M . Tech (Computer Science) Dissertation Series

MORPHOLOGICAL EDGE DETECTION

a dissertation submitted in partial fulfilment of the requirements for the M. Tech (Computer Science) degree of the Indian Statistical Institute.

BY

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Name	**Title of dissertation**
1. Girish C.V.S.R.	Morphological Edge Detection
2. Saurav Bhattacharyya	Study of Some Thinning Algorithms and Their Analysis.

(J.B. Chaudhuri)
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CONTENTS

ABSTRACT

Edge detection plays an important role in automatic machine recognition and automation etc. The information of an object is concentrated at its edges and corners. Detection of edges and corners is therefore an important task in image processing. There exist several spatial and frequency domain techniques for edge detection. Here the problem is approached with mathematical morphology as a tool. Mathematical morphology consists of set operations on an image with another image called structuring element. Originally developed for two tone images, the operations have been generalized for gray tone images also.

The gray tone image can be mapped to a 3-dimensional space where two of the dimensions denote the position (x,y coordinates) and the third dimension denotes the gray values of pixels in image. The mapping appears like a hilly terrain. The gray dilation and erosion have been used to detect the terrain surface discontinuities. The dilation and erosion residues estimate to what extent the original image is affected by dilation and erosion operations respectively. These residues are suitably thresholded to get the edges. This procedure of edge detection has been implemented in VAX - 8650 and the results are presented.

INTRODUCTION

Morphology is a set theoretic approach to image processing, especially in shape analysis.Originally developed for the binary images, it has been extended to the problems of gray tone images also. For two tone images the basic operations are defined in terms of the set operations like the UNION, INTERSECTION, SET DIFFERENCE etc. With the help of MAX/MIN operators, the concepts have been extended to the Gray scale images. The related discipline can be called as GRAY SCALE MORPHOLOGY.

Just as the algebra of convolutions plays a very important role in linear systems , the mathematical morphology takes the same role in shape analysis. As the shape is the prime carrier of information in machine vision, the morphological operations assume significant role in machine vision. With morphology as a tool, the identification and decomposition of object, features become easy. Morphological operations can simplify the image data, preserving their essential shape characteristics and eliminate the irrelevancies.

Most of the image processing operations are done by spatial domain or frequency domain techniques. Morphology takes on a slightly different approach, in that, it not only takes into account the pixel values but also the geometric component i.e, how the individual elements are related in spatial domain.

Here small sub-images of different shapes/topologies (gray values) are employed to probe the image and extract information regarding shapes etc.. These sub-images are called the STRUCTURING ELEMENTS or simply STELTS.

The stelts can be flat or non-flat depending on whether the gray values of points of the stelts are uniform or non uniform. Some of the commonly used stelts are Disks, Spheres, Cones etc. In this work flat stelts of size 3 * 3 of height (gray value) less than or equal to 15, have been used.

From the morphological point of view, the process of recognizing the objects in an image(treated as a set), is to find a set of discriminating criteria for selecting out the pixels of objects from the rest in the image space.

The union operator (X1U X2) is { x : x E X1 or x E X2 } for binary images. It is commutative and associative. For gray level images the union is interpreted as the maximum of the input images, taken point by point ,which has also commutativity and associativity properties. In case of binary images the union is the result of point by point ORing (binary) so that it can be implemented by the OR gates in hardware.This is equivalent to taking the disjunction of properties (criteria) of the two images.

The intersection operator ($X1 \cap X2$) is { x : x E X1 and X2} for binary images. It too is commutative and associative. For the gray level images it is interpreted as minimum of the input images at the points defined and have the same properties. The binary intersection can be interpreted as conjunction of the criteria defined for the input images. This can be easily implemented with AND gates in hardware.

The complement of a two tone image is defined as

$$g'(x,y) = |g(x,y) - 1|.$$

For gray tone images the complement is computed as

$$g'(x,y) = g_{max} - g(x,y)$$

where g_{max} is the maximum possible gray value.

DEFINITIONS:

DILATION:

Dilation (Binary images) is defined to be the union of all translations of the image, as defined by the points of stelt. Mathematically, it is represented as...

$$A \# B = \bigcup (A_b) \quad b \; E \; B \; (\text{ where } \# \text{ is dilation }).$$

In other words,the dilation value at each point of the image is '1' if the stelt hits any of the on(1) pixels of the image, with the origin of the stelt being placed at that point else the pixel is in off(0) state. It is also interpreted as an expanding operation, as the objects in the image tend to increase in their size.

EROSION:

The dual operation of the dilation is Erosion. It is defined to be the intersections of all the translations of the image according to the points of the stelt. Mathematically it is represented as

$$A \mathbin{@} B = (\bigcap A_{-b} \; , \; b \; E \; B \; (\text{where } @ \text{ is erosion}).$$

In other words,the erosion value at each point of the image is '1' if the stelt is fully covered by the on(1) pixels of the image, (the 'on' points in the stelt should be covered), with the origin of the stelt being placed at that point. Else the pixel is in off(0) state.

It is also interpreted as an shrinking operation, as the objects in the image tend to decrease in their size. These operations are also called as the MINKOWSKI operations. Erosion and Dilation are dual, increasing, non idempotent operations.

