[Reprinted from the March Supplement of the AMERICAN STATISTICAL JOURNAL]

APPLICATIONS OF STATISTICAL METHOD IN ENGINEERING

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I have been invited to indicate briefly what an engineer can do with the aid of statistical method that he cannot do without it.

There are many ways of doing this. I might discuss some of the detailed problems of a civil engineer, such as the determination of the maximum run-off of a flood area; the work of Westman of the Ontario Research Foundation in the application of statistical theory in ceramic engineering; the work of Hayes and Passano of the American Rolling Mills Company in the study of corrosion; the investigations of Wilson and others of the Forest Products Laboratory; the researches of Becker, Plaut, Runge and Daeves in the production of steel in Germany; or the work of any one of a number of others, including many of my colleagues in the Bell System who are applying statistical theory in the solution of their engineering problems. Applications have been made in every field of engineering.

However, to consider only the problems to which statistical method has been applied, would be to leave out of the picture some of the most important applications of the theory in engineering work. This is particularly true since industry is only now beginning to appreciate many of the important applications that can be made in connection with the control of quality of manufactured product. I have discussed elsewhere[1] in detail the nature of the applications of statistical method to research, development, design, production, inspection and supply so that any one interested in such details can easily avail himself of this information. Today I shall limit myself to a consideration of two fundamental concepts which characterize the present era of scientific development and necessitate the revision of previously accepted methods of presenting, interpreting and using all kinds of engineering data.

For a long time engineers based their developments upon two fundamental assumptions: (1) A physical quantity is a *true fixed value*, whereas measurements of this quantity differ from the true value because of ever present errors of measurement; (2) A physical law is a *functional* relationship of the mathematical type, whereas observed relationships are always influenced by ever present errors of measurement.

[1] W. A. Shewhart, "Economic Aspects of Engineering Applications of Statistical Methods," *Journal of the Franklin Institute*, Vol. 205, March, 1928, pp. 395–405.

In other words, even though engineers have heard much about statistical methods and their application in education, sociology, economics, etc., they have been inclined to stand aloof and say: "Well, these methods may be all right for the fellow who deals with such an inexact science as education or economics, but, thank goodness, we do not have to depend upon their use because we are dealing with the application of exact sciences, such as physics and chemistry."

Of course, they have been willing to admit that the theory of errors—a special form of the statistical method—has enabled them as engineers to do many things that they could not have done otherwise. For example, such a theory gives a rational basis for estimating the error of estimate of the assumed true but unknown quantity. With the development of this theory, it became possible to determine how many measurements should be made in order to reduce the error of an estimate of the assumed true value to any preassigned magnitude.

On the basis of these older assumptions, the theory had even more important applications than those just mentioned. Most physical and engineering measurements are indirect. The magnitude of the quantity to be measured is expressed in terms of measurements of some m other quantities to which it is assumed to be functionally related. In all such cases the theory of errors makes it possible to determine the effect of errors of measurement in any one of the m quantities upon the resultant error and to choose the best of several possible methods of measurement. This contribution has gone a long way toward increasing the efficiency of industrial research.

Scientists and engineers alike have made use of error theory in helping them to find or fix upon *the* objective functional relationship assumed to exist between two or more quantities or, in other words, to find or fix upon the objective laws of nature. Perhaps the most important criterion developed for this purpose is the method of least squares.

Thus we see that statistical method has played an important rôle in engineering from the very beginning of the period of industrial expansion based upon the applications of scientific principles. We are, however, living in the dawn of a new era; an era in which there is every reason to believe that the applications of statistical method in the past will be overshadowed by far more important applications in the future.

We hear distinct rumblings of a revolution in the camp of the "exact" sciences. The concept of exact is overthrown for the moment at least, and in its place statistical concepts hold sway.

But what of it? We as engineers have grown accustomed within recent years to cataclysmic upheavals in physical theory. Few of these have reached the height of general interest attained by the theory of relativity; and yet, after everything has been said and done, how many of us today have changed our engineering practices because of relativity? True it is, relativity has its place, but as yet it is not a useful tool for most of us. May it not be, therefore, that all of this new interest in statistical theory is but a tempest in a teapot so far as it touches us who are concerned primarily with those things which necessitate significant changes in our utilitarian mode of thinking?

Let us therefore consider two of the changes which have come into our fundamental scientific concepts which indicate that the applications of statistical method in engineering have come to stay—I refer to the statistical nature of physical properties and physical laws. Engineering development is based upon the use of such properties and laws. Hence, the substitution of statistical for "exact" concepts in this field is of vital engineering significance, as we shall now see.

