INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2013-2014) (Back paper)

MS (Q.E.) I Year

Macroeconomics I

Date 28.7.14 Maximum Marks: 100 Duration: 3 hours

1. Show that the steady state equilibrium in an OLG model can exhibit dynamic inefficiency. Analyse how a social planner can raise the steady state level of consumption and also the consumption in transition in the face of such inefficiency.

[25]

2. Show how equilibrium unemployment is sustained in the Shapiro- Stiglitz model. What do you think would happen to equilibrium unemployment if firms were to experience a technological progress (by this I simply mean that firms could produce more, say twice, the output they produced earlier, for any given combination of inputs).

[20+5]

3. Explain, in the light of Bent Hansen's model, why inflation is a quasi-equilibrium phenomenon **[25]**

4. (a) Show that, in the Mundell-Fleming model, with perfect capital mobility, the money supply is endogenous under a fixed exchange rate regime.

(b) How does a lowering of the pegged exchange rate (revaluation of the domestic currency) impact the endogenous variables in that model? Explain your answer.

(c) Suppose that, in a small country open economy macro model under imperfect capital mobility and flexible exchange rate regime, the money market is found to be in equilibrium. What additional condition needs to be satisfied so that the domestic bond market equilibrium is also ensured? Explain. **[5+15+5]**

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
SEMESTER II EXAMINATION (2013 – 14) – *BACK PAPER*
M. STAT. I and II Years & M.S. (Q.E.) I Year
Time Series Analysis / Time Series Analysis & Forecasting

Date: 30·7·14 Maximum Marks: 100 Time: 3 hours

*Answer **all** questions. Marks allotted to each question are given within parentheses.*

1. (a) Discuss what is meant by trend in a time series data. Show that the method of moving averages is an appropriate method for obtaining trend if the trend is linear.

   (b) Let $\{X_t\}$ be a process given by

   $$X_t = \mu + Z_t + \beta Z_{t-1},$$

   where μ is a constant and $\{Z_t\} \sim WN(0,1)$. Show that the ACF of this process does not depend on μ.

   (c) Let $\{Z_t\}$ be i.i.d. normal (0, 1) noise. Define $\{X_t\}$ as

   $$X_t = \begin{cases} Z_t \text{ if } t \text{ is even} \\ (Z_1^2 - 1)/\sqrt{2} \text{ if } t \text{ odd.} \end{cases}$$

   Show that $\{X_t\}$ is WN (0, 1), but not i.i.d. (0, 1) noise.

   [7+4+9 = 20]

2. (a) Show that the conditional expectation of X_{n+h}, given observations $x_n.x_{n-1}, x_{n-2}, \ldots,$ is the h-step ahead minimum MSE forecast of x_{n+h} made at origin n.

   (b) Find the 3-step ahead minimum MSE forecast at origin n of the following time series

   $$(1-0.6B)^2 \ x_t = (1+0.5B)a_t$$

   where a_t's are white noise with zero mean and unit variance.

   (c) What are out-of-sample forecasts? Discuss how out-of-sample forecasts are obtained in a given time series data.

   [6+7+7 = 20]

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BACK PAPER EXAMINATION: (2013-2014)

MSQE I and M.Stat II

Microeconomic Theory II

Date: 02.08.14 Maximum marks: 100 Duration: 3 Hours

Note: Answer all questions.

Note: $\mathbb{R}^\ell$ denotes the ℓ-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Throughout, a preference relation is assumed to be rational.

Q1. Let $\mathbb{R}_+$ denote the set of wealth and U be a twice-differentiable strictly increasing utility function on $\mathbb{R}_+$ for a decision maker. For any fixed amount of money ω and positive number ε, the probability premium is denoted by $\pi(\omega, \varepsilon, U)$ and the certainty equivalence and expected value for a lottery L are denoted by $CE(L, U)$ and $\mathbb{E}(L)$ respectively. Show that the following properties are equivalent:

(i) The decision maker is risk averse.
(ii) $\pi(w, \varepsilon, U) \geq 0$ for all $w \geq 0$ and all $\varepsilon > 0$.
(iii) $CE(L, U) \leq \mathbb{E}(L)$ for all lottery L. [18]

Q2. Suppose that $\succeq$ and ω are a continuous preference relation and an initial endowment of an agent, respectively. If the consumption set of the agent is $\mathbb{R}^\ell_+$, then show that the demand set $D(p, \omega, \succeq) \neq \emptyset$ for all prices $p \in \mathbb{R}^\ell_{++}$. [10]

Q3. Consider an economy $\mathscr{E} = \{N; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in N}\}$, where N is the set of agents containing n many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succeq_i$ and ω_i are the preference and initial endowment of agent i, respectively.

(i) Suppose that $\sum_{i \in N} p \cdot \omega_i \neq 0$ for some $p \in \mathbb{R}^\ell$. If an allocation is supported by the price p, then show that it is weakly Pareto optimal allocation. [10]

(ii) If $\succeq_i$ is continuous and monotone, then show that the set of Pareto optimal allocations is a compact subset of $\mathbb{R}^{n\ell}$. [12]

(iii) Let $N = \{1, 2\}$ and $\ell = 2$. Suppose that the preference relation $\succeq_i$ is represented by a utility function U_i for $i = 1, 2$. Given that

$$\begin{cases} \omega_1 = (2, 8), & U_1(x, y) = \min\{2x, y\}; \\ \omega_2 = (6, 0), & U_2(x, y) = \min\{x, 3y\}. \end{cases}$$

P.T.O

Find the set of Walrasian equilibrium of $\mathscr{E}$. [10]

(iv) Assume that $\succeq_i$ is continuous, strictly monotone for all $i \in N$ and $\sum_{i \in N} \omega_i \in \mathbb{R}^\ell_{++}$. If $x = (x_1, \cdots, x_n)$ is an allocation of $\mathscr{E}$ and $y \succeq_i x_i$ implies $p \cdot x \geq p \cdot \omega_i$ for all $i \in N$ and for some non-zero price p, then show that x_i is in agent i's demand set $D_i(p, \succeq_i, \omega_i)$ for all $i \in N$. [15]

(v) Suppose that $0 < \varepsilon < 1$. Show that there is closed ball B such that the excess demand set $\zeta(p) \subseteq B$ for all prices $p \in \mathbb{R}^\ell_{++}$ satisfying $\varepsilon \leq p^h \leq 1$ for any $1 \leq h \leq \ell$. [15]

(vi) Suppose that $\mathscr{E}_r$ is the r-fold replicated economy of $\mathscr{E}$ and $\succeq_i$ is convex for all $i \in N$. Let

$$x = (x_{11}, \cdots, x_{1r}, x_{21}, \cdots, x_{2r}, \cdots, x_{n1}, \cdots, x_{nr})$$

be a Walrasian allocation of $\mathscr{E}_r$. Show that the allocation $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$ of $\mathscr{E}$, defined by

$$\hat{x}_i = \frac{1}{r} \sum_{j=1}^{r} x_{ij},$$

is a Walrasian allocation of $\mathscr{E}$. [10]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination (2014–2015)

MS(QE) I

Mathematical Methods

Date : 27.08.2014 Maximum Marks : 60 Time : 2 hrs.

This paper carries 70 marks. Maximum you can score is 60. Precisely justify all your steps. Carefully state all the results you are using.

Group A

1. Let $Y = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0\}$.

   (a) Show that Y is a subspace of $\mathbb{R}^4$. [5]

   (b) Find a basis of Y. [5]

   (c) Does there exist a linear map $T : \mathbb{R}^2 \to \mathbb{R}^4$ such that $\mathcal{R}(T) = Y$? [5]

2. Let $W = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_1 = x_2, x_3 = x_4 = x_5\}$.

   (a) What is the dimension of W? [5]

   (b) If $\dim(W) = k$, find a linear map $T : W \to \mathbb{R}^k$ that is 1-1 and onto. [5]

3. Let V be a vector space and $A = \{x_1, x_2, \ldots, x_n\} \subseteq V$ be a generator for V. Suppose there is a vector $v_0 \in V$ such that v_0 can be expressed as a *unique* linear combination of elements of A. Show that A is a basis of V. [5]

4. Let X be a vector space and $T : X \to X$ be a linear map. Suppose $x_1, x_2 \in X$ are two nonzero vectors such that $Tx_1 = \lambda_1 x_1$, $Tx_2 = \lambda_2 x_2$ for some $\lambda_1, \lambda_2 \in \mathbb{R}$ and $\lambda_1 \neq \lambda_2$. Show that x_1, x_2 are linearly independent. [5]

Group B

5. Let $S \subseteq \mathbb{R}$ be nonempty and bounded above. Let $x_0 = \sup S$. Prove that there is a sequence $\{x_n\} \subseteq S$ such that $\lim_{n \to \infty} x_n = x_0$. [5]

6. (a) Use triangle inequality to show that [5]

$$\Big| \|x\| - \|y\| \Big| \leq \|x - y\| \quad \text{for all } x, y \in \mathbb{R}^k.$$

(b) Let $f : [a, b] \to \mathbb{R}$ be continuous at $x_0 \in [a, b]$. Show that $|f|$ is also continuous at x_0. [5]

7. If $c > 1$, prove that $\lim_{n \to \infty} \sqrt[n]{c} = 1$. [5]

8. Let $f : \mathbb{R} \to \mathbb{R}$ and there exists $M > 0$ such that

$$|f(x) - f(y)| \leq M|x - y| \quad \text{for all } x, y \in \mathbb{R}.$$

Show that f is a continuous function. [5]

9. Show that $[0, 1] \times [0, 1] = \{(x, y) \in \mathbb{R}^2 : 0 \leq x, y \leq 1\}$ is a compact subset of $\mathbb{R}^2$. [10]

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Mid-Semestral Examination : 2014-15

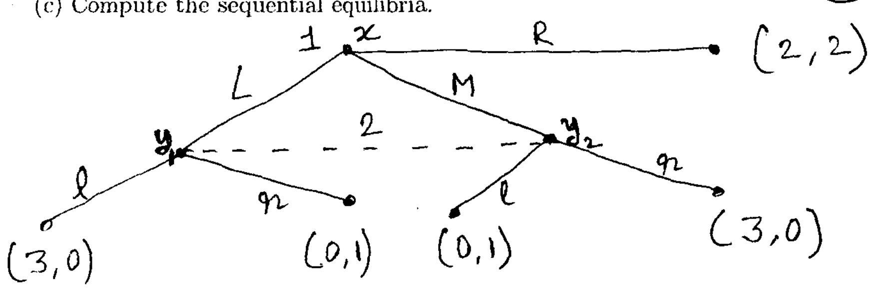
Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR

Subject Name: Game Theory I

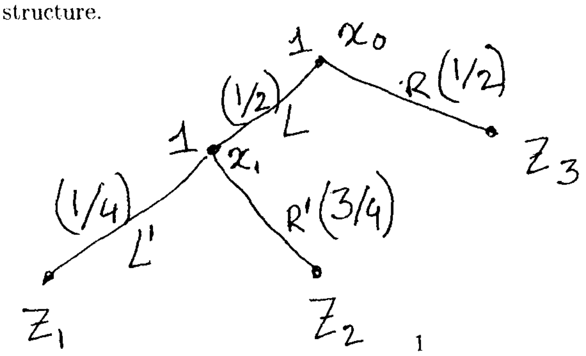
Date: 28-08-2014 Maximum Marks: 50 Duration: 2 hours

Problem 1. Let (b, β) be a consistent assessment in an extensive form game Γ. Show that (b, β) is Bayesian consistent. (5)

Problem 2. Consider the following extensive form game below.
(a) Determine the strategic form of this game.
(b) Compute the Nash equilibria and subgame perfect Nash equilibria. (10)
(c) Compute the sequential equilibria.



