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PEBBLE ORIENTATION IN RELATION TO CROSS-STRATIFICATION A STATISTICAL STUDY[1]

SHIBDAS BANDYOPADHYAY

Research and Training School, Indian Statistical Institute, Calcutta—35, India[2]

ABSTRACT

Through graphical comparison of pebble orientation and cross-bedding dip direction it has been shown earlier that elongated pebbles, in general, tend to be arranged transversely to the current direction on the topset and bottomset units having extremely low initial dips. They are arranged longitudinally on the foreset units having greater initial dips.

The above conclusions have been now verified by statistical tests. In all cases excepting one, long axes of elongated pebbles are parallel to the direction of foreset dip of cross-bedding. On topsets and bottomsets, the directions normal to the long axes of pebbles are parallel to the cross-bedding dip direction when tested at 5 percent level of significance. This confirms, in particular, the earlier view that for the paleocurrent interpretation, pebble orientation in the central part of foreset is as dependable as the foreset dip direction.

INTRODUCTION

Several contradictory observations by a number of workers exist on the orientation of elongated pebbles with respect to the direction of flow of the transporting agency (Sengupta, 1966). Recent laboratory (Johansson, 1963) and field observations (Sengupta, 1966) have now shown that the orientations of long axes of elongated pebbles are related, among other factors, to the initial slope of the depositing bed. The plunge and orientation of elongated pebbles therefore, varies in different portions of a cross-stratified unit. Graphical comparisons of the orientations of elongated pebbles from various parts of cross-stratified units and dip directions of the corresponding cross-stratified sets in the uppermost memeber of the Permo-Triassic, fluvial, Kamthi Formation near Bheemaram, India, led Sengupta to the following conclusions

1. Elongated pebbles, in general, tend to be arranged transversely to the current direction on the topset and bottomset units of cross-stratified units which have extremely low initial dips.
2. These pebbles are arranged longitudinally on the foreset units having greater initial dips.
3. A number of hydrodynamic factors (see Johansson, 1963) operating in the uppermost and lowermost portions of a foreset zone (i.e., near the foreset-topset and foreset-bottomset contacts) destroy the preferred orientations of pebbles in these zones.

This suggests that interpretation of paleocurrent direction from pebble orientation data should also take into consideration the location of the pebbles in the cross-stratified unit.

In view of the above suggestion, it was decided to undertake a statistical study along the following lines:

1. Statistically analyse the data presented by Sengupta (1966)
2. To collect and analyse statistically a fresh set of pebble orientation and cross-bed dip direction data from different parts of cross-bedded units.

Each of the above two studies was subdivided as follows:

1. Test of homogeneity of dip direction of cross-bedding foresets and the orientation of the longer axes of elongated pebbles.
2. Test of homogeneity of dip directions of cross-bedding topsets and bottom sets and the directions normal to the long axes direction of pebbles.

THEORY OF TESTS

The observations on cross-bedding dip directions and pebble orientations which can range from 0° to 360°, are considered as unit vectors on a plane. The observations are recorded (in the clockwise direction with respect to the North as the origin) as points on the circumference of a unit circle. At this point one has to remember that the variates are angular and unlike linear variates, the conventional statistical techniques cannot be used to analyse such data. For example, the mean or the standard deviation of a set of angular observations has no representative property for the observations, the usual t-statistic no longer has a t-distribution. This point has been stressed by a number of earlier authors (Watson, 1966; Sengupta and Rao, 1969; Rao, 1969). The theory behind the techniques adopted for the analysis and interpretation of the data in the present case, is as follows:

An angular or circular variate α on the circum-

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[1] Received September 28, 1970; revised December 22, 1970.

