INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2016-2017, First Semester
MS (QE) I
Mathematical Methods

Date: 05/9/16 Max. Marks 40 Duration: $2\frac{1}{2}$ Hours

Note: Answer all questions, maximum you can score is 40.

1. Prove or give counterexample:

   a) Let V be a vector space and $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ be a basis of V. Let U be a subspace of V such that $\mathbf{v_1} \in U$, $\mathbf{v_2} \in U$ but $\mathbf{v_3} \notin U$ and $\mathbf{v_4} \notin U$. Then $\{\mathbf{v_1}, \mathbf{v_2}\}$ is a basis of U.

   b) Trace of a nilpotent matrix is zero.

   c) There exists a nonsingular idempotent matrix which is not identity.

   d) $\mathbf{A}_{n \cdot n}$ is a singular matrix if and only if one of its eigenvalues is zero.

   e) Let V and W be two vector spaces and U be a subspace of V. If T is a linear map from U to W then there exists a linear map $\tilde{T}$ from V to W such that $\tilde{T}$ extends T, i.e., the restriction of $\tilde{T}$ to U is same as T.

   [2x5]

2. Let $\mathcal{P}_4(\mathbf{R})$ be the space of polynomials with real coefficients, and of degree at most 4. Let $U = \{p \in \mathcal{P}_4(\mathbf{R}) : p'(5) = 0\}$ where $p'(5)$ is the derivative of p evaluted at 5.

   a) Find a basis of U.

   b) Extend this basis to that of $\mathcal{P}_4(\mathbf{R})$.

   c) Find a subspace W of $\mathcal{P}_4(\mathbf{R})$ such that $\mathcal{P}_4(\mathbf{R}) = U \oplus W$.

   [4+3+3]

3. a) Find a rank factorization of the following matrix:

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 7 & 3 \\ 1 & 5 & 3 \end{bmatrix}$$

b) If $\mathbf{A}_{n \cdot m}$ is a matrix of rank r, and if $\mathbf{P}_{n \cdot n}$ is a non-singular matrix, then show that $\mathbf{PA}$ also has rank r.

[5+5]

4. Consider the following input-output matrix

$$\mathbf{A} = \begin{bmatrix} .1 & .2 & .1 \\ .5 & .3 & .2 \\ .3 & .2 & .4 \end{bmatrix}$$

and the consumer demand vector

$$\mathbf{c} = \begin{bmatrix} 10 \\ 20 \\ 20 \end{bmatrix}$$

Find the corresponding industry output.

[10]

5. Show without explicitly calculating the minimum, that the quadratic form $\mathbf{x^T A x}$ attains global minimum at $(0,0)^T$ if

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}.$$

[5]

INDIAN STATISTICAL INSTITUTE

MidSemestral Examination: (2016-2017)

M. S. (Q.E.) – I Yr.

Computer Programme and Application

Date: 06.09.2016 Maximum marks: 50 Duration: 2 hrs

1. Write brief answers. [20×2 = 40]
   a) What is the equivalent number in decimal system of the binary number 1010001?
   b) Write down the hexadecimal form of the binary number 10010.
   c) What are the basic hardware elements of a computer?
   d) Which part of the computer hardware does the fundamental processing?
   e) What fact is expressed by a statement like '2.5 GHz machine'?
   f) What is meant by a microprocessor's data width?
   g) Suppose two pieces of RAM chips have access times 40 nanoseconds and 60 nanoseconds. Is this information sufficient for comparing the actual speeds of the two RAM chips? Discuss.
   h) What do you understand by RAM latency?
   i) Why SRAM is more expensive than DRAM?
   j) What is the relation of FPU and GPU with ALU?
   k) Write two examples of system software.
   l) What is called a server? Name one computer server.
   m) What is the full form of HTTP? What is its use?
   n) Are the following two valid IP addresses? Justify your answer.
      (i) 192.168.256.12, (ii) 222.222.222.222
   o) Why IP is called a connectionless protocol?
   p) What is public cloud?
   q) Write advantages and disadvantages of flash memory over HDDs.
   r) What do you understand by a hierarchy of cash memories?
   s) What is the difference between 'high level language' and 'machine level language'?
   t) Give an example of a computational algorithm whose complexity is O(log n).

2. Draw a flow chart for the following algorithm. Write down its output. [8+2=10]

```
Step 1:  START = 1,
         INC   = 1,
         A(START) = 5,
         END = 20
Step 2: A(START+INC) = A(START+INC-1) + 10
Step 3: INC = INC + 1
Step 4: A(START+INC) = A(START+INC-1) + 5
Step 5: INC = INC + 1
Step 6: IF (INC > END) GO TO Step 8
Step 7: Go TO Step 2
Step 8: PRINT A
Step 9: STOP
```

3. Suppose A and B are two matrices of sizes $m \times n$ and $n \times p$. Write an algorithm to (i) read A, B, (ii) multiply A and B and (iii) print the result of the multiplication. [10]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2016-17

Course Name: M.S. (Q.E.) I YEAR

Subject Name: Game Theory I

Date: 8-09-2016 Maximum Marks: 40 Duration: 3 hours

Problem 1. Risk-discounted expectation of a random variable X is defined as: $\mathbb{E}(X) - \sqrt{V(X)}$ where $\mathbb{E}(X)$ and $V(X)$ are the expectation and variance of X respectively. Suppose utilities from mixed-strategies are calculated using risk-discounted expectation (instead of just expectation). Then, what happens to the

a) existence of mixed-strategy equilibrium, and

b) minmax theorem?

(10+10)

Problem 2. Consider the auction model where types are distributed independently and identically following uniform distribution over $[0, 1]$. Suppose the highest bidder gets the object and pays the average of highest and second-highest bids. Suppose further that everybody pays the lowest bid. Find all (if any) BNE of this game.

(10)

Problem 3. Consider the incomplete information bargaining game where $N = \{1, 2\}$ and $T_i = \{1, 0.5\}$ for $i = 1, 2$ with independent and identical distribution $(1/3, 2/3)$. The game goes as follows: First, player 1 makes an offer $(\alpha, 1 - \alpha)$ and player 2 accepts or rejects it. If player 2 accepts, then the game ends with utility vector $(\alpha, 1 - \alpha)$, otherwise player 2 makes a counter offer $(\beta, 1 - \beta)$. If player 1 accepts it, then the game ends with utility vector $(t_1\beta, t_2(1 - \beta))$ where $t_1 \in T_1$ and $t_2 \in T_2$, otherwise the game ends with utility vector $(1/4, 1/4)$. Formulate this as a game with incomplete information and find all BNE of this game. What happens when $T_i = \{\delta_1, \delta_2\}$ for $i = 1, 2$?

(10)

MS(QE) Statistics paper

I YEAR

Maximum marks: 100

Time: 3 hours

09/09/16

Class notes, books, calculator (but not mobile phone) are allowed.

1. Write down 20 data points where the top 20 percentile is less than the mean. Compute the mean deviation of this data. (15)
2. A data set containing 20 univariate values, has mean 7.2 and standard deviation 2.5. Later it was found that a data point 27 was recorded , by mistake, as 72. Find the true mean & standard deviation. (15)
3. Consider the following bivariate data (X, Y): (5,23), (7,53), (2,2), (10, 95), (2,5), (4,15), (6,37),(14,200),(3,10),(1,2).

   Fit a linear regression line of Y on X & also Y on X**2. Which one is a better fit? Justify. (15+15+5)
4. A district has 10 villages with number of households: 15, 100, 37, 29, 32, 18, 80, 37, 29,7 respectively. Draw a sample of size 3 using SRSWOR and PPSWOR. (10+10)
5. Consider the two regression lines: 3X+2Y+7=0 and 2X+4Y+1=0. One is Y on X & the other is X on Y, but it is forgotten as to which equation is for what. Can you determine? (15)

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2016-17

Course: Masters in Quantitative Economics Year I

Subject: Microeconomics I

Date: 13th September 2016 **Maximum Marks**: 40 **Duration:** 3 hours

ANSWER ALL FOUR QUESTIONS.

1. Prove that a (deterministic) choice function with complete domain satisfies Houthakker's WARP, if and only if it also satisfies Sen's α and β conditions. (10 marks)

2. Prove that (for a deterministic demand function) Samuelson's Weak Axiom of Revealed Preference is equivalent to Samuelson's Demand Inequality, given that the consumer always spends her entire wealth. (10 marks)

3. (i) Prove, for a deterministic supply function, that the Consistent Firm Choice and Non-reversibility conditions are, together, equivalent to the Weak Axiom of Profit Maximization. (5 marks)

   (ii) Construct a deterministic supply function that satisfies both cost minimization and the law of supply, but violates profit-maximization, for the general case of n commodities, $n \geq 2$. (5 marks)

4. In a world with exactly three goods, construct a *tight* and (non-trivially) stochastic demand function that satisfies the (stochastic) non-positivity of the own-price substitution effect, but violates the Weak Axiom of Stochastic Revealed Preference. Does your example satisfy Samuelson's Inequality in its stochastic version? Explain your claim. (10 marks)

Indian Statistical Institute

Mid-Semester Examination

Course Name: MSQE 1st Year & MStat 2nd Year 2016

Subject: Basic Economics

Full Marks -40 Time: 2 hrs

Answer all questions. Date : 14.09.2016

1. The following data regarding a firm in a given year are specified below:
(All figures are in lakhs of rupees)

Revenue earned from the sale of (5/6)th of output (The remaining part was unsold)	600
Raw materials purchased from other firms	100
Unused part of raw materials	20
Interest paid to households	5
Payment made to a labour contractor for supplying labour	5
Travelling and hotel expenses of the officials of the company	1
Land purchased by the company for construction of an additional shed	100
Wages and salaries	2
Depreciation	1
Dividend paid	100
Tax on profit	1
Subsidy received by the firm	2
Donations made to the chief minister's relief fund	1

From the data given above compute the firm's contribution to

(i) NDP
(ii) National Income
(iii) Personal Income, Private income
(iv) Aggregate final expenditure [5x4 = 20]

2. (i) Is it possible for an economy to absorb more than what its purchasing power can command of the world NDP? Explain your answer. Also discuss the financial aspects.

