A METHOD OF ESTIMATING THE REDUCTION IN BIRTH RATE BY STERILISATION OF MARRIED COUPLES

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SUMMARY. Adopting a generation approach retrospectively, and using the principles of multiple decrement tables, and on the basis of a given condition of eligibility, a given range of age of wife to come under the scope of sterilisation and a given rate at which sterilisation is sought to be performed, the proportion of unsterilised married couples at various ages of wife in the reproductive period and at various intervals from the initial year of sterilisation are obtained, from which the birth rate (All-India rural) at successive intervals can be calculated. Assuming that married couples with ages of wife between 20 and 44 and with at least 3 children are eligible for sterilisation, 7 sterilisations per thousand population per annum would bring down the birth rate by only about 20% in 10 years, but with an increased rate of 14 per thousand, the drop would be about 40% in 10 years. Such a drop however involves about 5 million sterilisations annually in the rural sector.

1. Sterilisation as a method of reducing the birth rate seems to have been receiving increasing attention in this country in recent times. This is perhaps due to the growing realisation of the practical difficulties in the way of an effective implementation of the Family Limitation Programme and hence to search for an alternative or additional and more effective method of controlling fertility. One might wonder if the difficulties in promoting a sterilisation programme and its adoption by the people might not be as great; but it is not proposed in this paper to discuss them or how they may be overcome, nor to go into the merits (or defects) of sterilisation from its social, psychological, political and other aspects. What is attempted in this paper is to develop a methodology by which given a certain rate of sterilisation or the numbers of sterilisation performed year to year, and in the absence of any other method of reducing fertility, the drop in the birth rate in a period of years may be estimated. This would also give an idea as to the adequacy or otherwise of the sterilisation currently performed or expected to be performed in the future, or to the extent to which a suitable sterilisation programme must be pushed in order to reach the level of adequacy.

2. Considering the level of fertility in the country in terms of the fertility performance of married women, we do not in this paper distinguish between male and female sterilisations. Sterilisation of a wife at a certain age, or that of her husband, puts the wife out of reproductive action in either case, and will therefore be considered simply to be sterilisation of a married woman. Sterilisation is also taken to be permanent, on the basis of available medical evidence. In this respect it differs from widowhood where the cessation of reproduction lasts only till a possible remarriage. Further, if the number of sterilisations of married couples performed in a year is classified by age of wife, and the procedure is repeated year after year, we could then trace the same generations of married women undergoing sterilisation through successive ages and successive years of time.

3. In any year T, we have a distribution of married women by age in the reproductive period (say 15-44) and assuming there had been no sterilisations, the sum of the marital fertility rate at each age weighted with the proportion of married women at that age, through the reproductive period, would provide the general marital fertility rate, all ages taken together.

In symbols,

$$I = \sum_{x=15}^{44} \pi_x \cdot i_x,$$

where π_x = ratio of the number of married women at age x to the number of all married women (15–44),

i_x = marital fertility rate at age x,

I = general marital fertility rate

and

$$\sum_{x=15}^{44} \pi_x = 1. \quad \ldots \quad (1)$$

4. Now suppose that sterilisation had been going on for some years prior to the year T, on married women of ages, say y and above ($15 < y < 44$). If we do not take into account the fresh sterilisations that will take place in the year T and consider only the cumulative effect of the sterilisations that had taken place in the years up to and including the year $T-1$, the married women in the year T at all ages between 15 and y are unsterilised and will contribute fully to the general fertility rate, while those aged $y+1$ and above, subject as they had been to sterilisation from earlier ages and possibly through varying periods of years from different initial ages, will not be at their full strength for reproductive action, for which the existing proportions unsterilised will have to be taken into account. Thus if this proportion at age x is u_x, we have

$$I' = \sum_{x=15}^{y} \pi_x \cdot i_x + \sum_{x=y+1}^{44} u_x \cdot \pi_x \cdot i_x. \quad \ldots \quad (2)$$

The above approach is a generation approach, as indicated earlier. It will be seen that the body of married women (15–44) in any year consists of separate generations corresponding to the individual ages, each generation subject to sterilisation to an extent appropriate to itself. Assuming that the age-specific fertility rates (i_x) and the proportions of married women at different ages (π_x) are known, the main problem is to estimate u_x. It is also necessary to fix the earliest age y at which sterilisation is to take place.

5. A method, simple in theory, would be to arrive at a standard distribution of women according to marital status and also according to sterilised or unsterilised status, by constructing a series of inter-connected multiple decrement tables (assuming that

the experience is not subject to migration), such as (1) spinsters', (2) married women's (unsterilised and also sterilised), (3) widows' (unsterilised and also sterilised) tables, each subject to decrements appropriate to itself (sterilisation is a decrement for the married women's (unsterilised) table), and to appropriate increments from other tables showing at each possible age, the number living as well as the numbers corresponding to its own decrements.

If $R =$ the basic radix of the spinsters' table at the earliest age at marriage z, and P and Q the tabular numbers living in the married women's (unsterilised) and married women's (sterilised) tables respectively, at age $z+t(z+t > y > z)$, then since P and Q both arise out of R, we have out of a total number of married women at age $z+t = P+Q$, the number unsterilised $= P$.

Hence,
$$u_{z+t} = P/(P+Q).$$

While the above method is of general application, covering communities having various marriage and remarriage experiences, the construction of the tables is laborious, and is possible only when the independent rates of decrement (the rates of the common decrements may again differ in different tables) are known and the appropriate dependent rates deduced and applied to build up the tables.

6. It is at once evident that with the Indian population, such a series of tables cannot be constructed for want of data to provide the rates of decrement, if not for anything else. Nevertheless, in India, where remarriages of widows are comparatively few, and marriages are usually completed by a comparatively early age, a considerable simplification of the procedure outlined above is possible.

Assuming that y, the age at which it is decided to start sterilisation is also the age above which marriages or remarriages are not supposed to take place, we have at age y, a number of married women, say P_y, subject to mortality, widowhood and sterilisation, each putting the corresponding decrement out of the experience of reproductive married women. The body of lives is subject to no other decrements, nor to any increments by marriages of spinsters and remarriages of widows above age y. It is also supposed to be free from migration. Thus if P_{y+t} is the number of married women who have attained age $y+t$, surviving all the three decrements, and P'_{y+t} the corresponding number surviving the first two only (i.e., mortality and widowhood, sterilisation occurring but not considered as a decrement), both P_{y+t} and P'_{y+t} having arisen out of P_y, we have

$$u_{y+t} = P_{y+t}/P'_{y+t}. \quad \dots \quad (3)$$

If α, β and γ, represent the decrements of mortality, widowhood and sterilisation respectively, the probability of a married woman attaining age $y+t$, surviving all the three decrements,

$${}_tp_y = {}_t(\alpha p)_y \cdot {}_t(\beta p)_y \cdot {}_t(\gamma p)_y,$$

where the factors are the independent probabilities of surviving the separate decrements.

Similarly,

$$ {}_tp'_y = {}_t(\alpha p)_y \cdot {}_t(\beta p)_y, $$

where sterilisation is ignored, provided of course that the rates of mortality and widowhood of a married woman are independent of whether she or her husband is sterilised or not. From available medical evidence, the health of a person is not significantly affected by sterilisation. We can therefore assume that sterilisation does not affect married women's mortality and widowhood rates.

We thus have

$$ P_{y+t} = P_y \cdot {}_t(\alpha p)_y \cdot {}_t(\beta p)_y \cdot {}_t(\gamma p)_y, $$

and

$$ P'_{y+t} = P_y \cdot {}_t(\alpha p)_y \cdot {}_t(\beta p)_y. $$

Therefore

$$ u_{y+t} = {}_t(\gamma p)_y. \quad \ldots \quad (4) $$

7. We thus need to have a table of ${}_t(\gamma p)_y$ at successive integral values of t. Remembering that the values are those of survival factors where the single decrement of sterilisation (and no mortality or widowhood) is in operation, the case is similar to that of a Life Table, where the single decrement is one of mortality. Proceeding on the lines of construction of a Life Table, we start with a suitable radix l_y at age y and applying the independent probabilities or rates of sterilisation at successive ages, we can build up a table of values of l_{y+1}, l_{y+2} etc., so that ${}_t(\gamma p)_y = l_{y+t}/l_y$ from the above table, which may be called a sterilisation-survivorship table.