[2] Present address: Department of Statistics, University of Minnesota, Minneapolis, Minnesota, USA.

ference of a unit circle has Circular Normal distribution with parameters ν and k, the polar direction and parameter of concentration respectively [denoted by CN(ν, k)] if the density function of α, $f(\alpha)$ is given by

$$f(\alpha) = \frac{\exp k \cos(\alpha - \nu)}{2\pi \cdot I_0(k)} 0 \leq \alpha < 2\pi$$

where $0 \leq \nu < 2\pi$, $0 \leq k < \infty$ and $I_0(k)$ is a Bessel function of purely imaginary argument.

Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ and $\beta_1, \beta_2, \cdots, \beta_m$ be n and m independent observations from $CN(\nu_1, k_1)$ and $CN(\nu_2, k_2)$ respectively. (1)

Let us further assume

$$k_1 = k_2 = k \text{ (unknown).} \qquad (2)$$

The assumption (2) is because of two reasons: Our primary interest is to estimate paleocurrent direction and to verify the relation between cross-bedding dip direction and pebble orientation. Secondly, Sengupta's data do not permit to test the validity of (2) which the fresh data does and justifies.

The vector resultant, R, of any set of observations:

$$R^2 = [(\text{total of cosine of angles})^2 + (\text{total of sine of angles})^2].$$

TEST OF EQUALITY OF POLAR DIRECTIONS

For testing the hypothesis, H_0, that the two sets of observations are from the same CN population one has

$$H_0: \nu_1 = \nu_2.$$

Two different test procedures for testing H_0 are given below:

1. When k is large, the likelihood ratio test yields a test based on the statistic S_1, given by

$$S_1 = \frac{(n + m - 2)(R_1 + R_2 - R)}{(n + m - R_1 - R_2)} \qquad (3)$$

where R_1 is the sample resultant based on first set of n observations, R_2 is the sample resultant based on the second set of m observations and R is the resultant of all observations (i.e., $n+m$ observations).

The above statistic S_1 was suggested by Watson (1956). S_1 has an F distribution with 1 and $(m+n-2)$ degrees of freedom when H_0 is true.

2. When k is small, that is, when the approximations

$$\hat{k} = \frac{2R}{(n + m)}$$

and

$$\log I_0(\hat{k}) = \frac{R^2}{(m + n)^2}$$

are valid, where $\hat{k}$ is the estimate of k based on $(m+n)$ observations, the log likelihood ratio yields

$$S_2 = \left(\frac{2}{m + n}\right) [(R_1 + R_2)^2 - R^2] \qquad (4)$$

which has, under H_0, χ^2 distribution with 1 degree of freedom (see Rao, 1969).

Hence the test procedure is to reject H_0 if S_1 exceeds upper 5 percent (or 1 percent) point of $F_{1,m+n-2}$ (when k is large) or reject H_0 if S_2 exceeds upper 5 percent (or 1 percent) point of χ_1^2 (when k is small).

PROCEDURE FOR CALCULATION OF S_1 AND S_2:

1. Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be observations on pebble orientation and $\beta_1, \beta_2, \cdots, \beta_m$ be the observations on cross-bedding dip directions from any particular location. If the pebble orientation data is either from topset or from bottomset we change α's, on physical consideration, by adding 90° (or subtracting 90°) to each of the observations since in these cases pebble orientation is proposed to be normal to cross-bedding dip direction.

2. The sine and cosine values of all the angles are then obtained from a standard table.

3. Then the following quantities are calculated:

For pebble orientation:

$$C_p = \cos\alpha_1 + \cos\alpha_2 + \cdots + \cos\alpha_n$$
$$S_p = \sin\alpha_1 + \sin\alpha_2 + \cdots + \sin\alpha_n$$

For cross-bedding direction:

$$C_X = \cos\beta_1 + \cos\beta_2 + \cdots + \cos\beta_m$$
$$S_X = \sin\beta_1 + \sin\beta_2 + \cdots + \sin\beta_m$$

And for the total data:

$$C = C_p + C_X$$
$$S = S_p + S_X$$

The resultants from pebble orientation data, cross-bedding dip direction data and the combined data are given by R_1, R_2, and R respectively, where

$$R_1^2 = C_p^2 + S_p^2, \qquad R_1 = \sqrt{R_1^2}$$
$$R_2^2 = C_X^2 + S_X^2, \qquad R_2 = \sqrt{R_2^2}$$
$$R^2 = C^2 + S^2, \qquad R = \sqrt{R^2}$$

4. S_1 and S_2 are calculated from the above values of R_1, R_2 and R by using (3) and (4). Since S_1 and S_2 are both symmetric functions of R_1 and R_2, it really does not matter which set we call α's and which set we call β's.