Problem 3. Determine all mixed strategies that are outcome equivalent with the behavioral strategy represented in the following one-player extensive form structure. (5)



Problem 4. Justify your answer by a proof or a counterexample.
(a) Existence of Nash Equilibrium in pure strategies implies existence of subgame perfect Nash Equilibrium in pure strategies.
(b) Nash Equilibrium in behavioural strategies always exists. (Remember: The set of behavioural strategies is a strict subset of the set of mixed strategies.) 10

Problem 5. There are n individuals who witness a crime. Everybody would like the police to be called. If this happens, each individual derives satisfaction $v > 0$ from it. Calling the police has a cost of c, where $0 < c < v$. The police will come if at least one person calls. Hence, this is an n-person game in which each player chooses from $\{C, N\}$: C means 'call the police' and N means 'do not call the police'. The payoff to person i is 0 if nobody calls the police, $v - c$ if i (and perhaps others) call the police, and v if the police is called but not by person i.
(a) What are the Nash equilibria of this game in pure strategies? In particular, show that the game does not have a symmetric Nash equilibrium in pure strategies (a Nash equilibrium is symmetric if every player plays the same strategy).
(b) Compute the symmetric Nash equilibrium or equilibria in mixed strategies.
(c) For the Nash equilibrium/equilibria in (b), compute the probability of the crime being reported. What happens to this probability if n becomes large? 10

Problem 6. Consider a seller of a used car and a potential buyer of that car. Suppose that quality of the car, θ, is a uniform draw from $[0, 1]$. This quality is known to the seller, but not to the buyer. Suppose that the buyer can make an offer $p \in [0, 1]$ to the seller, and the seller can then decide whether to accept or reject the buyer's offer.
Payoffs are as follows:

$$\begin{aligned} u_S &= p \quad \text{if offer accepted} \\ &= \theta \quad \text{if offer rejected} \end{aligned}$$

$$\begin{aligned} u_B &= a + b\theta - p \quad \text{if offer accepted} \\ &= 0 \quad \text{if offer rejected} \end{aligned}$$

Assume that $a \in [0, 1)$, that $b \in (0, 2)$, and that $a+b > 1$. These assumptions imply that for all θ, it is more efficient for the buyer to own the car.

(a) Show that the unique BNE is for the buyer to offer $p = a/(2 - b)$ and the seller to accept if and only if $p \geq \theta$.
(b) Note that if $a = 0$, then trade never occurs despite it the fact that there are always gains from trading. Explain this from game theoretic point of view. 10

Mid-semester Examination (2014-15)

M. S. (Q. E.) I Year

Computer Programming & Applications

Date: 29.08.14 | Full Marks: 60 | Duration: 2 hours

Answer as many questions you like, but you may at most score 60

1. Write a C code by which one can compute the Highest Common Factor (HCF) of two given integers. 8

2. The Fibonacci sequence may be expressed as: 0, 1, 1, 2, 3, 5, 8, 13, 21...............and so on (i.e. the next number is the sum of the previous two numbers). Write a C program to generate this sequence up to a specified number of terms. 8

3. Write a C program to find the factorial of a number using a function recursively. 8

4. (a) Convert the octal number $(2001.523)_8$ to binary. (b) Convert the hexadecimal number (63C.7) to binary (c) Convert the following binary number $(1001110.000111)_2$ to octal and hexadecimal systems. 3+3+6

5. Write a C program using a function to swap the values stored in two variables. 6

6. Write a C code for generating all single or double-digit prime numbers. 8

7. Write brief notes on the following:

   (a) Memory Registers (b) CPU (c) Program Counter (d) Accumulator 4x4

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: 2014-15
M. S. (Q.E.) .I Year
Basic Economics

Date: 29.08.2014 **Maximum Marks: 40** **Duration: $2\frac{1}{2}$ Hours**

Answer any four

1 Analyse how automatic attainment of full employment equilibrium is ensured in the classical macroeconomic model.

[10]

2 In the simple Keynesian model, derive the investment multiplier. Also show that balanced budget multiplier is equal to unity.

[5+5]

3 How do you derive IS curve and LM curve in Keynesian model? What role do they play?

[7+3]

4 In the presence of constant returns to scale and in the absence of technical progress, steady state growth equilibrium is a 'No growth' equilibrium. Examine the validity of this statement.

[10]

5 Distinguish between

a) Grow national product and grow domestic product
b) Transaction demand for money and speculative demand for money
c) Marginal propensity to consume and average propensity to consume

[3+5+2]

6 Prove the following propentie of the demand function.

(i) Price elasticity of demand is equal to the sum of income elasticity of demand and crom elasticity of demand.

(ii) Income elasticity of demand for all the commodities can not be greater than unity.

[5+5]

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MID-SEMESTRAL EXAMINATION: (2014-2015)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 02.09.2014 Maximum marks: 40 Duration: 2 Hours

Note: Answer **all** questions.

Note: Throughout, $\mathbb{R}^L$ is the L-dimensional Euclidean space. Let

$$\mathbb{R}^L_+ = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i \geq 0 \text{ for all } 1 \leq i \leq L\}$$

and

$$\mathbb{R}^L_{++} = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i > 0 \text{ for all } 1 \leq i \leq L\}.$$

Q1. Let $\succsim$ be a *preference relation* over a non-empty set of alternatives X. Suppose also that $\succ$ and $\sim$ are the *strict preference relation* and the *indifference relation*, respectively, associated with $\succsim$. Prove or disprove the following statements. In case of disprove, give an example to show that the statement is incorrect.

(i) If U and V are utility functions representing $\succsim$, then the function $f := \min\{U, U+V\}$, defined by $f(x) = \min\{U(x), U(x)+V(x)\}$ for all $x \in X$, is also a utility function representing $\succsim$.

(ii) If $U : X \to \mathbb{R}$ is a function, then "U is a utility function representing $\succsim$" is equivalent to "$x \sim y \Leftrightarrow U(x) = U(y)$ for all $x, y \in X$".

(iii) If $\succsim$ is rational, then the choice structure generated by $\succsim$ satisfies *WARP*.

(iv) Suppose that X is a countable set. Then $\succsim$ can be represented by a utility function.

(v) Suppose that X is a finite set and U is a utility function representing $\succsim$. For any non-empty subset B of X, let $|B|$ denote the number of elements in B. Assume that $\mathscr{B}$ denotes the set of non-empty subsets of X. Define a choice rule $\mathbb{C}(\cdot)$ on $\mathscr{B}$ by

$$\mathbb{C}(B) := \left\{x \in B : U(x) > U(y) \text{ for all } y \in D \text{ for some } D \subseteq B \text{ with } |D| > \frac{1}{2}|B|\right\}.$$

Then $(\mathscr{B}, \mathbb{C}(\cdot))$ satisfies *WARP*. [20]

Q2. Answer **all** questions.

(i) Show that if a demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$ satisfies *WARP* then it is homogeneous of degree zero.

(ii) Show that for a demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$ satisfying Walras's law, *WARP* holds if and only if *WARP* holds for all compensated price changes.

(iii) Show that the demand point $x(p, \omega)$ for a price-wealth combination (p, ω) is not necessarily a demand point for a price-wealth combination (p', ω'), where $\omega' = p' \cdot x(p, \omega)$. [12]

Q3. Answer **all** questions.

(i) Suppose that X is a non-empty set of alternatives and $(\mathscr{B}, \mathbb{C}(\cdot))$ is a choice structure of X that satisfies *WARP*. Show that $\succ^*$ and $\succ^{**}$ are identical, that is, for any $x, y \in X$, $x \succ^* y \Leftrightarrow x \succ^{**} y$, where $\succ^*$ is the *strict revealed preference relation* and $x \succ^{**} y \Leftrightarrow [x \succeq^* y \text{ and } y \not\succeq^* x]$.

(ii) Show that *WARP* is not a sufficient condition to ensure the existence of a rationalizing preference relation. [8]

End Semestral Examination: (2014)
MS (QE) I Year (First Semester)
Computer Programming and Applications

Date: 29.10.14 Maximum Marks: 100 Duration: 3 hours

Answer as many questions as you wish. But you may score at most 100.

1. Write brief notes on any three of the following: (i) low and high level languages (ii) flowcharts and pseudocodes (iii) pointer arithmetic (iv) do-while loop 5+5+5=15

2. (a) Write a C program to use some of the keys on the keyboard to play a piano. You may use the *beep (frequency in Hertz, time in ms)* function for its implementation.
   (b) Write a C program to verify whether a particular year is a leap year or not. 8+7=15

3. (a) How is a string different from any integer array?
   (b) Explain with example what do you mean by the ASCII character set.
   (c) Write a C program to identify whether a character entered from keyboard belongs to the upper or the lower case of alphabets.
   (d) Write a C program to count the number of words in a string and print each of the words in a separate line. 3+3+5+9=20

4. (a) What do you mean by (i) calling a function by value and (ii) calling a function by reference? Show how you can swap two integers stored against two different variables, using each of these two methods.
   (b) Explain with example the bubble sort algorithm and hence write a C program to implement the same in an array of integers. (4+4+8)+(5+9)=30

5. (a) Write C programs to (i) add two square matrices and (ii) calculate the trace (sum of the diagonal elements) of the resultant matrix.
   (b) Write a suitable C program to print out all prime numbers less than 1000, where the code implements the following algorithm from the Eratosthenes' sieve method (around 240 BC):
   (i) Create a list of consecutive integers from 2 to n: 2, 3, 4,..., n (n=1000 in the present case).
   (ii) Initially, let p equal 2, the first prime number.

......contd. on page 2

(iii) Strike out from the list all multiples of p greater than p.