That is,

$$ u_{y+t} = l_{y+t}/l_y. \quad \ldots \quad (5) $$

8. To find the rates of sterilisation, $(\gamma q)_y, (\gamma q)_{y+1}$ etc., or to construct the table even without having to find the rates as we shall see presently, we have first to consider the question of eligibility, arising, for instance, from the decision that only such married women at any age as have at least two or three living children should be considered for sterilisation. The proportion of such women increases with age and if the proportion at age y is Z_y and the proportion of eligible women sterilised in the $(t+1)$-th year from the beginning of the sterilisation programme is S_t (which is supposed independent of age), the number sterilised at age y in the first year $= l_y \cdot Z_y \cdot S_0$ and $l_{y+1} = l_y(1-Z_y \cdot S_0)$. Provided that the normal value of Z_{y+t} is not affected by sterilisation at earlier ages, the number normally eligible at age $y+1 = l_y \cdot Z_{y+1}$, but as the number already sterilised at age $y = l_y \cdot Z_y \cdot S_0$, the number remaining available for sterilisation at age $y+1 = l_y(Z_{y+1}-Z_y \cdot S_0)$. Therefore, the number sterilised at age $y+1 = l_y(Z_{y+1}-Z_y \cdot S_0) \cdot S_1$, and

$$ \begin{aligned} l_{y+2} &= l_{y+1} - l_y(Z_{y+1}-Z_y \cdot S_0) \cdot S_1 \\ &= l_y(1-Z_y \cdot S_0) - l_y(Z_{y+1}-Z_y \cdot S_0) \cdot S_1 \\ &= l_y[1-Z_{y+1} \cdot S_1 - Z_y \cdot S_0(1-S_1)]. \end{aligned} $$

Also, the number normally eligible at age $y+2 = l_y \cdot Z_{y+2}$, but the number already sterilised

$$= l_y \cdot Z_y \cdot S_0 \text{ (at age } y) + l_y(Z_{y+1} - Z_y \cdot S_0)S_1 \text{ (at age } y+1)$$
$$= l_y[Z_{y+1} \cdot S_1 + Z_y \cdot S_0(1-S_1)].$$

Therefore the remaining number available for sterilisation at age $y+2$

$$= l_y[Z_{y+2} - Z_{y+1} \cdot S_1 - Z_y \cdot S_0(1-S_1)].$$

Proceeding as above, it can be shown that

$$l_{y+t} = l_y[1 - Z_{y+t-1} \cdot S_{t-1} - Z_{y+t-2} \cdot S_{t-2}(1-S_{t-1}) + \ldots$$
$$- Z_y \cdot S_0(1-S_1)(1-S_2) \ldots (1-S_{t-1})],$$

and $\qquad = l_y \cdot u_{y+t}$ by equation (5).

In the same way, the remaining number available for sterilisation at age $y+t$

$$= l_y[Z_{y+t} - Z_{y+t-1} \cdot S_{t-1} - Z_{y+t-2} \cdot S_{t-2}(1-S_{t-1}) - \ldots$$
$$- Z_y \cdot S_0(1-S_1)(1-S_2) \ldots (1-S_{t-1})],$$
$$= l_y \cdot e_{y+t},$$

where e_{y+t} is the proportion available for sterilisation at age $y+t$.

The number not eligible at age $y+t$

$$= l_y(u_{y+t} - e_{y+t}) = l_y(1 - Z_{y+t}),$$

which is otherwise obvious. Also, the total number including the number not eligible, the number already sterilised and the number now available for sterilisation, at age $y+t$

$$= l_y[(1-Z_{y+t}) + (Z_{y+t} - e_{y+t}) + e_{y+t}]$$
$$= l_y,$$

as it should be.

It will be noticed that $(\gamma q)_y = Z_y \cdot S_0$, $(\gamma q)_{y+1} = (Z_{y+1} - Z_y \cdot S_0)S_1/(1 - Z_y \cdot S_0)$ and so on. It is therefore not necessary to extract these rates and construct the table as indicated in Section 7. On the other hand, as explained above, a sterilisation-survivorship table of l_{y+t} can be built up directly with an arbitrary radix l_y and with values of Z and S at integral ages and in successive years respectively, from which the values of u_{y+t} and e_{y+t} may be obtained.

9. The formulae in the previous section are derived by tracing a generation of married women currently aged y forward through successive years. Equation (2) however requires that a generation currently aged x should be considered in retrospect, giving the values of u_x after cumulative sterilisation through an appropriate

period already elapsed. If, therefore, u_x^t and e_x^t are the values of the functions in the $(t+1)$-th year from the start of the sterilisation programme, we have, from the expressions already derived in the previous section,

$$u_x^0 = 1,$$

$$u_x^t = 1-Z_{x-1}\cdot S_{t-1}-Z_{x-2}\cdot S_{t-2}(1-S_{t-1})-\dots Z_{x-t}\cdot S_0(1-S_1)\dots(1-S_{t-1}),$$

$$\text{for} \quad 1 \leqslant t \leqslant n, \text{ where } x-y=n, \qquad \dots \quad (6)$$

$$u_x^t = 1-Z_{x-1}\cdot S_{t-1}-Z_{x-2}\cdot S_{t-2}(1-S_{t-1})-\dots Z_{x-n}\cdot S_{t-n}(1-S_{t-n+1})\dots(1-S_{t-1})$$
$$\text{for} \quad t > n. \qquad \dots \quad (7)$$

Equation (6) represents what may be called the variable state, where t increasing from 1 to n, the initial age of sterilisation, $x-t$ diminishes from $x-1$ to $x-n(y)$, there is an additional negative term at each successive step (the total number of such terms at each step $= t$) and sterilisation of the women aged x had always started from the first year of the programme. Similarly, equation (7) together with the expression for u_x^n from equation (6) represents what may be called the steady state, where t increasing from n, the initial age of sterilisation is always y, the total number of negative terms is always n, and sterilisation of women aged x in the $(t+1)$-th year had started in the $(t-n+1)$-th year.

9.1. Expressions for e_x^t can be derived simply by replacing 1 on the right hand side of equations (6) and (7) by Z_x ($\because$ $u_x = 1-Z_x+e_x$). Also e_y is always $= Z_y$.

9.2. From equation (2), the general marital fertility rate in the $(t+1)$-th year,

$$I_t' = \sum_{x=15}^{y} \pi_x\cdot i_x + \sum_{x=y+1}^{44} u_x^t\cdot\pi_x\cdot i_x. \qquad \dots \quad (8)$$

The number available for sterilisation per thousand population in the $(t+1)$-th year,

$$E_t = 1000\cdot\pi^{(m)}\cdot \sum_{x=y}^{44} \pi_x\cdot e_x^t. \qquad \dots \quad (9)$$

where $\pi^{(m)}$ = ratio of the number of married women (15-44) to the total population (male and female). The number of sterilisations per thousand population in the $(t+1)$-th year,

$$N_t = 1000\cdot\pi^{(m)}\cdot S_t \sum_{x=y}^{44} \pi_x\cdot e_x^t. \qquad \dots \quad (10)$$

10. From the expressions for u_x^t and e_x^t in the previous section, we may arrive at the following general results.

10.1. With a set of higher (lower) values of Z_x(the values of S_t remaining the same), u_x^t is smaller (greater). Also, with the same set of values of Z_x, u_x^t is smaller at a higher age, as Z_x is an increasing function of x.

10.2. Equation (6) can be written as

$$u_x^t = 1-Z_{x-1}+Z_{x-1}(1-S_{t-1})(1-S_{t-2}) \dots (1-S_0)+(Z_{x-1}-Z_{x-2})\cdot S_{t-2}(1-S_{t-1}) +(Z_{x-1}-Z_{x-3})\cdot S_{t-3}(1-S_{t-2})(1-S_{t-1})+ \dots +(Z_{x-1}-Z_{x-t})\cdot S_0(1-S_1) (1-S_2) \dots (1-S_{t-1}).$$

Similarly, equation (7) can be written as

$$u_x^t = 1-Z_{x-1}+Z_{x-1}(1-S_{t-1})(1-S_{t-2}) \dots (1-S_{t-n})+(Z_{x-1}-Z_{x-2})\cdot S_{t-2}(1-S_{t-1}) +(Z_{x-1}-Z_{x-3})\cdot S_{t-3}(1-S_{t-2})(1-S_{t-1})+ \dots +(Z_{x-1}-Z_y)\cdot S_{t-n}(1-S_{t-n+1}) \dots (1-S_{t-1}).$$

As the function Z_x increases with x, in either expression for u_x^t, all terms after the second are positive for all values of $S < 1$.

