

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2017-18

B. Stat. II Year Analysis-III

Date: 04/09/2017 Maximum Marks: 40 Duration: 3 Hours

Note: Give proper justification to all your answers. State clearly all the results you are using.

(1) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{x^3}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

(a) Prove that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are bounded functions in \mathbb{R}^2 . Is f continuous?

(b) Find all $v \in \mathbb{R}^2$ such that the directional derivative $D_v f(0, 0)$ exists.

(c) Let $u : \mathbb{R} \rightarrow \mathbb{R}$ and $v : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable functions such that $u(0) = 0, v(0) = 0$, and $u'(0)^2 + v'(0)^2 \neq 0$. Define $g : \mathbb{R} \rightarrow \mathbb{R}$ by $g(t) = f(u(t), v(t))$. Show that g is differentiable on \mathbb{R} .

(d) Show that f is not differentiable at $(0, 0)$. [10]

(2) Show that there exists an open set $U \subset \mathbb{R}^4$ containing $(1, 2, 0, 1, -1)$ and an open set $W \subset \mathbb{R}^2$ containing $(1, -1)$ with the following property:

For every $(s, t) \in W$ there is a unique (u, v, w) such that $(u, v, w, s, t) \in U$, and

$$\begin{aligned} u^2 - (v + w)^2 + s^2 t + st^2 + 3 &= 0, \\ -u - v - w + e^{s+t} + 2 &= 0, \\ u + v + w^2 + (s + t)^2 - 3 &= 0. \end{aligned}$$

Define the (u, v, w) so obtained to be $g(s, t)$. Conclude that g is continuously differentiable on W and compute $g'(1, -1)$. [10]

(3) Consider the function $f(x, y) = (x - 3)(y - 2)(2x + 3y - 6)$.

(a) Make a sketch indicating the set of points (x, y) where $f(x, y) \geq 0$.

(b) Identify the saddle points of f from this sketch.

(c) Does f have an absolute minimum or an absolute maximum on the whole plane? Justify your answer.

(d) Find all points where the first order partial derivatives of f are zero.

(e) Find all points where f has a local maximum or a local minimum. [12]

(4) Find the point $(x, y, z) \in \mathbb{R}^3$ which is closest to the origin and satisfies the equations $2y + 4z = 5$ and $4x^2 + 4y^2 = z^2$. [10]

INDIAN STATISTICAL INSTITUTE, KOLKATA
MIDTERM EXAMINATION: FIRST SEMESTER 2017 -'18
B. STAT II YEAR

Subject : Elements of algebraic structures
Time : 2 hours
Maximum score (after scaling) : 30

DATE: 05/09/2017

Attempt all the problems. Justify every step in order to get full credit of your answers. All arguments should be clearly mentioned on the answerscript. Points will be deducted for missing or incomplete arguments.

- (1) If a group G is not abelian, then show that $\frac{G}{Z(G)}$ is not cyclic. [5 marks]
- (2) If an group G with order 1250, has a subgroup H with order 125, then show that there exists a nontrivial left H -coset which is also a right H -coset. [10 marks]
- (3) (i) If a element a in a group G has order $n \in \mathbb{N}$, and $k \in \mathbb{N}$, then show that $o(a^k) = \frac{n}{\gcd(n, k)}$.
(ii) If G is a cyclic group of order $n \in \mathbb{N}$, and $l \in \mathbb{N}$, then show that the cardinality of $\{g \in G : o(g) = l\}$ is $\varphi(l)$ where φ is the Euler's totient function.
(iii) Prove that $n = \sum_{d|n} \varphi(d)$ for all $n \in \mathbb{N}$. [15 marks]
- (4) Let A_5 denote the subgroup of even permutations in S_5 .
(i) Show that any element of A_5 must be conjugate to exactly one of $(1, 2, 3, 4, 5)$, $(1, 2)(3, 4)$ and $(1, 2, 3)$.
(ii) Show that A_5 is *simple*, that is, it has no nontrivial normal subgroup. [10 marks]
- (5) Show that no group of order 24 is simple. [10 marks]

Indian Statistical Institute
Semester 1 (2017-2018)
B. Stat 2nd Year
Midsemestral Examination
Probability Theory 3

Wednesday 6.9.2017, 2:30-4:30 PM

Total Points $6 \times 5 = 30$

Answers must be justified with clear and precise arguments. More than one uncrossed answers to the same question or part of a question will not be entertained and only the first uncrossed answer will be graded. If you use any theorem or result proved in class state it explicitly.

1. Consider a sequence of random variables X_n , with X_n having the following distribution function

$$\begin{aligned} F_n(x) &= 0, x < \frac{1}{n}, \\ &= 1, x \geq \frac{1}{n}, \end{aligned}$$

$n = 1, 2, \dots$. Describe the random variable X such that X_n converges in distribution to X and with F denoting the distribution function of X , discuss if $F_n(x) \rightarrow F(x)$ for $x \notin C(F)$ where $C(F)$ denotes the set of discontinuity points of F

2. Suppose X_1, X_2, \dots are i.i.d. Exponential random variables with parameter 1. Is it possible that for some appropriate sequence of real numbers a_n ,

$$Y_n = \max(X_1, X_2, \dots, X_n) - a_n$$

converges in distribution to a nontrivial random variable? You need to argue using the distribution function of Y_n .

3. Suppose ϕ is the characteristic function of a random variable. Is $Re(\phi)$ a characteristic function? Is $Im(\phi)$ a characteristic function? In both cases give proper justification for your answer.
4. Suppose X_1, X_2, \dots are i.i.d. Poisson (λ) random variables. Find the characteristic function of

$$\frac{X_1 + \dots + X_n - n\lambda}{\sqrt{n}}$$

in closed form and find the limit of the characteristic function obtained above as $n \rightarrow \infty$. P. T. O

5. An infinite sequence of i.i.d. Bernoulli trials with $P(1) = p, P(0) = q = 1 - p$, is observed. Let A_n be the event that a run of 1's of length n is observed in trials $2^n + 1, 2^n + 2, \dots, 2^{n+1}$. If $p < 1/2$ then zeroes occur more frequently so that $P(A_n \text{ occurs infinitely often}) = 0$. Show this using the following steps (no other solution will be accepted):

Consider the above trials $2^n + 1, 2^n + 2, \dots, 2^{n+1}$ renumbered without loss of generality as $1, 2, \dots, 2^n$. Let $B_{n,k}$ be the event that a run of 1's of length n is observed in blocks of trials with center $kn - (k - 1)$ and length $2n - 1$, i.e. in trials $(k - 1)n - (k - 2), \dots, kn - (k - 1), \dots, (k + 1)n - k, k = 2, 3, \dots, k_n = \lfloor 2^n / (n - 1) \rfloor$. $B_{n,1}$ is the corresponding event for the first block $\{1, 2, \dots, 2n - 1\}$. These blocks of trials are overlapping but clearly A_n implies $\cup_{k=1}^{k_n} B_{n,k}$.

Find $P(B_{n,k})$ exactly in closed form in terms of p, q and n , and if $p < 1/2$ prove that $P(A_n \text{ occurs infinitely often}) = 0$. Each answer must be justified with rigorous arguments.

INDIAN STATISTICAL INSTITUTE

Mid Semester Examination, 1st Semester, 2017-18

Statistical Methods III, B.Stat 2nd Year

Date: September 7, 2017

Time:2 hrs

This paper carries 30 marks. Answer all questions.

1. Consider the usual regression model:

$$y_i = \alpha + \beta x_i + e_i; \quad i = 1, 2, \dots, n$$

where, x_i s are fixed and e_i s are i.i.d. with mean 0 and variance σ^2 . Suppose $\hat{\alpha}$ and $\hat{\beta}$ are the least squares estimators of α and β , respectively.

(a) Show that $\hat{\beta}$ is the BLUE for β .

(b) Compute the mean squared error of $\hat{\alpha}$.

(c) If $Q = \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta})^2$, obtain an unbiased estimator of σ^2 in terms of Q . [5+4+3]

2. Suppose that a trait Y conditioned on three genetic profiles is distributed as $N(\mu, \sigma^2)$, $N(0, 2\sigma^2)$ and $N(-\mu, \sigma^2)$, respectively. The three genetic profiles are in the proportions p^3 , $3p(1-p)$ and $(1-p)^3$, respectively. If data are available on the trait values of n randomly selected individuals from the population, describe a suitable EM algorithm to obtain the maximum likelihood estimators of p , μ and σ^2 . Show all computational steps clearly. [10]

3. Suppose X_1, X_2, \dots, X_n is a random sample from an exponential distribution with mean λ . Examine whether:

(a) $nX_{(1)}$ is a consistent estimator of λ .

(b) the sample median is the UMVUE for the population median.

[3+5]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2017-18

Course Name: B.Stat. II Year

Subject Name: Physics I

Date: 08.09.2017

Maximum Marks: 40

Time: 2 hours 30 minutes

Group - A : Classical Mechanics

Answer any **two** questions. All questions carry 10 marks

1. Consider the cylindrical coordinate system ρ, ϕ, z with relations with Cartesian coordinates

$$x = \rho \cos \phi, \quad y = \rho \sin \phi, \quad z = z.$$

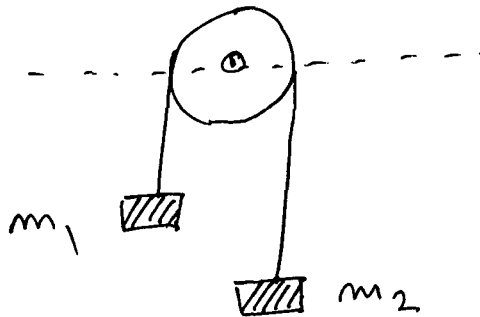
(i) Express unit vectors $\hat{\rho}, \hat{\phi}, \hat{z}$ in terms of unit vectors in Cartesian coordinates $\hat{i}, \hat{j}, \hat{k}$.

(ii) Show that $\hat{\rho}, \hat{\phi}, \hat{z}$ are orthogonal to each other.

(iii) Find the kinetic energy of a free particle of mass m in cylindrical coordinate system ρ, ϕ, z . [5+2+3]

2. (i) Derive the Lagrangian equation of motion for generalized coordinates $q_a, a = 1, \dots, n$.

(ii) Consider an Atwood's machine (see figure) where two masses m_1 and m_2 are connected by an inextensible string of length l . The string passes over a fixed frictionless pulley and the masses hang over the pulley in gravity (g acceleration due to gravity) and can move up or down. Construct the Lagrangian of the system and find the acceleration of the masses. [4+6]



3. Consider a particle of mass m in three dimensions moving in a potential $V = K \cos^2 \theta e^{-r^2}$ where K is a constant.

(i) Find the Lagrangian in spherical polar coordinates r, θ, ϕ where the variables have conventional meaning.

(ii) Find the equations of motion.

(iii) Identify cyclic coordinates. [4+5+1]

Group - B : Thermodynamics and Statistical Mechanics

Answer any **four** questions

1. 2 moles of an ideal gas is taken through a cycle abc consisting of the following processes: $a \rightarrow b$, isothermal expansion to three times its volume, at 20°C ; $b \rightarrow c$, compression to original volume at constant pressure of 10^5Pa ; $c \rightarrow a$, doubling of pressure at constant volume.
Plot the cycle on a P-V diagram. Calculate the work done, the heat transferred and the change in internal energy for each part of the cycle. [1+4]
2. Show that for an ideal gas, the efficiency of a Carnot cycle depends only on the two temperatures between which it operates. [5]
3. Write the Clausius and Kelvin-Planck statements of the second law of thermodynamics. Show that if the Kelvin-Planck statement is violated, then the Clausius statement is also violated. [2+3]
4. Let 30g of water be heated from 27°C to 87°C by placing it in thermal contact with a heat reservoir at 87°C . Calculate the change in entropy of the universe. What would be the result if the process was reversible? Give reasons for your answer. [3+2]
5. Two identical bodies of constant heat capacity C_p at temperatures T_1 and T_2 ($T_1 > T_2$) are used as reservoirs for a heat engine. If the bodies are at constant pressure, show that the amount of work obtainable is $W = C_p(T_1 + T_2 - 2T_f)$, where T_f is the final temperature attained by both bodies. Show that $T_f = \sqrt{T_1 T_2}$ when W is maximum. [2+3]

Indian Statistical Institute
B. Stat. II Year: Mid Semester Examination 2017-18
Microeconomics

Time: 2 hrs

Max Marks: 40

Date: 08.09.17

Answer as many questions as you can. The maximum you can score is 40.

1. Define completeness, transitivity, reflexivity, continuity and strong monotonicity of a preference relation. Show that under these conditions, the preference relation will have a functional representation. 10+6=16

2. Consider the direct utility function given by

$$u(x_1, x_2) = x_1^a x_2^b$$

(a) Find out the restrictions on a and b so that $u(\cdot)$ is strictly quasi-concave. Hence show that a quasi-concave function may not be concave but the converse is true.

(b) Set up the Lagrangian of maximising $u(\cdot)$ subject to the constraint that $p_1 x_1 + p_2 x_2 = M$. Find out the second order conditions for having a maximum.

(c) What can you say about the preference represented by $u(\cdot)$? Justify.

(d) How will the level sets of $u(\cdot)$ look like if $u(\cdot)$ is strictly quasi-concave? What is the implication for the optimisation problem in that case? Justify.

(e) Interpret the Lagrange multiplier in economic terms. 4+4+3+4+3 = 18

3. Consider an indirect utility function given by

$$v(p_1, p_2, M) = \frac{M}{p_1 + p_2}$$

(a) What are the demand functions?

(b) What is the expenditure function?

(c) What is the direct utility function? 3+4+4 = 11

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2017-2018

Course: B. Stat. III Year

Subject: Introduction to Sociology

Date: 08.09.17

Maximum Marks: 20

Duration: Two hours

The figures in the margin indicate full marks

Answer any four of the following questions:

Q 1. Choose the correct answers:

1 x 5 =5

(a) "Dynamics of Rural Sociology" was written by:

- (i) Ramkrishna Mukherjee
- (ii) Andre Beteille
- (iii) Parthanath Mukherji

(b) Who is the father of Sociology?