(iv) Find the first number remaining on the list greater than p (this number is the next prime); let p equal this number.

(v) Repeat steps (iii) and (iv) until p^2 is greater than n.

(vi) All the remaining numbers on the list are prime. (7+3)+10=20

6. (a) Explain with suitable examples, how a *structure* differs from an *array*.

(b) Write C programs to (i) insert a *new* element at any position in an array (ii) delete *duplicates* in an array. 6+(7+7)=20

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2014-15
M. S. (Q.E.) .I Year
Basic Economics

Date: 30.10.2014 **Maximum Marks: 60** **Duration: 3 Hours**

Answer any three questions.

1 What do you mean by Pareto-efficient allocation ? Derive the conditions of Pareto-efficiency and explain how perfect competition ensures efficient allocation.

[2+10+8]

2 a) Explain how IS curve and LM curve are derived and how they describe equilibrium in a closed economy.

b) Does an IS-LM equilibrium in an open economy necessarily ensure balance of payment equilibrium? Explain your answer.

[10+10]

3. a) What do you mean by a dual economy? How does a classical dual economy model differ from a neoclassical dual economy model?

b) What do you mean by surplus labour? Analyse, in terms of Lewis model, how economic development takes place in the presence of surplus labour.

[4+4 + 4+8]

4. a) Discuss how comparative cost theory explains trade between two countries.

b) Indirect taxes lead to inefficient allocation of resources – Do you agree with this view ? Explain your answer.

c) Discuss various principles of taxation.

d) Analyse how the aggregate supply curve is derived in Keynesian macro model.

[4 x 5]

5. a) Distinguish between

(i) Gross domestic product and Gross national product
(ii) Real national income and Nominal National income
(iii) Marginal propensity to consume and average propensity to consume.

b) Analyse how real rate of interest is determined in the classical macro economic model. How does an expansion of money supply affect the real as well as nominal interest rate in that model?

[3x3 + 7+4]

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INDIAN STATISTICAL INSTITUTE

203, B. T. ROAD, KOLKATA - 700108

Semester I Examination

M.S.(Q.E.) 1st Year (2014-15)

STATISTICS

Date: 03/11/2014 Maximum Marks: 70 Time: 3 hours

[Answer **Part I** and **Part II** on separate answer scripts. Marks allotted to each question are given in parentheses]

Part I

[Answer *as many as you can.* The maximum you can score is 45].

1. (a) Let $x_1, x_2, \ldots, x_n$ be independent observations from a zero-inflated Poisson distribution with the probability mass function (p.m.f.) given by

$$f(x) = \alpha \ e^{-\lambda} \ \frac{\lambda^x}{x!} \ + \ (1-\alpha) \ I(x=0); \quad x = 0, 1, 2, \ldots,$$

where $\lambda > 0$, $\alpha \in (0,1)$ and I denotes the indicator function. Describe how you will find method of moment estimates for α and λ. [6]

(b) Let f_θ be the probability density function (p.d.f.) of a bivariate uniform distribution inside a square S_θ with vertices $(\theta, 0), (0, \theta), (0, -\theta), (-\theta, 0)$. Assume that $X_1, X_2, \ldots, X_n$ are independent and identically distributed with p.d.f. f_θ.

(i) Find the maximum likelihood estimator of θ. [6]

(ii) Is this estimator consistent? Justify your answer. [4]

2. (a) If $X_1, X_2, \ldots, X_n$ are independent and identically distributed with probability density function (p.d.f.)

$$f(x) = \frac{2}{\pi \ [4 + (x-\theta)^2]}; \quad -\infty < x < \infty,$$

describe how you will find the maximum likelihood estimator (MLE) for θ using the Fisher's scoring method. Approximate the variance of this estimator for $n = 100$. [6+2]

(b) Let $x_1, x_2, \ldots, x_{10}$ be 10 independent observations from a normal distribution with mean μ and variance σ^2. The parameters μ and σ both are unknown, but it is known that $\mu \in [-2.5, 1.5]$ and $\sigma \in [1, 2]$. If $\sum_{i=1}^{10} x_i = 17.5743$ and $\sum_{i=1}^{10} x_i^2 = 56.8791$, find the maximum likelihood estimates of μ^2 and σ^2. [4+4]

3. Suppose that $X_1, X_2, \ldots, X_n$ are independent and identically distributed with probability density function (p.d.f.)

$$f(x) = p^2 \, \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} \; + \; 2pq \, \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-2\mu)^2} \; + \; q^2 \, \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-3\mu)^2},$$

where $p, q, \sigma > 0$ and $p + q = 1$. Describe how you will use the Expectation-Maximization (EM) algorithm to find

(a) the MLE of p when μ and σ^2 are known [6]

(b) the MLEs of μ and σ^2 when p is known. [10]

Part II

[Answer *as many as you can.* The maximum you can score is 25].

1. (a) Define weak convergence and strong convergence of a sequence of random variables. [4]

   (b) Prove that strong convergence implies weak convergence. Is the converse true? Give reasons in support of your answer. [6]

   (c) Find the characteristic function (c.f.) of a $N(\mu, \sigma^2)$ variable. Is it possible that the c.f. of a random variable may not exist. Justify your answer. [6]

2. (a) Suppose that a random variable X follows a Bernoulli distribution, where $P(X = 1) = \theta = 1 - P(X = 0)$. For a random sample of size 10, consider testing the null hypothesis $H_0 : \theta \leq \frac{1}{2}$ *versus* the alternative hypothesis $H_0 : \theta > \frac{1}{2}$. Use the critical region $\{\sum x_i \geq 6\}$. Find the power function of this test. [6]

   (b) Discuss briefly the relationship between confidence interval and statistical test. [5]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2014-15

Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR
Subject Name: Game Theory I
Date: 6-11-2014 Maximum Marks: 50 Duration: 2 hours

Problem 1. Consider the following version of prisoner dilemma game.

1/2	C	D
C	4,4	0,6
D	6,0	2,2

(a) Is it true that the outcome (C,C) forever can be a subgame perfect equilibrium of the infinitely repeated prisoners dilemma game? If yes, provide the equilibrium strategies. Find the restriction on the discount factor δ.
(b) What do you think about the other Subgame Perfect Equilibria that are possible in this repeated prisoners dilemma game (use folk theorem)? Plot graphically: (i) The set of subgame perfect Nash Equilibria payoffs, (ii) The set of feasible payoffs.
(c) Consider the following tit-for-tat (TFT) strategy (row player version)
First round: Play C.
Second and later rounds: If the history from the last round is (C,C) or (D,C) play C. If the history from the last round is (C,D) or (D,D) play D.
(i) Is it true that TFT supports (C,C) forever as an NE in the Infinitely Repeated PD? - Justify your answer by a proof.
(ii) Is it true that TFT as an equilibrium strategy is Subgame Perfect?
- Justify your answer by a proof.

(20)

Problem 2. Consider the Independent Private Values auction model with n bidders and value distribution function F.
(a) Find the optimum bidding strategies in first price auction.
(b) Do these strategies form an equilibrium? If yes, is it unique? Justify your answer.
(c) Calculate the expected revenue using the optimum bidding strategy (do not use revenue equivalence).

(15)

Problem 3. Justify by a proof or counter example:
(a) Evolutionary stable strategy always exits for a 2×2 symmetric game.
(b) Evolutionary stable strategy always exits for a symmetric game that has a pure strategy Nash equilibrium.
(c) Evolutionary stable strategies always constitute a Nash Equilibrium.

(10)

Problem 4. We know that the Nash bargaining solution is the *unique* bargaining solution that satisfies the axioms: (i) Pareto Efficiency, (ii) Symmetry, (iii) Invariance to Equivalent, (iv) Payoff Representations Independence of Irrelevant Alternatives. Provide examples to show that all these four axioms are necessary for the uniqueness of the Nash bargaining solution.

(15)

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION: (2014-2015)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 10.11.2014 Maximum Marks: 60 Duration: 3 Hours

Note: Answer any **five** questions.

Note: Throughout, $\mathbb{R}^\ell$ is the ℓ-dimensional Euclidean space. Let

$$\mathbb{R}^\ell_+ = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $\succeq$ be a rational preference relation over $\mathbb{R}^\ell_+$.

(i) Assume that $L(x) = \{y \in \mathbb{R}^\ell_+ : x \succeq y\}$ and $R(x) = \{y \in \mathbb{R}^\ell_+ : y \succeq x\}$ for all $x \in \mathbb{R}^\ell_+$. If $L(x)$ and $R(x)$ are closed subsets of $\mathbb{R}^\ell_+$ then show that $\succeq$ is continuous.

(ii) Give an example of a discontinuous rational preference relation.

(iii) Show that $\succ$ is transitive. [6+3+3]

Q2. Answer **all** questions.

(i) Show that the demand function $x : \mathbb{R}^\ell_{++} \times \mathbb{R}_{++} \to \mathbb{R}^\ell_+$ satisfies *WARP* if and only if for any wealth level ω and prices p, p' we have $p' \cdot x(p, \omega) > \omega$ if $p \cdot x(p', \omega) \leq \omega$ and $x(p, \omega) \neq x(p', \omega)$.

(ii) Prove or disprove: If $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ is a utility function then $V : \mathbb{R}^\ell_+ \to \mathbb{R}$, defined by $V(x) = \frac{U(x)}{1+U(x)}$, is a utility function.

(iii) Suppose that $e : \mathbb{R}^\ell_{++} \times \mathbb{R} \to \mathbb{R}^\ell_+$ is the *expenditure function* associated with a continuous utility function U on $\mathbb{R}^\ell_+$. Show that $e(p, \cdot)$ is strictly increasing for all $p \in \mathbb{R}^\ell_{++}$ and $e(\cdot, u)$ is non-decreasing in p_j for all $1 \leq j \leq \ell$ and $u > U(0)$. [3+3+6]

Q3. Answer **all** questions.

(i) Prove or disprove: If U is a continuous utility function representing a preference relation $\succeq$ over $\mathbb{R}^\ell_+$ then $\succeq$ is continuous.

(ii) Let $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ be a continuous utility representation of a locally non-satiated preference $\succeq$ over $\mathbb{R}^\ell_+$. Given a $(p, w) \in \mathbb{R}^\ell_{++} \times \mathbb{R}_{++}$, the *utility maximization problem* (UMP) of the consumer is the following:

$$\sup\{U(x) : x \in B(p, \omega)\}$$

where $B(p, w) = \{x \in \mathbb{R}^\ell_+ : p \cdot x \leq w\}$ is the budget set for (p, w). The expenditure minimization problem (EMP) for any given $p \in \mathbb{R}^\ell_{++}$ and any $u > U(0)$ is the following:

$$\inf\{p \cdot x : x \in \mathbb{R}^\ell_+, U(x) \geq u\}.$$

Show that if x is the optimal solution of the UMP when $\omega > 0$, then x is the optimal solution of the EMP when the required level of utility is $U(x)$. Show further that the minimum expenditure level in this EMP is ω.