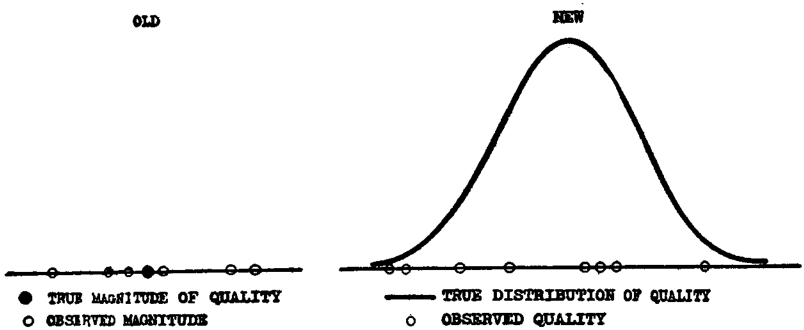
A. *Statistical Nature of Physical Properties.*—What engineer is not interested in the physical properties of materials? What engineer does not have about him a table of the so-called physical and chemical constants? Yet these so-called constants are really *not* constant. This idea of constancy holds over from the older concept of a physical quantity as a true, fixed, objective value.

Let us see why this new concept is of importance. In the first place it makes it necessary for us to revise our picture of a property of a material. To make our discussion specific, let us fix our attention on a metal bar. What is the density of the metal in that bar? If we take this question to mean point to point density, the answer is indeterminate. Density has meaning only as a statistical average of indeterminate point to point densities. In fact, the density of material of this kind is not an objective constant but rather an objective distribution function.

Suppose we were to take 1,000 similar bars and were to break them to determine their tensile strength. We would find that the observed values differed not alone because of error of measurement, but primarily because the tensile strength of bars, as nearly alike as we know how to make them, is a distribution function and not a fixed true value. We cannot say, therefore, that the tensile strength—or any other physical property of a material—is a certain fixed value. Instead all that we can ever hope to say is that a certain proportion of a given kind of material, essentially the same so far as we can determine, will have a tensile strength—or other physical property—lying within a specified range. The difference between the old and the new concept is shown schematically in Chart I.

Since objective physical properties are distribution functions, stand-

CHART I



ards for such properties should be distribution functions. Now, of course, if the coefficients of variation for most physical properties were small, the engineering importance of the statistical nature of such properties would not be so great as it is under the conditions that actually exist, as we shall show by a simple example.

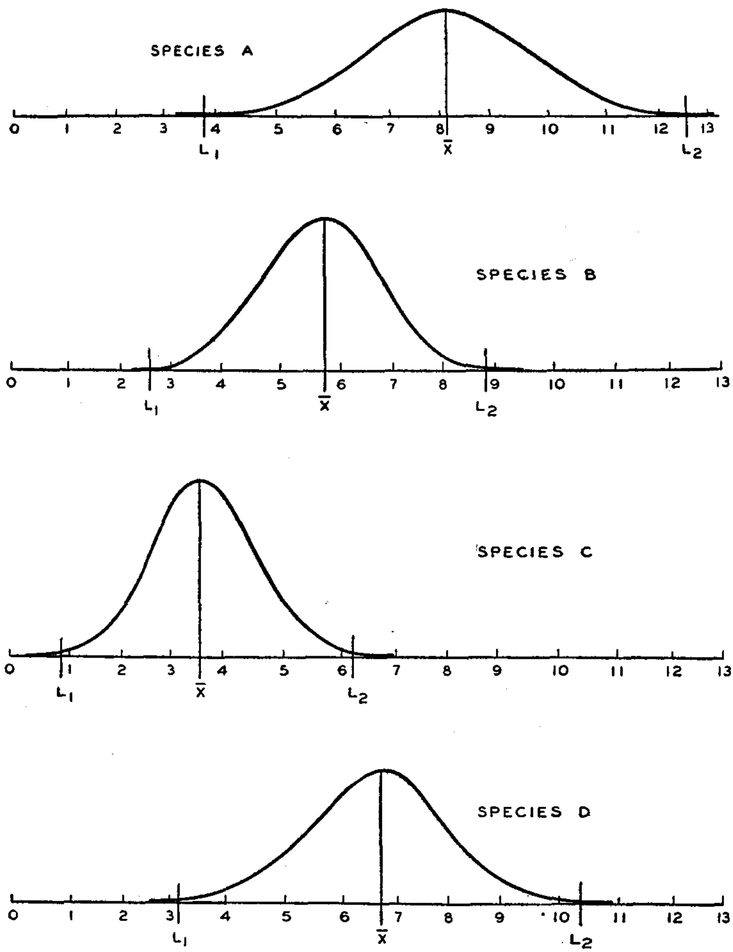
An important property of material is that indicating strength. Thus for wood an engineer is interested in its modulus of rupture. If he turns to an engineering handbook or almost any other source, he usually finds a single figure recorded for each species of wood. One such table of data taken from a standard book is shown below. Is the engineer to draw the conclusion from such a table that every specimen of long-leaf pine, for example, has a greater modulus of rupture than a specimen of any other species cited in this table? So far as the table is concerned, he might justly draw such a conclusion.

