STATISTICAL ANALYSIS OF CROSS-BEDDING AZIMUTHS FROM THE KAMTHI FORMATION AROUND BHEEMARAM, PRANHITA—GODAVARI VALLEY[1]

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with an appendix

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SUMMARY. Azimuths of cross-bedding foresets having sense and magnitude can be treated as vectorial quantities. Conventional statistical tests being not applicable to those circularly distributed vectorial data, these azimuth readings have been analysed with the help of special tests developed for the purpose.

Resultant vectors of 1001 cross-bedding azimuths from the Upper Permian (or Lower Triassic) Kamthi rocks, believed to be of fluviatile origin, indicate a general northerly paleo-direction of sediment transport. High consistency ratio values for the cross-bedding readings from the point-bar and channel-bar deposits which constitute the Middle and Upper Kamthi units, indicate that the flow of the Kamthi river was predominantly unidirectional during the later phase of Kamthi sedimentation. In the Lower Kamthi, cross-bedding measurements were made in the sand lenses interpreted to be the cut-off meander channels of the river, which were preserved within the interchannel flood plain deposits. Comparatively higher dispersion of cross-bedding azimuths noted in this unit is apparently the result of river meander.

Vector resultants of cross-bedding azimuths in the Lower, Middle and Upper Kamthi units are 30°06′, 0°55′ and 342°27′ respectively. Statistical analysis reveals that the paleocurrents in the three Kamthi units were very significantly different. This leads to the conclusion that the northwesterly shift of the Kamthi paleocurrent with time, is a significant one.

1. Introduction

It is a common experience that a sand bed underlying a flowing sheet of water is often thrown into a series of ripples. Continuous erosion of materials on the troughs of these ripples and deposition of these materials on the steeper, lee-sides, cause the ripple patterns to migrate in the direction of the flowing water. Movements of these ripples in turn, transport the whole sheet of sand in the direction of the current. In ancient sandstones, records of such successive ripples are often found to be preserved in the form of layerings lying oblique to the principal surface of accumulation. These oblique layerings are commonly known as cross-beddings and the

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directions (azimuths[1]) of inclinations of these cross-bedding foresets provide clues to the paleo-direction of sediment transport.

Many local fluctuations in current directions are known to occur during sedimentation in a flowing river. Because of these fluctuations, it is felt that statistical averages of cross-bedding azimuths rather than the individual dips themselves are better representatives of the overall paleocurrent direction. Azimuths of cross-bedding foresets have sense as well as magnitude and can be treated as vectorial quantities. Analysis of these vectorial data, circularly distributed on all quadrants of compass dial, provide challenging problems to the statisticians since conventional statistical techniques, designed primarily for linearly distributed data, are not applicable to the circularly distributed measurements.

The present paper discusses the techniques adopted for analysis and interpretation of cross-bedding data from the Upper Permian (possibly extending to Lower Triassic) Kamthi formation around Bheemaram, in the Pranhita-Godavari valley, Andhra Pradesh. The paleocurrent system responsible for sedimentation in the ancient Kamthi river system has been worked out from this analysis.

Depositional environment of the Kamthi sediments, as deduced from a study of the lithology and primary sedimentary structures of these rocks, has been discussed elsewhere (Sengupta, 1966). The Kamthi formation of the area has been divided into three units on lithological basis. Profusely cross-bedded, coarse sands and gravels of the Upper and Middle Kamthi units mainly represent deposits of the point-bars and channel-bars of the Kamthi river. Massive, and generally structureless, sandy siltstones and fine grained sandstones of the Lower Kamthi represent inter-channel flood plain deposits. Lenses of cross-bedded sandstone found interbedded with the Lower Kamthi siltstones are interpreted to be remnants of the cut-off meander channels of the Kamthi river within the flood basin deposits.