(iii) Give an example of an economy to show that a weakly Pareto optimal allocation is not necessarily a Pareto optimal allocation if all consumers have increasing utility functions and exactly one consumer has a discontinuous utility function.

[2+5+5]

Q4. Let $Y \subseteq \mathbb{R}^\ell$ be a production set.

(i) Show that if y is profit maximizing for some $p \in \mathbb{R}^\ell_{++}$ then y is efficient. Does the same conclusion hold if the condition $p \in \mathbb{R}^\ell_{++}$ is replaced with $p \in \mathbb{R}^\ell_+ \setminus \{0\}$?

(ii) Find the additive closure of $Y = \{(-x, y_1, y_2) : x, y_1, y_2 > 0, y_1 + y_2 \leq x^3\}$.

(iii) Show that for a single-output technology, Y is convex if and only if the production function f is concave. [4+3+5]

Q5. Answer **all** questions.

(i) Find the cost function of a single output technology whose production function is given by $f(z) = z_1 + z_2$, where $z \in \mathbb{R}^{\ell-1}_+$ for $\ell \geq 3$.

(ii) Suppose that Y denotes the aggregate production set of production sets $Y_1, \cdots, Y_m$. If π and π_i are profit functions for $i = 1, \cdots, m$ then show that $\pi(p) = \sum_{i=1}^m \pi_i(p)$ for $p \in \mathbb{R}^\ell_{++}$.

[6+6]

Q6. Answer **all** questions.

(i) Show that if U is a linear utility function, then

$$U(\hat{L}) = \sum_{i=1}^m U(L_i)$$

is satisfied for every compound lottery $\hat{L} = [q_1(L_1), \cdots, q_m(L_m)]$.

(ii) Let $\mathscr{L}$ denote the set of simple lotteries over $\mathcal{O} = \{a, b, c\}$. Define a preference relation $\succeq$ over $\mathscr{L}$ by

$$L_1 \succeq L_2 \text{ if either } [p_3 < p_3'] \text{ or } [p_3 = p_3' \text{ and } p_2 < p_2'],$$

where

$$L_1 = [p_1(a), p_2(b), p_3(c)] \text{ and } L_2 = [p_1'(a), p_2'(b), p_3'(c)].$$

Show that preference relation violates the continuity axiom of the Von Neumann-Morgenstern expected utility representation theorem.

(iii) Consider an economy with I consumers and J firms. Each consumer's preference is represented by a quasi-linear utility function $U_i(m_i, x_i) = m_i + \phi_i(x_i)$, where m_i denotes the numeraire and $\phi_i(x_i) = 2x_i^{\frac{1}{2}}$. Firm j's cost function is defined by $c_j(q_j) = \frac{q_j^2}{2}$. Assume that agent i's initial endowment is $(a_i, 0)$, where $a_i > 0$. Find the sufficient condition for the existence of a partial equilibrium and then find an equilibrium price. [4+4+4]

INDIAN STATISTICAL INSTITUTE

Semestral Examination (2014–2015)

MS(QE) I

Mathematical Methods

Date : 14.11.2014 Maximum Marks : 100 Time : $3\frac{1}{2}$ hrs.

This paper carries 120 marks. Maximum you can score is 100. Precisely justify all your steps. Carefully state all the results you are using.

1. In each of the following a map T is defined by the given formula. In each case, determine if T is linear. And if T is linear, describe its null space and range. [10]

   (a) $T(x,y) = (e^x, e^y), \quad (x,y) \in \mathbb{R}^2.$

   (b) $T(x,y) = (2x - y, x + y), \quad (x,y) \in \mathbb{R}^2.$

   (c) $T(x,y,z) = (x + z, 0, x + y), \quad (x,y,z) \in \mathbb{R}^3.$

   (d) $T(x,y,z) = (x + 1, y + 2, z + 3), \quad (x,y,z) \in \mathbb{R}^3.$

2. Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

   (a) Find the characteristic polynomial of A. [2]

   (b) Find the eigenvalues of A, and at least one eigenvector for each eigenvalue. [5]

   (c) Show that for any $n \geq 1$, [8]

$$A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 1 & 0 \\ 0 & n & 1 \end{bmatrix}.$$

   (d) Write A^{-1} as a quadratic polynomial in A, and hence, find it. [10]

   [Hint : Cayley-Hamilton Theorem.]

3. Find the inverse of the matrix [10]

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 9 & 5 & 4 \end{bmatrix}.$$

4. Does there exist $\alpha \in \mathbb{R}$ such that the following system has at least one solution? If such an α exists, find all possible solutions : [10]

$$\begin{array}{rcrcrcr} x & + & 3y & - & 2z & = & -2 \\ -x & - & 5y & + & 3z & = & 2 \\ 2x & - & 8y & + & 3z & = & \alpha \end{array}$$

5. Find all critical points and hence, all the local and global maxima and minima of

(a) $f(x) = 8x^5 - 15x^4 + 10x^2$ on $\mathbb{R}$. [10]

(b) $g(x) = x - \sin 2x + \frac{1}{3}\sin 3x$ in $[-\pi, \pi]$. [10]

Recall that : $\sin 2x = 2\sin x\cos x$, $\cos 2x = 2\cos^2 x - 1$, $\sin 3x = 3\sin x - 4\sin^3 x$, $\cos 3x = 4\cos^3 x - 3\cos x$. Also,

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

6. Find and classify the critical points (if any) of the function [20]

$$f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2.$$

7. Find the absolute minima and absolute maxima of $f(x, y) = 2x^2 - y^2 + 6y$ on the closed disc of radius 4, i.e., $x^2 + y^2 \leq 16$. [25]

[Hint : The interior and the boundary of the region should be treated separately.]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination (Back Paper): 2014-15

Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR
Subject Name: Game Theory I
Date: 29/12/14 Maximum Marks: 50 Duration: 2 hours

Problem 1. Compute all correlated equilibria in the game

1/2	c	s
c	-10,-10	5,0
s	0,5	-1,-1

by using the definition of correlated equilibrium.

(15)

Problem 2. A buyer wants to buy a car but does not know whether the particular car he is interested in has good or bad quality (a lemon is a car of bad quality). About half of the market consists of good quality cars. The buyer offers a price p to the seller, who is informed about the quality of the car; the seller may then either accept of reject this price. If he rejects, there is no sale and the payoff will be 0 to both. If he accepts, the payoff to the seller will be the price minus the value of the car, and to the buyer it will be the value of the car minus the price. A good quality car has a value of 15,000, a lemon has a value of 5,000. (a) Set up the extensive as well as strategic form of this game. (b) Compute the subgame perfect equilibrium or equilibria of this game.

(15)

Problem 3. Consider the First-Price Sealed-Bid Auction.
(a) Show that $(b_1, ..., b_n) = (v_2, v_2, v_3, ..., v_n)$ is a Nash equilibrium in this game.
(b) Show that, in any Nash equilibrium of the game, a player with the highest valuation obtains the object. Exhibit at least two other Nash equilibria in this game, apart from the equilibrium in (a).
(c) Show that bidding ones true valuation as well as bidding higher than ones true valuation are weakly dominated strategies. Also show that bidding lower

than ones true valuation is not weakly dominated.
(d) Show that, in any Nash equilibrium of this game, at least one player plays a weakly dominated strategy.

(15)

Problem 4. State and prove the revenue equivalence theorem in the Independent Private Values auction model.

(15)

INDIAN STATISTICAL INSTITUTE

BACK PAPER EXAMINATION: (2014-2015)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 01/01/15 Maximum Marks: 100 Duration: 3 Hours

Note: Answer **all** questions.

Note: Throughout, $\mathbb{R}^\ell$ is the ℓ-dimensional Euclidean space. Let

$$\mathbb{R}^\ell_+ = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. State the weak axiom of revealed preference ($WARP$) for a demand function $x : \mathbb{R}^\ell_{++} \times \mathbb{R}_{++} \to \mathbb{R}^\ell_+$. Show that if x satisfies the $WARP$ then for any compensated price change from (p, w) to $(p', w') = (p',\ p' \cdot x(p, w))$, the following inequality holds:

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0. \qquad [10]$$

Q2. A preference relation $\succeq$ over $\mathbb{R}^\ell_+$ is called *monotone* (resp. *weakly monotone*) if $x, y \in \mathbb{R}^\ell_+$ and $x - y \in \mathbb{R}^\ell_{++}$ (resp. $x - y \in \mathbb{R}^\ell_+ \setminus \{0\}$) together imply $x \succ y$ (resp. $x \succeq y$). Show that if $\succeq$ is locally non-satiated, rational and weakly monotone then it is monotone. [10]

Q3. Suppose that $(\mathscr{B}, \mathbb{C}(\cdot))$ is a choice structure over the set of alternatives X such that

(i) $WARP$ is satisfied and

(ii) $\mathscr{B}$ includes all subsets of X up to three elements.

