Location Estimation in a Terrain and Related Problems

Report Sumitted

By

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CERTIFICATE

This is to certify that the thesis entitled "**Location Estimation in a Terrain and Related Problems**" is submitted by **Amit Tripathi** in the partial fulfilment of the degree of **M. Tech.** in **Computer Science** at Indian Statistical Institute, Kolkata. It is fully adequate, in scope and quality as a dissertation for the required degree.

The thesis is a faithfully record of bona fide research work carried out by Amit Tripathi under my supervision and guidance. It is further certified that no parts of this thesis has been submitted to any other university or institute for the award of any degree or diploma.

Dr. Sandip Das
(Supervisor)

Countersigned
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Date:

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Place : Kolkata
Date : Amit Tripathi

Dissertation report

Contents

Abstract

In this report we present an efficient algorithm for finding Location of an object explorer in a given terrain with the help of some known sensor nodes under the assumption that all the reflections are specular. We present a geometric algorithm for location estimation, the running time of the algorithm is $O(n \log n + K)$ with $O(n)$ space, where n is the number of reflecting surfaces in the terrain and K is a constant which depends on the complexity of the terrain.

Keywords: Interference, Fading, Specular reflection, Diffused reflection, Transmission - delay, Signal diversity, Scattering, Time of arrival, Distance of arrival, Angle of arrival.

1 Introduction

General geolocation estimation problem is defined as the problem of finding the location information of an unknown object/sensor node by visibility or by communicating with the object under some known parameters. For example, given the geography of area under consideration with some known sensor nodes and objective is to locate unknown object explorer.

Generally geolocation estimation problem can be viewed in two frames: first is geolocation identification in indoor environments and second is geolocation identification in outdoor environments (due to difference between propagation characteristics of the indoor and outdoor environments).

Because of the nature of the environment, indoor radio channels suffer from extremely serious multi-path conditions that make outdoor propagation models non-applicable for indoor. In addition, indoor radio propagation models are not suitable for geolocation applications where time of arrival (TOA) is the important parameter to measure [1]. In addition, a model is necessary to model the distance error measured from the estimated time of arrival (TOA).

The applications of the problem in indoor environments range from commercial and residential services. For example, to track people with special needs, navigating policemen, firefighters and soldiers to help them complete their missions[1]. Problem in outdoor settings also has a series of applications e.g. in sensor networks, in military and rescue operations, Robot motion planning etc.. Because of this, the characterization of the performance of geolocation algorithms is an important issue.

Most of the location identification schemes can only identify a location area comprised of a number of *cells*, or at best a single cell, which may extend over an area of a few square kilometers. It is essential to know exact location for immediate services, So far, there is no practical location management scheme that is capable of finding the exact location with such an accuracy [26].

The global positioning system (GPS)[2, 3] is perhaps the most widely publicized location sensing system. Unfortunately, GPS does not scale well in dense urban areas or in indoor locations. Modeling of the radio propagation(or Electromagnetic Signal propagation) environment helps in providing a more accurate location estimate by mitigating the effect of errors.

In this report, we proposed an algorithm with complexity $O(n \log n + K)$ (Where n is the number of reflecting surfaces in the terrain and K is a constant which depends on the complexity of the terrain) to solve the problem of location identification for an object explorer, in a terrain known to us with the help of some sensor nodes. Idea of our algorithm is that we reduce the set of possible locations in successive steps. We assumed that all the reflections are specular (Of course most of the reflections from rough surfaces are diffused even then we can place good reflectors at some known locations having prior knowledge of terrain). We also assumed that explorer is reachable by at most one reflection.

2 Problem Definition

Given m source sensor nodes $S_1, S_2, S_3, ..., S_m$ and n specular line reflectors $R_1, R_2, R_3, ..., R_n$ of finite length such that m is constant w.r.t. n. Let measured distances from $S_1, S_2, S_3, ..., S_m$ to object explorer P are $d_1, d_2, d_3, ..., d_m$ respectively. Find the location of P.

