INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) I Year : 1992-93

# A PROFICIENCY CUM DIAGNOSTIC TEST IN ENGLISH

NAME: ______________________________

TOTAL MARKS :100

TIME : 90 Minutes

## SECTION - 1

## READING (30 Marks)

1. Read the following passage. Underline the first sentence in each paragraph and the last sentence of the passage. Can you identify the topic ?

(2 Marks)

A black hole is a region of space created by the total gravitational collapse of matter, It is so intense that nothing, not even light or radiation, can escape. In other words, it is a one way surface through which matter can fall inward but cannot emerge.

Some astronomers believe that a black hole may be formed when a large star collapses inward from its own weight. So long as they are emitting heat and light into space, stars support themselves against their own gravitational pull with the outward thermal pressure generated by heat from nuclear reactions deep in their interiors. But if a star eventually exhausts its nuclear fuel, then its unbalanced gravitational attraction could cause it to contract and collapse. Furthermore, it could begin to pull in surrounding matter, including nearby comets and planets, creating a black hole.

2. Read the following passage and identify the main idea. What would be a good title for this passage ?

(6 Marks)

For more than a century, despite attacks by a few opposing scientists, Charles Darwin's theory of evolution by natural selection has stood firm. Now, however, some respected biologists are beginning to question whether the theory counts for major development such as the shift from water to land habitation. Clearly, evolution has not progressed steadily but has progressed by radical advances. Recent research in molecular biology, particularly in the study of DNA, provides us with a few possibility. Not only environmental but also genetic codes in the underlying structure of DNA could govern evolution.

p.t.o.

3. First read this passage. Then read the questions following the passage, and make inferences. Can you circle the evidence for your inference in the reading passage ?

(8 Marks)

When an acid is dissolved in water, the acid molecule divides into two parts, a hydrogen ion and another ion. An ion is an atom or a group of atoms which has an electrical charge. The charge can be either positive or negative. If hydrochloric acid is mixed with water, for example, it divides into hydrogen ions and chlorine ions.

A strong acid ionizes to a great extent, but a weak acid does not ionize so much. The strength of an acid, therefore, depends on how much it ionizes, not on how many hydrogen ions are produced. It is interesting that mitric acid and sulphuric acid become greatly ionized whereas boric acid and carbonic acid do not.

(i) What kind of acid is sulphuric acid ?
(ii) What kind of acid is boric acid ?

4. Read the following course description :

American English Phonetics. Fall. 5 hours. Three lectures, two laboratory periods.

Prerequisite: English 205, Linguistics 210 or equivalent. A study of American English pronunciation, designed for advanced international students. Professor Ayers.

Now go through the following questions and choose the <u>one best</u> answer, a,b,c or d to each question.

(6 Marks)

(i) From this course description, we know that the class meets

a. two hours a day
b. three hours a week
c. five hours a day
d. five hours a week

(ii) In order to take American English Phonetics it is necessary to

a. take English 206 first
b. know the material from English 205 or Linguistics 210
c. have permission from Professor Ayers
d. pass an examination

Contd.....

(iii) Students who take this course should expect to -

a. study British English
b. be taught by international students
c. study English 205 and linguistics 210 at the same time
d. use a language laboratory twice a week.

5. Read the following instructions.

Take two tablets with water, followed by one tablet every eight hours, as required. For maximum night time and early morning relief, take two tablets in twenty four hours.

For children six to twelve years old, give half the adult dosage. For children under six, consult your physician. Reduce dosage if nervousness, restlessness, or sleeplessness occurs.

Now go through the items and select the most appropriate answer from each of the following.

(8 Marks)

(i) The label on this medicine bottle clearly warns not to take more than -

a. 24 tablets a day
b, 8 tablets a day
c. 6 tablets a day
d. 3 tablets a day

(ii) We can infer by this label that

a. the medicine could cause some people to feel nervous
b. children may take the same dosage that adults take
c. one may not take this medicine before going to bed
d. the medication is a liquid

(iii) If one cannot sleep, it is suggested that he

a. take two tablets before going to bed
b. take less than two tablets before going to bed
c. stop taking the medicine
d. consult a doctor

(iv) Evidently the medicine

a. may be dangerous for small children
b. cannot be taken by children under 12 years old
c. may be taken by children but not by adults
d. may be taken by adults but not by children.

Contd.....

## SECTION - 2

## VOCABULARY AND STRUCTURE

(30 Marks)

1. Read each of the following sentences, paying special attention to the underlined words. Substitute another word or phrase for the underlined word without changing the meaning of the sentence.
(10 Marks)

(i) In a trust, property is administered for the beneficiary by another person called a trustee.

______________________________

(ii) Calligraphy evolved as an art form in the Far East.

______________________________

(iii) There are over one million species of animals on earth.

______________________________

(iv) The last day of registration on a college campus is always very hectic.

______________________________

(v) Hostages go through such an ordeal that it takes them a long time to recover even after they have been released.

______________________________

2. In each of the following sentences, a word or phrase is underlined. Below each are four other words or phrases. You are to choose the one word or phrase which would best keep the meaning of the original sentence if it where substituted for the underlined word.
(10 Marks)

(i) City taxes are based on an estimate of the value of one's property.
(a) appraisal (b) forecast (c) diagnosis (d) outline

(ii) People who live in the country enjoy a rustic life style.
(a) slow (b) difficult (c) simple (d) happy

(iii) The remnants of the Roman Empire can be found in many countries in Asia, Europe and Africa.

(a) effects (b) small pieces (c) buildings (d) destruction

Contd....

(iv) It is difficult to discern the sample that is on the slide unless the microscope is adjusted

(a) discard (b) arrange (c) determine (d) debate

(v) Proximity to the court house makes an office building more valuable.

(a) interest in (b) similarity to (c) nearness to
(d) usefulness for

3. Study the following sentences. There are errors in the sentences. Point out the errors by underlining the words and then write out the correct sentences.

(10 Marks)

(i) As soon as the bus stopped, he went out.

(ii) No matter he tried hard, he never succeeds in passing.

(iii) According to me, we should spend more money on education.

(iv) My friend is doing a MSc in Civil engineering.

(v) There were above a hundred people in the crowd.

Contd.....