Show that there is a rational preference relation $\succeq$ that rationalizes $\mathbb{C}(\cdot)$ relative to $\mathscr{B}$. [15]

Q4. Consider a preference relation over $\mathbb{R}^2_+$ represented by the utility function $U(x, y) = \sqrt{x} + \sqrt{y}$. Find the demand functions for the commodities 1 and 2 as they depend on prices and wealth. [10]

Q5. Show that if the Walrasian demand function $x : \mathbb{R}^\ell_{++} \times \mathbb{R}_{++} \to \mathbb{R}^\ell_+$ satisfies the strong axiom of revealed preference, then there is a rational preference relation $\succeq$ that rationalizes x, that is, such that for all (p, ω), $x(p, \omega) \succ y$ for every $y \neq x(p, \omega)$ with $y \in B(p, \omega) = \{z \in \mathbb{R}^\ell_+ : p \cdot z \leq \omega\}$. [15]

Q6. Consider an economy with I consumers, J firms and ℓ commodities. Let ω_i and θ_{ij} denote the consumer i's initial endowment and share from firm j for all $1 \leq i \leq I$

and $1 \leq j \leq J$, respectively. Assume that $\mathbb{R}^\ell_+$ is the consumption set of consumer i and the preference of consumer i, defined over $\mathbb{R}^\ell_+$, is continuous and strictly monotone. Show that any weakly Pareto optimal allocation is a Pareto optimal allocation. [15]

Q7. Let $\mathcal{O} = \{A, B, C, D\}$ and $\succeq$ be a *preference relation* on the set of compound lotteries over $\mathcal{O}$. Suppose also that $\succ$ and $\sim$ are the strict preference relation and the indifference relation, respectively, associated with $\succeq$. Assume that $\succeq$ satisfies the von Neumann-Morgenstern axioms and

$$C \sim \left[\frac{3}{7}(A), \frac{4}{7}(D)\right], \quad B \sim \left[\frac{3}{5}(A), \frac{2}{5}(D)\right], \quad A \succ D.$$

Let

$$L_1 = \left[\frac{1}{5}(A), \frac{3}{10}(B), \frac{4}{10}(C), \frac{1}{10}(D)\right] \text{ and } L_2 = \left[\frac{2}{5}(B), \frac{3}{5}(C)\right].$$

Determine which of the following is true: (a) $L_1 \succ L_2$; (b) $L_2 \succ L_1$; and (c) $L_1 \sim L_2$. [5]

Q8. Show that if $Y \subseteq \mathbb{R}^\ell$ is a convex production set then its additive closure is $\bigcup\{nY : n \geq 1\}$ where for any positive integer n, $nY = \{ny \in \mathbb{R}^\ell : y \in Y\}$. [10]

Q9. Given a $(p, w) \in \mathbb{R}^\ell_{++} \times \mathbb{R}_{++}$, the *utility maximization problem* (UMP) of the consumer is the following:

$$\sup\{U(x) : x \in B(p, w)\}$$

where $B(p, w) = \{x \in \mathbb{R}^\ell_+ : p \cdot x \leq w\}$ is the budget set for (p, w) and $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ is a utility function representing a preference relation $\succeq$. Show that the function $v : \mathbb{R}^\ell_{++} \times \mathbb{R}_{++} \to \mathbb{R}$, defined by

$$v(p, w) = \sup\{U(x) : x \in B(p, w)\},$$

is quasi-convex. [10]

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203, B. T. ROAD, KOLKATA - 700108

Semester I Examination **(BACK PAPER)**

M.S.(Q.E.) 1^{st} Year (2014-15)

STATISTICS

Date: 2/01/2015 | Maximum Marks: 100 | Time: 3 hours

[Answer **Part I** and **Part II** on separate answer scripts. Marks allotted to each question are given in parentheses]

Part I: 64 Marks

Answer *all* questions.

1. Prove or disprove the following statements.

   (a) Uniformly minimum variance unbiased estimator (UMVUE), if it exists, is unique. [5]

   (b) Maximum likelihood estimator (MLE), if it exists, is unique. [4]

   (c) MLE is always consistent. [5]

   (d) The variance of a UMVUE cannot exceed than the Cramer-Rao lower bound [4]

   (e) The variance of an MLE cannot be smaller than the Cramer-Rao lower bound. [4]

   (f) MLE, if it is unique, it is a function of the minimal sufficient statistic. [4]

   (g) The family of distributions $\mathcal{F} = \{U(0, \theta);\ \theta \geq 1\}$ is not complete [4]

2. (a) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with probability density function (p.d.f.) $f(x) = e^{-(x-\theta)};\ x \geq \theta$.

   (i) Check whether f belongs to a one-parameter exponential family. [2]

   (ii) Check whether $X_{(1)}$ and $S = \sum_{i=2}^{n}(X_{(i)} - X_{(1)})$ are independnet. [4]

   (b) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed as $N(\mu, \sigma^2)$ variates, where μ and σ^2 both are unknown. Find the uniformly minimum variance unbiased estimator (UMVUE) for μ^3. [6]

3. A box contains M white balls and some black balls. If n balls are chosen from this box at random and x of them are found to be white, estimate the number of black balls in the urn using (i) the method of moments and (ii) the method of maximum likelihood. [4+6]

4. Out of 400 donors in a blood donation camp, 127 had blood group A, 109 had blood group B, 123 had blood group O and the rest had blood group B. Based on this information, how will you find the maximum likelihood estimates of the frequencies for A, B and O alleles ? If

it is known that the allele frequency for O is 0.5. find the maximum likelihood estimates of the allele frequencies for A and B ? [6+6]

Part II: 36 Marks

Answer *all* questions.

1. (a) Find the characteristic function of a standard normal deviate, and hence prove the Khinchine's theorem. [10]

   (b) State and prove the Lindeberg-Levy theorem. [8]

2. (a) Explain the concepts of Type I and Type II errors and probability value (p-value). [8]

   (b) State the assumptions and the two hypotheses for testing the equality of two means from two populations, and then describe the test. [10]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: 2014-15

M.S. (Q.E.) I

Environmental Economics

Date: 23 February, 2015 **Maximum Marks: 60** **Duration: 2 hrs.**

1. (a). It is known that impacts of emissions by polluting firms on ambient pollution concentrations differ by location of firms. In that situation how should an efficient emission fee for each firm be set by a regulator following equimarginal principle?

   Two identical firms spend money for pollution abatement. Marginal costs from emitting an amount e_i by i^{th} firm are given by $-14 + 7e_i$. Two firms differ in their impact on ambient pollution concentrations. One unit of emission results 2 units by firm 1 and 3 units by firm 2 of ambient pollution. Marginal damage is assumed to be same as total ambient pollution. Find the appropriate amount of emission at firm level and at ambient level.

   (b). Show how the decision of a regulator to choose an appropriate regulatory measure out of emission fee and quantity regulation is affected when pollution generated by individual firm is not observable to the regulator but total ambient pollution level is observable to him.

   [9+5+8=22]

2. Consider a small country facing market failure for producing a good with adverse environmental impact leaving uncontrolled and prospect of trade liberalization, in which its own actions do not affect the rest of the world.
   (a). Show how does the welfare of small country change if the country shifts from autarchy to open trade.
   (b). Show the relative efficiency of trade and environmental policies in reducing the environmental degradation.

   [16+12=28]

3. Write short note on the following:
   a. Tradable pollution permits.
   b. In a perfectly competitive market effects of tax and subsidy on pollution generation in long run.

   [5+5=10]

INDIAN STATISTICAL INSTITUTE
MID-SEMESTRAL EXAMINATION: 2014-2015
M.S.(Q.E.) I year
Theory of Finance I

Note: Clearly explain the symbols you use and state all the assumptions you need for any derivation.

Date: 23/02/2015 Maximum Marks: 100 Time: 3 hours

1. State and Prove a result which shows that if the relative risk aversion measure is increasing, then the wealth elasticity of demand for the underlying risky prospect is less than one. (2+15=17)

2. Define certainty equivalent and explain it graphically. Develop the necessary sufficient condition for the absolute cost of risk to be translation invariant. (2+3+13 = 18)

3. Show that if expected dividend remains constant over time, then for an indefinite period a share can be evaluated like a consol. (10)

4. Demonstrate rigorously that the Macaulay duration of a bond portfolio is a weighted average of durations of bonds in the portfolio. (20)

5. Show that in a one-step binomial model existence of a risk neutral probability is equivalent to non-arbitrage. (20)

6. (a) Give a graphical exposition of the simple binomial model when it is extended to 3 periods using the same probabilistic formulation ,by specifying the stock values and respective probabilities..

(b) Clearly demonstrate the existence of a transformation of the utility function under which the certainty equivalent remains invariant.

© Determine the modified Macaulay duration of a perpetuity.

(d) Do you agree or disagree with the following statement: " Incomleteness of an asset market means non-attainability of all contingent claims." ? In either case, justify your answer. (6+3+3+3)

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MID-SEMESTRAL EXAMINATION: (2014-2015)

MSQE I and M.Stat II

Microeconomic Theory II

Date: 24.02.2015 Maximum marks: 40 Duration: 2 Hours

Note: Answer **all** questions.

Note: Throughout, $\mathbb{R}^\ell$ is the ℓ-dimensional Euclidean space. Let

$$\mathbb{R}^\ell_+ = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $\mathbb{R}_+$ denote the set of wealth and U be a twice-differentiable strictly increasing utility function for a decision maker. For any fixed amount of money ω and positive number ε, the probability premium is denoted by $\pi(\omega, \varepsilon, U)$. Show that if the decision maker is risk neutral then $\pi(\omega, \varepsilon, U) = 0$. [6]

Q2. Show that the strictly increasing transformation of a linear utility function on the set of compound lotteries over a finite set of alternatives is not necessarily a linear utility representation of the same preference. [6]

Q3. Consider a preference $\succsim$ over $\mathbb{R}^2_+$ defined by

$$x \succ y \text{ if } x_1 + x_2 \geq 1 \text{ and } y_1 + y_2 < 1;$$

$$x \sim y, \text{ otherwise.}$$

Verify whether the above preference relation is upper or lower semi-continuous. [5]

Q4. Show that the budget correspondence $B : (\mathbb{R}^\ell_+ \setminus \{0\}) \times \mathbb{R}^\ell_{++} \rightrightarrows \mathbb{R}^\ell_+$ is lower hemi-continuous. Does the same conclusion hold if the space of wealth $\mathbb{R}^\ell_{++}$ is replaced with $\mathbb{R}^\ell_+$? [8]

Q5. Let $\succsim$ be a preference relation over $\mathbb{R}^\ell_+$. Prove or disprove the following statements. In case of disprove, give an example to show that the statement is incorrect.

(i) Assume $\succsim$ is continuous and $x \succ \omega$ implies $p \cdot x \geq p \cdot \omega$ for $p, x, \omega \in \mathbb{R}^\ell_+$. If $p \cdot \omega > 0$ and $x \succ \omega$, then $p \cdot x > p \cdot \omega$.

(ii) Assume $\succsim$ is continuous and $x \succ \omega$ implies $p \cdot x \geq p \cdot \omega$ for $p, x, \omega \in \mathbb{R}^\ell_+$. If $p \cdot \omega = 0$ and $x \succ \omega$, then $p \cdot x > p \cdot \omega$. [4+4]

Q6. Answer any **one** question.

(i) Show that the demand set $D(p, \omega, \succeq) \neq \emptyset$ for a price $p \in \mathbb{R}^{\ell}_{++}$, initial endowment $\omega \in \mathbb{R}^{\ell}_{+}$ and an upper semi-continuous rational preference $\succeq$ over $\mathbb{R}^{\ell}_{+}$.

