ESTIMATION OF ALL-INDIA BIRTH AND DEATH RATES, 1941-50

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SUMMARY. The paper describes variants of the differencing and reverse survival methods, where with the knowledge of the correct population of India by sex and age at the 1941 or 1951 census, as the case may be, and applying on it an appropriate life table for the period, 1941–50 and for each sex, the All-India birth and death rates, 1941–50 are derived together, without having to obtain one of them first and then the other through the observed growth rate from the 1941 and 1951 census populations. The variant of the differencing method makes use of the 1941 age table population and that of the reverse survival method of the 1951 age table population, these populations being the respective census populations by sex and age, corrected for age bias except in the age-group, 0–4, but not corrected for under-reporting, which is heavy at the young ages. The respective age table populations are corrected, by adjustment for under-reporting at the young ages only, and by different methods in the two cases, the errors at the other ages together with the residual errors at the young ages being considered not large enough to affect the age-structures and hence the derived rates materially. The life tables used are (i) *All-India Life Tables*, 1941–50 (Males and Females) and (ii) *U.N. Model Life Tables*, No. 31 (Females) and No. 30 (Males), which are considered close to the Indian situation, 1941–50. The All-India birth and death rates, 1941–50 (means of the respective rates by both methods) are 42.5 and 31.0 respectively by the *All-India Life Tables*, and 42.1 and 30.4 respectively by the *U.N. Model Life Tables*.

1. There seems to be an element of uncertainty about the birth and death rates of the various States of India during 1941-50, as would appear from the "combined final review of all available material" and an attempt at reconciliation between the estimates from different sources.[1] This uncertainty would naturally linger on the All-India rates also, if obtained as the weighted mean of the finally adopted rates for the States.[2] For, if for a decennium $(GR)_s$, $(FMR)_s$, $(BR)_s$ and $(DR)_s$ are respectively the observed growth rate, the fresh migration rate (estimated), the birth rate and death rate for a State, we have

$$(GR)_s - (FMR)_s = (BR)_s - (DR)_s \qquad \ldots \quad (1)$$

where $(BR)_s$ (say) has been estimated first by the reverse survival method and $(DR)_s$ obtained as the balancing item in (1), and

$$\Sigma\, W_s.(GR)_s - \Sigma W_s.(FMR)_s = \Sigma\, W_s.(BR)_s - \Sigma W_s.(DR)_s,$$

where W_s is the mean population of the State during the decennium and the summation covers all the States. Since $\Sigma W_s.(FMR)_s = 0$, fresh migration for All-India being assumed nil,

$$\Sigma\, W_s.(GR)_s = \Sigma\, W_s.(BR)_s - \Sigma W_s.(DR)_s.$$

Dividing by the mean population of All-India during the decennium, we have

$$(GR) = (BR) - (DR)$$

where (GR) is necessarily the observed growth rate for All-India, and (BR) and (DR) the All-India birth and death rates computed respectively from the State birth and death rates.

[1] Estimation of birth and death rates of India during 1941–50, *Census of India*, 1951, Paper No. 6 of 1954, Government of India, pp. 11–18.

[2] *ibid.*, Table 7, p. 18.

Thus, for All-India, the excess of the computed birth rate over the computed death rate must exactly reproduce the observed growth rate. This does not mean however that the computed birth and death rates are individually correct. For if there had been an error in estimating $(BR)_s$, $(DR)_s$ obtained as indicated above, would also be subject to the same error in sign and magnitude, so that the sum of the weighted errors (which need not be zero) would be the same in sign and magnitude in both $\Sigma W_s.(BR)_s$ and $\Sigma W_s.(DR)_s$. In this paper, an attempt has been made to estimate the All-India birth and death rates, 1941-50, directly on the basis of All-India census figures, where using the *All-India Life Tables*, 1941-50 and variants of the differencing and reverse survival methods, the birth and death rates are derived together, without having to obtain, as indicated above, one of them first, and then the other through the observed growth rate.

2. The application of the usual forms of these methods on a population free from migration rests on the following requirements :

(i) The total populations at two consecutive decennial censuses as also the population up to 10 years of age at the latter census are correctly known for each sex.