(ii) Suppose in an economy in a given year the central bank had to sell foreign exchange worth Rs. 20,000 crore from its stock to hold the exchange rate at the target level. Some firms in the domestic economy borrowed from foreign financial institutions Rs. 30,000 crore, while some foreign firms borrowed Rs.12,000 crore from domestic financial institutions. In additions foreigners purchased shares of domestic companies worth Rs.8000 crore. Domestic residents purchased land abroad worth Rs.200 crore. Domestic government's budget deficit was Rs.500 crore. Domestic households' expenditure on produced goods and services was Rs.90,000 crore of which Rs.10,000 was spent on buying houses from construction companies. From the data given above, compute the economy's private disposable income in the given period [8+12=20]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2016-17

Course Name: M.S. (Q.E.) I YEAR
Subject Name: Game Theory I
Date: 15-11-2016 Maximum Marks: 50 Duration: 3 hours

Problem 1. Justify your answer by a proof or a counterexample.

(a) Let b be a behavioral strategy-tuple in an extensive form game. Then, there is a *unique* belief system β such that (b, β) is consistent.

(b) Let G be a zero-sum game with at least three strategies for each player. Suppose G does not have any pure strategy Nash equilibrium. Then, every correlated Nash equilibrium is a mixed strategy Nash equilibrium.

(c) Let G be the game as follows:

$$\begin{array}{cc} & \begin{array}{cc} C & B \end{array} \\ \begin{array}{c} C \\ B \end{array} & \begin{pmatrix} 1,1 & 3,0 \\ 0,3 & 2,2 \end{pmatrix} \end{array}$$

Consider the infinitely repeated game $G^{\infty}(\delta)$ with discount factor δ. Then, for all $\delta \geq 0.5$, there is a mixed strategy Nash equilibrium of $G^{\infty}(\delta)$ where the pay-off of each player at each stage is strictly higher than 2.

(d) Every symmetric game has a symmetric Nash equilibrium. (4 × 10)

Problem 2. Consider the following game:

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & J \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ J \end{array} & \begin{pmatrix} -1,1 & 1,-1 & 1,-1 & -1,1 \\ 1,-1 & -1,1 & 1,-1 & -1,1 \\ 1,-1 & 1,-1 & -1,1 & -1,1 \\ -1,1 & -1,1 & -1,1 & 1,-1 \end{pmatrix} \end{array}$$

Justify your answer by a proof or a counterexample.
(i) This is a symmetric game.
(ii) In every mixed strategy Nash equilibrium of this game, each player assigns the same probability to the actions 1, 2, and 3. (2+8)

Indian Statistical Institute

Semester I Examination 2016-2017

M. S. (Q. E.) - I year

Subject: Computer Programming and Applications 18.11.2016

Full Marks: 100 Duration: 3 hrs.

(Answer all questions)

1. (i) What is the main task of Internet Protocol? (ii) Write one or two sentences describing the functioning of Internet Protocol. (iii) What is the first major version of this protocol? (iv) What is its next major version? (v) What are the most complex aspects of this protocol? (vi) What is an IP address? (vii) Can you assign an IP address to a computing device according to your own choice? Explain using one or two sentences.
(2+2+1+1+2+1+1 = 10)

(i) When will you call a given set of instructions an algorithm? (ii) What measures are frequently used to characterize the effectiveness of a given algorithm? (iii) State a problem and its solution algorithm whose execution time grows linearly with the size of the input. (5+2+3 = 10)

3. (i) What is the maximum value that can be stored in an unsigned char type variable of C programming language? (ii) What is the minimum value that can be stored in an unsigned long type variable of C programming language? (iii) Which operator or function of C language can be used to know the size of a data type? (iv) What is the difference between global ana local variables of C? (v) What is the difference between array and structure data types of C? ($5 \times 2 = 10$)

4. (i) Will both the following two programs behave similarly? Justify.

(*a*)
```
#include <stdio.h>
extern int i = 10;
main(){
printf("%d\n", i);
}
```

(b)
```
#include <stdio.h>
main(){
extern int i = 10;
printf("%d\n", i);
}
```

(ii) What are the differences between the use of an array and a pointer?
(iii) Write a C function to compute the length of a character string without using any string library function. (2 + 3 + 5 = 10)

5. (i) What is the use of structure data type in C language? (ii) Write an example of a structure type C variable having an array as its member. (iii) Write an example of an array of structure type variable in C programming language. (2 + 4 + 4 = 10)

6 (i) Will the following two programs behave identically? Justify.

```
#include <stdio.h>
#define MAX 3
 int main () {
  int var[3] = {50, 100, 200};
  int i=0;
    do{
    printf("Value %d =  %d\n", i, var[i] );
    } while(++i<MAX);
  return 0;
 }
```

```
#include <stdio.h>
#define MAX 3
int main () {
  int var[3] = {50, 100, 200};
  int i=0;
    while (++i<MAX){
    printf("Value %d =  %d\n", i, var[i] );
       }
  return 0;
}
```

P.T.O

(ii) What will be the output of the following program?

```
#include <stdio.h>
#define MAX 3
 int main () {
  int var[3] = {50, 100, 200};
  int i, *ptr;

  ptr=var;
  for ( i = 0; i < MAX; i++) {
    printf("Value %d = %d\n", i+1, *ptr++ );
  }
  return 0;
}
```

(4 + 6 = 10)

7. Write a C program which will take two character strings and an integer denoting a position of the first string as command line arguments and create an output string (to be printed on the screen) by inserting the second string into the provided position of the first string. Thus, *if* the first and second strings have '*m*' and '*n*' numbers of characters, the output string should have '*m+n*' characters. As for example, when you enter "*a* ape pl 3" at the command prompt, your program's output should be "apple", where "*a*.exe" is the executable file. (20)

8. Write a C program which will take two integer numbers as its input and print their GCD and LCM as its output. (10)

9. Write a C program, which will take two integer numbers '*n*' and '*r*' as its input and print the value of ${}^{n}P_{r}$. Your program should use a recursive function call. (10)

Indian Statistical Institute
First semester Examination 2016
Course Name: MSQE First Year and M - Stat Second Year
Subject Name: Basic Economics

Date of Examination: 18/11/2016 **Maximum Marks – 60** **Duration: 2.5 Hours**

Answer all questions

1. Consider the following information regarding a simple Keynesian model for an open economy with government: $mpc_{yd} = 0.8, m = 0.1, t = 0.5, i = 0.6, \rho = 0$ and $\overline{T} - \overline{R} = 40$.

(i) State the assumptions needed to regard $(Y - T + R)$ as personal disposable income.

(ii) Using the expression for personal disposable income given in (i), compute the required value of $(a + \overline{T} + \overline{G} + \overline{X} - \overline{M})$ that will make the equilibrium Y equal to 1600 units.

(All notations in the above question have their usual meanings and all relations in the model are linear) [6 + 16 = 22]

2. (i) Consider a Simple Keynesian Model for an open economy without government activities. Suppose 20 per cent and 60 per cent of aggregate planned consumption expenditure and aggregate planned investment expenditure, respectively, are spent on imported goods. Suppose the marginal propensity to consume and marginal propensity to invest (net) with respect to NDP (Y) are 0.8 and 0.3, respectively. Given the above information, answer the following questions under the assumption that all the relations in the model are linear:

a. Compute the marginal propensity to import with respect to Y and check if the equilibrium is stable.

b. Compute the autonomous expenditure multiplier in this model.

(ii) Suppose in a simple Keynesian model for a closed economy with government C = 0.75Y, G = 0 and I = 2000 – 100i, i_c= 8, m = 1 and m_d = 1 (where i_c denotes the repo rate and m_d and m are the relevant mark-ups; all other symbols have their usual meanings). Write down the equation of the aggregate demand function. Derive the equilibrium value of Y. To attain full employment, Y has to rise by 2000 units. By how much the central bank has to lower the repo rate to achieve full employment? **[7+3+12=22]**

3. (i) Consider a simple Keynesian model for a closed economy with government. Present the model in terms of saving and investment. Indicate the two components of aggregate planned saving.

(ii) The tax function is given by $T = \overline{T} + tY$. Suppose the government raises $\overline{T}$. What impact is it likely to have on private/personal saving, revenue deficit of the government and total saving of the economy?

(iii) Suppose the government intends to reduce revenue deficit by reducing $\overline{G}$. What kind of impact is it likely to have on Y and revenue deficit? In the light of the results derived, comment on the Fiscal Responsibility and Budget Management Act (2003) of Government of India.

(Assume government transfers to be a lump sum in answering the question). **[6+8+8=22]**

M. S. (QE) I Yr.
STATISTICS

MAXIMUM MARKS : 100

Books and Notes are allowed Date: 21-11-2016

1. Following are the measurements of heights of soybean plants in a field, a different random selection each week.

Age in weeks	(x)	1	2	3	4	5	6	7
Height in centimeter	(y)	5	13	16	23	33	38	40

Assuming a quadratic regression of Y on X, what will be the expected height after 8 weeks?