Therefore $$u_x^t > (1-Z_{x-1}).$$

But if $S_{t-1} = 1$(the maximum possible value), all terms after the second are reduced to zero, so that the lowest possible value of $u_x^t = 1-Z_{x-1}$. Similarly, the lowest possible value of $e_x^t = Z_x-Z_{x-1}$.

Now, S_{t-1} is the value of S in the previous (t-th year), so that for u_x and e_x to attain their respective minimum values, S must be unity in the previous year.

10.3. The minimum value of I'_t is obtained by substituting u_x^t in equation (8) by $1-Z_{x-1}$, and the minimum value of E_t by substituting e_x^t in equation (9) by Z_x-Z_{x-1}. It follows, therefore, that for given set of values of Z_x (corresponding to the condition of eligibility adopted), the lowest possible values of I' and E can be obtained outright as

$$I'_{\min} = \sum_{x=15}^{y} \pi_x \cdot i_x + \sum_{x=y+1}^{44} (1-Z_{x-1})\cdot \pi_x \cdot i_x \qquad \dots \quad (11)$$

and

$$E_{\min} = 1000.\pi^{(m)} \cdot \sum_{x=y}^{44} \pi_x (Z_x - Z_{x-1}) = p_0. \qquad \dots \quad (12)$$

10.4. From the description of the variable and steady states in Section 9, it follows that for the purpose of applying equations (8), (9) and (10) through successive years from the first, we should require a set of sterilisation-survivorship tables with initial ages ranging from y to 43, each starting from the first year (and so drawn up with the same set of values, S_0, S_1, ...) and continuing up to age 44, and also another set of such tables, each with initial age y and continuing up to age 44 but starting from the second, third etc., year, and so using different sets of values of S.

10.4.1. If $S_0 = S_1 = \ldots = S$, u^t_x and e^t_x both diminish with increasing t in the variable state but remain constant in the steady state. The sterilisation-survivorship tables will be the same as described in Section 10.4, but with a constant value of S operating throughout. The tables with initial age y and starting from successive years are in fact one table and will be the only one required, when after the lapse of $44-y$ years or more from the start of the setrilisation programme, the steady state will have been attained for each age between y and 44.

10.4.2. With a fixed value of S,

$$u^t_x = 1-Z_{x-1}\cdot S-Z_{x-2}\cdot S(1-S)-Z_{x-3}\cdot S(1-S)^2- \ldots -Z_{x-t}\cdot S(1-S)^{t-1},$$

(in the variable state).

$$\therefore \quad \frac{du^t_x}{dS} = -Z_{x-1}-Z_{x-2}\cdot S(1-2S)-Z_{x-3}(1-S)(1-3S)-\ldots-Z_{x-t}(1-S)^{t-2}(1-S_t).$$

If $S < 1/t$, all the terms are negative, but if S continuously increases and exceeds $1/t$, the terms from the right will start becoming positive and neutralising to the corresponding extent the total of the remaining negative terms. But, when S increases to 1, $\frac{du^t_x}{dS} = -(Z_{x-1}-Z_{x-2})$, which is negative. Hence, $\frac{du^t_x}{dS}$ always remains negative, decreasing however in absolute value as S increases to unity. Thus as S increases, u^t_x (and also e^t_x) diminishes, and their respective minimum values are $(1-Z_{x-1})$ and (Z_x-Z_{x-1}), a result already obtained (Section 10.2). Obviously, the same results hold, if deduced from the steady state, Z_{x-t} being replaced by Z_y and t by n.

11. Suppose the sterilisation programme is to work with a given uniform rate of sterilisation, say p per thousand population per annum, instead of a given set of values of S_t or a fixed value of S. We shall see presently that S is dependent on the year of sterilisation, but that the values of S_t for various values of t are given by and have to be determined from the given condition.

From equation (10)

$$1000\cdot\pi^{(m)}\cdot S_t \sum_{x=y}^{44} \pi_x\cdot e^t_x = p, \qquad \ldots \quad (13)$$

for all values of t.

We have $$e^t_x = Z_y,$$

for all values of t.

For $$x = y+1 \text{ to } 44,\; e^0_x = Z_x,$$

whence S_0 is obtained from equation (13),

and $$e^1_x = Z_x-Z_{x-1}\cdot S_0,$$

which is $< e^0_x$, so that from equation (13) $S_1 > S_0$.

For $x = y+1$, $e_x^2 = Z_x - Z_{x-1} \cdot S_1$, which is $< e_x^1$,

and for $x = y+2$ to 44, $e_x^2 = Z_x - Z_{x-1} \cdot S_1 - Z_{x-2} \cdot S_0(1-S_1)$,

which is also $< e_x^1$, so that from equation (13), $S_2 > S_1$.

For $x = y+1$, $e_x^3 = Z_x - Z_{x-1} \cdot S_2$, which is $< e_x^2$,

for $x = y+2$, $e_x^3 = Z_x - Z_{x-1} \cdot S_2 - Z_{x-2} \cdot S_1(1-S_2)$.

Comparing with the corresponding expression for e_x^2, in the terms involving Z_{x-1}, $S_2 > S_1$, in those involving Z_{x-2}, $S_1 > S_0$, but $(1-S_2) < (1-S_1)$, so the terms should be nearly equal, which may mean that $e_x^3 < e_x^2$, and

for $x = y+3$ to 44, $e_x^3 = Z_x - Z_{x-1} \cdot S_2 - Z_{x-2} \cdot S_1(1-S_2) - Z_{x-3} \cdot S_0(1-S_1)(1-S_2)$.

Comparing with the corresponding expression for e_x^2, besides the considerations made in respect of age $y+2$, there is an additional negative term in e_x^3 involving Z_{x-3}, which makes it more reasonable that $e_x^3 < e_x^2$. Thus, considering all ages, S_3, obtained from equation (13) should be $> S_2$.

The procedure may be continued to higher values of t, and on considerations made above, we could expect that the increase in S in the early years would be continued till S ultimately would reach unity. But the above treatment is not rigorous, and an exception to the progression of S till it reaches unity can be obtained as follows.

11.1. If in the $(t+1)$-th year, S_t is to be $= 1$, omitting to mention the limits, $1000 \cdot \pi^{(m)} \Sigma\ \pi_x \cdot e_x^t$ must be $\leqslant p$. But the lowest possible value of the quantity $= 1000 \cdot \pi^{(m)} \cdot \Sigma\ \pi_x(Z_x - Z_{x-1}) = p_0$ (equation (12)). Therefore, if $p < p_0$, the quantity must always be greater than p, and S must always be less than unity (which means that the increase in S or the diminution of $\Sigma\ \pi_x \cdot e_x$ in the early years must be arrested later on) and the number of sterilisations would be continued at the rate of p per thousand population per annum. Also, $1000.\pi^{(m)} \cdot \Sigma\ \pi_x \cdot e_x^t$ must always be greater than p_0, because e_x^t can fall to $Z_x - Z_{x-1}$ only when S had become unity in the previous year (Section 10.2). Similarly, u_x^t must always be greater than $1 - Z_{x-1}$ and I' (and also the birth rate) can never attain its lowest possible value.

11.2. On the other hand, if $p > p_0$, $1000 \cdot \pi^{(m)} \cdot \Sigma\ \pi_x \cdot e_x^t$ may fall below p and S_t becomes equal to 1. In the next, i.e., the $(t+2)$-th year the quantity becomes $1000 \cdot \pi^{(m)} \cdot \Sigma\ \pi_x(Z_x - Z_{x-1})$, which is still lower than p (and $= p_0$), and $S_{t+1} = 1$. That is, if S has increased to unity in the $(t+1)$-th year, it will remain steady at that value thereafter. At the same time, the number of sterilisations falls below p in the $(t+1)$-th year, falls further to p_0 in the $(t+2)$-th year and remains steady at that value thereafter. It follows therefore that only when the given rate of sterilisation p is greater than p_0, S can ultimately reach and remain steady at unity, I' (and also the birth rate) falling to and remaining constant at its lowest possible value, and the number of sterilisations per thousand population per annum also falling to and attaining a constant level less than the rate intended to be maintained through the years. The period that should elapse before this transition takes place depends on the magnitude of p, and on the particular set of Z-values given by the condition of eligibility. For instance, a higher value of p, and /or lower values of Z which would correspond to lower values of e, would make for larger values of S, which would reach unity sooner and thereby shorten the period. During this period, the diminution of $\Sigma\ \pi_x \cdot e_x$ is exactly counterbalanced by the corresponding increase in S, so that the number of sterilisations remains at the given rate of p per thousand population per annum.