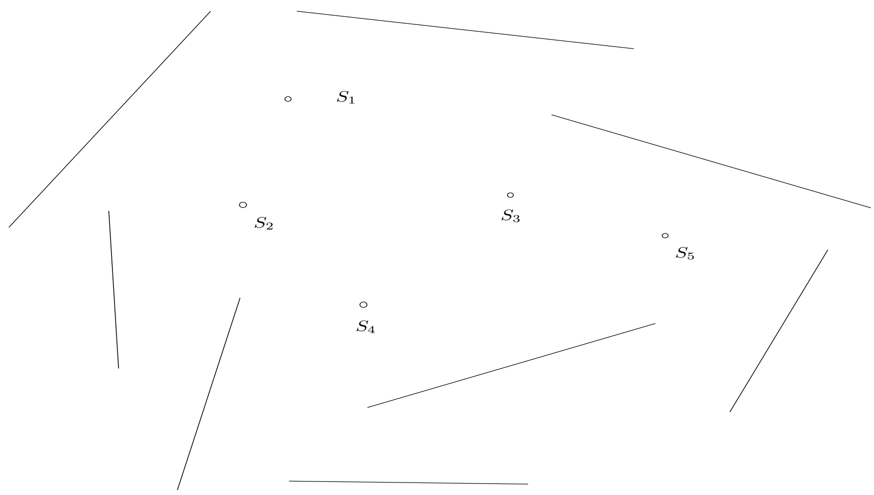

Figure 1: Multiple sources and multiple reflectors

3 Related Work

Generally two different location schemes have been extensively investigated [6] − [8]: one is the time based scheme, such as to measure the time of arrival (TOA) or time difference of arrival (TDOA) of incoming signals, the other is to measure the angle of arrival (AOA), which involves the use of an antenna array. Because

TOA/TDOA and AOA approaches have their own advantages and limitations, a hybrid TDOA/AOA mobile location scheme is proposed in [9], [10].

Most of the geolocation systems perform distance measurements for location identification. These measurements can be performed in a variety of ways, such as Angle of Arrival (AOA), Time of Arrival (TOA) Or Received Signal Strength(RSS) [1]. Of these, the TOA technique is the most popular for accurate geolocation systems. As the name implies, TOA-based systems use the time of arrival of the first path to estimate distance. However, the unique nature of the indoor propagation environment creates certain challenges in estimating the TOA of the first path accurately [11]. For TOA formulation, the direct (LOS) path which connects the transmitter and receiver is needed to calculate the corresponding range between them. However, in real wireless environment, especially in dense urban scenarios, the LOS path is often blocked and the communications is conducted through reflected, diffracted and scattered rays due to the interaction with objects in the propagation environment. In term of TOA estimation, this phenomenon leads to the positive bias in the TOA estimation and finally causes errors in location estimation. According to the measurement conducted in [11], the mean and standard deviation of range errors are on the order of $513m$ and $436m$ respectively.

The major error sources in the mobile location include Gaussian measurement noise and non-line-of-sight (NLOS) propagation error, the latter being the dominant factor [6]. To protect location estimates from NLOS error corruption, NLOS error mitigation techniques have been investigated extensively in the literature $[12] - [21]$.Several authors have, however attempted for mitigating the effect of NLOS errors $[5, 12, 15, 18, 22 - 24]$.

Models for distance measurement errors [11][24] are based on TOA-estimation techniques, and have characterized the distance errors as a function of the bandwidth of the system used to gather TOA measurements, and in the presence of Line-of-Sight (LOS), and Obstructed LOS (OLOS) propagation conditions.

Some authors [25, 26] provide algorithms for Location estimation in terms of region either with no definitive error bound or only with some probabilistic error.

In this report our objective is to find location of the explorer in a given terrain with the help of some known sensor nodes. As we have prior knowledge of the terrain, without loss of generality, we can take n-reflectors/n-reflecting surfaces in the terrain with known positions. We assume that all the reflections are specular and strength of signals is such that explorer is reachable by at most one reflection. There are some (constant w.r.t n) known (in terms of location) sensor nodes in the system.