[16]

2. Suppose a random variable X, has the following density function.

$$f(x) = \left(\frac{3}{7}\right) x^2 \text{ if } 1<x<2$$
$$= 0 \text{ otherwise}$$

a) Is it a valid density function?
b) Find out the distribution function of X.
c) Find out the mean and median of X.

[16]

3. Let $X \sim Poi(\lambda)$, λ finite, unknown.
Consider the following estimators S(x) and T(x)

$$S(x) = 1 \text{ if } x \text{ is even}, \quad = 0 \text{ if } x \text{ is odd}$$

$$T(x) = 1 \text{ if } x \text{ is even}, \quad = -1 \text{ if } x \text{ is odd}$$

Are these two estimates unbiased for $e^{-2\lambda}$? Justify.
Which is the better one? Justify.

[20]

P.T.O

4. Suppose that X_1, X_2, X_3,X_4 are iid from a N(θ, 4) population where θ is unknown

Test H_o : θ=2 vs. H_1 : θ=5

Consider the following procedures.

1. Reject H_o if $\frac{X1+X3}{2} > 4.2$

2. Reject H_o if $\frac{X1+X2+X3+X4}{4} > 4.1$

Find Type I & Type II error probabilities for both the tests and compare.

[16]

5. A population has 1000 individuals and their average spending on movies is to be estimated in such a way that the estimate is NOT more than 10% far away from the actual value, with probability at least .90. The CV of the population for this attribute is known to be 40%. An SRSWOR of size n has to be drawn. What should be the minimum possible value of n?

[12`]

6. Consider the following RBD with the yields given as in the table :

Blocks\Varieties	A	B	C	D
1	10	5	8	7
2	20	7	15	12
3	30	4	12	14
4	30	2	10	12

Write down the ANOVA table and find out if the effect of all 4 variables are the same?

[20]

INDIAN STATISTICAL INSTITUTE
Final Examination: 2016-17

Course name: MSQE I
Subject name: Microeconomics I
Date: 24.11.2016
Maximum marks: 60
Duration: 3 hours

Answer all questions

Q1. Suppose there is a finite set of prizes Z. Consider a decision maker who prefers lottery p to lottery q over this set iff $U(p) \geq U(q)$. In each of the three cases given below, determine if the preferences satisfy the von Neumann-Morgenstern Independence axiom. Argue rigorously.

a) A number $v(z)$ is attached to each prize $z \in Z$ and
$U(p) = min_{z \in Z}\{v(z)|p(z) > 0\}$. (10)

b) A number $v(z)$ is attached to each prize $z \in Z$ and
$U(p) = \sum_{z \in Z} p(z)v(z)$. (10)

c) $U(p) = \alpha \sum_{z \in Z} zp(z) - \beta \sum_{z \in Z}\{z - \sum zp(z)\}^2 p(z)$, $\alpha, \beta > 0$. (10)

Q2. Consider a Cournot oligopoly with n identical firms. Firms have an increasing cost function $C(q) = cq^2, c > 0$, where q is a firm's output. The inverse demand function is $p = a - bQ$, where p is price, Q is industry output, and a and b are positive.

a) What quantities do the firms produce in equilibrium? (5)

b) What is the equilibrium price? (5)

c) What is the consumers' surplus in equilibrium? (5)

d) Suppose n, the number of firms, grows without bound. Does the equilibrium price converge as n tends to infinity? If so, what level does the price converge to? (5 + 10)

INDIAN STATISTICAL INSTITUTE
Semester Examination: 2016-2017, First Semester
MS(QE) I
Mathematical Methods

Date: 28-11-16 Max. Marks 60 Duration: $3\frac{1}{2}$ Hours

Note: 1. Answer all questions

2. Total Marks: 68. Maximum you can score: 60.

1. a) Let

$$A := \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

Find its characteristic polynomial; its eigenvalues and eigenspace corresponding to each eigenvalue. Show that A is diagonalizable.

[5]

b) Show that

$$A := \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

is not diagonalizable.

[5]

c) Use diagonalization to find A^{200} where

$$A := \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

[5]

P. T. O

2. Let I and II be two commodities whose respective demands q_1 and q_2 are given by

$$q_1 = 4p_1^{-2}p_2^{4/3}y^2; \quad q_2 = 3p_1p_2^{-3/2}y^3$$

where p_1 and p_2 are unit prices of I and II respectively and y is the income. Suppose at present $p_1 = 5$, $p_2 = 3$ and $y = 10$. If p_1 decreases by .1, p_2 increases by .15 and income y falls by .2, then find the changes in demands q_1 and q_2.

[10]

3. a) If $\{x_n\}_{n=1}^{\infty}$ is a sequence such that $x_{n+1} = \sqrt{2 + \sqrt{x_n}}$ and $x_1 = \sqrt{2}$, then show that $\{x_n\}_{n=1}^{\infty}$ converges to a limit l (say) and $0 < l < 2$.

b) Find the lim sup and lim inf of the sequence $\{x_n\}$ where $x_1 = 0$, $x_{2k} = \frac{x_{2k-1}}{2}$ and $x_{2k+1} = \frac{1}{2} + x_{2k}$ $\forall k = 1, 2, \ldots$

c) Show that if $\sum_{k=1}^{n} a_k$ converges and if $a_k \geq 0$ $\forall k$ then $\sum_{k=1}^{n} \sqrt{a_k}/k$ converges.

[4x3]

4. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function and $U \subset \mathbf{R}$ an open set. Then show that $f^{-1}(U)$ is an open set. Hence show that $f(K)$ is compact if $K \subset \mathbf{R}$ is a compact set.

[5+5]

5. Give Kuhn-Tucker formulation of the following optimization problem:

Maximize $f(x, y, z) = xyz + z$, subject to the constraints $x^2 + y^2 + z \leq 6$, $x \geq 0,\ y \geq 0,\ z \geq 0$.

[5]

6. a) Give Euler's equation for the following optimization problem and then solve it:

Maximize $\int_0^1 [2(x'(t))^2 + 5tx(t)]dt$, subject to $x(0) = .1$ and $x(1) = .4$ where $x(t)$ is a real-valued twice continuously differentiable function defined on real line.

b) Solve by optimal control: Maximize $\int_0^1 (2x(t) + 3u(t))dt$, subject to $x'(t) = 1 - 2(u(t))^3$, $x(0) = 1$, where $x(t)$ is the state variable and $u(t)$ control variable, twice continuously differentiable. [8+8]

INDIAN STATISTICAL INSTITUTE
Back paper Examination: 2016-17

Course name: MSQE I
Subject name: Microeconomics I
Date: 30 · 12 · 2016
Maximum marks: 100
Duration: 3 hours

Answer all questions

Q1. a) Consider the following (lexicographic) preference relationship $\succ$ defined on R^2_+. For all $x, y \in R^2_+$, $[x \succeq y \text{ iff } x_1 > y_1 \text{ or } (x_1 = y_1 \text{ and } x_2 \geqslant y_2)]$. Show that $\succeq$ is an ordering. (10)

b) Explain whether the following statement is true, false or uncertain: if a binary relation $\succeq$ defined over a set S is an ordering, then for every non-empty subset A of S, the choice set $C(A, \succeq)$ must be non-empty. (10)

Q2. a) Prove that, for a tight stochastic demand function, the Weak Axiom of Stochastic Revealed Preference is equivalent to Samuelson's Inequality. (10)

b) State and establish the demand theorem from this result. (10)

Q3. a) Suppose given a set of prizes Z a decision maker prefers lottery p to lottery q if $U(p) \geq U(q)$, where

$$U(x) = \sum_{z \in Z} zx(z) - (\sum_{z \in Z} zx(z))^2 - \sum_{z \in Z}\{z - \sum zx(z)\}^2 x(z).$$

Do these preferences satisfy the von-Neumann Morgenstern Independence axiom? (10)

b) Suppose $U(.)$ from above is modified as follows:

$$U(x) = \sum_{z \in Z} zx(z) - (\sum_{z \in Z} zx(z))^2 - \alpha \sum_{z \in Z}\{z - \sum zx(z)\}^2 x(z), \alpha > 0$$

How do your conclusions from (a) above change if $\alpha \neq 1$? Why? (10 + 10)

Q4. Consider an asymmetric Cournot oligopoly with three firms 1, 2 and 3 facing an inverse demand function is $p = a - bQ$, where p is price, Q is industry output, and a and b are positive. Firms have linear cost functions, with firm i's cost of producing amount q being $c_i q$. Suppose $c_1 > c_2 = c_3 > 0$, so firms 2 and 3 are low cost firms, while firm 1 is a high cost firm.

a) Under what conditions will firm 1 produce a positive amount in equilibrium? (5)

b) Show that in any equilibrium with positive industry output, firm 1's production will be less than that of firms 2 and 3. (5)

c) Assuming all firms produce a positive amount, what are the equilibrium production levels for the three firms? (10)

d) In such an interior equilibrium, what is the equilibrium price? (5)

e) Show that any such interior equilibrium is inefficient in the sense that alternative production vectors could be found simultaneously increasing consumers' surplus and joint firm profit, at least one strictly. (5)

Indian Statistical Institute

Mid-semester Examination 2017

M.S.(Q.E)-I Year

Course name: **Political Economy**

Subject name: **Economics**

Date: **20 February 2017**

Maximum marks: **50**

Duration: **2 hours**

1. This question pertains to indirect aggregation of individual preferences in the context of a model of representative democracy as discussed in class. The standard assumptions of such a model remain the same, namely, there are n individuals in the economy each having well-behaved preferences over policy denoted by $\{\succsim_1, \succsim_2, \cdots, \succsim_n\}$. Moreover these preferences satisfy *Extremal Restriction* (ER). There are two candidates/parties/representatives, A and B, who try to maximize objective functions u_A and u_B. Let the objective function candidate A be as follows:

$$u_A(x_A, x_B) = \#\{i : x_A \succ_i x_B\} + \frac{1}{2}\#\{i : x_A \sim_i x_B\}.$$