2. Sampling

In an area of about 60 square miles covering the Kamthi outcrop near Bheemaram, 1052 cross-bedding azimuths have been measured (Sengupta, 1966). Of these measurements, only 1001 measurements have been used for statistical analysis. Readings lying within highly faulted areas have not been taken into consideration, since the original cross-bedding directions in these places were disturbed by later movements.

In this type of data collection, it is difficult to follow a rigid sampling procedure in the field, since sampling is guided mostly by the availability of suitable outcrops. The outcrops were sampled as encountered, and at the same time efforts

[1]Azimuths in this work refer to dip-directions of cross-bedding foresets measured clockwise from north (0°).

were made to collect a good number of samples within each square mile of the area covered. The Survey of India one-mile grids on topographic sheet 56 N/9 (scale : 1 inch to 1 mile) were used for this purpose and a pattern of grids was selected so as to cover the whole Kamthi outcrop. Within each square mile the cross-bedding azimuths were grouped and represented in the form of a circular histogram at the centre of the grid. It was later pointed out by Professor P. C. Mahalanobis that for proper statistical investigation, it is imperative to record the original (ungrouped) data within each grid.

It is known from statistical studies (Potter and Olson, 1954) that the variability of cross-bedding direction is usually greater between than within the sedimentation units. Following this observation, generally only one azimuth was measured from each cross-bedding set to obtain a statistically representative sample. Sampling was done in three stages. Initially, the whole area was covered by a few, widely spaced traverses. This was followed, during successive field seasons, by filling of gaps and collection of additional information in areas which proved to be of particular interest during reconnaissance survey.

3. Computation of resultant directions

It is well known that for cross-bedding azimuths circularly distributed on a compass-dial on either side of true north (360°), the usual method of arithmetic averaging leads to erroneous conclusions. Various graphical and mathematical techniques have been developed for obtaining the resultant direction of circularly distributed data (Reiche, 1938; Curray, 1956 and Pincus, 1956 in Potter and Pettijohn, 1963, p. 264). In the present work, following the procedure recommended by Potter and Pettijohn (1963) for grouped data, azimuths of the resultant vectors ($\hat{\gamma}$) have been determined by algebraic summation of the sines and cosines of the individual azimuths as follows :

$$V = \sum_{i=1}^{n} n_i \cos x_i \; ; \quad W = \sum_{i=1}^{n} n_i \sin x_i ; \quad \hat{\gamma} = \tan^{-1} W/V,$$

where, x_i = mid point azimuth of the i-th class interval,

n_i = number of observations in each class,

n = total number of observations in each square mile grid,

$\hat{\gamma}$ = azimuth of the resultant vector.

Table 1 and Figure 1 give the frequency distribution of the cross-bedding azimuths for the three Kamthi units (Upper, Middle and Lower).

TABLE 1. FREQUENCY DISTRIBUTIONS AND VECTOR RESULTANTS OF CROSS-BEDDING AZIMUTHS IN THE THREE KAMTHI UNITS

azimuth class interval	mid-point azimuth (x_i)	Lower Kamthi (n_i)	Middle Kamthi (n_i)	Upper Kamthi (n_i)
0°— 19°	9.5°	14	50	75
20°— 39°	29.5°	14	62	75
40°— 59°	49.5°	11	33	15
60°— 79°	69.5°	13	9	25
80°— 99°	89.5°	9	1	7
100°—119°	109.5°	16	3	3
120°—139°	129.5°	–	–	3
140°—159°	149.5°	4	–	–
160°—179°	169.5°	–	–	–
180°—199°	189.5°	3	–	–
200°—219°	209.5°	4	2	21
220°—239°	229.5°	–	8	8
240°—259°	249.5°	–	–	24
260°—279°	269.5°	–	11	16
280°—299°	289.5°	6	5	30
300°—319°	309.5°	7	20	75
320°—339°	329.5°	1	53	90
340°—359°	349.5°	21	41	107
total $N = 1001$		$N_1 = 123$	$N_2 = 298$	$N_3 = 580$
$W = \sum_{i=1}^{n} n_i \sin x_i$		$W_1 = 38.00125$	$W_2 = 3.54924$	$W_3 = -115.61271$
$V = \sum_{i=1}^{n} n_i \cos x_i$		$V_1 = 50.60586$	$V_2 = 220.20580$	$V_3 = 365.35602$
$R = \sqrt{W^2+V^2}$		$R_1 = 63.200$	$R_2 = 220.234$	$R_3 = 383.217$
$\hat{\gamma} = \tan^{-1} W/V$		$\hat{\gamma}_1 = 36°00'$	$\hat{\gamma}_2 = 0°55'$	$\hat{\gamma}_3 = 342°27'$