- (i) August Comte
- (ii) Ramkrishna Mukherjee
- (iv) Darwin

(c) What is the basis for the demarcation of the family in to matriarchal and patriarchal?

- (i) Organization
- (ii) Authority
- (iii) Residence

(d) The villagers maintain their general behaviouristic patterns because:

- (i) They are trained by their families to do so
- (ii) They are more cultured
- (iii) They generally follow traditions, customs and folkways more resolutely

P.T.O

(e) "Hunting and "Food-gathering" were the primary pursuits of most of the tribesmen in the primitive ages because:

- (i) People were ferocious and hunting and food gathering were their past times
- (ii) People did not know any other alternative to earn their living
- (iii) None of these

Q 2. What is Rural Sociology? What is meant by self-sufficient Village? 2+3 =5

Q 3. Briefly discuss about the importance of Statistics in conducting Social Research. 5

Q 4. What is case study? Explain its use in social research. 2+3 =5

Q 5. Discuss Durkheim's theory of suicide. 5

Q 6. Write short notes on any two: 2.5 x 2 =5

- a) Globalization.
- b) Positivistic stage of societal development.
- c) Agrarian society.

Indian Statistical Institute**Mid-Semester Examination: 2017-18****Course Name: B. Stat II, Subject name: Mol. Biology****Date: 8/9/2017****Maximum Marks: 40;****Duration: 2.0 hrs*****All questions carry equal marks, answer any five***

1. Write the metabolic steps in which genetic defects might lead to galactosemia, albinism, alkaptonuria and phenylketonuria. [8]
2. Distinguish between DNA replication and transcription. (use cartoons for better presentation). [8]
3. (a) Generally, number of base pairs in the same gene is different in bacteria and human: why? (b) Human and bacteria chromosomal DNAs contain about 10^9 and 10^6 base pairs respectively, but the time required for their replication is same: explain. (consider rate of replication is same in both cells). [4+4]
4. How you will determine molecular weight of hemoglobin and its different sub-units/chains. (4+4)
5. (a) Distinguish protein from DNA chemically and functionally.
(b) State the functions of the three different types of RNA used in protein synthesis. [4+4]
6. Write the metabolic reactions/steps in TCA cycle which generates NADH. How many ATPs will be generated from these NADHs. [8]
7. How this fatty acid $[\text{CH}_3(\text{CH}_2)_2\text{CH}=\text{CH}(\text{CH}_2)_2\text{COOH}]$ will be metabolized to generate ATP? [8]

INDIAN STATISTICAL INSTITUTE
Mid – Semestral Examination: 2017-18
B.Stat III Geology Elective
Maximum Marks: 40
(Answer all questions)

Date: 08/09/2017

Time: 2 hours

1. The largest known snake fossil *Titanoboa cerrejonensis* lived _____ to _____ million years ago. (1+1)
2. _____ and _____ are the examples of two living fossils. (1+1)
3. Increase in _____ concentration in atmosphere is causing global warming triggering the melting of _____ that will eventually raise global sea level by few meters. (1+1)
4. Magma generation by decompression melting happens in (1)
 - a) Convergent plate boundary.
 - b) Divergent plate boundary.
 - c) Hot spots.
 - d) All of the above.
5. Which of the following mineral is not present in Granite? (1)
 - a) Quartz.
 - b) Feldspar.
 - c) Biotite and/or muscovite.
 - d) Olivine.
6. Which of the following igneous rock has lowest melting temperature? (1)
 - a) Basalt.
 - b) Diorite.
 - c) Granite.
 - d) Gabbro.
7. Which of the following rocks is Mafic? (1)
 - a) Granite.
 - b) Diorite.
 - c) Andesite.
 - d) Gabbro.
8. Name the rock that floats in water. What type of rock it is igneous, metamorphic or sedimentary? (1+1)
9. You will look for fossils in (1)

- a) Igneous rocks.
- b) Metamorphic rocks.
- c) Sedimentary rocks.
- d) Migmatites.

10. Which of the prismatic minerals will align themselves perpendicular to the maximum stress direction during metamorphism? (1)

- (a) Hornblende, Tourmaline.
- (b) Muscovite, Biotite.
- (c) Olivine, Quartz.
- (d) Both (a) and (c).

11. Foliation is defined by the parallel alignment of the mineral(s) perpendicular to the maximum stress direction? (1)

- (a) Hornblende, Tourmaline.
- (b) Muscovite, Biotite.
- (c) Olivine, Quartz.
- (d) Both (a) and (c).

12. In paired metamorphic belt (example: Sierra Nevada belt and Franciscan belt) you will expect to get (1)

- a) High P and T; High P and Low T metamorphism.
- b) High P and T; High P and High T metamorphism.
- c) Low P and T; High P and Low T metamorphism.
- d) Low P and T; Low P and High T metamorphism.

13. Marble is metamorphosed form of (1)

- (a) Limestone.
- (b) Quartzite.
- (c) Quartz sandstone.
- (d) Bituminous coal.

14. Which of the following process is most likely to preserve pre-historic land plants? (1)

- (a) Dead plants are heated and dried by the tropical sun.
- (b) Eruption of lava flow covers and heat the plant matter.
- (c) Sea level rises and plant matter is buried by sediments.
- (d) Chemical and biological weathering occurs, causing organic decay.

15. Which of the following is the correct order for the composition of minerals as the temperature decreases according to Bowen's Continuous Reaction series? (1)

- (a) Calcic plagioclase (anorthite), Sodic plagioclase (albite), Potassium feldspar (orthoclase).
- (b) Sodic plagioclase (albite), Potassium feldspar (orthoclase), Calcic plagioclase (anorthite).
- (c) Potassium feldspar (orthoclase), Calcic plagioclase (anorthite), Sodic plagioclase (albite)
- (d) All the above.

16. Which of the following mineral has sheet like structure? (1)

- (a) Augite.
- (b) Hornblende.
- (c) Biotite.
- (d) Orthoclase.

17. Is the statement true “In the Bowen’s discontinuous reaction series amphibole is formed at a higher temperature than pyroxene.”? (1)

18. When there is sufficient space and time for the crystal to grow a crystal with well formed crystal faces is called _____ (1)

19. What is the hardness of Quartz according to Moh’s Scale of hardness? (1)

- (a) 5.
- (b) 6.
- (c) 9.
- (d) 7.

20. Metamorphism occurs due to change in (1)

- (a) Rock type.
- (b) Pressure.
- (c) Heat.
- (d) Heat and pressure.

21. If slate is exposed to more heat and pressure it changes into (1)

- (a) Phyllite.
- (b) Limestone.
- (c) Gneiss.
- (d) Garnet.

22. Extrusive Igneous Rocks contain (1)

- (a) Small crystals.
- (b) No crystals.
- (c) Large crystals.
- (d) Small or no crystals.

23. Process by which sedimentary rocks are arranged in layers is known as (1)

- (a) Stratification.
- (b) Composition.
- (c) Deposition.
- (d) Erosion.

24. At high temperature the early-formed olivine crystals react with silica in the remaining liquid magma and convert into (1)

- (a) Feldspar.
- (b) Quartz.
- (c) Pyroxene.
- (d) Does not react.

25. Porphyritic texture is identified by the presence of _____ crystals, also known as _____, that are embedded in a matrix of smaller crystals, called as _____ (1+1+1)

26. Aphanitic texture is identified by the presence of crystals that are _____ in size and indicates _____ rate of cooling. (1+1)

27. Mechanical weathering increases _____ and _____ thus facilitating chemical weathering and sediment transport. (1+1)

28. Biological effects induce (1)

- a) Only mechanical weathering.
- b) Only chemical weathering.
- c) Both Mechanical and chemical weathering.
- d) None.

30. Previously formed rocks incorporated in magma as it nears the surface is known as (1)

- a) Xenolith.
- b) Vesicles.
- c) Crystals.
- d) Groundmass.

31. "Metamorphic rock cannot be metamorphosed again." Is this statement true? If not then give one example. (1+1)

32. What is the chemical formula of Quartz? (1)

INDIAN STATISTICAL INSTITUTE, KOLKATA
FINAL EXAMINATION: FIRST SEMESTER 2017 -'18
B. STAT II YEAR

Subject: Elements of algebraic structures

Date: November 20, 2017

Duration: 3 hours

Time: 2:30 PM to 5:30 PM

Maximum score (after scaling): 70

Attempt all the problems. Justify every step in order to get full credit of your answers. All arguments should be clearly mentioned on the answerscript. Points will be deducted for missing or incomplete arguments.

- (1) (a) Let R be a commutative ring and $N = \{x \in R : \exists n \in \mathbb{N} \text{ s.t. } x^n = 0\}$. Prove that N is an ideal. (N is referred as *nilpotent ideal* of R .)

(b) In part (a), set $R = \frac{\mathbb{F}[x]}{\langle p(x) \rangle}$ for a field \mathbb{F} and a nonzero polynomial $p \in \mathbb{F}[x]$.

Show that the nilpotent ideal of R is nonzero if and only if p is divisible by the square of some non-constant polynomial. (*It will be easier to do the 'if' part first.*)

[4+16=20 marks]

- (2) Show that the polynomial $x^4 + 2x + 2$ is irreducible over \mathbb{Q} .

[10 marks]

- (3) Let R be a unital commutative ring and S be a subset of R . Show that the gcd of S exists if and only if the intersection of all principal ideals containing the set S , is principal.

[8 marks]

- (4) (a) Let k_1, \dots, k_m and l_1, \dots, l_n be two (possibly not strictly) increasing sequences in \mathbb{N} such that $\sum_{i=1}^m k_i = \sum_{j=1}^n l_j$. For any prime p , show that the groups $\bigoplus_{i=1}^m \mathbb{Z}_{p^{k_i}}$ and

$\bigoplus_{j=1}^n \mathbb{Z}_{p^{l_j}}$ are isomorphic if and only if the two sequences are identical.

OR

Prove that every finite abelian p -group G , has a unique increasing sequence k_1, \dots, k_m in \mathbb{N} such that G is isomorphic to the group $\bigoplus_{i=1}^m G_i$ where G_i is a cyclic group of order p^{k_i} .

(*Hint: For an abelian group G and $s \in \mathbb{N}$, the set $G(s) := \{x \in G : x^s = e\}$ is a subgroup of G . If G_1 and G_2 are isomorphic abelian groups, then for all s , $G_1(s)$ and $G_2(s)$ are also isomorphic.*) ATTEMPT THIS AFTER ANSWERING ALL OTHER QUESTIONS. Continued to the next page ...

2

(b) Find all abelian groups of order 3240 up to isomorphism. Justify every step and clearly state the results that you use. [10+10=20 marks]

(5) Prove that $\sin 1^\circ$ is algebraic over \mathbb{Q} . (*Hint:* $(\cos 1^\circ + i \sin 1^\circ)^{90}$) [10 marks]

(6) Find the splitting field for the polynomial $x^4 + 4$ over \mathbb{Q} [10 marks]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

B. Stat. II Year Analysis-III

Date: 23/11/2017 Maximum Marks: 60 Duration: 3 Hours

Note: Give proper justification to all your answers. State clearly all the results you are using.

(1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x + 2x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Show that $f'(0) = 1$, f' is bounded in $(-1, 1)$, but f is not one-to-one in any neighbourhood of 0. [10]

(2) Let H be the parallelogram in \mathbb{R}^2 with vertices $(1, 1), (3, 2), (4, 5), (2, 4)$. Find the affine map T which sends $(0, 0)$ to $(1, 1)$, $(1, 0)$ to $(3, 2)$, and $(0, 1)$ to $(2, 4)$. Use T to convert the integral

$$\int_H e^{x-y} dx dy$$

to an integral over the unit square I^2 in \mathbb{R}^2 and evaluate the integral. [12]

(3) Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a C^1 -function satisfying $f(x) \neq 0$ for all $x \in \mathbb{R}^n$. Let w be a 1-form in \mathbb{R}^n such that $d(fw) = 0$. Show that $w \wedge dw = 0$. [8]

(4) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the map defined by

$$T(u, v) = (u^2 - v^2, 2uv) = (x, y), \text{ say.}$$

Consider the 2-form w in the xy -plane

$$w = \frac{x dy - y dx}{x^2 + y^2}, (x, y) \neq (0, 0).$$

Find the 2-form w_T in the uv -plane. Note that w_T is also denoted by T^*w . [10]

(5) Given a k -form w in \mathbb{R}^n , define an $(n - k)$ -form \tilde{w} as follows. If w is a basic k -form, that is, if $w = dx_{i_1} \wedge \cdots \wedge dx_{i_k}$, define $\tilde{w} = \text{sign}(\sigma) dx_{j_1} \wedge \cdots \wedge dx_{j_{n-k}}$, where $i_1 < \cdots < i_k$, $j_1 < \cdots < j_{n-k}$ and σ is the permutation that maps $(1, 2, \dots, n)$ to $(i_1, \dots, i_k, j_1, \dots, j_{n-k})$. Now extend this definition linearly to a general k -form.

(a) Let f, g, h be continuous functions on \mathbb{R}^3 and $w = f dx_1 \wedge dx_2 + g dx_1 \wedge dx_3 + h dx_2 \wedge dx_3$ be a 2-form in \mathbb{R}^3 . Compute \tilde{w} .

