 and n_2 are respective refractive indexes inside and outside the guide, and d is the thickness for slab and radius for the circular cylindrical guide. For asymmetric slab guides, n_2 is the refractive index of the substrate and that of the superstrate is $n_3 < n_2$. The asymmetry is defined by a parameter

$$\alpha = [K^2 d^2 n_1^4 (n_2^2 - n_3^2)/n_3^4 V^2].$$

The normalized propagation constant is defined as $b = (\gamma d/V)^2$ where

$$\gamma d = [V^2 - k^2 d^2]^{1/2}, \quad k = [n_1^2 K^2 - \beta^2]^{1/2}.$$

The sensitivity of the normalized propagation constant b with respect to V is given by (1)

$$S_{b,V} = (2kd)/(bV^2) \tag{3}$$

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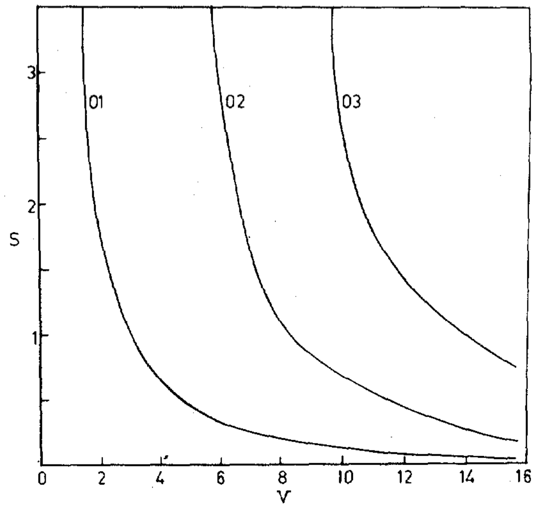

Fig. 1. Propagation constant sensitivity for slab guide.

where kd is a solution [4] of

$$\tan kd = m_1^2 k(m_3^2 \gamma + m_2^2 \delta)/(m_3^2 m_2^2 k^2 - m_1^4 \gamma\delta) \tag{4}$$

with $m_j = n_j$ for the TM mode and $m_j = 1$ for the TE mode of slab guide and

$$\delta = (\beta^2 - n_3^2 K^2)^{1/2}.$$

The corresponding equation for circular weakly guiding structures [5], i.e., for $(n_1 - n_2)/n_2 = \Delta \ll 1$, is

$$kJ_{n+1}(kd)/J_n(kd) = i\gamma H_{n+1}(i\gamma d)/H_n(i\gamma d) \tag{5}$$

where J_n and H_n denote, respectively, the Bessel and Hankel functions of order n and the first kind.

Another useful parameter, especially for circular guides used in optical transmission, is the group delay. The group delay per unit length τ_g is defined as

$$\tau_g = (V/cK)(d\beta/dV)$$

where c is the free space wave velocity. Ignoring material dispersion and using the weakly guiding approximation, we get

$$\tau_g = (n_2 \Delta/c) d(Vb)/dV$$

of which $d(bV)/dV$ is defined as normalized group delay Γ. The sensitivity of Γ with respect to V is then given by

$$S_{\Gamma,V} = [\sqrt{b}\, d(\gamma d)/d(Vb) - b/V] S_{b,V} \tag{6}$$

where $S_{b,V}$ and γd are given, respectively, by (3) and (5).

The plot of $S_{b,V}$ against V is shown in Fig. 1 for the slab guide with $\alpha = 1$ and in Fig. 2 (solid lines) for the circular cylindrical guide for the first three propagating (TE for slab

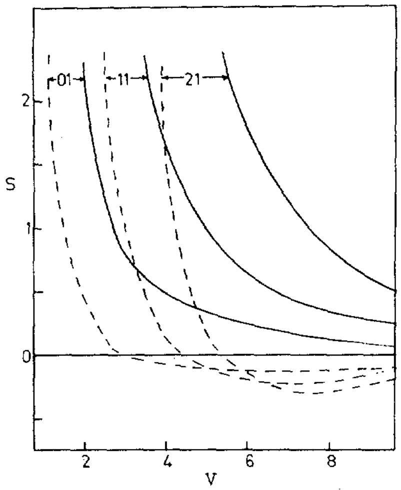

Fig. 2. Propagation constant and group delay sensitivity for circular cylindrical guides. ——— Propagation constant sensitivity; ----- group delay sensitivity.

and LP for circular guide) modes in each case. It is seen that the sensitivity is a decreasing function of V and becomes only a function of n_1 when the waveguide is overmoded. Also plotted in Fig. 2 (broken lines) is the sensitivity of Γ for the first three LP modes of circular cylindrical guides. The interesting fact to be noted is that apart from vanishing at $V = \infty$, the sensitivity is negligible around certain V's characteristic of the mode. From Fig. 2 it is seen that the sensitivity of Γ of 01, 11, and 21 LP modes are negligible around $V = 3.1$, 4.3, and 5.2, respectively. The fact may be useful in fabrication and use of single mode guides.

The knowledge of sensitivity is useful also in the design of experiments. It gives an idea about the sensitivity required for other components of the system and the possible error the experiment may suffer. One may chose the proper guide for the specific requirement to meet for his experiment.

For example, let a single mode fiber be specified by the manufacturer as $V = 3.5$ at $\lambda = 1$ μ with ± 0.01 percent tolerance. Then the fluctuation in the normalized propagation constant that the fiber will suffer is ± 0.0065 percent. The corresponding fluctuation in normalized group delay per unit length is ± 0.0005 percent. This value, though small, will add up to a large value if the fiber is quite long. It will play an important role in the distortion of long distance image transfer through single mode fibers.

The sensitivity of power with fluctuations in V is less meaningful since fluctuations in V give rise to intermode coupling and radiation of power. However, the dispersive sensitivity of power of ideal guide can be found in a similar manner as above.

The analysis may be carried out for more general anisotropic, semiconductor, or other guides following the same approach.

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