11.3. In any given case, for the determination of the successive values of S, as explained in Section 11, by the repeated application of equation (13), the value of e_x in any year is first found by a sterilisation-survivorship table starting from the appropriate initial age and constructed by using the values of Z from that age up to x (inclusive) and of S from the year corresponding to the initial age up to the previous year. The same table also gives the value of u_x, so that I' in that year can be calculated from equation (8). It will be noticed that the sterilisation-survivorship tables are exactly the same as described in Section 10.4, but that the tables instead of being completely built up at the outset with a given set of values of S_t are built up in steps, after determining the next value of S_t in the series.

11.4. Assuming for simplicity that sterilisation starts from the earliest reproductive age (15) and ignoring for the purpose of demonstration, the complications arising out of the fact that marriages would take place after that age, equation (13) would be

$$S_t \cdot \sum_{x=15}^{44} \pi_x \cdot e_x^t = p/1000 \cdot \pi^{(m)} = p', \text{ say.}$$

It might appear that since the body of married women (15-44) is depleted in respect of reproductive action by $100 \cdot p'\%$ every year, the proportion unsterilised of this body of women at the end of t years, would be $1-tp'$ and I'_t would be $(1-tp').I$ without, therefore, the use of the procedure outlined in the previous sections. The argument does not however take into account the fact that the body of married women (15—44) at the end is not altogether the same generations as that in the beginning, fresh generations entering into the reproductive period at the earlier ages and existing generations going out of it at the older ages. Each generation aged x at time t has therefore to be treated separately giving to it the appropriate period of exposure to sterilisation, which is not necessarily t years at every age. Secondly, the result gives a fall in I, which is independent of any condition of eligibility, but the proportion unsterilised at age x must depend on the values of Z at the previous ages involved, given by the condition of eligibility imposed.

Writing U^t for $1-tp'$, we have

$$U^t = 1 - S_0 \sum_{x=15}^{44} \pi_x \cdot e_x^0 - S_1 \sum_{x=15}^{44} \pi_x \cdot e_x^1 - \dots - S_{t-1} \sum_{x=15}^{44} \pi_x \cdot e_x^{t-1}$$

$$= \sum_{x=15}^{44} \pi_x [1 - S_0 \cdot e_x^0 - S_1 \cdot e_x^1 - \dots - S_{t-1} \cdot e_x^{t-1}] = \sum_{x=15}^{44} \pi_x \cdot A_x, \quad \text{say,}$$

which is independent of any condition of eligibility. This has to be compared with $\sum_{x=15}^{44} \pi_x \cdot u_x^t$ or $\sum_{x=15}^{44} \pi_x \cdot B_x$, writing B_x for u_x^t.

If $n = x-15$, for $n > t-1$ (i.e. $n \geqslant t$),

$$A_x = 1-Z_x \cdot S_0-[Z_x-Z_{x-1} \cdot S_0]S_1-[Z_x-Z_{x-1} \cdot S_1-Z_{x-2}S_0(1-S_1)]S_2 \ldots$$
$$-[Z_x-Z_{x-1} \cdot S_{t-2}-Z_{x-2} \cdot S_{t-3}(1-S_{t-2})-\ldots-Z_{x-t+1}-S_0(1-S_1)\ldots(1-S_{t-2})]\times S_{t-1}$$
$$= 1-Z_x \cdot S_{t-1}-[Z_x-Z_{x-1}]S_{t-2}-[Z_x-Z_{x-1} \cdot S_{t-2}-Z_{x-2} : S_{t-1}(1-S_{t-2})]S_{t-3}-\ldots$$
$$-[Z_x-Z_{x-1} \cdot S_1-Z_{x-2} \cdot S_2(1-S_1)\ldots Z_{x-t+1}(1-S_1)\ldots(1-S_{t-2})S_{t-1}]S_0,$$

by rearranging terms.

Now, $$B_x = 1-Z_{x-1} \cdot S_{t-1}-Z_{x-2} \cdot S_{t-2}(1-S_{t-1})-Z_{x-3} \cdot S_{t-3}(1-S_{t-2})(1-S_{t-1})-\ldots$$
$$-Z_{x-t} \cdot S_0(1-S_1)(1-S_2)\ldots(1-S_{t-1}),$$

from equation (6).

We have, therefore,

$$A_x = B_x-[Z_x-Z_{x-1}]\, S_{t-1}-[(Z_x-Z_{x-2})-(Z_{x-1}-Z_{x-2})S_{t-1}]S_{t-2}$$
$$-[(Z_x-Z_{x-3})-(Z_{x-1}-Z_{x-3})S_{t-2}-(Z_{x-2}-Z_{x-3})S_{t-1}(1-S_{t-2})]S_{t-3}-\ldots$$
$$-[(Z_x-Z_{x-t})-(Z_{x-1}-Z_{x-t})S_1-\ldots-(Z_{x-t+1}-Z_{x-t})(1-S_1)\ldots(1-S_{t-2})\times S_{t-1}]\times S_0$$
$$= B_x-\{(Z_x-Z_{x-1}) \cdot S_{t-1}+[Z_x-Z_{x-1} \cdot S_{t-1}-Z_{x-2}(1-S_{t-1})]S_{t-2}$$
$$+[Z_x-Z_{x-1} \cdot S_{t-2}-Z_{x-2}(1-S_{t-2})S_{t-1}-Z_{x-3}(1-S_{t-2})(1-S_{t-1})]S_{t-3}+\ldots$$
$$+[Z_x-Z_{x-1} \cdot S_1-Z_{x-2}(1-S_1)S_2-\ldots-Z_{x-t}(1-S_1)(1-S_2)\ldots(1-S_{t-1})]S_0\}$$
$$= B_x-Q_x,$$

where Q_x is positive, since by reducing it by replacing $Z_{x-1} \ldots Z_{x-t}$ by Z_x, Q_x becomes equal to zero.

Thus, $$B_x = A_x+Q_x,$$

and $$\sum_{x=15+t}^{44} \pi_x \cdot B_x = \sum_{x=15+t}^{44} \pi_x \cdot A_x + \sum_{x=15+t}^{44} \pi_x \cdot Q_x,$$

where $\sum_{x=15+t}^{44} \pi_x \cdot Q_x$ is a positive quantity. If the condition of eligibility be such that $Z = 1$ for all values of x,

$$\sum_{x=15+t}^{44} \pi_x \cdot B_x = \sum_{x=15+t}^{44} \pi_x \cdot A_x,$$

because $Q_x = 0$, either from the expression for Q_x or by comparison of the expressions for A_x and B_x.

4

If $$n < t-1 \text{ (i.e., } n < t),$$

$$A_x = 1-Z_x\cdot S_0-[Z_x-Z_{x-1}\cdot S_0]\, S_1-[Z_x-Z_{x-1}\cdot S_1-Z_{x-2}\cdot S_0(1-S_1)]S_2-\ldots$$
$$-[Z_x-Z_{x-1}\cdot S_{n-1}-Z_{x-2}\cdot S_{n-2}(1-S_{n-1})-\ldots-Z_{x-n}\cdot S_0(1-S_1)\ldots(1-S_{n-1})]S_n$$
$$-[Z_x-Z_{x-1}\cdot S_n-Z_{x-2}\cdot S_{n-1}(1-S_n)-\ldots-Z_{x-n}\cdot S_1(1-S_2)\ldots((1-S_n)]S_{n+1}-\ldots$$
$$-[Z_x-Z_{x-1}\cdot S_{t-2}-Z_{x-2}\cdot S_{t-3}(1-S_{t-2})-\ldots-Z_{x-n}\cdot S_{t-n-1}(1-S_{t-n})\ldots (1-S_{t-2})]\times S_{t-1}$$
$$= \{1-Z_x\cdot S_{t-1}-[Z_x-Z_{x-1}\cdot S_{t-1}]S_{t-2}-[Z_x-Z_{x-1}S_{t-2}-Z_{x-2}(1-S_{t-2})S_{t-1}]S_{t-3}-$$
$$\ldots-[Z_x-Z_{x-1}\cdot S_{t-n+1}-Z_{x-2}(1-S_{t-n+1})S_{t-n+2}-\ldots-Z_{x-n+1}(1-S_{t-n+1})\ldots (1-S_{t-2})S_{t-1}]S_{t-n}\}$$
$$-\{[Z_x-Z_{x-1}\cdot S_{t-n}-Z_{x-2}(1-S_{t-n})S_{t-n+1}-\ldots-Z_{x-n}(1-S_{t-n})\ldots(1-S_{t-2})S_{t-1}]$$
$$\times S_{t-n-1}+\ldots+[Z_x-Z_{x-1}\cdot S_1-Z_{x-2}(1-S_1)S_2-\ldots Z_{x-n}(1-S_1)\ldots(1-S_{n-1})S_n]S_0\},$$