4 System Model

General geolocation system proposed by *K. Pahlavan, X. Li, and J. P. Makela*[1] consists of three main parts: a number of location sensing devices, a positioning

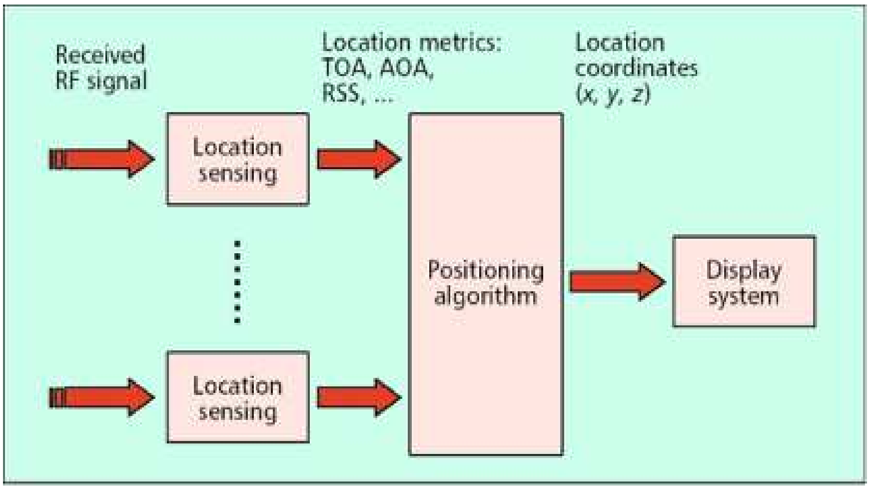

Figure 2: a functional block diagram of wireless geolocation system.

algorithm, and a display system (Fig.1.). The location sensing devices measure the metrics related to the distance between a mobile terminal (MT) and a known reference point (RP) such as TOA, angle of arrival (AOA), received signal strength (RSS), etc. The positioning algorithm processes the metrics reported by the location sensing elements, to estimate the location coordinates of MT. The display system illustrates the location of the MT to the users.

We communicate with the explorer by sending electromagnetic signals. When the signals (coming from different paths) combined at a point, the phase differences between the various rays having traveled paths of different lengths give rise to an interference pattern likely to cause fading or significant degradation of the signal. This fading due to the multiple paths may lead to significant degradation, in terms of both the quality of the signal received and system performance. Furthermore, the location of the fading changes over time depending on changes in the environment such as the presence of new objects or the passage of people. But to combat fading, we can have an antenna system for the transmission of electromagnetic signals [4].

For a signal an LOS path or direct path, is the straight line connecting the transmitter (source) and receiver (unknown object). NLOS signals occur due to multi-path conditions in which received signals come from either reflected, diffracted or scattered paths, thus introducing excess path lengths in the actual Euclidean distance between the transmitter and the receiver. The NLOS error is defined to be the excess distance traversed compared to the direct path and is always positive [5, 6].

In this report our objective is to design a positioning algorithm to locate the explorer.

Definition

Consider a source node S with its location and a mirror like (specular) line reflector $MM^{'}$ of infinite length such that length of perpendicular drawn from S to $MM^{'}$ is strictly less than a known real constant d. A signal is orginated from S and recieved at some point P after getting exactly one reflection from $MM^{'}$ such that measured distance from S to P via one reflection is d, then the locus of P is given by the curve Φ.

Lemma 1

Assuming axis are transformed such that S is origin and cartesian equation of $MM^{'}$ is $Y = c$,
In cartesian form, curve Φ can be described by the equation:

$$x^2 + (y - 2c)^2 = d^2 \tag{1}$$

where $-\sqrt{d^2 - c^2} <= x <= \sqrt{d^2 + c^2}$ and $(2c - d) <= y <= c$.
and in polar form, curve Φ can be described by the equation :

$$R^2 - 4cR\sin\phi = (d^2 - 4c^2) \tag{2}$$

where $sin^{-1}\left(\frac{c}{d}\right) <= \phi <= sin^{-1}\left(\frac{-c}{d}\right)$ and (R, ϕ) are polar co-ordinates.

Proof :

Draw a ray from S intersecting $MM^{'}$. Find a point say $P^{'}$ on the ray such that $P^{'}S = d$ and $P^{'}$ lies opposite side of S w.r.t $MM^{'}$. Let $SP^{'}$ intersects with $MM^{'}$ at L (such points will always exist as length of perpendicular drawn from S to $MM^{'}$ is strictly less than d). Since length of $MM^{'}$ is infinite so there