We can define $u_B(x_A, x_B)$ likewise. Let $G := (\{A, B\}, \{X, X\}, \{u_A, u_B\})$. Consider the mixed extension of G and denote it by G'. Then show that

(i) G' has a Nash equilibrium. **(5 points)**

(ii) Any Nash equilibrium (p_A, p_B) of G' has support$(p_k) \subsetneqq X^*$, $k = A, B$, where X^* is the set of Condorcet winners in X. **(15 points)**

2. This question deals with lobbying under three possible states of the world, namely θ_L, θ_M and θ_H, $\theta_L < \theta_M < \theta_H$, as discussed by Grossman and Helpman. Consider a single policy variable and a single lobby to answer the following questions:

(i) Let $\theta_L = 1, \theta_M = 7$, and $\theta_H = 11$. Recall $\delta > 0$ to be the size of divergence between the ideal policies of the policy maker and the lobby. What is the range of δ under which a full-revelation/fully-separating equilibrium is sustainable? **(14 points)**

(ii) Let $\theta_L = 1, \theta_M = 7$, and $\theta_H = 11$. What is the range of δ under which a partial-revelation/semi-separating equilibrium is sustainable? **(14 points)**

(iii) What happens to the range of δ that can be sustained in equilibrium when you move from an equilibrium in (i) to an equilibrium in (ii)? **(2 points)**

INDIAN STATISTICAL INSTITUTE
MID-SEMESTRAL EXAMINATION: 2016-2017
M.S. (Q.E.) I, II Years and M. Math. I Year
Game Theory II

Date: 20.02.2017 Maximum Marks: 100 Time: 3 hours

1 (a). When do you say that a pay-off vector is reasonable? Interpret your definition.

(b) Demonstrate the relationship between the core and reasonable set of a game?

© Prove or disprove the following: The converse of 1(b) is true as well. (2+2+6+8)

2. (a) Clearly provide an argument in favor of a stable set as a solution concept. Define a stable set of a general n-person game. Prove a sufficient condition for uniqueness of a stable set of a game. (You must define and prove here all necessary preliminaries that you require for proving this condition.)

(b) Prove or disprove the following: The only stable set of the 3-person majority game is {(0.5, 0.5,0), (0.5,0,0.5), (0,0.5,0.5)}.

(2+2+6+ 6)

3. (a)Define a market game rigorously by giving necessary preliminaries. Why should such a game exist? Provide precise arguments.

(b)Explain the role of the market.

©. Show that the core of a market game is non-empty. (Clearly state any result that you use here.)

(d) Use the same result that you employed in Question No. 3© for demonstrating non-emptiness of the core of a market game to establish that the core of the non-liability game related to the chemical firms- laundry game is empty.

(2+2+2+9+4)

4. In a 3-person buyer-seller game, the seller owning the good thinks that it is worthless to him. Of the two buyers, the first thinks that it is worth 100 and the other thinks that it is worth 50. Determine the characteristic function and the core of the game. (7+10)

5. (a) Show that the concept 'strategic equivalence' is an equivalence relation.

P.T.O

(b) An essential game has a unique representation in terms of a particular type of game. Identify the 'type' and demonstrate your claim. (6+8)

6. Do you agree or disagree with the followings? :

(a) The core of a game is a non-convex set.

(b) Superadditivity property of a game remains invariant under strategic equivalence. (5+7)

7. Give examples of non-negative valued strict subadditive and strict superadditive functions defined on the non-negative orthant of the real line. Establish your arguments indicating that the two different functions you consider satisfy respective properties. (2+2)

INDIAN STATISTICAL INSTITUTE

Mid-semestral Examination (2016-2017)

MS(QE) I

Microeconomics II

Date: 21.02.2017 Maximum Marks: 100 Duration: 3 hours

(1) Show that if a preference relation R on the commodity space $X \subset \Re^L$ can be represented by a utility function, then R must be rational. **(10)**

(2) Define continuity of a preference relation R on the commodity space $X \subset \Re^L$. Show that if $u(.)$ is a continuous utility function representing R on X, then R must be continuous. **(5+10=15)**

(3) Suppose that $f(.)$ is the production function associated with a single-output technology, and let $Y \subset \Re^L$ be the production set of this technology. Show that Y satisfies constant returns to scale if and only if $f(.)$ is homogeneous of degree one. **(12)**

(4) Suppose that the production set $Y \subset \Re^L$ is convex. Then every efficient production $y \in Y$ is a profit maximizing production for some non-zero price vector $p \geq 0$. **(13)**

(5) Consider an economy consisting of I consumers (indexed $i = 1, \ldots, I$), J firms (indexed $j = 1, \ldots, J$) and L commodities (indexed $l = 1, \ldots, L$). Each consumer i is characterized by a consumption set $X_i \subset \Re^L$ and a *rational* preference relation R_i defined on X_i. Each firm j has the production possibilities summarized by the production set $Y_j \subset \Re^L$. We assume that Y_j is non-empty and closed. The initial resources of commodities in the economy, that is, the economy's endowments are given by a vector $\hat{\omega} = (\hat{\omega}_1, \ldots, \hat{\omega}_L) \in \Re^L$. Thus, the basic data on preferences, technology, and resources for this economy are summarized by $(\{(X_i; R_i)\}_{i \in I}, \{Y_j\}_{j \in J}, \hat{\omega})$. Answer the following questions.

(a) Define the set of feasible allocations. Also define Pareto optimal and weak Pareto-optimal allocations. By imposing appropriate restrictions on the preferences, state and prove an equivalence result between Pareto-optimality and weak Pareto-optimality. **(2+4+14=20)**

(b) State and prove the second fundamental theorem of welfare economics by giving all the relevant definitions. **(30)**

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination : 2016-17
M.STAT. I YEAR, M.S.(QE) I and II YEAR
Optimization Techniques

Date: 22 February 2017 Maximum Marks: 75 Duration: $2\frac{1}{2}$ hours

---

Notation have usual meaning.

This paper carries 85 marks. However, maximum you can score is 75.

---

1 Two cities generate waste and their waste are sent to incinerators (furnaces) for burning. Daily waste production and distances among cities and incinerators are as below:

	Waste produced (ton/day)	Distance to incinerator (in km.)	
		A	B
City 1	500	30	20
City 2	400	36	42

Incineration reduces each ton of waste to 0.2 tons of debris, which must be dumped at one of the two landfills. It costs $3 per kilometer to transport a ton of material (either waste or debris). Distances (in km) among the incinerators and landfills are given below.

	Capacity (ton/day)	Incineration Cost ($/ton)	Distance to landfill (in km.)	
			Northern	Southern
Incinerator A	500	40	35	38
Incinerator B	600	30	51	48

Formulate a linear program that can be used to minimize the total cost of disposing waste of the cities. [15]

2 Let $\bar{\mathbf{x}}$ be an extreme point of a convex set $S(\subseteq R^n)$. Then $\bar{\mathbf{x}}$ lies on the boundary of S. [8]

3 Find all the extreme points and extreme directions of the polyhedral set given by:

$$\begin{aligned} x_1 + 2x_2 + \ x_3 &\leq 10 \\ -x_1 + 3x_2 \qquad &= \ 6 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

[12]

[P.T.O.]

4 Consider a maximization linear programming problem with extreme points $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ and $\mathbf{x}_4$, and extreme directions $\mathbf{d}_1$, $\mathbf{d}_2$ and $\mathbf{d}_3$, and with an objective function having cost vector $\mathbf{c}$ such that $\mathbf{c}^T\mathbf{x}_1 = 5$, $\mathbf{c}^T\mathbf{x}_2 = 7$, $\mathbf{c}^T\mathbf{x}_3 = 4$, $\mathbf{c}^T\mathbf{x}_4 = 7$, $\mathbf{c}^T\mathbf{d}_1 = 0$, $\mathbf{c}^T\mathbf{d}_2 = -3$ and $\mathbf{c}^T\mathbf{d}_3 = 0$. Is this problem unbounded or has an optimal solution? Hence, depending upon your answer, give either the set of all rays or the set of all optimal solutions. **[8]**

5 The following is the current simplex tableau (second iteration) of a given linear programming problem in canonical form with the objective to maximize $2x_1 - 3x_2$. The two constraints are of $\leq$ type with non-negative right-hand-sides. In the tableau, x_3 and x_4 are slack variables.

	x_1	x_2	x_3	x_4	RHS
z	b	1	f	g	6
x_3	c	0	1	$\frac{1}{5}$	4
x_1	d	e	0	2	a

Find the unknowns a through g above. **[12]**

6 Suppose that simplex algorithm terminates declaring Big-M problem as unbounded with all the artificial variables are at zero level in the current solution. Show that the original problem of interest is unbounded. **[10]**

7 State and prove the complementary slackness theorem. **[10]**

8 Consider the linear program (P): min $\mathbf{c}^T\mathbf{x}$ subject to $\mathbf{A}\mathbf{x} = \mathbf{b}_1$, $\mathbf{x} \geq 0$. Suppose that $\mathbf{u}_1$ is an optimal solution of P and $\mathbf{v}_1$ is optimal for its dual. Further, let $\mathbf{u}_2$ is an optimal solution of P when $\mathbf{b}_1$ is changed to $\mathbf{b}_2$. Show that

$$\mathbf{v}_1^T(\mathbf{b}_2 - \mathbf{b}_1) \leq \mathbf{c}^T(\mathbf{u}_2 - \mathbf{u}_1).$$

[10]

———*** xXx ***———

Indian Statistical Institute
Economic Development
Mid-Sem Examination
MSQE I & II

Date: 23.2.17

Time 2 hours

Answer both questions. Each question carries 20 marks.

