USE OF LAWS OF CHANCE

IN

INDUSTRIAL DEVELOPMENT

By

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USE OF LAWS OF CHANCE IN INDUSTRIAL DEVELOPMENT

SCOPE OF DISCUSSION

There are many aspects of industrial development. I shall choose but one as a basis for my discussion this evening. It is, however, one in which marked progress has been made in recent years through the application of laws of chance; one in which there is already an organized effort being made on a national and even an international front to bring to fruition the potential advantages of the application of the laws of chance; and one in which such progress has already made possible better standards of living for all of us. I refer to the economic control of quality under conditions of mass production.

An essential characteristic of Man, differentiating him from other animals, is his ability to predict and within limits to control the workings of nature. Among the earliest evidence of such control are the tools of our ancestors - tools which helped them to attain preconceived ends, tools which helped them to kill wild animals for food, to build their shelter from the elements, to obtain that with which to clothe themselves - in general, to satisfy their wants. Man is a creature of wants and the history of industry is largely a record of his struggle to produce physical things of such quality as to satisfy these wants. Tonight I wish to consider briefly some of the important steps in this conquest to control physical phenomena in the production of physical things to satisfy human wants, and to indicate the rôle played by laws of chance in this development.

An understanding survey of the progress made in controlling our environment must take into account two factors: (a) the change in mental picture and (b) the change in ~~the~~ human needs. For example, we have passed through stages in our development when Man perhaps did not dream of control but like the ape was content to use things as he found them and a stage where his attempt to control was through magic on the one hand or sacrifices to capricious gods on the other. Then came the stage of the deductive philosopher, scientist, or sage, who depended upon deductive reason, and finally the stage of observation and hypothesis, or scientific method. For our present purpose, we shall concern ourselves primarily with this latter stage to which progress, as we shall see, is largely confined.

STEPS TOWARDS MASS PRODUCTION

The beginnings: Let us imagine ourselves at a play showing some of the important steps in human progress toward the attainment of economic control of quality of product. The curtain rises. We view a landscape of some 1,000,000 or more years ago. We see our ancestors living much as do other animals for some 700,000 years and there is little evidence of attempts at control, except for the production of crude stone implements[1] such as those shown in Fig. 1.

Fig. 1 - Earliest Implements 1,000,000 ± B.C.

The earliest records of man's existence are the flint implements found at Bramford, England in 1909. Not admitted as evidence, however, until many years later. For example, Osborne did not admit them until 1921. These date back more than 1,000,000 years to the Pliocene age.

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1) From <u>Man Rises to Parnassus</u> by Henry Fairfield Osborne, Princeton Univ. Press, 1928

It is now 8,000 B.C. As the curtain rises on the third scene we see something quite spectacular from the viewpoint of development - the fitting together of piece-parts[2]. We see a workman fashioning holes in the stone implements[3] of Fig. 3, into which he later fits handles.

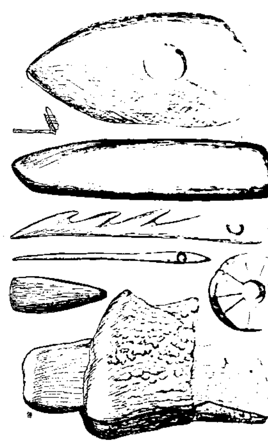

Fig. 3 - Parts to be Fitted with Handles

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1. From Early Steps in Human Progress by Harold J. Peake, J. B. Lippincott Co., 1933.

2. Some maintain that handles were fitted to fist hatchets several thousand years before, but there is no proof that handles were fitted to hatchets until the time here indicated. Cf. Introduction to Sociology, Davis, Barnes and others, D.C.Heath Co., 1931, p. 31.

3. From Peake, op. cit.

The second scene opens. It is 300,000 B.C. The basic industries of man are now hunting, fishing and the gathering of herbs, roots and berries. We see here and there the first real attempts at control in the form of production of wooden, bone, and stone one-piece implements. Typical examples are shown in Fig. 2. Such tools include the fist hatchet, knives, chisels, planers, scrapers, and the like. During this scene covering a period up to 8,000 B.C. there is nothing very spectacular as viewed from where we are today.

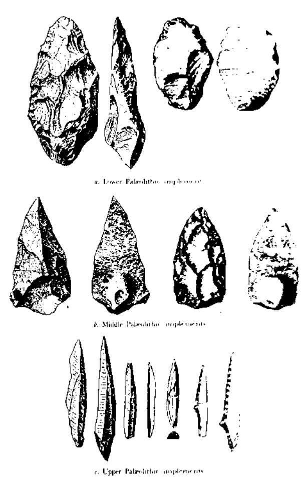
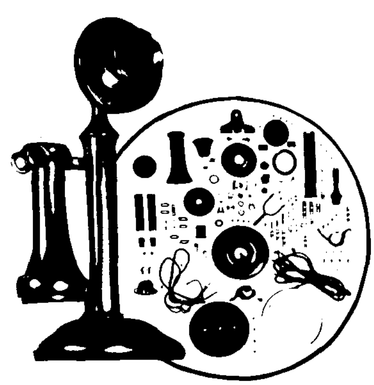


Fig. 2 - Paleolithic Implements - 300,000 B.C. to 10,000 B.C.

How interesting it is to see how many "tries" he makes in getting a handle that will just fit snugly into the crude hole.

We cannot help contrasting this very simple task of our early ancestors with the more complicated problems of fitting parts together under the mass production of today. For example, the telephone desk stand is not so simple as it looks, Fig. 4. To make it requires 201 parts and to



Not so simple as it looks

Fig. 4 - Not So Simple as it Looks

connect it with another requires upwards of 110,000 other parts. The annual production of such parts often runs into the millions. The manufacturing organization of the Western Electric Company buys for use in the Telephone plants more than 25,000 different items of materials and supplies and these come from more than 15,000 suppliers located in many different countries.

The next two figures (Figs. 5 and 6) show the rather complicated structure of the modern hand-set.

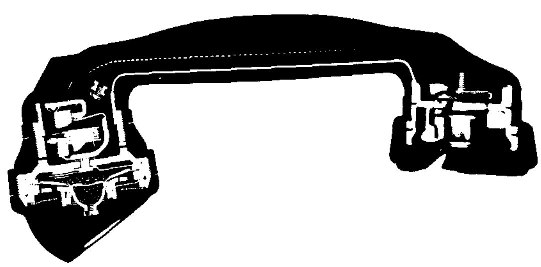
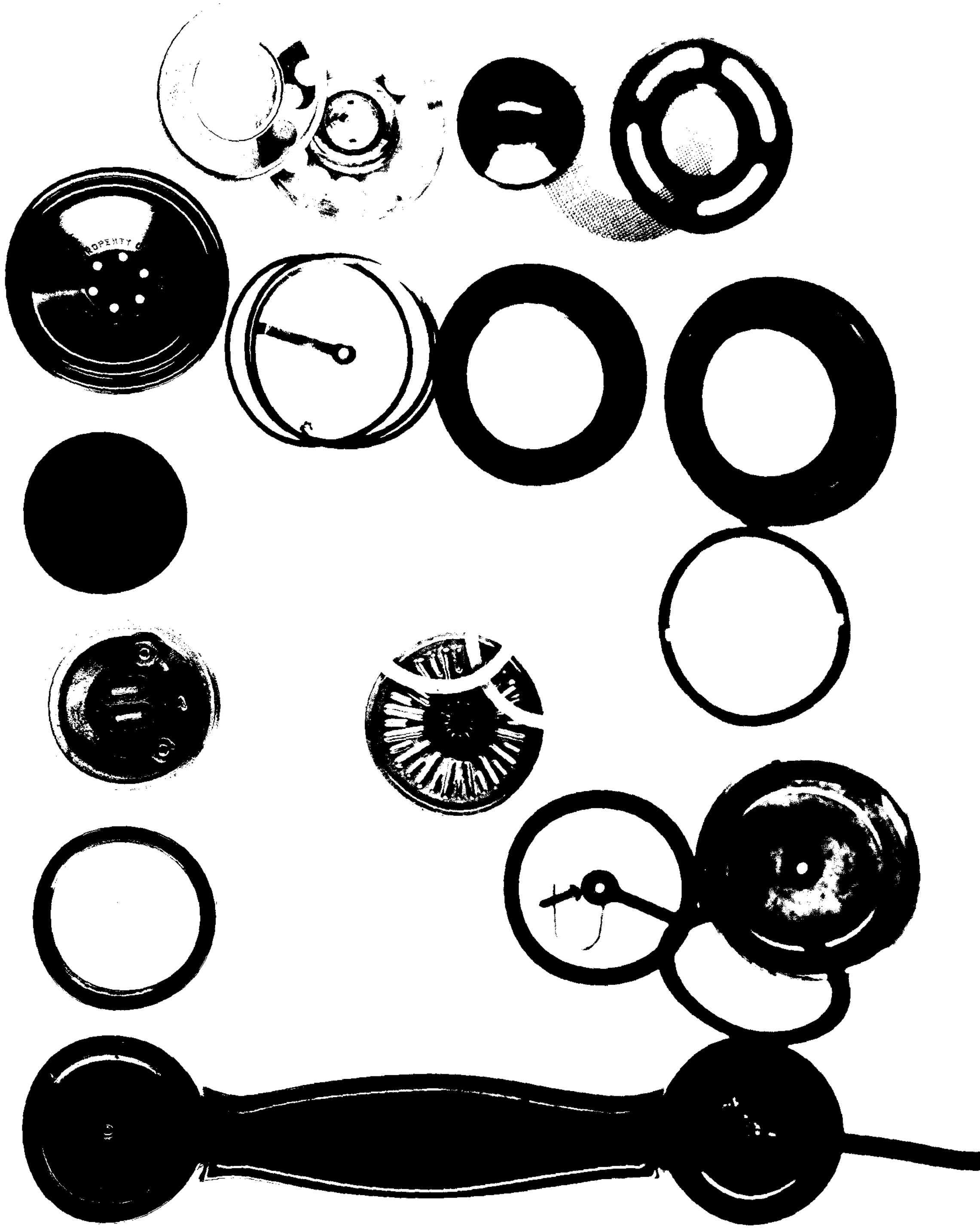


Fig. 5 - Handset Assembled

Fig. 6 - Handset Piece-Parts



Up to this time we are struck by the fact that, for the most part, each man makes his own tools. This situation is soon to change. About 3000 B.C. we see clouds of war gathering and now artisans gather into groups to produce parts to be fitted together into bows and arrows. We are witnessing what is perhaps man's first effort to produce interchangeable parts.