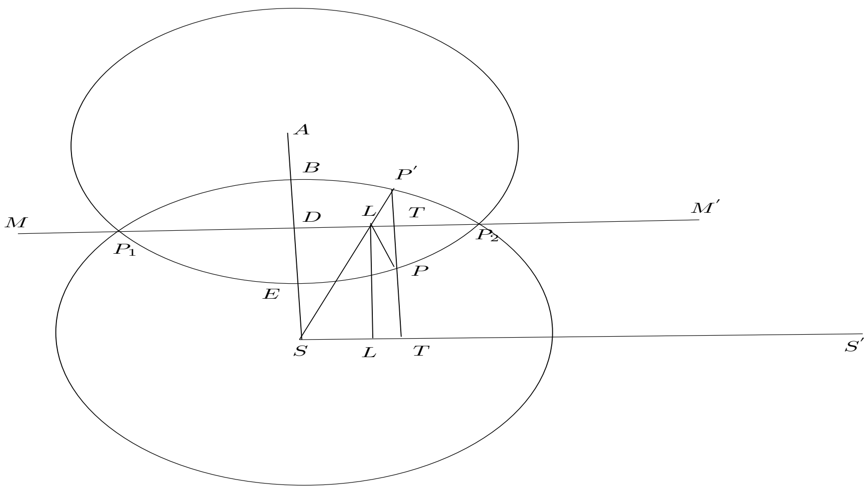

Figure 3: Formation of possible location curve w.r.t single source and single reflector

exist two points P_1 and P_2 such that $SP_1 = d = SP_2$. Segment P_1P_2 is called complete reflector. Note that locus of $P^{'}$ is the arc P_1P_2 of the circle

$$x^2 + y^2 = d^2$$

lying opposite side of S w.r.t. $MM^{'}$. Note that P is nothing but mirror image of $P^{'}$ w.r.t. $MM^{'}$ (triangles $P^{'}LT$ and PLT are congruent), therefore locus of P is the mirror image of locus of $P^{'}$ w.r.t. $MM^{'}$ i.e. locus of P is the arc P_1P_2 of the circle

$$x^2 + (y - 2c)^2 = d^2$$

lying in the side of S w.r.t. $MM^{'}$. Note that co-ordinates of P_1 and P_2 are $(-\sqrt{d^2 - c^2}, c)$ and $(\sqrt{d^2 - c^2}, c)$ respectively. Co-ordinates of A, B, D and E are $(0, 2c), (0, d), (0, c), (0, 2c - d)$ respectively. Therefore locus of P is given by

$$x^2 + (y - 2c)^2 = d^2,$$

where $-\sqrt{d^2 - c^2} <= x <= \sqrt{d^2 + c^2}$ and $(2c - d) <= y <= c$

Let $x = R\cos\phi$ and $y = R\sin\phi$,then locus of P in Polar form can be written as

$$R^2 - 4cR\sin\phi = (d^2 - 4c^2),$$

where $sin^{-1}\left(\frac{c}{d}\right) <= \phi <= sin^{-1}\left(\frac{-c}{d}\right)$.

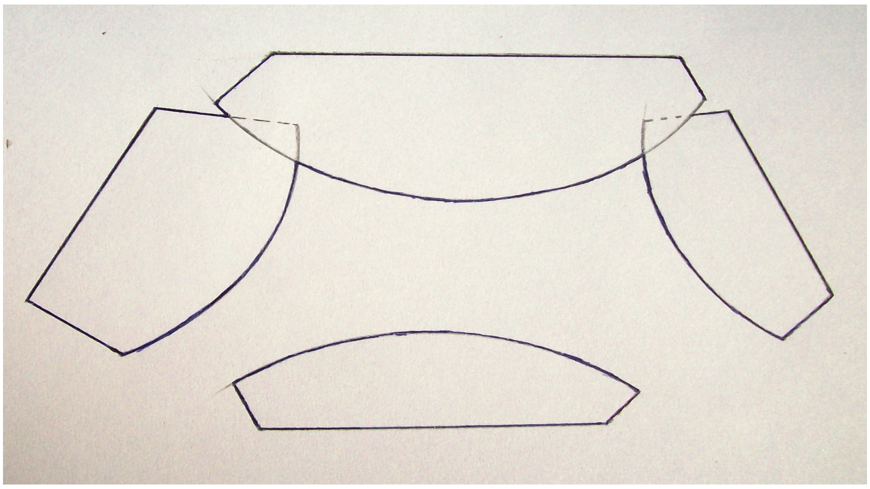

Figure 4: Examples of possible location curves w.r.t single source and multiple reflector