TABLE I

	Modulus of rupture
Cyprus	7110
Douglas fir	8280
Eastern hemlock	6685
Loblolly pine	7870
Long-leaf pine	8380
Norway pine	5173
Red spruce	5900
Redwood	6980

Chart II shows why, in such a case, the average does not tell the whole story. This chart gives approximate standard distribution functions for the modulus of rupture of round timbers from four species. It is apparent that the variability is large compared with the mean modulus of rupture. In fact some pieces from each species will have

CHART II



the same modulus of rupture. Hence, what the engineer needs is a working approximation to this standard distribution function that will tell him approximately the proportion of the pieces of material of a given species that may be expected to have a modulus of rupture within any two fixed limits. Certainly a single value of the modulus of rupture for any one of the four species shown in Chart II falls far short of giving the desired information.

What is true in the case of the modulus of rupture of wood is true of many of the most important physical properties of materials—the statistical nature of physical properties is of engineering significance.

Speaking of the modulus of rupture of telephone poles naturally brings up the sampling problem involved in giving assurance that the standard of quality of a material is being maintained, particularly in those cases where, as in the determination of the modulus of rupture of telephone poles, the test is destructive.

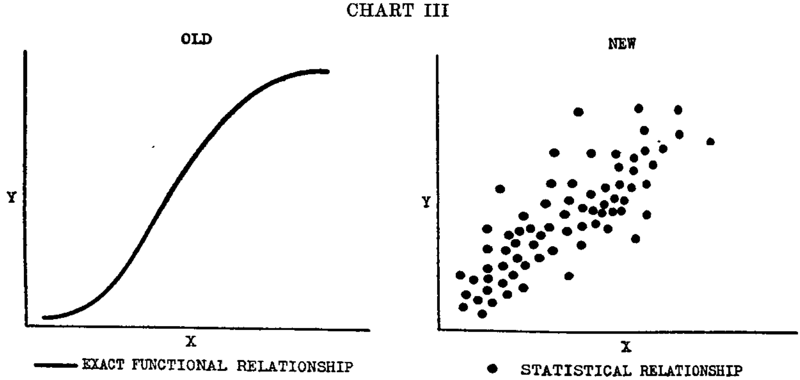
To be specific, how would you choose a sample of 100 poles from a pole yard for the purpose of giving us the best information as to the quality of the poles in this yard in respect to modulus of rupture? To answer this kind of question, the engineer finds it necessary not only to make use of available sampling theory but to extend this theory. Without such theory, he has no rational basis of assurance that the quality of product is being maintained.

When we stop to think that some of the most important qualities of everything that we use must be maintained upon a sampling basis because of the destructive nature of tests, we get a glimpse of the importance of sampling theory of this kind. Three specific illustrations are the fuse that protects your home, the steering rod of your car and everything that you eat.

B. *Statistical Nature of Physical Laws.*—No longer do we believe that relationships between physical quantities are functional in the mathematical sense. Instead, we think of them as being *statistical.* The contrast between the old and the new concept is indicated schematically in Chart III.

As a specific illustration, let us consider the relationship between tensile strength and hardness for some metal such as steel. The older concept assumed the existence of a functional relationship between hardness and tensile strength, represented by a curve showing a one-to-one correspondence between these two properties, as schematically illustrated in the left half of Chart III. Observed deviations from

CHART III



this hypothetical curve were attributed to errors of measurement. In fact, many calibration curves of tensile strength in terms of hardness are based upon this older concept.

Today, however, we look at this situation in a different light. No longer do we believe that there is a one-to-one correspondence between such properties. Instead, we believe that there is only a statistical relationship of pairs of values of two such quality characteristics corresponding to all possible samples of what we assume to be essentially the same material. This situation is represented schematically in the right half of Chart III. No longer then, in such a case, are we free to treat the deviations of an observed set of points from any curve of best fit as errors of measurement.