Lots of other interesting things take place in the remainder of this scene. One named Roger Bacon (1214-1294) flits on and off the stage, and leaves us with the thought that "We must first observe" in order to get at an understanding of nature that will in the end make a measure of control possible. Then comes the Father of modern physics, Galileo (1564-1642). But what is that we see going on at one corner of the stage? What! It is that notorious gambler of France, Chevalier de Mere, playing a game of chance with some of his friends. After a while he seeks out the famous mathematician Pascal and tries to get some advice on how to control his winnings through the application of PROBABILITY[1]. Pascal's efforts mark the

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1) Of course games of chance are of older origin.

A. Dice

North American Indians said to have played dice since 1636 - 300 years. Dice probably originated in the orient. Dice similar to those used to-day have been used since the earliest times being found among the ancient relics in Egypt, the Mediterranean and the Far East. They were certainly used prior to written records. (From Encyclopedia Brittanica) Helen Walker in N.Y.Times 12/19/35 says that dice on which the numbers on the opposite faces add up to seven have existed for at least 2000 years.

B. Cards

Origin obscure. Widely taken view is that they come from Asia. When they first became known in Europe is uncertain but certainly were known between 1240-1400 A.D. The Spaniards introduced them to the American Indians. (From Encyclopedia Brittanica)

C. Probability

Before 1600 - only 6 or 8 recorded problems. These in the Orient.

Earliest European reference in Comment on Dante's Divine Comedy concerning different throws with 3 dice. (1477).

Lacia Pacioli first writer to introduce a gambling problem into a work on mathematics (1494).

Cardan published (1663) a sort of gambler's handbook.

First scientific work on probability began 1654. Pascal and Fermat from studies in the History of Method pub. by William and Wilkins Co. 1929.

beginning of the development of the mathematical theory of probability which was later to play such an important role in the application of the laws of chance. Had engineers then taken a tip from this early attempt at applying laws of chance, perhaps my story would have been quite different from what it is going to be. Why didn't they do so? we may ask. The answer is that the need was not yet felt, as we shall soon see. They had not as yet been faced with the problems of mass production.

Shortly before the curtain goes down on this scene we see another attempt to produce interchangeable piece-parts on a quantity basis. This time, however, the parts are for muskets and the requirements for fit are much more stringent than was the case for bows and arrows. We may not be far off if we think of this attempt at mass production of interchangeable parts as pretty much the starting point of mass production as we know it to-day.

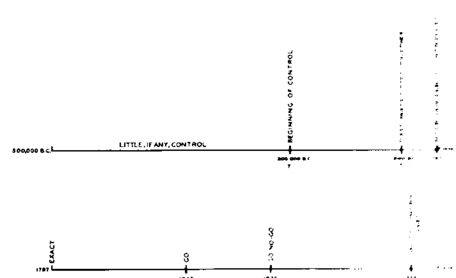

Fig. 7 - Important Steps in Control

Fig. 7 summarizes schematically what we have just been seeing. How long it took man to get under way.

Two Steps in Attaining Control Within Limits

Next we shall sketch two important steps in attaining control of quality within limits as is required in the mass production of interchangeable parts.

So far as I have been able to determine, the important concept behind the mass production of 1787 was that of making piece-parts to exact size so that they could be assembled at random without any rejections. This and succeeding attempts to attain exactness ended, as might have been expected, in the realization that it is impossible to make piece-parts to exact size. It was, of course, more or less to be expected that this first attempt would be to make things in accord with an exact science because the spirit of exactness in science was a controlling element throughout the greater part of the nineteenth century.

By 1840 or thereabouts another important step had been taken in the aim of production. No longer did they try to do the impossible by trying to make things exactly alike. Instead they simply tried to make piece-parts with dimensions or qualities that did not exceed some value. This could be done but it turned out to be an expensive process because the machinist tried to approach as close as possible but not to exceed the go limits on the quality characteristics. Hence to overcome this difficulty another important step was taken about 1870, this time the introduction of the "go no-go" limits on the quality characteristics of the piece-parts.

Roughly speaking, the controlling idea was that any quality characteristic of a piece-part was satisfactory so long as it fell within these limits.

From the viewpoint of the use of laws of chance in industrial development, it is of interest to note (a) how the first attempts to make interchangeable parts (1787) to exact dimensions met with failure because of chance or unknown causes and (b) how the first step (1840) to overcome the difficulties led on to the adoption of the "go no-go" tolerance. Here we see the engineer trying to get around the effects of chance causes of variability but still <u>thinking for the most part in terms of an exact science composed of exact laws</u>. There is no evidence that engineers were even thinking about laws of chance.

<u>Control Within Limits Based Upon Concept of Chance Causes</u>. During the nineteenth century, however, certain phenomena were being observed in the physical sciences the study of which was destined to change scientific faith in the possibility of attaining exactness. In 1827 the botanist Brown,

A CLOSE-UP OF MOLECULAR MOTION APPEARING ABSOLUTELY IRREGULAR YET CONTROLLED WITHIN LIMITS

Fig. 8

for example, observed the insistent movement of small particles, later called the Brownian movement. There also grew up in the period following, particularly during the latter part of the nineteenth century, the realization that the microscopic properties of matter - those properties and qualities which we can observe - are fundamentally dependent upon properties or qualities of microscopic and sub-microscopic atoms and molecules. As the twentieth century opens, physicists were beginning to accept the statistical nature of many of the observables of nature, or, in other words, the statistical nature of the pointer readings of the physicist and engineer. They were beginning to think about laws of chance.

During this same period there was another set of events which was of considerable importance as indicating the birth of movements which were later to bear fruit in the application of laws of chance to the problems of mass production. The Royal Statistical Society was founded in 1834; the American Statistical Association in 1839; just about a century ago. These institutions were to bring together social scientists and others interested in discovering order or law in the fluctuations of social phenomena.

One cannot, however, understand the significance of the problems of mass production until he gets a picture of the inter-relatedness of industrial groups within a nation and even of industrial groups throughout the world. Every company or corporate group must depend in some way upon materials or fabricated parts supplied by others. The problem of securing interchangeability is therefore not simply one that can be solved within a corporate body but it is one that must be attacked with cooperation upon a national and even an international scale. One of the first steps in this direction was the establishment of the first international standardizing agency for weights and measures in 1875 and the first national standardizing body in 1887, but the Great War brought to a point the need for industrial standardizing bodies as is evidenced in Fig. 9.

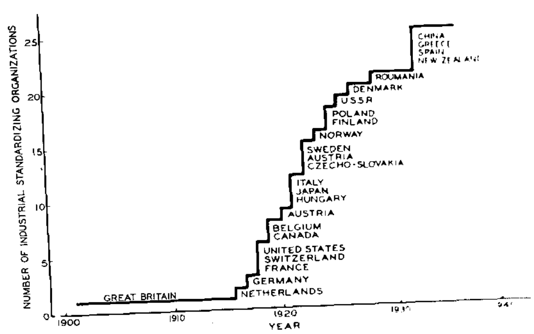

Fig. 9 - Growth of Standardizing Organizations

Thus as the twentieth century opens we find the stage set for the developments which we are to describe.