Aliter :

In the context of Fig.2., equation of $MM^{'}$ is $Y = c$, which can be written as

$$r = \frac{c}{\sin\theta} \tag{3}$$

where (r, θ) is any point on $MM^{'}$. Let S be the pole and horizontal through S be the initial line. Let $P = (R, \phi)$ be any point on the required curve and the corresponding point of reflection on $MM^{'}$ is $L = (r, \theta)$. Therefore

$$SL = r, SP = R, \angle S^{'}SL = \theta \text{ and } \angle S^{'}SP = \phi$$

In $\triangle SLL^{'}$,

$$LL^{'} = r\sin\theta \text{ and } SL^{'} = r\cos\theta$$

In $\triangle SPT^{'}$,

$$PT^{'} = R\sin\phi \text{ and } ST^{'} = R\cos\phi$$

In $\triangle P^{'}LT$,

$$P^{'}T = (d-r)\sin\theta \text{ and } LT = (d-r)\cos\theta$$

Now

$$ST^{'} = SL^{'} + L^{'}T^{'} = SL^{'} + LT = r\cos\theta + (d-r)\cos\theta = d\cos\theta$$
$$R\cos\phi = d\cos\theta \tag{4}$$

and

$$PT^{'} = P^{'}T^{'} - (P^{'}T + TP) = P^{'}T^{'} - 2P^{'}T = d\sin\theta - 2(d-r)\sin\theta = (2r-d)\sin\theta$$

using equation 3, we have

$$R\sin\phi = 2c - d\sin\theta \tag{5}$$

Eliminating θ from equations 4 and 5 we get,

$$R\sin\phi = 2c - \sqrt{d^2 - R^2\cos^2\phi}$$

$$R^2 - 4cR\sin\phi = (d^2 - 4c^2),$$

where $sin^{-1}\left(\frac{c}{d}\right) <= \phi <= sin^{-1}\left(\frac{-c}{d}\right)$.

Definitions

- $E = (0, 2c - d)$ is called *root* of the curve Φ given by equation (1).
- $A = (0, 2c)$ is called *center* of the curve Φ given by equation (1).
- Line $Y = (d - 2c)$ is called *directrix* of the curve Φ given by equation (1).
- $D = (0, c)$ is called *focus* of the curve Φ given by equation (1).
- Line $X = 0$ is called *axis* of the curve Φ given by equation (1).
- $P_1 = (-\sqrt{d^2 - c^2}, c)$ and $P_2 = (\sqrt{d^2 - c^2}, c)$ are called *end points* of the curve Φ given by equation (1) and segment P_1P_2 of the reflector is called *complete reflector.*
- Curve Φ is called *minimum distance curve* or simply *MDT curve* w.r.t. a given source and a given reflector.
- Given a source and a reflector AB of finite length which may or may not complete, with MDT curve say PQ such that P corresponds to reflection point A and Q corresponds to reflection point B. Join A and P, B and Q and hence form a simple closed curve $APQB$.The closed region bounded by this simple closed curve $APQB$ is called ***region of absence w.r.t single source and single reflector*** in the sense that inside this region no explorer is present.
- Circular part of boundary of a region of absence is called ***possible location curve***.

Observations

1. For each point on P_1P_2, where $P_1 = (-\sqrt{d^2 - c^2}, c)$ and $P_2 = (\sqrt{d^2 - c^2}, c)$, there is a unique point P on the curve Φ satisfying shortest distance criterian (path from S to P via one reflection is shortest).
2. Length of complete reflector P_1P_2 is $2\sqrt{d^2 - c^2}$.
3. Curve Φ is a circular arc and symmetric about its axis.

Definition

A Jordan curve is a plane curve which is topologically equivalent(i.e. homeomorphic image) to the unit circle i.e. it is simple and closed.

Result [***Edelsbrunner et al***[53]]

Maximum combinatorial complexity of finding union of n closed Jordan regions bounded by Jodan curves is $\Theta(n)$.

Lemma 2

Given a source node S with its location and n ($n \geq 3$) number of mirror like(specular) line reflectors of finite lengths such that each of the reflector lie completely in the interior of a circle with center S and radius d. A signal is orginated from S and recieved at some point P after getting exactly one reflection from some reflector such that measured distance from S to P via one reflection is d and d is smallest among all such distances, then possible location curves for P can be found in $O(n)$ time with $O(n)$ space.