(ii) A life table (for each sex) representing the mortality experience of the population between ages 0 and 10 is available, as essential for the working of the reverse survival method,[3] and also for obtaining the corrections to be applied to deaths at ages 5 and above in the differencing method.[4]

(iii) The ratio of total deaths to those at ages 5 and above during the decennium is available or can be estimated, in the differencing method.

It will be noticed that unless (i) is satisfied, we cannot say that the population at the earlier census is reduced by mortality to *exactly* that at ages 10 and above at the next census (as required for the differencing method), nor that the population at the next census at ages between 0 and 10 years are the survivors of *all* the births that occurred during the intercensal period (as required for the reverse survival method).

Turning to the case of India (for which net migration during the decennium, 1941-51 can be ignored for all practical purposes,[5] we have the age tables both for 1941 and 1951 censuses, which have been adjusted for age bias, but the total populations remaining unaltered after adjustment, the tables are not free from under-reporting, which apart from other age-groups possibly affected, is noticeable in the age-group, 15-24[7] and substantial in the young age-groups. Furthermore, population in the age-group, 0-4 (for both sexes and both in 1941 and 1951) is left unadjusted and is merely that given by the census enumerations. The age table populations therefore do not satisfy requirement (i) mentioned above, so that taken in conjunction, they would not yield the *correct* growth rate in the first instance, nor could they be used to find the *correct* death rate by the usual differencing method and thence the birth rate by addition to the growth rate, or the *correct* birth rate by the usual reverse survival method and thence the death rate by subtraction of the growth rate from the birth rate.

[3] *ibid.*, p. 39.
[4] *ibid.*, p. 34.
[5] *ibid.*, pp. 12-13.
[6] Age tables, *Census of India* 1951, Paper No. 3 of 1954, Government of India.
[7] Life Tables, *Census of India* 1951, Paper No. 2 of 1954. Government of India, 15-17.

3. In the modified versions of these methods, referred to earlier, our requirements are (i) the correct population at *only* one end and (ii) a life table representing the mortality experience of the population during the period. In the present case, these are provided by the 1941 or 1951 age table population with adjustment for under-reporting at the early ages (the errors at the other ages together with that at the young ages even after adjustment being considered not large enough to affect the age-structure and hence the derived rates materially), and the *All-India Life Tables*, 1941-50. The adjustment of the population at one end with application of the life table thereon, in connection with the differencing method (the theory of which is given in Section 5) is described in Section 4, and that in connection with the reverse survival method (the theory of which is given in Section 7) is described in Section 6. The adjustment of the population is however made by different methods in the two cases.

4. Starting with the Age Table population, 1941, we apply the survival factors from the *All-India Life Tables*, 1941-50 to arrive at the populations in the corresponding higher age-groups in 1951, as in Table 1 given below. The adjustment of the population, 0-4 in 1941 consists in replacing it by the one derived from the age table population, 10-14 in 1951 (which by its excess over the projected population in the same age-group, points to a substantial undercount in the age-group, 0-4 in 1941), by the reverse application of the appropriate survival factors, and this would be a practical and perhaps a close estimate of the true population, 0-4 in 1941. The altered values and the altered totals, used in the application of the method, are entered in brackets at appropriate places in Table 1.

5. *Theory of the differencing method.* The variant of the differencing method applied in this paper has the feature that the decennial deaths and births are derived together, whereas in the usual method, it is the decennial deaths that are derived. Further, the ratio of the decennial deaths at all ages to those at ages 5 and above (see Section 2(iii)) is derived from the data and has not to be estimated from other sources. The following symbols are defined :

$P^{y(m/f)}_{x+}$, $P^{y(m/f)}_{x/x+z}$ are respectively the populations (male/female) aged x and above and in the age-group, x to $x+z$, in the year y.

$D^{y/y+n(m/f)}_{x+}$, $D^{y/y+n(m/f)}_{x/x+z}$ are respectively deaths at ages x and over, and between ages x and $x+z$ of males/females, during the period, year y to year $y+n$.

In the above, for persons, (m/f) will be omitted.