GRAY SCALE DILATIONS & EROSIONS:

These operations are best illustrated by considering the image as well the stelt(gray scale) as a landscape with the surface height being proportional to the intensities of the pixels. The stelt is then passed below the surface(foreground) or over it (imagined to be background) according to whether it is for erosion or dilation (grayscale).The locations wherein the stelt can be centered with its surface entirely below (above) the surface of the image with the stelt placed at the surface level at that location of the landscape will remain unaltered. Those locations where the stelt has to be pushed down (above) the surface of the landscape(image) in order to fit entirely in the foreground (background) will be modified to the extent the stelt is pushed below or above the surface(Erosions and Dilations respectively) in optimal sense (ie. minimum push to be given).

To state in a more mathematical way, the gray value of a pixel after erosion will be the maximum value for which the stelt centered at that point and surafce level will fit entirely in the foreground. Similarly, the dilation value of a pixel is the maximum value for which the stelt, turned upside down, at that location does not fit entirely within the background above the surface of the landscape.

Thus, the erosion value is the minimum of the surface translated by all the points of the stelt and conversely the dilated value of the pixel as the maximum of the surface. These operators are considered to be basic and the other morphological operators are defined in terms of these basic operators. For example, the closing operation is defined as a dilation followed by the erosion of the image and the opening is defined as the erosion followed by the dilation of the image.

The expressions for grayscale dilation and erosion bear a marked similarity to the convolution integral frequently encountered in image processing with sums and differences replacing multiplication and min/max replacing summation. The grayscale dilations, erosion are non invertible unlike the other linear transformations. Also, they remove the grayscale information unlike their binary counterparts.

It is noteworthy that turning the element upside down for dilation and not for erosion seems a bit surprising as reflection is for erosion not for dilation. In order to find out where a positive valued element fits under the surface (for erosion), we have to translate the image in the opposite direction (i.e., to reflect the stelt), and take the Min. On the other hand, in case of dilation, one intends to find where the element barely fits in the background. Here, the surface is reflected once to get the inverted stelt.

After getting the inverted surface, the problem reduces to finding where the stelt fits entirely. As discussed for erosion, we have to invert once again the already inverted stelt. Thus, two reflections get cancelled in case of dilation.

RESIDUES

The residue operations give us an estimate of the extent to which the original set is affected by the morphological operations such as erosions and dilations.

In case of the binary images, these residues are defined as

$$X \text{ (ero\_residue) } S = X \cap (X \# S')^{c}$$

$$X \text{ (dil\_residue) } S = X \cap (X + S')^{c}$$

For grayscale dilations and erosions the resiues are defined by replacing the intersections and unions with clipped subtractions and additions. The subtraction is clipped in the sense that whenever the difference goes out of context (like negative values etc.) then an appropriate value is assigned (like zero when the difference goes negative etc.) for clipped subtraction. Some other definitions are also conceived for clipped subtraction like:-

$$\text{MIN} [X1(x,y), (X1(x,y) > X2(x,y))] \text{ or}$$

$$\text{MAX} [0, (X1(x,y) - X2(x,y))].$$

We followed the definition given earlier. Analogously we have the clipped union as MIN [g_{max} , (X1(x,y) + X2(x,y))]. Here X1 and X2 are two sets which are to be evaluated for differences (clipped) and g_{max} is the maximum pixel value allowed in the gray tone image.

PART I

MORPHOLOGICAL EDGE DETECTION

We define an edge as the boundary between two regions with relatively distinct gray level properties. It is assumed that the regions in the question are sufficiently homogenous so that the transition between two regions can be determined onthe basis of gray level discontinuities alone.

Basically, the idea underlying most edge detection techniques is the computation of a local derivative operator. The Fourier analysis is widely known, exploiting the fact that high frequencies are predominant at the discontinuites and hence suitable computation can be made to detect the edges. But a lot of computation has to done for computing the convolutions etc.

The other techniques involve the computation of the gradients over a finite region since edges are likely to be concentrated over the region where there is maximum gradient value. The size of the mask and the information required determine whether it is local or regional operator. Local operator uses the information of the neighbouring piexls to detect the edges. Regional operator uses the global information, such as the average pixel values etc. to detect the edges. The morphological operations such as the dilations and erosions can be thought of as local operations as we have employed stelts of size 3 * 3.

Dilations and Erosions of grayscale images can be computed directly from gray level functions for images and stelts. The dilated grayscale image is computed as the maximum of the sum of the gray levels of stelt S(x,y) with each point of I(x,y).

$$A(x,y) = \max_{i,j} \; [\; I(x-i,y-j)+S(i,j) \;] \quad \text{(gray scale dilation)}$$

The minimum level above which the stelt can be placed (above zero level) over the terrain surface (image) such that terrain surface of the stelt does not overlap, determines the dilation value of that pixel in the image. This is interpreted in the figs 1 & 2. The gray scale dilation sweeps stelt across the (X,Y) dimensions and modifies the gray values in the Z dimension.

Now the dilation residue which computes the extent to which each pixel value is affected by the dilation is calculated. The dilation residue computes the difference between each pixel value and the value of local maximum within the region swept by the stelt i.e. the maximum climb from that pixel value within the area covered by the stelt. Here one observation is that the pixels values near the edges will have a greater climb than those of pixels which are in the uniform region ie. non-border pixels. Thus by suitably thresholding and thus converting the dilation residue to binary image with the pixels having the climb greater than the threshold value are recognized as the edge pixels having the value 1 and the rest having the value 0.