Proof :

For each reflector, find region of absence w.r.t. S, with the help of lemma 1 this can be done in $O(n)$ time using $O(n)$ space. Find the union U of these n regions of absence. Note that any two closed curves must intersect in an even number of points (assuming non-degenerate configuration). Edelsbrunner et al[53] have shown that the maximum combinatorial complexity of finding union of n closed Jordan regions bounded by Jodan curves is $\Theta(n)$. Moreover if pair of boundaries intersect in at most two points then boundary of the union contains atmost $(6n - 12)$ intersection points provided $n \geq 3$ and if pair of boundaries intersect in four or more points then boundary of the union may contain $\Omega(n^2)$ intersection points in worst case(Kedem et al [54]).Therefore in $O(n)$ time, $O(n)$ number of possible location curves can be found having $O(n)$ number of pairwise intersection points.

Definition :

Union U is called ***region of absence w.r.t single source and n reflectors***.

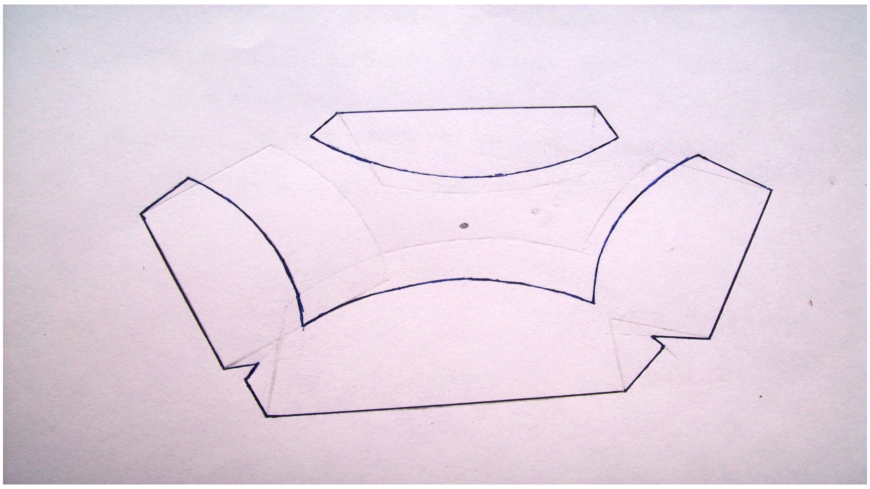

Figure 5: Region of absence w.r.t. single source and multiple reflectors

Result $[\boldsymbol{Chazelle}[49]]$

Intersection of two polygons having a total of n edges and K intersection points can be constructed in $O(n \log n + K)$ time and $O(n)$ space (theorem 7.8.1 pp. 268 *J.O' Rourke*).

Lemma 3

Given m sources $S_1, S_2, S_3, ..., S_m$ and n specular line reflectors $R_1, R_2, R_3, ..., R_n$ of finite lengths such that m is constant w.r.t. n. Let measured distances from $S_1, S_2, S_3, ..., S_m$ to some point P are $d_1, d_2, d_3, ..., d_m$ respectively and each of the reflector lie completely inside the interior of each of the circles of radius d and center at any source, where $d = \max\{d_1, d_2, d_3, ..., d_m\}$. If K is an upper bound to number of possible locations for P then possible locations for P can be found in $O(n \log n + K)$ time with $O(n)$ space.

Proof :

With the help of lemma 1 and 2, for each source we can find region of absence w.r.t. n reflectors in $O(n)$ time. So we have m non-convex polygons as regions of absence. Intersection points between these m non-convex polygons are the possible locations for P. If number of intersection points is K then we can find intersection points in $O(n \log n + K)$ time using $O(n)$ space [55]. Note that K depends on the complexity of the terrain.