by rearranging terms. The first expression in { } brackets above

$$= [1-Z_{x-1}\cdot S_{t-1}-Z_{x-2}\cdot S_{t-2}(1-S_{t-1})-\ldots Z_{x-n}\cdot S_{t-n}(1-S_{t-n+1})\ldots(1-S_{t-1})]-Q'_x,$$

where Q'_x is a positive quantity, and $= 0$, when $Z = 1$ (as before), $= B_x-Q'_x$ by equation (7). And the second expression in { } brackets above $= Q''_x$, say, where Q''_x is a positive quantity, both when Z_x is a function increasing with x (because reduced by replacing $Z_{x-1}, \ldots Z_{x-n}$ by Z_x, it remains positive) or $Z = 1$ for all values of x. We have however to compare the values of Q''_x in the two cases, and for that purpose, we may consider any one of the $t-n$ expressions (in [ ] brackets) included in Q''_x.

The last of such expressions

$$= [Z_x-Z_{x-1}\cdot S_1-Z_{x-2}\cdot(1-S_1)S_2\ldots -Z_{x-n}(1-S_1)\ldots(1-S_{n-1})\,.\,S_n]\,S_0$$
$$= Z_x(1-S_1)(1-S_2)\ldots(1-S_n)\cdot S_0+[(Z_x-Z_{x-1})S_1+(Z_x-Z_{x-2})(1-S_1)S_2+\ldots$$
$$+(Z_x-Z_{x-n})(1-S_1)\ldots(1-S_{n-1})S_n]S_0.$$

If $Z = 1$, the above $= (1-S'_1)(1-S'_2)\ldots(1-S'_n).S'_0$, using dashed symbols to denote the consequent lower values of S. Comparing the second expression (with $Z = 1$) with the first term of the first expression (with Z_x values), each is the product of larger and smaller factors, while there is an additional positive term in the first expression. Since again, Q'_x is positive with Z_x values and $= 0$ with $Z = 1$, we could expect that $Q_x = Q'_x+Q''_x$ would be greater than when $Z = 1$.

We have $$A_x = B_x-Q'_x-Q''_x.$$

Therefore, $$B_x = A_x+(Q'_x+Q''_x) = A_x+Q_x,$$

and $$\sum_{x=15}^{15+t-1} \pi_x\cdot B_x = \sum_{x=15}^{15+t-1} \pi_x\cdot A_x+\sum_{x=15}^{15+t-1} \pi_x\cdot Q_x.$$

Finally $$\sum_{x=15}^{44} \pi_x \cdot B_x = \sum_{x=15}^{44} \pi_x \cdot A_x + \sum_{x=15}^{15+t-1} \pi_x \cdot Q_x + \sum_{x=15+t}^{44} \pi_x \cdot Q_x ,$$

and for $Z = 1$,

$$\sum_{x=15}^{44} \pi_x \cdot B_x = \sum_{x=15}^{44} \pi_x \cdot A_x + \sum_{x=15}^{15+t-1} \pi_x \cdot Q_x + 0$$

where $\sum_{x=15}^{15+t-1} \pi_x \cdot Q_x$ is smaller than before. Since $\sum_{x=15}^{44} \pi_x \cdot A_x$ is independent of any condition of eligibility, its value is the same in both the above expressions (being equal to $1-tp'$).

Hence $$\sum_{x=15}^{44} \pi_x \cdot u_x^t > \sum_{x=15}^{44} \pi_x . u_x^t (Z = 1) > (1-tp') \ldots \qquad \ldots \quad (14)$$

Further $$\sum_{x=15}^{15+t-1} \pi_x \cdot Q_x \to 0 \cdot \text{ as } t \to 0.$$

Therefore, in the early years of sterilisation, $\sum_{x=15}^{44} \pi_x \cdot u_x^t (Z = 1)$ will approximately correspond to $(1-tp')$ but its excess over the latter would increase with increasing years.

From the inequality (14),

$$\bar{u} > \bar{u}(Z = 1) > (1-tp'), \qquad \ldots \quad (15)$$

where $\bar{u}$ is the average value of u_x^t with π_x as weights.

Now, $$I_t' = \sum_{x=15}^{44} \pi_x \cdot u_x^t \cdot i_x = \bar{\bar{u}} \cdot \sum_{x=15}^{44} \pi_x \cdot i_x,$$

where $\bar{\bar{u}}$ is the average value of u_x^t with $\pi_x \cdot i_x$ as weights. Considering the curves of π_x and $\pi_x \cdot i_x$, and also the relative variations of u_x^t with x, for Z_x values and for $Z = 1$ respectively, we can say that the inequality (11.3) holds when $\bar{u}$ is substituted by $\bar{\bar{u}}$.

Hence, $$I_t' > I_t' \ (Z = 1) > (1-tp') \cdot I \text{ (Section 15.1)}, \qquad \ldots \quad (16)$$

and the inequality between the last two is small in the early years but greater as the period of sterilisation increases.

In the above analysis, S is less than unity in both the cases, namely, with Z_x values and with $Z = 1$, but it can be shown that the inequality (16) holds also when S attains unity in one or both of them.

12. In applying the method outlined above on the rural population of India, to minimise arithmetic, we shall work in quinquennial age-groups, 15-19, 20-24, ... 40-44, using the central ages 17, 22, ... 42 for purposes of reference. For instance, if f_x is applicable to individual age x, the function applicable to the age-group $x-2$ to $x+2$

will be denoted by the symbol f'_x. Sterilisation will be assumed to start with the age-group 20-24, i.e., on the average from age 22, and to cover all higher age-groups. This age (22) is also the maximum age at marriage* so that the method described above is applicable. An adjustment for marriages if they occur after this age can be made but it will be small in any case and has not been shown in this paper.

Modifying equations (8), (9) and (10) to be applicable to the age-groups, we have

$$I'_t = \sum_{x=17}^{42} u'^t_x \cdot \pi'_x \cdot i'_x, \qquad \dots (17)$$

where $u'_{17} = 1$,

$$E_t = 1000 \cdot \pi^{(m)} \cdot \sum_{x=22}^{42} \pi'_x \cdot e'^t_x, \qquad \dots (18)$$

and

$$N_t = 1000 \cdot \pi^{(m)} \cdot S_t \cdot \sum_{x=22}^{42} \pi'_x \cdot e'^t_x \qquad \dots (19)$$

where the summations are in steps of 5. The following table gives the values of the basic functions (except Z) and the sources drawn upon for applying appropriate methods for deriving them are also mentioned below.

TABLE 1. PROPORTION OF MARRIED WOMEN IN AGE GROUPS

(All-India Rural)

age-group	central age, x	proportion of married[1] women (15-44) in age-group (%) $\pi'_x.10^2$	marital fertility rate[2] $i'_x.10^3$	$\pi'_x.i'_x.10^3$
(1)	(2)	(3)	(4)	(5)
15—19	17	18.4	180	33.1
20—24	22	19·5	244	47.6
25—29	27	20.7	215	44.5
30—34	32	16.0	173	20.2
35—39	37	14.1	128	18.0
40—44	42	10.4	79	8.2
		100.0		180.6

[1] *Source: Census of India*, 1951, Vol. 1, Part IIA, Demographic Tables, pp. 236-246.

[2] The National Sample Survey, *Vital Rates*, (1962): Seventh Round, October, 1953-March, 1954, 54 Cabinet Secretariat, Government of India Table 4.7, column (4), p. 20.

*See Registrar General, India (1962): *Vital Statistics of India for* 1960, Ministry of Home Affairs, New Delhi, p.XXIII.