## SECTION - 3

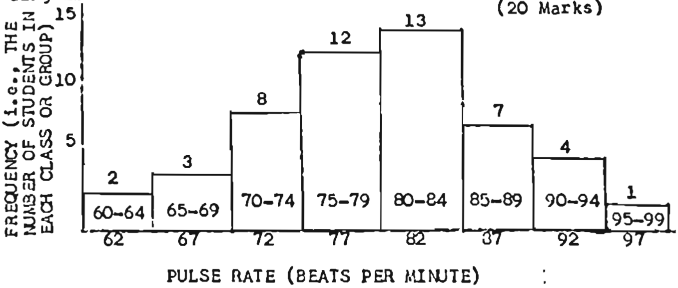
## WRITING

(40 Marks)

1. Look at the following block diagram. Here you have pulse rates of 50 students.

   Write a paragraph describing the information in the block-diagram

   (20 Marks)




2. You have done very well in your examination and you have got the news of your selection to the statistics course at the Indian Statistical Institute. Write about your performance and success to your father.

   (20 Marks)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1992-93
Vectors and Matrices
Semestral-I Examination

Date : 30.11.1992 Maximum Marks : 100 Time : 3 Hours

Answer as much as you can. The whole paper carries 110 marks but the maximum you can score is 100. Marks allotted to each question are shown in brackets. State clearly the theorems you use.

1. Prove that the statement "A is linearly independent, B is linearly independent and $B \cap \bar{A} = \emptyset \Rightarrow A \cup B$ is linearly independent" is false in general and is true if B has only one element. Here $\bar{A}$ denotes the linear span of A.

[6+5= 11]

2.(a) State the modular law for $d(S+T)$ where d denotes dimension and S,T are subspaces of a vector space.

[3]

(b) If S and T are subspaces of a vector space V such that $d(S) = 2$, $d(T) = 3$ and $d(V) = 4$, give the possible values for $d(S \cap T)$, prove that each of these is attained for some S,T and V and that no other value can be attained.

[11]

3. Let A be the matrix of the linear transformation $f: V_1 \to V_2$ with respect to the ordered bases $\mathfrak{X}$ and $\mathfrak{Y}$ and C the matrix of $h : V_2 \to V_3$ with respect to $\mathfrak{Y}$ and $\mathfrak{Z}$. Find (give proof) the matrix of hof with respect to $\mathfrak{X}$ and $\mathfrak{Z}$.

[10]

4. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Find the span S of $\{I, A, A^2, \ldots\}$ in the vector space $F^{2x2}$ and a complement T of S. Find the projection of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ into S along T.

[6+7+5=18]

p.t.o.

5.(a) Prove that for any matrix $A$, the row rank of $A$ equals the column rank of $A$.

[10]

(b) Show that $\rho\begin{bmatrix} A & B \\ 0 & C \end{bmatrix} \geq \rho(A) + \rho(~~B~~\,C)$ and that strict inequality is possible, where $\rho$ denotes rank.

[8]

6.(a) Prove that an mxn matrix $A$ has a right inverse iff $\rho(A)=m$

[10]

(b) Prove that if an mxn matrix $A$ has a unique right inverse then $A$ is square.

[9]

(c) Let $A$ and $B$ be nxn matrices such that $AB$ is a diagonal matrix with non-zero diagonal entries. Show by an example that $A$ and $B$ need not commute. However, if all the diagonal entries of $AB$ are equal, show that $A$ and $B$ commute.

[10]

7. Assignment.

[10]

# INDIAN STATISTICAL INSTITUTE
## BSTAT I : 1992 - 93
## SEMESTRAL I EXAMINATION

## COMPUTATIONAL TECHNIQUES AND PROGRAMMING I

Date : 2 December 1992 Time : 3 hrs.

*Question 6 is compulsory. You can answer any part of any questions 1 – 5. Maximum you can score of these is 70.*

1. How would you classify computers ? Describe different parts of a digital computer. Describe, in brief, two external storage devices. Describe the different types of files, which can be stored on each of them.

[ 5 + 8 + 6 + 6 = 25 ]

2. Convert the following decimal numbers to their binary forms, using binary arithmetic. Describe the algorithm you are using.

(i) −99.625 (ii) 128.0 (iii) −107 (iv) 0.1

Show how these numbers are stored internally in VAX/VMS system. Assume that the integer numbers are stored in two bytes and real numbers are stored in four bytes.

[ 10 + 5 + 10 = 25 ]

3. Consider the following FORTRAN statements

```
        DIMENSION A(10), B(3,3), C(2,3,2)
        EQUIVALENCE (A(10), B(3,3)), (A(10), C(2,3,2))
        N = 3
        OPEN (UNIT=1,FILE='TESTB1.DAT',STATUS='OLD')
        OPEN (UNIT=2,FILE='TESTB1.OUT',STATUS='NEW')
        READ (1, 111) C
111     FORMAT(6F2.0)
        WRITE(2, 222) ((B(I,J), J = 1,3), I = 1, 3)
222     FORMAT(1X, < N > F6.1)
        END
```

The file 'TESTBI.DAT' contains the following records (from first column onwards)

Record 1 : 1␢ 2␢ 3␢ 4␢ 5␢ 6␢ 7␢ 8␢ 9␢ 1 0␢

Record 2 : 1 1␢ 1 2␢ 1 3␢ 1 4␢ 1 5␢ 1 6␢ 1 7␢

Record 3 : 2␢ 2␢ 3␢ 3␢ 4␢ 4␢ 4␢ 5␢ 5␢ 5␢

Record 4 : 2 1␢ 2 2␢ 2 3␢ 2 4␢ 2 5␢ 2 6␢ 2 7␢

Show the allignment of the three arrays. What will be the content (indicate the columns) of the file 'TESTBI.OUT' when the above program is run on a computer.

[ 5 + 10 = 15 ]

4. Given an integer number $N$, write a FORTRAN program to find the sum of the digits of $N$.

[ 10 ]

5. In a survey of patients admitted in different hospitals, the following data were collected on each patient :

   - (i) Name of the Hospital
   - (ii) Paitent ID and Bed number
   - (iii) Name and Address of the patient
   - (iv) Date and Time of admission
   - (v) If dead, Date and Time of death
   - (vi) If released, Date and Time of release
   - (vii) Name of the disease
   - (viii) Cured completely/to be readmitted after a period

   Define FORTRAN data structures so that

   (i) Date can be treated as a record consisting day, month and year, where day and year are integers but month is the first 3 characters of the name of the month.