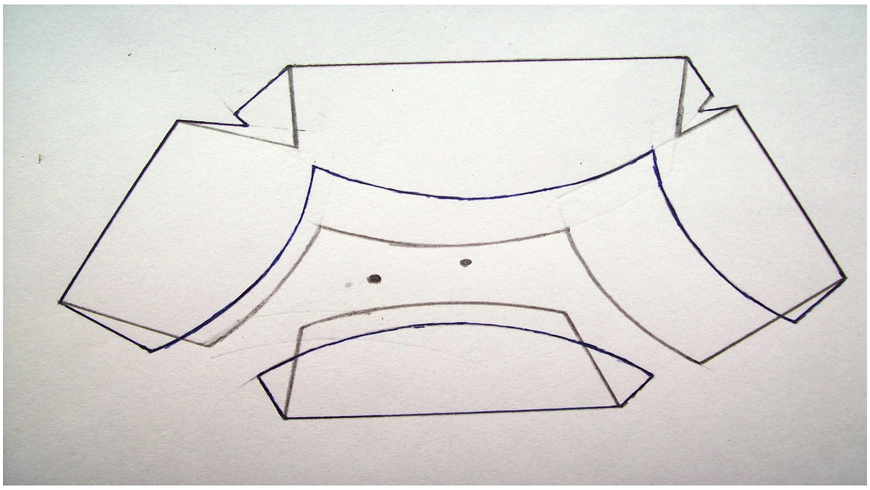

Figure 6: Region of absence w.r.t. two sources and multiple reflectors

Result [*Bentley, Ottman*[47] and *Clarkson, Shor and Mulmuley*[56][57]]

K intersections in a collection of n Jordan arcs in a two dimensional real plane can be reported in $O((n+K)\log n)$ time and expected running time can be improved up to $O(n\log n + K)$.

Lemma 4

Given m sources $S_1, S_2, S_3, ..., S_m$ and n specular line reflectors $R_1, R_2, R_3, ..., R_n$ of finite lengths such that m is constant w.r.t. n. Let measured distances from $S_1, S_2, S_3, ..., S_m$ to some point P are $d_1, d_2, d_3, ..., d_m$ respectively and each of the reflector lie completely inside the interior of each of the circles of radius d and center at any source, where $d = \max\{d_1, d_2, d_3, ..., d_m\}$. Then number of possible locations for P can be counted and reported in $O(n\log n + K)$ time.

Proof :

For each pair of source and reflector, find possible location curve with the help of lemma 1. Hence we have mn number of circular arcs. Note that m is constant w.r.t. n. If K is the number of intersection points for these mn arcs then these intersection points i.e. possible locations for P can be counted and reported in $O(n\log n + K)$ time [56][57](*Clarkson , Shor and Mulmuley*).

Result [*Rei*[58]]

Whether any pair of m convex n-gons intersects can be detected in $O(m\log m\log n)$ time, and whether they all share a common intersection point can be detected in $O(m\log^2 n)$ time.

Lemma 5

Given m sources $S_1, S_2, S_3, ..., S_m$ and n specular line reflectors $R_1, R_2, R_3, ..., R_n$ of finite lengths such that m is constant w.r.t. n. Let measured distances from $S_1, S_2, S_3, ..., S_m$ to some point P are $d_1, d_2, d_3, ..., d_m$ respectively and each of the reflector lie completely inside the interior of each of the circles of radius d and center at any source, where $d = \max\{d_1, d_2, d_3, ..., d_m\}$ then whether m sources are sufficient to locate P uniquely or not, can be decided in $O(n)$ time.

Proof :

For each pair of source and reflector, find region of absence with the help of lemma 1. Hence we have mn number of regions of absence. Each region of absence is a convex region. Now mn convex r-gons shares a common point or not can be detected in $O(mn\log^2 r)$ time [58]. Since m is constant w.r.t. n and $r = 4$ in our case therefore time complexity is $O(n)$.

5 Algorithm

Consider one source and n-refletors, in worst case, suppose all the reflectors intersects the circle of center as the source and radius as the measured distance. Then taking parts (created by due to intersection of circle and reflector) of reflectors as individual reflectors, in worst case we have $2n$ reflectors So now we have $O(n)$ reflectors in the interior of the circle and remaining outside the circle. Now consider part of reflector as an individual reflector if it is visible from the source. Hence possibly, increasing number of reflectors but still $O(n)$ because $O(n)$ reflectors has $O(2n)$ end points so shooting rays from source to all the $O(2n)$ end points we will have $O(2n)$ visibility sectors which implies that in worst case resulting number of reflectors will be double to the original number of reflectors. So without loss of generality, let all the n-given reflectors are circularly arrranged around the source. Now algorithm goes along following steps :

