Contribution of Statistics to the Science of Engineering

## By

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INTRODUCTION

THE potential contributions of statistics to the science of engineering are an important national asset; an asset of interest to all of us because it makes possible the most efficient and effective use of natural resources and human effort to satisfy human wants; an asset, however, that has for years remained frozen and is only now beginning to be utilized; an asset that can be used to the full only when engineers and others learn to use it as they have learned, in the past, to use the product of the scientist.

Much has appeared in the literature to indicate some of the contributions of statistics to date in the field of engineering and manufacturing. My object is not so much to review what has been done as to survey the potential contributions of statistics to the science of engineering. In doing this, I shall follow the old advice that the easiest way to reach the top is to go to the bottom of things, and I shall go to the bottom of the difference between engineering with, and without, statistics.

Let us recall how the applied scientist has wrought so many wonders for you and me to enjoy. He has done much of this with a comparatively simple but extremely powerful tool, namely, scientific method based upon the concept of physical laws of nature that assume perfect or certain knowledge of a set of facts and then state exactly what will happen at any future time. This method consists of three essential steps: hypothesis, experiment, and test of hypothesis. The fundamental difference between engineering with and without statistics boils down to the difference between the use of a scientific method based upon the concept of laws of nature that do not allow for chance or uncertainty and a scientific method based upon the concept of laws of probability

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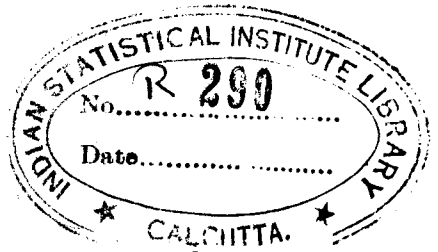
as an attribute of nature. When viewed in this way, the potential contributions of statistics become quite simple indeed. Statistically scientific method is in fact a fundamental discipline that includes all of the customary scientific method based upon the concept of exact laws as a limiting case in which repetitive operations of any given kind always give identically the same results.

All that I shall try to do here is to sketch in broad outline how the statistician has helped to refine scientific method by overhauling and reworking each of its three fundamental steps and then to indicate briefly how this new tool can be used by the engineer. In place of hypotheses based on exact laws, the new scientific method introduces statistical hypotheses. The introduction of statistical hypotheses makes it necessary to conduct the experiments in such a manner that the statistical hypotheses may be tested, and the new method provides means of testing the statistical hypotheses with which it starts.

Needless to say, scientists have long realized that their discovered "laws" do not always fit observed phenomena exactly. All that they have claimed is that the scientific method based upon the concept of exact laws has enabled them to make remarkable progress in understanding the world and to attain knowledge that could be used by engineers and applied scientists. Deviations from the assumed exact laws were simply dismissed as errors by the pure scientist and allowed for in factors of safety by the engineer. However, we are now coming to realize that many of the errors of the pure scientist and factors of safety of the engineer are more properly designated as factors of ignorance. We are beginning to see that we must refine our scientific methodology if we are to minimize these factors of ignorance in the scientific explanations of the world and if we are to make the most efficient and economic use of natural resources in the development of things to satisfy human wants. Statistically scientific method provides the scientist with an improved tool by which to extend his knowledge, and the engineer with a means by which to extend his useful service to mankind.

BASIC ENGINEERING PROBLEM

The engineer's job is to devise and develop the operations that, if carried out, will produce things that people want.[1] To do this,

[1] This is also the job of the applied scientist in the development of ways and means of making everything that we use.

he must be able to make things that have quality characteristics lying within previously specified tolerance ranges. Hence, a basic engineering problem is to devise an operation of using raw and fabricated materials that, *if* carried out, will give a thing wanted. The specified tolerance ranges for the quality characteristics of the thing wanted define a target for the engineer. He devises an operation and predicts that, if carried out, it will hit the target, but, since he does not have certain or perfect knowledge of facts and physical laws, he cannot be certain that a given operation will hit its target; in fact the best that he can hope to do is to know the *probability* of hitting the target. Here then is one fundamental way in which probability enters into everything that an engineer does.

Furthermore, if the thing produced fails to meet tolerance requirements, the engineer is penalized in one way or another. For example, if the quality of any piecepart fails to meet its tolerance requirements, a loss is incurred through rejection or modification of the defective part; if the time-to-blow of a protective fuse fails to meet its tolerance range, loss of property and even loss of life may result; if the time-to-blow of a fuse in a shell fails to meet its tolerance range, the shell may burst prematurely and kill members of the gun crew and, in any case, the round of ammunition will fail to fulfill its function of destruction within the ranks of the enemy. This means that when the engineer undertakes to use probability theory it is essential that he thoroughly understand the conditions under which its use will lead to valid predictions.

In what follows we introduce the term *operation* to include *any* experimental procedure designed to produce a previously specified result. In this sense, a production process is an operation, and a method of measuring is also an operation. Furthermore, an engineering operation or a production process may almost always be broken down into component operations. It should also be noted that even if only one thing of a kind is to be made, the operation devised by the engineer for producing this one thing is presumably capable of being repeated again and again so that the one thing to be produced may be thought of as but one of a class of an indefinitely large number of things that might be produced by repeating the operation again and again at will under the same essential conditions. In this way we may reduce the basic engineering problem of devising an operation to hit previously specified tolerance ranges to one that can be treated statistically, in that

3

statistical theory treats of the properties of certain kinds of repetitive operations.

To illustrate, let us consider a simple example in which only one quality characteristic of a thing is specified, let us say the length of a piecepart or the time-to-blow of a fuse. Let us symbolize this quality characteristic by X. The operation for producing a thing of quality X within specified limits,[2] if repeated again and again, would give rise to an indefinitely long sequence of values of quality X if taken in the order in which the things were produced, and these may be represented symbolically as follows:

$$X_1, X_2, \ldots, X_i, \ldots, X_j, \ldots, X_n, X_{n+1}, \ldots, X_{n+k}, \ldots \quad (1)$$

Hence, every operation may be characterized not only by a word description but also by the characteristics of the potentially infinite sequence (1) corresponding to an indefinitely large number of repetitions. In fact, we shall again and again make use of the characteristics of such a sequence in what follows. If an operation is developed to produce only one thing of a kind, then the engineer is interested only in the first term of this potentially infinite sequence (1) but if the engineer is interested in developing an operation or production process to turn out an indefinitely large number of things of the same kind, then he is interested in all terms in this sequence.

In what follows, we shall try to see how statistics can help the engineer to solve his basic engineering problem of developing an operation that, if carried out, will produce an object with qualities that lie within previously specified tolerance limits.

BASIC CONTRIBUTION OF CLASSICAL STATISTICAL THEORY

Basic Statistical Hypothesis. As a background for viewing the contribution of statistics to the solution of the basic engineering problem, we may state the fundamental hypothesis of applied classical statistical theory in the following way:

> *Hypothesis I. Some repetitive operations exist that obey laws of probability. These are called random. The probability that such a random operation will give a previously specified event, as for example, the occurrence of a value of X within a previously specified tolerance range, is a definite number associated with that event.*

[2] Or any operation of measurement of some objective quality characteristic.

