

# INDIAN STATISTICAL INSTITUTE

**Course Name:** M.Tech. (QROR)

**Year:** 1<sup>st</sup> year

**Subject Name:** Programming Techniques and Data Structures

**Date:** 05.09.16

**Maximum Marks:** 60

**Duration:** 2 hrs 30 min

---

Answer any 6 questions.

1. (a) Provide storage size and value range of the data types *int* and *char*.  
(b) Give examples of 2 linear and 2 non linear data structures in C.  
(c) Write a C program that takes an array of 10 integers as input and returns an array of the same size such that each index of the output array contains the product of all the elements in the array except the element at the given index.  

[3+2+5=10]
  
2. (a) What are the differences between *malloc()* and *calloc()*?  
(b) Write a C program to store *n* integers in dynamically allocated memory location. Let user decide the value of *n*. Use *realloc()* when user exceeds the limit. Finally print all the inserted integers.  

[4+6=10]
  
3. (a) Write a C program for insertion sort.  
or,  
Write a C program for selection sort.  
(b) Analyze the worst case time complexity of binary search or merge sort.  

[5+5=10]
  
4. (a) Write the steps and operator stack status for converting the following infix expression into its postfix form:  
$$A * (B + C * D) + E$$
  
(b) Write a C program to find the sum of the main diagonal of a user defined 10 x 10 matrix.  

[6+4=10]
  
5. (a) What are some of the programming best practices?  
(b) Write a C function that returns the position of a given integer in a singly linked list of integers. Report 'NOT FOUND' if the integer is absent in the list.  
(c) Write a C function that inserts a given integer at the beginning of an existing singly linked list  

[3+4+3=10]
  
6. (a) What is priority queue?  
(b) What is the use of *sizeof()* operator?  
(c) Can a program be compiled and executed without *main()*?  
(d) What is *dequeue*?

(e) When does *malloc()* return NULL?

[2+2+2+2+2=10]

7. (a) Write a C code for printing the number of lines in a user specified text file.  
(b) Write the steps (or pseudo-code) to check if a singly linked list is palindrome.

[4+6=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : 2016-17( First Semester)

Course Name: M. TECH. (QROR) I Yr.

Subject Name : Electrical and Electronics Engineering

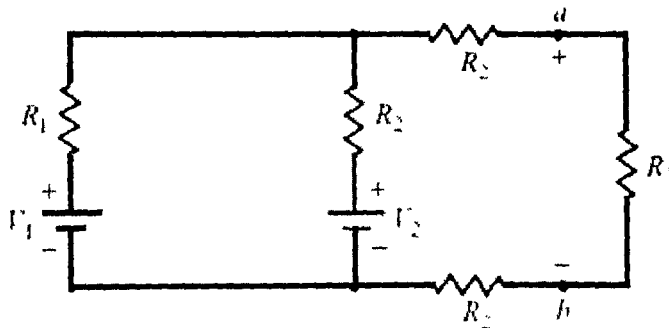
Date :06.09.2016

Maximum Marks : 50

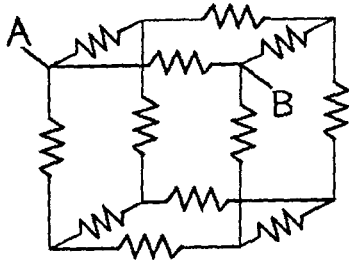
Duration :2Hrs

Answer any 5 questions.

1. What is an OP-AMP? Explain the operations of an OPAMP as an adder and an integrator. [2+4+4=10]
2. For the circuit, given below,  $V_1 = 8V$ ,  $V_2 = 16V$ ,  $R_1 = 4 \text{ Ohms}$ , and  $R_2=6 \text{ Ohms}$ . Find the Thevenin equivalent for the network to the left of terminals a,b. Assume that the internal resistances of the batteries are 0. [10]



3. A cube is formed by joining equal wires, each of resistance 1 Ohm. The cube is shown in figure below. Calculate the equivalent resistance between the points A and B. [10]

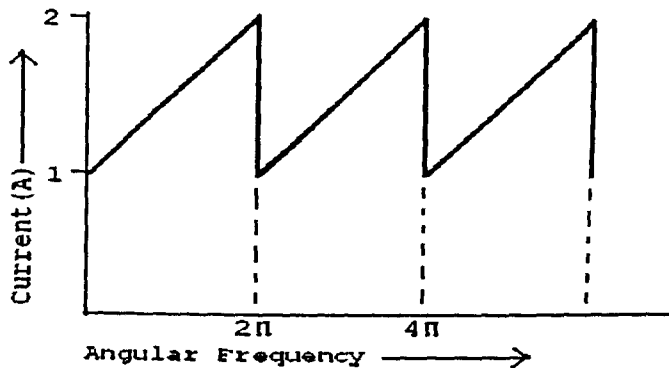


4. a) If  $f(t)$  is a complete response of a circuit, then define the steady state response and the transient response of that circuit.

b) A series RC circuit is excited by a battery of e.m.f.  $E$  at time  $t=0$ . Find the steady state current and the transient current in that circuit. [3+7=10]

5. a) Explain with a diagram how AC voltage can be generated for a rectangular coil having  $N$  turns rotating in a uniform magnetic field with an angular velocity of  $\omega$  radian/second.

b) Calculate the average value of current represented in figure below. [6+4=10]



6. State and prove Thevenin's theorem. [10]

7. a) Explain the maximum power transfer theorem.

b) Suppose, three resistances  $R_1$ ,  $R_2$ , and  $R_3$  are connected in delta formation and the delta formation is equivalent to a star formation comprising of resistances  $x$ ,  $y$ , and  $z$ . Find  $R_1$ ,  $R_2$ , and  $R_3$  individually in terms of  $x$ ,  $y$ , and  $z$ . [5+5=10]

**INDIAN STATISTICAL INSTITUTE**  
**Mid-Semester Examination 2016-17**

Course Name: M. Tech (QROR)

Subject Name: Statistical Methods – I

Date: 06/09/16

Maximum Marks: 100

Duration: 3 hours

Note: Answer all the questions

1. There are two sets of observations containing  $n_1$  and  $n_2$  observations with mean  $\bar{x}_1$  and  $\bar{x}_2$  and standard deviation  $\sigma_1$  and  $\sigma_2$  respectively. Derive the expressions for the mean and standard deviation of the combined set of  $n_1$  and  $n_2$  observations.

[3+7]

2. (a) Describe the different types of data with two examples each  
(b) Represent the frequency distribution of each of the above type of data graphically. Name the plots. Indicate the best measure of central tendency in each case.  
(c) Define skewness and kurtosis and their measures.

[4+8+6]

3. (a) Construct the histogram using the following data

Half-hourly record of generation by a power station during 19/9/01 (10 hrs.) to 21/9/01  
(1.30 hrs.) under normal operating condition

102.8	105.2	103.2	104.0	105.2	104.8	105.6	105.0	105.0	104.0	104.0
105.2	106.0	106.4	103.2	104.2	102.0	103.6	103.8	105.0	105.2	105.2
106.0	105.0	103.0	103.2	103.0	103.0	104.2	105.8	105.4	104.8	104.8
105.2	105.2	106.0	104.0	104.2	103.8	104.4	104.0	102.2	103.4	104.4
104.4	104.2	104.8	106.2	106.4	104.8	102.8	103.6	104.8	104.4	104.8
104.0	104.0	104.0	104.0	104.0	104.4	104.0	102.6	103.0	104.8	102.8
104.0	103.4	103.6	104.0	104.0	103.4	106.0	104.4	104.4	102.4	102.8
105.0	105.2	105.2								

- (b) Using the same data, construct another histogram with the first class interval as 101.7-102.7. Compare the histogram with that obtained in (a) above and identify the peculiar problem that is present in the data (Hint: examine the last digit of the data carefully).

[10+15]

4. Suppose that you are using sampling to estimate the total number of words in a book that contains illustrations.  
 (a) Is there any problem of definition of the population? (b) What are the pros and cons of (i) the page and (ii) the line as a sampling unit?

[3+2x4]

5. Let  $m$  be an estimate of  $\mu$ . The loss due to the error in the estimate is given by the quadratic function  $L = k(m - \mu)^2$ . What criteria should be used in selecting the estimate to minimize  $L$ ? Why?

[2+4]

6. The table below shows the number of inhabitants in each of the 197 cities of a state. Calculate the standard error of the estimated total number of inhabitants in all the 197 cities for the following two methods of sampling: (a) a simple random sample of size 50, (b) a sample that includes the five largest cities and is a simple random sample of size 45 from the remaining 192 cities.

Size Class (1000's)	f	Size Class (1000's)	f	Size Class (1000's)	f
50-100	105	550-600	2	..	..
100-150	36	600-650	1	1500-1550	1
150-200	13	650-700	2	..	..
200-250	6	700-750	0	1600-1650	1
250-300	7	750-800	1	..	..
300-350	8	800-850	1	1900-1950	1
350-400	4	850-900	2	..	..
400-450	1	900-950	0	3350-3400	1
450-500	3	950-1000	0	...	..
500-550	0	1000-1050	0	7450-7500	1

[15+15]

# INDIAN STATISTICAL INSTITUTE

M.Tech (QR-OR) 1<sup>st</sup> Year (S Stream)

Session: 2016 - 2017

MID-SEMISTRAL EXAMINATION

Subject: Engineering Drawing

Date of Exam: 07 .09.2016.

Max. Marks : 40

Time : 1hrs 30 mins

Note: (a) Answer any two questions.

(b) Write your Name and Roll no. at one corner of the drawing sheet.

(c) Marks allotted to each questions are indicated in the bracket.

1. Draw a diagonal scale of R.F. =  $3/100$  showing metres, decimeters and centimeters, and to measure upto 5 metres. Show the length of 3.69 metres on it. [20]
2. Draw the projections of a hexagonal pyramid, base 50 mm side axis 70 mm long, having its base on the ground and one of the edges of the base inclined at  $45^\circ$  to the V.P. [20]
3. Inscribe a regular heptagon and a hexagon in a circle having its radius 15 mm. [20]

**Indian Statistical Institute**  
**Mid-Semestral Examination : 2016-17**  
**M-TECH(QR&OR) -- 1<sup>st</sup> YEAR (E - STREAM)**  
**PROBABILITY-1**  
**{Answer all the questions}**

Date: 07.09.16

Full marks: 75

Time: 2½ hours

**[Symbols have their usual meaning]**

1. a) State and prove Continuity theorem.  
b) In a college 4% of male-students and 1% of female-students are taller than 6 feet. 60% of college students are female. If a student is selected at random and found to be taller than 6 feet what is the probability that she is a female-student?  
[12+8= 20]
2. a) Consider a sequence of independent Bernoulli's trials with success probability  $p$ . Define a random variable  $X$  as # trials to get  $r$ th success. Find the pmf of  $X$ ,  $E(X)$  and  $Var(X)$ .  
b) Find the mode of Hypergeometric distribution.  
[12+8=20]
3. a) Consider  $r$  indistinguishable balls randomly distributed in  $n$  cells. What is the probability that exactly  $m$  cells remain empty?  
b) Two absent minded roommates A and B forget their umbrellas in some way or another. A always takes umbrella when he goes out, while B forgets to take umbrella with probability  $\frac{1}{2}$ . Probability that each of them forgets his umbrella at a shop is  $\frac{1}{4}$ . After visiting 3 shops they return home. Find the probability that they have only one umbrella after their return.  
[12+8=20]
4. a) If you permute the word STATISTICS what is the probability that the three T's will always come side by side?  
b) Explain the difference between 'pairwise independent events' and "mutually independent events". Give an example to show that three events A,B, C are pairwise independent but not necessarily mutually independent.  
[8+7=15]



# INDIAN STATISTICAL INSTITUTE

**Mid-Semester examination: 2016-17**

**Course Name:** M.Tech (QR & OR) 1<sup>st</sup> YEAR (E & S Streams)

**Subject:** Operations Research-I

**Date of Exam:** 08.09.16      **Total Marks:** 80      **Duration:** 2½ hrs.

*Answer all the questions.*

1. (a) A fuel manufacturing company wants to mix two fuels ( $A$  and  $B$ ) for its trucks to minimize cost. It needs no fewer than 3,000 litre to run its trucks during the next month. It has a maximum fuel storage capacity of 4,000 litre. There are 2,000 litre of fuel  $A$  and 4,000 litre of fuel  $B$  available. The mixed fuel must have an octane rating of no less than 80. The octane rating is the weighted average of the individual octanes, weighted in proportion to their respective volumes. Fuel  $A$  has an octane of 90 and costs Rs. 80 per litre. Fuel  $B$  has an octane of 75 and costs Rs. 60 per litre. Formulate this product mix problem as a LPP.

(b) Show graphically, with the help of a 2-variable LP problem, how infeasibility and redundancy are occurred.

*[Mark feasible region, constraint lines with its direction.]*

**[10+8=18]**

2. If the set of all feasible solutions  $K$  of a LPP is a Convex Polyhedron, then at least one extreme point must be obtained which is optimal.

**[12]**

3. Solve the following assignment problem by Hungarian Method (minimize total cost). The owner's objective is to assign the three projects to the workers in a way that will result in the lowest total cost to the shop. Note that the assignment of people to projects must be on a one-to-one basis. Calculate the lowest total cost  $f$  of the project.

Workers	Project		
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>
Arjun	22	28	12
Bobby	16	20	22
Rajesh	18	24	14

[8]

4. (a) What is the characteristic of basic feasible solution of a transportation problem (T.P.)? Describe two unbalanced situations of a T.P? What is the basic difference between North-West Corner Rule (NWCR) and lowest cost method for solving a T.P.?