(b) Show that $\widetilde{(\tilde{w})} = (-1)^{k(n-k)}w$. [10]

(6) Let $0 < a < b$. Let $\Phi(r, \theta) = (r \cos \theta, r \sin \theta)$ for $a \leq r \leq b$, $0 \leq \theta \leq 2\pi$. Let $w = x^3 dy$. Compute both $\int_{\Phi} dw$ and $\int_{\partial\Phi} w$ to verify that they are equal. [15]

INDIAN STATISTICAL INSTITUTE
Final Examination, 1st Semester, 2017-18
Statistical Methods III, B.Stat 2nd Year

Date: November 27, 2017

Time: 3 hours

**This is an open notes examination. The paper carries 55 marks.
Answer all questions.**

1. (a) Suppose X is an observation from Geometric (p). Show that the variance of any unbiased estimator of $\log_e \frac{p}{1-p}$ based on X cannot be less than $1/(1-p)$.
- (b) Suppose T_1 and T_2 are distinct unbiased estimators of a parameter θ . If T_1 is uncorrelated with $(T_1 - T_2)$, show that T_2 cannot be the UMVUE for θ .
- (c) Assume that the number of times a machine becomes non-functional in a day is distributed as Poisson with mean λ . Consider the following data collected on 50 days chosen at random:

Frequency	Number of days
0	25
1	18
≥ 2	7

Use the Fisher's Scoring Method to obtain the maximum likelihood estimate of λ . Show all computations clearly with one cycle of iteration. [4 + 4 + 8]

2. Suppose X_1, X_2, \dots, X_n is a random sample from $N(\mu, \sigma^2)$.
 - (a) Obtain the mean squared error of the maximum likelihood estimator of μ/σ .
 - (b) Suppose $n = 3$ and Z is defined as $\frac{\bar{X}^2}{\frac{1}{n} \sum_{i=1}^n X_i^2}$. Develop a level α test for $H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$ in terms of Z . [8 + 7]

3. (a) It is claimed that the lowest airline ticket prices are at most doubled during Diwali compared to the lowest prices two weeks before Diwali. A reporter, who suspects that the factor of price increase during Diwali is even higher, collected the following data on the lowest prices before (LPB) and during (LPD) Diwali for five different flights between Kolkata and Delhi:

Flight	LPB	LPD
1	2480	4780
2	2650	5520
3	2530	5060
4	2270	4600
5	2560	5140

Based on the above data, do you think that the reporter's suspicion is justified? State your assumptions clearly.

- (b) Suppose we want to estimate the difference in the prevalence of Type 2 Diabetes among obese and non-obese individuals in a population using an asymptotic 95% equal tail confidence interval. What is the minimum combined sample size required such that the error in estimation based on the confidence interval is at most 0.04? Suppose the 95% confidence interval obtained from the above data is (-0.05, 0.03). Without any further computation, can you conclude at level 0.01 whether the prevalence of Type 2 Diabetes are same in the two groups? [7 + 7]
4. Suppose you are interested in testing whether the skills of two fielders are similar in hitting the stumps from a distance of 30 metres. It was found that the first fielder required n_1 attempts to achieve r_1 hits while the second fielder required n_2 attempts to achieve r_2 hits. Develop a suitable level α test for the above hypothesis. What would be your conclusion at level 0.05 if $n_1 = 7$, $r_1 = 4$, $n_2 = 5$ and $r_2 = 3$? [10]

Indian Statistical Institute
Semester 1 (2017-2018)
B. Stat 2nd Year
Semestral Examination
Probability Theory 3

Wednesday 29.11.2017, 2:30-5:00 PM

Total Points: 70

Answers must be justified with clear and precise arguments. More than one uncrossed answers to the same question or part of a question will not be entertained and only the first uncrossed answer will be graded. If you use any theorem or lemma proved in class state it explicitly.

1. (a) Suppose X_1, X_2, \dots, X_n are i.i.d. Bernoulli ($1/n$). Find the limiting probability distribution as $n \rightarrow \infty$, of $X_1 + X_2 + \dots + X_n$ by computing its characteristic function and taking its limit. 10 + 10 = 20 pts.
(b) Suppose X_1, X_2, \dots are independent random variables on (Ω, \mathcal{F}, P) with $X_k = \pm\sqrt{k}$ each with probability $1/2$. Writing $S_n = X_1 + \dots + X_n$, compute the variance of S_n and denoting $s_n^2 = \text{Var}(S_n)$ find the limit of the characteristic function

$$Ee^{it(S_n/s_n)}, t \in \mathbb{R}.$$

Note that the random variables X_i are not identically distributed. You may use the following estimate: for $|\theta| \leq 1$ one has $\cos \theta > 0$ and $\cos \theta = e^{\log \cos \theta} = e^{-\theta^2/2 + O(\theta^4)}$.

2. Suppose X_1, X_2, \dots are i.i.d. Uniform $(0, 1)$ on (Ω, \mathcal{F}, P) Consider $M_n = \max\{X_1, X_2, \dots, X_n\}$ which is nondecreasing almost surely and bounded by 1. 10 + 10 = 20 pts.
(a) Show that M_n converges to 1 in probability, i.e. show that for all fixed $\epsilon > 0$ one has $P(M_n < 1 - \epsilon) \rightarrow 0$ as $n \rightarrow \infty$.
(b) M_n is nondecreasing and bounded above, so converges almost surely. However does $M_n \rightarrow 1$ almost surely to 1? (Hint: the events $\{M_n < 1 - \epsilon\}$ may be helpful.)
3. (a) In problem 1 (b), note that $s_n/n \rightarrow \text{const.}$. Does the WLLN hold, i.e. does S_n/n converge in probability to 0? 5 + 10 = 15 pts.

(b) Let X_2, X_3, \dots be a sequence of independent random variables such that

$$P(X_n = \pm n) = \frac{1}{2n \log n}, P(X_n = 0) = 1 - \frac{1}{n \log n}.$$

Writing $S_n = \sum_{k=2}^n X_k$, does S_n/n converge to 0 in probability? (Hint: If you use Chebyshev's lemma, it may be useful to consider whether a certain function is an increasing or decreasing function of its argument.)

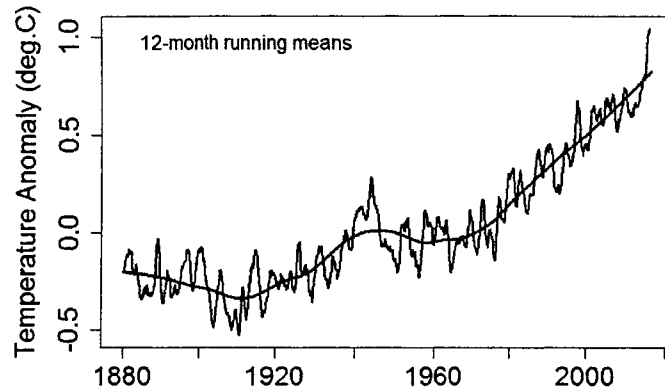
4. Suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are independent Poisson processes with rates λ_1 and λ_2 respectively.
- (a) Show that $\{N_1(t) + N_2(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1 + \lambda_2$.
- (b) Show that the probability that the first event of the combined process comes from $\{N_1(t), t \geq 0\}$ is $\lambda_1/(\lambda_1 + \lambda_2)$. 10 + 5 = 15 pts.

Indian Statistical Institute
First Semester Examination: 2017-18
Course Name: B. Stat. III Year
Subject Name: Geology

Date: 1st December 2017 Maximum Marks: 100 Duration: 3 Hours.

(Answer any ten questions)

Q1. The following figure shows the global temperature anomaly from year 1880 till present.



a) What is/are the reason(s) for the increase in global temperature anomaly from 1880 till present? What does negative global temperature anomaly signify? (2+2)

b) In long term do you think increase in global temperature will affect ocean current and ocean acidification. Explain in brief. (2+2+2)

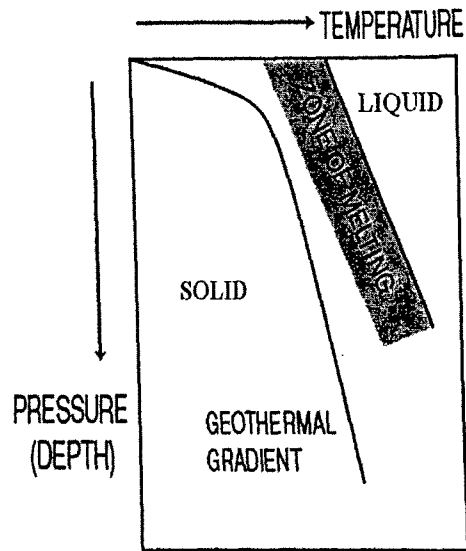
Q2. Average chemical compositions for few elements of four igneous rocks namely Rock 1, Rock 2, Rock 3 and Rock 4 are given in the table below.

Oxide (wt %)	Rock 1	Rock 2	Rock 3	Rock 4
SiO ₂	71.3	57.94	49.2	42.26
Al ₂ O ₃	14.32	17.02	15.74	4.23
Fe ₂ O ₃	1.21	3.27	3.79	3.61
MgO	0.71	3.33	6.73	31.24
Na ₂ O	3.68	3.48	2.91	0.49

a) Plot SiO₂, Fe₂O₃, MgO, Al₂O₃ and Na₂O wt% for all the four rocks (Y-axis will be the concentration and X-axis will be the 4 rock types Rock1, Rock 2, Rock 3 and Rock 4). (5)

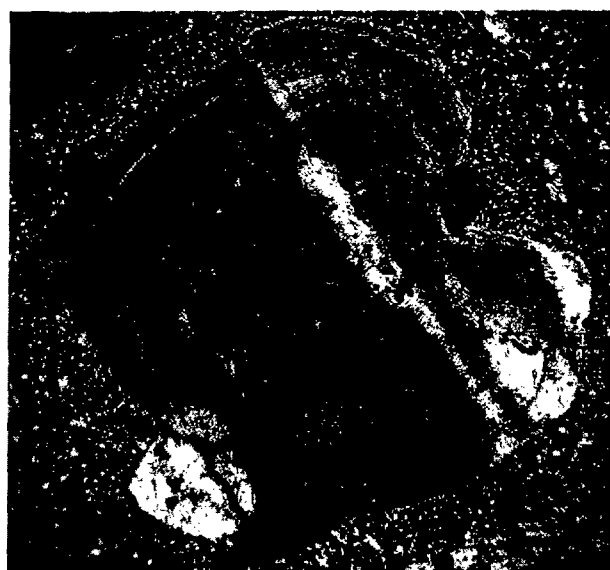
b) Out of four which rock has the highest and the lowest temperature of formation? Which of these rocks you will expect to get olivine? What kind of plate tectonic settings it will form? (1+1+1+2)

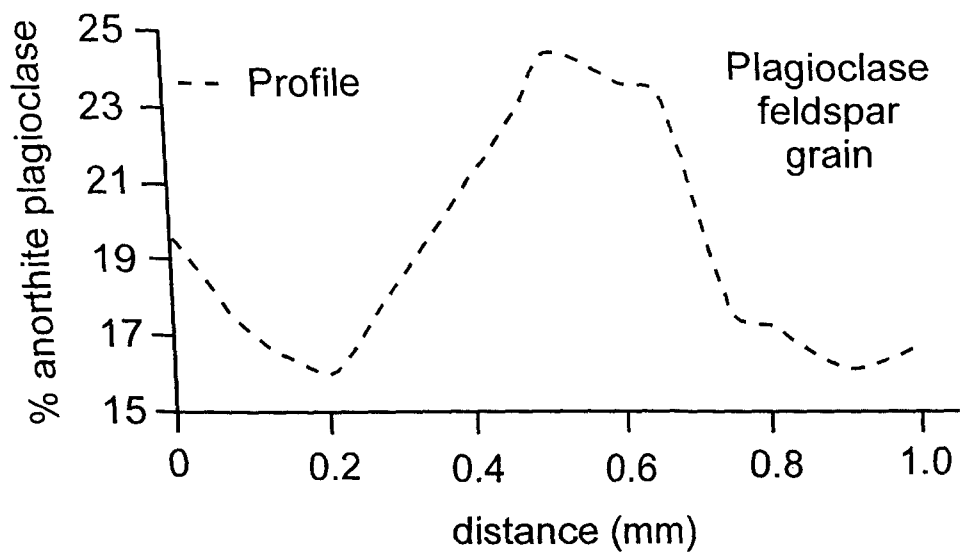
Q3. Study the following figure.



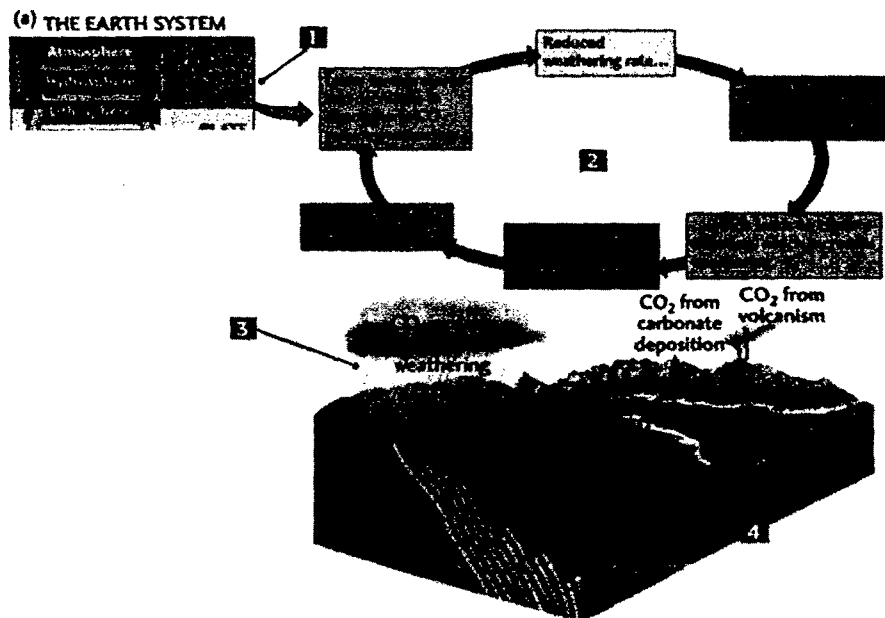
- a) What is geothermal gradient? (1)
- b) Instead of a sharp melting temperature why rocks have a zone of melting to melt completely? (3)
- c) What are the processes that will push the geothermal gradient such that it reaches the zone of melting and magma formation starts? Associate each of the melting process with its plate tectonic setting. (3+3)

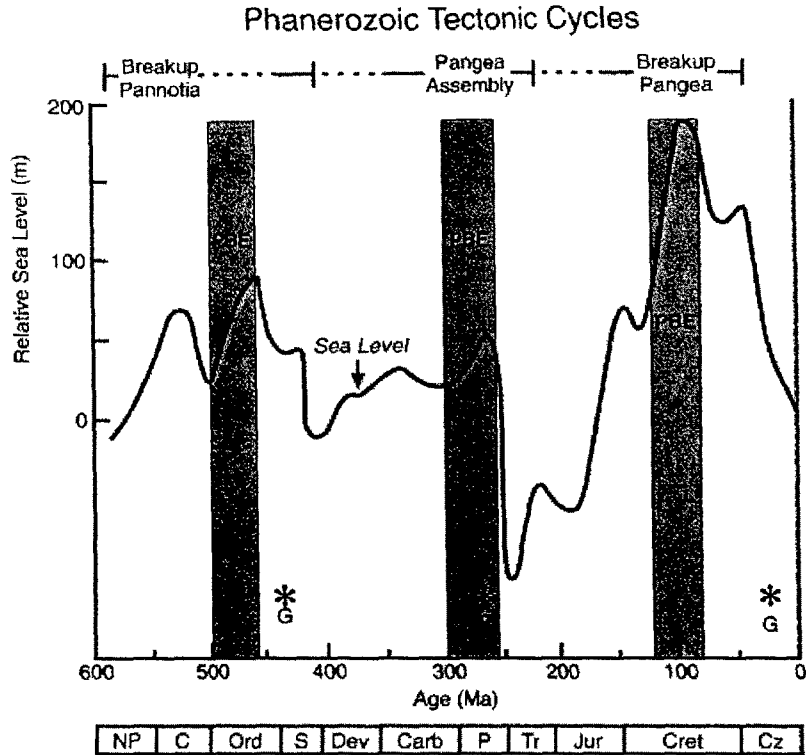
Q4. Recall Bowen's reaction series. Under microscope you observed a zoned plagioclase feldspar grain as given below. Spot analysis of the plagioclase feldspar grain along a profile (black dashed line 1mm in length) gives you the anorthite percent (calcic concentration) in the grain.