$b_{m/f}$ = average annual number of male/female births, during 1941/1946,

$1+s$ = sex ratio at birth = ratio of male births to female births,

$1+k = D^{41/51}_{0+}/D^{41/51}_{5+}$, writing 41 and 51 for 1941 and 1951 respectively,

$1+r$ = mean $P^{Q}_{10/14}$ (during 1946/1951)/mean $P^{Q}_{10/14}$ (during 1941/1946),

$l^{(m/f)}_x$, $L^{(m/f)}_x$ are life table functions for males/females, at age x, and ${}_4L^{(m/f)}_x = \sum_{t=0}^{4} L^{(m/f)}_{x+t}$.

Determination of $D^{41/51}_{0+}$ and b_f. We have,

$$D^{41/51}_{0+} = P^{41}_{0+} - P^{51}_{10+},$$

as a first approximation $= a_{..}$

TABLE 1 : PROJECTION OF 1941 POPULATION† TO 1951 BY SEX AND AGE-GROUPS

age-group	population 1931† (00's)	adjusted population 1941 (multiplying factor = 1.056065)*	projected population 1946	projected population 1951
(1)	(2)	(3)	(4)	(5)
		All-India : males		
0— 4	202,224	213,562 (254,441)		
5— 9	208,762	220,466	187,819 (223,771)	
10—14	173,045	182,747	205,906	175,415 (208,993)**
15—19	145,230	153,372	172,944	194,861
20—24	132,165	139,575	145,467	164,030
25—29	127,091	134,216	131,489	137,040
30—34	116,939	123,495	125,132	122,590
35—39	100,307	105,931	113,509	115,014
40—44	84,283	89,008	95,793	102,646
45—49	70,034	73,960	78,670	84,667
50—54	55,712	58,835	63,001	67,013
55—59	41,991	44,345	47,613	50,984
60—64	29,738	31,405	33,322	35,777
65—69	18,559	19,600	21,270	22,569
70—	26,058	27,519	23,728	22,660
		1,618,036	1,445,663	1,295,266
		(1,658,915)	(1,481,615)	(1,328,844)
		(All-India : females)		
0— 4	203,114	214,502 (246,413)		
5— 9	197,324	208,387	182,726 (209,900)	
10—14	156,271	165,032	194,192	170,279 (195,610)**
15—19	134,751	142,306	157,724	185,593
20—24	130,298	137,603	136,709	141,521
25—29	125,116	132,131	130,144	129,298
30—34	110,288	116,450	121,816	119,984
35—39	91,707	96,849	104,852	109,683
40—44	75,999	80,260	85,785	92,874
45—49	62,891	66,417	69,979	74,797
50—54	50,340	53,162	56,745	59,789
55—59	39,251	41,452	44,037	47,005
60—64	29,209	30,847	32,386	34,406
65—69	18,771	19,823	22,100	23,202
70—	23,112	24,408	24,314	25,514
		1,529,629	1,363,509	1,223,945
		(1,561,540)	(1,390,683)	(1,249,276)

* Ratio of the final population total (both sexes combined c.f. Final Population Totals, *Census of India* 1951, Paper No. 1 of 1952), to the total from the Age-Tables.

** The figures within brackets is the population, 10–14 in 1951 (see footnote 6).

† see footnote 6.

This however *includes* deaths between 0 and 5 eyars, arising out of $P^{41}_{0/4}$ which for males

$$= P^{41(m)}_{0/4}.(1-5{}_5^{(m)}/_5L_0^{(m)}) = \beta_m,$$

and for females $= P^{41(f)}_{0/4}.$

(a similar expression involving female functions) $= \beta_f,$

and for males and females together $= \beta_m+\beta_f = \beta,$

and *excludes* deaths between 5 and 10 years arising out of births during 1941/1946, which for females

$$= b_f.{}_5L_0^{(f)}/1_0^{(f)}.(5{}_0^{(f)}/_5L_0^{(f)}-1)$$

and for males $= (1+s).b_f.$

(a similar expression involving male functions), and for males and females together

$$= \gamma.b_f,$$

taking $s = .05$. Subtracting β and adding $\gamma.b_f$,

$$D^{41/51}_{5+} = \alpha-\beta+\gamma.b_f, \qquad \ldots (1)$$

where $\alpha, \beta,$ and γ are known constants.