(ii) Consider an utility function $U : \mathbb{R}^3_+ \to \mathbb{R}$ defined by

$$U(x, y, z) = \frac{xz}{(1+y)^2}.$$

Show that the demand set $D(p, \omega, \succeq) \subseteq \{(x, 0, z) : x, z \in \mathbb{R}_+\}$ for any price $p \in \mathbb{R}^3_{++}$ and initial endowment $\omega \in \mathbb{R}^3_+$. Take a sequence $\{p_n : n \geq 1\}$ of prices, where $p_n = (2, \frac{1}{2n}, \frac{1}{2n})$ for all $n \geq 1$. If (x_n, y_n, z_n) is a demand point at price p_n, then find the limit of $\{y_n : n \geq 1\}$. 7

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2014-2015)
MS (Q.E.) I Year
Macroeconomics I

Date: 26 · 02 · 2015 | Maximum Marks 40 | Duration 3 hours

Answer all questions

Group A

1. Examine the validity of the following statements in the context of Solow growth model:
   a) The result of the existence of unique stable steady state growth equilibrium is valid even if the production function satisfies non- constant returns to scale.
   b) The steady state equilibrium can explain a positive rate of growth of per-capita income only if there is Harrod neutral technical progress.

(5+5)

2. Consider a 2 X 2 dynamic system with following equations of motion

$$\dot{K_1} = 3K_2 - 2K_1 - 15$$

$$\dot{K_2} = K_1 + K_2 - 10$$

Where K_1 and K_2 are capital stocks of two sectors.

   a) Construct the phase diagram.
   b) Find out the steady state equilibrium.
   c) Examine the nature of stability of this equilibrium.

(5+2+3)

Group B

3. In an appropriate new Keynesian model derive the long run multiplier of a balanced budget increase in government expenditure and show that with variety effect absent, such multiplier is smaller than what would obtain in the short run:

(10)

4. Show that in the Blanchard and Kiyotaki model, a coordinated reduction in all prices and wages, beginning from a situation of monopolistic equilibrium, will raise real profits and also the utility.

(10)

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: 2014-2015
MS (Q.E.) I Year
Time Series Analysis and Forecasting
Date: 27th February, 2015 Maximum Marks 35
Duration 2 hours

All notations are self-explanatory. This question paper carries a total of 35 marks. Marks allotted to each question are given within parentheses.

1. Construct an example of a strong stationary but not weak stationary process. Construct an example of a weak stationary but not strong stationary process. **[5+ 5 =10]**

2. Let $\{X_t, t = 1, 2, \ldots\}$ be a sequence of random variables with mean zero and with unit variance. Define $Z_t = X_t X_{t-1}$. Examine whether Z_t is weakly stationary or not for the following cases:

   (a) X_t is white noise, (b) X_t is martingale difference with finite time invariant variance, (c) X_t is i.i.d process. **[5+ 3+2 =10]**

3. Let $\{X_t, t = 1, 2 \ldots,\}$ be a sequence of independent normal random variables with mean zero and with unit variance. Consider the following process:

   $W_t = 10 + 1.5W_{t-1} - 0.56W_{t-2} + X_t$.

   (a) Show that the process is stationary. (b) Find the mean and variance of the process W_t. **[5+ 3+7 =15]**

INDIAN STATISTICAL INSTITUTE

203, B.T. ROAD, KOLKATA – 700108

M.S.(Q.E.) 1st Year (2014 – 15)

Econometric Methods I

Mid-semester Examination

Date: 02 March 2015 | Maximum Marks: 100 | Time: 3 hours

This question paper carries a total of 110 marks. You can answer any part of any question. But the maximum that you can score is 100. Marks allotted to questions are given within parentheses.

1) (a) In a regression model $y_i = \alpha + \beta x_i + \varepsilon_i$ if the sample mean $\bar{x}$ of x is zero, show that $\text{cov}(\hat{\alpha}, \hat{\beta}) = 0$, where $\hat{\alpha}$ and $\hat{\beta}$ are the ordinary least squares estimators of α and β, respectively.

(b) Let e_i be the residuals in the least squares fit of y_i against $x_i (i = 1,2,\ldots.n)$. Show that and $\sum_{i=1}^{n} x_i e_i = 0$. What happens to $\sum_{i=1}^{n} e_i$ i.e., is it (i) always zero, (ii) zero under some condition(s), and (iii) never zero? Justify your answer.

[6+6 = 12]

2) (a) In a multiple linear regression model, explain in what sense(s), the term 'linear' is used. Give two regression specifications where this assumption is relaxed. Provide explanations in support of your answer.

(b) Consider a standard multiple regression model $y = X\beta + \varepsilon$, where the notations have their usual meanings. Derive the ordinary least squares estimator and then show that it has minimum variance in the class of linear unbiased estimators.

[6 + 14 = 20]

3) (a) In a multiple regression model, prove that $R^2 \leq \bar{R}^2$.

(b) State and prove the Frisch-Waugh-Lovell theorem.

[6 +14 = 20]

4) (a) Under the assumption of classical regression model with k regressors along with normality of the errors, suppose that $q(\leq k)$ linear restrictions $R\beta = r$ are to be tested based on n observations (the notations have their usual meanings). Derive the test statistic and show that it follows a $F\ (q,\ n-k)$ distribution under the null hypothesis.

(b) Suppose the following two restrictions on the coefficient parameters of a multiple regression model are required to be tested

(i) $\beta_3 + \beta_4 = 1$, and (ii) $\beta_2 = \beta_3 = \beta_4 = 0$.

Express these two separate restrictions in the usual notation $R\beta = r$. What would be the distribution (t or F) of the two underlying test statistics? Justify your answers.

(c) Suppose you decide to investigate the relationship given in the null hypothesis $\beta_1 + \beta_4 = 1$ and $\beta_5 = 1$ in a multiple regression with 5 regressors. What would constitute the restricted regression? Suppose the regression is carried out in a sample of 96 quarterly observations, and the residual sums of squares for restricted and unrestricted are 102.87 and 91.41, respectively. Perform the test for the null hypothesis. What is your conclusion?

[14 +10 + 10 = 34]

5) (a) Explain the term 'parameter structural stability'?

(b) A financial econometrician thinks that the stock market crash of October 1987 fundamentally changed the risk-return relationship given by the CAPM equation. He decides to test this hypothesis using a Chow-test. The model is estimated using monthly data from January 1980 – December 1995, and then two separate regressions are run for the sub-periods corresponding to data before and after the crash. The model i.e., the CAPM is

$$r_t = \alpha + \beta R_{mt} + u_t$$

so that the excess return on a security at time t is regressed upon the excess return on a proxy for the market portfolio at time t. The results for the 3 models estimated for shares in British Telecom (BT) are as follows:

1981 M1 – 1995 M 12

$$r_t = 0.0215 + 1.491 r_{mt} \qquad RSS = 0.189, T = 180$$

1981M1 – 1987M10

$$r_t = 0.0163 + 1.308 r_{mt} \qquad RSS = 0.079, T = 82$$

1987M11 – 1995M12

$$r_t = 0.0360 + 1.613\, r_{mt} \qquad RSS = 0.082, T = 98$$

where RSS stands for the residual sum of squares and T the number of observations.

What are the null and alternative hypotheses that are being tested here, in terms of α and β. Perform the test. What is your conclusion?

(c) What might Ramsey's RESET be used for? What is to be done if it is found that the RESET has rejected the null hypothesis? Explain your answers.

[4+12+8 = 24]

INDIAN STATISTICAL INSTITUTE

203, B.T. ROAD, KOLKATA – 700108

M.S.(Q.E.) 1st Year (2014 – 15)

Semester II Examination

Econometric Methods I

Date: 23.04.2015 Maximum Marks: 100 Time: 3 hours

[This question paper carries a total of 115 marks. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.]

1. (a) State clearly the assumptions of the classical linear regression model $y = X\beta + \varepsilon$, where the symbols have their usual meanings. Show that the best linear unbiased estimator of β is $\hat{\beta} = cy$ where $c = (X'X)^{-1}X'$, and obtain the sampling variance of $\hat{\beta}$.

   (b) A three-variable linear regression model $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$ is estimated on the basis of 9 observations, and the following results are obtained:

   $$\hat{\beta}_1 = 1.36, \quad \hat{\beta}_2 = 0.11, \quad \hat{\sigma}_\varepsilon^2 = 12.92,$$

   $$\text{and } X'X = \begin{bmatrix} 650 & -112 \\ -112 & 648 \end{bmatrix}$$

   where $\hat{\beta}_1, \hat{\beta}_2$ and $\hat{\sigma}_\varepsilon^2$ are the least squares estimates of β_1, β_2, and the disturbance variance σ_ε^2, respectively, and $X'X$ is the corrected sum of squares and product matrix of x_1 and x_2. Using these computations, test the null hypothesis $\beta_1 + \beta_2 = 1$ against the alternative $\beta_1 + \beta_2 > 1$.

   [12 + 10 = 22]

2. Suppose $Y : \{Y_t, t = 1,2,\ldots,T\}$ constitutes a time series of quarterly observations on a variable which is known to contain an additive seasonal component. It is known that this variable is dependent on a set of explanatory variables $X_1, X_2, \ldots, X_K$. Hence the following linear regression equation is set up to obtain the deseasonalized values of the dependent variable in question:

   $$Y = X\beta + D\alpha + \varepsilon$$

   where Y, X and D are the vector/matrix of observations on the dependent variable, its explanatory variables, and the dummy variables introduced for capturing the seasonality, respectively.

   Obtain the deseasonalization operator M by the least squares procedure, such that $\hat{Y} = MY$ will be the vector of estimates of the deseasonalized values of Y. Also examine if the deseasonalization operator that you obtain is unbiased in the sense that $E(MY) = X\beta$.

   [10 + 5 = 15]

P.T.O

3. (a) Discuss briefly the problem of multicollinearity in linear regression model, and then demonstrate its major consequences.

(b) State the intuition behind the ordinary ridge regression estimator of the regression coefficients of a p-variable linear regression model $y = X\beta + \varepsilon$, and then show that the mean squared error of this estimator is given by

$$\sigma_\varepsilon^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \beta' (X'X + kI)^{-2} \beta$$

where λ_i's are the eigenvalues of the matrix $X'X$, σ_ε^2 is the error variance, and k is a positive scalar involved in the ordinary ridge regression estimator.

[10 + 10 = 20]

4. (a) Consider the following model with two regressors:

$$y_t^* = \alpha z_t + \beta x_t^*$$

$$x_t = x_t^* + u_t$$

and $$y_t = y_t^* + v_t$$

where z_t is an observable variable subject to no error, x_t^* is an unobservable true variable, x_t and y_t are observed or measured values of the true variables x_t^* and y_t^*, respectively, and u_t and v_t are mutually and serially independent errors of observations distributed independently of x_t^* and y_t^* with mean zero and variances σ_u^2 and σ_v^2, respectively. Obtain expressions for the asymptotic bias of the ordinary least squares estimators of β and α, and then show that $\hat{\beta}$, the ordinary least squares estimator of β, is biased downward while for $\hat{\alpha}$, the ordinary least squares estimator of α, it can be either way.