[4+4+2 = 10]

- (b) Determine an initial basic feasible solution (b.f.s) by Minimum Cost method and test for the optimal solution for the following T.P.

		Distrn. Centres				Supply
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
Plants	P <sub>1</sub>	19	30	50	10	7
	P <sub>2</sub>	70	30	40	60	9
	P <sub>3</sub>	40	8	70	20	18
Demand		5	8	7	14	34

[5+15 = 20]

5. Prove the following two results of duality:
- (a) If  $x^*$  is any feasible solution (f.s.) of primal and  $w^*$  is any f.s. of dual such that  $c'x^* = b'w^*$ , then  $x^*$  is optimal f.s. of primal and  $w^*$  is optimal f.s. of dual.
- (b) If the primal has unbounded solution, then dual is infeasible.

[6+6 = 12]

**INDIAN STATISTICAL INSTITUTE**

Mid-Semester Examination: 2016-17

Course Name: **M. Tech. (QR&OR) I year**

Subject Name: **Quality Management & Systems**

**Date: 09 September 2016**

**Maximum Marks: 60**

**Duration: 2 hours**

---

**Note: Answer any three questions.**

1. Answer the following

(a) Define Quality Management.

(b) State the eight dimensions of quality identified by Garvin. Explain the dimensions briefly.

(c) Apart from these eight dimensions, what other dimension has been suggested by Deming?

[4 + 12 + 4 = 20]

2. What are the different views of quality? Define each of them briefly. Define measurement and state the different scales of measurement.

[15 + 5 = 20]

3. Define, in brief, the customer satisfaction model as proposed by Kano. Explain the model with a real life example.

[10 + 10 = 20]

4. What is MBNQA? State the different award categories under MBNQA. State the different criteria for performance evaluation, along with brief description on each of them.

[4 + 4 + 12 = 20]

5. Answer the following

(a) Why service quality is important? State different dimensions of service quality.

(b) What is ISO 9001? State the quality management principles on which ISO 9001 builds.

[10 + 10 = 20]

# INDIAN STATISTICAL INSTITUTE

Course Name: M.Tech. (QROR)

Year: 1<sup>st</sup> year

Subject Name: Programming Techniques and Data Structures

Date: 24.10.16

Maximum Marks: 60

Duration: 2 hrs 30 min

---

Answer any 6 questions.

1. (a) Describe the advantages and disadvantages of using linked lists.  
(b) Name two non-linear data structures.  
(c) Given an array of 50 integers ranging between 0 to 30, report the unique integers present in the array.  

[4+1+5=10]
  
2. (a) With a suitable example show the use of *realloc()*.  
(b) Define a structure for storing student information including roll number, name and address. Give the users options to a) insert details of new students and b) view the existing records.  

[3+7=10]
  
3. (a) Write a C program for bubble sort.  
or,  
Write a C program for insertion sort.  
(b) What is the definition of Big O notation? What is the worst case time complexity of merge sort?  

[5+5=10]
  
4. (a) Write the steps and operator stack status for converting the following infix expression into its postfix form:  
$$(A+B * C-D)/(E * F)$$
  
(b) Write a C program to check if a user inserted 10\*10 matrix is symmetric.  

[6+4=10]
  
5. (a) Write a pseudocode for checking if two strings are anagrams of each other (made of same characters).  
(b) Write a C program to swap two numbers using pointers.  

[5+5=10]
  
6. (a) What would *sizeof(int)* and *sizeof(char)* return?  
(b) What is a doubly ended queue?  
(c) What are differences between stack and queue?  
(d) Draw a binary tree by inserting 1 to 10 in their original sequence.  
(e) Write the syntax of *calloc()*.  

[2+2+2+2+2=10]

Debarshi Sen Gupta  
7/10/2016

7. (a) Write a C code for printing every alternative line in a user specified text file.  
(b) Given a singly linked list write a pseudocode to delete its middle element. Print the linked list, before and after deletion. Assume that there are an odd number of nodes in the singly linked list.

[4+6=10]

Debarika Sinha  
11/10/2016

# INDIAN STATISTICAL INSTITUTE

## Mid-Semester (Supplementary) Examination 2016-17

Course Name: M. Tech (QROR)

Subject Name: Statistical Methods – I

Date: 25.10.16

Maximum Marks: 100

Duration: 3 hours

Note: Answer all the questions

1. Consider the following as the 100 consecutive observations (coded) made over time on certain characteristic of a product.

Sample No.	Observation	Sample No.	Observation	Sample No.	Observation	Sample No.	Observation	Sample No.	Observation
1	7.5	21	6.4	41	0.9	61	9.5	81	3.2
2	7.8	22	-0.3	42	9.1	62	4.8	82	1.3
3	-0.2	23	8.0	43	6.2	63	0.5	83	9.8
4	6.4	24	7.5	44	0.0	64	9.1	84	4.8
5	8.4	25	0.2	45	8.1	65	6.3	85	0.5
6	0.1	26	6.8	46	7.6	66	0.2	86	9.1
7	4.8	27	8.6	47	0.0	67	7.9	87	6.1
8	9.1	28	0.3	48	6.8	68	7.5	88	0.2
9	1.0	29	4.9	49	8.8	69	0.3	89	8.3
10	3.6	30	9.3	50	0.2	70	7.1	90	7.1
11	10.2	31	0.9	51	5.4	71	8.7	91	-0.3
12	1.9	32	3.6	52	9.7	72	0.3	92	7.0
13	2.2	33	10.2	53	1.4	73	5.3	93	8.4
14	10.4	34	2.3	54	3.9	74	9.5	94	0.1
15	3.4	35	2.5	55	9.9	75	1.2	95	5.3
16	1.2	36	9.6	56	2.2	76	3.6	96	9.6
17	9.6	37	3.6	57	2.5	77	9.7	97	0.8
18	4.7	38	1.5	58	10.1	78	2.2	98	4.0
19	0.7	39	9.6	59	3.1	79	2.7	99	9.7
20	8.8	40	5.1	60	1.1	80	10.0	100	2.0

(a) Draw a run chart (plot of the observations over time) of the first twenty observations and make your observations. Suggest a statistical model for the data (use  $t$  and  $Y$  to denote the sample number and the observations respectively).

(b) Draw a first order lag plot (scatter diagram of  $Y_t$  vs.  $Y_{t-1}$ , where  $Y_t$  is the  $t^{\text{th}}$  observation) using the first twenty five observations and offer your comments.

(c) Draw a histogram using all the 100 observations and offer your comments.

(d) Summarize your learning, if any, based on the results obtained in (a)-(c) above. Assume that the patterns seen the run chart and the lag plot holds good for all the observations.

[4 + 5 + 6 + 12 + 8 = 35]

2. (a) Describe the different types of data with two examples each  
 (b) Represent the frequency distribution of each of the above type of data graphically. Name the plots. Indicate the best measure of central tendency in each case.  
 (c) Define skewness and kurtosis and their measures.

[4+8+6=18]

3. Suppose that you are using sampling to estimate the total number of words in a book that contains illustrations.

- (a) Is there any problem of definition of the population? (b) What are the pros and cons of (i) the page and (ii) the line as a sampling unit?

[3+2x4=11]

4. Find the correct answer. Also comment briefly on all the options to justify your choice. Suppose the mean score of the final examination is 24 (of 40), with a standard deviation of 1.5. If you get a 21, how well do you do (relative to the rest of the class)?

- i. Very poorly--perhaps the lowest score.  
 ii. Not well, but somewhere in the C's (lowest grade is E).  
 iii. OK--about average.  
 iv. Nicely--better than the median.

[6]

5. The table below shows the number of inhabitants in each of the 197 cities of a state. Calculate the standard error of the estimated total number of inhabitants in all the 197 cities for the following two methods of sampling: (a) a simple random sample of size 50, (b) a sample that includes the five largest cities and is a simple random sample of size 45 from the remaining 192 cities.

Size Class (1000's)	f	Size Class (1000's)	f	Size Class (1000's)	f
50-100	105	550-600	2	..	..
100-150	36	600-650	1	1500-1550	1
150-200	13	650-700	2	..	..
200-250	6	700-750	0	1600-1650	1
250-300	7	750-800	1	..	..
300-350	8	800-850	1	1900-1950	1
350-400	4	850-900	2	..	..
400-450	1	900-950	0	3350-3400	1
450-500	3	950-1000	0	...	..
500-550	0	1000-1050	0	7450-7500	1

[15+15]

**M-TECH(QR&OR) -- 1<sup>st</sup> YEAR (E-STREAM)**

**SESSION: 2016-2017**

**MID-SEMESTRAL(SUPPLEMENTARY) EXAMINATION**

**PROBABILITY**

**{Answer all the questions}**

**Date: 26.10.2016**

**Full marks:50**

**Time: 1½ hours**

1. a) State and prove Poincare's theorem  
b) The population of Nicosia is 75% Greek and 25% Turkish. 20% of Greek and 10% of Turks speak English. A visitor to the town meets someone who speaks English. What is the probability that he is a Greek?

[12+8=20]

2. a) Fit a negative binomial distribution to the following data.

x	0	1	2	3	4	5
freq	213	128	37	18	3	1

- b) Let  $X \sim \text{Bin}(10, 1/3)$ . Find its mean, variance, mode, a measure of skewness (no proof is required. Write down the formula only)

[10+10=20]

3. Ten letters are placed at random in ten envelopes. Find the probability that each letter will be placed in a wrong envelope.

[10]



**INDIAN STATISTICAL INSTITUTE**  
**Mid-Semester examination (Supplementary): 2016-17**

**Course Name:** M.Tech (QR & OR) 1<sup>st</sup> YEAR (E & S Streams)

**Subject:** Operations Research-I

**Date of Exam:** 27.10.16

**Total Marks:** 80

**Duration:** 2½ hrs.

*Answer all the questions.*

1. (a) An indigenous mobile manufacturer produces two brands of mobiles. Long-term projections indicate an expected demand of at least 100 brand-I and 80 brand-II mobiles each day. Because of limitations on production capacity, no more than 200 brand-I and 170 brand-II mobiles can be made daily. To satisfy a contract, a total of at least 200 mobiles must be despatched each day. Each brand-I mobile sold, results in a \$2 loss, but each brand-II mobile produces a \$5 profit. Formulate the optimization problem to maximize the net profit.

(b) Show graphically how many of each brand should be made daily to maximize net profit? What is the value of expected net profit to be maximum?

*[Mark all extreme points, feasible region, constraint lines and use line of the objective function to find out the optimal solution.]*

[10+8=18]

2. Prove that replacement of a basis vector forms another basis.

[12]

3. Solve the following assignment problem by Hungarian Method (minimize total cost).  
A Project work consists of four major jobs (J1, J2, J3, J4) for which four contractors (A,B,C,D) have submitted tenders. The tender amounts (in lakh of Rs.) are given below. Find the optimal assignment which minimizes the total cost of the project (each contractor must be assigned one job).

	Contractor			
Jobs	A	B	C	D
J1	10	24	30	15
J2	16	22	28	12
J3	12	20	32	10
J4	9	26	34	16

[8]

4. (a) What is the characteristic of basic feasible solution of a transportation problem (T.P.)? Under what conditions a b.f.s of a T.P. is said to be non-degenerate? What is the basic difference between North-West Corner Rule (NWCR) and lowest cost method for solving a T.P.?

[4+4+2 = 10]

- (b) Determine an initial basic feasible solution (b.f.s) by Minimum Cost method and test for the optimal solution for the following T.P.

		Distrn. centres				Supply
		D1	D2	D3	D4	
Plants	P1	5	4	6	14	15
	P2	2	9	8	6	4
	P3	6	11	7	13	8
Demand		9	7	5	6	27

[5+15 = 20]

5. Prove the following two results:

(a) If  $x$  is any f.s. of primal and  $w$  is any f.s. of dual, then  $c'x \leq b'w$ .

(b) The set of vectors  $a^1 = (2,-1,0)$ ,  $a^2 = (3,5,1)$  and  $a^3 = (1,1,2)$  forms a basis in  $E_3$ .

[6+6 = 12]

# INDIAN STATISTICAL INSTITUTE

First Semestral Examination: 2016-17

**Course Name:** M.Tech (QR & OR) 1<sup>st</sup> YEAR (E & S Streams)

**Subject:** Operations Research-I

**Date of Exam :** 15-11-2016

**Max Marks:** 75

**Duration:** 3 hrs.

**Assignment:** 25 marks

*Question no. 7 is compulsory . Use of scientific calculator/ RMMR table is allowed.  
Answer as many questions as you can.*

1. (a) Explain the physical interpretations of a dual problem considering its objective function, dual variables, dual constraints and its coefficients.

(b) Solve the following problem using Simplex method after converting it to its dual:

$$\text{Maximize } 3X_1 + 2X_2$$

Subject to

$$X_1 + X_2 \geq 1$$

$$X_1 + X_2 \leq 7$$

$$X_1 + 2X_2 \leq 10$$

$$X_2 \leq 3$$

$$X_1, X_2 \geq 0.$$

[6+9=15]

2. As a Project Manager, find the following

- Critical Path and expected project completion time;
- Least Cost schedule and the corresponding project cost;
- 12 days schedule and the corresponding project cost.

Given: Indirect cost per day = Rs. 40/-

The information on activities, their durations for completion and corresponding cost figures are tabulated in the next page.

Activity Node	Normal duration (days)	Crash duration (days)	Cost (Rs.)	Crash Cost (Rs.)
(1,2)	8	6	100	160
(1,3)	4	2	150	350
(2,4)	2	1	50	90
(2,5)	10	5	100	400
(3,4)	5	1	100	200
(4,5)	3	1	80	100

Comment on the solutions obtained in b) and c).