Scientists were beginning to think in terms of laws of chance and industry had developed to a stage where the need for controlling chance variations was beginning to be appreciated on a national and international scale. It is but natural therefore that since 1900 we should find attempts at applying statistical and probability theory to engineering problems springing up on a very broad front and in many countries. Out of such studies developed the theory of economic control of quality of manufactured product centered about the Quality Control Chart and its use. This chart was first introduced in 1924. Whereas in 1870 or thereabouts, engineers introduced the go no-go tolerances or limits on the quality of pieces of product, the control chart in its simplest form introduces an aimed-at value somewhere between these tolerances and two so-called control limits spaced one on either side of the aimed-at value but (usually at least) within the tolerance ranges as schematically shown in Fig. 10. The function of the control limits is to mark the range of variation in quality which

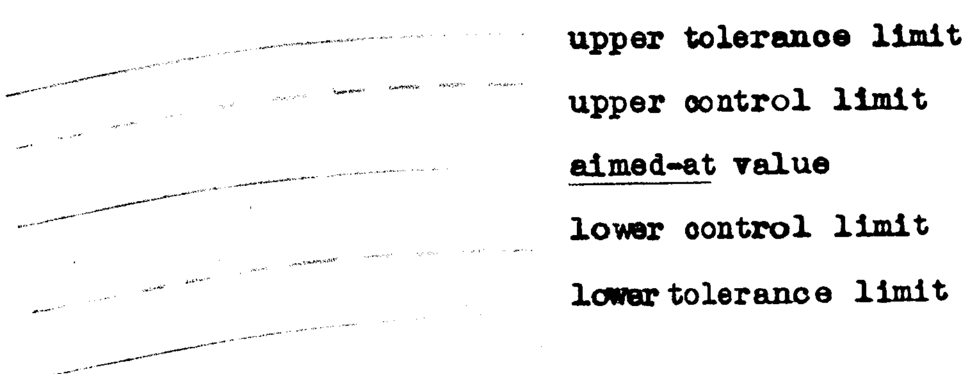

Fig. 10

economically should be left to chance. If variations fall outside these limits, action is to be taken to discover and remove, if possible, the causes of such variation even though the observed qualities lie within the tolerance limits. With the introduction of the quality control chart, engineers began a concerted study of the chance fluctuations of quality with a view to making use of laws of chance to reduce such variability to an economic minimum. This is in contrast with the objective of the introduction of the go and of the go no-go gauges which was simply to get around the difficulties caused by chance or unknown causes. Tuesday 18.04.66

Before passing on to a consideration of the economic need of making use of laws of chance, let us locate some of the important changes in the progress of ideas on the time scale of Fig. 7. First the beginning of the reign of exact science starting let us say with the Galileo 1564-1642 and running up to perhaps 1857 by which time the atomic theory of matter had been put on a scientific basis. From 1857[1]

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1) 400BC "Greek philosopher pictured a world made out of minute particles or atoms which were in constant motion".

95BC Lucretius pictured gases somewhat as they are pictured to-day.

These speculations were then no more than guesses.

1857AD Kronig and Clausius proposed a Kinetic Theory of gases based on an experimental footing.

Three stages in development may be distinguished.

1. Speculative opinion
2. Scientific basis for differentiating between opinion.
3. General agreement as to main foundation.

For Kinetic Theory of Gases by Leonard B. Loeb, McGraw-Hill Book Co. N.Y. - 1927

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to 1900 we witness the extension of the statistical theory of microscopic properties which has continued to develop with leaps and bounds since 1900. Then on the probability side we start with Pascal's work of 1654, and find that around the turn of the century there is already a very active development taking place in mathematical statistics. Is it not rather striking how short a time lag there has been between the development of the theory of statistical laws of chance and their application in industry in the control of quality?

FIVE IMPORTANT REASONS FOR APPLYING LAWS OF CHANCE

From the time (1787) of the first attempt at mass production of interchangeable parts with comparatively close requirements on the degree of fit, the effect of unknown or chance causes of variation introduced difficulties. As already noted, such causes made it impossible for engineers to attain the goal of exactness fixed, implicitly at least, in this 1787 attempt. Likewise, chance or unknown causes of variation made it necessary to proceed from the "go" step of 1840 to the "go no-go" step of 1870. But there were several reasons for trying to go beyond the "go no-go" tolerance. We shall consider five such reasons, each of which constitutes a reason for using laws of chance.

1. To minimize the cost or rejection - In practice, the "go no-go" tolerance limits are generally set so close together that parts are sometimes produced that must be rejected as not having qualities falling within the tolerance limits. Rejections are costly. Hence they should be minimized. It may be of interest to consider the significance of this type of problem in a large industry such as the telephone industry in this country. As already noted, to connect your telephone with another may involve the use of something like 110,000 different kinds of piece-parts. Each of these kinds, under conditions of normal production, are manufactured in lots, some of which run into the thousands and even millions per month. Imagine if you will, raw materials gathered literally from the four corners of the earth and poured into the funnel, Fig. 11, representing schematically the production process. In general there are several intermediary steps at which inspection is carried on to weed out the trouble which would otherwise be found

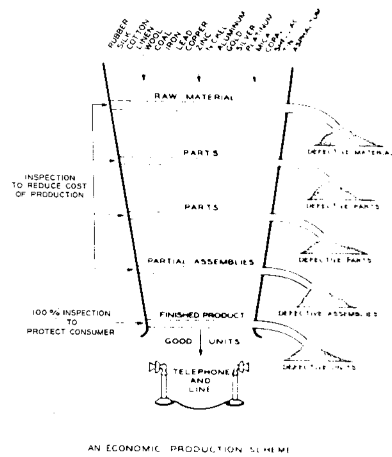

Fig. 11 - An Economic Production Scheme

in the final inspection of the finished product. The inspection at these various stages results in throwing out numbers of defectives represented schematically by the piles shown to the right of the figure. Hence one of the important problems is that of trying to find the economic minimum for the sizes of these piles. This problem exists even if one makes 100% inspection at each of the steps.

Of course, if one could discover and eliminate all chance causes of variability, it would be possible to reduce the percentage rejection in each and every case to zero. However, to detect the presence of findable causes of variability and to remove these adds to the cost of production and always tends to counterbalance savings resulting from the reduction in the numbers of rejections. Hence in trying to minimize the numbers of rejections there arises the problem of determining when variations in the quality of the piece parts should be left to chance.

2. <u>To minimize cost of inspection</u> - At intermediary steps in production, Fig. 11, it would usually be prohibitive from the viewpoint of cost to employ 100% inspection. If, however, sampling is to be resorted to, one obviously must decide how to sample and how large a sample to take.

It is obvious, however, that the answers to these questions depend upon the nature of the chance variations in the quality of product. In fact, it seems quite reasonable even without very much study to expect that the amount of sampling to be done to attain a certain end will be a minimum under those conditions where the fluctuations follow a law of chance. Now this more or less natural hunch is justified on the basis of both theory and practice and hence we come to see how the problem of minimizing the cost of inspection requires that an attempt be made to reduce the unknown or chance variations to those which follow a discoverable law.

3. <u>To minimize tolerances</u> - Imagine that you were designing something in which you were using malleable iron

castings and that one of the most important quality characteristics of such castings from the viewpoint of your job was tensile strength. If you were to look in the Proceedings of the American Society for Testing Materials for 1931 you would find there the results of some 20,000 tests on materials supplied by some seventeen different investigators as to the tensile strength of castings of this character. The average of these 20,000 tests is 54,040 pounds per square inch. You would also see, however, that tests vary all the way from 45,000 to about 63,000 pounds per square inch. Hence if you were interested in designing so as to escape failure, you would apparently have to take a tensile strength figure of something less than 45,000 pounds per square inch.

A study of the data, however from the seventeen sources gives the results indicated in the upper part of Fig. 12

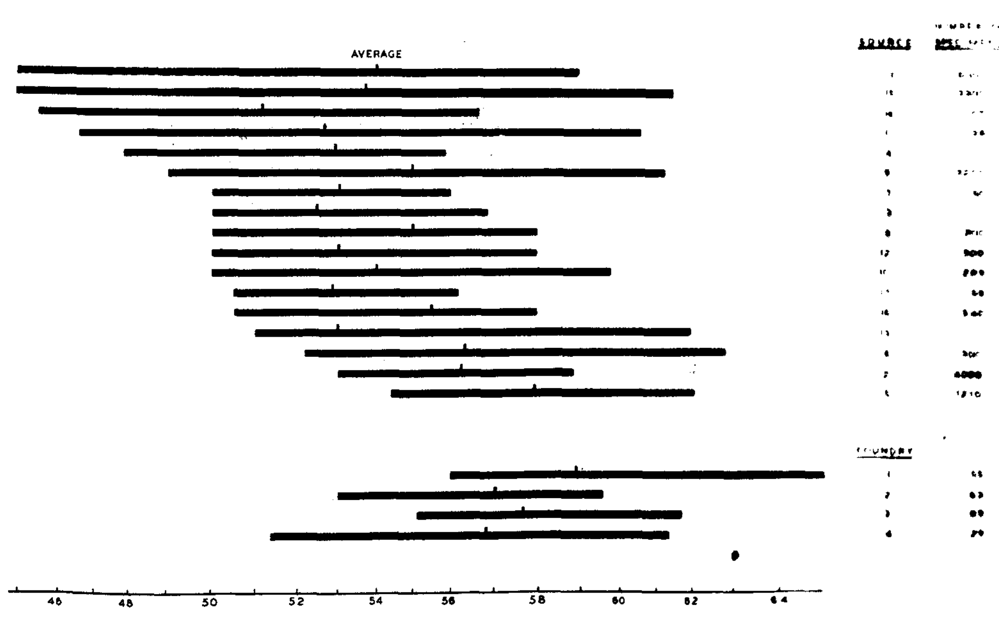

Fig. 12

Note that they differ quite a little in average and also in range of variation. Obviously if one could use the material from Source 5 with the assurance that it would not some time change over or become similar to any one of the other sources, he could make an appreciable saving through the reduction in the amount of material required in order to meet a given safety factor.

This typifies the economic desirability in many instances of reducing to an economic minimum the variability in the quality of a material or piece-part. The importance of such effort is indicated by a recent statement[1] of the Director of Research of the United States Steel Corporation, Dr. John Johnston, to the effect that it was his belief that one of the greatest sources of saving to come out of research in the steel industry in the near future would be that of increasing the homogeneity of the qualities of the material turned out by any given process.