If we know the law of probability or chance that controls a given operation, we may use the mathematical distribution theory of the statistician to describe how statistics of samples of size n given by successive repetitions of such an operation will be distributed. Likewise, if we know that an observed sample has been given by a random operation, the statistician has established valid rules of procedure for using the sample as a basis for estimating the parameters in the law of probability underlying the random operation that gave the sample. In other words, the applied mathematical statistician has provided us with rules for making valid predictions if we know the law of probability and with efficient rules of discovering the functional form of such a law, including the values of the parameters, if we simply know that it exists.

Of course, the work of the mathematician is purely formal; for example, what he calls the operation of drawing samples of size n at *random* consists essentially of acting upon some given mathematical law of chance or distribution function in accord with previously specified mathematical rules. Hence, if such a statistical hypothesis is to be of any value in engineering or applied science, it is necessary to know what is meant in an operationally verifiable manner by drawing at random. This necessitates our study of the second or experimental step in a statistically scientific method.

Basic Statistical Experiment or Operation of Drawing at Random. Unless an experimentalist knows what it means to draw a sample at random, he is not in a position to make use of statistical hypotheses because he cannot get the data with which to make valid tests of them and, having accepted a statistical hypothesis as valid, he does not know what kind of events he can predict with validity. Without such knowledge, he would be somewhat like a physicist who knew all about mathematical physics but did not know how to distinguish and measure the physical properties appearing in his equations. For example, an engineer may wish to select a random sample of 50 pieces of a new kind of product from the first 1,000 pieces produced. Very often he will ask under such conditions if he can take every twentieth piece as it is produced. Sometimes the engineer will propose other schemes but, in general, it is found that none of those proposed can be used with much assurance of giving a random sample. Hence the starting point in the use of statistical theory is a clear understanding of what is to be taken as the meaning of a random operation.

Let me begin with a description of two operations that I shall

5

choose to call random.[3] Let us assume that we have N physically similar chips on each of which is written a number and that these N chips are placed in a bowl. A blindfolded experimentalist thoroughly mixes the chips in the bowl, draws a number, and has his assistant record the number. The chip is then returned to the bowl and the blindfolded experimentalist, after thorough mixing of the chips, again draws one and has the number recorded. This random operation of drawing a number can theoretically be repeated again and again under the same essential conditions so as to give an infinite sequence like (1).

The other important operation for our present study is the following one of randomizing a finite set of N numbers. In this case the blindfolded experimentalist follows the same procedure as described above except that he does not return a chip to the bowl after it has been drawn. The N numbers drawn in this manner may be written down by the assistant in the order drawn. The operation of putting the N chips in a bowl, thoroughly mixing them, drawing them one at a time, and writing the numbers down in the order observed, may be repeated again and again at will so that the result of the operation of drawing one sequence is but one of the infinite number of sequences that might be obtained by repeating the operation again and again.[4] As an example, Fig. 1a records one such random drawing of the 144 values of thickness of inlay given in Table 1.

Of course, you will note many elements in my description of the experiment that are not operationally definite. What, for example, are symmetrical chips? What is thorough mixing? How can the experimentalist maintain all other conditions essentially the same? However, this kind of indefiniteness in the operation of drawing at random is much the same as exists in defining any experimental procedure.

The engineer without statistical training is likely to ask what

[3] A more comprehensive discussion of the difference between the mathematical concept of random and the operationally verifiable meaning of random has recently been given elsewhere.

Cf. W. A. Shewhart, *Statistical Method from the Viewpoint of Quality Control*, Graduate School of the Department of Agriculture, Washington, D. C. From the more technical viewpoint there presented, the two random operations here may be treated as one. For our present purpose they can best be considered separately.

See also the discussion of what the mathematician defines as random in the paper by Prof. Wilks, also contributed to this symposium.

[4] Of course there would only be N different possible orders.

6

TABLE 1

Thickness in Arbitrary Units of Inlay on 144 Relay Springs

55	13	45	27	33	59	43	24	43	52	39	62
72	31	38	36	36	45	26	30	28	42	38	50
93	37	23	18	34	41	2	28	38	53	49	70
54	38	58	43	59	47	17	71	28	53	38	60
30	43	52	10	75	38	28	57	23	42	35	36
41	38	35	26	59	56	51	37	18	29	35	65
40	36	50	8	43	39	19	43	39	43	22	50
45	21	50	53	41	57	42	58	52	17	30	75
31	30	23	39	57	22	35	33	43	39	24	90
25	34	68	47	59	43	3	62	31	48	67	96
30	36	53	48	48	33	27	27	36	58	54	80
23	41	26	34	42	20	27	26	42	59	73	88

there is of significance to him about the sequence in Fig. 1a. Well, the answer is that there is something about that sequence that is of very great importance to him. In the first place, most sequences
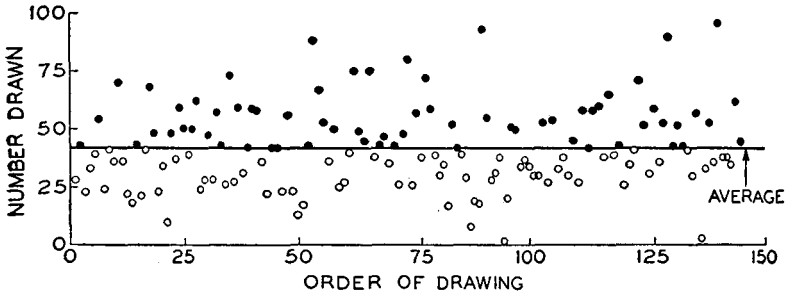

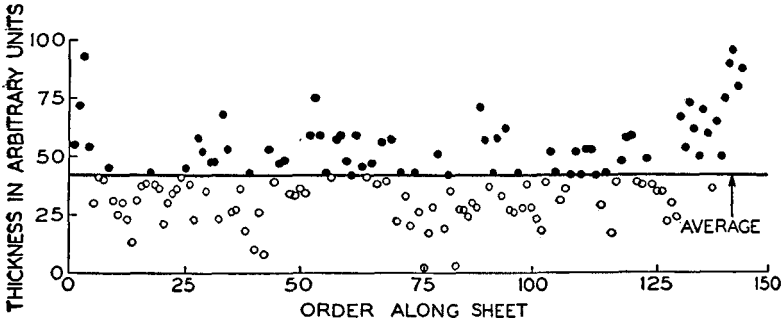FIG.1a SEQUENCE OF NUMBERS DRAWN AT RANDOM

FIG.1b SEQUENCE OF VALUES OF THICKNESS OF INLAY ON RELAY SPRINGS

obtained by a random operation possess certain characteristics that almost no sequence of results from repetitive engineering

7

operations is found to possess until after assignable causes have been eliminated through the application of the operation of statistical control. Furthermore, if anyone not an experienced statistician were to try to write down a lot of sequences of 144 different numbers or if he were to try to arrange the 144 numbers in Table 1 in what he would instinctively call random order, most of these sequences would fail to possess the characteristics possessed by almost all of the class of sequences obtained by drawing the 144 numbers again and again at random as described above.[5] Many of my colleagues have told me that their first real feeling for the meaning of a random operation came after they had tried to juggle with their eyes open, as it were, a set of numbers into what they would instinctively call random order only to find that the sequences thus obtained did not possess the characteristics possessed by most of the sequences obtained by drawing from a bowl with their eyes shut.[6] Before we can go further in characterizing the quantitative differences between such sequences, we must consider first the problem of testing a statistical hypothesis and later that of testing the hypothesis that an operation is in a state of statistical control.