[5+4+6=15]

3. What are the basic assumptions of an EOQ model? Derive optimal parametric values (order qty, ordering interval and cost) of the EOQ model when several items can be procured with the same ordering cost. Mention the basic differences between  $(t_p, S)$  policy and  $(t, S_p)$  policy. The notations have standard meaning.

[2+8+2=12]

4. (a) Explain how to estimate safety stock when demand and lead time of items are uncertain and stock out cost is unknown.

(b) State all the assumptions of a quantity discount inventory model and derive its optimal order quantity for each discount level.

[7+8=15]

5. Define Markov process in terms of a stochastic process. In general, what type of Markov process a Queue model is and why? Under the assumption of Poisson arrivals and exponential distribution of the service time, find the steady state probability of  $n$  persons in the queue system with single server.

[2+3+7=12]

6. Derive the expression for the average system length and the average queue length for Poisson arrivals and exponential distribution of the service time for the single server queue. What is the probability that the system length  $\geq 4$  given that the rate of arrival is 3 per hour and service rate of 5 per hour. What is the probability that the customer has to wait?

[3+3=6]

7. Reliance Digital sells and services several brands of home appliances. Past sales for a particular model of Laptop have resulted in the following probability distribution for demand:

Demand per week	0	1	2	3	4
Probability	0.2	0.35	0.20	0.20	0.05

The lead time, in weeks, is described by the following distribution:

Lead time (week)	1	2	3
Probability	0.10	0.30	0.60

Based on cost considerations as well as storage space, the company has decided to order 10 of these each time an order is placed. The carrying cost is Rs. 100 per week for each unit that is left in the inventory at the end of the week. The stock out cost is set at Rs 5,000 per stock out. The company has decided to place an order whenever there are only 2 laptops left at the end of the week. Simulate 10 weeks of operation for Reliance Digital with currently 5 units in inventory.

What would be the weekly carrying cost under this situation?

[12+3=15]

# INDIAN STATISTICAL INSTITUTE

M. Tech (QR - OR) 1<sup>st</sup> Year (S Stream)

Session: 2016-2017.

SEMESTER EXAMINATION

Subject: Workshop – 1 (Engg. Drawing)

Date of Exam: 18.11.16

Max. Marks : 60

Time : 3:00 hrs

Note: (a) Answer question No.1 (compulsory) and any other three questions

(b) Write your Name and Roll no. at one corner of the drawing sheet.

(c) Marks allotted to each question are indicated in the bracket.

1. Sketch a sectional front view of a Socket and spigot joint. Use suitable dimensions to complete the drawing. [18]

2. The pictorial drawing of a machine part is given below. Draw the top view and the front view of it. Insert all the dimensions in the views. Use first angle projection method. Ref. Fig - 1. [14]

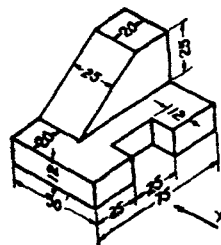


Fig - 1.

3. Show the following parts on a sketch of the threaded end of a screw.

- i) Core diameter
- ii) Outside diameter
- iii) Crest
- iv) Flank
- v) Depth
- vi) Pitch
- vii) Nominal diameter

[14]

4. A cube of 35 mm long edges is resting on the ground on one of its faces with a vertical face inclined at  $30^\circ$  to the V.P. It is cut by a section plane, inclined at  $60^\circ$  to the V.P. and perpendicular to the H.P., so that the face which makes  $60^\circ$  angle with the V.P. is cut in two equal halves. Draw the sectional front view, top view and the true shape of the section. [14]

5. Show by means of sketches (i) a muff coupling (ii) a half – lap coupling. [14]

6. Show by means of sketches the shapes of the following rivet- heads (i) cup (ii) pan (iii) counter sunk. [14]

INDIAN STATISTICAL INSTITUTE  
First-Semester Examination : 2016-17  
M-TECH(QR&OR) -- 1<sup>st</sup> YEAR (E-STREAM)

**PROBABILITY**

**Note : Answer any FIVE questions**

**[Symbols have their usual meaning]**

**Date: 18. 11.16**

**Full marks:100**

**Time: 3 hours**

1. a) State and prove De Moivre's central limit theorem. Explain its application in the field of SQC.  
b) State and prove Chebyshev's lemma.  

[12+2+6=20]
2. a) Derive the p.d.f of  $\chi^2$  distribution with n degrees of freedom. Find a measure of skewness of  $\chi^2$  distribution and comment on its shape.  
b) Let  $(x_1, x_2, \dots, x_n)$  be a sample of size n drawn from  $N(\mu, \sigma^2)$ . Let  $\bar{x}$  and  $s^2$  be the corresponding sample mean and the sample variance. Find the sampling distribution of  $\bar{x}$  and  $s^2$  and show that they are independent.  

[10+10=20]
3. a) Let X be a random variable with distribution function  
$$F_x(x) = P(X \leq x)$$
  
Then prove that  
i) F is monotonic non decreasing function and  
ii) F is continuous to the right.  
b) Let X and Y be two random variables such that  $Y = \log X$ . Let  $Y \sim N(\mu, \sigma^2)$ . Find out the distribution of X. Find E (X) and Var (X).  

[8+12=20]
4. a) Let  $X_i \sim \text{iid } P(\lambda_i)$ ,  $i = 1, 2$ . Let  $Y = X_1 + X_2$ . Find the distribution of Y. Find the conditional distribution of  $X_1$  given  $Y = y$ .  
b) Let  $X \sim \text{Bin}(n, p)$ . Prove that  $P(X \leq k) = \frac{1}{B(n-k, k+1)} \int_0^q z^{n-k-1} (1-z)^k dz$   
c) Suppose  $X \sim N(\mu, \sigma^2)$ . Let  $Y = X^2$ . Find the p.d.f of Y.  

[10+6+4=20]
5. a) State and prove Poincare's theorem.  
b) The pap test makes a correct diagnosis with probability 90%. Given that the test is positive for a person what is the probability that he really has the disease? Assume that one in every 1000 people, on average, has the disease.  
c) Three students A, B, C can solve a problem of Mathematics with probability  $\frac{1}{2}$ ,  $\frac{3}{4}$  and  $\frac{1}{4}$  respectively. What is the probability that the problem will be solved?  

[10+6+4=20]

6. a) Let  $X_i \sim \text{iid } N(0,1)$ ,  $i = 1(1)n$

Let  $X_{(1)} = \text{Min}(X_1, \dots, X_n)$

Find the distribution of  $X_{(1)}$

b) Define correlation coefficient ( $\rho_{XY}$ ) between two random variables X and Y. Let U and V be two other random variables such that,

$U = a + bX$  and  $V = c + dX$ .

Let  $\rho_{UV}$  be the correlation coefficient between U and V. Find the relationship between  $\rho_{UV}$  and  $\rho_{XY}$ . Justify your answer.

c) The individual monthly wages (in rupees) obtained by a group of people is normally distributed with mean 20000 and standard deviation 3000. If three people are taken randomly from the group what is the probability that exactly two of them are getting monthly salary more than Rs 26000/- ?

[6+8+6=20]



INDIAN STATISTICAL INSTITUTE

First-Semester Examination: 2016-17

Course Name: M. TECH. (QROR) I Yr.

Subject Name: Electrical and Electronics Engineering

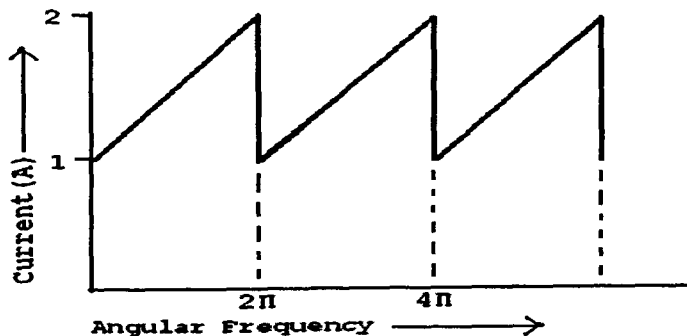
Date: ~~21/11/~~ .2016

Maximum Marks: 96

Duration: 3Hrs

Answer any 6 questions.

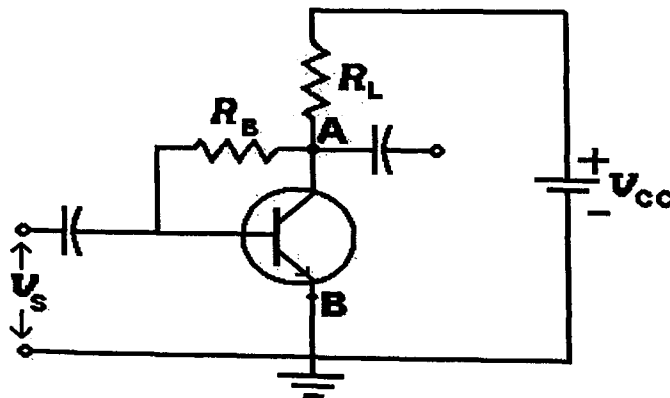
- Draw the equivalent circuit of a transformer and show that in ideal situation the ratio of the primary to the secondary voltage is equal to the primary-to-secondary turns ratio.
  - Show that the mutual inductance  $M$  between the primary coil (inductance  $L_1$ ) and the secondary coil (inductance  $L_2$ ) of a transformer is  $\sqrt{L_1 L_2}$ . [(3+8)+5=16]
- Explain with a diagram how AC voltage can be generated for a rectangular coil having  $N$  turns rotating in a uniform magnetic field with an angular velocity of  $\omega$  radian/second.
  - Calculate the average value of current represented in figure below. [10+6=16]



- A DC voltage  $E$  is applied across a series RLC circuit. Find the steady state current in the circuit when  $(R^2/4L^2) > (1/LC)$  and draw the respective curve of current w.r.t. time. [13+3=16]
- Define control system, control lag, load variable and dead time.
  - Derive the output of a PID (Proportional plus Integral plus Derivative) control action for a step type error signal. Explain

with a figure how the output changes with the error signal and time. [(2+2+2+2)+(5+3)=16]

5. Using OPAMP explain the operations of a differentiator, a differential amplifier, a non-inverting amplifier and an integrator. [4+4+4+4=16]
6. a) Draw the block diagram of a negative feedback amplifier and derive an expression for closed-loop gain in terms of feedback ratio and transfer gain.  
 b) An amplifier with negative feedback has a closed-loop gain of 100. Open-loop gain variation of 10% is expected owing to production limitations. Determine the value of open loop gain and feedback fraction  $\beta$  for which closed-loop gain will only vary by 1%. [(3+4)+(4+5)=16]
7. a) Derive the octal equivalent of  $(25.75)_{10}$  .  
 b) Subtract 01111 from 1000 using 2's complement method and verify the result by repeating the same subtraction using 1's complement method.  
 c) Show that an OR gate can be realized with diodes and resistances.  
 d) State the De Morgan's theorem. [3+5+6+2=16]
8. a) Draw the circuit diagram of an amplifier using a p-n-p transistor in common collector mode of operation.  
 b) Find the relation between the parameters  $\alpha$  and  $\beta$  of a transistor.  
 c) An n-p-n transistor, shown in the figure below, is used in common-emitter amplifier mode with  $\beta=49$ ,  $V_{CC}=10V$ , and  $R_L=2k$  Ohms. If a 100k Ohms resistor,  $R_B$ , is connected between the collector and the base of the transistor then calculate the quiescent collector current and the collector to emitter voltage drop between points A and B. [4+5+7=16]



**INDIAN STATISTICAL INSTITUTE**  
**Semester Examination 2016-17**

Course Name: M. Tech (QROR)

Subject Name: Statistical Methods – I

Date: 21.11.2016

Maximum Marks: 100

Duration: 3 hours

Note: Answer all the questions

1. Consider the following frequency distribution of the life (in minutes) of a certain type of battery.

Class Interval	frequency
220-240	5
240-260	16
260-280	10
280-300	25
300-320	12
320-340	40
340-360	10
360-380	20
380-400	2

- (a) Compute the mean and variance of the battery life  
(b) Examine the pattern of variation of distribution and offer your comments
- [(3+4)+5]
2. Let the random variable  $Y$  be normally distributed with parameters  $\mu$  and  $\sigma$ . Find the maximum likelihood estimate of  $\sigma$ .
- [10]
3. A life test is planned with ten units for a maximum duration of 320 hours. The time of failures of the seven units that failed during the test period are noted. Thus we have the test data is as follows:

<u>Time</u>	<u>Frequency</u>
15	1
21	1
75	1
122	1
150	1
182	1
286	1
$\geq 320$	3

Assume that the distribution of failure time is exponential (with only one parameter). Derive the expression for maximum likelihood estimate of the mean of the distribution and hence find the estimate for the test data as above.

[7+7]

P.T.O

4. We have observations ( $y$ ) on  $n (=40)$  samples drawn randomly from a truncated exponential distribution with mean  $m$  (known) and truncation point  $t$  (unknown);  $1.5m \leq t \leq 2.2m$ ,  $y \leq t$ . Write a simulation algorithm to find a suitable estimate of  $t$ . [Hint: use the method of moment estimation without bothering too much about the quality of the estimate obtained.]

[22]

5. Write an algorithm for simulating 200 random numbers from a Poisson distribution.

[12]

6. In a study of the possible use of sampling to cut down the work in taking inventory in a stock room, a count is made of the value of the articles on each of 36 shelves in the room. The values ( $y$ ) to the nearest dollar are as follows.