Similar discussion holds for the dual operation ie. the grayscale erosion and the erosion residue. Here the maximum operator is replaced by the minimum operator , the background being replaced by the foreground in the discussion. FIG 3.

$$A(x,y) = \min \left[I(x-i,y-j) - B(-i,-j) \right] \text{ (gray scale erosion)}$$

ALGORITHM

The algorithm to detect edges is as follows..

```
   Procedure edge_detect:

Input    : Gray tone images.
Output   : Binary images containing the edges.

  begin

    1.   initialize the stelt points (to define the surface of
         the stelt).

    2.   Perform the grayscale dilation (erosion) to  probe  the
         image.

    3.   Compute the dilation residues (erosion residues)
         and  get the maximum climb(drop) of the pixels.

    4.   Threshold the residue image  thus  converting
         the residues  to binary images (here the on pixels will
         be the edge pixels).

     end of edge_detect.
```

PART II..

HIT AND MISS TRANSFORM

As mentioned earlier the erosion and dilation operators have been the basic operators to be reckoned, with in morphological opeartions. The hit and miss transform (HM Tranform) is a natural operation to select out pixels and probe certain spatial characteristics etc.

The HM transform is defined as follows

Let J,K be two stelts satisfying $J \cap K = \vee$.

The HIT and MISS tranform is defined to be ..

$A \,@\, (J,K) = (A \ddagger J) \cap (A^c \ddagger K)$., where @ denotes the h.t transform operation, $\ddagger$ is the binary erosion operation and A^c is the complement set of A

The interpretation is obvious,in that the first erosion operation on A with J selects out the pixels which confirm the distribution of pixels in the stelt J and similarly the second erosion with K on complement set of A. The intersection of these erosions will select out those pixels which satisfy spatial orientations expressed by the distribution of points of stelts J, K in foreground and background respectively.

The HM transform has been employed in detecting sharp bends and corners. Here HIT and MISS transform has been employed with stelts of size 2*2.Basically it selects out the pixels which satisfy spatial orientations expressed by the distribution of points of stelts J, K in foreground and background respectively.

2*2 stelts have been taken and suitable arrangements of pixels to detect specific directionals, sharp bends are effected. One such series of stelts used, for example, are as follows..

1	1
1	0

J

0	0
0	1

K

Thus H.T is employed to detect such pixels and further investigation is done to locate sharp bends/corners.

-:CONCLUSIONS AND RESULTS :-

The edge detector designed here worked well with real images (standard). As mentioned in chapter number two the residue & thresholding operation can be thought of as gradient operation. Hence the properties and characteristics of the gradient operations, such as noise sensitivity, are applicable to the residue edge detector.

In figures 4 & 5, the edges from the encoded image of Abraham Lincoln are extracted with varying threshold values and the values of the stelts points as indicated below the figures.

In figures 6 & 7, the edges from the encoded image of a boy are extracted with varying threshold values and the values of the stelts points as indicated below the figures.

In figures 8 & 9, the edges from the encoded image of plane are extracted with varying threshold values and the values of the stelts points as indicated below the figures.

In figures 10 & 11, the edges from the encoded image of the great portrait of Monalisa are extracted with varying threshold values and the values of the stelts points as indicated below the figures.

In figure-12, the detection of northeast and southwest corners are depicted.

Thus we find that threshold plays an important role in deciding whether a pixel belongs to the edge. In addition we can also supply some of the global information (such as average pixel value) so that the decision threshold can be well tuned.An observation to be noteworthy is that the dilation effects the pixels on the transition region where as the erosion effects the pixels on the either side of the transition region.