Basic Test of Statistical Hypothesis. To determine whether any operation such as that of drawing from a bowl, as illustrated by the data in Fig. 1a, gives a sequence with the characteristics of one defined as random by the mathematician, we may choose one or more of the indefinitely large number of criteria that have been or may be established mathematically. If the operation gives a sequence that fails to meet the chosen criteria, the hypothesis is rejected, but if it meets the criteria, the hypothesis is accepted. Needless to say, no experimental test will prove or disprove a

[5] Since there are $\underline{|N}$ different possible orders in which N numbers may be drawn, one might argue that any arrangement whatsoever that one chooses to make of the N numbers is a random arrangement in that it would be one of the possible $\underline{|N}$ orders obtained by a random operation of drawing. The important point to note is that I do not speak of a random number or a random arrangement except in the sense of a number or arrangement given by a random operation. What I am contrasting is the class of arrangements given by the operation of arranging the numbers in what one intuitively may feel is a random manner with the class of arrangements given by the random operation of drawing from a bowl. For a more comprehensive discussion of this point cf. Shewhart, *op. cit.*

[6] The reader may wish to try for himself different orders of the data in Table 1 and see if his trials pass the three criteria considered later in discussion of statistical control theory.

statistical hypothesis, and in fact the test thereof constitutes a *rule of behavior* that must be justified upon the basis of extensive experience showing that in the long run we shall not be too often wrong. In fact, the test of any statistical hypothesis is subject to the following two kinds of errors known as errors of the first and second kinds: (1) sometimes the hypothesis will be rejected even though true and (2) sometimes the hypothesis will be accepted even though false.[7]

Experience reported in the literature from many different sources justifies the conclusion that we may use with confidence the deductive distribution theory of the statistician to predict the distribution of any observed statistic of samples of size n or of any one of the many characteristics, such as lengths of runs-up and runs-down, of the infinite sequence of numbers that we may expect to get by repeating again and again without limit the operation of drawing a number from a bowl. Certain other operations as, for example, the use of tables of random sampling numbers also give results that have been found by experiment to possess the properties predicted by the mathematical statistician.

Such studies show that a few specific kinds of operation exhibit properties described by the mathematical statistician as random. The ability to randomize a set of numbers or a set of objects by means of some distinguishable physical operation provides the scientist with a powerful technique for making valid predictions, and we shall now see how this can be used by the engineer.

Four Uses of the Basic Contributions of Classical Statistical Theory. (1) Obviously the method of testing a statistical hypothesis can be used for testing the hypothesis that a sample assumed to be random came from an assumed law of chance, or it can be used to test the hypothesis that two samples, both of which are random, came from the same law of chance. Such is the nature of statistical tests of significance of observed differences between two or more

[7] J. Neyman and E. S. Pearson have contributed many important papers on testing statistical hypotheses starting with one in *Biometrika,* July 1928, "On the Use and Interpretation of Certain Test Criteria for Purposes of Statistical Inference." Some of their latest contributions are given in *Statistical Research Memoirs,* Vol. 1, 1936, and Vol. 2, 1938, Cambridge University Press, London. Also see "On the Problem of the Most Efficient Tests of Statistical Hypotheses," by J. Neyman and E. S. Pearson, *Philosophical Trans. Royal Society of London,* Series A, Vol. 231, pp. 289–337. It is to be noted that all such tests depend upon the assumption that the sample used in testing is random. Later in testing the hypothesis of statistical control, we start with testing the hypothesis that the sample is random.

samples of data, but it should be noted that these tests are valid only *if* the samples tested are produced by random operations.

(2) If one after another of the operations of the engineer, such as an operation of measurement of some physico-chemical property or that of producing a given kind of product, could be shown to satisfy the hypothesis that it obeyed a law of chance,[8] then each such law and the estimates of the parameters therein could be quite simply obtained by means of well-established statistical procedures. Thereafter an engineer would be justified in using the distribution theory of the statistician in predicting the outcome of any future repetitions of that operation with the same degree of assurance that he has in using statistical distribution theory in predicting the outcome of future drawings from a bowl universe. This constitutes a goal highly to be desired from the viewpoint of design, for then it would be possible to use mathematical distribution theory in establishing the economical overall tolerance limits in terms of those for raw materials and pieceparts.

However, as long ago as 1924, abundant evidence, since substantiated on an even larger scale,[9] was obtained to show that few, if any, operations of the engineer obey laws of chance, even when carried out under presumably the same essential conditions. Two courses of action were open. One was to use available statistical technique as a *curative* measure in the sense of screening the product obtained by repeating a given production operation when this operation does not obey a known law of chance. The other was to develop a statistical technique to be used as a *preventive* measure in the sense of providing a means of detecting and eliminating assignable causes of variability that need not be left to chance.

(3) Next, then, let us see how classical statistical theory provides a means of screening the results already obtained by repeating an operation, as in the production of a product of a given kind. The basis for the technique lies in the empirically established fact that the statistician has justified the use of certain opera-

[8] Needless to say, in practice we can never be *sure* that an operation obeys a law of chance.

[9] Cf. *Economic Control of Quality of Manufactured Product*, W. A. Shewhart, New York, D. Van Nostrand and Company, 1931; *Statistical Method from the Viewpoint of Quality Control*, W. A. Shewhart, Graduate School of the Department of Agriculture, Washington, 1939; and *The Application of Statistical Methods to Industrial Standardization and Quality Control*, E. S. Pearson, British Standards Institution, London, 1935.

tions, like drawing from a bowl or from a set of random numbers, by means of which to randomize a set of objects. With this operation of randomizing, one may break up any quantity of product into a number of lots and establish the most economical plan of sampling these lots to insure both that the producer's risk of having a lot of satisfactory quality rejected will not exceed some previously specified value and that the consumer's risk of accepting a defective lot will not exceed some previously specified value irrespective of the quality of the product in the different lots. These producer and consumer risks correspond to the errors of the two types always involved in testing a statistical hypothesis and were introduced into commercial use within the Bell System as early as 1925 in the development of sampling plans for screening product.[10]

(4) Too much emphasis cannot be laid upon the practical importance of the fact that tests of statistical hypotheses are strictly valid only for random samples. This fact is taken into account in the design of sampling plans for screening, and it must also be taken into account in designing an experiment to test the significance of observed differences in the results obtained by submitting the results obtained by one operation to different subsidiary operations. For example, a given kind of product may be subjected to laboratory or field tests under different conditions. Unless the operation of producing the product obeys a law of chance or, in other words, is in a state of statistical control, it is necessary to randomize the samples submitted to the subsidiary operations in order to obtain a valid test of the significance of the observed differences. If this is not done, observed differences between the results of two or more subsidiary operations may have arisen from assignable differences in the results of the first operation. Application of the operation of randomization is particularly important in the comparison of new designs, new materials or alloys, study of contact phenomena under different conditions, corrosion of materials under different atmospheric conditions, and field trials of equipment, to mention only a few.[11]

[10] Cf. "Method of Sampling Inspection," by H. F. Dodge and H. G. Romig, *The Bell System Technical Journal*, Oct., 1929.