- a) Is this plagioclase feldspar grain an example of continuous or discontinuous reaction series? (1)
- b) Which part (periphery or core/centre) of the plagioclase feldspar grain forms at higher temperature? (1)
- c) Name one mineral each that forms at high temperature and at low temperature from magma? (1+1)
- d) The plagioclase enriched in calcium is called as anorthite. Name the plagioclase feldspar that is enriched in sodium. (1)
- e) How you will correlate atmospheric CO₂ budget with global temperature anomaly, chemical weathering and tectonic cycles through time. [Hint: Figures given below] (5)

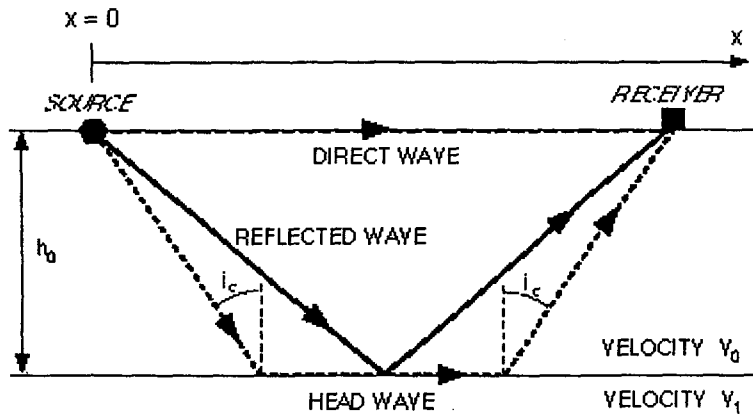




Q5. a) Contrast seismic P and S waves.

(1)

b) For a two layer seismic profiling you record arrival times of direct wave; reflected wave and head wave, as shown in the figure below. For direct wave the arrival time $t = \frac{x}{V_0}$.



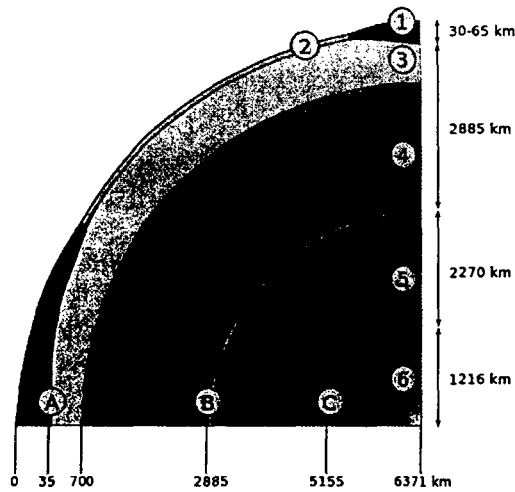
Now derive the following:

i) For reflected wave arrival time is $t = \frac{2}{V} \sqrt{d^2 + \frac{x^2}{4}}$; and

(2)

ii) For head wave it is $t = \frac{x}{V_1} + 2h_o \left[\frac{1}{V_0^2} - \frac{1}{V_1^2} \right]$ (4)

c) Internal structure of Earth is shown below. Identify A, B, C (3)



Q6.a) State the parameters used to describe the orientation of an inclined bed. Illustrate your answer? (5)

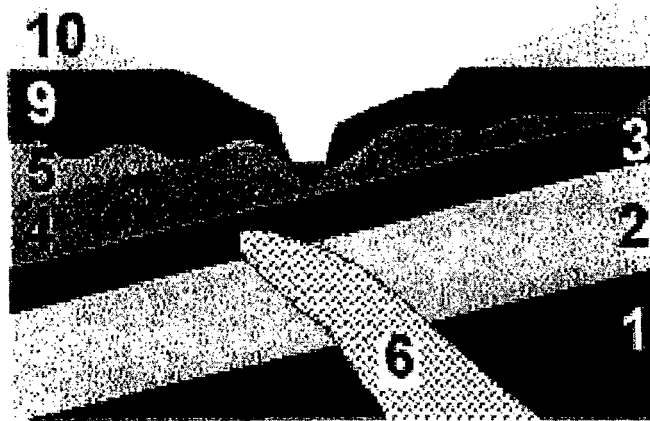
b) Describe parts of a fold in profile section with illustration? (5)

7. a) What are the different lithostratigraphic units? (2)

b) How you will define formation? (3)

c) Write a brief note on biostratigraphy. (2)

d) Name the different types of unconformities. (3)



8. a) Study the above figure and describe the stratigraphic succession? (5)
- b) Mention a name of a “system” in peninsular India that was deposited during Cambrian Period (1)
- c) Where do you expect Productus Beds and Syringothyris Limestone to be exposed? When they were deposited? (2)
- d) Are there any Mesozoic outcrops in coastal India? If yes, then what are the ages and in which parts of India? (2)
9. a) What is one of the great advantages for paleontologist over other branches of natural history? (1)
- b) What are the bases of the names of Phanerozoic geologic periods? (9)
10. a) What is the Cambrian explosion or Cambrian radiation? What are the factors? (7)
- b) The dinosaurs began their long evolutionary history almost ----- years ago. They became extinct about ----- years ago. Man as such appeared ----- years ago. (3)
- 11.a) What is termed as half-life in radiometric dating method? (2)
- b) Name four modern dating methods? Discuss in brief any two dating methods. (8)

12) Discuss with a schematic diagram about the nature of sediments in different depositional environments. (10)

13. a) What are the important factors in formation of the fossils ? (2)

b) Of what materials are hard parts composed? (3)

c) What are the major life modes of bivalves? (3)

d) Arrange the following sutural pattern with the advent of time.

Goniatitic, ammonitic, nautilitic and ceratitic. (2)

14. a) What are the different Raupian parameters? Explain with diagram. (5)

b) Write brief note on 'Corals as Geochronometer'. (5)

INDIAN STATISTICAL INSTITUTE

First Semester Examination : (2017-2018)

B. Stat. II Year

Physics I

Date: 01.12.17 Duration: 3 hrs Maximum Marks: 60

Group A: Classical Mechanics

Attempt as many questions. Total marks cannot exceed 30.

1. (a) Using variational principle find the equation for the shortest path between two fixed points in $x - y$ plane.
(b) Find Lagrange's equations of motion using variational principle for a system of N coordinates q_i , $i = 1, \dots, N$.

[3+2=5]

2. A particle of mass m is moving in a central potential $V(r) = -K/r$ where K is a constant.

- (a) Write down the Laplace-Runge-Lenz vector.
- (b) Show that the vector does not change with time.
- (c) Find the equation for the orbit.

[1+2+2=5]

3. Consider two variables $A(q_i, p_j)$, $B(q_i, p_j)$ where q_i, p_j are a set of N generalized coordinates and momenta.

- (a) Express the Poisson bracket $\{A(q_i, p_j), B(q_i, p_j)\}$ in terms of q_i, p_j Poisson brackets. If q_i, p_j obey canonical Poisson brackets, how will $\{A(q_i, p_j), B(q_i, p_j)\}$ appear?
- (b) Show that if $A(q_i, p_j)$ and $B(q_i, p_j)$ are two conserved quantities then

$$\{A(q_i, p_j), B(q_i, p_j)\}$$

is also a conserved quantity.

- (c) Assuming that the set q_1, q_2, p_1, p_2 satisfy canonical Poisson brackets, show that the following set of transformations to the set Q_1, Q_2, P_1, P_2 is a canonical transformation:

$$Q_1 = q_1; \quad Q_2 = p_2; \quad P_1 = p_1 - 2p_2; \quad P_2 = 2q_1 - q_2.$$

[2+3+5=10]

4. Consider two masses m and M moving in the central potential $V(r) = -\frac{GmM}{r}$.

- (a) Write Lagrangian for the system. Introduce coordinates such that the problem is reduced to a one body problem.
 (b) Define areal velocity and show that it is a conserved quantity.
 (c) From effective potential point of view discuss in a qualitative way the existence of unbounded and bounded orbits for different (constant) energy values.
 (d) Find the equation for the orbit for a given energy E and angular momentum l .

[3+2+4+1=10]

5. (a) Show that the transformations

$$q = \sqrt{\frac{2P}{m\omega}} \sin Q; \quad p = \sqrt{2m\omega P} \cos Q$$

are canonical if Q, P obey canonical Poisson brackets.

- (b) Express the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

in terms of Q, P variables and solve the Hamiltonian equations of motion.

[3+2=5]

Group B: Thermodynamics and Statistical Mechanics

Attempt as many questions. Total marks cannot exceed 30.

1. The pressure on 1 Kg of copper is increased reversibly and isothermally from 20 atm. to 1020 atm. at 10°C . Taking the density $\rho = 9 \times 10^3 \text{ Kg/m}^3$, volume expansivity $\beta = 5 \times 10^{-5} \text{ K}^{-1}$, and isothermal compressibility $\kappa = 8 \times 10^{-12} \text{ Pa}^{-1}$, calculate:

- (a) heat transfer during the process
- (b) the work done on or by the system
- (c) the change in internal energy of copper.

[3+3+1]

2. State and prove Liouville's Theorem in the context of ensemble theory. Apply it for the classification of stationary ensembles.

[5+2]

3. A single particle of mass m and energy $\epsilon \leq E$, where $E = p^2/2m$ is enclosed in a volume V .

- (a) Determine (asymptotically) the number of microstates available in energy range ϵ to $\epsilon + d\epsilon$.
- (b) If the energy varies from 0 to ∞ , obtain the partition function of the system.
- (c) If there are N such identical non-interacting particles in volume V , what is the total partition function of the system?

[3+3+1]

4. The expression for entropy S of an ideal gas having N particles, each of mass m , contained in a volume V at temperature T is $S = Nk \ln(V) + \frac{3}{2}Nk[1 + \ln(\frac{2\pi mkT}{h^2})]$, where k is the Boltzmann constant.

- (a) If two samples of the same ideal gas at same temperature T and same particle density (N/V) are mixed together, find the change in the entropy of the system before and after the mixing.
- (b) Explain the paradox or fallacy on physical grounds.
- (c) Modify the given expression for S in such a way that the process described in part (a) becomes explicitly reversible.
- (d) A system in a micro canonical ensemble is in a degenerate energy state with multiplicity 2. Evaluate its entropy?

[3+1+2+1]

5. Find the relative root-mean-square fluctuation in the number of particles N of a system in grand canonical ensemble. When is the fluctuation negligible? Mention a physical phenomenon when the fluctuation becomes large.

[5+1+1]

Indian Statistical Institute

First Semester Examination: 2017-18

Course: Mol. Biology, B. Stat 2nd year,

Date: 1/12/2017

Maximum Marks: 50;

Duration: 2.0 hrs

All questions carry equal marks, answer any five

1. In the table below, both genotypes and phenotypes for the pattern *baldness* (i.e. on the top of head) are shown: explain the phenomenon with the help of alleles, genotypes and biological environment. [10]

Genotype	Female phenotype	Male phenotype
BB	Baldness	Baldness
Bb	Normal hair	Baldness
bb	Normal hair	Normal hair

2. A cross between snapdragon trees having pure Red (genotype WW) and pure white (genotype ww) flowers were performed. Trees with pink color flowers (genotype Ww) were produced after F1 generation. These trees with pink color flower were self-fertilized to get F2 generation trees, and, as a result, 62 red, 131 pink color and 57 red colored trees were produced. Prove whether this observation follows Mendel's Law of segregation? ($\chi^2_{0.05} = 3.84, 5.99$ and 7.81 for 1, 2 and 3 degrees of freedom, respectively). [10]
3. A red colored flower tree (genotypes AA, bb) was crossed with a white color flower tree (genotypes aa, BB) and resultant purple colored flower trees (genotypes Aa, Bb) were self-fertilized to get (F2) trees having purple, red and white color flowers. What would be the proportion of trees with different flower colors after F2 generation? Assume, recessive a and b allele products are non-functional, epistasis between gene products for flower colors and independent assortment of genes from one generation to next. [10]

4. A man, who is color blind and possesses "O" blood group, has children with a woman, who has normal color vision and "AB" blood group. The woman's father had color blindness. X-linked and autosomal genes determine color blindness and blood group, respectively. (a) What are the genotypes of the man and the woman? (b) What proportion of the children will have color blindness and "B" type blood group? (c) What proportion of their children will have color blind and "O" type blood group? Give the answer drawing a pedigree. [4+3+3]

5. If both the parents of five children are carriers of sickle cell anemia, an autosomal recessive disease caused by a mutation in globin gene, then find out the chances that (a) all children will be normal (b) four children will be normal and one will be affected by disease (c) at least three children will be affected by disease and (d) the first child will be a normal girl. [2.5 x 4]

6. (a) Within a cycle of polymerase chain reaction (PCR), "initially the temperature of the reaction is raised to 95°C, then lowered down to 55°C and finally raised to 72°C": Answer with reasoning why the temperature changes are necessary for the reaction mentioned above. [7]

(b) Mention the effects of frameshift, nonsense and missense mutations in a gene on the fate of protein function synthesized from the gene. [3]

7. (a) Nucleotide compositions of three different viruses were found to be: (a) 35% A, 35% T, 15% G and 15% C; (b) 35% A, 15% T, 20% G and 30% C; (c) 35% A, 30% U, 30% G and 5% C. Mention the physical nature of the nucleic acid present in these viruses. [4]

(b) If a diploid cell with genotypes (A/a, B/b, D/d) in three different chromosomes, undergo mitotic and meiotic cell divisions, what will be the genotypes and allele compositions in the resultant diploid and haploid cells? [6]

INDIAN STATISTICAL INSTITUTE
Backpaper Examination, 1st Semester, 2017-18
Statistical Methods III, B.Stat 2nd Year

Date: 01/01/2018

Time: 3 hours

The paper carries 100 marks. Answer all questions.