   (ii) Time can be treated as a record consisting hours, minutes and seconds, each is to be treated as integer.

   (iii) Entire data on a patient can be treated as one record.

   State, clearly, any assumption you make in defining the structures.

[ 10 ]

6. Assignment [ 30 ]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1992-93
Probability Theory and its Applications I
Semestral-I Examination

Date : 4.12.1992 Maximum Marks : 100 Time : 3 Hours

The paper carries 110 marks. Maximum one can score is 100.

1. Consider the following famous line from Shakespeare

FAITHFUL FRIEND FROM FIENDISH FOE

consisting of 5 words. One word is selected at random. Let V denote the number of vowels in the selected word and C denote the number of consonants in the selected word (repetition is counted).

Determine the Joint Probability distribution of V and C and their marginal distributions. Find E(V), E(C), E(VC), Var(V), Var(C), Cov(V,C) and $\rho(V, C)$. Are V and C independent ?

[5+2+2+2+3+3+3+3+2 = 25]

2. Let X be a random variable which assumes values 1, 2, 3, ..... If $P(X>m+n \mid X>m) = P(X>n)$ for $m,n \geq 0$, establish that X has the geometric distribution $\{pq^k, k=0,1,2,....\}$ where

$p = P(X=1)$ and $q = 1-p$. (Hint: Write the given condition as $P(X>m+n) = P(X>m)\,P(X>n)$ for all $m,n \geq 0$).

[20]

3. Let X be a random variable whose possible values are 1, 2, 3, ..... with $p_n = P(X=n)$. Prove that E(X) exists if and only if $\sum_{n=1}^{\infty} P(X \geq n) < \infty$. If $p_n = pq^{n-1}$, $n \geq 1$ for some $0<p<1$ and $q = 1-p$, check whether E(X) exists by the result above. Also obtain E(X) if it exists.

[10+5=15

4. Consider Polya Urn model as follows : An urn contains b black and r red balls. A ball is drawn at random. It is replaced and moreover, c (>0) balls of the colour drawn are added. The procedure is repeated for each subsequent drawing.

(i) Given that the second ball was black, what is the probability that the first was black ?

(ii) Find the probability of a black ball at the third trial.

[6+9=15]

p.t.o.

5. A book has n pages. The number of misprints per page is assumed to follow Poisson distribution with parameter $\lambda$. Estimate the probability that at least one page will contain more than k misprints, k being a fixed positive integer. [15]

6. Let $X_1$ and $X_2$ be Poisson random variables with parameters $\lambda$ and $\mu$ respectively, $(\lambda, \mu > 0)$. If $X_1$ and $X_2$ are independent,

(i) find the probability distribution of $Y = X_1 + X_2$

(ii) find the distribution of $X_1$ given that $X_1 + X_2 = n$, a positive integer. [8+12=20

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) I Year : 1992-93

Statistical Methods I

Semestral-I Examination

Date : 9.12.1992 Maximum Marks : 100 Time : 3 Hours

Answer all questions. Notations used are as usual.

1.(a) The table given below represents the frequency distribution of intelligence quotients of school children from 5 to 14 years of age:

| Class Intervals | Number of children |
|---|---|
| 55 - 64 | 3 |
| 65 - 74 | 21 |
| 75 - 84 | 78 |
| 85 - 94 | 182 |
| 95 -104 | 305 |
| 105 -114 | 209 |
| 115 -124 | 81 |
| 125 -134 | 21 |
| 135 -144 | 5 |

Compute the median for the data by direct computation and also from the ogive (less than type) and compare the results. Also obtain the Inter Quartile Range from the ogive.

(b) The Department of Education predicted a C.V. of less than 10% for this data. Do the data support this prediction ?

[6+6+4+10 = 26]

2.(a) If the ith raw moment is denoted by $\alpha_i$, show that $\alpha_i^2$ is not larger than the a.m. of $\alpha_{i-1}^2$ and $\alpha_{i+1}^2$.

(b) If, in particular the mean is zero show that

$$1/\mu_2 \geq 2\beta_1 - \beta_2^2/\mu_2$$

where the symbols have their usual meanings.

p.t.o.

2.(c) A student has calculated the following quantities for $N = 100$ observations :

$\Sigma x_i = -114$, $\Sigma x_i^2 = 1030$, $\Sigma x_i^3 = -1590$, $\Sigma x_i^4 = 1424$.
Explain with reasons if these are possible.

[8+6+6 = 20]

3.(a) Let (A) denote the number of individuals possessing attribute A. In a population of N individuals, if if $\frac{(A)}{N}$, $\frac{(B)}{N}$ and $\frac{(C)}{N}$ are x, 2x and 3x respectively while $\frac{(AB)}{N}$, $\frac{(BC)}{N}$ and $\frac{(AC)}{N}$ are all equal to y, show that the value of neither x nor y can exceed $1/4$.

(b) An assistant in a sweets shop recorded that of 998 people who purchased sweets during the 3 days of Pujas, 313 took Rosogolla, 526 took Sandesh while Chumchum was popular with 471. Only 43 persons bought Rossogolla as well as Sandesh while 148 bought both Sandesh and Chumchum . The combination of Rosso-golla and Chumchum was preferred by 87. A very low 26 only purchased all the three sweets. Examine if the assistant had a correct record.

[9+7 = 16]

4.(a) Derive a formula in terms of rank differences for computing the correlation coefficient between ranks in two series of observations for N individuals, stating assumptions you need to make. When is this coefficient equal to unity ?

(b) A class teacher wishes to take the best two class test scores out of three held during a semester, for computation of the final grade. He then decides to take those two test scores for which $\rho_{ij}$, the correlation coefficient between the $i^{th}$ and $j^{th}$ test scores is the largest. Explain with an illustration of this is a good procedure.

contd.....

(c) Suppose that the teacher had decided to take those two tests which are independent and found that $\rho_{23} = 0.54$, $\rho_{12}=0.82$ and $\rho_{13} = 0$ based on which he decided to choose tests 1 and 3. Explain if he is right.

[(6+2 )+4+5 = 17]

5.(a) The scores in plus two examination of 22 students who belong to two boards are as follows :

$Board_1$ : 83, 82, 80, 83, 77, 70, 85, 70, 84

$Board_2$ : 95, 92, 90, 89, 87, 87, 79, 81, 72, 79, 64,73,85.