Here we meet the problem of reducing to an economic minimum the variability that must be left to chance that we first met in trying to reduce the number of rejections to a minimum.

4. <u>To make prediction possible</u> - It is pretty obvious that if one is to make use of materials, he must be able to predict at least within limits the qualities of such materials.

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1. Mechanical Engineering, February, 1935

This is basic in all design. Likewise if one is to figure out, let us say, the cost of producing a certain kind of piece-part with tolerances on the quality characteristics, it is necessary for him to be able to predict the number that are likely to be defective and the amount of inspection that will be required, as these are important factors in the cost. These are but a few of the countless number of instances where it is necessary to be able to predict, at least within limits - in fact, <u>the whole concept of control rests upon the possibility of successful prediction within limits</u>. If engineers could not predict the quality characteristics of materials and piece-parts better than economists can predict the trend of stock market prices, you can well imagine the difficulties that would arise!

Hence the problem that confronts the engineer is to know his chance cause system sufficiently well that he can predict. This means that he must be able to differentiate between the kind of chance cause system ~~that apparently holds, for example, in the case of some economic series~~, and those ~~which hold under~~ the conditions ~~where the greatest assurance of predictability can be attained.~~

5. <u>To give maximum quality assurance at given cost</u> - All of us have at some time or other been forewarned that we cannot eat our cake and have it too. Much the same situation arises when we try to test certain qualities of many of the

most important things produced by industry. The test for quality is often destructive and must therefore be applied on a sampling basis. This means that we must infer the quality of the whole from tests of the quality on a sample. For a fixed size, it is therefore important to get the greatest possible quality assurance. Examples with which each and every one of you are familiar are the chemical and other kinds of tests on food supplies, oils and gasoline; the test for bacterial count in the water of the lake where you spend your vacation, or of the water supply which you use for domestic purposes; the tests of the clothes you wear, and so on indefinitely.

Now, this problem of giving assurance at minimum sampling cost reduces pretty much to the same kind of fundamental problem as did that of determining the best way of sampling to find and remove causes of variability in order to minimize the cost of rejections. In passing, it is of interest to note that in reducing rejections to a minimum we are really trying to eliminate findable causes of variability from the chance system controlling the quality of product not yet made whereas in giving quality assurance we are interested in inferring with a fixed degree of assurance the quality of a lot of products already made from the quality of a sample drawn from this lot. In both cases, however, it is desirable to approach as closely as possible to a condition where the variability in quality is one following a law of chance.

Thus we have made a rapid survey of at least five needs, 1) Minimize cost of rejection, 2) Minimize cost of inspection, 3) Minimize tolerances, 4) Make, prediction possible, 5) Maximum assurance, that are important not only from the viewpoint of economics but also from the viewpoint of giving quality assurance; needs that touch every one of us either directly or indirectly and needs which cannot be satisfied understanding the nature and usefulness of the laws of chance.

WHAT ARE LAWS OF CHANCE?

Before trying to go any further in our discussion, we had better try to find out what is the nature of the difference between the most respectable kind of chance cause system that we can ever hope to attain under practical conditions and the more common kind of chance cause system wherein it is often practically impossible to justify any attempt at prediction even in the probability sense.

Law-Abiding or Constant Systems of Chance Causes - We usually assume that one of the characteristic differences between an exact law and a law of chance is that one makes it possible to predict what may always be expected to happen while the other makes it possible to predict what may be expected to happen in the long run. For example, if I hold this penny above the table and let it go, it will always fall to the table. If, however, I flip the penny in the customary way, I am not justified in saying that a head will always turn up. I can only say that <u>in the long</u> run a head will turn up about 50% of the number of times that the penny is thrown. In the first case the penny, as we say, always falls in accord with the law of gravity. In the second place, the frequency of occurrence of heads is controlled by a so-called law of chance.

The way in which the frequency p of occurrence of the throw of a head behaves in the long run or, as we say, approaches as a statistical limit some value approximately equal to 1/2, is a very important concept in practical applications. Fig. 13 shows experimental results for 1000 such throws. In this case the observed fluctua~~tion~~ to begin with started at unity and after some oscillations crossed the .5 line at the tenth throw and thereafter oscillated back and forth across the .5 line several times. We can be pretty sure that any penny that in physical characteristics appears to be pretty much like the one used in this experiment will behave in very much this same way. The fluctuation in a given case may, of course, start from zero instead of unity but we are justified in believing that it will approach some value p' approximately equal to .5 as it did in the experiment shown in Fig. 13.

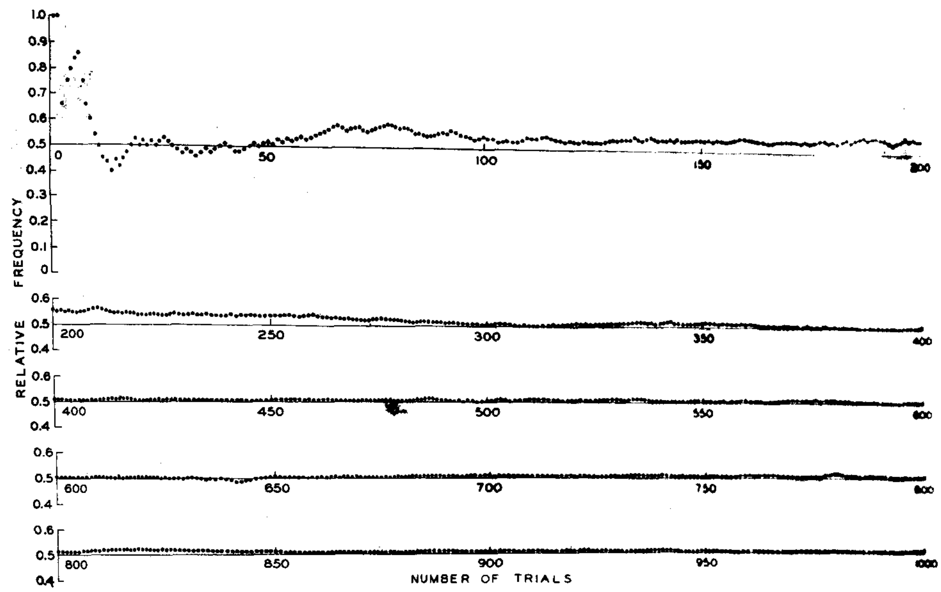


Fig. 13 - Statistical Approach to a Limit

As noted, we say, in such cases that the observed fraction p approaches as a statistical limit L_s some value p' as n is increased indefinitely and we symbolize this by

$$\underset{n \to \infty}{L_s} \qquad p = p' \qquad (1)$$

Since this concept of statistical limit is very important, let us consider another example. I have here a bowl of chips. If you were to examine them carefully, I think you would agree that they are very much alike physically. Suppose now that I were to write a finite number on each chip (not all numbers being the same) and then were to put the chips back into the bowl. Suppose I draw one with my eyes shut, record the number, replace the chip, stir the contents of the bowl, draw another chip with my eyes shut, record the number, and so on until I have drawn 26 such numbers. If now I take the averages of the first one, the first 2, the first 3, and so on, and plot the n averages thus obtained, I shall find them approaching as a statistical limit some number $\overline{X}'$ probably very nearly equal to the average of the numbers in the bowl. Fig. 14 shows such an approach

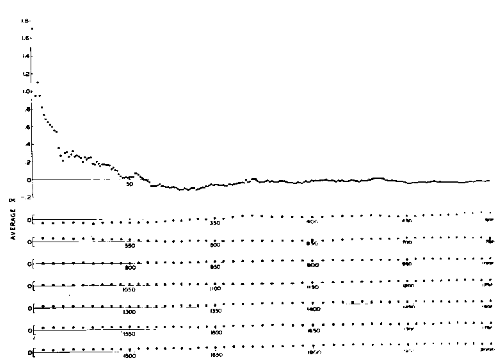


Fig. 14 - Statistical Approach of an Observed Average to a Limit

for the case where there were 998 chips with a normal distribution having an average equal to zero and a standard deviation equal to unity; that is, there is one chip marked -3.0, 1 marked -2.9, ... 40 marked 0, and so on. The arithmetic mean of the numbers on the chips is zero. The root mean square deviation of the numbers is 1.00. We can be reasonably sure that every time we repeat this experiment we will get much the same result. Of course, the start of the approach would sometimes be from above the limiting value as in the example and sometimes from below. We symbolize the statistical approach of the observed average $\bar{X}$ to some value $\bar{X}'$ as n is increased indefinitely by

$$\underset{n \to \infty}{L_s} \quad \bar{X} = \bar{X}' \qquad (2)$$

Now, let us assume that instead of recording the reading each time we simply note whether or not it lies within some preassigned limits X_1 to X_2. In n trials let n_1 be the number of times the observed number falls within these limits and let $p = \frac{n_1}{n}$. Then in such a case p will approach a limit as symbolized by (1).

Concept of Chance Cause

But why does a penny turn up head in one instance and tail in another? Why does a value of X fall within the range X_1 to X_2 in one instance and not within this range in another? These questions are usually answered by saying that there are likely many unknown causes of such phenomena. They constitute, however, a well behaved system in the sense that they provide a kind of regular behavior in the long run. But all chance cause systems are not so well behaved. Some as it were have little demons present which destroy all apparent regularity even in the long run.