[11] The contributions of R. A. Fisher (see references to follow) are of particular importance in such studies. A simplified treatment of the elementary principles is given in *Field Trials: Their Layout and Statistical Analysis*, by John Wishart, School of Agriculture, Cambridge, England, 1940.

It is important for the engineer to keep in mind when reading all the literature on the randomization of the results of the first operation, that the validity of the tests for significant differences between the effects of different kinds of subsidiary operations rests upon the condition that the latter must be in a state of statistical control although this limitation is not explicitly stated. Hence, to be sure of the validity of tests of significant differences in the effects of subsidiary operations, we must first show that these operations are in a state of statistical control. This caution is necessary because otherwise an engineer may accept the results obtained from small samples at their face value.

By and large, classical theory treats of random fluctuations. It is a theory that tells how phenomena in nature would happen *if* they happened at random as do the results of drawing from a bowl and certain molecular phenomena treated in kinetic theory and statistical mechanics. It tells us how to discover such laws of chance *if* they exist and how to use them for purposes of valid prediction when discovered. It tells us how to make valid test of the hypothesis that a random sample came from a given universe or two random samples came from universes that have certain characteristics in common. It also provides us with a very important experimental operation of drawing at random that can be used in drawing random samples from the results of repetitive operations already made.

It is not, however, a theory designed primarily to tell us whether or not observed phenomena happen at random; or how to attain a state of statistical control (or randomness) of the cause systems underlying the operations of the engineer if these are not already in such a state. Whereas classical statistical theory gives us a very useful curative operation of randomizing results already obtained by a repetitive operation, control theory attempts to give us a very useful preventive operation of modifying the cause system underlying a physical operation or production process until it becomes random in the sense of being in a state of statistical control.

BASIC CONTRIBUTION OF STATISTICAL CONTROL THEORY

That an ounce of prevention is worth a pound of cure holds for the application of statistics. For example, if a manufacturing process can be made to produce a quality of product distributed

in accord with a law of chance, one can then establish once and for all with the requisite degree of assurance by means of a sufficiently large sample, the probability q' that the quality of a piece of product will lie within any previously specified tolerance limits L_1 and L_2. Now, it may be shown rigorously that if one knew this probability q', sampling of lots would not tell us anything more about uninspected portions of the lots than we knew before we sampled them.[12] Hence *if* we could attain this idealized condition, the necessity of sampling would be completely eliminated and there would be no need of applying a screening process. Moreover, if it can be shown under such conditions that it is not possible to modify the manufacturing process further by simply removing assignable causes, then it follows that we have also minimized the percent to be rejected because of failure to meet the tolerance requirements. Let us now see how we may approach this idealized condition with its associated advantages.

Basic Hypothesis for Statistical Control Theory. Perhaps, in simplest terms, the fundamental hypothesis is that even though the occurrence of an engineering operation exhibiting a state of statistical control is as rare as the proverbial hen's tooth, it is feasible and often desirable to establish a scientific method of modifying an existing operation until it obeys a law of chance. We shall consider the hypothesis of control in three parts, the first of which is:

> *Hypothesis IIa: The maximum attainable degree of validity of prediction that an operation will give a value X lying within any previously specified tolerance limits is that based upon the prior knowledge that the probability of this event is q' or more generally upon the prior knowledge of the law of chance underlying the operation.*

This part of the hypothesis is in line with the definite abandonment of the causal laws of classical physics and chemistry in favor of an indeterministic theory that includes the idea of probability in the ultimate laws. It is also in line with current theories of knowledge of the world in that no way has yet been devised for arriving at certain knowledge. So far as the statistical theory of control is concerned, no special attempt is made to justify this part of the fundamental hypothesis. Instead it is simply taken over from modern physics and modern logic. As such it represents the limiting knowledge to which an engineer may hope to attain in giving assurance

[12] For an amplification of this point, see the paper contributed to this symposium by Captain Simon.

that his engineering operations will give results lying within speci-
fied tolerance ranges.

> *Hypothesis IIb: The maximum degree of attainable control of the
> cause system underlying any repetitive operation in the physical world
> is that wherein the system of causes produce effects in accord with a
> law of probability.*

Such a state of control has been termed a *statistical state* of control.
The hypothesis that such a state represents the limit to which one
may hope to go in controlling a given operation by finding and
removing a few assignable causes of variation was originally sug-
gested by the second law of thermodynamics and its interpretation
in kinetic theory and the theory of statistical mechanics. In much
the same way that entropy measures the degree of run-downness
of a physical system at a given energy level, so the degree of ap-
proach to a state of statistical control measures the run-downness
of the cause system underlying a given operation. In much the
same way that it would take a Maxwell Demon to reverse an other-
wise irreversible process, so it may be shown by a study of chance
cause systems that it would take the equivalent of this Demon to
modify a cause system already in a state of statistical control with-
out changing, as it were, the whole operation and hence the whole
system of causes.

On many occasions, I have heard an engineer say on reaching
a state of statistical control of some operation that, without chang-
ing the kind of operation, he was going to decrease the range of
variation still further by simply asking the men performing the
operation to take greater pains to reduce the variability, but I
have not yet witnessed one case where such effort succeeded. Hence
the engineer is pretty safe in taking the state of statistical control
as a limit to which he may go in reducing the tolerance range for
that particular operation.

> *Hypothesis IIc: It is assumed that some criterion or criteria may be
> found and methods developed for their application to the numbers ob-
> tained in a sequence of repetitions of any operation such that whenever
> a failure to meet the criterion or criteria is observed, an assignable
> cause of variability in the results given by the operation may be dis-
> covered and removed from the operation. It is further assumed that, by
> the removal of a comparatively small number of causes, a state of sta-
> tistical control is approached where the results of repetitions of the
> operation behave in accord with a law of chance.*

14

The development of an operation of statistical control and its use in justifying hypothesis IIc is an empirical contribution of mass production because only in such a process would it be economically feasible to make the wide range of trials of experimental techniques for the purpose of finding one that works satisfactorily.

Let us now consider the requirements in the way of experimental data for attaining control.