29, 38, 42, 44, 45, 47, 51, 53, 53, 54, 56, 56, 56, 58, 58, 59, 60, 60,

60, 60, 61, 61, 61, 62, 64, 65, 65, 67, 67, 68, 69, 71, 74, 77, 82, 85

The estimate of total value made from a sample is to be correct within \$200, apart from a 1 in 20 chance. An advisor suggests that a simple random sample of 12 shelves will meet the requirements. Do you agree? Explain. [ $\sum y = 2138$ ,  $\sum y^2 = 131682$ ]

[12]

7. Consider the following ten pairs of observations ( $x$ ,  $y$ ).

x	3	7	2	1	8	10	5	4	6	9
y	2	5	1	3	9	8	6	7	4	10

(a) Estimate the Pearson's correlation coefficient  $r_{xy}$

(b) Consider that the above values are ranks and obtain an estimate of the Spearman's rank correlation coefficient  $\rho_{xy}$ .

(c) Compare the above results and offer your comments.

[3+3+4]

8. Write short notes on the following:

(a) Normal Probability Plotting

(b) Stratified Random Sampling

[4+4]

# INDIAN STATISTICAL INSTITUTE

## Semestral Examination (Semester I)

Course Name: M.Tech. (QROR)

Year: 1<sup>st</sup> year

Subject Name: Programming Techniques and Data Structures

Date: 24/11/2016

Maximum Marks: 75

Duration: 2 hrs 45 min

Answer all 7 questions.

- (a) Write a pseudocode for merge sort.  
(c) Derive the best, average and worst case time complexity of linear search.

[5+5=10]
- (a) Present a simple technique for hashing when the data type is string. Give examples to demonstrate your hash method yields different values for anagrams.  
(b) Given the data appearing in the following sequence: 113, 117, 97, 100, 114, 108, 116, 105, 109, demonstrate how the hash table will look like when (i) quadratic probing, (ii) chaining is used for collision resolution. Consider the table size to be 11.  
(c) What is load factor?  
(d) What are some of the good features of a hash function?

[3+3+2.5+2.5=11]
- (a) Write a program to efficiently search for a user entered element in a sorted array.  
(b) Write the outputs of the following code snippets:

```
#include <stdio.h>
int main()
{
    int a = 1, b = 1, c;
    c = a++ + b;
    printf("%d, %d", a, b);
}
```

**and**

```
#include <stdio.h>
int main()
{
    int a = 0, i = 0;
    for (i = 0; i < 5; i++)
    {
        a++;
        if (i == 3)
            break;
    }
    printf("%d", a);
}
```

[6+2x2.5=11]

P.T.O

4. (a) What is asymptotic analysis of an algorithm?  
(b) Name some factors that influence an algorithm's execution time.  
(c) Justify: "Merge Sort is based on divide and conquer principle."  
(d) What is the use of free()?  
(e) You are given a pointer to a node (not the tail node) in a singly linked list. Write a C function for deleting the node from the list.

[3+2+2+2+3=12]

5. (a) Consider a set of numbers inserted in the following sequence: 50, 25, 10, 5, 7, 3, 30, 20, 8, 15. Draw the interim trees produced in successive steps for constructing an AVL tree. Show the rotations.  
(b) Write a C program to find factorial of a user specified positive integer using recursion.

[5+5=10]

6. (a) What is the utility of a height balanced tree?  
(b) Write a short note on clustering in the context of hashing.  
(c) Mention a few applications of the stack data structure.  
(d) What is quadratic probing?

[2.5+3+2.5+3=11]

7. (a) Define the structure for a single threaded binary tree. Explain the utility of each variable in it.  
(b) Write the pseudocode for inorder traversal of single threaded binary tree.  
(c) What is the time complexity for inorder traversal of a threaded binary tree?

[4+4+2=10]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2016-17

Course Name: M. Tech. (QR&OR) *I year*

Subject Name: Quality Management & Systems

Date: 28 November 2016

Maximum Marks: 100

Duration: 2 hours 30 Minutes

Note: This paper carries 75 marks. 25 marks is allotted for Assignment. *Answer any five questions.*

---

1. What are the eight dimensions of quality identified by Garvin? Explain the dimensions briefly. Apart from these eight dimensions, what other dimension has been suggested by Deming?  
[12 + 3 = 15]
2. Use Kano's model to explain Juran's insight that "customer satisfaction and dissatisfaction are not opposites". Elaborate your explanation with a real life example.  
[10 + 5 = 15]
3. Answer the following
  - (a) Explain the concepts of internal and external customers with examples.
  - (b) Define briefly the different steps for carrying out six sigma project of an existing process.
  - (c) What is Juran's trilogy? Explain briefly.[5 + 5 + 5 = 15]
4. Define QFD. Draw a schematic structure of "House of Quality" and explain its different components.  
[3 + 12 = 15]
5. Write short note on any three of the following terms.
  - (a) Emergency Preparedness
  - (b) Significant Environmental Aspects
  - (c) Adequacy Audit versus Compliance Audit
  - (d) Disposition of Nonconforming Product[5 x 3 = 15]
6. Following are the objective evidences of an internal audit conducted according to both ISO 9001: 2015 QMS and ISO 14001: 2015 EMS. For any five objective evidences, identify whether the observation is related to QMS or EMS and provide your brief justification on being classified as a nonconformance or not.
  - (a) An operator was using a work instruction detailing how the test should be carried out. However, the work instruction does not refer the approval authority.
  - (b) EMS Objectives are found to be measured routinely, but no improvement is observed.
  - (c) The Inspector in the Laboratory was referring to revision 01 of the test method TMLAB/06, but was not sure whether that is the latest revision.
  - (d) Testing procedure specifies that the temperature needs to be controlled within  $40 \pm 2^\circ\text{C}$ . However, a calibrated thermometer of least count  $1^\circ\text{C}$  is being used for this purpose.
  - (e) Cleaning of solvent drums has been identified as having Significant Environmental Impact, but no Operating Procedure has been developed.
  - (f) The company Quality Manual has not been revised since December 2014.[3 x 5 = 15]

INDIAN STATISTICAL INSTITUTE  
Back Paper Examination : 2016-17  
M-TECH(QR&OR) -- 1<sup>st</sup> YEAR (E-STREAM)

**PROBABILITY**  
**[Symbols have their usual meaning]**

Date: 29.12.2016

Full marks:100

Time: 3 hours

Note : Answer all the questions

1. a) State and prove Bonferroni's inequality.  
b) At an art exhibition there are 15 paintings of which 10 are original. A visitor selects a painting at random and before he decides to buy, he asks the opinion of an expert about the authenticity of the painting. The expert is right in 7 out of 10 cases on an average. Given that the expert decides that the painting is authentic what is the probability that it is really original?  

[10+10=20]
2. a) Let  $(X, Y) \sim N_2(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$ . Find  $E(X|Y)$  and  $\text{Var}(X|Y)$ .  
b) A pays Rs. 10 for each participation of the following game: 3 dice are thrown. If one six appears he gets Rs. 10. If two sixes appear he gets Rs. 20. If three sixes appear he gets Rs. 30. Otherwise he gets nothing. What is his expected gain?  

[12+8=20]
3. a) Let  $X$  follow Binomial distribution (with parameters  $n$  and  $p$ ) truncated at  $X = 0$ . Find the p.m.f and expectation of  $X$ .  
b) Explain when do you say that two random variables  $X$  and  $Y$  are independent? Prove that if  $X$  and  $Y$  are independent then  $\rho_{XY} = 0$ . Is the converse true? Justify your answer.  

[12+8=20]
4. a) Suppose  $X$  follows Exponential ( $\theta$ ). Let  $Y = X^{1/\beta}$ ,  $\beta > 0$ . Find the p.d.f of  $Y$ .  
b) Let  $X$  and  $Y$  be two random variables with  $E(X) = 5$ ,  $\text{Var}(X) = 2$ ,  $E(Y) = 10$ ,  $\text{Var}(Y) = 3$ , and correlation coefficient between  $X$  and  $Y$  be  $-0.4$ .  
Let  $Z_1$  and  $Z_2$  be two random variables defined as follows  
$$Z_1 = 3X + 5Y, \text{ and } Z_2 = 2X - 3Y.$$
Find  $E(Z_1)$ ,  $\text{Var}(Z_1)$ ,  $E(Z_2)$ ,  $\text{Var}(Z_2)$  and correlation coefficient between  $Z_1$  and  $Z_2$ . (state the results you have used).  

[6+14=20]
5. a) State and prove Central Limit Theorem due to Lindeberg and Levy.  
b) State and prove Chebyshev's inequality. What does it provide?  

[12+8=20]



INDIAN STATISTICAL INSTITUTE

Back Paper Examination: 2015-16  
M. Tech. (QR & OR), 1<sup>st</sup> Year, E Stream  
Subject: Statistical Methods I

Date: 30.12.2016

Duration: 3 hours

Note: Answer all the questions. Full marks are 100.

1. (a) Classify the four types of data. Give two examples of each type. (b) Draw the flow chart of the data collection process.

[(3+4) +5=12]

2. In a study of the possible use of sampling to cut down the work in taking inventory in a stock room, a count is made of the value of the articles ( $y$ ) on each of the 36 shelves in the room. The values to the nearest rupee are as follows.

29, 38, 42, 44, 45, 47, 51, 53, 53, 54, 56, 56, 56, 58, 58, 59, 60, 60

60, 60, 61, 61, 61, 62, 64, 65, 65, 67, 67, 68, 69, 71, 74, 77, 82, 85

It is required that the estimate of total value made from a sample be correct within Rs. 100.00, apart from a 1 in 20 chance. An advisor suggests that a simple random sample of 12 shelves will meet the requirements. Do you agree? (Given  $\sum y = 2138$ ,  $\sum y^2 = 131682$ )

[12]

3. Explain the inverse transform method for generating random numbers from a specified distribution using two examples.

[14]

4. Define the terms 'statistic', 'sampling distribution' and 'standard error'. Why do we need to study sampling distributions? Describe the methods for constructing 95% confidence intervals for the Binomial proportion  $p$ , when the sample size ( $n$ ) is small and also when  $n$  is large.

[6+3+10=19]

5. (a) Define point and interval estimates. Give examples. (b) Describe the three criteria normally used to judge the quality of an estimator. (c) Name three methods of estimation and describe any one of them with an example.

[4 + 6 + 6 = 16]

P.T.O

6. Assume the time taken ( $y$ ) by the police to arrive at a bank after the alarm starts follows exponential distribution. The following data on  $y$  (minutes) are obtained from 11 trial runs.

3.4, 2.4, 4.2, 10.2, 7.8, 3.1, 5.2, 7.2, 3.9, 2.9, 9.5

What is the probability that a robber entering the bank will be caught if he takes two minutes to finish off his business?

[12]

7. Suppose the process of police arrival as mentioned in question 6 above is modified in order to reduce the arrival time. Following data are obtained after the modification.

2.2, 3.1, 3.2, 6.3, 4.7, 2.8, 4.0

Construct a Q-Q plot to examine whether the process has really improved or not.

[15]

**INDIAN STATISTICAL INSTITUTE**  
**M. Tech (QR & OR) I Year 2016-2017**  
**Mid-Semestral Examination**

**Subject : Reliability - I**

**Date : 20.02.2017**

**Full Marks : 100**

**Duration : 3 hrs.**

1. a) In many situations the failure rate  $\lambda$  may vary from one unit to another. In a particular nuclear plant, this variation in  $\lambda$  was modelled by a lognormal distribution. If the median failure rate  $\lambda_M = 6.0 \times 10^{-5}$  failures per hour and a 90% error factor  $k = 3$  is assumed, then find the parameters of the lognormal distribution.

- b) Describe the shape of the hazard rate of the lognormal distribution.

[10+5=15]

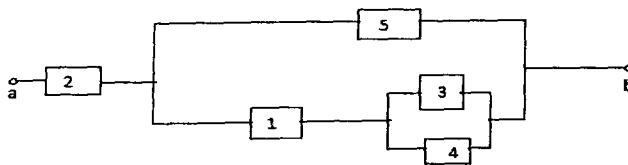
2. a) Define a *coherent system*.

- b) Let  $\phi$  be a coherent structure. Then show that  $\phi(x \cup y) \geq \phi(x) \cup \phi(y)$ .

- c) Give its interpretation.

[3+10+2=15]

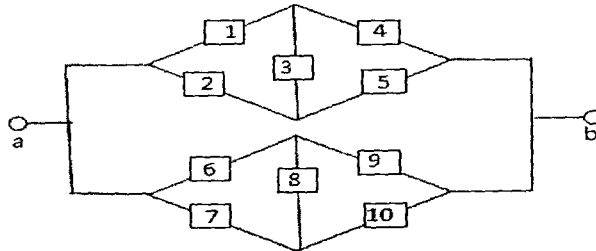
3. Consider the reliability block diagram given below:



- a) Write down its structure function.
- b) Find the minimal path sets and the minimal cuts of the system.
- c) Identify the critical vector with respect to component 4.
- d) Find the Birnbaum reliability importance of component 4

[3+6+1+5=15]

4. a) Define *modular decomposition* of a coherent system.
- b) Consider the coherent structure consisting of 10 components presented in the schematic diagram given below:



The components are mutually independent with common probability  $p$  of functioning and common probability  $q \equiv 1 - p$  of failing.

- (i) Carry out a modular decomposition of the above system.
- (ii) Write down its organizing structure.
- (iii) Draw an equivalent minimal cut representation of the given structure.
- (iv) Find the system reliability as a function of the component reliabilities.

[4+(10+1+3+7)=25]

5. Let  $X$  be a random variable with the following reliability function:

$$\bar{F}(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ e^{2-2t} & 1 \leq t \leq 2 \\ e^{-t} & t \geq 2 \end{cases}$$

Find its hazard rate and sketch the same. Mention, with reasons, the smallest nonparametric class to which this life distribution belongs. Define that class.