Little Demons of Chance - Assignable Causes - To illustrate, suppose you were the one making drawings with replacement from this bowl after the manner already described except that after each drawing and while your eyes were shut I substituted another set of similar chips with any set of (finite) numbers written thereon that I might choose. The kind of statistical regularity previously noted would (very likely) be destroyed. Under the circumstances, you do not know that I am changing the chips on you except as you may detect this from the results you observe. I would in effect be acting as an unknown or chance cause-as a kind of demon to disturb the previously observed order. Until you found me out as it were and caused me to remove my influence you would be pretty much out of luck trying to predict with much assurance from an observation of past results what you would expect to get in the future.

Most chance cause systems met with in one's every day life are full of these little demons or assignable causes which must be found and removed (or understood) before one can predict with much success. Witness as familiar examples the chance fluctuations in the stock market and the weather.

Practical Significance - Now, what bearing has all of this on the problems of mass production of piece-parts? The answer is very simple indeed.

If the variability in product is one that exhibits such regularity, we may think of it as being controlled by what are technically known as a constant system of chance causes, or by what we might term a law-abiding set of chance causes. If, however, certain chance causes enter in the production process, such as a more or less radical change in the nature of raw material, a change in process of machining, a change in personnel, or the like, they would tend to destroy any kind of observable regularity. Such cases tend to produce effects in many ways similar to those which I might produce by changing at will the chips in the bowl while you are sampling. These causes become, as it were, the little demons, the presence of which can be detected in the majority of cases by a study of the observed data, on the one hand, and a study of the manufacturing conditions on the other. So long as there exist unknown or chance causes of this character within the system of chance causes affecting a given result, such a result cannot be said to be controlled by a law of chance. Our practical problem therefore in using laws of chance in industry is pretty clearly cut out for us - the first thing we have to do is to get rid of the little demons, or assignable causes as we call them. Monday 08.05.06

HOW DETECT THE PRESENCE OF THE DEMONS OF CHANCE

One method of detecting the presence of the demons or assignable causes is through the analysis of the observed data. For example, it was early suggested that constant systems of chance causes - the law abiding kind - gave particular kinds of frequency distributions. If this were true and one knew the functional forms thus favored by such a chance system, then one could compare an observed distribution with these standard curves, and, "if the fit were good", might conclude that the cause system behind the observed results was operating in accord with a law of chance. To illustrate, let us consider the 204 measurements of resistance on as many pieces of a new kind of insulation given in Table 1.

5045	4635	4700	4650	[illegible]	3940	4570	4560	4450	4500	5075	4500
4350	5100	4600	4170	4335	3700	4570	3075	4450	4770	4925	4850
4350	5450	4110	4255	5000	3650	4855	2965	4850	5150	5075	4930
3975	4635	4410	4170	4615	4445	4160	4080	4450	4850	4925	4700
4290	4720	4180	4375	4215	[illegible]	4325	4080	3635	4700	5250	4890
4430	4810	4790	4175	4275	4845	4125	4425	3635	5000	4915	4625
4485	4565	4790	[illegible]	4275	[illegible]	4100	4300	3635	5000	5600	4425
4285	4410	4340	[illegible]	5000	4660	4340	4430	3900	5000	5075	4135
3980	4065	4895	2855	4615	4700	4575	4840	4340	4700	4450	4190
3925	4565	[illegible]	2920	4735	4310	3875	4840	4340	4500	4215	4080
3645	~~4190~~	4740	4375	4215	4210	4050	4310	3665	4840	4325	3690
3760	4725	5000	4375	4700	5000	4050	4185	3775	5075	4665	5050
3300	4640	4895	4355	4700	4575	4685	4570	5000	5000	4615	4625
3685	4640	4255	4090	4700	4700	4685	4700	4850	4770	4615	5150
3463	4895	4170	5000	4700	4430	4430	4440	4775	4570	4500	5250
5200	4790	3850	4335	4095	4850	4300	4850	4500	4925	4765	5000
5100	4845	4445	5000	4095	4850	4690	4125	4770	4775	4500	5000

Table 1

204 Observed Values of Insulation Resistances

Fig. 15 shows the result of fitting one of the curves often used in the past in the above sense to the observed data. The fit of the observed points looks quite

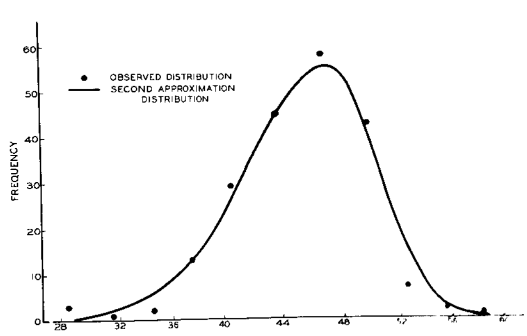


Fig. 15 - Observed Distribution Fitted to an Assumed Law of Distribution

good. There is, of course, a technical way of measuring the goodness of fit in such cases and this too indicates a pretty good fit.

For our present purpose three remarks about this method are sufficient.

1. It was a method in vogue around the beginning of the 20th century. It was for example, the method first called to my attention in engineering some 15 years ago and is one now often applied at least in some quarters, particularly in Germany.

2. It is a method which in application requires that we take large numbers of data and group them together into a frequency distribution without regard to any other available information about the separate observations such, for example, as the order in which they were taken.

3. It is a method that at least in my own field seldom works. In fact it is very easy to produce any given functional form of distribution by compounding those coming from different cause systems and hence a positive result cannot be very significant in itself.

In other words, this method puts all faith as it were in quantity of data ignoring the mind of the observer and its ability to give pertinent information about the data not contained in the numbers themselves.

There are some serious criticims of this method even though it did detect the presence of assignable causes. For example, it does not provide any method for detecting just when an assignable cause enters the chance system and so does not make it easy to discover and remove such causes even when present. Furthermore it does not provide a method of approaching the goal of a ~~lawful~~ law abiding system of causes in an orderly manner. Instead you take a lot of data, group them all together, and if in the end the test indicates the presence of assignable causes you have to do what you can to find and remove them with scarcely any help from the analysis of data and then start all over again and see if you were successful.

Now this is particularly serious when, as in our work, one almost invariably finds many assignable causes of variability in the initial stages of production. Furthermore, one must have a technique which is usable in the shop and in the purchase of materials for it is here that many of the most bothersome causes of variability enter.

Example - The answer to this situation is the quality control chart technique of attaining economic control introduced in 1924. A look at one ~~from~~ form of this kind of chart in action is now in order. In Table 1, the data were taken in the order beginning at the top of the first column, going down this one and then beginning at the top of the second column and so on. Grouping these into samples of 4 in the order in which the data were taken we get 51 sets of 4. Now we may calculate from the averages and standard deviations of these 51 samples of 4 certain limits within which the 51 averages may be expected to fall if there be no assignable causes present. In practice we look for an assignable cause when a point goes outside the limits. Fig. 16 shows eight averages outside the limits - eight suggestions of the presence of assignable causes. Certain assignable causes were later found and removed. It worked, whereas grouping all the data together (Fig. 15) gave no indication.

In contrast with the statements about the older method the following remarks are pertinent:

1. It is a method which makes use of many recent developments in mathematical statistics (distribution theory in particular).

2. It is a method which does not treat the observations simply as numbers irrespective of the pertinent information the observers may give about these numbers. In fact, no matter how many observations are available it asks the observer to break these up into rational subgroups which may be as small as four observations each.

3. An indication of the presence of an assignable cause is given by one or more such subgroups and hence the experimentalist is aided in his search for trouble because he knows about where it is quite likely to have entered.

4. It is a method which is self-corrective as causes are found and removed so that it allows one to approach in a regular way the goal of a lawful system of chance causes.

5. It is a method which involves the use of engineering judgment or the mind of the observer at every step. For example, the choice of form of control chart rests largely upon certain judgments of the observer. Thus he must balance the cost of looking for trouble when not present with the savings to be obtained by weeding out the trouble. He must also use his

judgment as to the kind of cause that is present as the nature of the chart depends upon the kind of cause.

6. It is a method which experience has shown to work with much smaller numbers of observations than the earlier method although its use does not involve a commitment to any total sample size.

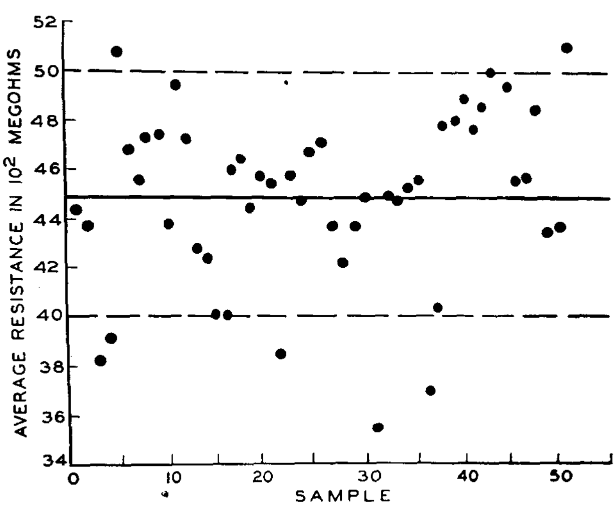


Fig. 16 - Control Chart for 204 Observations in Table 4 Grouped into Samples of 4 in the Order the Data were Taken

<u>Emphasis on Pertinent Information</u> - I have stressed the importance of relying not on quantity of data alone but also upon the pertinent information given by the experimentalist. Let us illustrate this point with the 204 observations.