(b) For the multiple linear regression model $y = X\beta + \varepsilon$ with $D(\varepsilon) = \sigma^2 I_n$ and $\underset{n\to\infty}{p\lim}(X'\varepsilon / n) \neq 0$, where notations have their usual meanings, derive the IV estimator of β. Also obtain the standard error of the IV estimator.

(c) Consider the following model for stationary $\{y_t\}$:

$$y_t = \Phi\, y_{t-1} + u_t,\ t = 1,2,\ldots,n$$

$$u_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \text{ where } \varepsilon_t \sim WN\,(0, \sigma_\varepsilon^2).$$

Show that the OLS method is not appropriate in estimating the parameters of this model. Could y_{t-2} and y_{t-3} be used as instruments? Justify your answer with derivations wherever necessary.

[10+8+12=30]

P·T·O

5. Sketch the feasible GLS procedure for estimating the model $y_i = x_i'\beta + \varepsilon_i$ where the variance of ε_i *is* $\sigma_\varepsilon^2 = e^{\alpha z_i}$ for some exogenously given variable z.

[8]

6. (a) Explain what is meant by "identification problem" in the simultaneous structural equations system.

(b) Derive the rank condition for identification of any equation in the simultaneous structural equations system.

(c) Prove that 2SLS method of estimation produces consistent estimator of the parameters of an identified simultaneous structural equations system.

[5+6+9 = 20]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2014-2015)
MS (Q.E.) I Year
Macroeconomics I

Date: 27·04·15 Maximum Marks 60 Duration 3 hours

Group A
Answer any two

1. Rate of economic growth in a two sector planned economy varies positively with the investment share allocated to the investment good producing sector.— Examine the validity of the statement in the light of the Feldman-Mahalanobis model. [15]

2. Derive the optimum income tax rate in the endogenous model developed by Barro and interpret the derived result. [12+3]

3. Consider a two sector economy with the following two equations of motion.

$$\dot{K}_1 = K_1 K_2 - 9$$

$$\dot{K}_2 = K_1 + K_2 - 10$$

Here K_i is the capital stock of the i th sector for i=1, 2.

a) Construct the phase diagram.

b) Examine the existence, uniqueness and stability of the steady-state equilibrium in this model. [7+2+2+4]

Group B
Answer all questions

1. Show how equilibrium unemployment is sustained in the Shapiro- Stiglitz model. What do you think would happen to equilibrium unemployment if the firms were to experience a technological progress (by this I simply mean that firms could produce more, say twice, the output they produced earlier, for any given combination of inputs).

[12+3]

P.T.O

2. a) In a flex price, monopolistically competitive equilibrium of the Blanchard-Kiyotaki kind; show that money is neutral.

Also show that in such a model the monopolistically competitive output is smaller than the competitive output.

b) Consider an economy with the representative agent having the utility function:

$$U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}, 0<\alpha, \gamma<1$$

Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i{}^{\rho}\right]^{1/\rho}$, $0<\rho<1$ and c_i is the consumption of the i^{th} variety. L is the labour supply, P is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. M is the money balances (and suppose M_0 is the initial endowment of money). The household budget constraint is given by:

$PC + w(1-L) + M = M_0 + w + \pi - T$ where w is the money wage rate and π is the economy wide profits and T is the taxes. Production of varieties is given by:

$$Y_i = 0 \; if \; L_i \leq F$$

$$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

Y_i is the output of i^{th} variety and L_i is the labour employed in the production of the i^{th} variety.

Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed n).

(i) Derive the multiplier of a balanced budget ($PG=T$) increase in government expenditure where G takes the form:

$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$ and g_i is the government consumption of the i^{th} variety.

(ii) What would be the effect of such increase in government expenditure on P?

[Hint: Try to write down the goods market equilibrium ($Y=C+G$) in a form which does not involve money balances. That would require a look into the money market equilibrium ($M = M_0$).]

[10+5]

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION: (2014-2015)

MSQE I and M.Stat II

Microeconomic Theory II

Date: 04/05/2015   Maximum marks: 60+10   Duration: 3 Hours

Note: Answer **all** questions.

Note: Let $\mathbb{R}^\ell$ denote the ℓ-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Answer **all** questions.

(i) Let $\mathbb{R}_+$ denote the domain of wealth and $N = \{1, 2\}$ denote the set of agents. Suppose that $U_i : \mathbb{R}_+ \to \mathbb{R}$ be a strictly monotonically increasing and twice differentiable function for $i \in N$. If agent 1 is more risk averse than agent 2, then show that there is a strictly increasing and concave function $V : Im(U_2) \to \mathbb{R}$ such that $U_1 = V \circ U_2$, where $Im(U_2) = \{U_2(x) : x \in \mathbb{R}_+\}$ is the image of U_2.

(ii) Consider the utility function $U : \mathbb{R}_+ \to \mathbb{R}$ defined by $U(x) = x^2$. Find the certainty equivalence and the probability premium for lotteries

$$L_1 = \left[\frac{1}{2}(25), \frac{1}{2}(5)\right] \text{ and } L_2 = \left[\frac{1}{2}(36), \frac{1}{2}(16)\right].$$

Compare the probability premium of L_1 and L_2. [6+4]

Q2. Consider an economy $\mathscr{E} = \{I; \mathbb{R}^\ell_+; (\succsim_i, \omega_i)_{i \in I}\}$, where I is the set of agents containing m many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succsim_i$ and ω_i are the preference and initial endowment of agent i, respectively. Suppose further that $\succ_i$ and $\sim_i$ are the strict preference and indifference relations associated with a rational preference relation $\succsim_i$ for all $i \in I$. A price is an element of $\mathbb{R}^\ell \setminus \{0\}$. Assume

$\mathscr{W}(\mathscr{E})$: the set of Walrasian equilibrium allocations of $\mathscr{E}$;

$\mathscr{C}(\mathscr{E})$: the core of $\mathscr{E}$;

$\mathscr{I}(\mathscr{E})$: the set of individually rational allocations of $\mathscr{E}$;

$\mathscr{P}(\mathscr{E})$: the set of Pareto optimal allocations of $\mathscr{E}$.

(i) Suppose that $\succeq_i$ is continuous and strictly monotone for all $i \in N$, and $\sum_{i \in I} \omega_i \in \mathbb{R}^\ell_{++}$. Show that every quasiequilibrium allocation is a Walrasian equilibrium allocation. [10]

(ii) Suppose that $\omega_i = \omega_j$ and $\succeq_i = \succeq_j$ for all $i, j \in N$. Let $\succeq_i$ be continuous, strictly convex and strictly monotone for $i \in N$. Show that $\mathscr{W}(\mathscr{E}) = \mathscr{I}(\mathscr{E}) = \mathscr{C}(\mathscr{E})$. Further, show that if $(x_1, x_2, \cdots, x_m) \in \mathscr{C}(\mathscr{E})$, then $x_i \sim \omega_i$ for all $i \in I$. [10]

(iii) Let $I = I_1 \cup I_2$ and $I_1 \cap I_2 = \emptyset$, where each I_i has at least two agents. Put

$$\mathscr{E}_1 = \left\{I_1; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in I_1}\right\} \text{ and } \mathscr{E}_2 = \left\{I_2; \mathbb{R}^\ell_+ : (\succeq_i, \omega_i)_{i \in I_2}\right\}.$$

Suppose that $\bar{y} = (y_i : i \in I_1)$ and $\bar{z} = (z_i : i \in I_2)$ are Walrasian equilibrium allocations of $\mathscr{E}_1$ and $\mathscr{E}_2$, respectively. Show that $(\bar{y}, \bar{z})$ is an allocation of $\mathscr{E}$. Prove or disprove $(\bar{y}, \bar{z}) \in \mathscr{C}(\mathscr{E})$. [8]

(iv) Assume that $N = \{1, 2\}$ and $\ell = 2$. Suppose that $\succeq_i$ is represented by a utility function U_i for $i = 1, 2$. Let

$$\begin{cases} \omega_1 = (2, 8), & U_1(x, y) = \min\{2x, y\}; \\ \omega_2 = (6, 0), & U_2(x, y) = \min\{x, 3y\}. \end{cases}$$

Find the set of Walrasian equilibrium allocations of $\mathscr{E}$. [6]

(v) If $\succeq_i$ is strictly convex for all $i \in I$, then show that $\mathscr{W}(\mathscr{E}) \subseteq \mathscr{P}(\mathscr{E})$. [6]

(vi) If $\succeq_i$ is continuous and monotone for all $i \in I$, then show that $\mathscr{P}(\mathscr{E})$ is a compact subset of $\mathbb{R}^{m\ell}$. [10]

Q3. Recall that a *production set* is a non-empty closed convex subset Y of $\mathbb{R}^\ell$ such that $Y \cap \mathbb{R}^\ell_+ = \{0\}$ and there is some $a \in \mathbb{R}^\ell_+$ such that $y \leq a$ for all $y \in Y$. Show that for any $p \in \mathbb{R}^\ell_{++}$, there is some $y_0 \in Y$ such that $p \cdot y \leq p \cdot y_0$ for all $y \in Y$. Does the same conclusion hold if $p \in \mathbb{R}^\ell_+ \setminus \{0\}$? [10]

INDIAN STATISTICAL INSTITUTE
Semestral Examination: 2014-2015
MS (Q.E.) I Year
Time Series Analysis and Forecasting

Date: $8-th$ May, 2015 Maximum Marks 50
Duration 2 hours

All notations are self-explanatory. This question paper carries a total of 50 marks. Marks allotted to each question are given within parentheses.

1. Consider the following autoregressive process:
$W_t = \rho W_{t-1} + X_t, \quad \rho < 1, t = 1,2\ldots,T$, where $\{X_t, t = 1,2\ldots,\}$ is a sequence of $i.i.d$ random variables with mean zero and with positive variance σ^2. Assume that $\{X_t, t = 1,2\ldots,\}$ has finite fourth moments.

   (a) Propose a consistent estimator of ρ. Prove that your proposed estimator is consistent.
   (b) Show that your proposed estimator is asymptotically normal.
   (c) How will you test for $H_0 : \rho = 1$ against $H_0 : \rho < 1$? Derive analytically the asymptotic distribution of your test statistic.
   [10+15+18 =43]

2. Consider the following time series process:
$Y_t = \sum_{i=1}^{k}\{A_i sin(t) + B_i cos(t) + \epsilon_{it}\}$, where A_i, B_i and ϵ_{it} are $i.i.d$ random variables with mean zero and with positive variance σ^2. All these random variables are independent to each other. Examine if the time series process is stationary or not. [7]

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2014-2015
M.S. (Q.E.) I
Theory of Finance I

Date: May 11 2015 | Maximum Marks: 100 | Time: 3 hours

Note: Clearly explain the symbols you use and state all the assumptions you need for any derivation. The paper carries 103 marks. The maximum you can score is 100.