Compare the two boards explaining your criterion.

(b) A particular CABLE T.V. company sent out an investigator who found that the percentages of households in 3 regions receiving their network are 12% , 6% and 11% . He reported to the company that overall less than 10% receive their network. Comment.

(c) Based on a set of data on heights (x in inches) and weights (y in kgs.) of 100 persons, an anthropologist fitted the straight line $y = 1.67x - 50$. Will you be justified to use this relation to find the values of y when x = 63 and x = 96 ?

(d) The railways wanted to minimize the 'discomfort' for passengers of chair car. They defined 'discomfort' as the average deviation of knee heights and heights of the chairs. In a survey of 420 adult passengers, it was found that the average knee height was 45 cms. and the railways then ordered chairs with a height of 45 cm. for the next batch of chair cars. Comment on the decision of the railways with the relevant theory.

[5+5+4+7 = 21]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS) I YEAR 1992-93
FIRST SEMESTRAL EXAMINATION : REMEDIAL ENGLISH

**SECTION-I**

**READING**

DATE: 11.12.92

Marks: 20

1. Read the following passage and answer the ~~following~~ questions.

Although each baby has an individual schedule of development, general patterns of growth have been obtained. Three periods of development have been identified, including early infancy, which extends from the first to the sixth month; middle infancy, from the sixth to the ninth month; and late infancy, from the ninth to the fifteenth month. Whereas the new born is concerned with his or her inner world and responds primarily to hunger and pain in early infancy, the baby is already aware of the surrounding world. During the second month, many infants are awake more and can raise their heads to look at things. They also begin to smile at people. By four months, the baby is searching for things but not yet grasping them with its hands. It is also beginning to be wary of strangers and scream when a visiting relative tries to pick it up. By five months, the baby is grabbing objects and putting them into its mouth. Some babies are trying to feed themselves with their hands.

In middle infancy, the baby concentrates on practicing a great many speech sounds. It loves to imitate actions and examine interesting objects. At about seven months, it begins to crawl, a skill that it masters at the end of middle infancy.

p.t.o.

In late infancy, the baby takes an interest in games, songs & even books. Progress towards walking moves through standing, balancing, bouncing in place, and walking with others. As soon as the baby walks well alone, it has passed from infancy into the active toddler stage.

1. What is the main subject of this passage?

2. What would be a good title for this passage?

3. Tick the right answer.

(a) When does a baby take an interest in books?

A. After nine months.
B. At two months.
C. After five months.
D. In middle infancy.

(b) According to this passage, what would a six-month old baby like to do?

A. Smile at people.
B. Crawl on the floor.
C. Imitate actions.
D. Play simple games.

(c) What does grasp mean in the context of this passage?

A. Watch.
B. Understand.
C. Hold.
D. Catch.

(d) When does a baby become frightened of unfamiliar people?

A. In early infancy.
B. In middle infancy.
C. In late infancy.
D. In the toddler stage.

2. Each question or group of questions is based on a short passage or set of conditions. Select the best answer choice given by putting a tick mark against each of the following.

(1). Sarah: Only General Council members sit on the President's Cabinet.

Charles: That's not true. Dr.Grogan is a General Council member and she's not on the President's Cabinet.

Charles's response implies that he incorrectly interpreted Sarah's statement to mean that -

(a) all Cabinet members are on the General Council.
(b) Dr.Grogan sits on the President's Cabinet.
(c) all members of the General Council sit on the President's Cabinet.
(d) no General Council members are on the President's Cabinet.
(e) Dr.Grogan is not a General Council Manager.

(2) My father, my three uncles, and both my grandfathers became bald within five years after they began practicing law. I don't want to lose my hair, so I'm going to become a doctor.

Which of the following most closely resembles the reasoning used in the argument above?

(a) Everytime I drink coffee before going to bed, I have trouble falling asleep. I want to sleep well tonight so I'm going to take a sleeping pill.

(b) Everyone else got transferred out of our department within three years after starting work here. I don't want to work in another department, so I'm going to start working harder.

(c) The other punch press operators on my shift each were seriously injured on the job within a week after eating at Rosie's-Dinner. I want to maintain my safety record, so I'm going to eat at Harry's - Luncheonette.

(3) I'm afraid that Roger will never be an outstanding football player again. Last year he injured his knee and the doctors had to remove some of the cartage.

The argument above is based on which of the following assumptions?

I. One must have healthy knees to play football.

II. How well one plays football may be influenced by the condition of one's knees.

III. Healthy knees are necessary for a professional football career.

(a) I only
(b) II only
(c) I & II only
(d) II and III only
(e) I, II & III

(4) Our newModel EXT Superwash Automatic Dishwasher is the most luxurious dishwasher you'll aever own. It comes in any of fourteen decorator colours. It's so quiet you'll find yourself checking to see if its really on. Andbest of all, it comes in different widths and heights so that there'll be no need to redesign your present kitchen around it.

The argument above is most weakened by its failure to mention.

(a) the terms of the warranty
(b) how well the dishwasher washes dishes.
(c) the specific sizes available
(d) how much electricity the dishwasher uses.
(e) how many dishes the dishwasher holds.

## SECTION-II

## VOCABULARY AND SETENCE CORRECTION

Marks: 40

1. In each of the following questions, a related pair of words of phrases is followed by five lettered pairs of words or phrases. Select the lettered pair that best expresses a relationship similar to that expressed in the original pair.

(Marks: 30

Marks: 20

| | | | | |
|---|---|---|---|---|
| (1) | | PROLOGUE | : | PLAY |
| | (a) | Chapter | : | Novel |
| | (b) | Overture | : | Opera |
| | (c) | intermezzo | : | symphony |
| | (d) | epilogue | : | oration |
| | (e) | gesture | : | pantomime |

| | | | | |
|---|---|---|---|---|
| (2) | | THIRST | : | DRIVE |
| | (a) | inebriety | : | excess |
| | (b) | success | : | ambition |
| | (c) | indifference | : | passion |
| | (d) | taste | : | gusto |
| | (e) | smell | : | sense |

| | | | | |
|---|---|---|---|---|
| (3) | | MARATHON | : | STAMINA |
| | (a) | relay | : | independence |
| | (b) | hurdle | : | perseverance |
| | (c) | sprint | : | celerity |
| | (d) | jog | : | weariness |
| | (e) | ramble | : | directness |

| | | | | |
|---|---|---|---|---|
| (4) | | ANNOTATE | : | TEXT |
| | (a) | exact | : | law |
| | (b) | prescribe | : | medication |
| | (c) | caption | : | photograph |
| | (d) | abridge | : | novel |
| | (e) | censor | : | film |

Marks: 20

(5) SCULPTOR : STONE

(a) essayist : words
(b) painter : turpentine
(c) composer : symphony
(d) logger : timber
(e) etcher : acid

2. In each sentence a word or phrase is underlined. Below each sentence are four other words or phrases. You are to choose the one word or phrase which would bestkeep the meaning of the original sentence if it were slubstituted for the underlined word.