Basic Experimental Data for Attaining Control. Let us consider the second or experimental step in the scientific method of using control theory. The crucial difference between the experimental technique in control work and that in applying the classical theory of statistics to provide a screen is that in control work we pay attention to the condition C_i under which an operation gives a value X_i, whereas in the screening operation we ignore this condition; in control work, the experimentalist must keep his eyes very much open, but in screening product he must, as it were, keep them blindfolded; and in control work, interest centers in controlling the product not yet made through modification of the underlying cause system, but in the screening process, interest centers in the product already made.

To symbolize this situation let us rewrite sequence (1) and attach to each value X_i its associated symbol for condition C_i. Also let us divide the sequence into two parts representing that already observed and that observable in the future. Then we have

$$X_1, \ X_2, \ \ldots, X_i, \ldots, \ X_j, \ \ldots, \ X_n, \ \bigm| \ X_{n+1}, \ \ldots, \ X_{n+k}, \ \ldots$$

$$C_1 \quad C_2 \qquad C_i \qquad C_j \qquad C_n \ \bigm| \ C_{n+1} \qquad C_{n+k} \qquad (2)$$

$$\underbrace{\qquad\qquad \text{Past} \qquad\qquad}_{} \qquad\qquad \underbrace{\text{Future}}_{}$$

$$\text{Present}$$

The screening process applies to the X's of the past and eliminates the information contained in the C's through the operation of drawing samples at random. It also completely ignores the results of future repetitions. In contrast, the control process focuses attention on both the X's and the C's of the past in the hope of detecting and removing assignable causes in the C's of the past so that these causes will not enter into future repetitions of the operation. Of course, in practice the screening and control processes may be carried on simultaneously and the data obtained in the screening process may also be used for the purpose of detecting lack of control. For example, inspection samples of lots taken in the order of

15

their production have long been used in some industries in applying the operation of statistical control shortly to be described.

Just as the batter must keep his eye on the ball first to make a hit and afterwards to determine what to do, so does the experimentalist have to keep his eye on the condition underlying the performance of any operation first to be able to apply this information most effectively in testing the hypothesis that he can do something to change the condition, and afterwards to determine what to do if the test is positive. By keeping his eye on the condition associated with each of n repetitions of an operation, the engineer may distinguish the following three situations:

(1) In the absence of any a priori reason for distinguishing between any two conditions, he may judge them to be essentially the same as may be symbolized by the equivalence

$$C_i \approx C_j. \tag{3}$$

Operations of drawing with replacement from a bowl illustrate a situation where the experimentalist is practically forced to conclude that the conditions are essentially the same from drawing to drawing. However, even under such circumstances, it is possible to arrange the results obtained by repeating an operation again and again in the same order as the operation was repeated.

Now, if the repetitions of the operation do not obey a law of chance or, in other words, do not arise under a state of statistical control even though the experimentalist considers them to be essentially the same, as symbolized by (3), the sequence of values of X, when taken in the order that they were observed, is not likely to pass the criteria of randomness established by the statistician. In fact, the order of observation in such instances provides the only quantitative basis for testing the hypothesis that the observations came from a state of statistical control, and in practice it is usually found that this order indicates lack of randomness. Hence the statistical control engineer should insist that the record of the order of repetitions be preserved even though the experimentalist judges the conditions to be essentially the same.

(2) The engineer may have a priori reasons for believing that the conditions do not remain essentially the same, as can be symbolized by

$$C_i \neq C_j. \tag{4}$$

For example, he may surmise that there are erratic effects or possibly trends superimposed upon the effects of a law of chance. In

such a situation, the experimentalist may be in a position to suggest certain ways of ordering the results of n repetitions of an operation, independently of the magnitudes of the associated values of X, and solely upon his knowledge of the conditions under which the observations were taken, in order to reveal, if present, trends or erratic effects that he considers likely to exist.

For example, in some recent work, interest centered in attaining a state of statistical control of the variation in thickness of a rolled inlay of contact metal on a particular kind of relay spring. These springs were cut from a long strip and there were a priori reasons for expecting both trends and erratic effects in the thickness along the strip. Hence the springs were numbered in the order that they were cut from the original strip so that the measurements of thickness on the individual springs could be arranged in the order of their original position in the strip. These observations have already been given in Table 1. The order in which the observations were arranged along the strip is that of the table, beginning at the left and reading down the columns. The set of 144 ordered measurements is given in Fig. 1b and shortly we shall see how useful this experimental order is in giving clues to the presence of assignable causes.[13] Hence, because of the importance of order, the statistical control engineer must insist that the experimentalist suggest orders in which to arrange the results of a series of n repetitions when he does not have any a priori reason for believing that any two repetitions have been carried out under the same essential conditions.

(3) The engineer may have a priori reasons for believing that the conditions may be divided into rational groups such that within each group the conditions are essentially the same but such that the conditions for each group are not essentially the same as those for any other group. In this situation, the engineer must insist that the experimentalist indicate the groupings and also the observed order within each group so that this order as well as the differences between the groups may be tested for indications of assignable causes.

Thus far we have considered three kinds of information that the statistical control engineer needs to know about the conditions associated with the repetitions of an operation so as to make the quantitative results useful in testing the hypothesis that the opera-

tion is in a state of statistical control. However, the contribution of the statistician does not necessarily end with the attainment of such a state, because even then the variance and hence the economic tolerance range may exceed the desired value.

Accordingly, it may be desirable to change the whole operation or some part thereof. For example, even though the variations in the thickness of the inlay of contact material shown in Fig. 1b were found to have risen from a state of statistical control, the variance might still be so large that many units of product would have to be rejected in assembly. Under such conditions, there would be no use trying to find and remove assignable causes of variability because these would have presumably been removed. If because of too large variance, the whole operation of producing the inlay is to be changed, the job is primarily one for the engineer, but if only some of the component parts of the operation such as the kind of inlay or fuse metal used, the rolling process, method of inserting the inlay, or heat treatment of the material are to be changed in order to reduce the overall variance, the statistician may be of great service in showing how to obtain at minimum cost the data necessary for analyzing the total variance of the operations into the component parts associated with the component parts of the operation of production.[14]

If some one of the component variances is significantly larger than the others, then it may be most economical to change, if possible, the corresponding component of the operation of production. In any case, the technique of analysis of variance makes it possible to estimate the maximum reduction in total variance that may be expected as a result of changing any component of an operation in a state of statistical control even though no substitute for this component is known at the time. It is very important, however, for the engineer to note that, in accord with hypothesis IIb, the problem of reducing the variance in the results of an operation already in a state of statistical control by changing one or more of the component operations is a fundamentally different[15] problem from that of eliminating assignable causes

[14] The principles underlying the design of efficient experiments for analyzing variance have been set forth by R. A. Fisher and his co-workers. See, for example, *Statistical Methods for Research Workers*, 7th ed., 1938, and particularly *The Design of Experiments*, 2nd ed., 1937, Oliver and Boyd, London. Also see *The Methods of Statistics*, by L. H. C. Tippett, 2nd ed., Williams and Norgate, London, 1937, for applications in the cotton industry.

[15] This significant practical difference has, in general, been overlooked by many students of statistical theory.

without changing the component part of the operation such as kind of inlay used, the rolling process, and the like in the example considered above.