1. When do you say that a future liability can be perfectly immunized by a bond with respect to interest rate risk? Clearly develop a necessary and sufficient condition for this to hold. (10)

2. (a) Determine the payoff function of butterfly-price spread strategy and plot the graph of this function. (6)

   (b) Repeat the above steps for a bottom straddle. Make a systematic comparison between the two hedging policies. (6+3)

3. State the First Fundamental Theorem of Mathematical Finance. Demonstrate the theorem rigorously by proving all necessary preliminaries. (18)

4. Determine the Black-Scholes pricing formula analytically for a European call option. You must prove any result that you use here. Clearly explain why the Vega of a European call option is positive. (17+3)

5. Demonstrate the von-Neumann utility equivalence of second order stochastic dominance relation. Is your ordering complete and transitive? Give reasons in support of your answer. (12+3)

6. Identify a sufficient condition for the Arrow-Pratt absolute measure of risk aversion to be decreasing in a two-asset self- financed portfolio in a two-period structure, given that one asset is risk-free. (10)

7. Show that given the state-contingent probabilities, the certainty equivalent is an increasing and continuous function of state-contingent outputs. (6)

8. In a two-asset minimum-risk portfolio with only risky assets, identify the unique condition under which there is no short-selling, given that the assets are not perfectly correlated (9)

INDIAN STATISTICAL INSTITUTE

Second Semester Examination : (2014 - 2015)

MS(QE) I Year

Environmental Economics

Date : May 11, 2015 **Maximum Marks : 100** **Duration : 3 hrs.**

Group A
Answer any three.

1. (a) 'The Coase Theorem tells us that there is no reason to regulate externalities as long as property rights are clearly defined.' Explain with an example.
(b). Suppose there is a paper mill by the side of a river discharging waste water into the river. In the downstream, there is a fishery whose production is affected by the polluted river water. Suppose the cost function of producing the quantity of paper P is $C_p = P^2 + 8$ and that of producing fish product F is $C_F(F, P) = F^2 + FP + 4$. The per unit prices of fish and paper are Rs. 12.0 and Rs. 14.0, respectively. What will be the production and profit of each firm if
(i) two firms are independent,
(ii) two firms are merged,
(iii) paper mill has the right to pollute and
(iv) fishery has the right to get clean water.

[15+10=25]

2. Consider a non-renewable resource, say coal, whose stock is given at X_0. Let p_t and q_t be respectively the price and quantity of extraction in period t. Assume constant marginal cost of extraction and the rate of interest to remain fixed at r for all periods. Consider two types of firms, viz., competitive and monopoly.

(a) State and derive the optimal extraction policy of each type of firms.

(b) Assume a linear (inverse) demand curve: $p_t = p^b - \beta q_t$ where β is a positive constant and p^b is the back stop price.

(i) Starting from 0^{th} period what will be the optimal time (T) when the firms will stop extraction?
(ii) Compare the results graphically.

[12+8+5=25]

3. In a single species competitive fishery the initial stock of fish is X_0 and production function is $q = \theta XE$, where q is the harvest rate, θ is a positive constant and E is effort involved in catching fish. The simple cost function is $C(X,q) = c(X)q =$ wE, where $c(X)$ is average cost and w is the cost per unit of effort. The biological growth function is given by $g(X) = \gamma X(1-\frac{X}{K})$, where K is carrying capacity level and γ is a positive constant. The firm's objective is to maximize present value of net benefit over an infinite time horizon when discount rate is r. Competitive price is p which is constant.

(a) Determine the 'Fundamental Rule ' for the management of fishery.

(b) Determine the optimal harvest rate, optimal stock and optimal effort level in this dynamic model for a sole ownership firm.

[15+10=25]

4. (i). Explain why the forest is considered as a separate type of renewable resource.
(ii). Suppose, the 'best next use' of land is rotational forestry for an infinite period of time. (a) Find the condition for determining the optimal rotation period (years) of a forest, when the owner of the forest considers only the timber supply and follows the even-aged management strategy. (b) Explain the role of discount rate (rate of interest), stumpage price and establishment cost in determining the optimal rotation age and supply of timber in the market?

[5+10+10=25]

5. Develop a methodology to measure the benefits from protecting East Calcutta Wetland, the services of which are not sold in the market. Outline the theoretical basis for developing the methodology.

[20+5=25]

Group B

1. Explain any of the following:
(a) Unit Cost method of measuring resource scarcity.
(b) Environmentally adjusted Net National Product (ENP).

[15].

2. Home assignment.

[10]

INDIAN STATISTICAL INSTITUTE

BACK PAPER EXAMINATION: (2014-2015)

MSQE I and M.Stat II

Microeconomic Theory II

Date: 13.07.15 Maximum marks: 100 Duration: 3 Hours

Note: Answer **all** questions.

Note: Let $\mathbb{R}^\ell$ denote the ℓ-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $\mathbb{R}_+$ denote the domain of wealth and U be a twice-differentiable strictly increasing utility function for a decision maker. For any fixed amount of money ω and positive number ε, the probability premium is denoted by $\pi(\omega, \varepsilon, U)$ and the certainty equivalence for a lottery L is denoted by $CE(L, U)$. Show that the following properties are equivalent:

(i) The decision maker is risk lover.

(ii) $\pi(w, \varepsilon, U) \leq 0$ for all $w \geq 0$ and all $\varepsilon > 0$ with $\omega - \varepsilon \geq 0$.

(iii) $CE(L, U) \geq \mathbb{E}(L)$ for all lottery L. [15]

Q2. Consider an economy $\mathscr{E} = \{N; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in N}\}$, where N is the set of agents containing n many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succeq_i$ and ω_i are a rational preference and an initial endowment of agent i, respectively.

(i) If $\succeq_i$ is continuous, convex and strictly monotone for all $i \in N$, then show that an allocation is a Walsian equilibrium allocation if and only if it is an Edgeworth equilibrium. [20]

(ii) Let $N = \{1, 2\}$ and $\ell = 2$. Suppose that the preference relation $\succ_i$ is represented by a utility function U_i for $i = 1, 2$. Given that

$$\begin{cases} \omega_1 = (2, 1), & U_1(x, y) = (y + 1)e^x; \\ \omega_2 = (2, 3), & U_2(x, y) = xy. \end{cases}$$

Find the set of Walrasian equilibrium of $\mathscr{E}$. [10]

(iii) Suppose that $\mathscr{E}_r$ is the r-fold replicated economy of $\mathscr{E}$ and $\succ_i$ is continuous, convex and strictly monotone for all $i \in N$. Let

$$x = (x_{11}, \cdots, x_{1r}, x_{21}, \cdots, x_{2r}, \cdots, x_{n1}, \cdots, x_{nr})$$

be a core allocation of $\mathscr{E}_r$ for some $r \geq 1$. Show that the allocation $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$ of $\mathscr{E}$, defined by

$$\hat{x}_i = \frac{1}{r}\sum_{j=1}^{r} x_{ij},$$

is a core allocation of $\mathscr{E}$. [15]

(iv) If $\{p_k : k \geq 1\} \subseteq \mathbb{R}^{\ell}_{++}$ satisfies $p_k \to p \in \mathbb{R}^{\ell}_{++}$, then show that there exists a bounded subset B_i of $\mathbb{R}^{\ell}_{+}$ such that the demand set $D_i(p_k, \omega_i, \succeq_i) \subseteq B_i$ holds for each $i \in N$ and $k \geq 1$. [15]

(v) If an allocation is both a Walrasian equilibrium allocation and a quasiequilibrium allocation, then show that it is a Pareto optimal allocation. [10]

(vi) Let $\succeq_i$ be continuous and strictly monotone for all $i \in N$. Show that an allocation $(x_1, \cdots, x_n)$ is a core allocation if and only if there are no coalition S and a set of vectors $\{y_i \in \mathbb{R}^{\ell}_{+} : i \in S\}$ such that

$$\sum_{i \in S} y_i \leq \sum_{i \in S} \omega_i$$

and $y_i \succeq_i x_i$ for all $i \in S$ and $y_i \succ_i x_i$ for some $i \in S$. [15]

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M.S.(Q.E.) 1st Year (2014 – 15)

Semester II Examination (BACK PAPER)

Econometric Methods I

Date: 16.07.15 | Maximum Marks: 100 | Time: 3 hours

Answer **all** questions

[*Marks allotted to questions are given within parentheses*]

1. (a) Discuss the importance of R^2 in a fitted regression. Why is $\bar{R}^2$ also considered? Give explanations for your answer.

   (b) State and prove the Frisch-Waugh-Lovell theorem for the standard multiple regression model.

   [8+12 = 20]

2. (a) Describe the Breusch-Pagan test for heteroscedasticity.

   (b) Consider estimating a two-variable regression model with autocorrelated disturbances as follows:

   $$y_t = \alpha + \beta x_t + \varepsilon_t, \; t = 1,2,\ldots,n$$

   and $$\varepsilon_t = \delta + \phi \varepsilon_{t-1} + a_t, \quad a_t \sim WN\,(0,\sigma^2).$$

   Discuss how you would obtain an efficient estimator of β. Justify your claim that the estimator is efficient.

   [8+12 = 20]

3. (a) Obtain the reduced-form equations from the simultaneous structural equations system, and then argue why, unlike the latter, the reduced-form equations has no problem in estimating its parameters by the least squares method.

   (b) Suppose an investigator is asked to work with the following two-equation system:

   $$r_{11} Q_t + r_{21} P_t + \beta_{11} x_{1t} + \beta_{21} x_{2t} = u_{1t}$$
   $$r_{12} Q_t + r_{22} P_t + \beta_{12} x_{1t} + \beta_{22} x_{2t} = u_{2t}\,.$$

   (i) Discuss the identification problem of this system of equations.

   (ii) If it is given *a priori* that $\beta_{21} = \beta_{22} = 0$, discuss the identification status of each equation under these restrictions. If the system is not identified, modify your restrictions suitably to make the system identifiable.

   (iii) For your identified model, which estimation method would you suggest? Give justification in support of your answer.

   [10+10 = 20]

4. (a) Discuss briefly the errors-in-variables problem in econometrics as well as its solution.

   (b) Consider a standard two-variable regression model where the regressor is subject to measurement error. Using an appropriate test procedure, derive a test statistic for testing the null hypothesis that the regression is indeed an errors-in-variables model.

   [10+10 = 20]

5. Discuss the limitations of linear probability model. Describe a method of estimation of the Logit model based on single observation on the dependent variable for each setting of the independent variables.

   [8+12 = 20]