M.Tech.(Computer Science) Dissertation Series

Partitioning Problems

A dissertation submitted in partial fulfillment of the requirements for the M.Tech.(Computer Science) degree of the Indian Statistical Institute

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Certificate of Approval

This is to certify that the dissertation thesis titled "Facility Location Problems" submitted by Mr. Nilesh Kumar, in partial fulfillment of the requirements for the M. Tech.(Computer Science) degree of the Indian Statistical Institute, Kolkata, embodies the work done under my supervision.

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Abstract

In this thesis we have proposed algorithms for partitioning a region in which we need to equalize the area of region with certain constraints. Given a set of facilities on a rectangular region, here our objective is to partition the region into equal area such that each partition contain one facility and furthest point from facility to its partition region is minimized. In case of two facilities we present an algorithm which equipartition the region (using at most one horizontal line segments) and minimize maximum distance between facility and its furthest point of concave region. If the partition region is separated by a single line segment then an another algorithm is given which present partition if it exits. For general case we propose a heuristics which equipartition the region. We are using Weighted Vornoi Diagram for the division of region and Secant Methods for updating weights.

Contents

Chapter 1

Introduction

Location models have been discussed by researchers from a wide variety of discipline, among them mathematicians, geographers, marketing and retail specialists. Virtually all location models have a common basic nature: a metric space in which customers(representing demand) are positioned, and in which facilities are to be located by a decision maker. Most authors classify models based on the space of the model(e.g.,Euclidean space or a network), the number of facilities to be located, the objectives the facility planners follow, and the type of behaviour customers are assumed to exhibit. This contribution takes the last route and focuses on consumer behavior in competitive facility location modeling. The problem of locating facilities in a manner so that they can effectively serve a set of clients has been the subject of much interest. While one could consider fairly general measures of effectiveness of a set of locations in serving the clients,one measure that is typically used is the distance between the client and the facility that is serving it.Since by opening a lot of facilities, we can be near every client, it also makes sense to take into account the number of facilities opened in judging the quality of a solution.These two measures, typically referred to as the service cost and the facility cost,can be combined in many was to obtain interesting variants to the general facility location problem.

In general, whenever more than a single facility is given in some space(regardless whether it already exists in space or it has to be located in the process), any location model needs a rule for assigning demand to the facilities. Here, we can distinguish between allocation models and choice models. In allocation models, the facility planner assigns customers to facilities, while in choice models, customers are free to choose the facility. The latter is the case when modeling competitive facility locations, particularly in the context of retail location. Here,planners must understand how consumers behave and how they make spatial choices.

Typical customer characteristics include the consumer's disposable income as expression of their buying power, the customer's age, and their level of education. All characteristics will determine what potential customers consume, enjoy and, more generally, their life-style. Facility attributes include the variety offered by a facility, such as the number of books in the library, the number and types of services provided by a medical facility or the goods offered in a store or a shopping mall. Finally, the spatial separation, i.e.,the distance between customer and a facility, will determine the level of support the facility will enjoy from the customer. In general, a customer will consider a facility less attractive the greater the distance between them. This is typically referred to as a transportation cost, a term that is to be understood in the wide sense, including actual transport costs, the time of transport aggravation due to long waiting times and similar factors.

In order to demonstrate the difference in location models based on different customer's behavior, consider the location of gas stations, given a typical suburban bedroom community and a major town, both connected by a road. First assume a case in which customer's always make a special trip from their home to the station. In such a case, the planner of a new station will locate his facility on either side of the road, as close as possible to the suburb in order to capture its demand. Suppose now, somewhat more reasonably, that will fill up their cars on the way to or from work. If they do so on the way to the town in the morning,they will prefer to have gas station on the right-hand side anywhere along the road, as left turns are more time-consuming and require more attention than right turns. More importantly, a station on the left side of the road requires the customers to cross on land after they turn out of the station and rejoin their traffic flow. The opposite will apply if customers can be expected to fill up their car on their way back home. As customers are usually more hurried on their way to work in the morning than on their way in the afternoon, they will tend to fill up in the afternoon, so that gas stations will often be located on the right side of the road as seen from town towards the suburb.

Our work partitioning the region in which some facilities are already located, based on some constraints like, area of all partitioned regions should be equal and/or use only horizontal or vertical line for partitioning, and/or minimize the maximum distance between facility and it's users. These kind of problems has lot of applications in the industry. If the population is uniformly distributed in the area then, area can give us how many people actually using that facility. As described earlier if a user has to travel more to get a facility then his attitude becomes negative towards using that. So based on these behavior, the solution of this problem has been given with some constraints.

1.1 Organization of the thesis

In this dissertation, the problem statement and motivation is formally briefed in *Chapter 2*. In *Chapter 3* we have explained related work and various facility location problems. *Chapter 4* focuses on our work. In *Chapter 5* we have presented results of algorithm discussed in *Chapter 5*. Finally in *Chapter 6* we explained conclusions and summary.

Chapter 2

Problem Definition and Motivation

Problem Definition

Given a region R and a set of facilities, we need to partition the region using horizontal and vertical line segment such that

- Each Partition contains exactly one facility.
- Area covered in each partition is same.
- Minimize the maximum distance from a facility to a point lie in the corresponding partition.

Motivation of the problem

Suppose a company has given dealership to fixed number of businessman and each businessman wants to cover equal number of customers(Assume that number of customer is uniformly distributed in the area). Problem is to partition the region such that distance covered by business representative to its furthest user and its business center should be minimum.

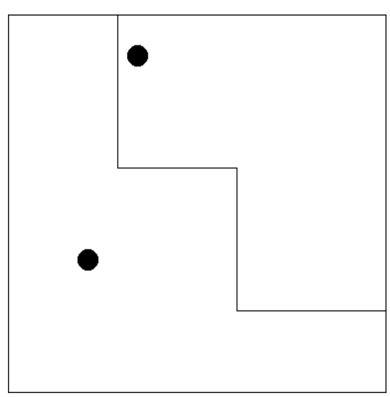

Figure 2.1: An example of desired partitioning

Chapter 3

Related Work

3.1 Weber Problem

The simplest form of facility location is the *Weber Problem*[1] , *we are to find the* **minisum** *point* (x^*, y^*) *which minimizes the sum of weighted Euclidean distances from itself to* n *fixed points with co-ordinate* (a_i, b_i). *The weight which are associated with the fixed points are denoted by* w_i.

The problem can be mathematically expressed as

$$\min_{x,y} \left\{ W(x,y) = \sum_{i=1}^{n} w_i d_i(x,y) \right\} \tag{3.1}$$

3.1.1 Solution Procedure

This problem dates back to the seventeenth century and many variations of it are still the subject of intense research nowadays.One solution of this problem is to solve it numerically,that has been done by *Weiszfeld*[1].

The Weiszfeld Algorithm: Differentiate and set the partial derivative equal to zero to obtain the first order conditions for optimality we have

$$\begin{aligned} \frac{\partial W(x,y)}{\partial x} &= \sum_{i=1}^{n} \frac{w_i(x-a_i)}{d_i(x,y)} = 0 \\ \frac{\partial W(x,y)}{\partial y} &= \sum_{i=1}^{n} \frac{w_i(y-b_i)}{d_i(x,y)} = 0 \end{aligned} \tag{3.2}$$

Euation 3.2 can be solved numerically to get desired solution.