Variability or scatter within each grid can be represented as the magnitude or length of the resultant vector (R), where, $R = \sqrt{V^2+W^2}$. A useful measure of concentration of azimuths is the consistency ratio (R/n of Reiche, 1938, p. 913), expressed in terms of percent, i.e., $L = (R/n) \times 100$. "L" has been termed "vector magnitude" by Curray and, "vector strength" by Pincus (both quoted by Pelletier, 1958, p. 1044).

Length of the vector resultant (R), and consistency ratio in percent (L) for each square mile area of the Kamthi outcrop have been worked out and the results for the 60 square mile area covered are summarised in Table 2.

TABLE 2. RESULTS OF CROSS-BEDDING ANALYSIS

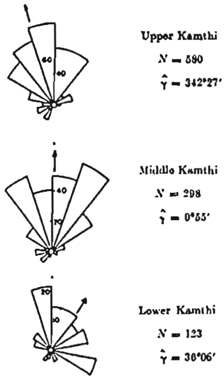
depositional environment and relative age	rock units	(N)	($\hat{\gamma}$)	(R)	($L = \frac{R}{N} \times 100$)
younger point-bars and channel-bars	Upper Kamthi	580	342°27′	383.21	66.07%
older point-bars and channel-bars	Middle Kamthi	298	0°55′	220.24	73.90%
cut-off meander channels within oldest flood plain deposits	Lower Kamthi	123	36°06′	63.21	51.30%

N = sample number

$\hat{\gamma}$ = azimuth of the resultant vector

R = length (magnitude) of the resultant vector

L = consistency ratio in percent



Figue 1. Distribution of cross-bedding azimuths in the three Kamthi units represented as circular histograms with 20° class-intervals. Dot indicates north. Arrow indicates direction of resultant vector.

4. Paleocurrent interpretation

The resultant vectors tabulated above indicate a general northerly direction of flow during Kamthi sedimentation. Within each Kamthi unit, however, certain amount of dispersion of current direction is noted around the grand vector resultant (Fig. 1). This dispersion is attributed to (i) local meanders in the stream channel, and (ii) variations in the direction of sediment transport within the meander bars. L-values for the Middle and Upper Kamthi units are fairly high (73.90% and 66.07% respectively), and it is concluded that the flow of the Kamthi river was essentially unidirectional during the later phase of Kamthi sedimentation.

In the Lower Kamthi sandstone lenses, which are believed to be remnants of meander cut-offs, a comparatively higher dispersion of current direction is indicated by the lower L-value (51.39%). Presumably, the arcuate current patterns of Lower Kamthi which show fairly larger dispersions, are the results of divergent directions of sedimentation on meander bars.

Statistical problem : Large dispersions of current directions and overlapping spreads of cross-bedding azimuths of the three Kamthi units lead to an interesting question, namely, whether the directions recorded from the three units actually belong to three significantly different populations. Confirmation of such a difference would obviously indicate that in spite of the overall northerly flow of the Kamthi river, the direction of sediment transport actually, significantly changed with time.