When assignable causes are present, the engineer may reasonably expect to find and eliminate these without changing the production operation or any of its component parts. For example, if assignable causes of variation are present in the process of producing the thickness of inlay on relay springs as shown in Fig. 1b, the engineer may find that they could be removed by further adjustment of the controls of component operations of rolling, heat treating, and the like. When assignable causes are shown not to be present, and it is found that one of the component operations, say the rolling operation, contributes a large share of the variance, it will usually be most economical to secure a reduction in variance by changing this component operation, perhaps by using another type of rolling mill.

Enough has been said, I hope, to indicate some of the things that an experimentalist must do in taking and recording data if he is to obtain data that can be used efficiently in testing the hypothesis of statistical control and in analyzing the total variance under controlled conditions into its component parts or, in other words, if he is to make progress in the removal of assignable causes of variability that need not be left to chance in almost every field of science and engineering, thereby extending the potential usefulness of raw and fabricated materials.

Even in the face of this situation, however, it is still customary engineering practice to neglect the importance of order and instead group together into a frequency distribution all data whether $C_i \approx C_j$ or $C_i \neq C_j$ and irrespective of whether or not subgroups of conditions might be suggested. Then most likely engineers will use the average of this distribution and in addition they may possibly plot the distribution as a frequency curve or ogive.

For example, in making field studies on new materials or designs, studies of the electrical properties of contact materials, or the corrosion of materials, the engineer's faith is likely to be pinned on the averages of large numbers of observations whereas such a procedure is almost sure to mask the very differences that he is looking for. In fact, we shall shortly see that where there are assignable causes present, as soon as we group data together we are almost certain to destroy thereby all clues to the presence of

these causes, the effects of which must be eliminated before we can make valid comparisons of materials, designs, and the like.

Basic Test of Statistical Control Hypothesis. The test of the hypothesis that a technique can be established for approaching a state of statistical control *must be empirical.* One sets up criteria of control and then searches for and eliminates, if possible, the causes producing the deviations that fall outside the limits fixed by the criteria. If such search reveals the presence of assignable causes and if, as these are removed, one approaches a state where the criteria of control are satisfied, we accept this evidence as an empirical justification of the hypothesis.

A satisfactory technique for attaining a state of statistical control has been described in considerable detail elsewhere[16] and consists of the following five essential steps that are referred to in control theory as the *operation of statistical control*:

(1) Specify in a general way how an observed sequence of n data is to be examined for clues to the existence of assignable causes of variability. For example, it is essential that the order in an observed sequence always be tested for randomness whether $C_i \approx C_j$ or $C_i \neq C_j$.

(2) Specify how the original data are to be taken and how they are to be broken up into subsamples upon the basis of human judgments about whether the conditions under which the data were taken were essentially the same or not.

(3) Specify the criterion of control that is to be used and indicate what statistics are to be computed for each subsample and how these are to be used in computing action or control limits for each statistic for which the control criterion is to be constructed. Three of the conditions that such criteria should satisfy are as follows: the limits in the criteria should be as nearly independent as possible of the functional form of the law of chance when the state of statistical control is attained; the criteria should in so far as possible minimize the error of accepting the hypothesis when false and should keep the error of rejecting the hypothesis when true less than some prescribed value fixed by economic considerations; and the criteria should indicate as closely as possible the condition under which the assignable causes enter the operation and as much as possible about the nature of these causes.

(4) Specify the action that is to be taken when an observed statistic falls outside its control limits. The general action required

[16] *Statistical Method from the Viewpoint of Quality Control, loc. cit.*

is to look for assignable causes whenever the criteria are not satisfied.

(5) Specify the quantity of data that must be available and found to satisfy the criterion of control before the engineer is to act as though he had attained a state of statistical control.

To illustrate the role played by statistical criteria in the process of detecting and removing assignable causes of variation and to show how important it is not to group data together so as to destroy all possibility of ordering them in terms of the conditions under which they were taken, let us see what happens when certain of these criteria are applied to the two sequences of Fig. 1. Will the criteria tell us anything about these two sequences that we cannot see by just looking at the sequences themselves? To comprehend the significance of order, it should be kept in mind that the data in Fig. 1a are the same as in Fig. 1b except that in Fig. 1a they appear in the order drawn from a bowl. Anything that we can find out about the presence of assignable causes by studying the order in Fig. 1b is thus lost just as soon as this original order is destroyed by grouping the data together. Will the criteria of control indicate the presence of assignable causes of variation in both Fig. 1a and 1b? Of course, we know that if they give an indication of such causes for the drawings from a bowl (Fig. 1a), this will likely constitute a false lead because experience shows that we should not expect to find any assignable causes in the operation of drawing from a bowl, particularly when carried out by an experienced observer.

As a starting point, let us apply the control chart criterion based upon averages of successive fours.[17] Fig. 2 shows what happens: No false lead is given for the drawings from the bowl, Fig. 2a, but the presence of assignable causes of variation in thickness is indicated, Fig. 2b.

[17] For a detailed statement of the method of constructing a control chart, see *Economic Control of Quality of Manufactured Product, loc. cit.*, pp. 309–313. It should be kept in mind that, for reasons that I have given in the literature, it is desirable to use small subgroups as is here done. See, for example, Chapter 1 of *Statistical Method from the Viewpoint of Quality Control, loc. cit.* Of course, the practical man wants to know what is a small sample, and to him it may be of interest to know that in much of my own work, particularly in laboratory research, I have found it desirable to use where possible subgroups of size four. One reason for not using a larger sample is that, as we shall soon see, runs of seven or more are very unlikely for a statistically controlled process. Hence, if we are to be sure that at least one subgroup will be completely within a run of seven, we should not use a subgroup larger than four.

Next let us see if other criteria may be found that will tell us something about the nature of assignable causes to be expected. For example, do you detect any evidence of causes producing trends in the data of Fig. 1b? Is there a downward trend at the left and an upward one at the right? Or would you expect to find that the assignable causes are of the type that produce discontinu-
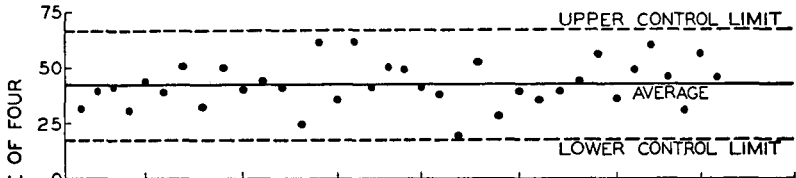

FIG. 2a CONTROL CHART FOR AVERAGES OF
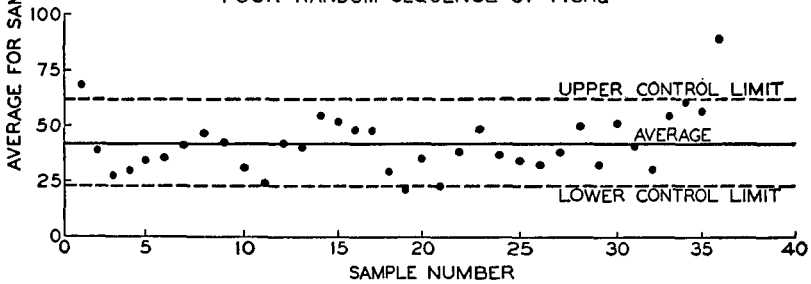FOUR—RANDOM SEQUENCE OF FIG. 1a

FIG. 2b CONTROL CHART FOR AVERAGES OF FOUR
SEQUENCE OF THICKNESS OF FIG. 1b

ous and erratic shifts in the expected values? Is there any evidence for believing that both kinds of causes are present? It should be kept in mind that we seek answers to these questions so that we may be better able to detect and remove the assignable causes.