3.2 Min-max-min Geometric Facility Location Problems

This is a type of facility location problems in which customers make some decision on whether they have some interest in using the facility or not.The facility is defined as a geometric object,and customers are not interested in using it if it is too far from their own location. We assume there are n customers.If we denote by x the facility to be located,then $C_x(i)$ is the cost incurred to the i^{th} customer if he uses the facility,and $C_{\bar{x}}(i)$ is the cost if he does not use the facility.A *min-max-min* facility location problem [1] is a problem of the form:

$$\min_{x} \max_{1\leq i\leq n} \min\{C_{\bar{x}}(i), C_x(i)\} \tag{3.3}$$

3.3 The k-facility location Problem

In the $k-$*facility* location problem, we are given a set of facility locations F and a set of clients C. For a facility $i \in F$, *its facility cost* $f_i \geq 0$ is the cost of *opening* that facility. We are also given the distances $c_{i,j}$ between $i, j \in F \cup C$ that satisfy metric properties. The distance $c_{i,j}$, between $i \in F$ and $j \in C$ is the cost of serving the client j by the facility i. We are given upper bound $k > 0$ on the number of facilities to be opened.The objective is to open at most k facilities in F such that sum of the facility costs of the open facilities and the cost of serving each client by the nearest open facility is minimized.

This problem is NP-hard. So it become more interesting for researchers to give heuristic solution of this problem which is very efficient. Several heuristics has been given for solving this problem.

Chapter 4

Our Contributions

In order to achieve the solution of the problem defined in chapter 2, initially we need to relax some constraints. In this way some new SubProblems has been arisen. Which needs to be solved. One of them can be defined as:

4.1 SubProblem 1

In a square region two facilities are already installed, can we partition the region into two equal parts using only one line segment such that each partition contain exactly one facility?

4.1.1 Technical Preliminaries

Partition Line :-A line segment which partition the region.
Partition Point :-A point on a boundary of the square from which partition line passes.
Conjugate Partition Point :-Other end of partition line for a given partition point.
Bisection Set :- A set of partition point which partition the region into two equal parts.

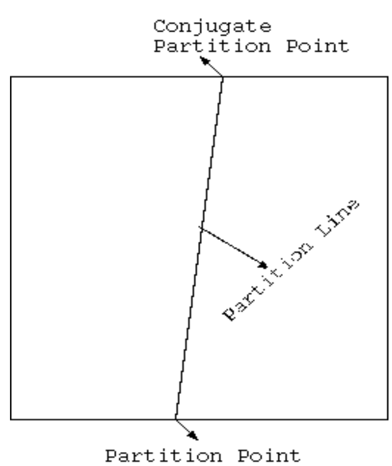

Figure 4.1: Illustration of definitions

$n-$**section Set :-** A set of partition point which partition the region into n equal parts.

Observation 1 *If Bisection Set exists then we can always partition a square region by a line segment in two equal parts, such that each region contains exactly one facility.*

Proof.

Let the side of the square is a, and two facilities are located at $F_1(\alpha_1, \beta_1)$ and $F_2(\alpha_2, \beta_2)$. Take a partition point $P_1(b, 0)$ on side OA so $P_2(b', a)$ is its conjugate partition point.
Area of OP_1P_2B is= $\frac{a^2}{2}$
$\Rightarrow \frac{a(b+b')}{2} = \frac{a^2}{2}$
$\Rightarrow b' = a - b$
Equation of line P_1P_2 is
$ax + (2b - a)y - ab = 0$
Since two facilites F_1 and F_2 should lie opposite to this line so we have
$a\alpha_1 + (2b - a)\beta_1 - ab > 0(< 0)$
$a\alpha_2 + (2b - a)\beta_2 - ab < 0(> 0)$
If these two inequalities are consistent for b we can get Bisection Set(i.e range of b). Hence by choosing one point from Bisection Set we can always partition the region from X-axis. If Bisection Set is empty then we will try partitioning in Y-axis and do the similar work.

If we get empty Bisection Set while partitioning from X-axis and Y-axis both, then partitioning is not possible at all. □

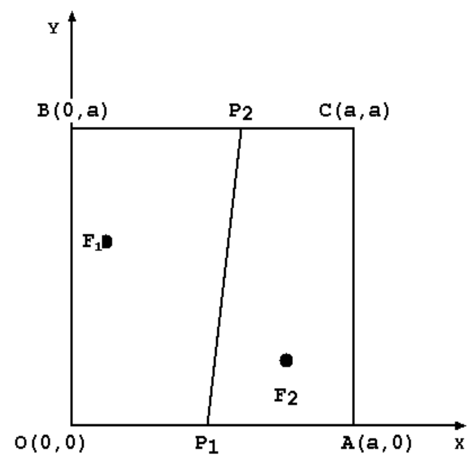

Figure 4.2: Partitioning using one line segment

Straight forward generalization of this sub problem with some assumptions is also possible. If we can assume that both partition points should lie on the opposite side of the square.Then with this assumption we have following observation.

Observation 2 *If $n-$section set exists then we can always partition a square region by a $n - 1$ line segments in n equal parts, such that each region contains exactly one facility.*

Proof.

Let the side of the square is a, and n facilities are located at $F_1(\alpha_1, \beta_1)$, $F_2(\alpha_2, \beta_2)$ ...$F_n(\alpha_n, \beta_n)$. Partition points on side OA are $P_1(b_1, 0)$, $P_2(b_2, 0)$ .... $P_n(b_n, 0)$, corresponding conjugate partition points are $Q_1(b'_1, a)$,$Q_2(b'_2, a)$....($Q_n(b'_n, a)$.For the i^{th} partition line P_iQ_i we have
Area of OP_iQ_iB is = $\frac{a^2 i}{n}$
$\Rightarrow \frac{a(b_i+b'_i)}{2} = \frac{a^2 i}{n}$
$\Rightarrow b'_i = \frac{2ai}{n} - b_i$
Equation of line P_iQ_i is $ax + (2b_i - \frac{2ai}{n})y - ab_i = 0$

Now Since two facilities F_i and F_{i+1} should lie opposite side of line P_iQ_i so we have

$$a\alpha_i + (2b_i - \frac{2ai}{n})\beta_i - ab_i > 0(< 0)$$
$$a\alpha_{i+1} + (2b_i - \frac{2ai}{n})\beta_{i+1} - ab_i < 0(> 0)$$

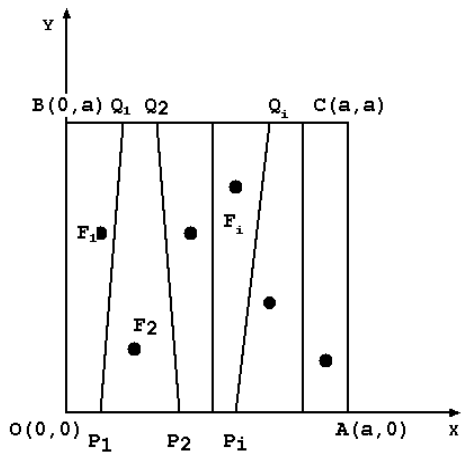

Figure 4.3: Partitioning using multiple line segments

But here we need to ensure that P_iQ_i and $P_{i-1}Q_{i-1}$ should not intersect in the middle, so Y-coordinate of intersection point of the lines should either ≤ 0 or $\geq a$. We are using $\geq a$ so in this way we get another inequality.

$$\frac{b_i - b_{i-1}}{2b_i - 2b_{i-1} - \frac{2a}{n}} \geq 1$$

If these three inequalities are consistent for all b_is we can get $n-$*section Set*(i.e ranges of b_i). Hence by choosing points from $n-$section Set we can always partition the region from X-axis. If $n-$*section Set* is empty then we will try partitioning in Y-axis and do the similar work.

If we get any empty $n-$section Set while partitioning from X-axis and Y-axis both, then partitioning is not possible at all. $\square$

Now on the basis of these observations we can give an algorithm that partition the region in n-equal parts. This algorithm return true if partitioning is possible otherwise false.