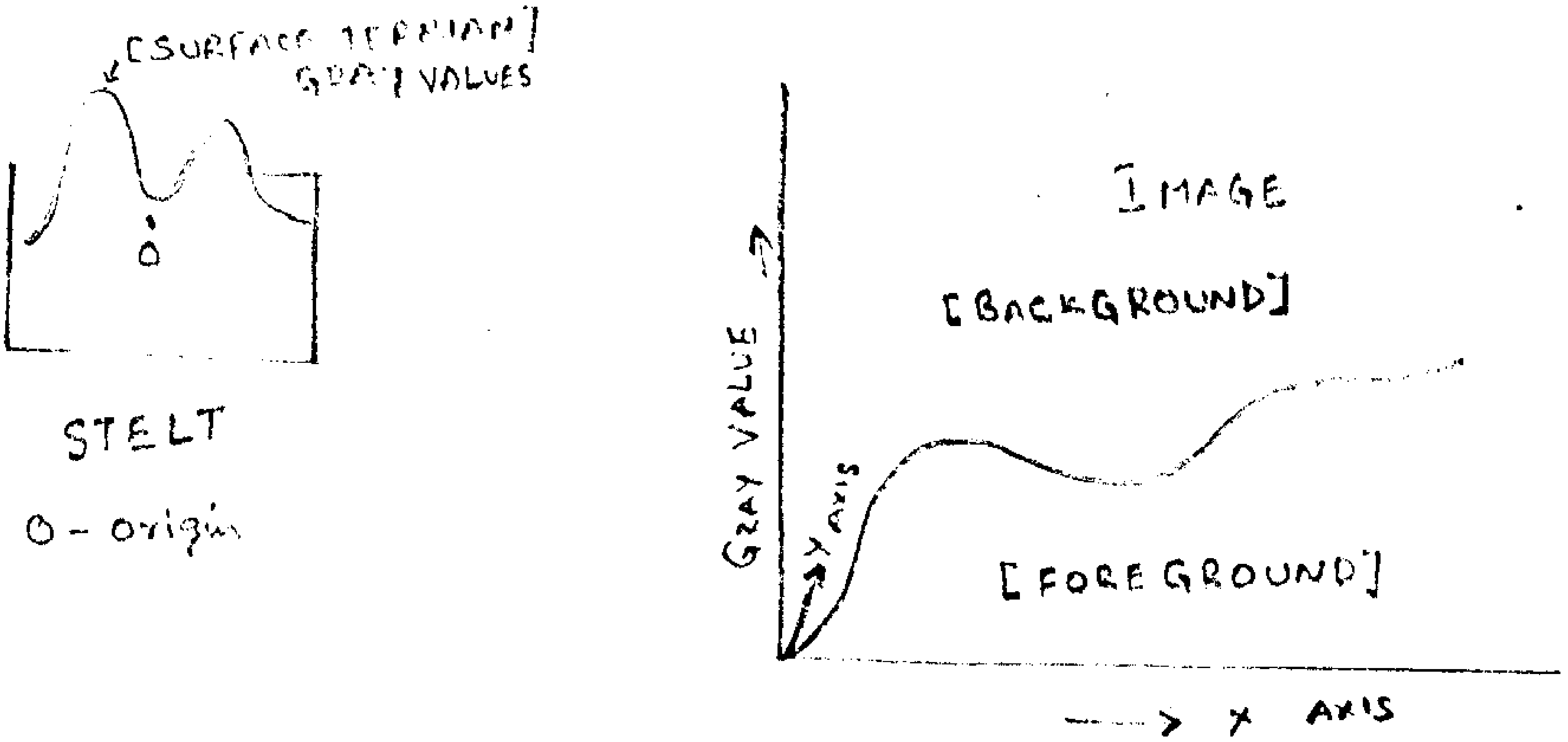


FIG 1.