The problem of comparison of circularly distributed data belonging to two or more populations have led to many interesting discussions (Court, 1952, p. 75; Steinmetz, 1962; Potter and Pettijohn, 1963, p. 262; Krumbein and Graybill, 1965, p. 130), since Student's-t and similar conventional statistical tests are not valid for data having circular distribution. Under very specific conditions again, when the range of the angular data does not exceed 57° on either side of the vector mean, it has been shown that the normal approximation to circular distribution can be satisfactorily applied (Agterberg and Briggs, 1963, quoted by Krumbein and Graybill, 1965, p. 130). Spreads of azimuths of the Kamthi cross-beddings greatly exceed 57° on either side of the vector mean. Therefore normal approximation to circular distribution cannot be applied in the present case.

Statistical analysis : Under the hypothesis that the Kamthi data follow circular normal distribution, maximum likelihood estimates of γ and κ (where γ is the azimuth of the resultant vector and κ is a parameter measuring concentration in the circular normal distribution, see Appendix) were obtained as follows :

TABLE 3. ESTIMATES OF THE PARAMETERS OF THE CIRCULAR NORMAL DISTRIBUTION

rock units	sample size	$\hat{\gamma}$	$\hat{\kappa}$
Upper Kamthi	580	342°27′	1.7895
Middle Kamthi	208	0°55′	2.2893
Lower Kamthi	123	30°06′	1.1910

Using these estimates, circular normal distributions were fitted to the respective samples. The χ^2 goodness of fit, comparing the observed frequencies with expected, gave significant values in all the three cases, indicating that circular normal distribution does not adequately fit these data. For the Kamthi data, therefore, maximum likelihood estimates and the standard errors computed from the inverse of the information matrix and likelihood tests of homogeneity of parallel estimates are not applicable. For these data therefore, special large sample tests (see Appendix by J. S. Rao) which are applicable to any distribution, were used. Estimates of measures of location and concentration and their standard errors valid for any distribution are given in Table 4.

TABLE 4. ESTIMATES OF LOCATION AND DISPERSION MEASURES WITH STANDARD ERRORS

rock units	$T = \frac{W/N}{V/N}$	$U = \frac{V^2+W^2}{N^2}$	$\hat{\gamma}$ azimuth of resultant vector	(R/n) 100 consistency ratio in percent
Upper Kamthi	-0.3164 ± 0.0323	0.4305 ± 0.0322	$342°27' \pm 6°49'$	$66.07 \pm 4.74\%$
Middle Kamthi	0.0161 ± 0.0440	0.5462 ± 0.0318	$0°55' \pm 2°31'$	$73.90 \pm 2.15\%$
Lower Kamthi	0.7524 ± 0.1863	0.2640 ± 0.0487	$36°06' \pm 1°41'$	$51.39 \pm 2.44\%$

Note: V and W respectively refer to the *sums* of cos and sin values from the sample and X and Y of the Appendix refer to corresponding *means* i.e. $X = \frac{V}{N}$ and $Y = \frac{W}{N}$.

For testing equality of azimuths of resultant vectors and concentration ratios of the rock units, the homogeneity test H, as developed by Rao (1965) and explained in the appendix to this paper, was used. The statistic H, when computed separately for T and U values of the samples, gave significant results in both cases, showing that the population directions of cross-beddings as well as their dispersions are significantly different in the three Kamthi units.

5. Conclusions

Overall direction of sedimentation throughout the Kamthi time was northerly. During Middle Kamthi sedimentation, the gross direction of flow was essentially towards due north ($\hat{\gamma} = 0°55'$ for 298 observations) and during Upper Kamthi, this direction changed to north-northwest ($\hat{\gamma} = 342°27'$ for 580 observations). High consistency ratio values for the Middle and Upper Kamthi (73.90% and 66.07% respectively) indicate that the flow of the Kamthi river was predominantly unidirectional during the later phase of Kamthi sedimentation.