Algorithm 1

STEP1:-For each point do

STEP2:-Solve the folowing inequalites for b_i

$$a\alpha_i + (2b_i - \frac{2ai}{n})\beta_i - ab_i > 0(< 0)$$
$$a\alpha_{i+1} + (2b_i - \frac{2ai}{n})\beta_{i+1} - ab_i < 0(> 0)$$
$$\frac{b_i - b_{i-1}}{2b_i - 2b_{i-1} - \frac{2a}{n}} \geq 1$$

STEP3a:-If solution is consistent for b_i go to STEP-1

STEP3b:-Otherwise go to step 5

STEP4:- return **true**

STEP5:-For each point do

STEP6:-Solve the folowing inequalites for b_i

$$(2b_i - \frac{2ai}{n})\alpha_i + a\beta_i - ab_i > 0(< 0)$$
$$(2b_i - \frac{2ai}{n})\alpha_{i+1} + a\beta_{i+1} - ab_i < 0(> 0)$$
$$\frac{b_i - b_{i-1}}{2b_i - 2b_{i-1} - \frac{2a}{n}} \geq 1$$

STEP7a:-If solution is consistent for b_i go to STEP-5

STEP7b:-Otherwise return **false**

STEP8:- return **true**

Time Complextity of Algorithm 1 is $O(n)$.

Now we can move further up and put more constraints, so we will get another sub problem. That will be discussed in next section.

4.2 SubProblem 2

If we will partition a square region in which two facilities are already installed, using exactly one horizontal and one vertical line segment, then one of partitioned region will be convex and other will be concave. As we know in convex polygon any inside point is visible form all the corner points but this is not true for concave polygon. So for finding distance from corner to any inside point in concave polygon we may need to consider geodesic distance. As we know geodesic distance between two points is not less than Euclidean distance between them. In next SubProblem we have minimized geodesic distance between facility and users. Before defining next SubProblem we have following observation.

Observation 3 *Given a square region in which two facilities are already installed, we can always partition the region into two equal parts using at most one horizontal and vertical line segments such* *that each partition contains exactly one facility.*

Proof. *Case 1 When two facilities lie on different quarters of the square*
This case is easy we can use either horizontal or vertical line segment for partitioning as shown in figure 4.4.
Case 2 When two facilities lie on same quarter of the square

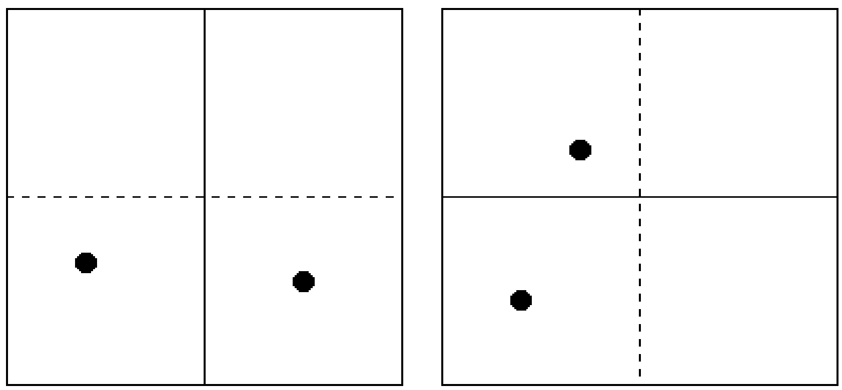

Figure 4.4: Partitioning when facilities are in different quarters

In this case we have following two sub cases as
Case 2a When $(x_1 > x_2 \wedge y_1 > y_2)$ *or* $(x_1 > x_2 \wedge y_1 = y_2)$ *see figure 4.5*
Clearly we have $FG \geq \frac{a}{2}$. Now suppose if area of $DFGC > \frac{a^2}{2}$, then area of $EFGA < \frac{a^2}{2}$. Hence we

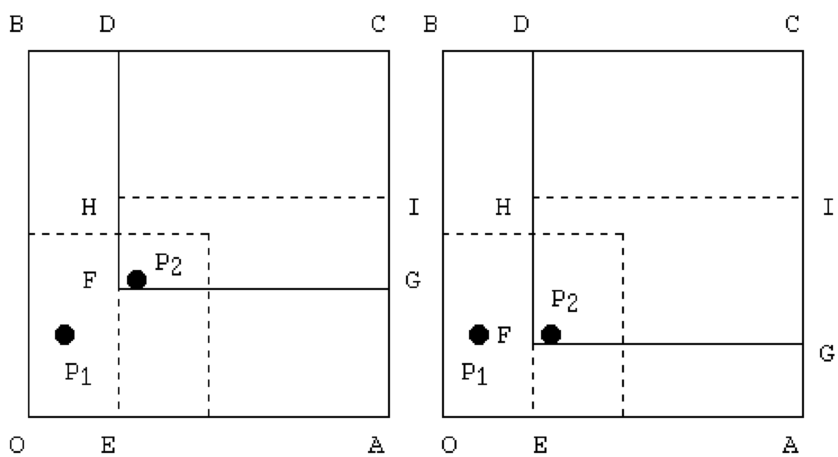

Figure 4.5: Partitioning using horizontal and vertical line segments

can find a point H between F and D such that area of $EHIA = \frac{a^2}{2}$.
But if area of $DFGC \leq \frac{a^2}{2}$, area of $EDCA \geq \frac{a^2}{2}$. If we will slide line FG towards EA we can get area of $CDFG = \frac{a^2}{2}$.
Case 2b When $(x_1 > x_2 \wedge y_1 < y_2)$ *or* $(x_1 = x_2 \wedge y_1 = y_2)$ *see figure 4.6*
Since $DE < \frac{a}{2}$ so area of $DEFB < \frac{a^2}{2}$ also area of $DGCB > \frac{a^2}{2}$.If we slide line EF towards GC we

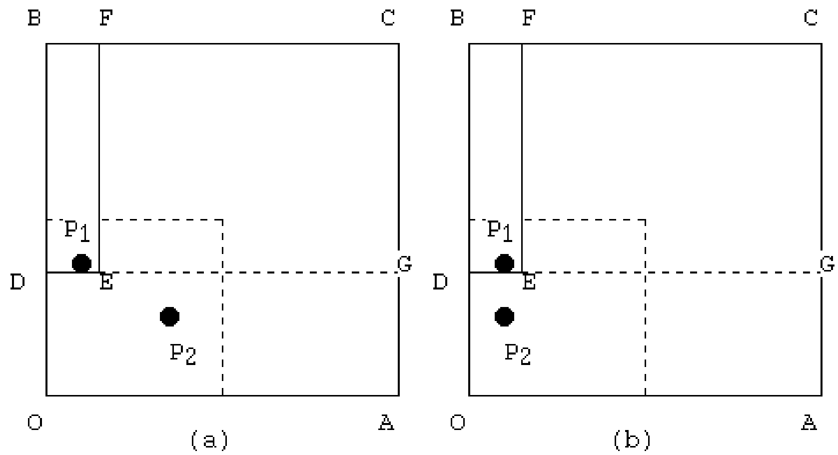

Figure 4.6: Partitioning using horizontal and vertical line segments

can get area of $DEFB = \frac{a^2}{2}$.
□

Now we can move to our next sub problem which can be defined as
In a square region two facilities are already installed can we partition the region using at most one horizontal and vertical line segment such that

- *Area in two parts should be equal.*
- *Each part contains exactly one facility.*
- *If possible then keep both parts convex(i.e. partition through either horizontal or vertical line segment).*
- *If convex partitioning is not possible then minimize the maximum distance from users to its facility in concave region.*

As we have discussed in observation 3, if facilities fall in different quarters then just a horizontal or vertical line segment is sufficient for partitioning. Interesting cases arise when they fall in same partition.