13. We now assume that it is decided that only such married women as have produced three or more live births are eligible for sterilisation. We then need to have a distribution of married women at each age according to the number of live births produced, to find Z_x. In the absence of direct knowledge for such a distribution we fall back on any possible estimates of the proportions of married women at different ages, in a position to produce the fourth or a higher order birth (having already produced three or more live births) and define those proportions as the values of Z_x. We have in the National Sample Survey (1962): distributions of births in a year by order of birth and age of mother, from which the values of Z_x are derived as follows. If b_x = the total number of births in the year, to married women aged x, b_x^k = the number of births (out of b_x) of the $(k+1)$-th order, i_x^k = fertility rate at age x of married woman at parity k, i_x = total fertility rate at age x of married woman, N_x = the total number of married women at age x, and N_x^k = the number (out of N_x) in a position to produce births of the $(k+1)$-th order, we have

$$b_x^k/b_x = N_x^k \cdot i_x^k / N_x \cdot i_x$$

$$N_x^k/N_x = (b_x^k/b_x) \cdot i_x/i_x^k.$$

and $$N_x^{3+}/N_x = (b_x^{3+}/b_x) \cdot i_x/i_x^{3+},$$

where 3+ means 3 and above, $\quad = b_x^{3+}/b_x$

approximately, on the assumption that i_x/i_x^{3+} is approximately = 1.

$$Z_x = b_x^{3+}/b_x \quad \text{approximately.}$$

The table in the National Sample Survey (1962): gives b_x^{3+}/b_x in age-groups 12-21, 22-26, 27-31, 32-36, 37-, from which the values of Z_x at individual ages and thence those of Z_x' (for quinquennial age-groups) are obtained. The values of Z_x' are given in the following table.

TABLE 2

(All-India Rural)

age-group	central age x	Z_x'	age-group	central age x	Z_x'	age-group	central age x	Z_x'
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
20—24	22	.182	27—31	29	.696	34—38	36	.939
21—25	23	.241	28—32	30	.758	35—39	37	.947
22—26	24	.309	29—33	31	.809	36—40	38	.953
23—27	25	.385	30—34	32	.851	37—41	39	.957
24—28	26	.466	31—35	33	.884	38—42	40	.960
25—29	27	.547	32—36	34	.909	39—43	41	.963
26—30	28	.626	33—37	35	.927	40—44	42	.965

It is to be noted that the normal values of Z'_x(as in the above table) are not affected by any sterilisations performed up to age $x-1$. At age $x-1$, the normal proportion eligible is Z'_{x-1} (3 or more live births) and the normal proportion not eligible is $1-Z'_{x-1}$ (0, 1 or 2 live births). In the year of age $x-1$ to x, a fresh live birth may occur in either category, but the increment from Z'_{x-1} to Z'_x is solely due to the fresh live-births to the 2 live-births women in the latter category. That is, the whole of the former category, Z'_{x-1} comes to be included in Z'_x, and it is immaterial if some or all of them are sterilised and prevented from producing a live birth in the year. It is also to be noted that the function Z'_x increases with x.

14. *Practical applications: S is constant.* In what follows, we shall assume that sterilisation had started in 1961 and would continue with a given value of S, independent of the age of the woman (since there is hardly any evidence to show that the preference of a married couple for sterilisation is selective of the age of the wife), and also of the year of sterilisation (Section 10.4.1). We further assume that the values given in Tables 1 and 2 would remain stable in time as the marriage patterns and the basic fertility performance in rural India may not vary to an appreciable extent in the not too distant future. We can then arrive at a series of forecast general (marital) fertility rates (as also birth rates) at successive quinquennia from 1961, for which the following scheme, indicated in Section 10.4.1 may be considered first.

TABLE 3

	year T (number of years from 1961)					
	1961(0)	1966(5)	1971(10)	1976(15)	1981(20)	1986(25)
	age groups : duration of sterilisation in years					
	20—24	20—24	20—24	20—24	20—24	20—24
IV	(0)	(0)	(0)	(0)	(0)	(0)
	25—29	25—29	25—29	25—29	25—29	25—29
III	(0)	(5)	(5)	(5)	(5)	(5)
	30—34	30—34	30—34	30—34	30—34	30—34
II	(0)	(5)	(10)	(10)	(10)	(10)
	35—39	35—39	35—39	35—39	35—39	30—39
I	(0)	(5)	(10)	(15)	(15)	(15)
	40—44	40—44	40—44	40—44	40—44	40—44
	(0)	(5)	(10)	(15)	(20)	(20)

In the table, each diagonal represents the progression of a different generation, indicating the age attained and the duration of sterilisation undergone (within brackets), at the end of each quinquennium. Taking a vertical column through a particular year T, the number of years of sterilisation undergone by a particular age-group is shown in brackets under that age-group, the left-hand end of the diagonal passing through it indicating the initial age-group for sterilisation in respect of that age-group. It will be seen from the diagonal lines that for calculating the u' functions for the different quinquennia, 4 independent sterilisation-survivorship tables will be required, viz., (I) starting from age-group 35-39, (II) starting from age-group 30-34, (III) starting from age-group 25-29 and (IV) starting from age-group 20-24, each continuing to age-group 40-44, from a suitable radix at its initial age-group.

Considering, however, married women (20-24) in 1966 or thereafter, they will have been in fact already subject to sterilisation for approximately 2 years on the average (20-0, 21-1, 22-2, 23-3, 24-4, average 2), so that along the diagonal lines through 20-24, the durations of sterilisation(within brackets) should be 2, 7, 12, 17, 22 instead of 0, 5, 10, 12, 20 respectively. To give effect to this at the age-groups affected in 1966 or thereafter, we have to construct an additional table, Table No. (V), starting from 20-24 but with a certain average proportion already sterilised. The u' functions at the age-groups affected will be smaller, causing a corresponding reduction in fertility.

After constructing the sterilisation-survivorship Tables, (I), (II), (III), (IV) and (V) with a given value of S, the u' and e' functions may be tabulated for convenience intho form given below, following the scheme of Table 3. The figures are taken from tables drawn up with $S = .1$.

TABLE 4. VALUES OF u' FOR VARIOUS INITIAL AGES AND DURATIONS OF STERILISATION ($S = .1$)

initial age-group	central age	duration of sterilisation in years 0(2)	5(7)	10(12)	15(17)	20(22)
20—24	22	1.000	.864	.633	.412	.261
		(.977)	(.851)	(.625)	(.407)	(.258)
25—29	27	1.000	.713	.469	.289	—
30—34	32	1.000	.629	.389	—	—
35—39	37	1.000	.608	—	—	—
40—44	42	1.000	—	—	—	—

TABLE 5. VALUES OF e' FOR VARIOUS INITIAL AGES AND DURATIONS OF STERILISATION ($S = .1$)

initial age-group	central age	duration of sterilisation in years 0(2)	5(7)	10(12)	15(17)	20(22)
20—24	22	.182	.411	.484	.369	.226
		(.169)	(.398)	(.476)	(.364)	(.223)
25—29	27	.547	.564	.406	.254	—
30—34	32	.851	.576	.354	—	—
35—39	37	.947	.573	—	—	—
40—44	42	.905	—	—	—	—

Note: The values in brackets in the above tables are taken from Table V.

14.1. Using the values from Table 4 the general fertility and birth rates can now be obtained when $S = .1$, as shown in the table below applying equation (17).

TABLE 6

$\sum_{x=17}^{42} u'^t_x . \pi'_x . i'^t_x . 10^3$	1961	1966	1971	1976	1981	1986
= general fertility rate (marital) =	180.6	155.1	147.5	145.6	145.2	145.2
	× .22	× .22	× .22	× .22	× .22	× .22
birth rate[1]	39.7	34.1	32.5	32.0	31.9	31.9

[1]To find the birth rate, we have to remember that the values of i are derived from group fertility rates (see Section 12 for source) which were not corrected for under-reporting of births. A study of the data from the same source (NSS 7th round, rural), revealed that under-reporting was of the order of 10% and further that $\pi^{(m)}$ was about 20%. Hence the factor .22(= 1.1×.2).

(See Gupta, P. B. and Malakar, C. R. (1963): Fertility differential with level of living and adjustment of fertility, birth and death rates. *Sankhyā*, Series B, 25, 23-48).

Similarly, using the values from Table 5, the number of sterilisations per 1000 population through the quinquennia is obtained from the following table (applying equation (12.3), where $S_t = .1$ for all values of t)

TABLE 7

	1961	1966	1971	1976	1981	1976
$\sum_{x=22}^{42} \pi'_x . e'^t_x =$	.526	.351	.288	.268	.266	.265
	×	×	×	×	×	×
S =	.1	.1	.1	.1	.1	.1
	.053	.035	.029	.027	.027	.027
	×	×	×	×	×	×
$1000 . \pi^{(m)}$ =	200	200	200	200	200	200
number of sterilisations per 1000 population	10.6	7.0	5.8	5.4	5.4	5.4

As indicated in Section 10.4.1 and represented in Table 3 the position will attain a stationary condition by 1986, the steady state having been reached for all ages. The u' and e' factors at all the age-groups will attain their respective constant values, all given by Table 3, i.e., the values in brackets in Tables 4 and 5 respectively, and the general marital fertility rate and also the number of sterilisations in the year their minimum values.