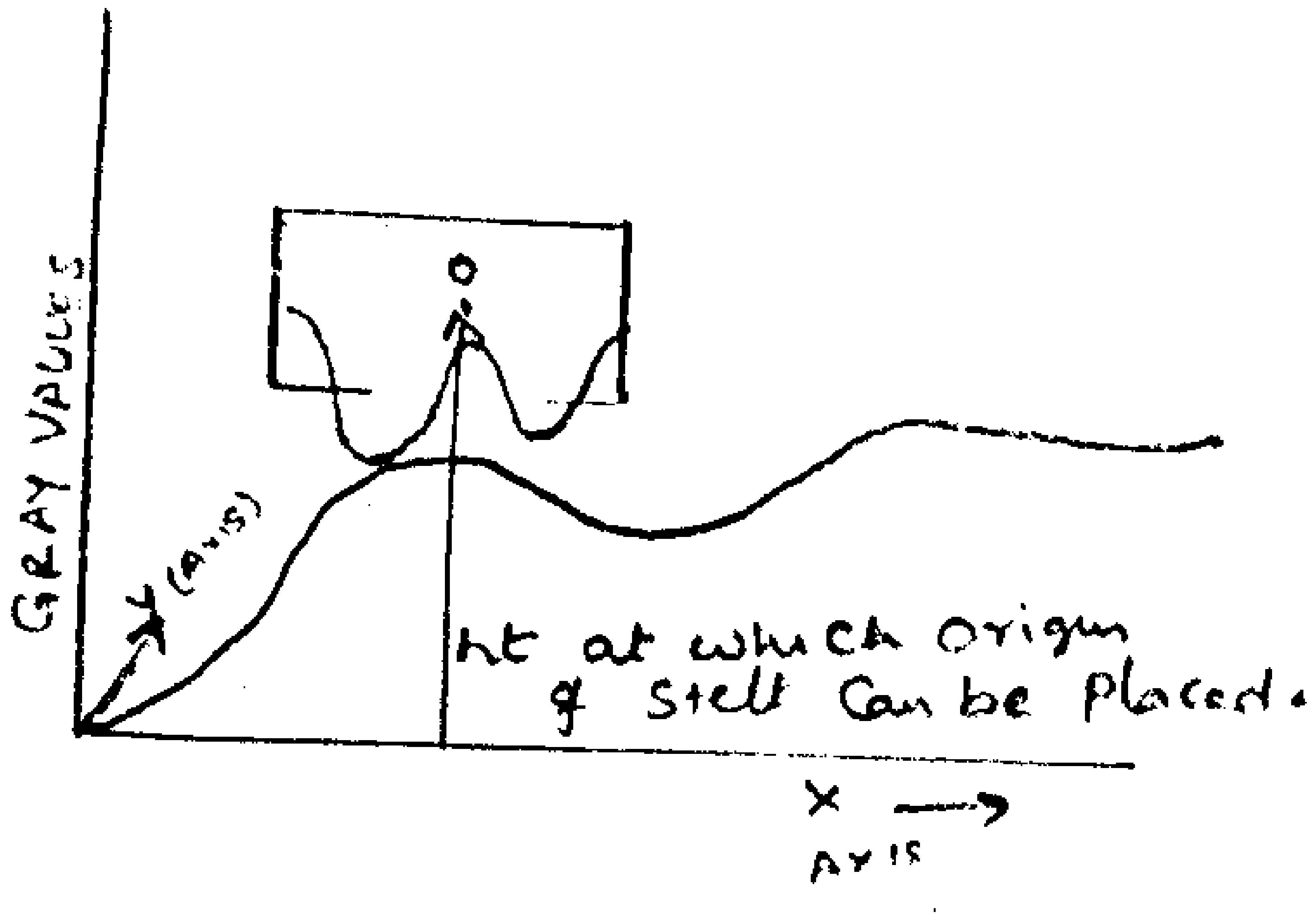
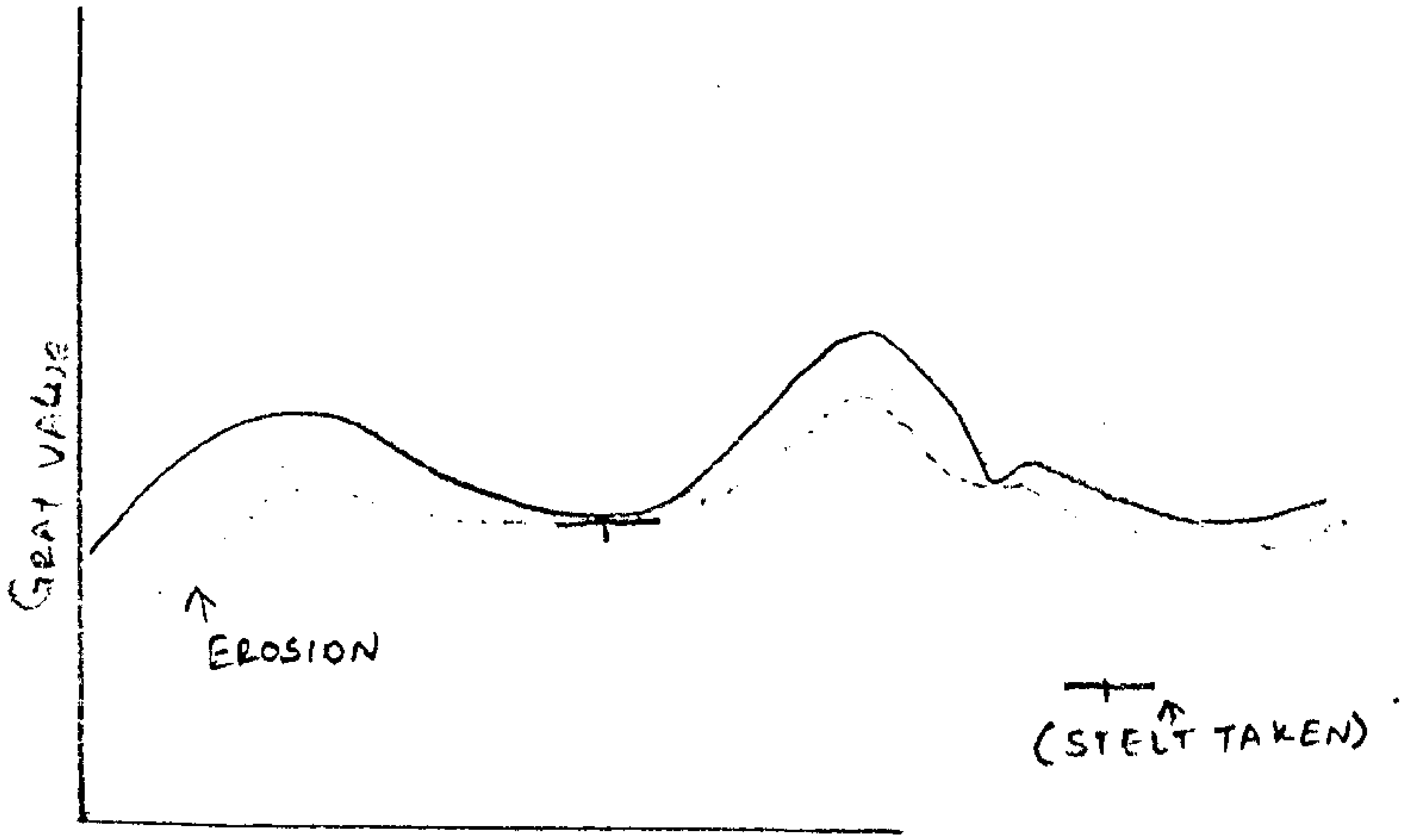


FIG 2.
GRAY SCALE DILATION SEQUENCE



GRAY SCALE EROSION

FIG 3.