Now two cases may arise (i) When a facility remain visible after partition.(see figure 4.7a). (ii) When a facility may not visible(see figure 4.7b). Let the coordinate of F_1 and F_2 are respectively (x_1, y_1) and (x_2, y_2), and length of the side is a. Let the coordinate of D is (x, y). Now for case (i) Area of $DECF$ gives relation between y and x coordinate i.e. $y = f(x)$ so the distances F_1E and F_1F is also some function of x more specifically $d(F_1, E) = \mu_1(x)$ and $d(F_1, F) = \mu_2(x)$.
Objective function can be given as

$$\begin{aligned} &\min_x \{\mu_1(x), \mu_2(x)\} \\ &s.t. \\ &F_1 \textit{ is outside and } F_2 \textit{ is inside of quadrilateral } OAFC. \end{aligned} \tag{4.1}$$

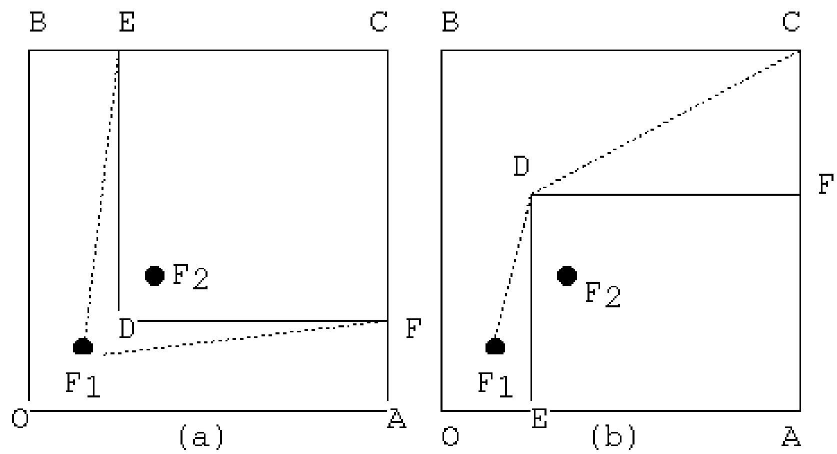

Figure 4.7: Illustration of visible and non-visible points after partitioning

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Equation 4.1 is a *Multiobjective Optimization*. There are several methods of solving these kind of equations. But we have used aggregate objective function (AOF) technique [2] for solving it.
For case (ii) From above discussion we conclude that distances will be function of x i.e $d(F_1, D) = \mu_1(x)$ and $d(D, C) = \mu_2(x)$.
In this case objective function is

$$\begin{aligned} &\min_x \{\mu_1(x) + \mu_2(x)\} \\ &s.t. \\ &F_1 \textit{ is outside and } F_2 \textit{ is inside of quadrilatral } EDFA. \end{aligned} \tag{4.2}$$

Equation 4.2 is a *Nonlinear Optimization*. We have used Mathematica function *NMinimize*[3] to solve this.
Solution of equation 4.1 or 4.2 gives partition point and hence the partition. Now we will list all possible partitions when facilities lie in a same quarter, as shown in figure 4.8. Here for example $3(ii)$ means second possibility of partition in case 3.
Now we are presenting an algorithm for *SubProblem 2*.

Algorithm 2

STEP-1:-Check in which case facilities fall into

STEP-2(a):-If it falls into case 1 (i.e. facilities are in different quarters)
    partition using either horizontal or vertical line and **return**

STEP-2(b):-If it falls into other cases except 1(i.e.facilities are in same quarters)
    for each possibility of a case
        solve equation 4.1 or equation 4.2.

STEP-3:For each possibility of a case do
        calculate the distances from facility to corner points in concave region
        find maximum distance among these

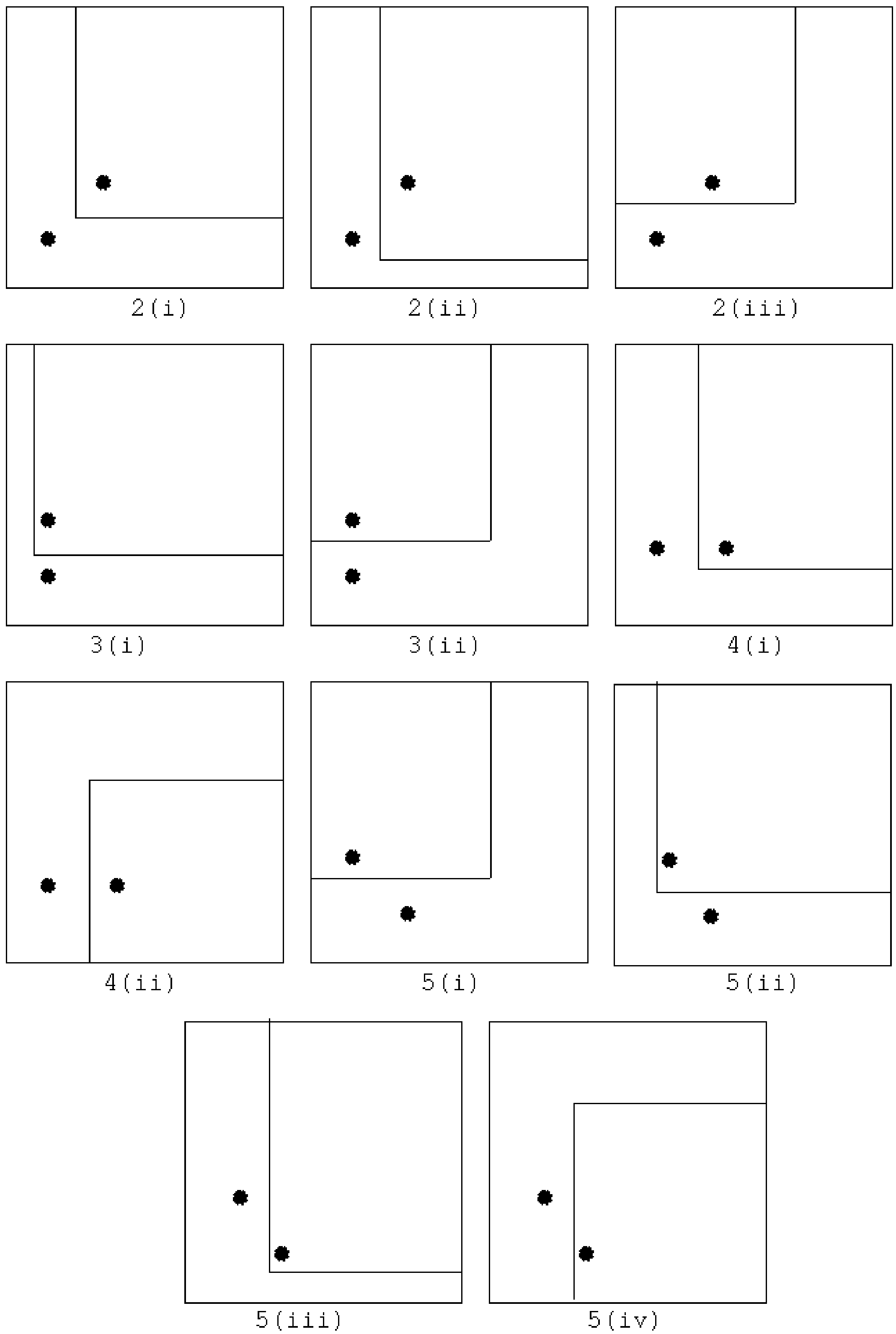

Figure 4.8: Different cases when facilities are in same quarters

STEP-4:Find minimum among the distance stored in each possibility and **return** *corresponding partition*

Analysis of Algorithm 2

Let C be the complexity of NMinimize[3] function then complexity of Algorithm 2 is shown in table 4.1

Now we can move to our next SubProblem.