Again, $D^{41/50(m)}_{5+} = P^{41(m)}_{5+}-P^{50(m)}_{5+}-\beta_m = \delta_1^{(m)}.$

Taking the mean $P^{(m)}_{5+}$ during 1941/1946 $= \frac{1}{2}(P^{41(m)}_{5+}+P^{50(m)}_{5+})$ and the mean $P^{(m)}_{5+}$ during 1941/1951 $= P^{50(m)}_{5+}$, the former is increased to the latter in the ratio,

$$P^{50(m)}_{5+}.2/(P^{41(m)}_{5+}+P^{50(m)}_{5+}) = i_m.$$

Hence,

$$D^{41/51(m)}_{5+} = \delta_1^{(m)}.i_m.2,$$

since the period, 1941/1951 is double the period, 1941/1946. Similarly,

$$D^{41/51(f)}_{5+} = \delta_1^{(f)}.i_f.2.$$

Hence $D^{41/51}_{5+} = 2(\delta_1^{(m)}.i_m+\delta_1^{(f)}.i_f) = \delta_1.$

Alternatively, for each sex we can obtain the group values of the central mortality rates from the life tables, and assuming the population in 1946 to be the mean population of the decennial period, 1941/1941 in each age-group, to which 10 times the group central mortality rate is multiplied we have, $D^{41/51}_{5+} = \delta_2$. Taking the mean of the two values,

$$D^{41/51}_{5+} = \tfrac{1}{2}(\delta_1+\delta_2) = \delta. \qquad \ldots (2)$$

From (1) and (2), $\alpha-\beta+\gamma.b_f = \delta.$

Therefore $b_f = (\delta-\alpha+\beta)/\gamma$

and is thus determined. Thus the total births during 1941/1951

$$= 5.b_f(2+r)+5.(1+s).b_f(2+r) = 5.b_f(2+r).(2+s) = B,$$

r being obtained from the data and s taken as .05.

Determination of k and $D^{41/51}_{0+}$. We have,

$$D^{41/51}_{0+} = (1+k).D^{41/51}_{5+},$$

so that $D^{41/51}_{0+5} = k.D^{41/51}_{5+} = k.\delta,$ and $D^{41/51}_{0/5}$

out of births only during 1941/1951

$$= k.\delta - \beta,$$

but this is also

$$= (1+s).b_f.[(5-{}_5L_0^{(m)}/l_0^{(m)})+(1+r).(5-{}_5L_0^{(m)}/l_0^{(m)})+{}_5L_0^{(m)}/l_0^{(m)}.(1-l_5^{(m)}/{}_5L_0^{(m)})]+b_f.$$

(a similar expression involving female functions)

$$= \mu.b_f,$$

where μ is a known constant (taking $s = .05$). Therefore $k.\delta - \beta = \mu.b_f$, whence $k = (\mu.b_f+\beta)/\delta$ and is thus determined.

Finally, $D_{0+}^{41/51} = (1+k).\delta = D$. The increase in the total population from 1941 to 1951 $= B-D = I$, so that $P_{0+}^{51} = P_{0+}^{41}+I$, and mean population between 1941 and 1951 $= \frac{1}{2}(P_{0+}^{41}+P_{0+}^{51}) = P$.

Therefore the decennial birth rate $= B/P.100\%$,

and the decennial death rate $= D/P.100\%$

the decennial rate of natural increase $= I/P.100\%$.

Further, average annual births during 1941/1946 $= b_f+(1+s).b_f = (2+s).b_f$ and mean $P_{15/44}^{(f)}$ during 1941/1946 $= \frac{1}{2}(P_{15/44}^{41(f)}+P_{15/44}^{46(f)}) = \bar{P}_{15/44}^{(f)}$.

Therefore the general fertility rate $= (2+s).b_f/\bar{P}_{15/44}^{(f)}.1000$.

Obviously, the same value of the general fertility rate will be obtained, if the second quinquennium, 1946/1951 is considered.