Suppose we put the 204 numbers on as many chips and put them in a bowl and mix them thoroughly. This would effectively destroy what the experimentalist was able to tell us about the order in which they were taken. Now let us draw one chip at a time without replacement and then apply the same control chart criterion to this new set of 51 averages. The results are shown in Fig. 17.

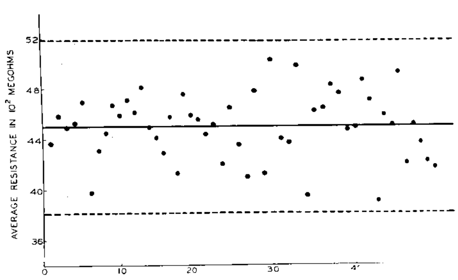


Fig. 17 - Control Chart for 204 Observations of Table 1 Grouped at Random into 51 Samples of Four

All indication of lack of control is lost!

In general, the experimentalist is able to suggest a better grouping than that merely following the order in which the data were taken. This is all to the good and simply indicates more than ever the significance of making use of the observer's judgment. In fact at the beginning of this section, we spoke of two methods of detecting assignable causes, first, that depending largely upon the observer's insight and second, that depending for the most part upon the analysis of data. The control chart method effectively combines the two and in this union there is strength - the strength required to detect in an economic way the presence of the little demons (assignable causes) that must be removed before we can attain a state of control based upon laws of chance.

Chart Must Not Give False Indications Too Often - Now, there is another angle from which we must consider the use of this chart. It should not indicate the presence of assignable causes more than an economic percentage of times when such causes are not present. For example, in our work this percentage is approximately .3%. Let us therefore apply the control chart to 400 observations drawn from a bowl where we know a law of chance does exist. Fig. 18 shows 100 averages of four for samples drawn from a bowl. The test behaves as expected.

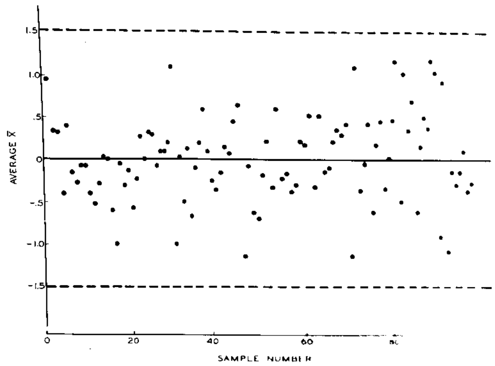


Fig. 18 - 100 Averages of Four for Samples Drawn From a Bowl

<u>How Detect the Demons of Chance</u> - In Conclusion years of experience have now given pretty definite evidence that it is possible to devise a technique for dividing data into rational sub-groups and for setting up certain aimed-at values and certain limits about these aimed-at values for any chosen statistic such that: a) whenever a point falls outside such limits it may be taken as an indication of need for action, and b) so long as points fall within such limits this fact may be taken as indicating that <u>All's Well</u>.

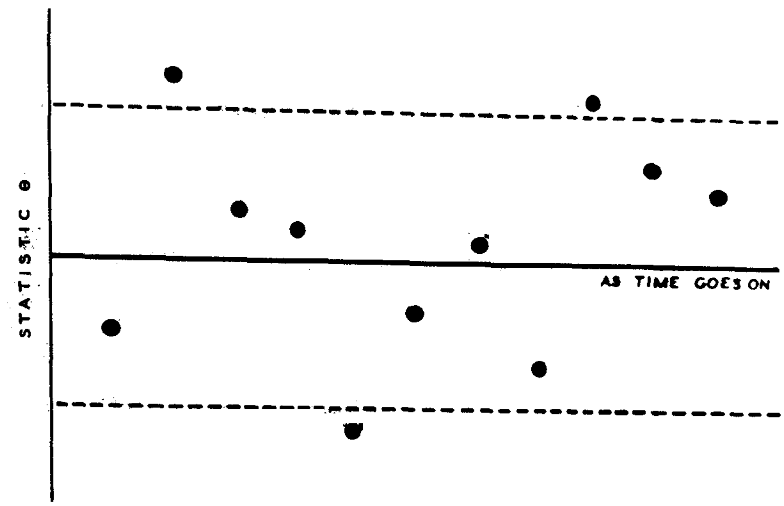
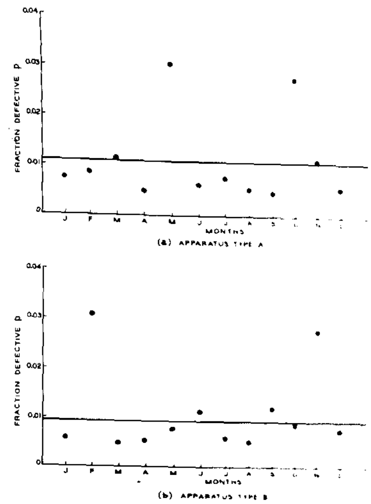


Fig. 19

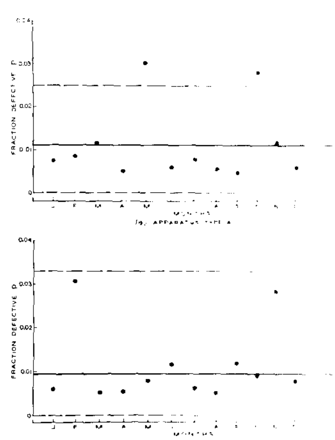
MAKING USE OF LAWS OF CHANCE

The proof of the pudding is in the eating so let us see how the quality control chart technique for approaching a condition of control works toward solving the five problems previously considered.

To Reduce Rejections to a Minimum - Tolerances are usually set after a study has been made of the variability observed in tool-made samples and are so placed that they just about include the observed spread. Now, under conditions of shop production there is usually a certain fraction p in every lot falling outside these tolerance limits. In general, it may be expected that the presence of assignable causes of variability will increase



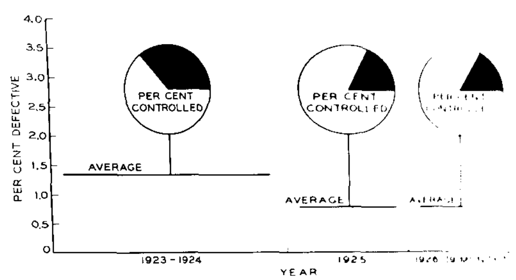
SHOULD THESE VARIATIONS BE [illegible]



SHOULD THESE VARIATIONS BE [illegible]
a – NO [illegible] YES

the range of variability and hence raise the fraction defective above what it would be if these causes were removed.

Fig. 20 shows the results of the first wholesale check on the theory. About thirty typical items used in the



EVIDENCE OF IMPROVEMENT IN QUALITY WITH

ig. 20 - Evidence of Reduction in Rejections as State of Control is Approached

telephone plant and produced in lots running into the millions per year were made the basis of this study. As shown in this figure, during 1923-24 these items showed 68 per cent control about a relatively low average of 1.4 per cent defective. However, as the assignable causes, indicated by deviations in the observed monthly fraction defective falling outside of control limits, were found and eliminated, the quality of product approached the state of control indicated by an increase of from 68 per cent to 84 per cent control by the latter part of 1926. At the same time the quality improved: in 1923-24 the average per cent defective was 1.4 per cent, whereas by 1926 this had been reduced to 0.8 percent.

This illustrates how assignable causes can be detected and removed in practice, how this results in a lowering of the number of rejections and how the technique set up is one that approaches a limiting economic percentage rejection for a given kind of process corresponding to the state where a law of chance controls the variability.

Can We Get Rid of the Demons for Good - It is one thing to detect the presence of assignable causes and to remove them, but won't others take their place? Experience shows that in the majority of cases when one has once succeeded in getting rid of assignable causes in a given process others do not often take their place. Witness, for example, the story told in Fig. 21.

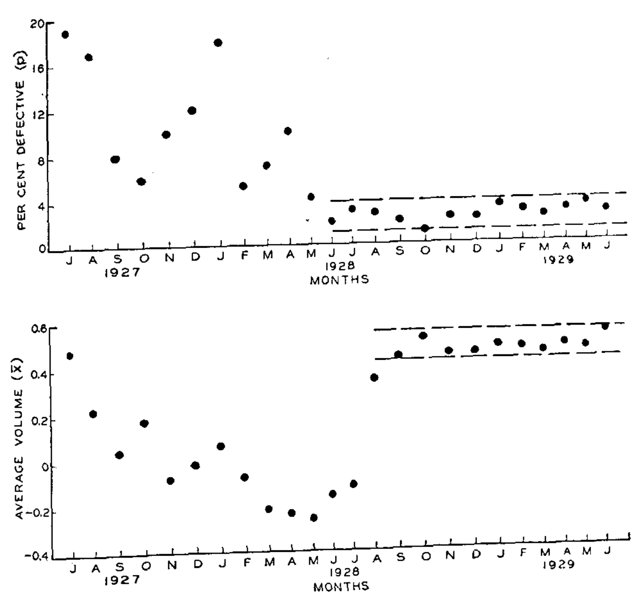


Fig. 21

Now, of course, it is desirable to detect the presence of assignable causes of variability as quickly as possible after they enter the process and before they have had a chance to affect much of the output and thus increase the number of rejections. As previously stated, the control chart technique is especially designed to do this. Let us, therefore, see how it works to this end. Fig. 22 shows the observed fraction defective in a certain kind of apparatus