STEP	Complexity
1	$O(1)$
2(a)	$O(1)$
2(b)	$C.O(1)$
3	$O(1)$
4	$O(1)$
Total	C

Table 4.1: Complexity of Algorithm 2

4.3 SubProblem 3

In this SubProblem we are looking for equipartition of the region having n *facilities* already installed. We will use *Weighted Voronoi diagram* and Secant method as our tool. Problem can be stated as

In a square-region n facilities are already installed, can we partition the region into n equal parts such that each part consists of exactly one facility?

Before giving the algorithm for this problem let us have a look on some technical preliminaries.

4.3.1 Technical Preliminaries

Voronoi Diagram(VD)[7]:-Let $S = f_1, f_2, ..., f_n$ be the set of facility installed in region R. VD of R is the subdivision of plane into $n-cells$ where i^{th} cell $V(f_i)$ can be defined as $V(f_i) = \{x|d(x, f_i) \leq d(x, f_j)\forall j \neq i\}$. Where $d(x, y)$ is Euclidean distance between x and y.

Weighted Voronoi Diagram(WVD):-Let $S = f_1, f_2, ..., f_n$ be the set of facility installed in region R. WVD of R is the subdivision of plane into $n - cells$ where i^{th} cell $V(f_i)$ can be defined as $V(f_i) = \{x|d(x, f_i) - w_i \leq d(x, f_j) - w_j \forall j \neq i\}$. Where $d(x, y)$ is the square of Euclidean distance between x and y and w_i is the weight associated with i^{th} region. An example of Weighted Voronoi Diagram has been shown in figure 4.9.

We have also observed that a weighted voronoi region may be empty or it it does not contain facility see figure 4.10. We call it Degenerate WVD(DWVD).

Let $f(x) = 0$ be an equation of x, x_i and f_i denotes value of x and $f(x)$ in i^{th} iteration. Now Secant Method [4] for solving this equation is given as $x_{i+1} = x_i - \frac{x_i - x_{i-1}}{f_i - f_{i-1}}$. We are using Secant

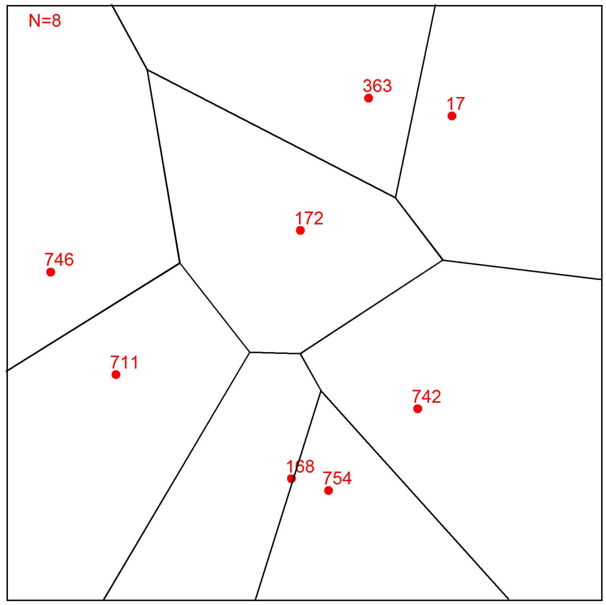

Figure 4.9: Example of weighted voronoi diagram

Method because of two reasons (i) we need not to calculate derivative like Newton-Raphson Method [4]. (ii) Need not to choose x_i and x_{i-1} such that $f(x_i)f(x_{i-1}) < 0$ like Regula Falsi Method [4], and its convergence is also better than Regula Falsi [4].

Computation of WVD defines region and its associated area. It means area of a region is a function of its weight, by changing weights appropriately one can get desired area. It is also important to note that change of weight is not only changing area of the corresponding region but it also changes areas of other regions. So we are using simultaneous system of equations for finding appropriate weights. Here function of weight $f(w_i)$=average area$-$area of the region i. In Secant Method two initial values and two function values are required to start iteration so for this purpose we are finding $w_i' = w_i - h$ and corresponding $f(w_i')$. Here value of h is usually chosen very small. While computing WVD there is a possibility that we may get DWVD. So calculation of WVD immediately followed by check for DWVD. Algorithm terminates without giving desired results when it finds DWVD. We have used ϵ to denote desired accuracy. Condition in while loop of STEP 1 in *Algorithm 3* ensures that areas in all the regions will be close to average area.
Algorithm 3 is not only useful in calculating equipartition region. One may change $f(w_i)$ appropriately and get desired area of region i, for example when users of facilities are not uniformly distributed across the region but we want equipartition of the region in terms of users. In this case also we can use *Algorithm 3* with suitable weight functions.

Algorithm 3

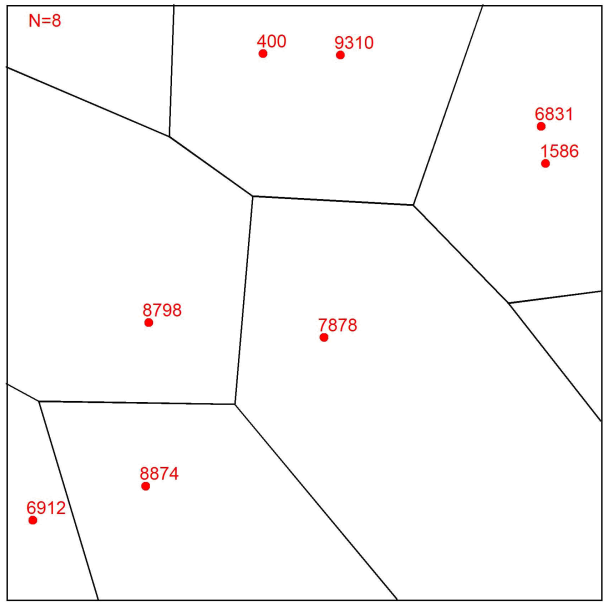

Figure 4.10: Example of weighted voronoi diagram

STEP 0:-for each facility i do

initialize w_i and assign 0 to $flag_i$

compute WVD

if WVD is degenerate then exit

for each i calculate $f(w_i)$ = average area−area of the region i

STEP 1:-while(any $f(w_i) > \epsilon$)

for each point i do

if($flag_i$==0)

$w'_i = w_i - h$

compute WVD

if WVD is degenerate then exit

$f(w'_i)$=average area−area of the region i

end if

$flag_i = 1;$

$w''_i = w_i - \frac{w_i - w'_i}{f(w_i) - f(w'_i)}$

$f(w'_i) = f(w_i)$

$w'_i = w_i$

$w_i = w''_i$

compute WVD

if WVD is degenerate then exit

$f(w_i)$=*average area−area of the region i*

end for

end while

Convergence of *Algorithm 3* is slow. In next chapter we will show results as well as total time to run the algorithm for some inputs. Some times algorithm diverges because of generation of DWVD. There are mainly three questions which remain unanswered (i) For better convergence what should be initial weights ?(ii) What will be relation between points, algorithm will surely diverges ? and (iii) What will be bound on number of iterations in case of algorithm converges ?

In spite of these *Algorithm 3* is the *F*irst algorithm which claims equipartition of a region. It is also directly applicable in the industry because it incurred one time cost in running algorithm but one can get exact idea of shape of regions. Hence slow convergence does not effect too much in this case.