6. In applying the reverse survival method, we start with the age table population, 1951, in which, as indicated earlier, the population in the age-group, 0-4 has to be corrected for under-reporting, and if necessary, in the age-group, 5-9 as well. Further, to ensure that the population aged 10 and above in 1951 should be the survivors of the 1941 population, the procedure adopted here is to obtain P_{0+}^{41} by multiplying P_{0+}^{51} by the ratio of these quantities as derived in the application of the differencing method.

To obtain $P_{0/0}^{51}$ for any sex correctly, cumulative population figures are drawn up at quinquennial ages, from 70 to 0, and P_{x+}^{51}/P_{25+}^{51} denoted by F_x, where $F_{25} = 1$ and the series of F_x should be a smooth one, provided the recorded population figures are correct. Thus, if a smooth graph is drawn through the tabulated series of F_x, any deviations from the graph at individual points could be taken as a pointer to the corrections needed at those points. To obtain a better estimate of the corrections, the graphical values are adjusted by inspection of their third differences, to ensure further smoothness. It has been found that both in the male and female tables, F_0 is substantially in defecit of the adjusted graphical value (F_0' say) and there are small defecits also at ages 5 and 15 in the male table. As the tabular value, F_0 must have been depressed on account of under-reporting over the whole table, it might be thought that F_0' would be the value corrected for such underreporting but, the correct value is even higher than F_0', as would appear for reasons given below.

A similar method is applied to the adjusted age-distribution of the 1961 population[8] the total population, male or female, being increased by .68% to correct for total under-reporting.[9] The series of F_x (omitting F_0) is smooth except for a small excess at age 5, but F_0, which is now the correct value since underreporting has been fully allowed

[8] Age Tables, *Census of India* 1961, Paper No. 2 of 1963, Part II, p. 35.

[9] *Census of India*, 1931, Vol. I, Part II-A(i), p. 41.

for, is considerably in excess of F'_0, which, as we have seen, falls in line with the rest of the smooth series. There is thus an inherent "roughness" at age 0, i.e., a deviation from the usual run of values, and F_0/F'_0 can be called the "index of roughness" at age 0, as an inherent feature of the age-distribution of the Indian population. Further, as this index should not vary appreciably from decade to decade, in a continuing stable age-distribution as of the Indian population, the index derived from 1961 is applied to F'_0 in 1951 to get the correct value of F_0, from which and the value of F_{10} (not requiring adjustment in the present case), the correct population in the age-group, 0-9 in 1951 is obtained by differencing of the corresponding cumulative population figures and used in the application of the reverse survival method. It may be noted here that the value of the index is found to be 1.023 both for males and females. The age table population, 1951 in total and in the age-group, 0-9 (for each sex), and the corresponding corrected populations, estimated by the process described above, are shown in the following table.

TABLE 2. CORRECTED AND AGE TABLE POPULATION, 1951 (in 00's) IN TOTAL AND IN THE AGE-GROUP, 0-9, BY SEX

population, 1951	males		females	
	total	age-group 0-9	total	age-group 0-9
(1)	(2)	(3)	(4)	(5)
age table	1,832,867	471,035	1,735,120	460,570
corrected	1,892,390	530,558	1,789,163	514,613

7. *Theory of the reverse survival method.* It will be noted that in this method as applied in this paper the decennial births as well as the deaths are derived together, instead of the former alone as by the usual method.

If $b_{f/m}$ = average annual number of female/male births, during 1941/1951 (contrast with the definition in Section 5),

$$P^{51(f)}_{0/9} = b_{f\cdot 10} L^{(f)}_0 / l^{(f)}_0, \quad \text{and} \quad P^{51(m)}_{0/9} = (1+s).b_{f\cdot 10} L^{(m)}_0 / l^{(m)}_0$$

so that $$P^{51}_{0/9} = a.b_f,$$

where a is a known constant (taking $s = .05$) and $b_f = P^{51}_{0/9}/a$. Therefore total number of births (male and female) during 1941/1951

$$= 10 \times 2.05 \times b_f = B.$$

Now, increase in the population in the decennium

$$I = P^{51}_{0+} - P^{41}_{0+},$$

and the mean population during 1941/1951

$$= \tfrac{1}{2}(P^{41}_{0+} + P^{51}_{0+}) = P.$$

Also, deaths during 1941/1951 $= B - I = D$.