1. Consider the model of occupational choice of Aghion and Bolton (1997) where agents can choose either to be self-employed with a low but certain income or to take up a risky project requiring a minimum amount of capital. In the latter project probability of success increases with privately observable efforts.
   (a) Find the wealth levels for which agents are credit rationed.
   (b) Show that if no one is credit rationed, all risky projects are funded in the long run, provided the rate of return on the project and the rate of savings (bequests) are high enough.
2. Show that if a small dose of noise is introduced into the History versus Expectations model of Krugman (1991), the problem of multiple equilibrium can be avoided.

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2016-17

MS (QE) I YEAR

Econometric Methods I

Date: 24 February 2017 Maximum Marks: 100 Duration: 3 hours

*[Note: Answer question 5 and any **two** from the rest of the questions]*

1. In the following simple linear regression model

$$y = X\beta + e = x_1\beta_1 + x_2\beta_2 + e,$$

where $e \sim$ i.i.d. $(0, \sigma^2)$, x_1 is a $(T\times1)$ vector of '1's, suppose $\hat{\beta}_1$ and $\hat{\beta}_2$ are LS estimators and $\bar{y}$ and $\bar{x}_2$ are the arithmetic means of y and x_2.

(a) Show that $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{x}_2$ and $\hat{\beta}_2 = \frac{\sum(x_{t2}-\bar{x}_2)(y_t-\bar{y})}{\sum(x_{t2}-\bar{x}_2)^2}$.

(b) Show that $(y-X\hat{\beta})'(y-X\hat{\beta}) = \sum(y_t - \bar{y})^2 - \frac{(\sum(x_{t2}-\bar{x}_2)(y_t-\bar{y}))^2}{\sum(x_{t2}-\bar{x}_2)^2}$.

(c) If you could select any values for the treatment variable, x_2; how could you choose the treatment values in order to minimize the sampling variance of $\hat{\beta}_2$?

(d) Show that $\text{Var}(\hat{\beta}_1) = \sigma^2[\frac{1}{T} + \frac{\bar{x}_2{}^2}{\sum(x_{t2}-\bar{x}_2)^2}]$,

$\text{Var}(\hat{\beta}_2) = \sigma^2/\sum(x_{t2} - \bar{x}_2)^2$,

and $\text{Cov}(\hat{\beta}_1,\hat{\beta}_2) = \sigma^2[-\bar{x}_2/\sum(x_{t2} - \bar{x}_2)^2]$.

(e) Simplify the expression of the idempotent matrix M. [8+8+2+12+10=40]

2. Define "perfect" and "absence of" multicollinearity and discuss their consequences. What are the effects of multicollinearity? How will you detect the existence of multicollinearity? Suggest some strategies to solve the problem of multicollinearity. Do you agree with the statement that "if all the simple correlations are small then the problem of multicollinearity will not arise"? Give explanations for your answer. [10+8+10+10+2=40]

3. Starting from the assumptions of first order autocorrelation of error terms in a linear regression model derive the dispersion matrix of the error vector and hence find the GLS estimate of the regression coefficient vector when the degree of correlation ρ is known. Discuss and compare different estimation procedures when ρ is unknown. [15+25=40]

[PTO]

4. Consider the regression model $y_t = \beta x_t + u_t$, $t = 1, 2, \ldots, T$.

(a) It is assumed that all the assumptions of CNLRM are satisfied. Find $\hat{\beta}_{OLS}$, its variance $V(\hat{\beta}_{OLS})$ and $\hat{V}(\hat{\beta}_{OLS})$.

(b) It was later discovered that all the assumptions of CNLRM are satisfied except for $E(u_t^2) = \sigma^2 x_t^2$. Find $\hat{\beta}_{GLS}$, its variance $V(\hat{\beta}_{GLS})$ and estimate of its variance $\hat{V}(\hat{\beta}_{GLS})$.

(c) Compare $V(\hat{\beta}_{OLS})$ with $V(\hat{\beta}_{GLS})$ and $\hat{V}(\hat{\beta}_{OLS})$ with $\hat{V}(\hat{\beta}_{GLS})$.

(d) Taking the same form of heteroscedasticity as in (b), find variance of $\hat{\beta}_{OLS}$ and estimate of its variance and name these as $V_G(\hat{\beta}_{OLS})$ and $\hat{V}_G(\hat{\beta}_{OLS})$.

(e) Compare $V_G(\hat{\beta}_{OLS})$ with $V(\hat{\beta}_{GLS})$ and $\hat{V}_G(\hat{\beta}_{OLS})$ with $\hat{V}(\hat{\beta}_{GLS})$. [8×5=40]

5. Write short notes on any two of the following:

(a) Durbin-Watson Test

(b) Problems with Economic Data and methods of refining data.

(c) Coefficient of determination R^2 and Adjusted R^2.

(d) Partial correlation coefficient.

(e) Different types of Economic Data. [10×2=20]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
MD-SEMESTRAL EXAMINATION: 2016 – 17
M.S.(Q.E.) 1st Year

Time Series Analysis & Forecasting

Date: 27.02.2017 | Maximum Marks: 50 | Time: 2 hours

This question paper carries a total of 60 marks. You can answer any part of any question. But the maximum that you can score is 50. Marks allotted to each question are given within parentheses.

1. (a) (i) Discuss what you understand by seasonality and noise in a time series.

   (ii) Describe an appropriate procedure for obtaining trend and seasonality of a non-stationary time series.

   (b) Let $\{x_t\}$ be a white noise process following normal distribution with mean μ and variance σ^2. Consider the time series $y_t = x_t\, x_{t-2}$. Determine the mean and the autocovariance function of $\{y_t\}$, and check whether it is weakly stationary. Is $\{y_t\}$ also strongly stationary? Justify your answer.

$[\overline{4+8}+8=20]$

2. (a) Find the ACF of the following process

   $$x_t = 2 + 1.3x_{t-1} - 0.4x_{t-2} + a_t\ \ a_t \sim WN(0,\sigma^2).$$

   (b) Find the coefficients $\phi_j, j = 0,1,2,\cdots$, in the time series representation $x_t = \sum_{j=0}^{\infty} \phi_j\, a_{t-j}$ of the ARMA (2, 1) process given by $(1-0.9B+0.18B^2)\, x_t = (1+0.4B)\,a_t$, where $\{a_t\} \sim WN(0,\sigma^2)$.

   (c) Let $\{x_t, t = 1,2,\cdots,n\}$ be the observed values of a time series at time points $1,2,\cdots,n$ and $\hat{\rho}(h)$ be the sample ACF at lag h. If $x_t = a + bt$, where a and $b\,(\neq 0)$ are constants, show that for each fixed $h \geq 1$,

   $$\hat{\rho}(h) \to 1 \text{ as } n \to \infty$$

[6+6+8 = 20]

P.T.O

3. (a) Let a time series $\{y_t\}$ be given by $y_t = \alpha + \Phi y_{t-1} + a_t$ where $a_t \sim$ white noise $(0, \sigma_a^2)$. It $\{x_t\}$ is another time series defined as $x_t = at^2 + s_t + y_t$ where a and b are constants and s_t is a seasonal component with period 12, then check whether $\Delta_{12}\,\Delta x_t$ is stationary or not under all possible values of Φ.

(b) Suppose that $\{x_t\}$ and $\{y_t\}$ are stationary processes satisfying $x_t - \alpha x_{t-1} = w_t$ and $y_t - \alpha y_{t-1} = x_t + z_t$

where $\{w_t\} \sim WN(0, \sigma^2), \{z_t\} \sim WN(0, \sigma^2), |\alpha| < |$ and w_t and z_t are uncorrelated. Find the ACF of $\{y_t\}$.

(b) Discuss briefly how sample ACF and sample PACF are used in correlogram analysis for deciding which of the standard stationary time series models is appropriate for a given time series.

[4+8+8 = 20]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2016-2017)
MS (Q.E.) I Year
Macroeconomics I

Date: 28·02·2017 Maximum Marks 40 Duration 3 hours

Group A
Answer any one

1. Describe the Ramsey-Solow model with its assumptions and equations. Show that the steady state equilibrium in this model is a saddle point. Why does this model fail to explain endogenous growth?

(12 + 6 + 2 = 20)

2. How does a 'learning by doing' model differ from a Ramsey-Slow model and how is it similar to a AK growth model? Describe this learning by doing model in terms of its equations and assumption; and then analyse how this model explains endogenous growth.

$(\overline{2+2} + 12 + 4 = 20)$

Group B
Answer all questions

1. In an appropriate new Keynesian model, derive the long run multiplier of a balanced budget increase in government expenditure and show that with variety effect absent, such multiplier is smaller than what would obtain in the short run.

(10)

2. Show that in the Blanchard and Kiyotaki model, a coordinated reduction in all prices and wages, beginning from a situation of monopolistic equilibrium, will raise real profits, and also utility.

(10)

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2016-2017)

MS (Q.E.) I Year

Macroeconomics I

Date: 21.04.2017 Maximum Marks 60 Duration 3 hours

Group A

Answer any two

1. (a) Analyse how efficiency wage hypothesis explains unemployment equilibrium in an one sector macro-static model.