fig. 4
Threshold = 6 , Stelt pts (gray) value = 12

fig. 5
Threshold = 8 , Stelt pts (gray) value = 13

fig. 6
Threshold = 7 , Stelt pts (gray) value = 17

fig. 7
Threshold = 6 , Stelt pts (gray) value = 12

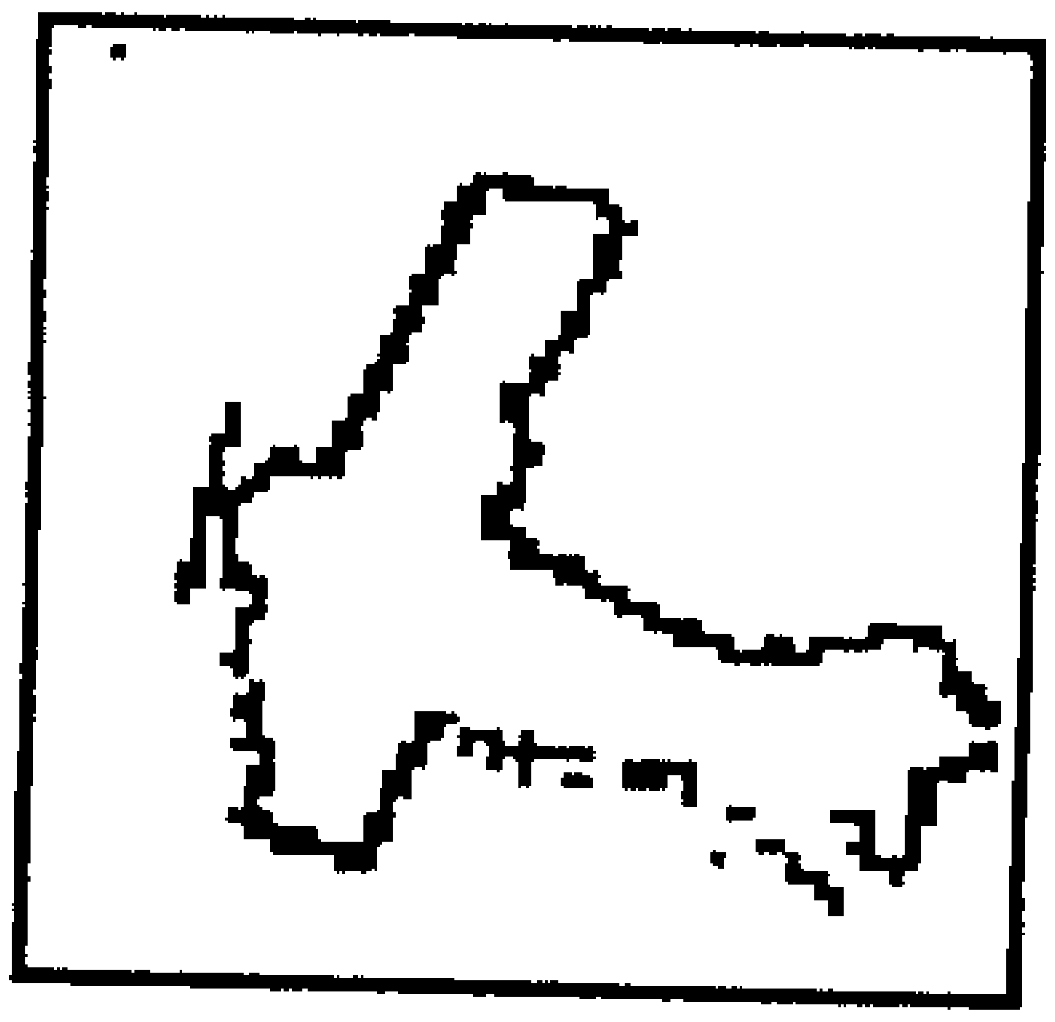


