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ON PRESERVATION OF SOME PARTIAL ORDERINGS UNDER SHOCK MODELS

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Abstract

Singh and Jain (1989) have proved some preservation results for partial orderings of life distributions assuming that shocks occur according to a homogeneous Poisson process. It is shown that their results hold under less restrictive conditions.

TP_2 FUNCTIONS; NON-HOMOGENEOUS POISSON PROCESS

1. Introduction

Recently Singh and Jain (1989) have proved some interesting results on certain partial orderings of life distributions of two devices subjected to similar shocks occurring according to a homogeneous Poisson process. In this note it is shown that their results hold under more general shock models. We use their notation and terminology.

Theorem 1. Let shocks occur according to a counting process such that $P[N(t)=k]$ is TP_2 on $R \times N^0$, where $N^0 = \{0, 1, 2, \ldots\}$. Then the results (i), (ii) and (v) of Theorem 2.1 of Singh and Jain (1989) continue to hold.

Proof. It follows from Karlin (1968), p. 17, that if $\phi_1(t, k)$ is TP_2 on $R \times N^0$ and $\phi_2(k, \Theta)$ is TP_2 on $N^0 \times R$, then

$$\sum_k \phi_1(t, k)\phi_2(k, \Theta) \quad \text{is } TP_2 \text{ on } R \times R.$$

Let

$$\phi_1(t, k) = P[N(t)=k], \qquad \phi_2(k, 1) = c_k, \qquad \phi_2(k, 2) = b_k$$

where

$$b_k = p_k,\ \bar{P}_k \text{ and } \sum_{i=k}^{\infty} P_i \text{ for parts (i), (ii) and (iii),}$$

and

$$c_k = q_k,\ \bar{Q}_k \text{ and } \sum_{i=k}^{\infty} Q_i \text{ for parts (i), (ii) and (iii), respectively.}$$

The assumed conditions are equivalent to saying that $\phi_2(k, \Theta)$ is TP_2 on $N^0 \times \{1, 2\}$. Hence the result.

Corollary 2. Let shocks occur according to a non-homogeneous Poisson process with a non-decreasing mean value function $m(t)$. Then the conclusions of Theorem 1 continue to hold.

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Proof. Since the function $\phi_1(t, k) = \exp(-m(t))\ [m(t)]^k/k!$, $k = 0, 1, 2, \ldots$ is TP_2 when $m(t)$ is non-decreasing, the proof follows from the above theorem.

Theorem 2.1 of Singh and Jain is thus a particular case of this corollary. Parts (iii) and (iv) of their Theorem 2.1 hold without any restrictions on the shock models.

References

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