1. Consider the regression model:

$$y_i = \beta x_i + e_i, \quad i = 1, 2, \dots, n$$

where x_i s are fixed and assume values 1, 0 or -1, while e_i s are random errors with mean 0 and the same variance. Explain whether it is possible that the least squares estimator of β is identical to the least absolute deviation estimator of β . [10]

2. Suppose a particular trait can be divided into four classes with probabilities $(\theta^2 + 2)/4$, $(1 - \theta^2)/4$, $\theta/4$ and $(1 - \theta)/4$. Based on a random sample of 100 observations, the frequencies of the four groups were obtained as 48, 27, 3 and 22, respectively. Explain how you would use the EM algorithm to estimate θ . Show all computational steps clearly with one cycle of iteration. [15]
3. Suppose X_1, X_2 and X_3 are three independent observations from the density $f(x) = \frac{1}{2\lambda} \exp\{-|x - \theta|/\lambda\}$; $-\infty < x < \infty$ where, θ and λ are unknown parameters. Examine whether the maximum likelihood estimators of θ and λ are unbiased. [20]
4. Suppose $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ represent the coordinates of a set of independent bivariate observations that are distributed uniformly in the region between two concentric circles with centre (0,0) and radii of the inner and the outer circles being r and r_0 , respectively, where r_0 is known. Denote the maximum likelihood estimator of r by \hat{r} .

- (a) Examine whether \hat{r} is consistent for r .
- (b) Consider estimators of the form $c\hat{r}$ to estimate r , where c is a real number. Which estimator in the above class has the minimum mean squared error? [15]
5. It is suspected that Vitamin B12 levels are lower in people consuming a vegetarian diet compared to those consuming a non-vegetarian diet. In a random sample of 8 people in the vegetarian group and 12 people in the non-vegetarian group, the Vitamin B12 levels were measured. Assuming that Vitamin B12 levels have a normal distribution in each of the two groups, the m.l.e.s of the mean and the standard deviation of Vitamin B12 levels in the vegetarian group were found to be 135.7 pmol/L and 12.6 pmol/L, respectively; while those in the non-vegetarian group were found to be 154.2 pmol/L and 13.4 pmol/L, respectively. Based on the above data, can you conclude at level 0.01 whether the suspicion is justified? State your assumptions clearly with suitable justification. [15]
6. Suppose X_1, X_2, \dots, X_n is a random sample from an exponential distribution with mean θ .
- (a) Consider a test for $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$ which rejects H_0 if and only if $\bar{X} > k$. If the size of the test is α , determine the value of k in terms of the quantiles of a chi-squares distribution. Draw a rough sketch of the power function of the above test. If n is large, show that the power of the above test at $\theta = \theta_1 (> \theta_0)$ can be approximated by $\Phi\left\{\frac{\sqrt{n}(\theta_1 - \theta_0) - \theta_0 z_\alpha}{\theta_1}\right\}$, where Φ and z_α are the c.d.f. and the $(1-\alpha)^{th}$ quantile, respectively, of a standard normal variate.
- (b) Suppose $n = 4$ and the observations are 2.8, 0.3, 0.9, 1.7. Obtain two 95% equal tail confidential intervals for θ , one based on the sample mean and the other based on the sample minimum.

[15 + 10]

Indian Statistical Institute

First Semester Examination: 2017-18

B. Stat. II Year

Microeconomics

Date: 01.12.2017

Max Marks: 60

Time: 3 hrs

Answer as many questions as you can. The maximum you can score is 60.

1. Suppose that Intel has a monopoly in the market for computer chips. In order to produce X computer chips, it costs Intel $C(X) = 2X^2$. The demand for computer chips is $X_D = 12 - 0.25P$.
 - a. Find the level of output that maximizes Intel's profits. What price is Intel charging?
 - b. Suppose the government knew the demand and production functions and it imposes a lump sum tax of 48 units. Find the price, output and profit.
 - c. If the government taxed Intel t for every unit of computer chips produced, what quantity would Intel choose as a function of t ? Find the choice of t that gives the government a tax revenue of 48. Find the output, price and profit. Interpret the results. [4+4+6]

2. Firm 1 and firm 2 are the only producers of spring water in the market. The market demand for spring water is given by $P = 70 - Q_1 - Q_2$. Firm 1 and firm 2 compete by choosing quantities Q_1 and Q_2 respectively. Each firm has a marginal cost of 10 and no fixed cost.
 - a. Suppose the two firms decide to collude and share the profit equally. Find the quantities and profits of the firms.
 - b. Suppose the government fixes the price at 10. Find the quantities and outputs of the firms.
 - c. Suppose the two firms choose quantities simultaneously. What are the equilibrium price, quantities, and profits of the two firms in this market?
 - d. Suppose only firm 1 has a chance to bribe the government and get the right to choose the quantity first. What is the maximum amount of money that firm 1 is willing to pay? If firm 1 gets to move first, what are the equilibrium quantities and profits of firm 1 and firm 2? [4+3+6+7]

P.T.O

3. Firms produce according to the production function

$$q = (K - 8)^{0.25} L^{0.25} \text{ where } K \text{ and } L \text{ are capital and labour.}$$

- (a) Assuming that the unit cost of capital (r) and the unit wage (w) are both equal to 1, derive the demands for inputs—capital and labor, respectively—as a function of output (q).
- (b) Find the long run total cost function.
- (c) What will be the price in the long run in this market? How much will each firm produce in this market in the long run.
- (d) How many firms will there be in this market in the long run? [4+3+4+3]

4. Suppose the demand function for corn is $Q_d = 10 - 2p$, and supply function is $Q_s = 3p - 5$. The government is concerned that the market equilibrium price of corn is too low and would like to implement a price support policy to protect the farmers. By implementing the price support policy, the government sets a support price of 4 and purchases the extra supply at this support price.

- (a) Calculate the original market equilibrium price and quantity in absence of the price support policy.
- (b) At the support price $p_s = 4$, find the quantity supplied by the farmers, find the quantity demanded by the market, and the quantity purchased by the government. [4+5]

5. Consider the following pay off matrix of a zero sum game.

Strategy I/II	L	C	R
U	5	8	4
M	-7	9	0
D	9	1	-2

- a. Compute the pure-strategy maximin payoff of player 1.
- b. What is the minimum expected payoff if Player I mixes between the strategies U and M with probability $\frac{1}{2}$ and $\frac{1}{2}$?
- c. What is the minimum expected payoff if Player I mixes all the strategies U, M and D with probability $\frac{1}{3}$ each?
- d. Show that no matter what mixed strategy Player I plays, Player II can hold I's expected payoffs at or below 4. [3+3+3+4]

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2017-18 (Backpaper)

B. Stat. II Year Analysis-III

Date: ~~26~~ / 12 / 2017 Maximum Marks: 100 Duration: 3 Hours

Note: Give proper justification to all your answers. State clearly all the results you are using.

- (1) Let S be an open subset of \mathbb{R}^n and assume that $f : S \rightarrow \mathbb{R}^m$ is differentiable on S . Let $x, y \in S$ be such that $L(x, y) := \{tx + (1 - t)y : 0 \leq t \leq 1\} \subseteq S$. Show that for every $a \in \mathbb{R}^m$, there exists $u \in L(x, y)$ such that

$$a \cdot [f(y) - f(x)] = a \cdot [f'(u)](y - x).$$

[10]

- (2) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by

$$f(x, y) = \begin{cases} \frac{xy^3}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Show that f is continuously differentiable and both $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ exist at $(0, 0)$ but are not equal.

[15]

- (3) Show that the system of equations

$$3x + y - z + u^2 = 0$$

$$x - y + 2z + u = 0$$

$$2x + 2y - 3z + 2u = 0$$

can be solved for x, y, u in terms of z ; for x, z, u in terms of y ; for y, z, u in terms of x ; but not for x, y, z in terms of u .

[15]

- (4) Find all the local maxima, local minima, and saddle points of the function

$$f(x, y) = x^4 + y^4 + 4xy.$$

[15]

- (5) Show that the mapping T defined by $T(x, y) = (y, x)$ cannot be written as a composition of two primitive mappings, in any neighbourhood of the origin.

[10]

- (6) Let I^2 be the unit square in \mathbb{R}^2 and let $\Phi : I^2 \rightarrow \mathbb{R}^4$ be the 2-surface in \mathbb{R}^4 defined by

$$\Phi(u_1, u_2) = (u_1, u_1 - u_2, 3 - u_1 + u_1 u_2, -3u_2).$$

Let $w = x_2 dx_1 \wedge dx_2 - x_4 dx_3 \wedge dx_4$ be a 2-form in \mathbb{R}^4 . Compute $\int_{\Phi} w$.

[15]

- (7) Suppose w and λ are exact k -forms. Is $w \wedge \lambda$ also exact? Justify your answer.

[10]

- (8) Let F be a vector field of class C^2 in an open set E of \mathbb{R}^3 and let Φ be a 2-surface of class C^2 in E . Prove that

$$\int_{\Phi} (\nabla \times F) \cdot n \, dA = \int_{\partial\Phi} (F \cdot t) \, ds.$$

[10]

INDIAN STATISTICAL INSTITUTE, KOLKATA
BACKPAPER EXAMINATION: FIRST SEMESTER 2017 -'18
B. STAT II YEAR

Subject : **Elements of algebraic structures**
Time : 3 hours
Maximum score (after scaling) : 100

Attempt all the problems. Justify every step in order to get full credit of your answers. All arguments should be clearly mentioned on the answerscript. Points will be deducted for missing or incomplete arguments.

- (1) For an ideal A in a commutative ring R , define $N(A) = \{x \in R : \exists n \in \mathbb{N} \text{ s.t. } x^n \in A\}$. Prove that
- (a) $N(A)$ is an ideal containing A ,
 - (b) $N(N(A)) = N(A)$.
 - (c) For the ring $R = \mathbb{Z}_n$, $n \in \mathbb{N}$, describe $N\{0\}$.
- [15 marks]
- (2) (a) In any principal ideal ring, gcd always exists.
(b) Show that $\mathbb{F}[x_1, x_2]$ is not a principal ideal ring for any field \mathbb{F} .
- [10 marks]
- (3) Let $\mathbb{K} = \mathbb{Q}(2^{\frac{1}{6}})$. Answer the following questions.
- (a) Find the degree $[K : \mathbb{Q}]$ with justification.
 - (b) Find the degree $[K : \mathbb{Q}(\sqrt{2})]$ with justification.
 - (c) Find the minimal polynomial of $2^{\frac{1}{6}}$ over $\mathbb{Q}(\sqrt{2})$.
- [15 marks]
- (4) Compute the degree of the splitting field of $X^4 + 4$ over \mathbb{Q} .
- [10 marks]
- (5) Let G be a group and H, K be two normal subgroups of G such that $H \cap K = \{1\}$. Prove that HK is a subgroup of G and $HK \simeq H \oplus K$.
- [15 marks]
- (6) Let M_n denote the ring of $n \times n$ matrices with entries from a field \mathbb{F} .
- (a) What are the two-sided ideals in M_n ?
 - (b) For a subspace V of \mathbb{F}^n , define $J_V = \{A \in M_n : \text{rows of } A \text{ belong to } V\}$. Show that J_V is a left ideal of M_n , and conversely any left ideal appears in this way.
- [20 marks]
- (7) Let G be a finite group and H be a subgroup of G such that $|H| = p^n$ for some prime p and $n \geq 1$. Prove that $[N(H) : H] \equiv [G : H] \pmod{p}$.
- [20 marks]

INDIAN STATISTICAL INSTITUTE

BACK PAPER

First Semester Examination : (2017-2018)

B. Stat. II Year

Physics I

Date: 28.12.17 Duration: 3 hrs Maximum Marks: 100

Group A: Classical Mechanics

1 Consider a particle of mass m moving in a potential $V(r) = \frac{K}{r}$ in three dimensions where r is the distance of m from the centre and k is a constant.

- (a) Write down the Lagrangian.
- (b) Derive the equations of motion in spherical polar coordinate r, θ, ϕ .
- (c) Find the cyclic coordinate.

[5+5+2=12]

2 Consider the Lagrangian

$$L = \frac{1}{2}m_1\dot{q}_1^2 + \frac{1}{2}m_2\dot{q}_2^2 + Kq_1q_2 + Gq_2^2.$$

m_1, m_2, K, G are constants and $\dot{q} = (dq)/(dt)$.

- (a) Write down the equations of motion for q_1 and q_2 .
- (b) Find the momenta p using $p_1 = \frac{\partial L}{\partial \dot{q}_1}$, $p_2 = \frac{\partial L}{\partial \dot{q}_2}$.
- (c) Derive the Hamiltonian using $H = p_1\dot{q}_1 + p_2\dot{q}_2 - L$.

