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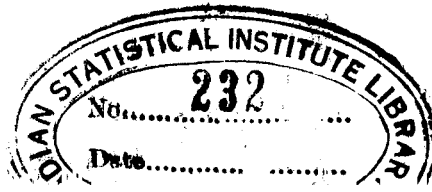
*Statistical Research Memoirs*, Volume II, edited by J. Neyman and E. S. Pearson. London: Department of Statistics, University College. December, 1938. 149 pp. 15s.

This is the second of a series of semi-periodical publications that are being issued by the Department of Statistics, University College, University of London. The first of this series, published in June 1936, was reviewed by S. S. Wilks in this journal, Vol. 31, No. 196, December 1936, pages 760-62. The present volume contains nine papers, dealing with problems of testing statistical hypotheses and statistical estimation along lines originally laid down by J. Neyman and E. S. Pearson in their paper published in *Biometrika*, *XXA*, 1928, pages 175-240, 263-94.

The first paper by P. L. Hsu contributes to the theory of "Student's" *t*-test as applied to the problem of two samples. It is pointed out that in deriving a statistical test to determine whether the two normal populations from which two samples have been drawn are alike or not, it is necessary to distinguish at least three different senses in which the problem may be interpreted. It is shown that the *t*-test gives a complete answer for only one of these. Hsu surveys the possibility of deriving certain tests for solving the problem in the other two senses. Although the results are of a somewhat negative character, they shed light on the nature of the problems involved and indicate clearly the significance of the alternative hypothesis in determining the choice of test.

The second paper, by Neyman and Pearson, contains an extension of their previous work on what they term unbiased critical regions of types *A* and *A*<sub>1</sub> and broadens the conception of the unbiased tests so as to cover the case where the hypothesis tested is simple but specifies the value of more than one parameter. Neyman, in the third paper, considers the necessary and sufficient condition for the probability law of a function of *n* random variables with an elementary probability law  $p(E/\theta)$  to be independent of  $\theta$ . The fourth paper by H. V. Allen presents a theorem concerning the necessary and sufficient conditions for linearity of regression under certain specified conditions.

By generalizing a theorem of Laplace, F. N. David in the fifth paper shows that the distribution of the means of samples drawn without replacement from a finite population tends to become normal as the sample size and the size of the population increases indefinitely. The author also discusses the limiting distributions of certain unbiased estimates arising through the use of the Friedman and Wilcoxon method of sampling human populations. The next paper is by P. L. Hsu and treats of the best unbiased quadratic estimate of the variance. The use of the general theory is illustrated by finding the best estimate of variance, subject to specified limitations, for a single sample, for *m* treatments in *n* blocks in a randomized block experiment, and for *k* samples.



In the seventh paper, F. N. David and J. Neyman discuss an extended form of the Markoff theorem on Least Squares. The justification of the use of the least squares method was originally based on the assumption that all variables are both independent and normally distributed. Markoff, W. F. Sheppard and, more recently (1935), A. C. Aitken have considered the problem of freeing the theory from these and other unnecessary assumptions. The present paper is contributed as a purely didactic one that offers a proof in terminology familiar to statisticians. In the next article W. Feller considers the problem of finding regions similar to sample space in the Neyman-Pearson sense and shows that in many instances such spaces either do not exist or if they do, they form a category to be called trivial. The volume closes with a discussion by P. C. Tang on the power function of the analysis of variance tests with tables and illustrations of their use in rational planning of experiments. This paper constitutes a further illustration of the importance of considering both kinds of error in testing a hypothesis.

To anyone interested in the modern development of theoretical statistics, this volume like its predecessor contains much material of fundamental importance. Likewise the practicing statistician will find this volume as well as all of the work of Neyman and Pearson on testing statistical hypotheses beginning with their original paper of 1928 to be of valuable service. All their work in this field is based on the concept of the use of a statistical test as an *operationally verifiable rule of behavior* that is subject to the following two kinds of error: (1) the rejection of a hypothesis when it is true, and (2) the acceptance of it when it is false. At least in the field of the control of quality in mass production these two kinds of errors are of outstanding importance. This was realized as early as 1924 in the adoption of what are termed consumer and producer risks as a basis for developing sampling plans for inspection by lots and also in the development of the operation of control involving the use of the control chart. These two kinds of errors must also be taken into account in determining the size of an adequate sample in testing the significance of an observed difference, as for example in using the "Student's" *t*-test.

It is extremely helpful to the practical statistician to have so much of the work on testing statistical hypotheses gathered together in one place as is being done in this series of memoirs. A more or less connected and critical story showing the limitations of different tests and the relationship between them based upon the acceptance of a minimum of general principles is a necessary background for intelligent use of statistics at least in the field of engineering and particularly in mass production where because of the repetitive nature of the processes, a slip in the use of statistical tests may be costly. Although the contributions in the present volume, as in the previous one, are pretty heavy going for the non-professional mathematician, they are exceedingly well written and will well repay a lot of careful study on the part of a broad audience of statisticians.

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