1. Check whether P is directly visible with the measured distance or P is invisible from the source, visibility can be decided and reported in $O(n)$ time for one source and n reflectors.
2. If P is (directly) visible from any of the source then location estimation is over.
3. If P is invisible from all the sources then with the help of lemma 1, for each source-reflector pair, find region of absence and possible location curve.
4. For each source, find regions of absence w.r.t. single source and n reflectors with the help of lemma 2.
5. Find intersection of regions of absence obtained in previous step using lemma 3 and find possible locations for P.
6. To locate P uniquely, supply minimum number of sources given by lemma 5.

6 Implementation efforts over region based location identification approach

In this section we are presenting details of implementation of existing region based approach[1].

6.1 Formal description

1. MATLAB implementation to generate a degree-controlled arbitrary graph over given number of nodes.
2. MATLAB implementation to generate region of residence around each beacon node.

[1] Refer [26]

3. C++ code using CGAL for getting viewed-region of residence for a non-beacon node i.e. Minkowski sum of region of residence of a beacon node with the circle having radius as its distance from the node to be identified.

4. C++ code using CGAL for Minkowski sum of two regions of residence.

5. C++ code using CGAL for intersection of two regions of residence.

6. C++ code using CGAL for finding convex hull of viewed-region of residence.

6.2 Implementation details

6.2.1 Error-bound description

Intersection of two convex polygons may involve following type of edge interactions:

- Intersection point of the two edges lie on both the edges.
- Edges will intersect under error bound in future and point of intersection will lie on exactly one of them.
- Edges will intersect under error bound in future and point of intersection will not lie any one of them.
- Edges are parallel under error-bound.
- Edges are co-incident.

Due to floating point operations in above cases we can report non-intersecting polygons as touching / intersecting / having one edge common polygons and vice-versa. Therefore we have to handle the error under given error-bound.

6.2.2 Error Handling

To take care of error that may occur in above cases under the given error-bound, our implementation will declare ***polygons do not intersect*** for any interaction of edges within given error-bound. This means that if two polygons intersect / touch / have a common edge, even then we will consider them as non-intersecting.

6.2.3 Files and description

1. MATLAB file graphgen.m generates a degree-controlled arbitrary graph which is stored in 20_graph.fig. Supporting files are init_nodes.m and degree_controlled_graph.m.

2. MATLAB file graphgen1.m generates a degree-controlled arbitrary graph in which, around each node region of residence has been generated, final figure is stored in file 20_graph.fig. Supporting files are init_nodes.m and degree_controlled_graph.m.

3. C++ code for Minkowski sum of region of residence of a beacon node with the circle having radius as its distance from the node to be identified is provided by the file exact_offset.cpp. Input is given by file and execution results are stored in files offset_P.txt, offset_Q.txt, offset_R.txt.

4. C++ code for Minkowski sum of two regions of residence is provided by the file sum_by_decomposition.cpp. Input is given by files and execution result is stored in a file ouput.txt.

5. C++ code for intersection of two regions of residence is provided by the file simple_join_intersect.cpp. Input is given by files and Execution results are stored in a file output.txt.

6. C++ code for finding convex hull of viewed-region of residence is provided by the file ch_example_from_cin_to_cout.cpp. Input is given by file and output is written into file.

7 Conclusion and Open Problems

We have presented a novel approach to the problem of location discovery in a terrain using computational geometric methods. In contrast to existing approaches for location discovery, proposed algorithm gives a bound on possible locations for an unknown sensor node such that possible locations can be counted and reported in $O(n \log n + K)$ time, where K depends on the complexity of the terrain which indicates that proposed algorithm is senstive to the complexity of the terrain.
A number of related problems are still waiting for the solution :

- Find an efficient and optimal placement strategy for placing source nodes, so that minimum number of source nodes are required for location discovery.

- Suppose n-number of source locations are given, find minimum number of reflectors providing optimal placement in order to locate unknown node efficiently.

- Can we reduce complexity of location discovery if sources can exchange the information?

- How to solve the problem of location discovery if nodes are mobile ?

- How to solve the problem of location discovery if there are some non-reflecting obstacles in the terrain.

- How to solve the problem of location discovery if all the reflections are diffused.

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