Chapter 5

Experimental Results

We have implemented all *S*ubProblems in java. In SubProblem 2 we are using package JLink for accessing Mathematica functions [3] in java. In this chapter mainly we are presenting results of SubProblem 3. All programs run in Intel dual core processor 2.66 GHz with 512 MB RAM.

In table 5.1 we have shown final weights and time taken to converge *Algorithm 3* for different number of points and with various desired accuracy. Here symbols are defined as NOP=No of Total Points, InitialFig= Initial figure reference, FinalFig= Final figure reference, PN= Point Number, x/y =x/y-coordinate of point, InitialWt =Initial Weights, FinalWt=Final Weights,Acurcy= Desired Accuracy.

We can see from table 5.1 some weights has been decreased and some has been increased at the

NOP	InitalFig	FinalFig	PN	x	y	InitalWt	FinalWt	Acurcy	Time (in secs)
2	figure5.2	figure 5.3	1	204	232	10000	12643	0.5	4815
			2	83	81	10000	7849		
2	figure5.2	figure 5.4	1	204	232	8000	9845	5	4149
			2	83	81	8000	6249		
2	figure5.2	figure 5.5	1	204	232	10000	14845	10	3147
			2	83	81	10000	7249		
4	figure5.6	figure 5.7	1	233	260	10000	13743	0.5	24736
			2	133	84	10000	8539		
			3	342	269	10000	9452		
			4	312	96	10000	10249		
4	figure5.6	figure 5.8	1	233	260	8000	9743	5	22539
			2	133	53	8000	8889		
			3	342	269	8000	9853		
			4	312	96	8000	6145		

NOP	InitalFig	FinalFig	PN	x	y	InitalWt	FinalWt	Acurcy	Time (in secs)
4	figure5.6	figure 5.7	1	233	260	10000	12443	10	18915
			2	133	53	10000	8845		
			3	342	269	10000	7552		
			4	312	96	10000	10389		
4*	figure5.1	-	1	309	221	10000	-	10	-
			2	114	84	10000	-		
			3	326	214	10000	-		
			4	260	183	10000	-		
4*	figure5.1	-	1	309	221	8000	-	20	-
			2	114	84	8000	-		
			3	326	214	8000	-		
			4	260	183	8000	-		
8	figure5.10	figure 5.11	1	15	347	10000	13743	0.5	38649
			2	283	278	10000	9763		
			3	140	81	10000	6515		
			4	372	38	10000	12789		
		-	5	78	55	10000	8539		-
			6	314	240	10000	8552		
			7	162	228	10000	11402		
			8	40	182	10000	9682		
8	figure5.10	figure 5.12	1	15	347	10000	11743	5	34447
			2	283	278	10000	9943		
			3	140	81	10000	7645		
			4	372	38	10000	14784		
		-	5	78	55	10000	6574		-
			6	314	240	10000	9874		
			7	162	228	10000	8741		
			8	40	182	10000	12451		
8	figure5.10	figure 5.13	1	15	347	10000	9763	10	28524
			2	283	278	10000	8954		
			3	140	81	10000	7985		
			4	372	38	10000	13547		
		-	5	78	55	10000	7845		-
			6	314	240	10000	9574		
			7	162	228	10000	7988		
			8	40	182	10000	15874		

Table 5.1:

time of convergence. Also in case of point 4*, algorithm diverges in both accuracy levels. Now we are showing some figures as referred in table 5.1. Number used in figures shows area of corresponding region.

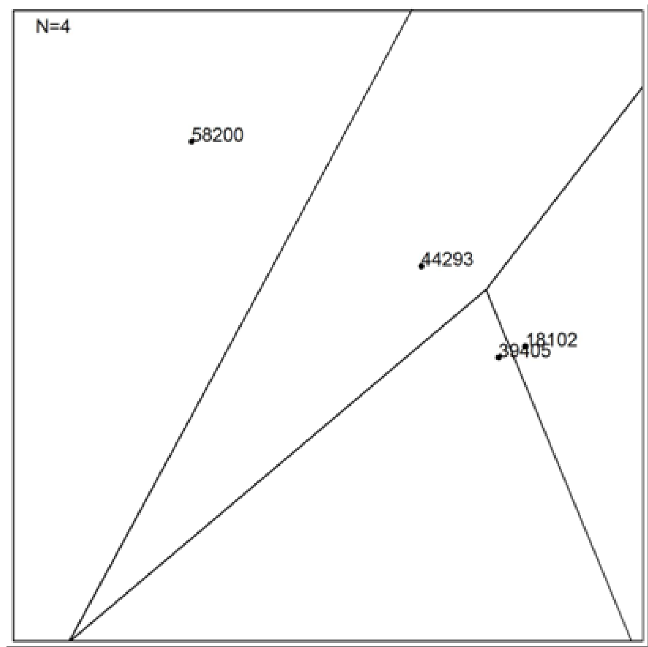

Figure 5.1: Initial shape of region containing four points for which algorithm did not converge