In the Lower Kamthi, 123 measurements were made in the sand lenses inferred to be the cut-off meander channels of the Kamthi river. These measurements indicate a northeasterly direction of flow ($\hat{\gamma} = 36°06'$). Comparatively higher dispersion ($L = 51.39\%$) noted in the Lower Kamthi measurements are presecumably the results of arcuate directions of flow at the river meanders and repeated oscillations of the river channel. It has been statistically proved that in spite of the repeated oscillations and many local changes, the westerly shift of the direction of sedimentation with time, within the Kamthi river (from northeasterly direction in Lower Kamthi to north-northwesterly direction during Upper Kamthi), is a significant one.

6. Acknowledgements

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The special problems involved in statistical analysis of circularly distributed data were pointed out by Dr. C. R. Rao and Dr. Jogabrata Roy. The tests used for comparison of polar vectors and dispersions were specially developed to suit the angular data by Mr. J. S. Rao, under the supervision of Dr. C. R. Rao. Mr. J. S. Rao is also responsible for the statistical analysis presented in Tables 3 and 4 and for the theoretical discussions in the Appendix. The author is grateful to Dr. C. R. Rao for taking very active interest in the work and also for critically reading the manuscript.

Appendix

LARGE SAMPLE TESTS FOR HOMOGENEITY OF ANGULAR DATA

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1. Introduction

Data on angles as random variables need special consideration since the usual linear distributions like the 'normal', do not, in general, provide the appropriate statistical models. So, certain special distributions have been developed for the purpose. One such important distribution is known as the 'circular normal distribution', which is essentially due to Fisher (1953), who developed the three-dimensional analogue of this. A random angle α, with reference to any arbitrary vector in two dimensions, is said to have a circular normal distribution, if it has the density

$$\frac{1}{2\pi I_0(\kappa)} \exp\left[\kappa \cos(\alpha-\gamma)\right] \qquad \ldots (1)$$

where γ is the population mean angle or the angle made by the polar vector, κ is a measure of concentration (a large value of κ means more concentration around the true direction, γ) and $I_0(\kappa)$ is a Bessel function of purely imaginary argument. A theoretical model which gives rise to the circular normal distribution is discussed in Rao (1965).

Suppose we have n observations $\alpha_1, \alpha_2, \ldots \alpha_n$ belonging to the circular normal distribution. Denoting by

$$X = \frac{1}{n}\sum_1^n \cos\alpha_i\ ,\ Y = \frac{1}{n}\sum_1^n \sin\alpha_i \quad \text{and} \quad \frac{R}{n} = \sqrt{X^2+Y^2},$$

it is known that the maximum likelihood estimates of the two parameters γ and κ, denoted by $\hat{\gamma}$ and $\hat{\kappa}$, are given by the equations

$$\hat{\gamma} = \tan^{-1}\left(\frac{Y}{X}\right) \qquad \ldots (2)$$

$$\frac{I_1(\hat{\kappa})}{I_0(\hat{\kappa})} = \frac{R}{n} \qquad \ldots (3)$$

Tables for obtaining $\hat{\kappa}$, for given $\frac{R}{n}$, are available in Gumbel, Greenwood and Durand (1953).

The quadrant in which $\hat{\gamma}$ of equation (2) lies is determined by the signs of X and Y.

Up to now, no satisfactory statistical procedures have been developed for the analysis of sample observations from a circular normal population. However, a number of approximate tests have been developed for different hypotheses regarding the parameters of this distribution by Watson (1956), Watson and Williams (1956) and Stephens (1962). Some further tests derived by the present author will appear elsewhere. In particular, tests for a given polar vector, for equality of dispersions or polar vectors in different populations are developed. These tests are not of general validity and the purpose of the present note is to show how a large sample test, called the H-test (homogeneity test ; see Rao, 1965) can be applied for testing the equality of dispersions or polar vectors, *even if the observations do not exactly conform to a circular normal distribution.*