[10+5=15]

6. a) Consider a parallel system with identical components each with reliability 0.8. If the reliability of the system is to be at least 0.99, find the minimum number of components in this system.
- b) Consider a series system of  $n$  components. The corresponding lifetimes  $T_i$  for  $i = 1, \dots, n$  are assumed to be independent and Weibull distributed:

$$T_i \sim \text{Weibull}(\alpha, \lambda_i) \quad \text{for } i = 1, \dots, n.$$

Show that the system has a Weibull distribution with the shape parameter being unchanged.

- c) Define *Mean Residual Life*.

[5+5+5=15]

**INDIAN STATISTICAL INSTITUTE**  
**Mid- Semester of Second Semester Examination: 2016-17**

Course Name : M. TECH (QR-OR)-I  
Subject Name : MECHANICAL ENGINEERING  
Date: 21/02/2017 Maximum Marks: 50 Duration: 1hr 30 min  
Note, if any :  
Date : 21.2.17

Answer question no. 1 and any *two* questions from the rest.

1. a) What is machining?  
b) Define machining accuracy.  
c) What is break even quantity in manufacturing?  
d) Write Taylor's principles of limit gauge design.  
e) Define machine tool. Give example. 2 X 5
  
2. a) Determine the type of fit for assembly 20 H7-f8. Given that  
 $i$  (micron) =  $0.45\sqrt[3]{D} + 0.001D$  where D is in mm,  
20 mm lies in the diameter steps of 18 mm and 30 mm,  
IT7=16i, IT8=25i, *fundamental deviation for 'f' shaft =  $-5.5(D)^{0.41}$*   
Also calculate the limits and tolerances of the hole and shaft.  
b) What is zero line? Explain bilateral and unilateral tolerances.  
c) Discuss the selective assembly or hole basis system. 12 + 4 + 4
  
3. a) Discuss the effects of surface roughness on performance of parts.  
b) Calculate the CLA value of a surface for the following data:  
Sampling length= 0.8 mm  
Vertical magnification of the graph= 14000  
Horizontal magnification of the graph= 100  
Areas above datum line are: 160, 90, 180 and 50 mm<sup>2</sup>  
Areas below datum line are: 95, 65, 170 and 150 mm<sup>2</sup>.  
c) Explain form error, surface waviness and surface finish.  
d) Explain ovality and lobedness of circular cross section of a job. 7 + 4 + 5 + 4
  
4. a) Draw the stress-strain diagram of a ductile material and explain it.  
b) Show the input-output model of manufacturing process and explain it.  
c) Distinguish between mechanism and machine. 10 + 7 + 3

INDIAN STATISTICAL INSTITUTE  
Mid -Semester Examination: 2016 – 17  
M. Tech (QROR), E-Stream, Semester II  
Statistical Methods – II

Date: 21/02/2017

Maximum Marks: 50

Duration:  $1\frac{1}{2}$  Hrs.

**Note: Answer Question 4 and any three (3) from the rest.**

1. a) Define and explain Simple and Composite Hypothesis.  
b) Define Uniformly Most Powerful Test.

[(3+3)+4=10]

2. Suppose a random sample of size  $n$  is collected from a normal distribution, where mean  $\mu$  is known and variance  $\sigma^2$  is unknown. Obtain the critical region for testing the following hypothesis

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu \neq \mu_0$$

using the likelihood ratio approach.

[10]

3. Let  $X$  denote the number of heads when four coins are tossed at random. 100 repetitions of the experiment resulted in the following data on number of heads:

$X$	0	1	2	3	4
# of Heads	8	17	41	30	4

Can it be assumed that binomial is a reasonable model for the distribution of  $X$ ? Use  $\alpha=0.05$

[10]

4. The firmness of a ceramic material is believed to be dependent on the Pressure (A), Temperature (B) and an Additive (C). A factorial design with two replicates, with each factors at two levels, are run to see their effect on the response. The results are shown in the following table.

Additive	Pressure 1		Pressure 2	
	Temperature		Temperature	
	1	2	1	2
1	14, 16	7, 11	18, 20	9, 10
2	4, 8	24, 32	6, 10	26, 34

The personnel involved in the experimentation are interested in studying the effect of following two possible interactions only:

- i) Pressure and Temperature, and
- ii) Temperature and Additive.

- a) Write and explain the linear statistical model suitable for this design.
- b) Assuming Pressure and Temperature to be fixed and Additive to be random, find the expressions for expected mean squares for the model components.
- c) Hence suitably analyze the data using analysis of variance. Use  $\alpha=0.05$ .

[(2+2)+6+10=20]

5. a) Write down the linear statistical model of a single factor fixed-effect design. Explain the terms and assumptions made.
- b) State the hypotheses to be tested in such a design.
- c) Show that under the null hypothesis  $MS_{Treatment}$  is an unbiased estimator of  $\sigma^2$ .

[2+2+6=10]

# INDIAN STATISTICAL INSTITUTE

## Second Mid-Semester Examination (2016 – 2017)

Course Name : M.Tech (QR & OR)  
Subject : Industrial Engineering and Management  
Date : 22/02/2017  
Maximum Marks : 40  
Duration : 90 minutes

---

### Question Paper

---

Answer all questions

1. With respect to the Table as given below the home builder wants to finish the following project (i.e. a house) within 30 weeks. In this context, find out the extra cost needed to complete the house by this time. [10]

Table: Normal Activity and Crash Data

Activity	Normal Time (Weeks)	Crash Time (Weeks)	Normal Cost (Rs)	Crash Time (Rs)
1-2	12	7	3000	5000
2-3	8	5	2000	3500
2-4	4	3	4000	7000
4-5	4	1	500	1100
4-6	12	9	50,000	71,000
5-6	4	1	500	1100
6-7	4	3	15,000	22,000

2. Explain, with proper justification, the different factors, required to set up a hospital.

[10]



3. Write short notes on *any four*:

[4X5=20]

- (a) Product layout
- (b) Productivity and its importance
- (c) Relation of Unemployment and Standard of living with productivity
- (d) Ineffective time and role of a worker
- (e) Inherent safety in plant layout
- (f) Drawback of profit to cost ratio

**INDIAN STATISTICAL INSTITUTE**  
**M. Tech. (QR OR), I Year**  
**Session: 2016-17, Semester II**  
**Semestral Examination**

**Subject : Elements of Stochastic Process**

**Date : 23/02/2017**

**Time : 2.30 hours**

**Maximum Marks : 60**

(1) Define the following

- (a) Stochastic Process
- (b) State Space
- (c) Parameter Space
- (d) Discrete time parameter Markov Chain

(4 x5) = [20]

(2) Show that for a discrete time parameter Markov Chain with stationary transition probability  $P(X_0 = i_0, X_1 = i_1, \dots, X_n = i_n) = P(X_0 = i_0)p_{i_0 i_1} \dots p_{i_{n-1} i_n}$

Where  $P = \left( (p_{ij}) \right)_{i,j \in I}$  denotes the one step transition probability matrix of the Markov Chain. [ 20 ]

(3) State and prove the Chapman – Kolmogorov Equation for a Markov Chain. Prove the results that you use. [ 10 ]

(4) Suppose we have two boxes, labeled 1 and 2 and  $d$  balls labeled  $1, 2, \dots, d$ . Initially some of these balls are in Box 1 and the others in Box 2. An integer is selected at random from  $\{1, 2, \dots, d\}$  and the ball labeled by that integer is removed from its box and placed in the other box. The procedure is repeated indefinitely with the selection being independent from trial to trial. Let  $X_n$  denote the number of balls in box 1 after the  $n^{\text{th}}$  trial.

Show that  $\{X_n | n=0, 1, 2, \dots\}$  is a discrete time parameter Markov Chain and find its state space and one-step transition matrix. [ 10 ]

**INDIAN STATISTICAL INSTITUTE**  
**Mid-Semester Examination: 2016-17**  
**Course: M.Tech (QR & OR) 1<sup>st</sup> YEAR (E & S Streams)**

**Subject: SQC**

**Date of Exam: 24/02/2017**

**Max. Marks: 100**

**Duration: 3 hrs.**

**Answer All Questions.**

- 1) Choose the correct alternative out of a, b, c, d and e wherever applicable.
- A. A process in a state of statistical control means
- I. Only common causes are present
  - II. Only special causes are present
  - III. The product will always meet its specifications
  - IV. The process is stable and only random or inherent sources of variation are present in the process
- a) I and II above
  - b) II and III above
  - c) III and IV above
  - d) IV and I above
  - e) None of the above
- B. The rational subgrouping for the control chart means
- a) A subgroup that is free from the assignable causes as much as possible
  - b) A subgroup that represents the homogeneous conditions as much as possible
  - c) A small group of consecutively produced parts from a production process
  - d) All of the above
  - e) To form subgroups by randomly taking samples from the entire production
- C. If a process is out of control, the theoretical probability that five consecutive points on an  $\bar{X}$  chart will fall on the same side of the mean is
- a)  $(1/2)^4$
  - b)  $2(1/2)^4$
  - c) Unknown
  - d)  $1/2(1/2)^4$
  - e)  $(1/2)^5$
- D. You are monitoring a critical characteristic which is in a state of statistical control for a long period of time. However, the inherent variation or common cause variation in the process is bigger than the allowable tolerance on the characteristic. The least favorable action that you should take is
- a) Live with the process as is and a certain level of defectives and implement a containment plan as it is not economical to change the process
  - b) Stop monitoring the characteristic as it is not capable

- c) Change the system, i.e., change the current manufacturing process to reduce the inherent variation
  - d) Convince the engineer to open or expand the tolerance and make it realistic based on the data from process capability if there are no adverse consequences to the customer
- E. A product characteristic is monitored using control charts, and the quality engineer finds that the process is not in a state of statistical control. Which one of the following actions is not preferable?
- a) A root cause analysis of the process parameters should be done to identify and remove the special causes of variation and bring the process in the state of statistical control
  - b) Analyze the data of individuals to see if the process meets the tolerance limits even with some special causes present
  - c) Perform a capability study whether the process is in statistical control or not and predict the capability
  - d) Do not give a very high priority to quality improvement efforts if the process meets the specifications, and if there are no adverse consequences
- F. A process has multiple machines producing the same part. e.g., a multicavity molding machine, multiple spindles, etc. Which of the following is not advisable for the different batches produced from each separate machine?
- a) All batches should be mixed and samples should be taken at random to monitor the process on a single control chart
  - b) Each subgroup in the case of a multicavity machine comes from its unique population of that cavity alone and all successive subgroups are taken from the same cavity only
  - c) Each cavity is treated separately and has its own control chart
  - d) All of the above
- G. The probability of rightly saying a process out of control is
- a)  $\beta$
  - b)  $\alpha$
  - c)  $1 - \beta$
  - d)  $1 - \alpha$
- H. The probability of rightly saying a process in-control is
- a)  $\beta$
  - b)  $\alpha$
  - c)  $1 - \beta$
  - d)  $1 - \alpha$
- I. The probability of wrongly saying a process in-control is
- a)  $\beta$
  - b)  $\alpha$
  - c)  $1 - \beta$
  - d)  $1 - \alpha$
- J.  $\%R \& R = \frac{R \& R}{TV} \times 100$ . Find the correct relational equation from the following:

- a)  $R \& R = \sqrt{TV^2 + PV^2}$
- b)  $PV = \sqrt{TV^2 + R \& R^2}$
- c)  $R \& R = \sqrt{TV^2 - PV^2}$
- d)  $TV = \sqrt{R \& R^2 - PV^2}$

K. The probability of wrongly saying a process out of control is

- a)  $\beta$
- b)  $\alpha$
- c)  $1 - \beta$
- d)  $1 - \alpha$

L. In a Pareto analysis if number of defects are translated into corresponding monetary loss

- a) a large number of defects generally represent a great amount of money lost
- b) a large number of defects may not represent a great amount of money lost
- c) a small number of defects always represent a great deal of money lost
- d) none of the above

M. The cause and effect or Fishbone diagram can be classified as:

- a) Cause enumeration, dispersion classification & tree diagram types
- b) Cause enumeration, dispersion classification & activity network diagram or arrow diagram types
- c) Dispersion classification, process classification & cause enumeration types
- d) Process classification, affinity diagram & tree diagram types

N. Based on their function, the check sheets can be classified as:

- a) Process decision program checks, defective item checks, defect location checks, defective cause checks, check-up confirmation checks
- b) Defect location checks, defective item checks, defective cause checks, production process distribution checks, check-up confirmation checks
- c) Affinity checks, check-up confirmation checks, production process distribution checks, defective cause checks, defect location checks
- d) Defect location check, defective item checks, defective cause checks, interrelationship checks, check-up confirmation checks

O. Variation in a process exists due to:

- a) Chance causes only
- b) Assignable causes only
- c) Either chance cause or assignable cause
- d) Both chance and assignable causes

[15×2 = 30]

- 2) The thickness of a printed circuit board is an important quality characteristic. Data on board thickness (in inches) are given in the following Table for 25 samples of three boards each.