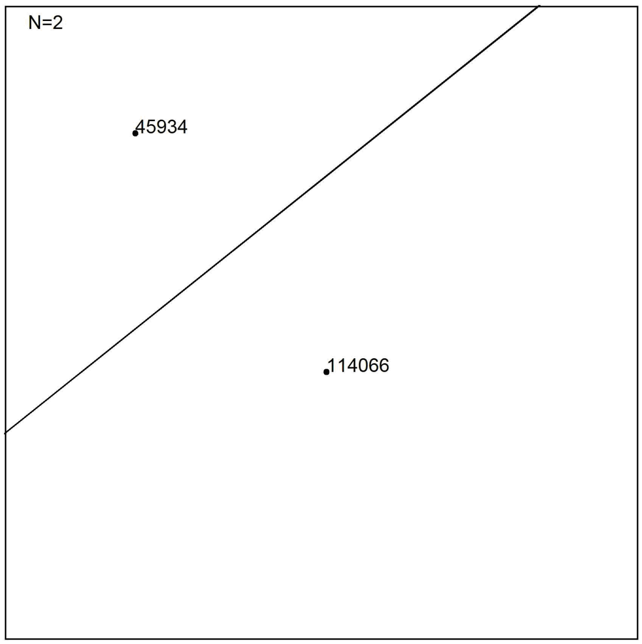

Figure 5.2: Initial shape in two points

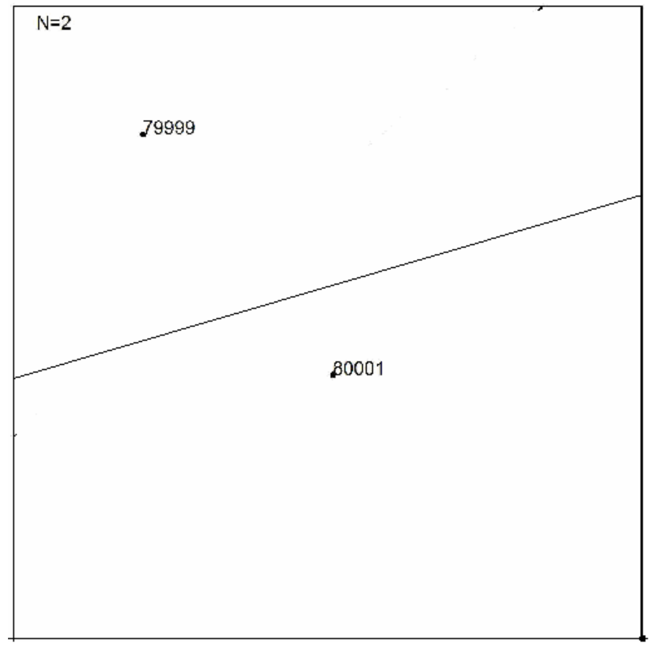

Figure 5.3: Final shape with accuracy 0.5

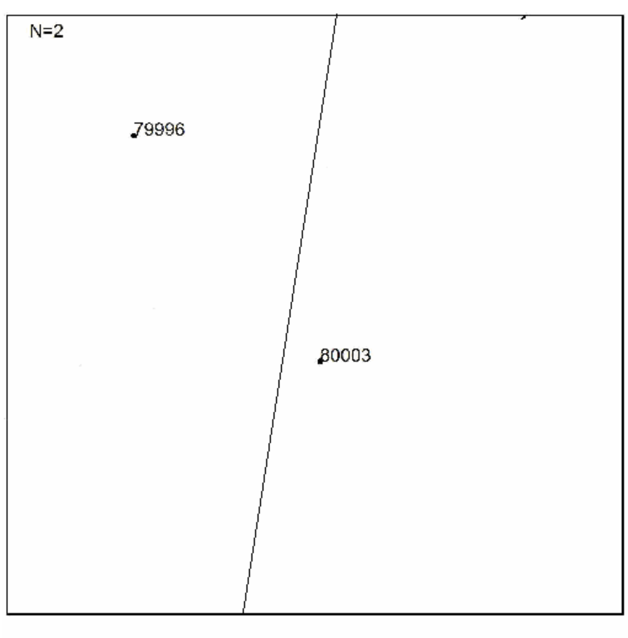

Figure 5.4: Final shape with accuracy 5

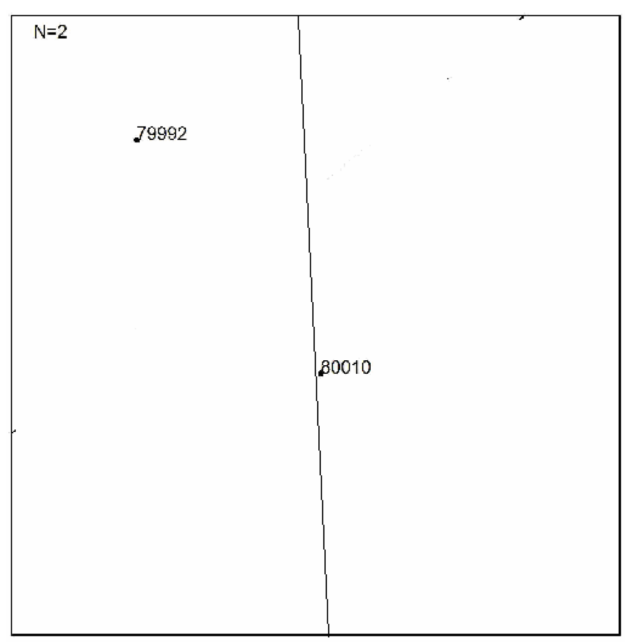

Figure 5.5: Final shape with accuracy 10

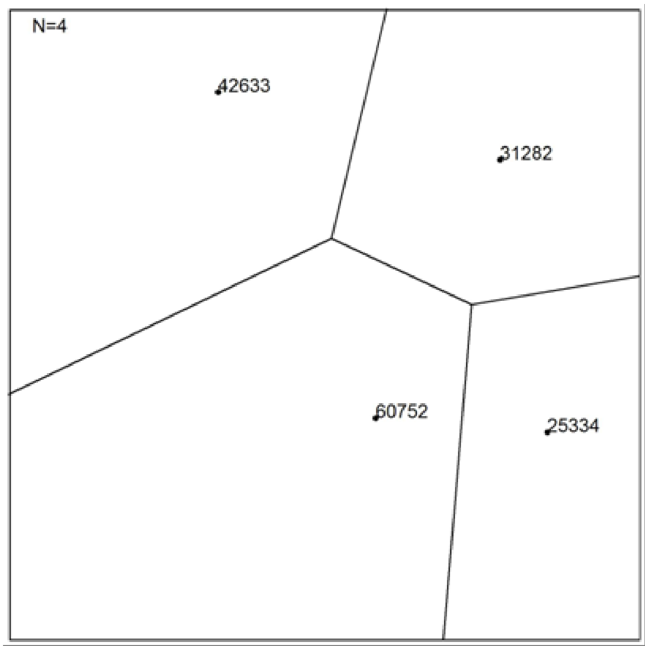

Figure 5.6: Initial shape in four points

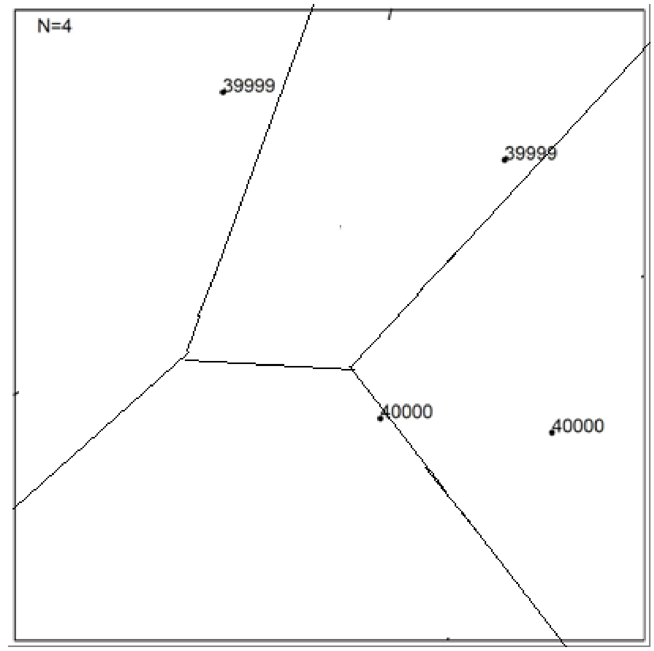

Figure 5.7: Final shape with accuracy 0.5

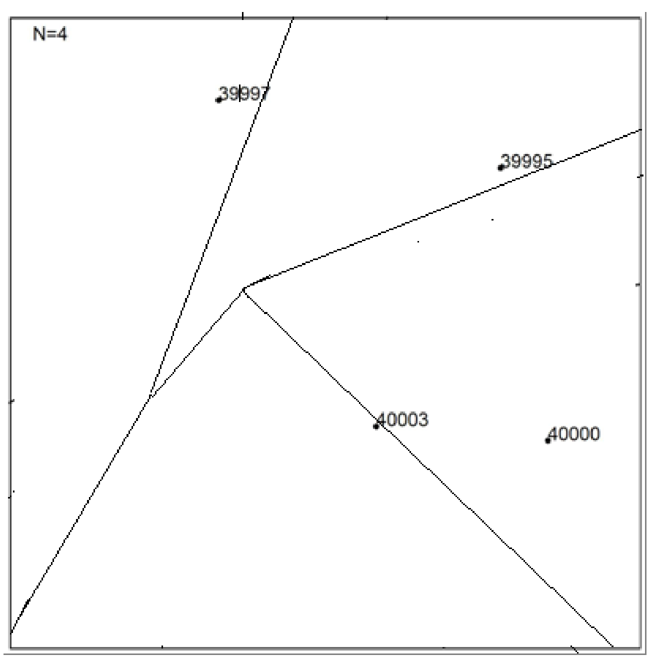

Figure 5.8: Final shape with accuracy 5

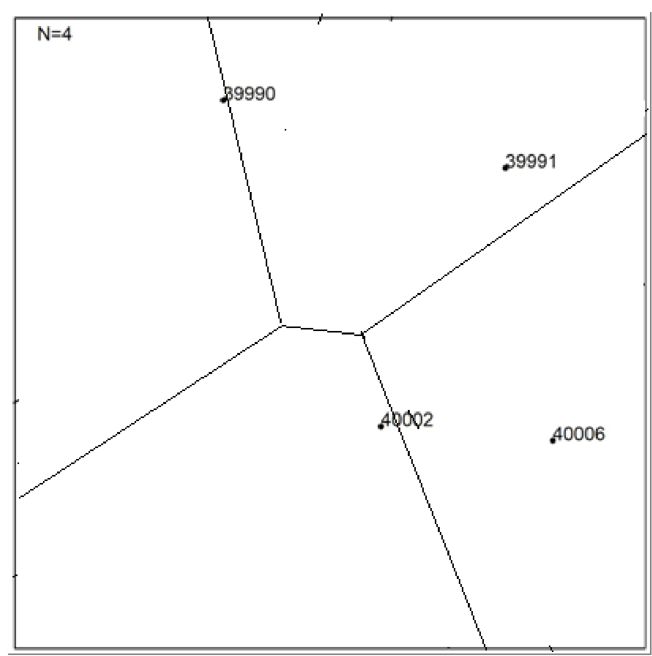

Figure 5.9: Final shape with accuracy 10

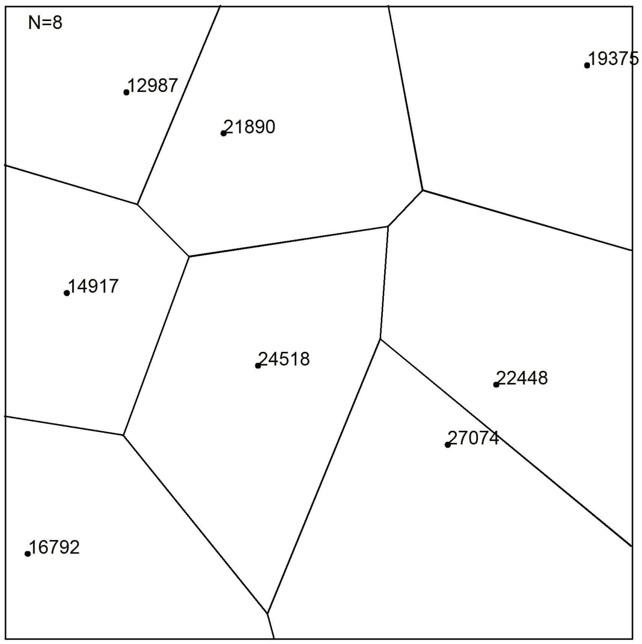

Figure 5.10: Initial shape in eight points

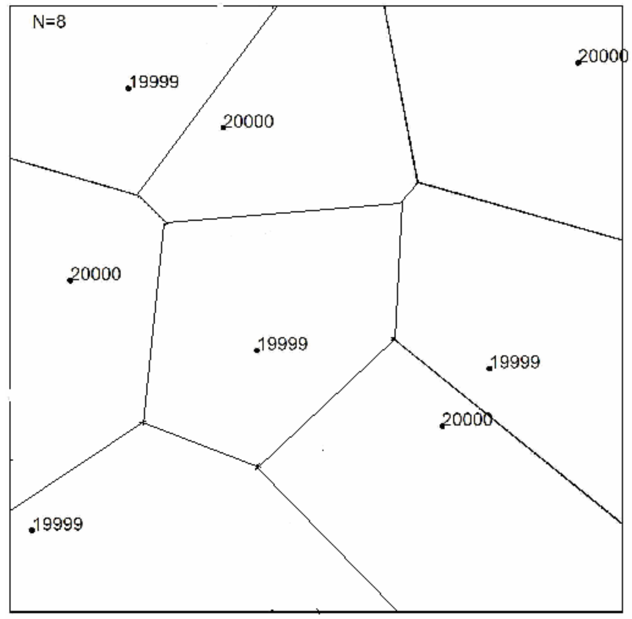

Figure 5.11: Final shape with accuracy 0.5

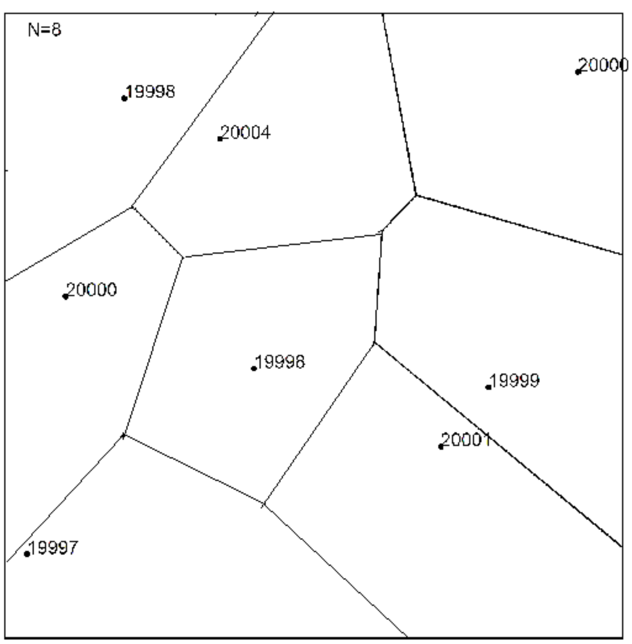

Figure 5.12: Final shape with accuracy 5

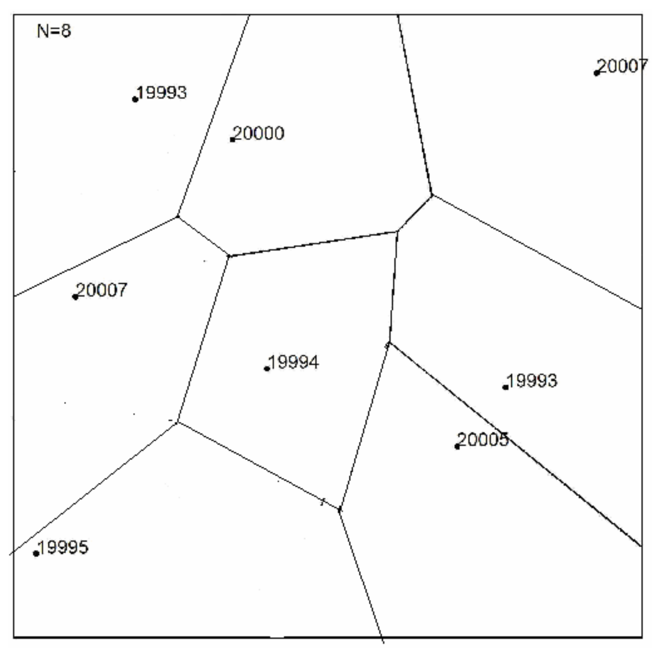

Figure 5.13: Final shape with accuracy 10

Chapter 6

Conclusions

In this thesis we have presented partitioning of a square region with different constraints. In SubProblem 1 we have seen that a single line segment can not always partition square region in two equal parts. While in SubProblem 2 we have marked that one horizontal and one vertical line segment is sufficient for partitioning a square region in two equal parts. We have also minimized the maximum distance between user and facility which resides in concave region. This was motivated from observation that usually a concave region residing user has to travel more than convex users. These two sub problems has been solved for particularly when number of facility is exactly two. Finally we have given a solution which equipartition the region containing n facilities, described in *Algorithm 3*. This algorithm has slow convergence as we have seen in results. Some time algorithm diverges as in case of DWVD. So a more efficient algorithm is required. In spite of these *Algorithm 3* is the First algorithm which claims equipartition of a region. It is also directly applicable in the industry because it incurred one time cost in running algorithm and one can get exact idea of shape of regions. Hence slow convergence does not effect too much in this case.

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