2. Testing for equality of polar vectors

Suppose we have k populations on angular variables. Let X_i and Y_i denote the *means* of 'cos' values and 'sin' values from a sample of size n_i from the i-th population. Further let $S_{cc}^{(i)}$, $S_{ss}^{(i)}$ be the sample variances of the 'cos' and 'sin' values and $S_{cs}^{(i)}$, the sample covariance between the 'cos' and 'sin' values, from the i-th sample. A consistent estimator of $\tan \gamma_i$, where γ_i is the polar angle in the i-th population, is provided by

$$T_i = \frac{Y_i}{X_i} \qquad \dots \quad (4)$$

with the asymptotic estimated variance

$$s_i^2 = \frac{1}{n_i}\left\{ \frac{S_{ss}^{(i)}}{X_i^2} + \frac{Y_i^2 S_{cc}^{(i)}}{X_i^4} - \frac{2Y_i\, S_{cs}^{(i)}}{X_i^3} \right\}. \qquad \dots \quad (5)$$

It is easily seen that the asymptotic distribution of $\sqrt{n_i}(T_i - \tan \gamma_i)$ is normal. Now let us consider the hypothesis

$$H_0: \quad \tan\gamma_1 = \tan\gamma_2 \dots = \tan\gamma_k = \tan\gamma \quad (\gamma \text{ unknown}). \qquad \dots \quad (6)$$

Since $T_1, T_2, \dots, T_k$ are independent, consistent estimators of the same quantity viz., $\tan\gamma$, under the hypothesis, based on samples of sizes $n_1, n_2, \dots, n_k$ respectively, we can use the statistic (given on p. 323 of Rao, 1965)

$$H = \sum_1^k \frac{T^2}{s_i^2} - \left(\sum_1^k \frac{T_i}{s_i^2} \right)^2 \Big/ \left(\sum_1^k \frac{1}{s_i^2} \right). \qquad \dots \quad (7)$$

This statistic can be used to test the hypothesis H_0 since if each T_i does not estimate the same quantity, the differences are reflected in H. Under certain very general conditions, this statistic H has a χ^2 distribution with $(k-1)$ degrees of freedom, under the hypothesis H_0 (Rao, 1965).

Rejection of H_0 leads us to conclude that the polar vectors are different. But on the other hand, even if H_0 is not rejected, it is possible that the true directions are different because our hypothesis does not distinguish between the pole and the antipole, since $\tan\gamma = \tan(\pi+\gamma)$. But this is not a draw-back since such wide differences can easily be found out by a simple examination of the data. The procedure for applying this test is very simple and consists in getting the values of T_i from (4), s_i^2 from (5) and then computing H from (7).

3. Testing for equality of dispersions

Let us consider again the same set-up as in the earlier section, i.e., we have k populations with X_i and Y_i denoting the means of 'cos' values and 'sin' values from the i-th sample. Let $S_{cc}^{(i)}$, $S_{ss}^{(i)}$ and $S_{cs}^{(i)}$ also have the same significance as in Section 2. We define the statistic

$$U_i = X_i^2 + Y_i^2 \qquad \dots \quad (8)$$

which is easily seen to be a measure of concentration (the reciprocal of U_i would measure the dispersion) from the i-th population with the asymptotic estimated variance

$$s_i'^2 = \frac{4}{n_i}\left\{ X_i^2\, S_{cc}^{(i)} + Y_i^2\, S_{ss}^{(i)} + 2X_iY_i\, S_{cs}^{(i)} \right\}. \qquad \dots \quad (9)$$

Under the hypothesis of equal dispersions, all these statistics $U_1, U_2, \dots, U_k$ are independent and consistent estimators of the same quantity so that we can use the test criterion H for homogeneity. Compute

$$H = \sum_1^k \frac{U_i^2}{s_i'^2} - \left(\sum_1^k \frac{U_i}{s_i'^2} \right)^2 \Big/ \left(\sum_1^k \frac{1}{s_i'^2} \right) \qquad \dots \quad (10)$$

which is distributed as a χ^2 with $(k-1)$ d.f. under the hypothesis of equality of dispersions.

These tests are applied to the data on the directions of paleocurrents and the results are given in the main paper.

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