Data on thickness of printed circuit board

Sample number	$X_1$	$X_2$	$X_3$
1	0.0629	0.0636	0.0640
2	0.0630	0.0631	0.0622
3	0.0628	0.0631	0.0633
4	0.0634	0.0630	0.0631
5	0.0619	0.0628	0.0630
6	0.0613	0.0629	0.0634
7	0.0630	0.0639	0.0625
8	0.0628	0.0627	0.0622
9	0.0623	0.0626	0.0633
10	0.0631	0.0631	0.0633
11	0.0635	0.0630	0.0638
12	0.0623	0.0630	0.0630
13	0.0635	0.0631	0.0630
14	0.0645	0.0640	0.0631
15	0.0619	0.0644	0.0632
16	0.0631	0.0627	0.0630
17	0.0616	0.0623	0.0631
18	0.0630	0.0630	0.0626
19	0.0636	0.0631	0.0629
20	0.0640	0.0635	0.0629
21	0.0628	0.0625	0.0616
22	0.0615	0.0625	0.0619
23	0.0630	0.0632	0.0630
24	0.0635	0.0629	0.0635
25	0.0623	0.0629	0.0630

- Investigate whether the process was in control or not based on the trial control limits and existence of non-random patterns of variation.
- Draw the  $\bar{X}$ -R chart and prescribe the central lines and control limits for future control purposes.
- Obtain the process capability index  $C_p$  of the process if the specification limits are  $0.0631 \pm 0.00047$ .
- Considering an underlying normal distribution find the percentage of non-conforming thickness measurements for the specification limits given above.

[10+10+4 +6= 30]

- 3) Consider an experiment in which three operators were asked to make two measurements on each of 10 different parts using the same measuring instrument. Each part is measured at the same location. The part specification is LSL = 20 and USL = 40. The results are shown below:

Part	Operator 1		Operator 2		Operator 3	
	Measurements		Measurements		Measurements	
	A-1	A-2	B-1	B-2	C-1	C-2
1	31.83	31.67	32.55	32.07	31.99	32.23
2	30.31	30.47	30.63	30.39	30.15	30.15
3	29.51	29.83	29.83	29.67	29.51	29.83
4	29.43	29.27	29.83	29.35	29.27	29.51
5	29.91	30.23	30.07	29.67	29.83	29.27
6	30.47	30.31	30.87	30.87	30.31	30.31
7	28.79	28.79	29.03	28.71	28.23	28.47
8	29.99	29.67	30.07	29.91	29.67	29.83
9	28.55	28.63	28.95	28.79	28.55	28.23
10	29.35	29.11	29.83	29.59	29.59	29.51

- a) Compute the P/T ratio based on the  $\bar{X}$ -R methodology and comment about the adequacy of gage capability.
  - b) Estimate the product variance.
  - c) How much percentage of gage variability can be attributed to product variability? [20+5+5 = 30]
- 4) Establish the parameters of the  $\bar{X}$ -S control chart considering that no standard is known for the process centering as well as the process standard deviation and that these must be estimated by analyzing past data.

[10]

**INDIAN STATISTICAL INSTITUTE**  
**M. Tech (QR & OR) I Year 2016-2017**  
**Semestral Examination**

**Subject : Reliability - I**

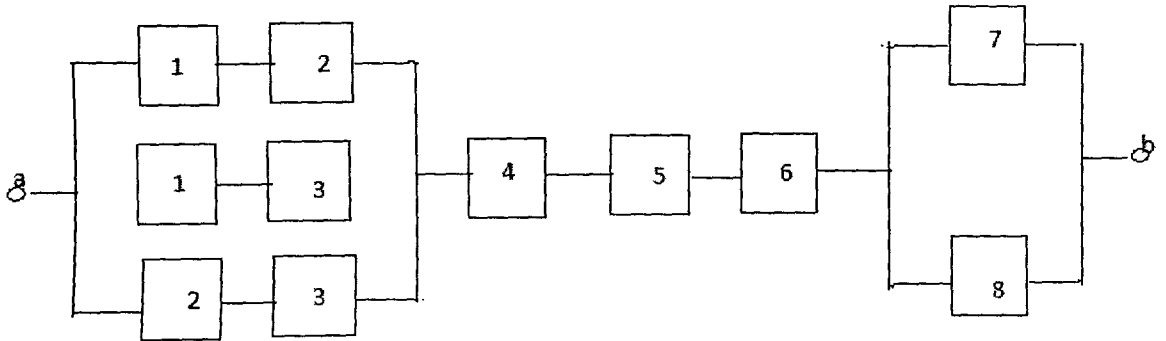
**Date : 17.04.2017**

**Full Marks : 100**

**Duration : 3 hrs.**

**Note: This paper carries 105 marks. You may answer as much as you can; but the maximum you can score is 100.**

1. Figure 1 below shows the reliability block diagram of a simplified automatic alarm system for gas leakage. In the case of gas leakage, “connection” is established between the points (a) and (b) so that at least one of the alarm bells (7 and 8) will start ringing. The system has three independent gas detectors (1, 2 & 3), configured as a 2-out-of-3 subsystem, is connected to a voting unit (4); i.e. at least two detectors must indicate gas leakage before an alarm is triggered off. Component 5 is a power supply unit, and component 6 is a relay.



**Figure 1:** Reliability block diagram of a simplified automatic alarm system for gas leakage

- a) Find the minimal path and cut sets.  
 b) Write down the structure function of the system.  
 c) Let the component reliabilities at time  $t_0$  be as follows:-

$$p_1 = p_2 = p_3 = 0.93$$

$$p_4 = 0.98$$

$$p_5 = p_6 = 0.97$$

$$p_7 = p_8 = 0.90$$

If the components are independent, find the system reliability at time  $t_0$ .

- d) Suppose that the state variables of the system are associated but not necessarily independent. Determine the tightest possible bounds for the system reliability.

[7+5+3+10=25]



2. a) The following data concern times to failure observed during testing of 25 of a particular engine component:

0.44, 2.41, 3.07, 3.08, 3.14, 3.20, 3.92, 4.29, 4.51, 4.98, 5.12, 5.59, 5.85, 5.96, 6.01.

The test was terminated at the time of the 15<sup>th</sup> failure.

- i. Find an estimate for the reliability at time  $t = 5$ .
  - ii. Write down the algebraic expression (clearly explaining your notation) for obtaining a 95% lower confidence limit for the reliability at time  $t = 5$ .
- b) If the strength  $S$  and the load  $L$  are both exponentially distributed and are assumed to be independent, then find  $P(S > L)$ .

[(10+5)+10=25]

3. a) Consider a 2-unit standby redundant system with a switch. Suppose unit 1 is the active unit and unit 2 is the standby unit. The active unit is under surveillance by a switch, which activates the standby unit when the active unit fails. The probability that the switch is unable to activate the standby unit is  $p$ . The active unit has a constant failure rate  $\lambda_1$ , while the failure rate of the standby unit when activated is  $\lambda_2$ . Assume that the failure rate of unit 2 in standby position is negligible. The three units operate independently. No repairs are carried out. Find the reliability of the system at time  $t$  and the mean time to system failure.

- b) Consider the block diagram shown in Fig 2. Find the reliability of the system if the reliability of each of the individual components is 0.9.

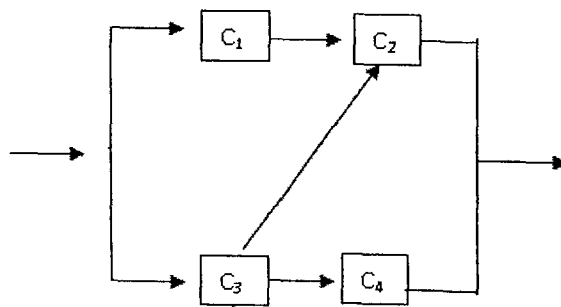


Figure 2: Block diagram

[(10+5)+10=25]

4. The first 8 observations in a random sample of 12 lifetimes from an assumed exponential distribution with mean  $\theta$  are, in hours,  
31, 58, 157, 185, 300, 470, 497, 673.
- i. Find the MLE of  $\theta$ .
  - ii. Find an exact two-sided 95% confidence interval for  $\theta$ .
  - iii. Find a 0.95 lower confidence limit on the reliability at  $t = 500$ .
  - iv. Test the hypothesis that  $\theta = 1000$  hours against the alternative that  $\theta < 1000$ .

[5 × 4 = 20]

5. The CDF for times to failure for a system is given by

$$F(t) = 1 - \frac{100}{(t + 10)^2}; \quad t \geq 0.$$

- a) What is its failure rate as a function of time?
- b) To which non-parametric class can this distribution belong?
- c) Suppose the system cannot run with reliability below 95%. What is the maximum time the system can be operated?

[4+1+5=10]

# INDIAN STATISTICAL INSTITUTE

Second Semester Examination 2016 - 17

M. Tech. (QR OR); I Year; E-Stream

## ELEMENTS OF STOCHASTIC PROCESSES

Date : 19.04.2017

Maximum Marks : 100

Time : 3 hours

### Notes:

- (i) Unless stated otherwise, "Markov Chain" will mean a discrete time parameter, time-homogeneous Markov Chain.
  - (ii) The symbols have their usual meanings.
  - (iii) Answer all questions.
- (1) Explain the following :
- (a) Irreducible Markov Chain.
  - (b) First Passage Probabilities.
  - (c) Recurrence and Transience.
  - (d) Essential and Inessential states of a Markov Chain.
  - (e) Positive and Null Recurrence. (5 × 5)=[25]
- (2) (i) State the Basic Renewal Equation for a Markov Chain (No proof is required).
- (ii) Show that  $f_{ij}^{(n+1)} = \sum_{\{k \in I | k \neq j\}} p_{ik} f_{kj}^{(n)}$  for all  $i, j \in I$  and  $n = 1, 2, \dots$  (2+13)=[15]
- (3) Let  $\{X_n | n = 0, 1, 2, \dots\}$  be a Markov chain with state space  $I$  and transition matrix  $P$ . Define  
 $N_j(n) =$  Number of visits to state  $j$  over the time span  $\{0, 1, 2, \dots, n\}$ .  
 $L_{ij}(n) = E(N_j(n) | X_0 = i)$ ;  $n = 0, 1, 2, \dots$  and  $i, j \in I$   
 $L(n) = \left( (L_{ij}(n))_{i, j \in I} \right)$ ;  $n = 0, 1, 2, \dots$  [20]  
Show that  $L(n) = P^0 + P^1 + P^2 + \dots + P^n$
- (4) (a) Show that  $g_{ij}(m+1) = f_{ij}^* g_{jj}(m)$  for  $m = 0, 1, 2, \dots$  and  $i, j \in I$ .  
(b) Using the result in (a), find an expression for  $P(N_j = m | X_0 = i)$  for  $m = 1, 2, \dots$  (8+12)=[20]
- (5) (a) Show that Recurrence is a class property.
- (b) Consider a Markov Chain  $\{X_n | n = 0, 1, 2, \dots\}$  with state space  $I$  and transition matrix  $P$ . Let  $i$  be a recurrent state. Denote by  $T_1$  and  $T_2$  the times of first and second visits to state  $i$  respectively.  
Show that  $P(T_1 = m | X_0 = i) = P(T_2 - T_1 = m | X_0 = i)$  for all  $m = 1, 2, \dots$  and interpret the result. (8+12)=[20]

**INDIAN STATISTICAL INSTITUTE**  
**Second End-Semester Examination (2016 – 2017)**

Course Name : M.Tech (QR & OR)  
Subject : Industrial Engineering and Management  
Date : 21.4.2017  
Maximum Marks : 60  
Duration : 180 Minutes

---

**Question Paper**

---

**Direction: Question 1 is compulsory. Attempt Any two questions from rest.**

1. (a) What do you mean by Capital budgeting? Explain the need and importance of Capital budgeting.
- (b) Represent a typical income statement of an organization.
- (c) A choice is to be made between two competing project proposals which require an equal investment of Rs 50,000 and are expected to generate net cash flows as under.

<u>Year</u>	<u>Project I</u>	<u>Project II</u>
	Rs	Rs
1	25,000	10,000
2	15,000	12,000
3	10,000	18,000
4	Nil	25,000
5	12,000	8,000
6	6,000	4,000

Which project proposal should be chosen and why? Evaluate the project proposals under discounted cash flow and payback period methods considering cost of capital as 10%.

[2+3+5+20]

2. (a) What do you mean by EOQ? What are the underlying assumptions of EOQ model?  
 (b) A company requires 30,000 pens in a year. The pens cost Rs 12 each. The transportation and ordering cost is Rs 200 per order. The annual cost of holding one pen in stock is estimated to be Rs 1.20.  
 A 2% discount is available on orders of at least 5,000 pens and a 2.5% discount is available if the order quantity is 7,500 pens or above.  
 Calculate the EOQ ignoring the discount and determine if it would change once the discount is taken into account? [2+3+10]
3. (a) Develop the relation of required number of observations to predict the true time within  $\pm 10\%$  precision and 95% confidence level?  
 (b) How is important the human factor engineering in manufacturing industries?  
 (c) Why time study is important to improve the productivity of an organization? [5+5+5]
4. One of the four ovens at a bakery is being considered for replacement. Its salvage value and maintenance costs are given in the table below for several years. A new oven costs Rs 80,000 and this price includes a complete guarantee of the maintenance costs for the first two years, and it covers a good proportion of the maintenance costs for years 3 and 4. The salvage value and maintenance costs are also summarized in the table.

Year	Old Oven		New Oven	
	Salvage Value at End of Year (Rs)	Maintenance Cost (Rs)	Salvage Value at End of Year (Rs)	Maintenance Cost (Rs)
0	20000	----	80000	----
1	17000	9500	75000	0
2	14000	9600	70000	0
3	11000	9700	66000	1000
4	7000	9800	62000	3000

Both the old and new ovens have similar productivities and energy costs. Should the oven be replaced this year, if the cost of capital equals 10%? [15]

# INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2016 – 2017  
M.Tech (QROR), First Year (E-Stream)

## Statistical Methods – II

Date: 24.04.2017

Maximum Marks: 100

Duration: 3 Hours

*Note: Answer only five questions.*

1. a) Write down the expression for simple linear regression model involving one independent variable, explaining the assumptions made.

b) Show that for such a model, expected value of error sum of square is  $E(SS_E) = (n - 2)\sigma^2$ .

c) A study was conducted to investigate the relationship between purity of oxygen ( $y$ ) produced in a chemical distillation process and the percentage of hydrocarbon ( $x$ ) that are present in the main condenser of the distillation unit. Summary quantities are:

$$n = 20, \quad \sum_{i=1}^{20} x_i = 23.92, \quad \sum_{i=1}^{20} y_i = 1843.21,$$
$$\sum_{i=1}^{20} x_i^2 = 29.2892, \quad \sum_{i=1}^{20} y_i^2 = 170044.5321, \quad \sum_{i=1}^{20} x_i y_i = 2214.6566$$

Calculate the least square estimates of the slope and intercept.