   (b) Do you obtain any positive efficiency wage from the following efficiency function? Explain your answer.

   (i) $h(w) = w$

   (ii) $h(w) = \sqrt{w}$

$(9 + \overline{3 + 3} = 15)$

2. In a unionized labour market model, wage is rigid and an unemployment equilibrium always exists for any level of capital stock – Explain the validity of this statement using an appropriate model.

(15)

3. Develop an endogenous growth model with a public input stating its assumptions; and derive the properties of the optimum income tax rate.

$(8 + 7 = 15)$

Group B

Answer all questions

1. Show how an unanticipated increase in money supply makes the short run exchange rate overshoot its long run equilibrium value in the Dornbusch model, with sluggishly adjusting commodity price and an asset market that is continuously in equilibrium. Give an

explanation, why high interest response of money demand serves to dampen the extent of overshooting.

(12+3)

2. a) In a flex price, monopolistically competitive equilibrium of the Blanchard-Kiyotaki kind; show that money is neutral.

Also show that in such a model the monopolistically competitive output is smaller than the competitive output.

b) Consider an economy with the representative agent having the utility function:

$$U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}, 0 < \alpha, \gamma < 1$$

Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i{}^{\rho}\right]^{1/\rho}$, $0 < \rho < 1$ and c_i is the consumption of the i^{th} variety.

L is the labour supply, P is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. M is the money balances (and suppose M_0 is the initial endowment of money). The household budget constraint is given by:

$PC + w(1-L) + M = M_0 + w + \pi - T$ where w is the money wage rate and π is the economy wide profits and T is the taxes. Production of varieties is given by:

$$Y_i = 0 \; if \; L_i \le F$$
$$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

Y_i is the output of i^{th} variety and L_i is the labour employed in the production of the i^{th} variety.

Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed n).

(i) Derive the multiplier of a balanced budget ($PG=T$) increase in government expenditure where G takes the form:

$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$ and g_i is the government consumption of the i^{th} variety.

(ii) What would be the effect of such increase in government expenditure on P?

[Hint: Try to write down the goods market equilibrium ($Y=C+G$) in a form which does not involve money balances. That would require a look into the money market equilibrium ($M = M_0$).]

[10+5]

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2016-2017
M.S. (Q.E.) I, II Years and M. Math. I Year
Game Theory II

Date: 24.4.17 Maximum Marks: 100 Time: 3 hours

Note 1: Answer Parts (A) and (B) in separate answer scripts. Clearly explain the symbols you use and state all the assumptions you need for any derivation. Marks will be deducted substantially for any mistake you make in definitions and statements of assumptions, whenever you need them.

Note 2: The paper carries 110 marks. You may attempt any part of any question. The maximum you can score is 100.

A

1. Let $h = \{h_1, h_2, h_3, h_4\}$ be a set of four house owners whose preferences on the set of houses $H = \{H_1, H_2, H_3, h_4\}$ are given in the following table:

Table I : House Matching

h_1	h_2	h_3	h_4
H_3	H_1	H_1	H_2
H_2	H_2	H_4	H_3
H_1	H_4	H_2	H_1
H_4	H_3	H_3	H_4

Clearly establish how the house owners can be made better off through exchange of houses. (8)

2. Is the core of a bankruptcy game non-empty? Demonstrate your claim rigorously. (10)

3. Show that the Shapley value of a game is invariant under strategic equivalence. (5)

P.T.O

4. Define a voting game. Establish a necessary and sufficient condition for non-emptiness of the core of such a game in terms of a blocker.

(6)

5. Consider the problem of allocating costs for providing some service to a set of individuals. Assume that the following conditions hold: (a) all non-users of the service do not pay for it but all users should be charged equally; (b) the total cost of using the service is the sum of capital and operating costs, and (c) the service provider will recover the entire cost from the customers. Clearly demonstrate that there is a unique solution to this cost recovery game.

(10)

6. An firm I owns a factory and each member of a set L of laborers owns only his own labor skills. Laborers can produce nothing on their own and members of any non-empty subset S of $s = |S|$ laborers can produce exactly s units of output if they work in the factory. Formulate a coalition form game that models this situation. Justify your formulation.

(6)

7. State and prove the Bondareva-Shapley theorem by defining all necessary concepts.

(12)

8. Let $N = \{A_1, A_2, A_3\}$ be a set of 3 firms producing a homogenous output whose price function is $10 - x_{A_1} - x_{A_2} - x_{A_3}$, x_{A_i} being the output of firm A_i. The maximum output a firm can produce is 3. The cost function of firm A_i is $(1 + x_{A_i})$. The worth of any non-empty coalition $S \in 2^N$ is defined as

$$v(S) = \max_{x_{A_i}, A_i \in S} \min_{x_{A_j}, A_j \notin S} \sum_{A_i \in S} x_{A_i}\left(10 - x_{A_1} - x_{A_2} - x_{A_3}\right) - \left(\sum_{A_i \in S}\left(1 + x_{A_i}\right)\right).$$

Determine the numerical value of worth of each non-empty coalition. Also identify the set of core elements and determine its Shapley value.

(26)

P.T.O

9. Show that a solution to the two-person bargaining problem satisfying the four Nash axioms is the Nash product.

(7)

B

1. Suppose a weighted majority game has 5 players with weights 7, 3, 8, 2 and 1, given that the quota is 14. Using an efficient method determine the number of winning coalitions in the game and the number of swings for the player having weight 2.

(4+4 = 8)

2. Describe the "men propose" variant of the Gale-Shapley algorithm. In this algorithm, is it possible that before termination there arises a round where a man does not have any woman to propose to?

(8 + 4 = 12)

Indian Statistical Institute

Final Examination 2017

Course name: **Political Economy**

Subject name: **Economics**

Maximum marks: **100**

Duration: **3 hours**

24-04-17

1. This question pertains to direct aggregation of individual preferences $\{\succsim_i\}_i$ over alternatives in some set X to obtain a social preference by applying the majority rule, $\succsim_{MR}$. For simplicity, you may assume $X \subseteqq \Re$, discrete and finite. Either prove or give a counterexample for **all** of the following four statements, clearly indicating whether you are proving or giving a counterexample: **(5 x 4 = 20 points)**

(i) If $\succsim_{MR}$ is intransitive over any $x, y, z \subseteqq X$, then exactly one of the following must hold:

(a) $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$

(b) $y \succsim_{MR} x \succsim_{MR} z \succsim_{MR} y$.

(ii) If $\succsim_i$ is complete $\forall i$, then $\succsim_{MR}$ is complete.

(iii) If individual preferences are *single-peaked* (SP), then $\succsim_{MR}$ is transitive. That is, SP $\implies \succsim_{MR}$ transitive.

(iv) Let X^* be the set of Condorcet winners. Then transitivity of $\succsim_{MR}$ is necessary for X^* to be non-empty. That is, $X^* \neq \phi \implies \succsim_{MR}$ transitive.

2. This question deals with lobbying as discussed by Grossman and Helpman. They conclude that "both the policymaker and the interest group may benefit from having lobbying not be free." Consider a single interest group, a single policy variable and two possible states of the world, to elucidate the above statement. **(20 points)**

OR

This question dwells on order-restricted preferences in the context of collective choice of tax-transfer schemes. There are n individuals, with preferences defined over two goods, a consumption good and leisure; let $c \in \Re_+$ denote units of the former and $l \in \Re_+$ those of the latter. Individual i's preferences over (c, l) are represented by a utility function of the Cobb-Douglas form

$$u_i(c, l) = c^{\alpha_i} l^{1-\alpha_i}$$

P.T.O

where $\alpha_i \in (0, 1)$. Suppose each individual has an endowment of 1 unit of time that can be allocated to leisure and work ($h = 1 - l$) at a wage rate $w > 0$, with the price of the consumption good normalized to 1. Assume further that the collective decision to be made is over a set of proportional tax/transfer schemes on earned income. Specifically, the set of possible tax/transfer schemes is $X \subseteq [0, 1] \times \Re$ with typical element (t, T) where $t \in [0, 1]$ is a proportional tax on labor income and $T \in \Re$ is a lump-sum transfer payment. Prove or argue otherwise that individual preferences over (t, T) schemes satisfy *Extremal Restriction*. (You may assume interior solutions throughout.) **(20 points)**

3. In the context of voter turnout, Grossman and Helpman conclude, "The paradox in voting is... the choice by a reasonably high percentage of eligible voters to bear the cost of voting." Elucidate the paradox. **(5 points)**

4. Consider participation games as discussed by Palfrey and Rosenthal in the context of strategic voting. According to them, "The conclusion is that pure strategy equilibria fail to exist except for a few very special cases." Assume c, the identical cost of voting of all citizens, to be less than 1/2, and coin-toss rule for breaking ties, to substantiate their conclusion. **(15 points)**

5. Consider the expected turnout in a "$q - k$" or "mixed-pure" equilibrium as discussed by Palfrey and Rosenthal. They conclude that "turnout is quite strongly correlated with the relative sizes of the minority and the majority." Elucidate their conclusion. **(5 points)**

6. This question pertains to a parameterized version of Feddersen and Sandroni's ethical voting model. Let the fraction of ethical agents in groups 1 and 2, $\tilde{q}_1$ and $\tilde{q}_2$ respectively, be independently and identically distributed as $U[0, 1]$. Let the fraction of the population in group 1 be deterministic and be given by $k \in (0, 1/2]$. Let cost of voting for each individual be random and be drawn from $U[0, \bar{c}]$. Let the payoff from 'doing one's part', D, be $> \bar{c}$. Let the social cost function be linear, that is $v(x) = x$. Recall w to be the parameter capturing the 'importance of election'. Also let parameters $\bar{c}, w$ and k satisfy $\frac{\bar{c}}{w} > \frac{1}{\sqrt{k(1-k)}}$. Then we can derive that the equilibrium fraction of ethical agents who vote in each group is given by:

$$\sigma_1^* = \sqrt{\frac{w}{\bar{c}}} \cdot \frac{1}{k^{1/4}(1-k)^{1/4}};$$

$$\sigma_2^* = \sqrt{\frac{w}{\bar{c}}} \cdot \frac{k^{1/4}}{(1-k)^{3/4}}.$$

Consider σ_1^* and σ_2^* (as given above) in answering the following questions:

(i) What can you conclude about the participation rates of the minority and the majority? **(3 points)**

(ii) What can you conclude about the chances of winning of the minority versus that of the majority? **(3 points)**

P.T.O

(iii) What is total expected turnout? How does it vary with

(a) the 'level of disagreement' in the economy;

(b) the 'importance of the election';

(c) the 'average cost' of voting?