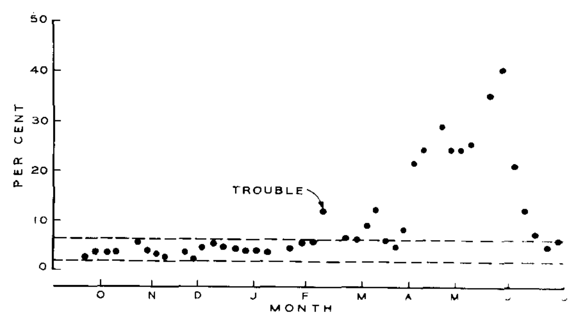


Fig. 22 - When Did Trouble Enter?

over a period of ten months. Beginning about April, the rejection became excessive as indicated. It is of interest, therefore, to see how this trouble could have been detected through the use of a control chart for fraction defective. Such a chart is shown in Fig. 23 and gives indication of the

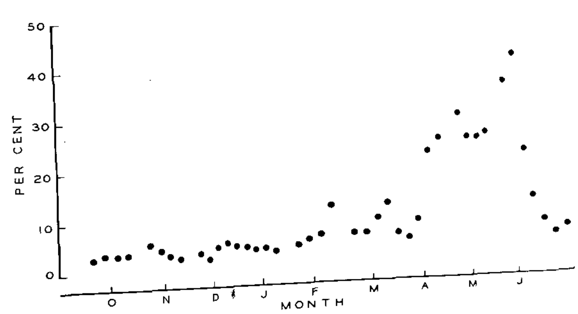


Fig. 23 - Even an Inefficient Control Chart Detected Trouble Eight Weeks in Advance

presence of an assignable cause eight weeks in advance. Investigation revealed that it was very likely that the assignable cause at this time was the same as that found to have caused the trouble beginning about the second week in April.

Now, it may be shown that the average is a more sensitive detector of assignable causes than is the fraction defective. It so happened that the quality of a few instruments of this same kind had been measured as variables over this same period. Applying the control chart technique to these averages we get an indication of trouble sixteen weeks in advance, Fig. 24.

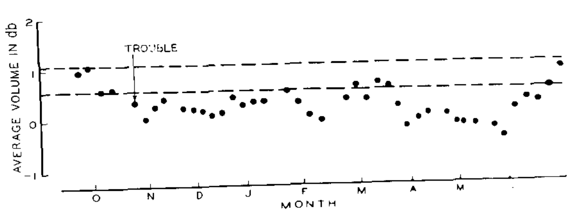
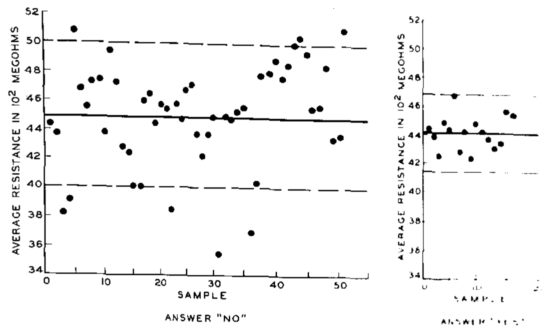


Fig. 24 - Efficient Control Chart Caught Trouble Sixteen Weeks in Advance

These results are but typical of those experienced right along in the application of the control chart method.

<u>To Reduce Amount of Inspection to a Minimum</u> - Inspection may be as we have just seen to catch trouble in the form of assignable causes and it may be for the purpose of giving quality assurance. Here we shall consider only the first of these two. How does the application of the control chart technique tend to reduce the amount of inspection needed? The answer is pretty simple. In general, there is only a comparatively small number of assignable causes that enter any production process. We have just seen how these can be detected and eliminated. Pretty soon we get to know these causes and how to prevent them from entering the process. When there is good evidence that this stage is reached, the required amount of inspection becomes a <u>minimum</u>.

<u>To Minimize the Tolerances</u> - As already noted, the effect of assignable causes is in general to introduce variability which can in most cases be effectively removed. Thus in the case of the 204 measurements of resistance, the assignable causes of variability were discovered with the result indicated in the right-hand side of Fig. 25. Here we see sixteen averages of four observations each all of which lie within the control limits, although these limits are much closer together than in the case of the 204 observations.



SHOULD THESE VARIATIONS BE LEFT TO CHANCE?

Fig. 25 - How the Removal of Assignable Causes Enables Us to Close up the Tolerances

It is of particular interest, of course, to note that this control chart method to be most effective should be used at the various stages of fabrication beginning with the production of raw materials. In this way it enables engineers to weed out troubles wherever they occur in a production process and in this way gradually to obtain in an economic manner a state of control where the fluctuations in quality follow a definite law of chance with minimum dispersion.

To Make Prediction Possible - A few minutes ago we were talking about the case where you were drawing with replacement a sample from a bowl of chips with the idea of trying to find out what was in the bowl under a condition where I was allowed to change the chips in the bowl without your knowledge. Presumably we are agreed that so long as I was playing the part of a little demon in this way, it would be pretty difficult for you to predict with much certainty the contents of the bowl after a given drawing from an observation of what had been drawn. In much the same way the little demons or assignable causes in a production process must be found and removed before we attain the state where we can predict with much assurance in the probability sense, ~~what may be expected~~. Now we have been considering the control chart technique which has been developed to detect the presence of such causes of variability so as to make possible their removal. Through this process it is feasible to attain as a limit a condition where the chance cause system behaves with certain regularity which we term a law of chance.

This situation is represented very nicely by the case of throwing a penny or drawing a sample from the bowl of chips, both of which we have already considered. We have seen, for example, that in each case the succession of throws presents a certain kind of statistical regularity. For example, in the case of a sample drawn from the bowl, the average approached as a statistical limit some value presumably pretty close to the average of the numbers on the chips.

Now, the next thing of interest to note is that the number and kinds of predictions that may be made increases as we know more and more about the distribution of the numbers in the bowl from which we are drawing. Likewise in the case of production where assignable causes have been eliminated we can make more and more kinds of prediction the more we know about the nature of the distribution function belonging to the chance cause system. The difficulty in this practical case is that we never know exactly what this distribution is and under the best of conditions we can only approach this knowledge in much the same manner as we approach a statistical limit.

Case 1 - Functional Form of Distribution Known - To illustrate what I have just said assume that the set of numbers in the bowl follows the well-known normal distribution although we do not know the values of either the average or the standard deviation belonging to this distribution. The work of "Student", a statistician in an industrial laboratory in Great Britain, in 1908 made it possible for us to make a certain kind of prediction under these conditions. For example, his work showed that if we draw a succession of samples from a normal universe and set up certain ranges in terms of the averages and standard deviations of these samples, we can predict how many of the ranges may be expected to include the true average of the distribution. Thus, if we take samples of four we can set up a range which will include this average in the long run 50% of the time. Fig. 26 shows one experiment in which one hundred samples of four were drawn from the normal universe. For each sample the average and standard deviation were calculated and ranges set up equal to the average plus and minus .44 times the standard deviation.

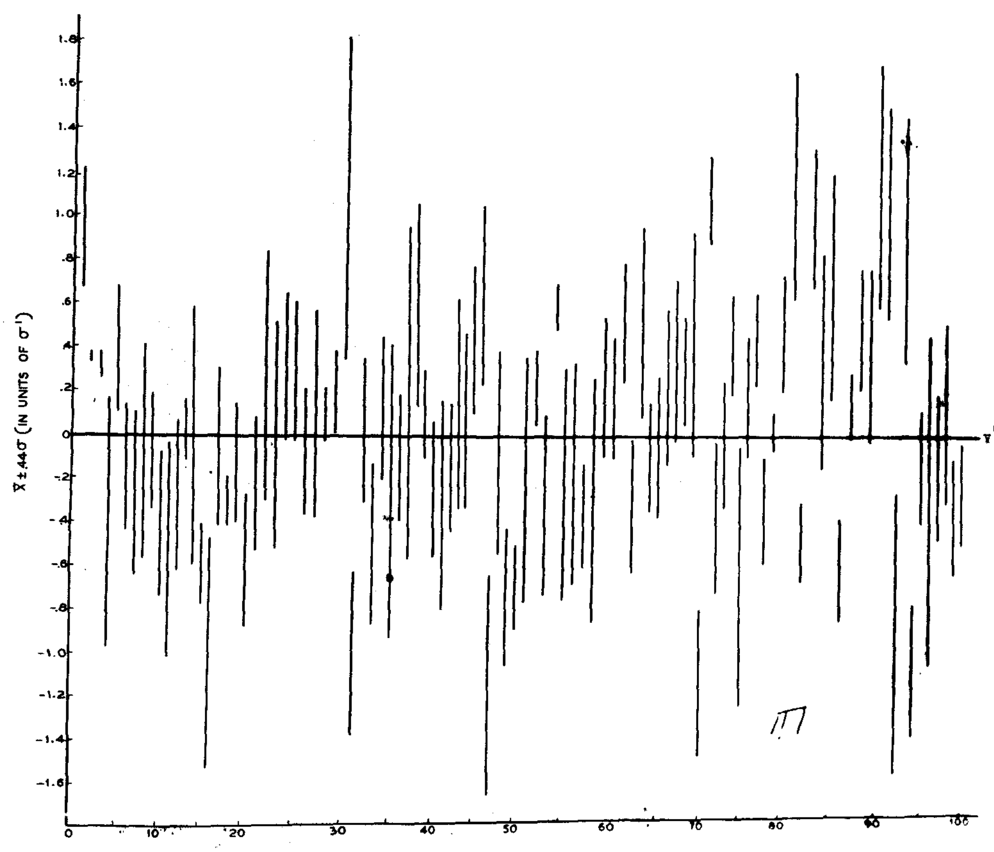


Fig. 26

In 51 out of 100 samples drawn, the range $\bar{X} \pm .44\ \sigma$ included the true average of the samples universe.