(1) Collection of opals and quartz are featured at the city Museum's annual exhibition of precious stones.

| | |
|---|---|
| (a) coins | (c) gems |
| (b) loot | (d) shells |

(2) A compound break is more serious than a simple one because there is more opportunity for loss of blood and infections.

| | |
|---|---|
| (a) bruise | (c) sprain |
| (b) burn | (d) fracture |

(3) A thrifty buyer purchases fruits & vegetables in season.

| | |
|---|---|
| (a) healthy | (c) disinterested |
| (b) careful | (d) professional |

(4) The value of an old item increases with time.

(a) an antique (c) an original
(b) a bonus (d) a facsimile

(5) Mark Anthony's eulogy of Caesar at his funeral is memorably recorded in a play by Shakespeare.

(a) biography (c) denunciation
(b) prayer (d) praise

3. Each question below consists of a word printed in capital letters, followed by five lettered words or phrases. Choose the lettered word or phrase that is most nearly opposite in meaning to the word in capital letters.

(1) MOURNFUL:

(a) informal (b) sympathetic
(b) private (d) appropriate
(e) joyous

(2) ORTHODOXY

(a) renown (b) trepidation
(c) unconventionality
(d) inquisitiveness
(e) remoteness

(3) ENTICE

(a) repel (b) authorize
(c) baffle (d) misplace
(e) diminish

(4) PROTRUSION

(a) deep recess

(b) strong dislike

(c) growing ~~[illegible]~~ scarcity

(d) illusion

(e) chaos

(5) FICKLE

(a) spotless

(b) industrious

(c) welcome

(d) urgent

(e) loyal

4. Study the following sentences. There are errors in the sentences. Point out the errors by underlining the words and then write out the correct sentences.

Marks: 10

(1) I always feel ashamed when I have to speak in public.

(2) He asked her a glass of water.

(3) In average about ten people die every day.

(4) On the bus he was sitting right before me.

(5) I am afraid I speak English very bad.

## SECTION-3 - WRITING

**Marks: 24**

1. Complete the following conversation on an interview between Mr.Mani & the Chairman.

Mr.Mani - Good morning sir

Chairman -

Mr.Mani - Yes, sir, I had reserved a berth & I had a comfortable night in the train.

Chairman - May I say, I think I possess the necessary qualifications, I'm interested in lay-outs & structural design. There is an emphasis on these things in the advertisement.

Chairman - [redacted]

p.t.o.

Mr.Mani - Yes, sir. For example, a gadget has been invented to save petrol.

Chairman -

Mr.Mani - Yes, sir. I'm a cricket player.

Chairman -

Mr.Mani - Thank you very much, sir.

2. Write about where and how you would like to spend your December vacation in about 150 words.

p.t.o.

# INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons. ) I Year : 1992-93
Calculus - I
Semestral-I Examination

Date : 7.12.1992 Maximum Marks : 100 Time : 3 Hours

Answer any five questions.

1. Find the following limits :

   (a) $\lim_{n \to \infty} \theta^{1/n}$, $\theta > 1$

   (b) $\lim_{x \to \infty} (1 + \frac{1}{x})^x$

   (c) $\lim_{x \to \infty} \frac{\log x}{x^a}$, $(a>0)$ (d) $\lim_{n \to \infty} (5^n+7^n)^{\frac{1}{n}}$

[5x4=20 ]

2.(a) Let f be a continuous function on $\mathbb{R}$ such that $f(0) = 2$ and $\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0$.

Prove that f attains its maximum value.

(b) Show that the function

$$f(x) = \frac{3x^3-x^2-1}{1+x^2}, \quad x \in \mathbb{R}$$

attains its maximum value at $x=1$.

(c) Find the supremum of the set

$$\{x \in \mathbb{R} : \text{for all } y \text{ in some neighbourhood of } x, \ 3y^2 - 10y + 3 < 0\}$$

[10+5+5=20]

3.(a) Show that if $\sum_{n=1}^{\infty} a_n$ converges, then for a given $\varepsilon > 0$, there exists N such that $\left|\sum_{p=1}^{\infty} a_{N+p}\right| \leq \varepsilon$.

p.t.o.

3.(b) Let $\sum_{n=1}^{\infty} a_n$ be convergent and, for some rearrangement $\pi$ of the natural numbers, suppose that $\sum_{n=1}^{\infty} a_{\pi(n)}$ exists and $\sum_{n=1}^{\infty} a_{\pi(n)} \neq \sum_{n=1}^{\infty} a_n$. What can you say of the convergence behaviour of $\Sigma a_n^+$, $\Sigma a_n^-$ and $\Sigma |a_n|$. Give reasons.

(c) Test for convergence:

( i ) $\sum_{n=1}^{\infty} \left( \frac{n^3 x^n}{n!} + \frac{1}{2^n} \right)$ (ii) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$

where x is a fixed real number.

[5+5+5x2= 20]

4.(a) Prove that for all $m \in \mathbb{N}$, $\sqrt{m} + \sqrt{m+1}$ is an irrational number.

(b) Show that

(i) For all $m \in \mathbb{N}$, $\frac{1}{\sqrt{m+1}} < 2(\sqrt{m+1} - \sqrt{m}) < \frac{1}{\sqrt{m}}$

(ii) $2(\sqrt{n}-1) < \sum_{k=1}^{n} \frac{1}{\sqrt{k}} < 2\sqrt{n} - 1$

(iii) If $a_n = 2\sqrt{n} - \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$, $n = 1,2,\ldots$ then $\{a_n\}$ converges.