14.2. A case of great importance arises when $S = 1$. In the first place, we have the maximum number available for sterilisation in the first year, and hence, with $S = 1$, the maximum possible number of sterilisations per annum, say p_m. Secondly, from Section 10.2, I' and E must fall to their respective lowest possible values in the second year and remain steady at them thereafter. Thus, we have also the values of I_{min} and $E_{min}(p_0)$ under the given condition of eligibility (Section 10.3). Applying equations (17), (18) and (19), we have the following table.

TABLE 6

	I' (1961) $= \sum_{x=17}^{42} \pi'_x . i'_x . 10^3$	I' (1962 and after)[1] $= \sum_{x=17}^{42} u'_x . \pi'_x . i'_x . 10^3$	$\sum_{x=22}^{42} \pi'_x . Z'_x$ (1961)	$\sum_{x=22}^{42} \pi'_x . o'_x$[2] (1962 and after)
	(1)	(2)	(3)	(4)
	180.6	105.8 (I'_{min})	.526	.037
	×	×	×	×
	.22	.22		
birth rate	39.7	23.3		
% drop (maximum possible)		41.3	$1000 . \pi'^{(m)} = 200$	200
number available for sterilisation per 1000 population			105.2	$7.4 (p_o)$
number of sterilisations per 1000 population			$105.2 (p_m)$	7.4

[1] The proportion unsterilised at age-group 20-24, i.e., $u'_{22} = 1/5\ [1+(1-Z_{20})+(1-Z_{21})+(1-Z_{22})+(1-Z_{23})]$ approximately $= .881$, from values of Z at individual ages from 20 to 23. $u'_{17} = 1$. For other values of x, $u'_x = 1 - Z'_{x-1}$, from columns (3), (6) and (9) of Table 2

[2] In 1962 and after, the proportion of women remaining available for sterilisation in age-group 20-24, i.e., $o'_{22} = 1/5\{Z_{20}+(Z_{21}-Z_{20})+(Z_{22}-Z_{21})+(Z_{23}-Z_{22})+(Z_{24}-Z_{23})\}$ approximately $= 1/5 . Z_{24} = .060$. For other values of x, $o'_x = Z'_x - Z'_{x-1}$, from columns (3), (6) and (9) of Table 2.

The implications of full sterilisation is therefore that in the first year, the number of sterilisations to be performed would be of the order of 100 per thousand population. The birth rate would fall outright from about 40 per thousand to about 23 per thousand and remain steady at that value with about 7 sterilisations per thousand population each year in subsequent years.

14.3. It will be remembered that in deriving the values of Z_x (Section 13), i_x/i_x^{2+} has been taken to be unity approximately. The preliminary results of a study of fertility rates according to parity and age of mother, in which the author is at present engaged, however show that i_x/i_x^{2+} is less than unity and progressively so, as x increases.

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If this feature is taken into account in the present connection, it will result in the lowering of the Z values, especially at the higher ages. At the same time, it is apparent that in the total of unsterilised married women at any age, the normal proportions exposed to risk in relation to i_x^{3+} and i_x^{0-2} respectively are disturbed by previous sterilisation, the latter predominating more and more with cumulative sterilisation and advancing age. The average fertility rate at age x with which the body of unsterilised married women will be credited will therefore be less than the normal value of i_x and the effect will be more pronounced at a higher age. These two factors will have opposite effects on the overall fertility rate (see equation (2)), for the diminution of the Z_x values will result in an increase in the u_x values (Section 10.1), but the latter will be associated with decreased values of i_x. Calculations have shown that the effects almost counterbalance one another and in this paper, the refinement mentioned above is omitted.

15. *Practical applications : Examination of sterilisation programmes, current or proposed*. We shall now examine, on the basis of our method, some of the sterilisation programmes current or proposed in this country, with a view to assess their effects on the fertility levels of the population in the coming years, and also to see whether the specific claims, if any, made in respect of them, are justified.

15.1. We shall first consider the sterilisation programme in Maharashtra State as outlined in the Government of Maharashtra (1963). In the programme "it is estimated that the rate of 7 sterilisations per thousand population per year for a period of 10 years will decrease the fertility rate by 50%." While the target is well defined, the methods for arriving at an estimate of the reduction in fertility are not clear. Further in the opinion of the Committee (Government of Maharashtra, 1963, p.29) which went into the details of the target, sterilisation in order to be effective, should be concentrated within the age-group 25-30 years in women and 30-35 in men, and therefore the selection of cases for sterilisation should be restricted to couples normally having 3 children. The condition of eligibility laid down above is expected to ensure greater incidence of sterilisation on the women of the fertile age-group 25-29, because the proportion eligible (having 3 children) in this age-group is expected to be greater than in any of the other age-groups, which are of lower fertility, except perhaps the age-group 20-24. At the same time, it may be thought inadvisable to exclude from sterilisation women having more than 3 children. If she belongs to a young age-group, it would mean that she is not prevented from further contributing to the birth rate, which she will, being fertile. If she belongs to an older age-group, she is at least more fertile than another of the same age-group with exactly 3 children and no more at that age, and it may be thought to be more gainful to sterilise the more fertile women. It seems to us that a better procedure would be to consider women with 3 children or more for sterilisation and it might after all produce a greater reduction in the birth rate (as we shall see later).

Let us first assume the condition of eligibility as stated in the Maharashtra programme. We would however consider the problem at the All-India level but on the Maharashtra basis, and secondly, in the absence of our knowledge as to the proportions of 3-children married women at the various age-groups, we would take the proportions of 3-live births married women as our values of Z, which may be obtained from *Vital Rates*, Table 4.13, p.30, of the National Sample Survey (1962), and which show a maximum at central age 29, a result similar to that expected with 3-children women as noted above. The number of sterilisations is 7 per thousand population per annum, and from equation (11.1) modified to apply to age-groups.

we have,

$$1000 \cdot \pi^{(m)} \cdot S_t \sum_{x=22}^{42} \pi'_x \cdot e'^t_x = 7 \text{ or } S_t \cdot \sum_{x=22}^{42} \pi'_x \cdot e'^t_x = .035 (\pi^{(m)} = .2).$$

Applying the procedure explained in Section 11.3, the birth rate can be calculated at the end of any given period, by applying equation (17). In the practical application of the procedure, it has to be remembered that, when at the higher ages, Z_x diminishes with increasing x, the normal decrement will be affected by sterilisation at an earlier age, since the decrement is mainly caused by a fresh birth, the woman concerned leaving the 3-births category and entering the 4-births one. As a fresh birth in this category is prevented by sterilisation, the actual decrement will be less, and an appropriate adjustment of the Z-values is therefore necessary. It is found that S increases to about unity in the fourth year, reaches and remains steady at unity in the fifth and subsequent years. The number of sterilisations per annum remains at 7 per thousand population up to the fourth year, falls to about 2 per thousand in the fifth year and remains steady at a slightly lower value thereafter. It is thus not possible to maintain the 7 per thousand rate beyond the fourth year, because of the reduction in the numbers of available women due to cumulative sterilisation, even though all such women are supposed to be sterilised. It is found also that the reduction in the birth rate is about 14% in 10 years, which is the maximum reduction possible in the given circumstances. Further, the adoption of the condition of eligibility as determined by exactly 3 children and not exactly 3 live births is not expected to make any material difference. The values of Z would be somewhat different with the maximum probably shifted to a slightly higher age, but as higher values of Z (lower u, Section 10.1) correspond to higher values of e and to lower values of S (higher u, Section 10.4.2), and vice versa, and as the cases are of comparable magnitudes, we may be justified in saying that the reduction in the birth rate in 10 years would still be of the order of 14%.

We now assume that the condition of eligibility is that the woman must have 3 children or more. As in the previous case, we adopt the alternative condition of 3 live births or more, corresponding to the Z-values in Table 2. It can be seen at once that the maximum drop possible in this case is about 41% (Table 8), so that a drop of 50% is out of the question. Secondly, since from the same table, $p_0 = 7.4$ and greater than $p(= 7)$, even the drop of 41% is not attainable (Section 11.1). However, proceeding as before, we find that S increases to a little over, .1 in 9 years, and that the reduction in the birth rate, though greater as anticipated earlier, is only about 17% in 10 years.