[4+4+4=12]

3 Using Poisson brackets

$$\{q, p\} = 1, \quad \{q, q\} = \{p, p\} = 0$$

calculate the following brackets:

- (a) $\{q, p^3\}$,
- (b) $\{\frac{1}{4}q^4, p\}$,
- (c) $\{q^3, q^5\}$.

[4+4+4=12]

- 4 Consider the Hamiltonian $H = p_1^2 + p_2^3 + \alpha q_1 p_1 + \beta q_2 p_2$, where α and β are constants. Using canonical Poisson brackets

$$\{q_i, p_j\} = \delta_{ij}, \quad \{q_i, q_j\} = \{p_i, p_j\} = 0$$

and using $\frac{\partial A}{\partial t} = \{A, H\}$ where A is any function of q_i, p_j find time derivative of A for

- (a) $A = q_1$
- (b) $A = p_2$
- (c) $A = p_1$
- (d) $A = q_1 q_2$.

[3+3+3+5=14]

Group B: Thermodynamics and Statistical Mechanics

- 1(a) State and explain the first law of Thermodynamics.
- 1(b) The molar internal energy of a monoatomic gas obeying van der Waal's equation is given by $\frac{3}{2}RT - \frac{a}{V}$, where a is a constant. If the gas is adiabatically allowed to expand freely into vacuum from volume V_1 to V_2 and T_1 is its initial temperature, what is the final temperature T_2 of the gas? What would be the relation between T_1 and T_2 if instead, an ideal gas was considered? [4+(4+2)]
- 2(a) A Carnot engine absorbs 200J of heat from a reservoir at 80C and rejects some heat to another reservoir at 10C. Calculate:
- (i) the amount of heat rejected
 - ii) the work done by the engine
 - (iii) the thermal efficiency
- 2(b) 100g of water is heated from 20C to 90C by placing it in thermal contact with a heat reservoir at 90C.
- (i) Is the process reversible or irreversible? Why?
 - ii) Calculate the change in entropy of the water (Given that $C_p = 4200JK/Kg$). [(2+2+1)+(1+1+3)]
- 3(a) Distinguish between macrostate and microstate of a thermodynamic system. Write down explicitly the macrostates and their corresponding microstates when a pair of dice is rolled. What is the probability of getting the macrostate 4?
- 3(b) Consider a free particle of mass m in volume V having momentum $p \leq P$, where P is some fixed value. Determine (asymptotically) the number of microstates lying between p and $p + dp$. [(2+3+1)+4]
- 4(a) Define ensemble average of a physical quantity. State the condition under which the ensemble becomes stationary?
- 4(b) Consider a system in contact with a heat reservoir at temperature T , in a canonical ensemble. Derive an expression for the probability P_r , that at any time t , the system is found to be in one of the states characterized by the energy value E_r . Does P_r depend on the physical nature of the reservoir? [(2+2)+(5+1)]
- 5(a) Consider a box with a partition in the middle. Argon gas at temperature T is filled in both the compartments. Let the number density of the two samples be same. Now the partition is removed. Is the entropy after mixing the same as before

mixing? Give reason for your answer. If now the partition is inserted back, do we get the same sample of the gas in each compartment that we started with? Why? What will happen if the number density of the two samples of Argon is not same?

- 5(b) Consider a one-dimensional harmonic oscillator in a canonical ensemble. Obtain an expression for the partition function. What would be the partition function if there are N such oscillators? $[(1+2+1+1+1)+(3+1)]$

Indian Statistical Institute

First Semester Back Paper Examination: 2017-18

B. Stat. II Year

Microeconomics

Date: 29.12.17

Max Marks: 100

Time: 3 hrs

1. The country of Economia has two industries. In the Clothing industry, the marginal product of labor is always 1. In the Steel industry, the marginal product of labor is $12L_S^{-1/2} - 2$, where L_S is the total number of workers employed in the Steel sector. The total supply of labor in Economia is fixed at $L_C + L_S = 25$, and the output price is 1 for both Clothing and Steel.
 - a. Suppose that the labor market is perfectly competitive. How many workers will be employed in the Clothing sector, and how many in the Steel sector? What wage rate will workers in each sector receive?
 - b. Suppose that workers in the Steel sector form a union, which acts as a monopolist in supplying labor to the Steel industry. The union chooses a level of employment that maximizes the total wages of its members. How many workers will the union allow to be employed in the Steel sector? How many will now be employed in the Clothing sector? What wage rate will workers in each sector receive?
 - c. If workers in Clothing had unionized in order to increase their wage rate instead of workers in Steel, what would have happened to employment and wages in each sector? Explain why.

[10+12+8]

2. Consider the market for apple juice. In this market, the supply curve is given by $Q_S = 10P_J - 5P_A$ and the demand curve is given by $Q_D = 100 - 15P_J + 10P_T$, where J denotes apple juice, A denotes apples, and T denotes tea.

- (a) Assume that P_A is fixed at 1 and $P_T = 5$. Calculate the equilibrium price and quantity in the apple juice market.

P.T.O

- (b) Suppose that a poor harvest season raises the price of apples to $P_A = 2$. Find the new equilibrium price and quantity of apple juice. Draw a graph to illustrate your answer.
- (c) Suppose $P_A = 1$ but the price of tea drops to $P_T = 3$. Find the new equilibrium price and quantity of apple juice.
- (d) Suppose $P_A = 1$, $P_T = 5$, and there is a price ceiling on apple juice of $P^* = 5$. What is the excess demand for apple juice as a result? Draw a graph to illustrate your answer.

[8+12+8+12]

3. You run a cost-minimizing firm with production function $f(L,K) = [\min\{L,K\}]^{1/3}$, L is labor and K is capital. Assume that you are a price-taker in the input markets: you pay w for each unit of labor you hire and r for each unit of capital (where w and r are set exogenously); and face no costs other than those from labor and capital.
- (a) Assuming that you can freely choose both labor and capital (i.e., the “long run problem”), derive expressions for your cost-minimizing conditional input demands, $L^*(r,w,Q)$ and $K^*(r,w,Q)$.
- (b) Write down an expression for your total cost function $TC(r,w,Q)$.
- (c) What will happen to your conditional demand for labor if there is an increase in the wage rate, assuming that r and Q remain the same?

[12+8+10]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : Semester II (2017-2018)

B. Stat 2nd Year

Stochastic Processes

Date: 19. 2. 18

Maximum marks: 40

Time: 2 hours.

Note: Answer all questions. Maximum you can score is 40.

1. Consider a discrete time Markov Chain with transition matrix P .

Prove that $P_{ii}^{m+n} \leq 1 + P_{ii}^m * P_{ii}^n - \max(P_{ii}^m, P_{ii}^n)$. [6]

2. Consider a discrete time Markov Chain with state space $\mathcal{S} = \{0, 1, 2, 3, 4, 5\}$, and the transition matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

- (a) Determine which states are recurrent and which states are transient. [3]
- (b) Find the absorption probabilities to different classes from each of the transient states. [6]
- (c) Find the mean time spent in each of the transient states. [4]
3. In a queuing chain, suppose the expected number of arrivals in unit time is less than 1.
- (a) Show that if the chain is irreducible, all states are recurrent. [10]
- (b) If the chain is not irreducible, find the states that are recurrent. [5]
4. Suppose in a Branching process, the initial population has 10 individuals. If each individual produces 0, 1 or 2 offsprings with respective probabilities 0.25, 0.25 and .5, independently of each other, find the probability that the population will die out. [10]

INDIAN STATISTICAL INSTITUTE

Statistical Methods IV

Mid Semester Examination

B II, Semester II, 2017-18

Date: 20.02.2018

Time: 2 hours

(Maximum you can score is 80)

1. Let X_1, X_2, \dots, X_n be i.i.d. Bernoulli(p) random variables.
 - (a) Describe the usual large sample test for testing the null hypothesis $H_0 : p = 0.5$ against the two sided alternative.
 - (b) For the same data and the same hypothesis, derive the likelihood ratio test. Is it equivalent to the test in part (a)?

[4+12=16]

2. Suppose X_1, X_2, \dots, X_n are independent, and $X_i \sim N(i\theta, \sigma^2)$, $i = 1, 2, \dots, n$. Let

$$k = \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad U = \sum_{i=1}^n \frac{iX_i}{k}, \quad \text{and} \quad V^2 = \frac{\sum_{i=1}^n (X_i - iU)^2}{n-1}.$$

- (a) Express U as a linear combination of X , and V as a quadratic form in X , where

$$X = (X_1, X_2, \dots, X_n)^T.$$

- (b) Show that $U \sim N(\theta, \sigma^2/k)$.
- (c) Show that $(n-1)V^2/\sigma^2$ has a central chi-square distribution with degrees of freedom $(n-1)$.
- (d) Show that U and V^2 are independent. [Use the result for independence between a linear combination of X and a quadratic form in X .]

[2+4+7+7=20]

3. An experimenter selected 86 individuals each from the population of lung cancer patients and healthy (control) individuals respectively. Each individual was further cross classified on the basis of whether he/she was a smoker or not. The observed layout is as follows. The aim is to determine whether there is a a relation between smoking and lunch cancer.

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination

B. Stat. - II Year (Semester - II)

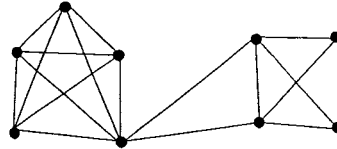
Discrete Mathematics

Date : 21/02/2018 Maximum Marks : 60

Duration : 2:00 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. Give an example of a Δ -free graph G such that $\chi(G) = 4$. [6]
2. Give an example of a graph having 8 vertices and 11 edges to justify that $R(3, 4) > 8$. [8]
3. Find intersection number of the following graph: [6]



4. Find all 3-regular graphs on 10 vertices having no Δ or C_4 as a subgraph. [8]
5. Suppose that d_1, d_2, \dots, d_n are integers with $d_1 \geq d_2 \geq \dots \geq d_n \geq 0$. Prove that, there exist a loopless graph (that is does not contain self loop edge), which may have multiple edges, with degree sequence d_1, d_2, \dots, d_n if and only if $\sum d_i$ is even and $d_1 \leq d_2 + d_3 + \dots + d_n$. [15]
6. Find all diameter 2 Δ -free graphs with minimum degree less than or equal to 2. Note that the diameter of a graph is the maximum eccentricity of any vertex in the graph. That is, it is the greatest distance between any pair of vertices. [12]
7. A secrete society has 7 members. They want to hold a series of meetings such that each meeting must have exactly 3 members and each pair of members must meet exactly once. Is this achievable? Justify. [10]
8. An independent set is a set of vertices in a graph, no two of which are adjacent. Consider a linear order π of the vertices in V . We consider an independent set I of any graph $G(V, E)$ depending on π as follows. Define $I = \{v \in V | u \in N(v) \text{ does not precede } v \in \pi\}$. Show that there exist a linear order such that $|I| \geq \sum_{v \in V} \frac{1}{d(v)+1}$. Here $d(v)$ denotes the degree of vertex v . [10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2017 - 18

Course Name : B.STAT. II & III Year

Subject : Differential Equations

Date: 22. 02. 2018 Maximum Marks: 45 Duration: 3 hrs.

Any result that you use should be stated clearly.

1. Find the general solution of the following equations:

a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 4x^2 + 6e^x$. [4]

b) $2x^2\frac{d^2y}{dx^2} + 10x\frac{dy}{dx} + 8y = 0$. [4]

c) $xy^2\frac{dy}{dx} + y^3 = x\cos x$. [4]

d) $\frac{dy}{dx} + 2y - 3z = x$, $\frac{dz}{dx} - 3y + 2z = e^{2x}$. [5]

2. (a) Prove that $\phi(x)$ is a solution of initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ on I iff it is a continuous solution of the integral equation

$$\phi(x) = y_0 + \int_{x_0}^x f(t, \phi(t))dt$$

on I .

(b) Consider the initial value problem $\frac{dy}{dx} = -\sqrt{|1 - y^2|}$, $y(0) = 1$. Does it admit a unique solution in the neighbourhood of 1? Justify your answer and find the solution(s). [4+(2+2)]

3. Deduce Euler's differential equation for extremization (stationarity) of the integral

$$I = \int_{x_1}^{x_2} f\left(x, y, \frac{dy}{dx}\right)dx,$$

using the method of variational calculus. [6]

4. Find the general solution of

$$(x - y)\frac{\partial z}{\partial x} + (x + y)\frac{\partial z}{\partial y} = 2xz$$

OR

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = 0.$$

[5]

P.T.O.