[(2+2)+11+5=20]

2. a) Show that for a multiple regression model involving  $p$  regressor variables, and  $n (> p)$  observations,  $(x_{i1}, x_{i2}, \dots, x_{ip}, y_i)$ ,  $i = 1, 2, \dots, n$ , the least square estimator of the regression coefficient vector  $\mathbf{b}$  is given by  $\hat{\mathbf{b}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$ , where  $\mathbf{y}$  is a column vector of order  $n$ ,  $\mathbf{X}$  is the design matrix of order  $(n \times (p + 1))$  and  $\mathbf{b}$  is column vector of order  $p + 1$ .

b) Show that the expression for variance of residuals for multiple linear regression is  $Var(e_i) = \sigma^2(1 - h_{ii})$ , where  $h_{ii}$  is the  $i$ -th diagonal element of the hat matrix,  $H$ .

c) Starting with multiple linear regression model, derive the expression for standardized regression model under unit normal scaling.

[10+5+5=20]

3. An experiment was performed to determine if either firing temperature or furnace position affects the baked density of a carbon anode. A factorial experiment was performed assuming both furnace positions and firing temperatures to be fixed and the experiment was replicated thrice. The tests were made in random order and the data below resulted.

Furnace Position	Firing Temperature ( $^{\circ}\text{C}$ )		
	800	825	850
1	570	1063	565
	565	1080	510
	583	1043	590
2	528	928	526
	547	1026	538
	521	1004	532

- a) Write down the appropriate statistical model and obtain the expected mean square for the model components.  
 b) Analyze the data and offer your conclusion.  
 c) Use Fisher's Least Significance Difference method to determine which level of firing temperature is significantly different from others.

[5+10+5=20]

4. a) What do you mean by Nonparametric Statistical Inferences? What are its advantages?  
 b) In a paint manufacturing process, drying agents are used to speed up the drying process. A chemical engineer, with a view to determine the effect two drying agents, selected 36 panels and painted 16 panels with paints containing drying agent 1 and 20 panels with paints containing drying agent 2 and recorded the drying time. Data thus obtained are given below.

Panel	Drying Agent		Panel	Drying Agent	
	1	2		1	2
1	68	73	11	65	72
2	64	62	12	59	60
3	68	66	13	78	78
4	82	92	14	67	66
5	58	64	15	65	68
6	80	87	16	76	77
7	72	77	17		72
8	65	70	18		86
9	84	88	19		72
10	73	79	20		97

Assume the two distributions to be continuous having same shape and spread. Do the data indicate that Drying Agent 1 results in reduced drying time? Use Mann-Whitney U test with  $\alpha = 0.05$ .

[(4+6)+10=20]

5. a) Age and percentage body fat were measured in 18 adults and is given in the table below:

Age	% Body fat	Age	% Body fat	Age	% Body fat
39	31.4	45	27.4	27	17.5
23	9.5	56	32.5	41	25.9
50	31.1	23	27.9	58	33.8
61	34.5	60	41.1	49	25.2
27	7.8	58	33.0	53	34.7

- i) Compute Kendal's  $\tau$ .
- ii) Do the data indicate a significant association between the variables?  
[Use the critical value for the test statistic as 39]
- b) To examine whether physical exercise alleviate depression, 24 equally depressed people were selected and were randomly divided into 3 groups of size 8 each. Each group was randomly subjected to one of the three exercise regime: no exercise, 20 minutes jogging per day and 60 minutes jogging per day. At the end of a month, each participant was asked to rate how depressed they feel now, on a Likert Scale that runs from 1 ("totally miserable") through to 100 ("ecstatically happy"). Data thus obtained is given below:

No Exercise	20 minutes jogging	60 minutes jogging
23	22	59
26	27	66
51	39	38
49	29	49
58	46	59
37	48	60
29	49	56
44	65	62

Use Kruskal-Wallis procedure to test whether the three exercise regime have same mean depression level or not. Use  $\alpha = 0.05$ .

$$[(8+2)+10=20]$$



6. a) Given below the height of six adult people in kg.

73    83    70    88    67    77

Can it be assumed that above data comes from a Normal population? Use Anderson Darling Test and critical value for Anderson Darling statistic as 0.752.

b) Six different machines are being considered for use in manufacturing rubber seals. The machines are being compared with respect to tensile strength of the product. A random sample of three seals from each machine is used to determine whether the mean tensile strength varies from machine to machine. The following are the tensile-strength measurements in kilograms per square cm  $\times 10^{-1}$ .

<b>Machine Number</b>					
<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
17.5	16.4	20.3	15.5	18.9	18.3
16.9	18.1	18.6	16.7	19.2	17.2
17.3	15.4	17.1	18.5	20.5	20.1

Perform the analysis of variance at the 0.05 level of significance and indicate whether or not the mean tensile strengths differ significantly for the six machines.

[12+8=20]

**INDIAN STATISTICAL INSTITUTE**  
**Second Semester Examination 2016-17**

**Course Name** : **M. TECH (QR-OR)-I**  
**Subject Name** : **MECHANICAL ENGINEERING**  
**Date:** 24.4.17 **Maximum Marks:** 100 **Duration:** 3 hours  
**Note, if any** :

*Answer any five questions.*  
*Assume suitable data if necessary.*

1. a) Define tolerance. What are unilateral and bilateral tolerances? State and explain the different types of fit in hole basis system. Give necessary sketches.  
b) Explain the different terms of assembly 30 H9/f6.  
c) What is meant by machining accuracy? Sketch to show some of the form errors (at least three).  
d) Explain economically feasible accuracy. 5 + 5 + 7 + 3
2. a) State Taylor's principle for limit gauge design. Sketch a limit plug gauge and label it.  
b) What are effects of surface roughness on the performance of a machined component?  
c) With the help of a block diagram discuss the manufacturing process. 8 + 6 + 6
3. a) Explain the following:  
i) Toughness  
ii) True strain  
iii) Hardness  
iv) Ductility  
b) Show the variation of net interatomic force with interatomic distance and explain it. Also discuss about the mechanics of slip in perfect crystal. 10 + 10
4. a) What are forward and backward extrusion processes? Explain them with necessary sketches.  
b) What is meant by High-Energy-Rate-Forming process? With suitable examples explain its necessity.  
c) Distinguish between hot working and cold working of materials. 8 + 6 + 6
5. a) Explain the rolling process. Discuss about the shape of the rolls.  
b) With help of neat sketch(es), explain the deep drawing process.  
c) Neatly sketch a drawing die and discuss the wire drawing process. 8 + 6 + 6
6. a) What is a taper? How is it expressed? Calculate the small end diameter of M80 X 150 taper.  
b) Calculate the change gears for cutting 12 tpi threads on a job in a lathe having 4 tpi lead screw. Give neat sketch of the set up. 10 + 10
7. a) Define cutting speed in metal cutting. State the factors on which it depends.

P. T. O

- b) Write the differences between up cut milling and down cut milling operations.
- c) A grinding wheel is specified as

300 X 80 X 20 W A-60-L-7-V 17.

Explain all the terms of it.

- d) Write the differences between
  - i) Turning operation and shaping operation
  - ii) Shaping machine and planing machine.

**5 + 5 + 5 + 5**

8. a) What is tool life? When operating with roughing cuts on mild steel at 20 m/min, a certain tool gave a tool life of 170 min between regrinds. Estimate the life of this tool on similar cuts at a speed of 24 m/min. Take the exponent of tool life as 1/8.
- b) Discuss the different modes of tool failure in metal cutting operation.
  - c) What are the different types of chips in metal cutting? Explain them.
  - d) Write the assumptions for developing Merchant' Circle Diagram.

**5 + 5 + 5 + 5**

**INDIAN STATISTICAL INSTITUTE**  
**Second Semester Examination: 2016-17**  
**Course Name: M. Tech. (QROR) I Year**  
**Subject Name: Statistical Quality Control**

Date: 27/04/2017

Maximum Marks: 100

Duration: 3 hours

[All Symbols Have Usual Meaning.]

**Answer Any Five Questions**

1. a) Consider a normally distributed quality characteristic of a process with  $\mu = 20$  and  $\sigma = 2$ . The lower and upper specification limits for the associated quality characteristic are  $LSL = 8$  and  $USL = 32$  respectively. Set up the parameters of an appropriate control chart for monitoring the mean with sample size  $n = 4$ , if the mean is allowed to drift to the tune of  $1.5\sigma$ .
- b) A process of interest is in a state of statistical control and generates continuous random measurements for a measured characteristic  $X$  from a normal distribution with a known expected value  $\mu$  and a known standard deviation  $\sigma$ . Consider that the measured characteristic has lower and upper specification limits ( $LSL$  and  $USL$  respectively). For such a process prove that  $C_{pk} = (1-k)C_p$ . Where  $C_p$  is process potential index,  $C_{pk}$  is process performance index and  $k$  is the process centering index.
- c) Explain how one can estimate process capability indices for many of the quality characteristics for which it is desirable to have a skewed data.  
[8+8+4 = 20]
2. a) A normally distributed process has specifications of  $LSL = 75$  and  $USL = 85$  on the output. A random sample of 25 parts indicates that the process is centered at the middle of the specification band and the standard deviation  $s = 1.5$ .
  - (i) Find a point estimate of  $C_p$ .
  - (ii) Find a 95% confidence interval on  $C_p$  and comment on the width of this interval.
- b) Assume that the quality characteristic of interest  $X \sim N(\mu, \sigma^2)$ . Let the lower and upper specifications for  $X$  be  $[L, U]$  and the potential process capability index of the process is  $C_p$ . If the process is operating at the mid-point of the specification limits then prove that the expected proportion of conforming items is given by  $2\Phi\left(\frac{3C_p}{2}\right) - 1$ .
- c) Prove that for an  $\bar{X}$ -chart with sample size  $n$  and  $k\sigma$  control limits, if the mean shifts from the in-control value  $\mu_0$  to another value  $\mu_1 = \mu_0 + \delta\sigma$ , then the probability of non-detection of the shift is  $\beta = \Phi\left(k - \delta\sqrt{n}\right) - \Phi\left(-k - \delta\sqrt{n}\right)$ .  
[(2+8)+6+4 = 20]

P.T.O

3. Suppose  $X$  is a measured characteristic with  $U$  and  $L$  as the upper and lower specification limits and  $\mu$  and  $\sigma$  are the mean and standard deviation of the process. The values of  $X$  outside these limits will be termed as nonconformance. Prove that the proportion of nonconformance (NC) is given by the formula

$$NC = \Phi\left[-3\left(2C_p - C_{pk}\right)\right] + \Phi\left(-3C_{pk}\right)$$

where  $\Phi(\bullet)$  is the cumulative distribution function of standard normal distribution.

[20]

4. In a plant producing pumps, one of the critical components is piston, the production process of which is found to be in statistical control. A key characteristic of the piston is its outside diameter (OD), the variation (s.d.) of which is found to be 0.2 mm based on an analysis of the past data. The upper and lower specification limits of the piston OD are 6.5 mm and 5.5 mm respectively. The cost of rejection due to under-specification and over-specification has been worked out to be Rs. 1000 per piston and Rs. 10 per piston respectively. The profit per piston is Rs. 100 provided the pistons conform to specifications. Considering the underlying distribution of OD to be normal, arrive at the economic process centre with appropriate formulation of the problem.

[20]

5. Consider the simple DC circuit components where the current ( $I$ ) flows across the points (a, b) through a linear path with specifications  $25 \pm 1$  amp overcoming a resistance ( $R$ ) with specifications  $4 \pm 0.06 \Omega$ . The voltage across the points (a, b) is required to be  $100 \pm 2$  V. Assume that  $I$  and  $R$  are normally and independently distributed and their upper and lower specification limits coincide with their respective natural tolerance limits.

- Find the probability of conformance to specification for voltage
- Compute the process capability index ( $C_p$ ) for voltage and comment.

[14+6=20]

6. The underlying distribution for an attribute sampling plan is binomial with sample size  $n$  and proportion defective  $p$ . The lot from which samples are taken, will be accepted if the number of defectives in the sample does not exceed the acceptance number  $c$ .

- Prove that the probability of acceptance is an increasing function of  $c$
- Prove that the probability of acceptance is a decreasing function of  $n$
- Prove that the probability of acceptance is a decreasing function of  $p$ . Also show that

the point of inflection for the associated operating characteristic curve is  $p = \frac{c}{n-1}$ .

[2+8+(6+4)=20]

INDIAN STATISTICAL INSTITUTE  
 Second Semester Examination (Back Paper): 2016 – 17  
 M. Tech (QROR), First Year (E-Stream)  
 Statistical Methods – II

Date: 12.07.17

Maximum Marks: 100

Time: 3 Hours

Note: Answer all questions.

1. (a) In a random sample of 85 automobile engine crank-shaft bearings, 10 have a surface finish roughness that exceeds the specification. Does this data present strong evidence that this proportion of crankshaft bearings' surface finish roughness exceeds 0.10? State and test the appropriate hypothesis using  $\alpha = 0.05$ .
- (b) Let  $X$  denote the number of flaws observed on a large coil of galvanized steel. Seventy-five coils are inspected and the following data are observed for the values of  $X$ .