fig. 8
Threshold = 6 , Stelt pts (gray) value = 13

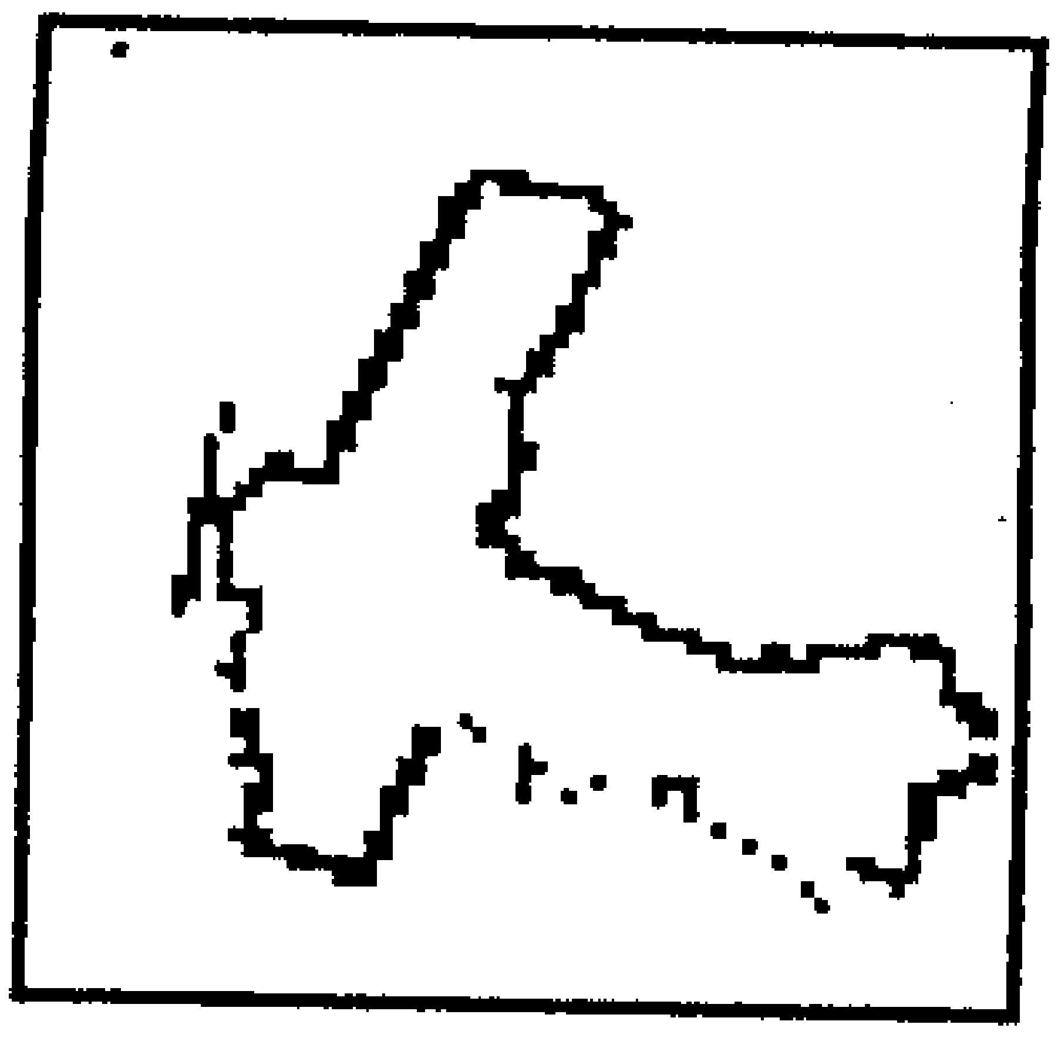


fig. 9
Threshold = 7 , Stelt pts (gray) value = 15

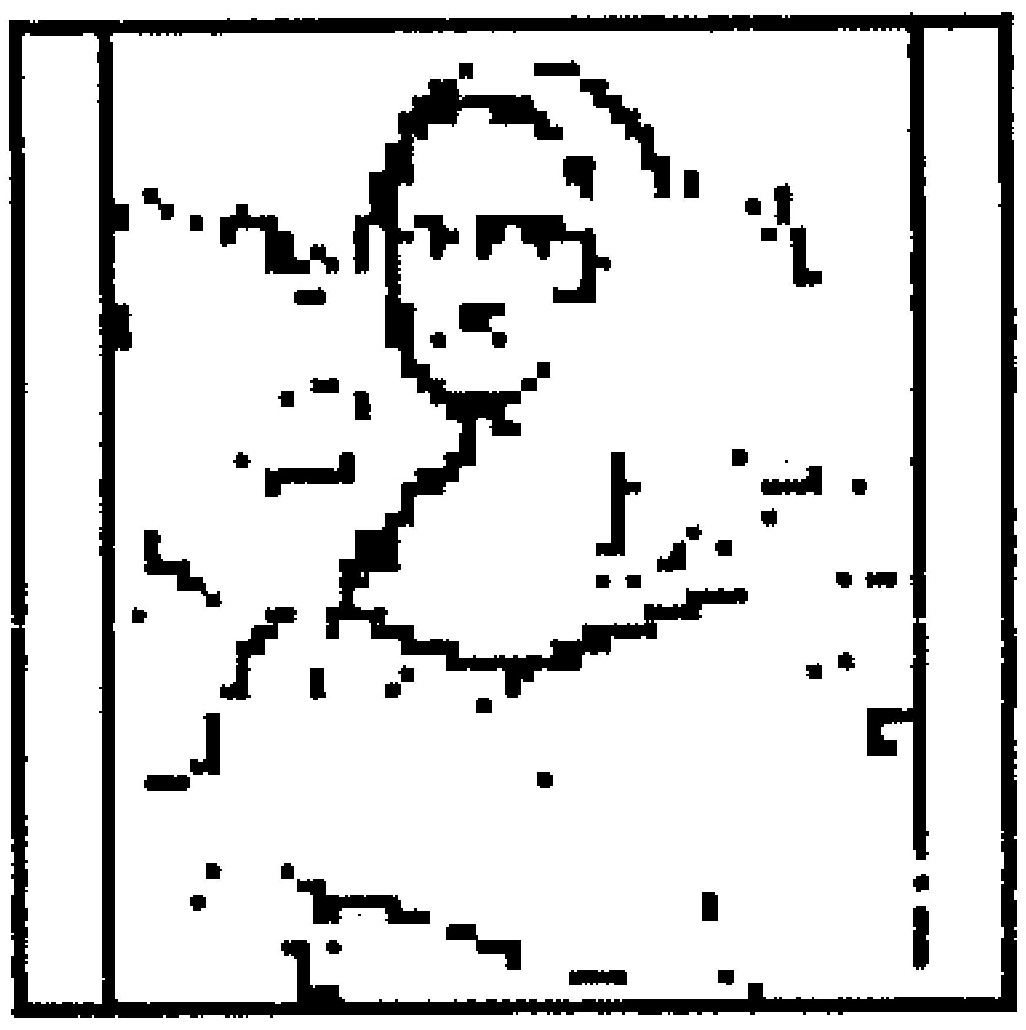


fig. 10
Threshold = 6 , Stelt pts (gray) value = 13