[6+(4+4+6) = 20]

5.(a) Let f be a differentiable function on (-1,1) with $f(0)=0$ and $|f'(x)| \leq |f(x)|$ for all $x \in (-1,1)$. Show that $f(x) = 0$ for all $x \in (-1,1)$.

(b) Let f be a continuous function on $\mathbb{R}$ s.t. for all $x \neq 0$, $f'(x)$ exists and $f'(x) \to 3$ as $x \to 0$. Show that $f'(0)$ exists.

[10+10 = 20]

6.(a) Let $R_n$ be the remainder after n terms in the Taylor-Mclaurin expansion of the function $1/(1-x)^2$. Show that $R_n < (n+1)x^n/(1-x)^2$ for $0<x<1$.

(b) If $0<x<.01$, show that $1+2x+3x^2$ is an approximate value of $1/(1-x)^2$ with an error less than $5/10^6$ in magnitude.

(c) State and prove a version of the L'Hospital's Rule.

[7+3+10 = 20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1992-93
Vectors and Matrices I
Semestral-I Backpaper Examination

Date : 28.1.1993 Maximum Marks : 100 Time : 3 Hours

Answer all questions. The whole paper carries 100 marks. Marks allotted to each question are shown in brackets.

1.(a) If S is a subspace of a finite-dimensional vector space, prove that every generating set of S contains a basis. [10]

(b) Find a complement of the linear span of the set $\{(2,0,1,3), (0,3,1,1), (2,-6,-1,1)\}$ in $\mathbb{R}^4$ containing $(4,0,8,0)$. [11]

2. Let $f : V_1 \to V_2$ be a linear transformation and $x_1, \ldots, x_k \in V_1$. Prove or disprove each of the following statements.

(a) If $f(x_1), \ldots, f(x_k)$ are linearly independent then $x_1, \ldots, x_k$ are linearly independent.

(b) If $x_1, \ldots, x_k$ are linearly independent then $f(x_1), \ldots, f(x_k)$ are linearly independent.

[13]

3.(a) If A is of order mxn and $Ax_1 = 0, \ldots, Ax_n = 0$ where $x_1, \ldots, x_n$ are linearly independent, show that $A = 0$. [7]

(b) If $x^TAx = x^TBx$ for all x and A and B are symmetric, prove that $A = B$. [7]

(c) If $A_{n\times n}$ ~~comments~~ commutes with every nxn matrix, prove that A is a scalar matrix. [10]

(d) Prove that $tr(AB) = tr(BA)$ whenever both AB and BA are defined. [7]

p.t.o.

4. The rank of a matrix is defined as the common value of row rank and column rank .

(a) Show that the rank of any submatrix of A cannot be greater than the rank of A.

[6]

(b) If rank (A) = r, show that for every k with $1 \leq k \leq r$, A h[as] a kxk submatrix which has an inverse.

[9]

5.(a) Prove that if A is a real square matrix such that $|a_{ii}| > \sum_{j:j\neq i} |a_{ij}|$ for all i then A has an inverse.

[12]

(b) Show that the statement " if A is a 3x3 matrix then there exists a scalar $\alpha$ such that $\alpha I+A$ is invertib[le]" holds over $\mathbb{R}$ but is false over GF(2).

[8]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1992-93
SEMESTRAL II EXAMINATION

Probability Theory and its Applications II

Date: 28.4.1993 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: There are 7 questions carrying 120 marks.
You can score at most 100.

1. Let X be an absolutely continuous random variable such that $E(X^2) < \infty$. Show that (i) $E(|X|)$ exists (ii) $\{E(|X|)\}^2 \leq E(X^2)$ and (iii) $|E(X)| \leq E(|X|)$.

(4+4+3) = [11]

2. (i) Let $0 \leq a_{nk} \leq 1$, $0 \leq b_k \leq 1$; $n, k \geq 1$. Suppose $\lim_{n \to \infty} a_{nk} = a_k$ for each k and $\sum_{k=1}^{\infty} b_k < \infty$. Show that

$$\lim_{n \to \infty} \sum_{k=1}^{\infty} a_{nk} b_k = \sum_{k=1}^{\infty} (\lim_{n \to \infty} a_{nk}) b_k.$$

(ii) Let X be a continuous random variable with distribution function F. Also suppose that Y is a discrete random variable with probability distribution $P(Y = y_n) = p_n$, $n \geq 1$ where $y_n$'s are real numbers. If X and Y are independent, prove that $Z = X + Y$ is a continuous random variable.

(10+10) = [20]

3. Random variables X and Y have the joint density function

$$f(x,y) = \cos x \cos y, \quad 0 \leq x, y \leq \pi/2.$$

Find the distribution function F(x,y). Also find Var(X), Var(Y) and Cov(X,Y). [10]

4. Let X and Y have a joint density f that is uniform over the region T, the interior of the triangle with vertices (0,0), (2,0) and (1,2). Find the marginal distributions of X and Y. Find $P(X \leq 1, Y \leq 1)$ and compare with $P(X \leq 1) P(Y \leq 1)$. Can you conclude that X and Y are independent ?

(5+4+8+8) = [25]

5. Suppose X, Y are independent random variables having Gamma distributions with parameters $(\alpha_1, \lambda)$, $(\alpha_2, \lambda)$ respectively $(\alpha_1, \alpha_2, \lambda > 0)$. Find the density of X+Y and identify the probability distribution. [10]

p.t.o.

6. (i) Let X be a random variable with distribution function F and density function f. Find $\int_{-\infty}^{\infty} F(x)\, f(x)\, dx$.

(ii) Assume that Y is a non-negative random variable with density function g. If E(Y) exists, show that

$$E(Y) = \int_0^{\infty} P(Y > y)\, dy .$$

Let the duration T of a certain type of telephone call is found to satisfy the relation

$$P(T > t) = \lambda e^{-\alpha t} + (1-\lambda) e^{-\beta t}, \quad t \geq 0;$$

where $0 \leq \lambda \leq 1$, $\alpha, \beta > 0$ are constants. Find E(T).

(5+9+6) = [20]

7. (i) The joint distribution function of X and Y is F(x,y). Define Z = max(X,Y), W = min(X,Y).

Show that Z and W are random variables. Also determine their distribution functions interms of F, $F_X$ and $F_Y$.

(ii) If X and Y are independent standard normal random variables, show that $E\left\{\max(X,Y)\right\} = \frac{1}{\sqrt{\pi}}$.