(14 points)

7. Consider the paper "Voting over flat taxes in an endowment economy" by Gouveia and Oliver, Economics Letters, 1996. Show that voters' induced preferences over tax parameters do not satisfy *Extremal Restriction*. **(15 points)**

OR

What can you conclude about violence during elections in a static one period game between two political parties following Chaturvedi ("Rigging Elections with Violence", Public Choice, 2005)? **(15 points)**

INDIAN STATISTICAL INSTITUTE

Semestral Examination (2016-2017)

MS(QE) I

Microeconomics II

Date: 26/4/2017 Maximum Marks: 100 Duration: 3 hrs.

Note: Answer all the questions.

(1) Suppose that $u(.)$ is a continuous utility function representing a locally non-satiated preference relation R on $X = \Re^L_+$. Show that the indirect utility function $v(p, w)$ is homogeneous of degree zero, increasing in w and non-increasing in p_l for any $l \in \{1, \ldots, L\}$. Also show that $v(p, w)$ is quasi-convex: that is, the level set $\{(p, w) : v(p, w) \leq v\}$ is convex for any v. **(12+5=17)**

(2) Suppose that the monotone preference R on $X = \Re^L_+$ is rational, continuous and homothetic so that the associated utility function $u(x)$ is continuous and homogeneous of degree one. Show that the Walrasian demand function $x(p, w)$ and the indirect utility function $v(p, w)$ are homogeneous of degree one in w. **(8+7=15)**

(3) Consider the labour market model of adverse selection, where the marginal (average) productivity of a worker is θ and that $\theta \in [\underline{\theta}, \bar{\theta}]$. Let the opportunity cost of accepting employment for a worker of type θ be $r(\theta)$. Assume that $r(\theta)$ is continuous and increasing and that there exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $r(\theta) < \theta$ for all $\theta \in [\underline{\theta}, \hat{\theta})$ and $r(\theta) > \theta$ for all $\theta \in (\hat{\theta}, \bar{\theta}]$. Show that a competitive equilibrium with unknown types will necessarily lead to Pareto inefficient outcome. **(8)**

(4) Show that in any sub-game perfect Nash equilibrium of the screening game with unknown worker-types, the following results are true.

(a) In any equilibrium both firms earn zero profits.

(b) No pooling equilibrium exists.

(12+8=20)

(5) Consider the labor market signaling model where the marginal productivity of a worker is $\theta \in \{2, 18\}$ and $Pr(\theta = 18) = 1/2$. The cost of education is $c(e, \theta) = \frac{e^2}{2\theta}$ for all $e \geq 0$. Let $u(w, e; \theta) = w - c(e, \theta)$ be the utility of a worker of type θ who chooses education level e and receives wage w. Assume that both worker types earn zero by staying home, that is $r(2) = r(18) = 0$.

(a) Consider the belief function

$$\mu^a(e) = \begin{cases} 1 & \text{if } e \geq e^*, \\ 0 & \text{if } 0 \leq e < e^*. \end{cases}$$

Find all possible values of e^* for which we can have a separating equilibrium. Justify your answer.

(b) Consider the belief function $\mu^b(e) = \frac{e - \max\{0, e-8\}}{8}$ for all $e \geq 0$. Can you find a separating equilibrium for the belief function $\mu^b(e)$? Justify you answer.

(6+14=20)

(6) Consider the labour market model where the effort level of the tenant is neither observable nor verifiable. Derive the second best contract. **(20)**

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
SECOND SEMESTRAL EXAMINATION 2016 - 17
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: 28-4-17 Maximum Marks: 100 Time: 3 hours

*This question paper carries a total of **120** marks. You can answer any part of any question. But the maximum that you can score is **100**. Marks allotted to questions are given within parentheses.*

1. (i) Derive the conditions (in terms of the parameters involved) for stationarity of an AR (2) process.
   (ii) Obtain the ACF of an ARMA (1, 1) process, and discuss its similarity with that of an AR (1) process.
   (iii) Suppose that a time series $\{X_t\}$ is defined as $X_t = \alpha + \beta \cos\theta t + Z_t$ for some known value of θ, where $\{Z_t\}$ is assumed to be stationary. Show that $\{X_t\}$ is nonstationary. Explain under what condition(s) it can be stationary.

[6+8+6 = 20]

2. (a) Discuss why the issue of power is very important for the original Dickey-Fuller test, and then describe briefly how this issue has been duly considered in the subsequent generalizations of this test.

   (b) Explain what you consider to be the major problem(s) of the ADF test if the test is carried out without any consideration to possible structural break(s) in the deterministic trend function of the time series. Also suggest how the ADF test can be generalized so that this issue is duly incorporated.

[8 + 12 = 20]

3. (a) Obtain a *h*-step ahead forecast of an ARMA (*p*, *q*) process, based on *n* observations, such that it satisfies at least one optimal property.

   (b) Find a 2-step ahead forecast for the following time series:

$$\Delta^2 (1 - 0.4B)x_t = a_t + 1.3a_{t-1}, \text{ where } \{a_t\} \sim WN\ (0,1).$$

   (c) Discuss how efficient out-of-sample forecasts can be obtained for a time series.

[7 + 7+6 = 20]

P.T.O

4. (a) Specify the transfer function noise model (TFNM) along with all the assumptions.

(b) Suppose the form of the transfer function of a TFNM is given by

$$\upsilon(B) = (\omega_0 - \omega_1 B - \omega_2 B^2)B^h / (1 - \delta B), \; |\delta| < 1.$$

Find the impulse response weights of the underlying TFNM implied by this transfer function.

(c) Consider an autoregressive-type process of order 2

$$x_t = \Phi_1 x_{t-1} + \Phi_2 x_{t-2} + a_t, \quad a_t \sim WN(0, \sigma^2).$$

Suppose that the following sample moments of $\{x_t\}$, based on 100 observations, were obtained for this process:

$$\upsilon_0 = 6.06, \; r_1 = 0.68, \text{ and } r_2 = 0.61$$

where υ_0, r_1 and r_2 stand for the sample variance and sample autocorrelation of lags 1 and 2, respectively. Obtain the estimates of the parameters Φ_1, Φ_2, and σ^2. Are these estimates feasible? Explain.

[4 +7+ 9=20]

5. (a) State the usual spectral representation of a stationary time series along with all the assumptions, and then show that this representation indeed satisfies all the conditions of a weakly stationary time series.

(b) Prove that the spectral density function, $f(\lambda)$, of a stationary time series is non-negative for all $\lambda \in [-\pi, \pi]$. Also find the spectral density function of an AR(1) process.

[10 + 10 = 20]

6. Suppose that the following SACF and SPACF have been obtained based on a sample of size 100 from a stationary time series.

Lag	1	2	3	4
SACF ($\hat{r}_k$)	0.63	0.51	0.32	0.19
SPACF ($\hat{\Phi}_{kk}$)	0.42	0.19	0.05	0.04

[Standard error of $\hat{r}_k$ may be approximately obtained as $\sqrt{\frac{1}{100}(1 + 2\sum_{v=1}^{k} \hat{r}_v^2)}$.]

Using Box-Jenkin's correlogram analysis, identify the underlying time series. Further, if the Portmanteau test statistic values at lags 2, 4, 6 and 8 i.e., Q (2), Q (4), Q (6) and Q (8) values – based on the residuals of the estimated model

P.T.O

obtained from the correlogram analysis above – have been computed as 14.22, 16.01, 19.29 and 22.32, respectively, what can you conclude about the white noise property of the noise term of the chosen time series?

[20]

Indian Statistical Institute

Semestral Examination

Second and Fourth Semesters, 2017

MSQE I & II

Economic Development

03.05.2017

Maximum Marks 60

Time 3 hours

Answer question 1 and any two from the rest.

1. Consider a political economy scenario where there are two parties: an incumbent and an opposition. The state of the economy is $\theta = e + \delta$ where e is the effort put in by the incumbent and δ is a random shock which is normally distributed with mean zero and variance σ^2. Each voter gets a signal s about θ, where s is uniformly distributed over the interval $[\theta - \varepsilon, \theta + \epsilon]$ and $\varepsilon > 0$ is a noise. A voter votes for the incumbent if the conditional expectation of θ given s is at least as large as some exogenously given standard $\bar{\theta}$. The incumbent chooses e to maximize $W = p(e, \bar{\theta}) - C(e)$ where $p(e, \bar{\theta})$ is the endogenously determined probability of winning the election and $C(e)$ is the cost of effort put in by the incumbent. Assume that $\frac{dC(e)}{de} = \bar{c}$, a constant.
   (i) Determine the equilibrium effort choice and show that it is unique.
   (ii) How does the equilibrium effort choice and the probability of winning in equilibrium change if there is an increase in $\bar{\theta}$?
   (iii) How does the equilibrium effort choice and the probability of winning in equilibrium change if there is an increase in the marginal cost of effort $\bar{c}$?