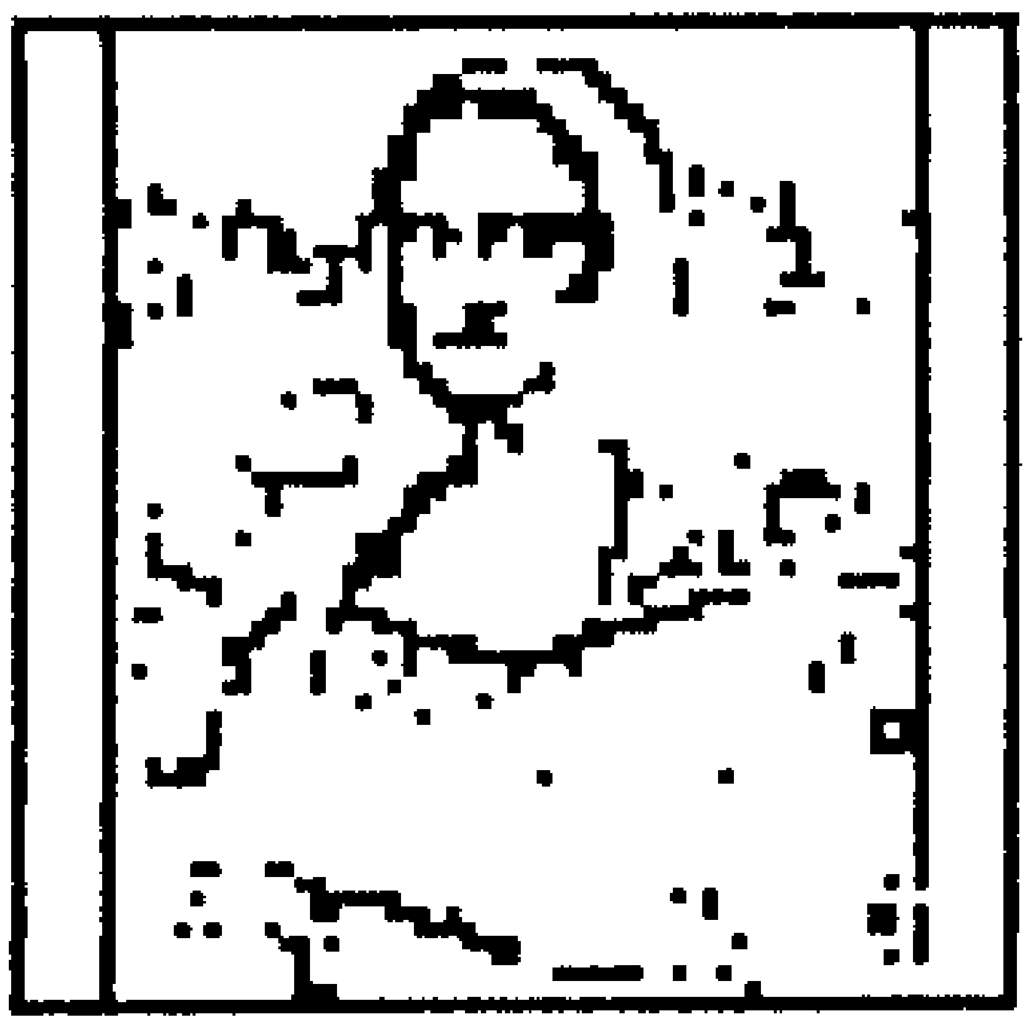


fig. 11
Threshold = 7 , Stelt pts(gray) value = 14

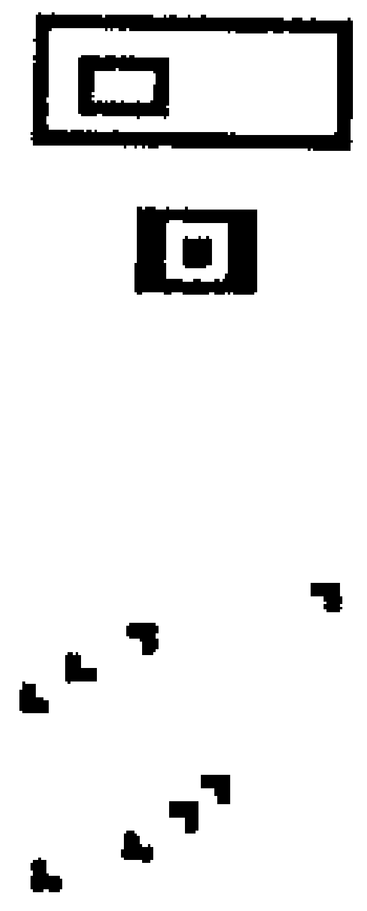


fig. 12
Detecting N.E. & S.W. corners:

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