(Hint: If the distribution function of a random variable is differentiable, then the derivative is a density for the same random variable).

[6 + (4+5) + 9] = [24]

---

:bcc:

# Indian Statistical Institute

BStat – I : 1992 – 93

Semestral Examination II

**Computational Techniques and Programming II**

Time : 3 hrs.

*You can answer any part of any question.*
*Maximum you can score is 100.*

1. (a) Write a short note on computational error.
   (b) How does floating point arithmetic differ from conventional arithmetic.
   (c) Let $a, b, c$ be floating point numbers. Analyze the error in computing the sum $a+b+c$ using floating point arithmetic and discuss your results. Will your results be true if cancellation occur ?

[ 8 + 5 + 12 = 25 ]

2. (a) Let $f(x_0), f(x_1), \ldots, f(x_n)$ be the functional values of $f(x)$ corresponding to values $x_0, x_1, \ldots, x_n$ of $x$.
   (i) Show that the divided difference of $f(x)$ of order $k, k = 1, 2, \ldots, n$ is symmetric function of its arguments.
   (ii) $\frac{d}{dx} f[x_0, x_1, \ldots, x_n, x] = f[x_0, x_1, \ldots, x_n, x, x]$
   (b) Assuming that the fourth divided difference of $f(x)$ is constant and there is a single error in one of the following values, correct the error using the divided difference table.

$$\begin{array}{lllll} f(0) = 1 & f(1) = 4 & f(-1) = 2 & f(3) = 64 & f(-4) = 209 \\ f'(0) = 0 & f'(1) = 9 & f'(-1) = -3 & & \\ & f''(1) = 20 & f''(-1) = 8 & & \\ & f'''(1) = 30 & f'''(-1) = -18 & & \end{array}$$

[ 6 + 6 + 8 = 20 ]

3. (a) Define spline function. What is a natural spline ? Describe in brief, what is meant by cubic spline interpolation.
   (b) Use the spline interpolation technique to find the cubics and an approximation to the value $\log_{10} 4.5$ from the values given below.

| $x$ | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
|---|---|---|---|---|---|---|
| $y$ | 0.0 | 1.0 | 4.0 | 7.0 | 15.0 | 23.0 |

Assume that $y_0''$ is a linear extrapolation of $y_1''$ and $y_2''$, $y_5''$ is a linear extrapolation of $y_3''$ and $y_4''$.

(c) Prove or disprove the following statement :

Addition of a distinct point will always increase the degree of the interpolating polynomial.

[ 10 + 15 + 5 = 30 ]

4. Find an approximation to the definite integral

$$\int_0^1 \frac{dx}{1+x^2}$$

by using

(a) the composite Simpson's 1/3 rule and five ordinates.

(b) the composite Gaussian *3-point* quadrature formula and two subdivisions.

Compare your results with the true value of the integral.

[ 4 + 10 + 4 = 18 ]

5. (a) Given a real symmetric matrix $A$ of order $n$, write an algorithm for Jacobi's method to compute all the eigenvalues and eigenvectors of $A$.

(b) Consider a real symmetric tridiagonal matrix $T = (t_{i,j})$ of order $n$ defined as

$$\begin{aligned} t_{i,i} &= \alpha_i, \quad i = 1, 2, \ldots, n \\ t_{i+1,i} &= \beta_{i+1}, \quad i = 1, 2, \ldots, n-1 \end{aligned}$$

Let $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ be an eigenvector of $T$ corresponding to the eigenvalue $\lambda$, ~~then~~ Show that

$$x_r = (-1)^{r-1} p_{r-1} / \beta_2 \beta_3 \ldots \beta_r, \quad r = 2, 3, \ldots, n$$

where, $p_r$ is characteristic polynomial of the $r^{th}$ order leading principal minor of $T$.

[ 15 + 10 = 25 ]

INDIAN STATISTICAL INSTITUTE | 1992-93 | 148 |
B.Stat. (Hons.) I Year : 1992-93
SEMESTRAL II EXAMINATION
Statistical Methods II

Date: 3.5.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1.(a) Lengths of iron ingots Y obtained in a factory are modelled by a statistician to have a Normal distribution with mean 165 and variance 9.

(i) Write down the probability density function of Y.

(ii) The engineer of the factory is not satisfied with the model since Y, here, cannot assume negative values while a normally distributed variable may assume positive as well as negative values. How do you justify the model assumed ?

(b) On locating an albino-type child a geneticist goes to the family and records how many other children in the family are of albino-type. He has collected data on 24 families of 5 children each and tabulated the frequencies. Suggest a suitable distribution for this data.

(c) The electricity board of a State has found that a small percentage of households (hh) consume not too much of electricity load while most of the hh's fall in the next higher categories of consumption. There are some hh's which consume quite a huge load but this number is rather small. Explain what statistical model would apply for this case and describe the model (mathematically).

(6+6+6) = [18]

2.(a) With the usual notation, show that

$$\sigma^2_{0.12\ldots p} = \sigma_0^2(1-\rho^2_{01})(1-\rho^2_{02.1})(1-\rho^2_{03.12})\cdots(1-\rho^2_{0p.123\ldots(p-1)}).$$

Explain briefly what you understand by the situation when $p = 2$.

(b) (i) Show that $\rho_{02.1} = \dfrac{\rho_{02} - \rho_{01}\,\rho_{12}}{\sqrt{(1-\rho^2_{01})(1-\rho^2_{12})}}$.

(ii) Write down the formula for $\rho_{03.12}$.

contd..... 2/-

(c) An anthropologist has predicted the index Y by the relation $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$ where $X_1$, $X_2$, $X_3$ are 3 measurements on 86 subjects (all measurements taken around the means). The corrected sums of squares and products matrix is calculated as:

| Y | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| 0.12692 | 0.03030 | 0.04410 | 0.03629 |
| | 0.01875 | 0.00348 | 0.00684 |
| | | 0.02904 | 0.00878 |
| | | | 0.02886 |

He is now interested in the (i) multiple correlation coefficient of Y on the other variables and (ii) the partial correlation coefficient between the variables $X_3$ and $X_2$, eliminating the effect of Y and $X_1$. Obtain these values.

((6+3) + (6+1) + 14+16) = [46]

3.(a) It has been observed in an office that there is a 10% chance that the lift does not work on a day. Calculate the probability that in a month of 20 working days, you have to walk up on utmost 2 days.