Recent studies covering a broad field of research problems have shown that two very simple criteria may be used successfully in helping to distinguish between causes producing trends and those producing discontinuous and erratic effects. A complete report on the application of such criteria cannot be given here and it must suffice to indicate in broad outline the simple nature of these criteria and how they have been found to work. Basically, what is done is to note two kinds of runs in any sequence: runs up and down and runs of numbers above and below the average of the sequence where runs up and down are defined as follows. In any sequence, certain numbers are greater than either of their im-

22

mediate neighbors and form, as it were, maxima in the sequence. In a similar way, we have minima. A run-down is the interval between a maximal number and the next succeeding minimal one, and the length of run-down has been defined in the literature as the number of numbers involved, including the maximal and minimal numbers. Corresponding definitions hold for runs-up.
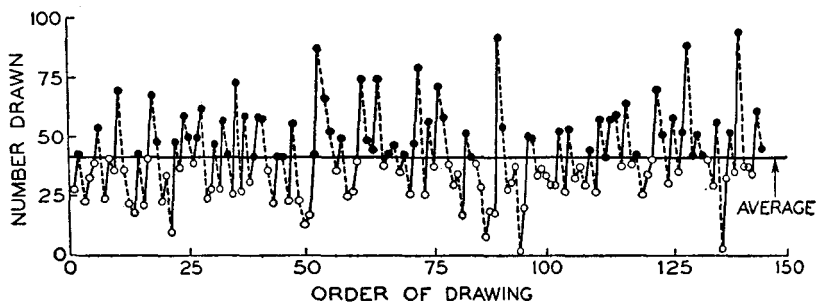

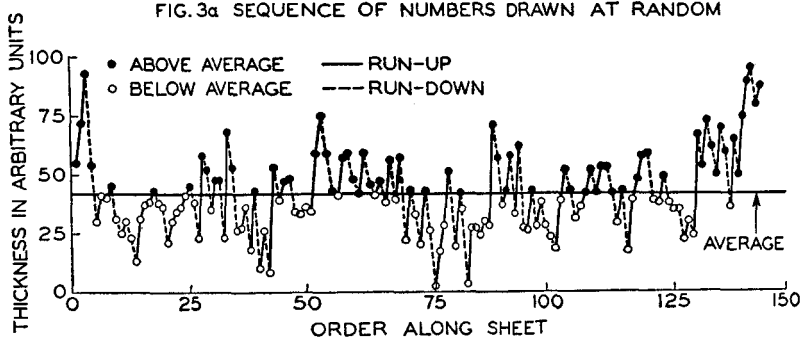FIG. 3a SEQUENCE OF NUMBERS DRAWN AT RANDOM

FIG. 3b SEQUENCE OF VALUES OF THICKNESS OF INLAY ON RELAY SPRINGS

In Fig. 3 where the data of Fig. 1 are replotted, runs-up are shown by solid lines connecting observed values, and runs-down by dotted lines. Also in Fig. 3 the points above the average of the sequence are filled in and those below are not.

W. O. Kermack and A. G. McKendrick[18] have recently given

[18] "Tests for Randomness in a Series of Numerical Observations," *Proc. of Roy. Soc. of Edinburgh*, Vol. LVII, pp. 228–240, 1937. If the total number of observed runs up and down be N, these authors show that the expected number of lengths 2, 3, 4 . . ., are $0.6250000N$; $0.2750000N$; $0.0791667N$; $0.0172619N$; $0.0030506N$; $0.0004547N$ and $0.0000587N$. See also R. A. Fisher's note, "On the Random Sequence," in the *Quarterly Journal of the Royal Meteorological Society*, July, 1926, p. 250. Of course, the probabilities here given apply to a frequency distribution of runs in an infinite sequence. By making use of results given by Kermack and McKendrick in a paper, "Tests for Randomness in a Series of Numerical Observa-

the probability of a run of any length l occurring in an observed set of N runs up and down from any infinite random sequence independent of the law of distribution of the variable. Likewise, W. G. Cochran[19] has recently given a formula for deciding whether sequences of similar meteorological events, such as runs of consecutive wet months, may be expected under a state of statistical control. If the result of an operation must be either E_1 or E_2, and if we know the probability p that an operation will give the event E_1, and the probability $(1 - p) = q$ that the operation will give the event E_2, then Cochran's work gives the expected number of runs of length r (counting runs of both E_1 and E_2) in any sequence of m repetitions of the operation. If now we call $\overline{X}$ the average of a sequence of m observed numbers and if we call event E_1 the occurrence of a number in the sequence greater than $\overline{X}$ and event E_2 the occurrence of a number less than $\overline{X}$, we may use Cochran's theory to compare the observed number of runs of any length r with the corresponding theoretical number to be expected upon the *assumption* that the observed fraction of the numbers greater than the average $\overline{X}$ is equal to the probability p that the underlying cause system will give a value of X greater than the number $\overline{X}$, considered simply as a number and not necessarily as the average.[20]

Now, if the assignable causes simply produce erratic shifts in the expected values, it is not likely that the distribution of lengths of runs-up and of runs-down will be much disturbed, but such shifts will tend to give extra long runs of numbers above and below average. However, if the assignable causes produce trends with

tions," *Proceedings of the Royal Society of Edinburgh*, Vol. LVII, Part III, No. 17, pages 352–356, it may be shown that there will not be appreciable differences in this frequency distribution so far as we are here concerned with its use even though we are dealing with an observed distribution of runs from a finite sequence of 144 numbers.

[19] Cf. "An Extension of Gold's Method of Examining the Apparent Persistence of One Type of Weather," in *Quarterly Journal of the Royal Meteorological Society*, Vol. LXIV, pp. 631–634, 1938. The number of runs of length r of the events E_1 and E_2 out of m trials is

$$f_{m,r} = 2(p^r q + p q^r) + (m - r - 1)(p^r q^2 + p^2 q^r) \text{ when } 1 \le r \le (m-1)$$

where $p + q = 1$.

[20] Of course, one might arbitrarily choose any value X_1 and any value p and use Cochran's theory to compare the observed number of runs of length r both above and below X_1 with the corresponding theoretical number to be expected upon the assumption that the probability of occurrence of a number greater than X_1 is p.

slopes large in comparison with the fluctuations produced by the superimposed causes acting at random, the distribution of lengths of runs-up and runs-down will be disturbed. Likewise, the distribution of runs of numbers above and below the average of the sequence may be somewhat modified.