Suppose that you have, let us say, 1,000 metal bars, all of which have been made under the same essential conditions so far as it is possible to do this. Suppose that these bars are to be used in a piece of apparatus where it is desirable to insure that the tensile strength lies between two preassigned limits Y_1 and Y_2. How would you divide the bars into two piles—one that meets the limits and one that does not?

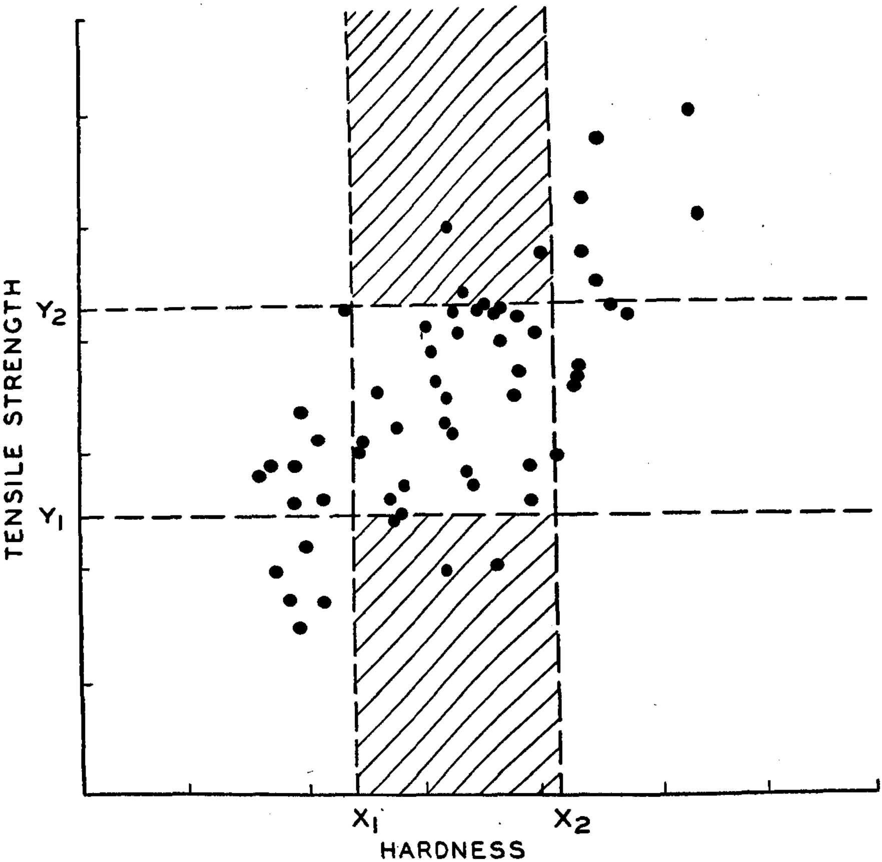
Obviously you cannot test these bars by breaking them, for then you would have no bars left. In fact you must resort to the use of some statistically correlated variable, such as hardness, as a measure of tensile strength. Immediately, however, you are confronted with the fact that the inherent indeterminateness of such a statistical measure makes it impossible to set up any two limits X_1 and X_2 on hardness such that one can be assured that the pieces of the material having values of hardness within these two limits will have tensile strengths within the specified range Y_1 to Y_2. The situation is shown schematically in Chart IV.

Even when the two variables, tensile strength and hardness, are *truly correlated*, the best that one can hope to do is to say that the probability that a piece of material having a hardness between X_1 and X_2 will have a tensile strength between Y_1 and Y_2 is some fixed value p'.

The establishment of inspection methods to assure the quality of product in such an instance not only necessitates the application of statistical theory now available in the literature but even of an extension of this theory.

We have seen that, whereas the philosophy of industrial development of the old era attributed importance to the statistical method only in the handling of errors of measurement, the philosophy of industrial development today takes as two of its fundamental postulates the statistical concept of physical property and the statistical concept of physical law. Furthermore, industry today is becoming more and more

CHART IV



aware of the fact that it must rely upon statistical method to furnish a rational basis for establishing economic standards of quality of raw materials and finished products; to assist in obtaining minimum overall variability in finished products; to provide ways and means of reducing to a minimum the cost of inspection and the cost of rejection; and to give to the consumer maximum assurance that the quality of product is being maintained.