[10+5+5]

P.T.O

2. In a model of endogenous growth and electoral democracy where only capital is taxed, show that an increase in the inequality in capital holding leads to a fall in the rate of growth.

[20]

3. Can group lending with joint liability mitigate the problem of moral hazard? Give your answer in terms of a suitable model.

[20]

4. Consider an economy where agents live for two periods. In the first period they can acquire t units of education where t is a choice variable and $t \epsilon [0,1]$. The total cost of acquiring t units of education is $C(t)$ with $C'(t) > 0, C''(t) > 0$. An agent acquiring t units of education in the first period earns $(1 - t)w_u$ in the first period and $w_u + t(w_n - w_u)$ in the second period. Assume that $w_n > 2w_u$. Consumption and bequests take place at the end of the second period. An agent chooses his bequest (b) and consumption (c) to maximize his utility given by $U = c^{\alpha} b^{1-\alpha}$. At the beginning of his life an agent is endowed with an inheritance x.

   (a) First assume that there is a perfect credit market where an agent can borrow or lend as much as he wants to and the borrowing rate is equal to the lending rate. Find the level of education t acquired by an agent as a function of his inheritance. Also derive the long term distribution of wealth in the economy, for any given distribution of initial wealth.

   (b) Now assume that the credit market is imperfect, that is, the borrowing rate is greater than the lending rate. Find acquired education t as a function of inheritance and the long term distribution of wealth.

[10 + 10]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2016-17

MS (QE) I YEAR

Econometric Methods I

Date: 5 May 2017 Maximum Marks: 100 Duration: 3 hours

Note: Answer question 1 and any **three** from the rest of the questions

1. State whether each of any **four** of the following statements is true or false giving proof if it is true and counter example if it is false.
   (a) It is not appropriate to compare the R^2 from the equation estimated by 2SLS with the R^2 from the OLS equation. The R^2 from the OLS will always be smaller than the R^2 from the 2SLS.

   (b) If the residuals in a regression model are not independently distributed with a common variance σ^2, the OLS estimates are always less efficient than the GLS estimates.

   (c) An estimation of the demand function for steel gave the price elasticity of steel as +0.3. This finding should be interpreted to mean that the price elasticity of supply is at least +0.3.

   (d) In a LS regression of y on x, observations for which x is far from its mean will have more effect on the estimated slope than observations for which x is close to its mean value.

   (e) Heteroscedasticity in the errors leads to biased estimates of the regression coefficients and their standard errors.

   (f) If a variable x is uncorrelated with z, the addition of z to a regression, in which x is used as an independent variable, will not change either the coefficient of x or the standard error of the coefficient.

   (g) In the model $y_t = \alpha + \beta x_{t-1} + e_t$, where $e_t \sim$ iid $N(0, \sigma^2)$; $\hat{\beta}_{LS}$ is unbiased and $\hat{\beta}_{ML}$ is different from $\hat{\beta}_{LS}$. [4×7=28]

2. Suppose in the following regression model

$$y = x_1\beta_1 + x_2\beta_2 + \ldots + x_K\beta_K + e,$$

   the variables $x_1, x_2, \ldots, x_K$ are multiplied by $c_1, c_2, \ldots, c_K$ respectively, where β_1 is the intercept. Compare the changes that will occur in the LS estimates of the regression coefficients and their variances and covariances. Also discuss what will happen to the prediction of y. In particular discuss the special case where $c_1 = 1$ and $c_i = 1/(\text{sd of } x_i)$, for $i = 2, 3, \ldots, K$. [24]

3. Examine the validity of the assumptions of CLRM under the presence of errors-in-variables in the regression set up. Discuss the identification problem in this model. Describe the methods of estimation in errors-in-variables model. [6+6+12=24]

P.T.O

4. Consider the following model

$$y_i = \alpha_1 + \alpha_2 D_i + \beta x_i + e_i,$$

where y = annual salary of a college professor,
x = years of teaching experience, and
D = dummy for the gender.

Find LS estimate of the regression coefficients and hence interpret the coefficients for each of the following three ways of defining the dummy variable.

a. D = 1 for female and 0 for male,
b. D =1 for male and 2 for female, and
c. D = 1 for male and -1 for female.

Is one method preferable to another? Justify your answer. [3×7+3=24]

5. (a) Explain with examples what is meant by under-, exact and over- identification of an equation in a SEM. State the rank and order conditions for identification of the equations in a SEM.

(b) Discuss the identification status of each of the following equations using rank and order conditions:

$$Y_1 = \alpha_1 X_1 + \alpha_2 X_2 + U_1,$$
$$Y_2 = \beta_1 Y_3 + \beta_2 X_1 + \beta_3 X_2 + U_2,$$
$$Y_3 = \gamma_1 Y_1 + \gamma_2 X_1 + \gamma_3 X_3 + U_3,$$

where Y variables are endogenous and X variables are exogenous. [15+3×3=24]

6. Write short notes on any **three** of the following:

(a) The problem of normalization in SEM.
(b) Linear Probability Model.
(c) Logistic regression.
(d) Testing significance of regression slope coefficient vs. testing significance of R^2 in simple linear regression model.
(e) Detection of Outlying Observations and Prediction by Dummy Variables.
(f) Stochastic Regressors. [3×8=24]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2016-2017) (Back paper)

MS (Q.E.) I Year

Macroeconomics I

Date 10·07·17 Maximum Marks: 100 Duration: 3 hours

Group A

Answer any two

1. (a) Derive the efficiency function of labour in the generalized efficiency wage model.

   (b) How does an increase in unemployment rate affect (i) unit cost of labour and (ii) efficiency of labour?

   (c) How is the equilibrium unemployment rate sensitive to capital accumulation in this model?

   $(10 + \overline{4 + 4} + 7 = 25)$

2. How does the 'learning by doing' model differ from the Ramsey-Solow model? How does this model explain endogenous growth?

   (25)

3. How does the theory of the firm in Bennasy-Malinvaud model differ from the traditional theory? How does this modified theory explain Keynesian unemployment in the fixed price equilibrium model?

   (25)

Group B

Answer all questions

1. Show that in an OLG model, introducing a 'Pay as you go' pension scheme will reduce the steady state per capita capital stock.

   What would be the effect of such a pension scheme on the steady state welfare?

   [25]

P.T.O

2. Assume that there are costs to adjusting prices (small menu cost) which makes non adjustment of prices/ wages optimal for firms/ laboures in the face of an expansionary policy brought about through an increased money supply. Under such circumstances show that a monetary expansion will expand output and will also raise welfare (evaluated in vicinity of the initial monopolistically competitive equilibrium).

[25]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108

SECOND SEMESTRAL BACK PAPER EXAMINATION 2016 - 17

M.S.(Q.E.) 1st Year

Time Series Analysis & Forecasting

Date: 12.07.17 Maximum Marks: 100 Time: 3 hours

Answer any ALL questions. Marks allotted to each question are given within parentheses.

1. (a) Find the mean and ACF of the following ARMA (1, 1) process.

$$x_t = 2 + 1.1x_{t-1} - 0.3x_{t-2} + a_t - a_{t-1},\ \ a_t \sim WN(0, \sigma^2).$$

(b) Let $\{Y_t\}$ be an ARMA plus while noise process given by $Y_t = X_t + W_t$, where $\{W_t\} \sim WN(0, \sigma_w^2)$, $\{X_t\}$ is an ARMA (*p*, *q*) process satisfying $\phi(B)X_t = \theta(B)Z_t, \{Z_t\} \sim WN(0, \sigma_z^2)$, and $E(W_s, Z_t) = 0$ for all *s* and *t*. Show that $\{Y_t\}$ is stationary. Also find the autocovariances of $\{Y_t\}$ in terms of σ_w^2 and the ACVF of $\{X_t\}$.

(c) Find the 3-step ahead minimum MSE forecast at origin *n* of the following series.

$$x_t = 2x_{t-1} - x_{t-2} + a_t - 0.4a_{t-1} + 0.3a_{t-2},\ \ a_t \sim WN(0, \sigma^2).$$

[6+8+6 = 20]

2. (a) Describe the ADF test for unit roots in a time series and comment on its power.

(b) Describe the Quandt-Andrews test for detecting the presence of a structural break in a time series.

[10+10 = 20]

P.T.O

3. (a) Suggest an appropriate procedure for obtaining seasonal indices in a monthly time series from which trend and cyclical components have been removed.

(b) Describe the HEGY test for testing the presence of seasonal and nonseasonal unit roots in a quarterly time series.

[8 + 12 = 20]

4. (a) Suppose the form of the transfer function of a transfer function noise model (TFNM) is given by

$$\upsilon(B) = (\omega_0 - \omega_1 B - \omega_2 B^2)/(1 - \delta B), \quad |\delta| < 1.$$

Find the impulse response weights of the underlying TFNM implied by this transfer function.

(b) Specify a simple intervention model along with all assumptions, and then discuss how different types of dynamic effects can be simulated using 'step' intervention.

[10 + 10 = 20]

5. (a) Define the spectral density function, $f(\lambda)$, of a stationary process and then show that it is nonnegative for all $\lambda \in [-\pi, \pi]$. Also find $f(\lambda)$ for an MA (1) process.

(b) State and prove the theorem on finding the spectral density function of a linear combination of stationary time series.

[10 + 10 = 20]