Therefore decennial birth rate $= B/P.100\%$,

decennial death rate $= D/P.100\%$

and decennial rate of natural increase $= I/P.100\%$.

Further, if $P^{41(f)}_{10/44}/P^{41(f)}_{0+} = t$ (from the basic populations in the differencing method), $P^{41(f)}_{10/44}$ (in the present method) may be taken as $P^{41(f)}_{0+}$ (estimated). t, so that mean $P^{(f)}_{10/44}$ during 1941/1951 $= \frac{1}{2}(P^{41(f)}_{10/44}$ (estimated)$+P^{61(f)}_{10/44}) = \bar{P}^{(f)}_{10/44}$. Then the general fertility rate $= (B/10)\bar{P}^{(f)}_{10/44} \times 1000$.

TABLE 3. ALL-INDIA RATES, 1941-50 BY THE TWO METHODS

method	birth rate (per mille per annum)	death rate (per mille per annum)	dicennial rate of natural increase (%)	general fertility rate (per 1000 females, 15/44) per annum
(1)	(2)	(3)	(4)	(5)
differencing	42.8	31.0	11.8	198
reverse survival	42.1	30.9	11.2	194
mean	42.5	31.0	11.5	196

It will be noticed in the above table that the death rate comes out to be almost identical by both methods. The birth rates also are close to one another, and so also the general fertility rates to which they correspond. We have thus tken the means from the above table as the estimates of the corresponding rates for All-India, 1941-50. It may be argued however that the use of the All-India life table values does not provide independent estimates of birth and death rates, since, the life tables had themselves been computed from census data by age differencing methods, and that the accuracy of the life table values and hence of the estimates of birth and death rates depends on the correctness of the adjusted age-distributions (in quinquennial groups) in the 1941 and 1951 censuses. A computation of life table values independently of the age distributions of the two censuses would meet these arguments and for this purpose, a selection of life tables which can be considered close to the Indian situation (1941-50) is made from the *U. N. Model Life Tables*, and additional estimates of birth and death rates obtained by employing the same methods, for comparison with those on the basis of the *All-India Life Tables*.

The selected Life Tables are No. 31 (Females) and No. 30 (Males)[10] which correspond to an infant mortality rate of about 235. This lies between the probable limits of 200 and 250, but there is no sufficient basis for arriving at a precise figure for the actual rate (Coale and Hoover, pp. 52-53). They also correspond to a higher value for the male expectation of life at birth, a feature peculiar to the Indian population, and the difference between the male and female values is of about the same magnitude as in the *All-India Life Tables*. Further, the value (for each sex) is somewhat smaller than in the *All-India Life Tables*, which is reasonable, as the infant mortality rates assumed are higher. The estimates of birth and death rates by the differencing method come out to be 41.8 and 31.1 respectively, and 42.4 and 29.7 respectively by the reverse survival method. Taking means, the birth, death and growth rates are 42.1, 30.4 and 11.7 respectively, which may be considered along with those obtained on the basis of the *All-India Life Tables*. It may be of interest to compare both these results with those obtained by quasi-stable population techniques, viz., 43.1 and 30.9 for the pre-1951 period (Coale and Hoover, Table 8, p. 44) and 43.9 and 30.8[11] or with those by the method discussed in Section 1 of this paper, viz., 39.9 and 27.4[12] as the birth and death rates respectively.

REFERENCE

COALE, A. J. and HOOVER, E. M. (1959): *Population Growth and Economic Development in Low-income Group Countries*. Oxford University Press.

[10] United Nations, ST/SOA/Series A/22, Model Life Tables for underdeveloped countries.

[11] Age Tables, *Census of India*, 1961, Paper No. 2 of 1963, Part I, Table 21, p. 58.

[12] Estimation of birth and death rates in India during 1941-50, Paper No. 6 of 1954, p. 39.

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