In this case the true average is zero and we see that this number is included within 51 of the ranges. This prediction, of course, is made for samples of four. It could have been made for any other sample size. Thus Fig. 27 shows the results of a similar experiment taking instead four samples of 1000 where the ranges are so constructed that we should expect them to cut the true value 50% of the time. Old Lady Luck smiled on us on this occasion and gave us ranges two of which cut the average. The significant thing to note

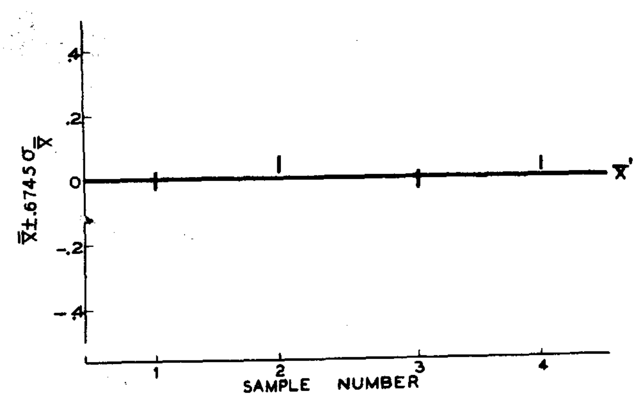


Fig. 27

In 2 out of 4 samples of 1000 the range $\bar{X} \pm .6745\,\sigma$ included the true average here is that the results shown in Fig. 19 could not have been predicted before the beginning of the twentieth century, whereas it so happens that the prediction for the larger size sample could have been made at least since the time of Gauss. (1777-1855).

Case 2 - Functional Form and Parameters Known - Now, if we know not only the functional form but also the parameters in this functional form, we can make still further predictions which will check just as well as those indicated.

What does this mean from the viewpoint of using laws of chance in industry? Well, in connection with the use of raw materials and piece-parts, it is desirable to be able to predict with as high a degree of assurance as is feasible the range of variation that is to be expected in the probability sense in the completed unit. This is necessary in order that we may make the most efficient use of the materials at our disposal. It is therefore desirable not only that we attain a condition where the chance variations in the qualities of piece-parts and raw materials behave in accord with a law of chance but also that we obtain as rapidly as possible information about the nature of the distribution of quality approached in any given case. Thursday 08.06.06. Friday 09.06.06

To institute a research that would discover before the start of production all of the assignable causes that might come in securing raw materials and in applying the shop processes would be prohibitive from an economic viewpoint. The beauty of the method briefly described here this evening is that it enables engineers to begin at the beginning and to proceed in a direct way to get more and more information during the process of production. It enables the engineer, as it were, to keep on the right track through the discovery and elimination of the little demons of trouble. As a result the assignable causes may be detected and eliminated early in the production stages and other data may be accumulated where desirable to indicate the nature of the distribution function associated with a given cause system. This makes possible a continuous approach toward the goal of the greatest efficiency in the design and use of engineering materials.

To Give Maximum Quality Assurance at Minimum Cost - What more would you want to know about the quality of a thing that you were going to use than that it was controlled in accord with the law of chance which gave you a certain distribution function? Now, everyone of us is interested in the quality of the things we buy, in our food and clothing, in the quality of our water supply and in the quality of all kinds of things that we use. In order to get a line on this quality we sometimes ask to see a sample. In fact, certain organizations are set up to do this kind of sampling.

I think perhaps enough has been said to indicate some of the reasons at least why we must be very careful in the interpretation of any such sample. We have just seen, for example, some of the ways in which the little demons or assignable causes may enter in such a way as to make prediction in a probability sense with any degree of assurance very uncertain. At the same time we have seen how it is feasible on the part of industry to find and remove causes of variability in the process and to build up information which will give perhaps the greatest possible assurance as to the quality of product to be made by this process. In other words, we may grasp here the significance of taking "a look at the record", as it were, of what has been done up to a given time in the process of attaining a state of control where the variability follows some law of chance. Just a few days before I left my office a beautiful example of one such a control record came to my desk. This was a case where early trouble had been experienced. Fig. 28 shows the recent consecutive quality record of approximately 250 samples of this product, both as to average and dispersion. Quite a record of control! Practically as good as drawing chips from a bowl where the chips are selected with care and are free from dirt. Possibly better than one could do with a deck of cards unless well glazed and free from dirt.

Such a record, in other words, provides the greatest possible assurance of quality of product and I think it is significant from this viewpoint to see that it is the kind of record that must be built up by industry itself in the process of production. Trying to interpret a single sample without this look at the record is somewhat like a fisherman trying to judge how many fish there are in a given lake after he has fished there only once. He could probably learn a lot more by comparing notes with some old fellow who had fished there all his life. It is significant that not only does this method provide the greatest possible quality assurance but at the same time provides a plan of production which gives the other four advantages which we have just considered.

NEED FOR COOPERATION OF INDUSTRIAL GROUPS

We have seen some of the advantages that a single industry can attain by reducing variability to a condition where it is controlled by a law of chance. As already noted, however, no industry can secure all of the possible advantages of such control unless it has the cooperation of other companies furnishing materials of one kind or another because the need for control in this sense extends throughout every stage of the production process from raw material to finished product. This need for cooperation is much the same as that which brought into existence national and international standardizing organizations. It is therefore more or less to be expected that such groups would be the first to attempt to secure on behalf of industry the advantages to be obtained through cooperation.

As early as 1929 the American Society for Testing Materials and The American Society of Mechanical Engineers called a conference to consider the problem of securing the advantages of the application of statistical theory and laws of chance in the problem of manufacturing. As a result of this early conference, a joint committee was established under the sponsorship of these two societies together with the American Mathematical Society and the American Statistical Association. In 1930 the American Society for Testing Materials organized a separate committee to treat of the specific problems confronting members of that group in the testing of raw materials and the writing of specifications for their quality. In 1932 a committee similar to the American joint committee was established under the auspices of the British Standards Institution of Great Britain and a short time after a committee was established in Germany under the Deutscher Normenausschuss. In 1933 the Royal Statistical Society organized its first section upon its one-hundredth anniversary- a section devoted to the special problems of the application of statistical theory to industry and agriculture, particularly that of quality control. Already much has been done through the cooperation of these groups on a national and international scale. As a result, applications are being made in many industries in several countries.

SUMMARY

Man started his sojourn on earth some 1,000,000 years ago. For something like 700,000 years he apparently did not even dream of control. It was nearly 990,000 years before he began to fit piece-parts together. By 1787 or only 149 years ago, he had felt the need for mass production

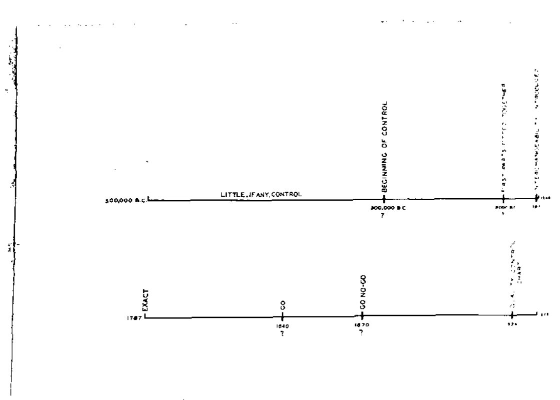


Fig. 29

of interchangeable parts and acquired the idea that he could make such parts "exactly" alike. He was acting under the influence of the then prevalent concept of exact science. He failed to attain his objective and introduced the concept of a "go" tolerance. Even yet he did not begin to think of chance cause systems as such - he simply introduced the "go" tolerance in an effort to get around the effects of chance causes. Again he met with difficulties which led him to adopt about 1870 the "go no-go" tolerance. Still he did not concern himself with the study of chance causes as such. By about this time or a little later, however, scientists were beginning to change their concepts. They were beginning to realize the necessity of thinking in terms of the effects of chance causes. But even up to the turn of the century there is scarcely any record of an attempt to discover and apply laws of chance in the field of engineering, at least to the control of quality. Soon thereafter, however, such studies began to spring up in various corners of the earth. In 1924 we have the introduction of the quality control chart technique based upon a study of chance cause systems. This, as we have seen, has gone a long way toward providing a technique for 1) minimizing the cost of rejections, 2) minimizing the cost of inspection, 3) minimizing the tolerances that must be allowed thus making more efficient use of materials with which an engineer must work, and 4) providing the maximum quality assurance at a given cost.

Chance or unknown causes of variability enter into everything we do. This fact is of particular importance in trying to effect economic control of the quality of manufactured product - control of the quality of the things each and every one of us use.

It is significant, therefore, ~~to each~~ that industry through the study of laws of chance ~~in cooperation with statisticians~~ is well on the road to the development of a technique of making full use of such laws ~~of chance~~; that the full advantage of such efforts can come only through cooperation of industrial groups and that such cooperation is being developed on a national and even an international scale through engineering and scientific organizations. Progress in this direction should make for higher standards of quality - a fact of interest to each and every one of us.