5. A pot of soup cooling in air at 0°C was initially boiling at 100°C and cooled 20° degree during the first 30 minutes. How much will it cool during the next 30 minutes? [4]
6. A tank contains 100 gallons of brine in which 40 pounds of salt are dissolved. It is desired to reduce the concentration of salt to 0.1 pounds per gallon by pouring in pure water at the rate of 5 gallons per minute and allowing the mixture (which is kept uniform by stirring) to flow out at the same rate. How long will this take? [5]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination 2017 – 2018

B. Stat. II Year

AGRICULTURAL SCIENCE

Date : 23.2.2018 Maximum Marks : 40 Duration : Three hours

(Attempt all the questions)

(Number of copies of the question paper required : 20)

1. a) Define the term 'regolith'.
b) What is the basic difference between prismatic and columnar structure? How soil structure can influence on plant growth?
c) Why Ca, Mg and S are treated as secondary nutrients?
d) Give the definition of micronutrients. What are the anionic micronutrients found in soil?
e) Mention the different forms of nitrogen present in soil.
f) When phosphorous in soil is fixed?
g) How available potassium ion concentration in soil varies?
1+3+1+1+1+1+2
2. a) Why bulk density values are lower than particle density values?
b) Calculate the percentage of pore space in a soil with a bulk density of 1.4 Mg/m³ and particle density of 2.6 Mg/m³.
c) An oven dry clod of soil weighing 132.5 g is saturated with water. If the weight of saturated clod is 172.5 g and particle density of the soil is 2.65 Mg/m³. Compute the bulk density and porosity of the clod.
d) Calculate pH of 0.001 M NaOH solution.
e) Why gravitational water is not available to plant?
f) Define 'hygroscopic coefficient'.
1+2+2+2+2+1
3. Write down the difference between:
a) Soil texture and soil structure b) Physical weathering and chemical weathering
c) Fertilizer and Manure d) Straight fertilizer and complex fertilizer
e) Mass flow and diffusion
2X5
4. Why Phosphorus is essential to a plant?
A farmer is willing to grow Potato in 1.5 hectares of land. The recommended doses of nutrients are 200 kg N + 120 kg P + 100 kg K. The 50% of the required N is to be given through Vermicompost (VC). Calculate the amount of VC, Urea, Single super phosphate and Murite of Potash are needed for the Potato crop.
3+7

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2017-18

Course Name : B. STAT. II Year (Second Semester)

Subject Name : Physics II

Date : 23/02/2017

Maximum Marks : 40

Duration : 3 hours

Use separate answer sheet for each group

Group A

Maximum Marks : 20

1. Find the Laplacian of the following functions: $\mathbf{v} = x^2\hat{\mathbf{i}} + 3xz^2\hat{\mathbf{j}} - 2xz\hat{\mathbf{k}}$
[2]
2. Let $\mathbf{F}_1 = x^2\hat{\mathbf{i}}$ and $\mathbf{F}_2 = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$. Explain how these vectors may be written as the gradient of a scalar or the curl of a vector.
[2+2]
3. Find the electric field inside a sphere which carries a charge density proportional to the distance from the origin, $\rho = k r$, for some constant k . Express your answer in terms of the total charge and radius of the sphere.
[4]
4. Find the potential a distance s from an infinitely long straight wire that carries a uniform line charge density λ . Compute the gradient of your potential and check that it yields the correct field.
[3+2]
5. A metal sphere of radius a carries a charge Q . It is surrounded, out to radius b , by linear dielectric material of permittivity ϵ . Find the electric field for the regions $a < r < b$ and $r > b$. Finally, determine the bound charge explicitly.
[3+2]
6. Suppose there are two long parallel wires a distance d apart, carrying currents I_1 and I_2 . Using the Lorentz force law find the force of attraction per unit length between them.
[5]

Group B

Maximum Marks : 20

1. Answer the following questions: $[2 + 2 + 2 + (1 \times 4)]$

Solve any two from (a), (b) and (c); (d) is compulsory.

a. Prof. Shanku proposed a special force between two particles located at \vec{r}_1 and \vec{r}_2 in a specific inertial reference frame \mathcal{A} is given by $F_{12} = C|\vec{r}_1 + \vec{r}_2|$, where C is constant. Is it invariant under spatial translation and rotation? Is it invariant under Galilean transformation?

b. According to Prof. Calculus, a certain turbulence created at \vec{r}_1 at time t_1 affects the air pressure at r_2 at time t_2 as $B \sin(|\vec{r}_1 - \vec{r}_2|^2 - c^2(t_2 - t_1)^2)$, where B is constant and c , as usual, is the speed of light in vacuum. Is this expression invariant under spatial translation? Is it invariant under Lorentz transformation?

c. An unstable high-energy particle enters a detector and leaves a track 1.05 mm long before it decays. Its speed relative to the detector was $0.99c$. How long would it have lasted before decay had it been at rest with respect to the detector? In other words, what is its proper lifetime?

d. True or False :

i. The speed of light in vacuum, according to a postulate of special relativity, only depends on the motion of the source.

ii. No experiment inside an isolated sealed lab in space can determine its velocity.

iii. No experiment inside an isolated sealed lab in space can determine its acceleration.

iv. A pingpong ball moving at a speed of $0.98c$ looks spherical.

2. Ravi, in his inertial frame \mathcal{A} , observes two light flashes (events) e_A and e_B at (t_A, \vec{r}_A) and (t_B, \vec{r}_B) respectively. Assume that $\vec{r}_A = x_A \epsilon_1$ and $\vec{r}_B = x_B \epsilon_1$.

i. What are the conditions on the respective co-ordinates which ensure

that event e_A precedes event e_B in every inertial reference frame?

ii. Another observer inertial frame \mathcal{A}' is moving with a velocity $\vec{v} = v\epsilon_1$ (along x_1) with respect to frame \mathcal{A} . Under what conditions would these two events appear simultaneous in \mathcal{A}' ?

iii. With similar assumptions, under what conditions would the two events occur at the same point in space in frame \mathcal{A}' ? [1 + 2 + 2]

3. A stick of *proper* length ℓ_0 is at rest in an inertial frame \mathcal{A} . It makes an angle θ with the x_1 axis in the $x_1 - x_2$ plane. An observer in another inertial frame \mathcal{A}' , which is moving with a velocity $\vec{v} = v\epsilon_1$, observes that the stick makes an angle φ with respect to the x'_1 axis. Let us assume that x_1 and x'_1 axes are aligned.

i. Estimate the angle φ .

ii. Determine the length of the stick ℓ' as measured by the moving observer.

iii. Suppose a laser torch is attached to the bottom end of the stick, which shines a light beam along the stick in \mathcal{A} . Would the observer in \mathcal{A}' see the light beam aligned? Explain (very) briefly. [2 + 1 + 2] *Hint: How does the velocity in the x_2 direction transform when one goes from frame \mathcal{A} to frame \mathcal{A}' or vice versa?*

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2017-18

Course Name: BStat Second Year

Subject Name: Economics II (Macroeconomics)

Date: 23.2.2018

Maximum Marks: 40

Duration: 2 Hours 15 Minutes

Answer all the following questions

1. Consider the following information regarding a firm in the domestic economy in a given period

Revenue earned by the firm	Rs.80000
Unsold output of the firm	2000
Raw materials purchased from other firms	30500
Unused portion of the raw materials purchased	2500
Wages and salaries paid to households of which	
Rs.1000 is paid to foreign technical experts for their advice	22000
Rent paid to another firm for hiring the office premises	500
Dividend paid to households of which 1/5 th goes to foreigners	10,000
Net indirect taxes paid	1500
Depreciation	3300

What is the firm's contribution to GDP and NI? Does the firm make any contribution to the final expenditure? [20]

2. (i) Is it possible for an economy to absorb more than what its purchasing power can command of the world NDP? Explain your answer. Also discuss the financial aspects.
- (ii) Suppose in an economy in a given year the central bank had to sell foreign exchange worth Rs. 20,000 crore from its stock to hold the exchange rate at the target level. Some firms in the domestic economy borrowed from foreign financial institutions Rs. 30,000 crore, while some foreign firms borrowed Rs.12,000 crore from domestic financial institutions. In addition, foreigners purchased shares of domestic companies worth Rs.8000 crore. Domestic residents purchased land abroad worth Rs.200 crore. Domestic government's budget deficit was Rs.500 crore. Domestic households' expenditure on produced goods and services was Rs.90,000 crore of which Rs.10,000 was spent on buying houses from construction companies. Firms' investment in the domestic economy was Rs.60000 crore and depreciation of the capital stock of the economy was Rs.500 crore. From the data given above, compute the economy's private disposable income in the given period. [6+14]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination : (2017-2018)

B. Stat 2nd Year

Introduction to Stochastic Processes

Date: 25. 4. 18

Maximum marks: 100

Time: 3 hours.

Note: This paper carries 110 marks. Answer as much as you can. Maximum you can score is 100.

You may use any result proved in class after clearly stating the result.

1. Consider an irreducible birth and death chain on the non-negative integers.
 - (a) Find a necessary and sufficient condition for the chain to be transient. [10]
 - (b) Find a necessary and sufficient condition for the chain to be positive recurrent. [12]
 - (c) Find a necessary and sufficient condition for the chain to be null recurrent. [2]
2. (a) The transition matrix of a Markov Chain is said to be doubly stochastic if the sum of the elements in each column is 1. For an irreducible and aperiodic chain having a doubly stochastic stationary transition matrix, find its stationary distribution. [6]
(b) Let Y_n be the sum of the outcomes of n independent rolls of a fair die. Using part (a) or otherwise, find

$$\lim_{n \rightarrow \infty} P(Y_n \text{ is a multiple of } 13).$$

[9]

3. Let, π be a stationary distribution of a Markov Chain with stationary transition matrix P .
 - (a) Show that if $\pi_i > 0$ and i leads to j , $\pi_j > 0$. [5]
 - (b) Suppose, i and j are two states such that $P_{ki} = cP_{kj}$, for all $k \in S$. Show that, $\pi_i = c\pi_j$. [5]
 - (c) Verify that for each subset A of S , $A \neq S$,

$$\sum_{j \in A} \pi_j \sum_{k \notin A} P_{jk} = \sum_{i \notin A} \pi_i \sum_{j \in A} P_{ij}.$$

[10]

4. A total of m white and m black balls are distributed among two urns with each urn containing m balls. At each stage, a ball is randomly selected from each urn, and the two selected balls are interchanged. Let X_n denote the number of black balls in urn 1 after the n th interchange.
 - (a) Derive the transition probabilities of the Markov Chain $X_n, n \geq 0$. [6]
 - (b) Find the limiting probabilities. Check whether the stationary chain is time reversible. [10+2]

[P.T.O]

5. (a) An insurance company pays out claims on its life insurance policies in accordance with a Poisson process having rate $\lambda = 5$ per week. If the amount of money paid on different policies are independent and exponentially distributed with mean \$2000, find the mean and variance of the amount of money paid by the insurance company in a four-week span? [8]
- (b) A store opens at 8 AM. From 8 am to 12 pm, customers arrive according to a Poisson process with a rate of 8 per hour. From 12 pm to 2 pm, arrival rate increases steadily at a linear rate from 8 per hour at 12 pm to 10 per hour at 2 pm, and from 2 pm to 5 pm, the arrival rate drops steadily at a linear rate from 10 per hour at 2 pm to 4 per hour at 5 pm.
- (i) Find the expected number of arrivals between 12:30 pm to 1:30 pm on a particular day.
- (ii) Find the probability that no customer arrives between 4 pm to 5 pm on a particular day. [5+5]
6. For a Yule process, derive the transition probability function $P_{ij}(t)$. You may use any result for pure birth process done in class after clearly stating the result. [15]

INDIAN STATISTICAL INSTITUTE

Statistical Methods IV

Semestral Examination

B II, Semester II, 2017-18

Date: 27.04.2018

Time: 3 hours

(Total points 111. Maximum you can score is 100)

1. (a) Let $Y \sim N_p(\mu, I)$, where μ is a p -dimensional vector and I is the $p \times p$ identity matrix. Given $p \times p$ idempotent matrices A and B , show that $AB = 0$ is a sufficient condition for the quadratic forms $X^T A X$ and $X^T B X$ to be independent.
- (b) Consider the one way ANOVA problem with k groups and n observations per group. Show that the *between groups sum of squares* and the *sum of squares due to error* are independent.

[12+12=24]

2. (a) Consider a one way ANOVA model with k groups and n observations per group. Assume that the usual conditions of the ANOVA model (normality, common variance, independence, etc.) hold, and suppose that $\mu_1, \mu_2, \dots, \mu_k$ are the theoretical group means. We want to test, at a suitable level α , the null hypothesis $H_0 : \mu_1 = \mu_2 = \dots = \mu_k$, against the alternative that the means are not all equal. Consider the likelihood ratio test for the above hypothesis. Derive the test, and show that it is equivalent to the usual analysis of variance F test.
- (b) Soyabeans were planted in four different blocks and under four different treatments (Arasan, Spergon, Semesan and Fermate). Out of a common, large number of plantings, the experimenter recorded the number of plantings which failed to sprout in each (block, treatment) combination. The results are given in the following table.

	Arasan	Spergon	Semesan	Fermate
Block 1	2	4	3	9
Block 2	6	10	5	7
Block 3	7	9	9	5
Block 4	11	8	10	5

- i. Write down a two way ANOVA model to describe the data with all the assumptions.
- ii. Test, at a suitable level α which you may choose, whether there is a significant treatment effect. Clearly state your hypotheses.
- iii. Test, at the same level as in the previous part, whether there is a significant block effect. Clearly state your hypotheses.

[12+(3+6+6)=27]

3. (a) Explain how, and under what conditions, the delete-1 jackknife is successful in reducing the bias of the estimator of the target parameter.
- (b) Suppose that X_1, X_2, \dots, X_n be a random sample from the distribution F . Suppose that the median functional is defined as $T(F) = F^{-1}(1/2)$. Then

$$T(F_n) = F_n^{-1}(1/2) = X_{(\lfloor n+1 \rfloor / 2)}.$$

where F_n is the empirical distribution function based on (X_1, X_2, \dots, X_n) . Let $(X_1^*, X_2^*, \dots, X_n^*)$ be a bootstrap resample drawn from the original sample, and let F_n^* be the empirical distribution distribution based on this resample, and $T(F_n^*)$ is the m -th order statistic of the bootstrap resample, where $m = (\lfloor n + 1 \rfloor / 2)$. Show that, conditional on the original sample

$$P(T(F_n^*) = X_{(k)}) = \sum_{j=0}^{m-1} \left\{ \binom{n}{j} \left(\frac{k-1}{n} \right)^j \left(1 - \frac{k-1}{n} \right)^{n-j} - \binom{n}{j} \left(\frac{k}{n} \right)^j \left(1 - \frac{k}{n} \right)^{n-j} \right\},$$

for $k = 1, 2, \dots, n$, where $X_{(k)}$ is the order statistic of the original sample.

[10+12=22]

4. Under appropriate assumptions, find the seasonally adjusted values for the following time series using an appropriate method. (That is, find the values of the series after the seasonal component have been removed).

Year	Quarter	Series
1	I	28.4
	II	33.8
	III	37.7
	IV	32.6
2	I	32.7
	II	37.0
	III	41.3
	IV	36.5
3	I	36.0
	II	41.3
	III	45.1
	IV	40.9
4	I	40.6
	II	33.2
	III	37.4
	IV	32.2

[20]

5. Suppose we have a sample X_1, \dots, X_n , from the family of distributions on the real line with density $f_\theta(x) = c(\alpha)e^{-|x-\theta|^\alpha}$, $\alpha > 0$. We may use the sample mean \bar{X}_n , or the sample median m_n to estimate the location parameter θ .