$X$	1	2	3	4	5	6	7	8
Observed Frequency	1	11	8	13	11	12	10	9

Based on these 75 observations, is a Poisson distribution an appropriate probability model? Perform a goodness-of-fit test with  $\alpha = 0.01$ .

[10+10=20]

2. The strength of paper ( $y$ ) used in the manufacture of cardboard boxes is related to the percentage of hardwood concentration ( $x$ ) in the original pulp. 16 samples are manufactured, each from different batch of pulp, in a pilot plant under controlled condition. The hardwood concentration of the pulp and the tensile strength of the resulting samples are measured. The data are shown below.

$x$	1.0	1.5	1.5	1.5	2.0	2.0	2.2	2.4
$y$	101.4	117.4	117.1	106.2	131.9	146.9	146.8	133.9
$x$	2.5	2.5	2.8	2.8	3.0	3.0	3.2	3.3
$y$	111.3	123.0	125.1	145.2	134.3	144.5	143.7	146.9

- a) Fit a simple linear regression model to the data.  
 b) Calculate coefficient of determination for the fitted model.  
 c) Test for lack of fit and significance of regression.  
 d) Find the 90% confidence interval on the slope.

[6+2+8+4=20]

3. a) Given below two sample data sets taken from two continuous populations. Use Kolmogorov-Smirnov test to check whether that the two populations from which the samples are taken are identical in nature or not. Use  $\alpha = 0.05$ .

Sr. No.	Set 1	Set 2	Sr. No.	Set 1	Set 2	Sr. No.	Set 1	Set 2
1	1.26	2.37	6	1.55	0.23	11	0.15	27.44
2	0.34	2.16	7	0.08	1.32	12	0.49	4.51
3	0.70	14.82	8	1.42	2.91	13	0.95	0.51
4	1.75	1.73	9	0.50	39.41	14	0.24	1.50
5	50.57	41.04	10	3.20	0.11	15	6.98	14.68

- b) The number of pounds of steam used per month by a chemical plant is thought to be related to the average ambient temperature (in °F) for that month. Last 12 months data are shown in the following table.

Month	Temp	Usage	Month	Temp	Usage
1	21	185.79	7	68	621.55
2	24	214.47	8	74	675.06
3	32	288.03	9	62	562.03
4	47	424.84	10	50	452.93
5	50	454.58	11	41	369.95
6	59	539.03	12	30	273.98

Calculate Kendall's rank correlation statistic and hence show that there exists a positive association between the variables.

[Value of T for  $n = 12$  and  $\alpha = 0.05$  is 28]

[10 + 10 = 20]

4. a) Show that if there are no tied ranks, Kruskal-Wallis test statistic  $H$  for testing the equality of medians of  $p$  groups of sample data can be represented by

$$H = \frac{12}{N(N+1)} \sum_{i=1}^p \frac{R_i^2}{n_i} - 3(N+1)$$

where  $R_{ij}$  is the rank of observation  $y_{ij}$ ,  $n_i$  is number of observation in the  $i$ -th group and  $N = \sum_{i=1}^p n_i$ .

- b) The mean axial stress of the alloys used in aircraft structures is being studied. Two alloys are being investigated. Alloy 1 is a traditional material, and alloy 2 is a new-aluminum-lithium alloy that is much lighter than the standard material. Ten specimen of each alloy type are tested, and the axial stress is measured. The sample data are given below.

Axial Stress in psi			
Alloy 1		Alloy 2	
3238	3254	3261	3248
3195	3229	3187	3215
3246	3225	3209	3226
3190	3217	3212	3240
3204	3241	3258	3234

Using Mann-Whitney U test and  $\alpha = 0.05$ , test the hypothesis that the means of the two stress distributions are identical

[10+10=20]

5. a) In an experiment conducted to determine which of the three missile systems is preferable, the propellant burning rate of 24 static firings was measured. Four different propellant types were used. The data on propellant burning rate, after suitably coding, are given in the following table.

Missile System	Propellant Type			
	1	2	3	4
1	34.0	30.1	29.8	29.0
	32.7	32.8	26.7	28.9
2	32.0	30.2	28.7	27.6
	33.2	29.8	28.1	27.8
3	28.4	27.3	29.7	28.8
	29.3	28.9	27.3	29.1

- i) Do the factors or their interactions affect the mean burning rate? Use  $\alpha = 0.05$ .  
 ii) Draw the average response diagram for significant effects.



5. b) A study was performed on a type of bearing to find the relationship of amount of wear ( $y$ ) to oil viscosity ( $x_1$ ) and load ( $x_2$ ). The following data were obtained.

$y$	$x_1$	$x_2$
193	1.6	851
172	22.0	1058
113	33.0	1357
230	15.5	816
91	43.0	1201
125	40.0	1115

- i) Estimate coefficients of the multiple linear regression equation

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon.$$

- ii) Predict wear when oil viscosity is 20 and load is 1200.

$$[(7+3)+(9+1)=20]$$

**INDIAN STATISTICAL INSTITUTE**  
**M. Tech (QR & OR) I Year 2016-2017**  
**Backpaper Examination**

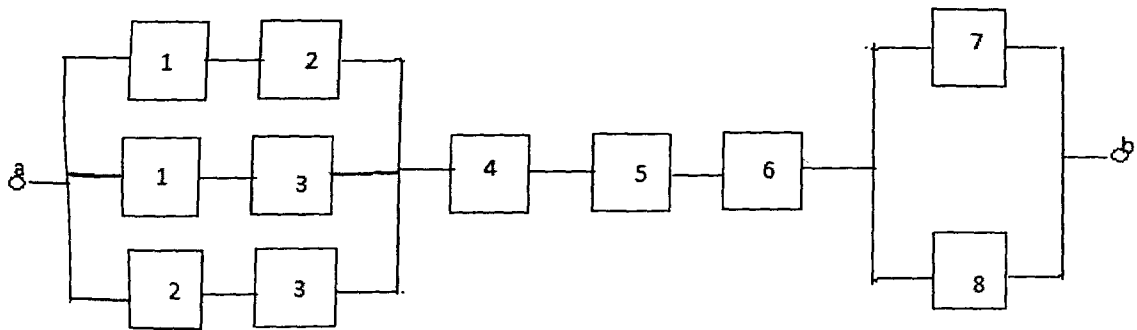
**Subject : Reliability - I**

**Date : 14.07.2017**

**Full Marks : 100**

**Duration : 3 hrs.**

1. Figure 1 below shows the reliability block diagram of a simplified automatic alarm system for gas leakage. In the case of gas leakage, “connection” is established between the points (a) and (b) so that at least one of the alarm bells (7 and 8) will start ringing. The system has three independent gas detectors (1, 2 & 3), configured as a 2-out-of-3 subsystem, and is connected to a voting unit (4); i.e. at least two detectors must indicate gas leakage before an alarm is triggered off. Component 5 is a power supply unit, and component 6 is a relay.



**Figure 1:** Reliability block diagram of a simplified automatic alarm system for gas leakage

- a) Find the minimal path and cut sets.
- b) Write down the structure function of the system.
- c) Let the component reliabilities at time  $t_0$  be as follows:-

$$p_1 = p_2 = p_3 = 0.93$$

$$p_4 = 0.98$$

$$p_5 = p_6 = 0.97$$

$$p_7 = p_8 = 0.90$$

- If the components are independent, find the system reliability at time  $t_0$ .
- d) Suppose that the state variables of the system are associated but not necessarily independent. Determine the tightest possible bounds for the system reliability.

[7+5+3+10=25]

2. a) The following data give remission times for a group of 21 patients who had been administered a particular drug for acute leukemia:

6, 6, 6, 6\*, 7, 9\*, 10, 10\*, 11\*, 13, 16, 17\*, 19\*, 20\*, 22, 23, 25\*, 32\*, 32\*, 34\*, 35\*.

The censored observations have been marked with a star (\*).

- i. Find an estimate for the survival at time  $t = 15$ , i.e.  $\hat{S}(15)$ .
- ii. Obtain an estimate of the variance of  $\hat{S}(15)$ .

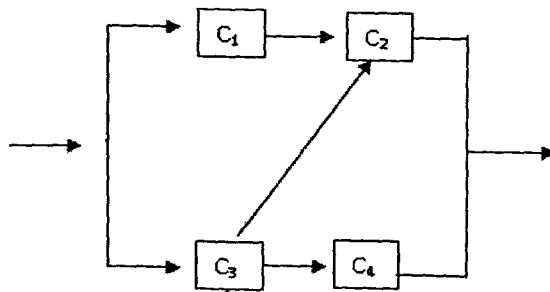
- b) An electronic circuit consists of 10 silicon diodes, 4 silicon transistors, 20 composition resistors and 10 ceramic capacitors, all connected in series. Assume that the wiring (printed circuit) and the solder connections are perfectly reliable, and that the components are independently exponentially distributed with failure rates (per million hours):

Silicon diode:  $\lambda_D = 2$ , Silicon transistor:  $\lambda_T = 10$ , Composition resistor:  $\lambda_R = 1$ , Ceramic capacitor:  $\lambda_C = 2$ .

What will be the distribution of the system life of this electronic circuit?

**[(8+7)+10=25]**

3. a) Write down the structure function corresponding to a system consisting of six components in such a way that the system functions if and only if components 1 and 6 both function, and at either 2 and 4 both function or 3 and 5 both function. Find the minimal path sets and minimal cut sets of this system.
- b) Consider the block diagram shown in Fig 2. Find the reliability of the system if the reliability of each of the individual components is 0.9.



**Figure 2: Block diagram**

**[(5+5+5)+10=25]**

4. Consider a situation in which pieces of equipment were installed in a system at different times. At a later date some of the pieces had failed and the rest were still in use. With the aim of studying the lifetime distribution of the equipment, the following data were collected:

**Table: Lifetimes for 10 Pieces of Equipment**

Sl. No	1	2	3	4	5	6	7	8	9	10
<b>Installation Date</b>	11/6	21/6	22/6	2/7	21/7	31/7	31/7	1/8	2/8	10/8
<b>Failure Date</b>	13/6	-	12/8	-	23/8	27/8	14/8	25/8	6/8	-
<b>Lifetime (days)</b>	2	$\geq 72$	51	$\geq 60$	33	27	14	24	4	$\geq 21$

The first item was installed on June 11 and data were collected upto August 31. At that time, three items (SL. Nos 2, 4, 10) has still not failed and their failure times are therefore censored.

Assuming that the exponential distribution  $f(t; \theta) = \theta^{-1} \exp(-t/\theta)$  is an appropriate model, find an estimate of  $\theta$ . Also find a two-sided 95% confidence interval for  $\theta$ .

[8+7=15]

5. a) Consider a parallel system with identical components each with reliability 0.8. If the reliability of the system is to be at least 0.99, find the minimum number of components in this system.
- b) What are the different shapes that a failure rate can take? Give examples.

[5+5=10]

**INDIAN STATISTICAL INSTITUTE**  
**Back Paper Examination (2<sup>nd</sup> semester) : 2016 – 17**

**M. Tech. (QR OR); I Year; E-Stream**

**ELEMENTS OF STOCHASTIC PROCESSES**

Date : 07/08/2017

Maximum Marks : 100

Time : 3 hours

Notes:

- (i) Unless stated otherwise, "Markov Chain" will mean a discrete time parameter, time-homogeneous Markov Chain.
  - (ii) The symbols have their usual meanings.
  - (iii) Answer all questions.
- (1) Explain the following :
    - (a) Stochastic process.
    - (b) Markov process.
    - (c) Chapman-Kolmogorov equation.
    - (d) Stationarity of transition probabilities.
    - (e) First passage probabilities. (5 × 5)=[25]
  - (2) Let  $\{X_n | n = 0, 1, 2, \dots\}$  be a Markov chain with state space  $I = \{0, 1, 2\}$ , initial distribution  $\pi = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$ , and transition matrix  $P = \begin{bmatrix} 1/4 & 3/4 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/4 & 3/4 \end{bmatrix}$ .
    - (i) Compute  $P(X_0 = 0, X_1 = 1, X_2 = 1)$ .
    - (ii) Compute  $P(X_1 = 1, X_2 = 1 | X_0 = 0)$ .
    - (iii) Compute  $p_{01}^{(2)}$ . (6+6+8)=[20]
  - (3) (i) State the Basic Renewal Equation for a Markov chain (No proof is required).
    - (ii) Show that  $f_{ij}^{(n+1)} = \sum_{\{k \in I | k \neq j\}} p_{ik} f_{kj}^{(n)}$  for all  $i, j \in I$  and  $n = 1, 2, \dots$  (5+15)=[20]
  - (4) Let  $\{X_n | n = 0, 1, 2, \dots\}$  be a Markov chain with state space  $I$  and transition matrix  $P$ . Define
 

$N_j(n)$  = Number of visits to state  $j$  over the time span  $\{0, 1, 2, \dots, n\}$ .

$L_{ij}(n) = E(N_j(n) | X_0 = i)$ ;  $n = 0, 1, 2, \dots$  and  $i, j \in I$

$L(n) = ((L_{ij}(n)))_{i, j \in I}$ ;  $n = 0, 1, 2, \dots$

Show that  $L(n) = P^0 + P^1 + P^2 + \dots + P^n$ . [15]
  - (5) (a) Show that  $g_{jj}(m+1) = f_{jj}^* g_{jj}(m)$  for  $m = 0, 1, 2, \dots$  and  $i, j \in I$ .
    - (b) Using the result in (a), find an expression for  $P(N_j = m | X_0 = i)$  for  $m = 1, 2, \dots$  (8+12)=[20]