RESULTS AND DISCUSSIONS

The following three tables give the values of S_1 and S_2 for the data collected and graphically presented by Sengupta (1966, fig. 2; p. 365).

TABLE 1.—*Foreset pebble orientation Vs. Cross-bedding dip direction*

Location	n	m	S_1	S_2
1	13	1	0.12875	0.119426
2D	38	1	0.10044	0.08639
3	2	1	247.74021	5.15283
5	16	1	0.00152	0.00141
8	51	1	0.19687	0.12496

TABLE 2.—*Direction normal to pebble orientation Vs. Topset dip direction*

Location	n	m	S_1	S_2
2C	10	1	0.48768	0.34932
3	6	1	0.02698	0.012439
5	10	1	0.14098	0.15299
6A	4	1	0.48191	0.73662

TABLE 3.—*Direction normal to pebble orientation Vs. Bottomset dip direction*

Location	n	m	S_1	S_2
2A	14	1	0.07522	0.05581
3	18	1	0.00996	0.01113
4	20	1	0.42702	0.26368
5	15	1	0.49985	0.35952

The calculated S_1 and S_2 are, in all cases, less than the corresponding upper 1 percent point of F and χ^2. At 5 percent level of significance all the results excepting one at the foreset location 3 are insignificant. This supports the general validity of the earlier conclusions drawn on the basis of graphical comparisons.

This conclusion implies that in the central part of a cross-bedded unit the long axes of the elongated pebbles are as good as the foreset dip direction in supplying clues to the paleocurrent direction.

To reexamine the validity of the above statement the present author has collected a fresh set of data from five foreset exposures in the uppermost part of Kamthi Formation near Bheemaram, India. A set of three measurements on pebble orientation and cross-bedding dip direction was collected at random from each of these five exposures. Since the exposures were very close to one another, each set of 15 observations is assumed to come from the same population. The result of this analysis is tabulated below:

TABLE 4.—*Foreset pebble orientation Vs. Cross-bedding dip direction*

n	m	S_1	S_2
15	15	0.0294	0.01112

The calculated test statistic S_1 and S_2 are insignificant when these are compared with corresponding upper 5 percent point of F and χ^2. This conclusively proves that in the central part of the foreset unit, the orientation of the longest axis of the elongated pebble is essentially same as of the inclination of the cross-bedding foreset.

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REFERENCES

JOHANSSON, C. E., 1963, Orientation of pebbles in running water: a laboratory study: Geog. Annaler, Vol. 45, No. 2–3, p. 85–112.

RAO, J S., 1969, Some contributions to the analysis of circular data: unpublished Ph.D. thesis, Indian Statistical Institute, Calcutta.

SENGUPTA, S. AND RAO, J. S., 1966, Statistical analysis of cross-bedding azimuths from the Kamthi Formation around Bheemaram, Pranhita-Godavori Valley: Sankhyā, Ser. B., Vol. 28, p. 165–174.

SENGUPTA, S., 1966, Studies on orientation and imbrication of pebbles with respect to cross-stratification: Journal of Sed. Petrology, Vol. 36, No. 2, p. 362–369.

WATSON, G. S., 1956, Analysis of dispersion on a sphere: Monthly notices Royal Astron. Soc., Geophys. Suppl., Vol. 7, 153–159.

WATSON, G. S., 1966, The statistics of orientation data: Jour. Geology, Vol. 74, 786–797.