Of course, we should expect close agreement between theory and experiment for the lengths of runs of both kinds in Fig. 3a, but since the control chart Fig. 2b gave evidence of the presence of assignable causes, we may expect discrepancies between theory and experiment for the lengths of runs in Fig. 3b. The data for such comparisons are given in Table 2. As expected, there is close

TABLE 2

	Drawings from bowl				Measurements of inlay thickness			
	Runs above and below average		Runs up and down		Runs above and below average		Runs up and down	
Length of runs	Observed number	Expected number	Observed frequency	Expected frequency	Observed number	Expected number	Observed frequency	Expected frequency
1	42	36			29	36		
2	18	18	65	61	13	18	53	57
3	10	9	22	27	3	9	28	25
4	4	4	10	8	5	4	7	7
5	4	2	1	2	0	2	2	2
6	0	1	0	0	4	1	1	0
7	0	1	0	0	4	1	0	0
8	0	0	0	0	1	0	0	0

agreement for the drawings from the bowl. For the sequence of thickness measurements, there is excellent agreement between theory and experiment for the distribution of lengths of runs-up and runs-down but not for lengths of runs above and below average.[21] These results constitute good evidence for believing that the assignable causes underlying the measurements of thickness in Fig. 3b do not produce trends such as might be produced by lack of symmetry in the rolls but simply produce discontinuous and erratic shifts in expected values of thickness such as might be produced by slippage at the cleavage planes of the crystals in the inlay.

Not only does the application of these criteria to the observed

[21] Of course, the total number of expected runs above and below average will not usually be the same as the corresponding number of observed runs.

order indicate the presence of assignable causes and give a clue to their nature, but it also indicates when the assignable causes enter. For example, we see from the table that runs above and below the average of length greater than 7 are very unlikely,[22] and all we need to do is to note where these runs occur, keeping in mind that such indications are subject to errors of the first and second kind as described above.

Since all of this information about assignable causes is lost just as soon as we group the data together in the form of a frequency distribution, as is so often done by the engineer, we see how very important it is to pay attention to order in taking and recording data. For example, years of experience have shown that Nature almost never gives us a sequential order that passes even the control chart criterion and is almost certain not to give us one that passes all three criteria and that the failure to meet such criteria almost always is traceable to an assignable cause. Yet, if we were to take these same data without reference to observed order as might be done by shaking them up, drawing them at random, and then applying the three criteria, almost all of them would slip through the net, thereby failing to indicate the presence of assignable causes that must be detected and removed in order to attain the advantages of a state of statistical control.

Since, as noted above, experience has shown that one can almost always find and remove assignable causes when indicated by such criteria and thereby approach a state of statistical control beyond which it is not possible to go except by some fundamental change in the operation itself, we have good grounds for accepting the basic hypothesis of statistical control theory as stated at the beginning of this section.

SUMMARY OF POTENTIAL CONTRIBUTIONS OF STATISTICS TO THE SCIENCE OF ENGINEERING

The basic contribution of statistics to the science of engineering is an *improved scientific method to fit the world of probability in which we live*. Classical theory contributes the hypothesis of a repetitive operation obeying a law of chance, the knowledge of which combined with the knowledge of formal mathematical distribution theory enables an engineer to make valid predictions of the out-

[22]The expected number is actually 0.5682 instead of the rounded value of unity given in Table 2.

come of future repetitions of the operation. Statistical control theory contributes the hypothesis that it is humanly possible to remove assignable causes of variability in the repetitive operations of the engineer until such operations approach a state of maximum control and obey laws of chance, a knowledge of which provides maximum assurance that the results of repeating an operation will fall within previously specified tolerance limits. To test these two hypotheses, statistical theory provides the necessary experimental techniques outstanding among which are (a) the operation of randomization and (b) the operation of statistical control.

Broadly speaking, statistical theory treats of repetitive operations and provides the engineer with a method of regulating such operations to his best interest. Fundamentally, the engineer's job is to devise operations that, if carried out, will give results within previously specified tolerance limits. Sometimes the operation, like that of building a bridge, is to be carried out only once or at most a few times, and sometimes the operation, like that of mass production, is to be carried out an indefinitely large number of times. Inherently, all such operations are potentially repetitive, and in this sense, differ only in the number of repetitions carried out. It has long been recognized that one of the most revolutionary principles ever introduced into manufacturing was that of interchangeability dating back at least to Eli Whitney in 1798. The introduction of that principle prompted the engineer to consider the advantages of introducing repetitive operations into production processes, and the contribution of statistics to engineering may be thought of as a means of maximizing the advantages to be attained by interchangeability.

The basic contributions of statistics to scientific method make possible the attainment of the following objectives that are not otherwise attainable and that are of interest to all of us:

(1) Even before a repetitive operation has reached a state of statistical control, the application of statistical theory makes possible the establishment of sampling plans that will screen at minimum cost the output of such an operation so as to meet previously specified tolerance requirements and previously specified producer and consumer risks.

(2) The use of statistical theory provides efficient experimental techniques based upon the operation of randomizing the results obtained from one operation or production process that is not in

a state of statistical control before submitting them to two or more subsidiary operations or treatments for the purpose of comparing the effects of these subsidiary operations, as in the case of field tests and the like. Such a procedure minimizes the chance of concluding that observed differences in the effects of the subsidiary operations are significant when in fact they came about because of assignable differences in the results of the first operation.

(3) The operation of statistical control provides an experimental technique for minimizing tolerance ranges and maximizing the assurance that the product turned out by a given process will meet its tolerance requirements. Such an operation makes possible the most efficient use of limited quantities of raw material and provides the maximum degree of refinement attainable by any production process. Preliminary studies indicate that the operation of statistical control also provides a useful technique for eliminating assignable causes of variability in certain kinds of human effort as, for example, typing and other forms of transcription. Both strategically and commercially, industrial groups and even nations often need every increment of efficiency in the use of limited quantities of raw materials and human effort that can be provided through the application of the operation of statistical control. Likewise they often need maximum refinement in quality through elimination of assignable causes, not only in pursuit of the arts of peace but also in time of war. As one example, the attainment of maximum homogeneity and hence minimum tolerance ranges in the properties of raw and fabricated materials may extend the potential carrying capacities of ships in the air and on the sea. Needless to say both the engineer and the consumer of the engineer's or manufacturer's products stand to gain through the increased assurance that the products will be found to meet their tolerance requirements.

(4) The operation of statistical control provides a technique for modifying and coördinating the three fundamental steps in the process of mass production, namely, specification, manufacturing, and inspection, so that the maximum number of pieces of product having a quality within specified tolerance limits can be turned out at given cost. It does this by showing how to minimize the cost of inspection and the cost of rejection.

In conclusion it may be said that statistical theory plus mass production provides a means of maximizing our physical comforts in time of peace and our strategic factors in time of war.

28