It may be of interest to examine the effect of waiving all restrictions about eligibility in the Maharashtra programme as applied to All-India. In this case, $Z = 1$ at all ages, and the maximum possible reduction in the birth rate is of the order of 75%, though not attainable with only 7 sterilisations per thousand population per annum, because, here also, p_0 is greater than 7. But all the same, the drop in the birth rate in a given period of years would be greater than when eligibility is given by 3 or more live births, or 3 or more living children, or for that matter, by any condition which produces a set of Z_x values (less than 1) increasing with increasing x (Section 11.4). The usual calculations however show that the drop in the birth rate in 10 years would be only about 22%, greater of course than before, but still substantially less than 50%, in spite of the advantage derived from sterilisation performed without regard to the previous reproductive history of a married couple.

Finally, the application of the simple formula discussed in Section 11.4, which is independent of any condition of eligibility, would produce a drop of 35% in 10 years, which is still considerably short of 50%, in spite of the fact that the simple formula exaggerates the drop in the birth rate increasingly with time and also that the exaggeration, as expected, is found to be substantial in a period of 10 years. The potentiality of Maharashtra Scheme, when applied to All-India, with 7 sterilisations per thousand population (or say 2.5 million sterilisations in the rural sector) per annum, therefore falls far short of envisaging a 50% reduction in the birth rate in 10 years, and the claim for such a reduction does not appear to be justified.

15.2. Considering next the sterilisations that have been performed on an All-India basis, we have the following figures from *Family Planning News*, Vol. 4, No. 12, December, 1963.

TABLE 9

(All-India Sterilisations)

year	male	female	total	increase to next year
(1)	(2)	(3)	(4)	(5)
1956	2,879	5,256	8,135	6,452
1957	3,760	10,797	14,557	13,972
1958	10,676	17,853	28,529	11,708
1959	16,630	23,607	40,237	22,293
1960	37,373	25,157	62,530	35,071
1961	60,196	37,405	97,601	31,913
1962	96,418	33,096	129,514	

From the above, sterilisations have been increasing year to year but the increase has not been steady. For instance, the increase was of the order of 35,000 (1960 to 1961), but it fell to about 32,000(1961 to 1962). Adopting a liberal view, we take the number of sterilisations to be 100,000 in 1961 and suppose that the number would go on increasing by 50,000 per year, say for 10 years. Taking the Indian population in 1961 at 440 million, the number of sterilisations in 1961 works out at .23 per thousand population. Assuming this to hold for the rural population separately (which is possibly an overestimate), the number increases to about 1.4 per thousand population in 1971, ignoring population growth (if population growth is taken into account, the figure would be smaller). But let us take it at a still higher figure of 2 per thousand population and strengthen the basis further by supposing that it would be in operation every year during the decennium, 1961-71. We propose to go another step ahead and assume that no restrictions about eligibility are imposed, which, as we have seen, would produce a greater reduction in the birth rate. The usual calculations show that the drop in the birth rate in 10 years would be about 6%. The actual drop must be appreciably smaller and therefore insignificant in the context of the present population problem in India.

16. In the previous section, we have examined in some detail the prospects of a couple of sterilisation programmes. In Section 1, the question was posed as to the extent a suitable sterilisation programme should be pushed in order that it may attain the level of adequacy. It has been noticed that the fundamental functions operating to reduce the birth rate are Z and S. When the range of ages (of married women) to come under the scope of sterilisation has been decided on, the values of Z_x will be exactly determined by the conditions of eligibility imposed. For any such set of values of Z_x (provided Z_x increases with x), there is a maximum drop in the birth rate possible as also the lowest possible number available for sterilisation per thousand population in a year (p_0), which can be calculated outright from equations (11) and (12) respectively. It follows also that the maximum drop with a set of higher values of Z_x is larger. In a given case, the number of sterilisations per thousand population per annum (p), provided it is greater than p_0, only determines the time that must elapse for S to reach unity and the maximum drop to occur. A larger number would accelerate the fall, till, when it equals p_m, S is unity at the outset and the maximum drop takes place in one year (Section 14.2). It is therefore wrong to suppose that simply by increasing the number of sterilisations per thousand population per annum, we could produce any desired fall in the birth rate in a given time. In fact, the limit is set by the condition of eligibility imposed, resulting in a particular set of values of Z_x. Suppose now that adequacy is defined by a drop of k% in the birth rate in t years. The condition of eligibility must therefore be such that it corresponds to a set of Z values, for which the maximum drop possible is not less than k%. Granted that such a condition is provided for in the sterilisation basis, the required number of sterilisations per thousand population per annum (greater than p_0), under which the time taken would be t years for the desired fall to take place, may be found by a few trials,

following the procedure outlined in the previous section. This would then provide an answer to the question posed above.

The current view of adequacy seems to be that the drop in the birth rate should be 50% in 10 years. Assuming that the range of ages is 20-44, we have seen that eligibility determined by three or more live births corresponds to a maximum drop of about 41%. The condition of eligibility must therefore be liberalised so as to correspond to a set of sufficiently higher values of Z_s. Such a condition may be given by say two or more live births, for which the maximum drop is likely to exceed 50%, and the appropriate number of sterilisations per thousand population per annum may then be determined in order that the desired fall of 50% in the birth rate may occur in 10 years.

If, however, we are satisfied with a drop of 40%, say in 10 years, eligibility may be determined by three or more previous live births, and we have to determine the appropriate number of sterilisations per thousand population per annum, greater than 7.4, which will produce the desired result. By a few trials, it is found that with a rate of 14 per thousand population per annum, S reaches unity in 11 years, the rate falling to 11 per thousand in the 11-th year and to 7.4 per thousand thereafter. And the birth rate also falls by about 41% after 11 years of sterilisation and would remain at the reduced level, provided sterilisation is continued at 7.4 per thousand population per annum.

But while the level of adequacy as accepted above is thus reached, it is important to recognise the implications of the programme. For one thing, the number of sterilisations per annum (considering rural India only) has to be of the order of 360.13^6 $\times$.014 or say 5 million (ignoring population growth), as against only about 100,000 in the whole of India in 1961. Secondly it implies the gradual increase in the popularity of the programme to the extent that ultimately all eligible couples will offer themselves for sterilisation. If this does not happen, and since sterilisation is essentially on a voluntary basis, the number of sterilisations that could be performed must fall below the desired level, with a consequent check on the fall in the birth rate.

17. For ready reference and comparison the results of the previous sections are tabulated below.

It may be interesting to compare the results on bases (b) and (c). The condition of eligibility is the same, but whereas on basis (c), S is constant at .1, on basis (b), it increases steadily, exceeding .1 only in the 9-th year. The drop in the birth rate on basis (b) is therefore less during the first 10 years, but greater thereafter than that on basis (c).

The above results are represented graphically in the figure.

TABLE 10. BIRTH RATES (ALL-INDIA RURAL) AT SUCCESSIVE QUINQUENNIA FROM 1961 ON VARIOUS BASES OF STERILISATION, AND PERCENTAGE DROP IN BIRTH RATE IN 10 YEARS

(Birth rate in 1961 = 39.7)

basis of sterilisation (per annum)	number of years from 1961					percentage drop in 10 years
	5	10	15	20	25	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) 7 per 1000 eligibility: 3 live births	34.5	34.3	34.2	34.2	34.2	13.6
(b) 7 per 1000 eligibility: 3 or more live births	35.2	32.8	31.4	30.4	29.6	17.4
(c) 7 per 1000 eligibility: all	34.0	31.1	29.4	28.4	27.8	21.7
(d) 14 per 1000 eligibility: 3 or more live births	30.5	23.3 (in 11 years)	23.3	23.3	23.3	41.3 (in 11 years)
(e) $S = .1$ eligibility: 3 or more live births	34.1	32.5	32.0	31.9	31.9	18.1

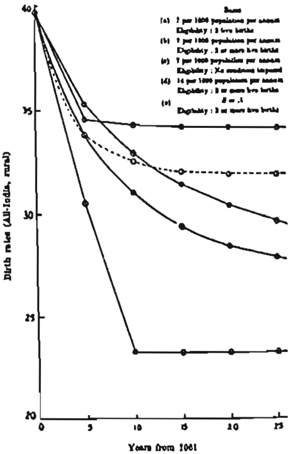

Figure. Forecast birth rates (All India, rural) at successive quinquennia from 1961 on various bases of sterilisation. (Birth rate in 1961-39.7)

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