(a) Find the constant $c(\alpha)$.

(b) For what values of α , if any, is the asymptotic variance of m_n smaller than the asymptotic variance of \bar{X}_n ?

[6+12=18]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

B. Stat. - II Year (Semester - II)

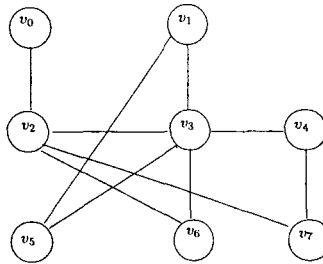
Discrete Mathematics

Date : 02.05.2018 Maximum Marks : 75

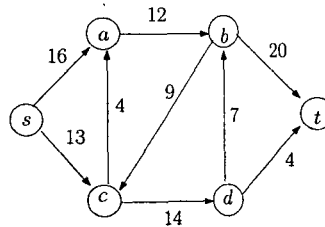
Duration : 3:30 Hours

Note : You may answer any part of any question, but maximum you can score is 75.

1. Draw a depth first search tree and breadth first search tree of the following graph considering v_0 as root. [5+5=10]



2. Compute maximum flow between s and t of the following network. [10]



3. Let $G(V, E)$ be a graph with $V = \{u_1, u_2, \dots, u_n\}$. The number of ways to colour vertices of G with colours $\{1, 2, \dots, k\}$ is denoted by $P_G(k)$. For graph $G(V, E)$, with $|E| = 0$, $P_G(k) = k^n$.

- (a) Show that for the complete graph K_n , $P_{K_n}(k) = k(k-1) \dots (k-n+1)$.
(b) For all trees T having n vertices, does all $P_T(k)$ have the same value? If so write down its value, otherwise give an example of two trees of same size but different $P_T(k)$ values. [4+(4+6)=14]

4. A graph is called outerplanar if it has a drawing in which every vertex lies on the boundary of the outer face. Using 4 colour theorem for planar graph or otherwise, prove that any outerplanar graph is 3-colorable. [10]

P.T.O

5. Show that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G . [15]
6. A vertex of graph G is simplicial if its neighbourhood in G induces a clique. A simplicial elimination ordering is an order v_n, v_{n-1}, \dots, v_1 such that v_i is a simplicial vertex of the graph induced by v_1, v_2, \dots, v_i for each i .
- (a) Construct a connected graph $G(V, E)$ which has a perfect elimination ordering, where $|V| > 4$.
- (b) If possible, construct a connected graph $G(V, E)$ that has a perfect elimination ordering containing an induced chordless cycle of size 5.
- (c) For non-adjacent vertices a and b , $S \subseteq V \setminus \{a, b\}$ is a separator of a, b if induced graph of $V - S$ partition into components where a and b are in different components. If no subset of S is a separator of a, b , then it is minimal. If G does not have an induced chordless cycle of size greater than equal to 4, then prove that minimal separator is a complete subgraph. [4+5+12=21]
7. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ for $n > 1$ with $a_0 = 0$ and $a_1 = 1$. [10]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination : 2017-18

Course Name : B. STAT. II Year (Second Semester)

Subject Name : Physics II

Date : 04/05/2018

Duration : 3 hours

Use separate answer sheet for each group

Group A

Maximum Marks : 30

1. If \mathbf{B} is uniform, show that $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$ works. Please note that the symbols have their usual meaning. [3]
2. Find the vector potential a distance s from an infinite wire carrying a current I . [4]
3. An infinite solenoid (n turns per unit length, current I) is filled with a linear magnetized material of susceptibility χ_m .
 - (i) Find the magnetic field inside the solenoid .
 - (ii) Find the bound surface current for the paramagnetic and diamagnetic medium. [3+1+1]
4. A coil 17 cm in diameter and wound with 3 turns of wire is placed with the plane of the coil at right angles to a magnetic field of 3.8×10^{-2} T. What e.m.f. is induced in the coil if :
 - (a) the field is halved in 0.27s,
 - (b) the field is reversed in 0.27s,
 - (c) the coil is rotated through an angle of 90° about its diameter in 0.27s, and
 - (d) the coil is rotated through an angle of 180° about its diameter in 0.27s? [2+2+2+2]

P.T.O

5. Two tiny wire loops, with areas a_1 and a_2 are situated at a distance r apart. Treating the loops as tiny dipoles find their mutual inductance.

[4]

6. Write down the Maxwell's equations in materials so that these involve only free charges and currents. Explain all the terms in the equations.

[4]

7. (a) Define and explain Poynting's vector.

(b) What is the work energy theorem of electrodynamics.

(c) A copper ring and a wooden ring of the same dimensions are placed in magnetic fields so that there is the same change in magnetic flux through them. Compare the induced electric fields for both the cases.

(d) How would you position a flat loop of wire in a changing magnetic field (keeping the direction fixed) so that there is no induced e.m.f. in the loop?

[2+2+1+1]

Group B

Maximum Marks : 30

1. Answer the following questions : [3 + 3 + (1 × 4)]

Solve any two from (a), (b) and (c), you may attempt all three for bonus marks; (d) is compulsory.

(a) A car crossed a junction while the traffic signal was red. Police chased the car and eventually stopped it. On interrogation, the driver claimed that the signal, as observed by him, was green and cited Doppler shift as the reason. Estimate the speed of the car while it crossed the junction. Assume the wavelength of red and green light to be 700 nm and 500 nm respectively ($1 \text{ nm} = 10^{-9} \text{ m}$).

(b) As the outlaws escape in their car, which travels at a speed $\frac{3}{4}c$, the police office fires a bullet from the pursuit car, which only travels at $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. Does the bullet reach the target? Would it reach the target according to Galilean relativity?

(c) At Large Hadron Collider (LHC), a (typical) proton of mass m_p ($938 \text{ MeV}/c^2$) travels in a circular tunnel of circumference $s \simeq 27 \text{ km}$ at a speed $v_p \simeq 0.999999990c$, i.e. about 11 km/hr slower than the speed of light. It takes less than 90 microseconds for a proton to travel around the main ring, resulting in about 11000 revolutions per second. Assuming the proton is moving under the influence of a constant magnetic field (perpendicular to the plane of its motion), estimate the magnetic field.

Hint: $F = \frac{dp}{dt}$, use the relativistic mass and momentum.

(d) True or False :

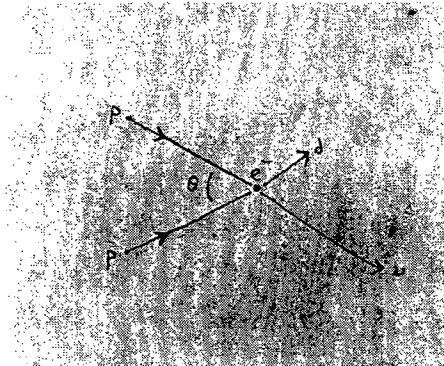
i) The scalar product of two 4-vectors transforms as a vector under Lorentz transformation.

ii) An elastic collision process is observed by two different (inertial) observers. Both these observers would measure the same total (relativistic) energy for this process.

iii) Rest mass of an object depends on its relative motion with respect to

the observer.

iv) Under Lorentz transformation, electric and magnetic fields transform as vectors.



2. In the Sun, one of the processes in the *He* fusion chain is $p + p + e^- \rightarrow d + \nu$, where p , e^- , d , ν denote proton, electron, deuteron and neutrino respectively. Make the approximations that rest mass of the deuteron is $2m_p$ ($m_p =$ rest mass of the proton). Also electron and neutrino have negligible rest mass compared to that of the proton, you may ignore m_e , m_ν .

(a) For the arrangement shown in the figure, where (in the lab frame) the two protons have the same energy $\gamma m_p c^2$ and impact angle θ , and the electron is at rest, calculate the energy E_ν of the neutrino in the rest frame of the deuteron in terms of θ , m_p and γ .

Hint: You may use the invariance of the norm of the total 4-momentum in these two frames.

(b) For the special case where the deuteron remains at rest in the lab frame and $\theta = 30^\circ$, solve for γ and calculate the energy of all particles (the deuteron, the neutrino, one of the protons) in terms of the rest mass of the proton m_p .

[5 + 5]

Answer any two from (3), (4), (5)

3. An electron (e^-) with mass m_e and (relativistic) momentum p_e hits a positron (e^+) at rest. Note that e^+ is the *anti-particle* of e^- , and has the same mass m_e and opposite electric charge. They *annihilate* producing it two photons (γ). One of the photons emerges with an angle of θ with respect to the incident electron. What is its energy? Can the e^+e^- pair annihilate into just one photon?



$$E_1 + E_2 = E_e + m_e c^2 \quad [5]$$

$$E_1 \cos \theta + E_2 \cos \phi = p_e c$$

4. A photon (with frequency f) emitted at the surface of a star will lose energy when it escapes from the star's gravitational field. This can be simply estimated by assuming that a photon possesses an *effective mass* m determined by its energy. The frequency shift Δf when it escapes from the surface of the star to infinity is given by,

$$\frac{\Delta f}{f} = -\frac{GM}{Rc^2},$$

$$E_1 \sin \theta = E_2 \sin \phi$$

$$\Rightarrow \sin \phi = \frac{E_1 \sin \theta}{E_2}$$

with $\Delta f \ll f$, where G = gravitational constant, c = velocity of light, R = radius of star and M = mass of star. Thus, the *red shift* of a known spectral line measured a long way from the star can be used to measure the ratio M/R .

An unmanned spacecraft, launched from earth, approaches the star radially. The photons emitted from He^+ ions on the surface of the star are monitored via resonance excitation of a beam of He^+ ions in a test chamber inside the spacecraft. Resonance absorption of an incoming photon occurs only if the He^+ ions are given a velocity towards the star to *compensate exactly for the gravitational red shift*. Assume that the spacecraft is very far away from the star. Calculate the (radial) velocity of the He^+ beam (with respect to the star) required for this resonant absorption. [5]

5. A neutral pion (π^0) of rest mass m_π and (relativistic) momentum $p = \frac{3}{4}m_\pi c$ decays into two photons (γ). One of the photon is emitted in the same direction as the pion and the other in the opposite direction. Find the (relativistic) energy of each photon. [5]

Indian Statistical Institute
End-Semester Examination 2018
Course Name: B.Stat Second Year
Subject: Economics II (Macroeconomics)

Date: 04/05/2018

Maximum Marks: 60

Time: 135 Minutes

Answer the following questions

1. (i) Consider a Simple Keynesian Model for an open economy with government. Start with an initial equilibrium situation. Now, suppose the government increases its expenditure on produced goods and services, G , by 500 units and finances it by collecting lump sum tax of the same amount. Marginal propensity to consume with respect to disposable income is given to be $(4/5)$ in the model and 20 percent of the additional G is spent on imported goods. Find out the impact of this policy-mix on the NDP of the economy.

(ii) Suppose, in a Simple Keynesian Model (due to some exogenous technological progress) resources required to produce each unit of output falls to three-fourth of its previous level. What impact is this change likely to produce on the equilibrium level of GDP as well as on the level of employment in this model? Explain your answer. [15+5=20]

2. Consider an IS-LM model with government expenditure and tax. Following a change in the income tax rate, the equilibrium in this model is found to change from $(Y=3000, r = 4\%)$ to $(Y=3500, r = 6\%)$. It is given that a unit increase in GDP (Y), given other things, changes the demand for real balance by 0.25 units. Find out the response of the demand for real balance to a change in r , given other things. (Note that the symbols used here have their usual meanings. All relations in the model are linear.) (20)

3. Suppose, CRR and the currency-deposit ratio in an economy are 0.5 and 0.5 respectively. Now, following a loan extended by the central bank to the government, the stock of high-powered money goes up by 500 units in the economy.

(i) Find out the resulting impact on money supply when the excess demand for commercial bank loans is 150 units in the economy.

(ii) How does your answers to 3(i) change if the excess demand for commercial bank loan were 400 units instead. (10+10 = 20)

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2017 – 2018)

B. Stat II Year

Agricultural Science

Date 04/05/2018

Maximum Marks 40

Duration 3:00 hours

(Attempt any four questions)

(Number of copies of the question paper required 20)

1. What are the various agrometeorological parameters that influence evapotranspiration? Calculate the Pan evaporation with the following data:
 - a) Water added : 64 cm
Rainfall for the past 24 hours : Nil
 - b) Water added : 25 cm
Rainfall for the past 24 hours : 5.2 mm
 - c) Water removed : 67 cm
Rainfall for the past 24 hours : 10.8 mm

(4+6)
2. Write the different types of rice with their suitable varieties. Briefly describe the cultural practices associated with the transplanted irrigated rice.

(3+7)
3. Name the weather parameters related to crop production? Write down the names of the apparatus used for measuring/estimating different weather components with their units. Write in brief about the measuring cylinder used to measure the rainwater received by the manual rain gauge. Calculate the amount of water to receive by Potato crop grown on 1.5 ha. of land if the rainfall was 32 mm.

(2+4+2+2)
4. What are the growth related and yield attributing characters of rice. Estimate the yield per hectare of rice crop from the following data.
 - (i) Average no. of tillers/hill – 25, (ii) Average no. of panicles/hill – 22, (iii) Average no. of seeds/panicle – 50, (iv) Average panicle length - 14 cm, (v) Test weight - 28 g.

(4+6)
5. What is intercropping? Rice-Black gram intercropping experiment was done in 2:1 and 2:2 row replacement series system and yield data are given in the following table. The spacing of rice and black gram are 20 x 10 cm and 30 x 10 cm. In your opinion, which combination is the best intercropping system ?

(3+7)

Cropping System	Rice yield in kg/ha	Black gram yield in kg/ha
Rice Sole	4123	-
Black gram Sole	-	7435
Rice+Black gram (2:2)	3344	4325
Rice+Black gram (2:1)	3278	2987

6. Write short notes on:

- a) Monsoon onset
- b) Moisture availability index
- c) Mixed cropping
- d) Sub-surface irrigation

(2.5 x 4)