(b) On the basis of the performance in examinations, students lose their stipends (L) if they score less than 400 and continue to get them (C) if the scores are between 400 and 500 and get a prize in addition to the stipend (P) if they score 500 or more. In one semester, the percentages of the categories L, C and P were respectively 23, 62 and 15. Under a suitable model for the scores, find the mean and the standard deviation of the scores of (i) all students and (ii) students of the category (P).

(4 + (6+8)) = [18]

4.(a) Draw a random sample of size <u>five</u> from the Cauchy distribution with density

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, \quad -\infty < x < \infty .$$

(b) Let X have a Uniform distribution in the interval (0,1). Based on a random sample of size 2, consider

$$\frac{1}{2} Y = -(\log X_1 + \log X_2).$$

Derive the distribution of Y, stating clearly all the results you have used.

(6 + (9+3)) = [18]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1992-93
BACKPAPER SEMESTRAL II EXAMINATION

| 1992-93 | 138(b) |
|---|---|

Vectors and Matrices II

Date: 21.6.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions. The paper carries 100 marks.

1.(a) Show that every matrix A can be reduced to a matrix in normal form. Explain how you would get non-singular matrices P and Q such that PAQ is in normal form. [14]

(b) Using the matrices P and Q in (a), how do you get a g-inverse of A ? Give proof. [6]

2.(a) Show that a general solution of a consistent system $Ax = b$ can be obtained as a particular solution plus a general solution of $Ax = 0$. [7]

(b) Show that if a system $Ax = b$ over $\mathbb{R}$ has at least two distinct solutions then it has infinitely many solutions. [4]

3. Assuming the result on the expansion of a determinant by a row, obtain a formula for the inverse of a non-singular matrix in terms of its minors. Deduce Cramer's rule for solving $Ax = b$ when A is non-singular. [17]

4. Using generalised Gram - Schmidt orthogonalization process, find an orthonormal basis of $\mathcal{C}(A)$ where $A = \begin{bmatrix} 2 & 6 & 4 \\ -1 & 1 & 1 \\ 0 & 4 & 3 \\ 1 & -5 & -4 \end{bmatrix}$.

Hence find the orthogonal projector into $\mathcal{C}(A)$. [18]

5.(a) Prove that algebraic multiplicity $\geq$ geometric multiplicity for any eigen value. [8]

(b) Prove that A is diagonalisable iff all eigen values of A are regular. [10]

6. Show that a real symmetric matrix A is p.d. iff $A = B^T B$ for some real upper triangular non-singular matrix B. [16]

---

:bcc:

# INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) I Year: 1992-93

BACKPAPER SEMESTRAL II EXAMINATION

Statistical Methods II

Date: 22.6.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1.(a) Scores Y obtained in a test are modelled by a teacher to have a Normal distribution with mean 50 and variance 9.

(i) Write down the probability density function of Y and the moment generating function of Y.

(ii) A student doubts the model since Y, here, cannot assume negative values while a normal variable may take both positive and negative values. How do you clear the doubt of the student ?

(b) Give an example of a truncated Poisson model and obtain the mean and variance for this model.

(c) What is the 'memory-less property' of an exponential distribution ? Give a practical illustration.

(8+5+5) = [18]

2.(a) Using the standard notation, show that

$$\sigma^2_{0.12\ldots p} = \sigma^2_0(1-\rho^2_{01})(1-\rho^2_{02.1})(1-\rho^2_{03.12})\ldots(1-\rho^2_{0p.123\ldots(p-1)}).$$

Explain briefly what you understand by this relation when p = 2.

(b) (i) Prove that

$$\rho_{02.1} = \frac{\rho_{02} - \rho_{01}\,\rho_{12}}{\sqrt{(1-\rho^2_{01})(1-\rho^2_{12})}}$$

(ii) Using the above, or otherwise, write down the formula for $\rho_{03.12}$.

(c) An anthropologist has predicted the index Y by the relation $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$ where $X_1$, $X_2$, $X_3$ are 3 measurements

contd..... 2/-

on 86 subjects. (The measurements are taken around the mea[n]. The corrected sums of squares and products matrix is calcul[a]ted as:

| Y | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| 0.12692 | 0.03030 | 0.04410 | 0.03629 |
| | 0.01875 | 0.00848 | 0.00684 |
| | | 0.02904 | 0.00878 |
| | | | 0.02886 |

He is now interested in the (i) multiple correlation coefficient of Y on the other variables and (ii) the partial correlation coefficient between the variables $X_3$ and $X_2$, eliminating the effect of Y and $X_1$. Obtain these values.

[(6+3) + (6+1) + 14+16] = [46

3.(a) It has been observed in an office that there is a 10% chan[ce] that the lift does not work on a day. Calculate the probab[i]lity that in a year of 200 working days, one has to walkup on utmost 5 days, stating your assumptions clearly.

(b) On the basis of the performance in examinations, students lose their stipends (L) if they score less than 400 and continue to get them (C) if the scores are between 400 and 500 and get a prize in addition to the stipend (P) if they score 500 or more. In one semester, the percentage of the categories L, C and P were respectively 23, 62 and 15. Under suitable model for the scores, find the mean and the standard deviation of the scores of (i) all students and (ii) students of the category (P).

[4 + (6+8)] = [18

4.(a) (i) Find the (cumulative) distribution function for the Cauchy distribution with density

$$f(x) = \frac{1}{\pi}\cdot\frac{1}{1+x^2}, \quad -\infty < x < \infty.$$

(ii) Draw a sample of size 5 from the above.

(b) Let $X_1$ and $X_2$ be a random sample from a Uniform distributio[n] on the interval (0,1). Derive the distribution of $Y = -2(\log X_1 + \log X_2)$ stating clearly all the results you have used.

[(3+3) + (9+3)] = [18

| 1992-93 | 118(b) |
|---|---|

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1992-93
BACKPAPER SEMESTRAL II EXAMINATION

Calculus II

Date: 23.6.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1.(a) Let f be a bounded monotone function on [a, b]. Prove that f is a Riemann-integrable function.

(b) Let $f(x) = \sin \frac{1}{x}$, $0 < x < 1$.

Prove that f is not a uniformly continuous function.

(6+8) = [14]

2.(a) Let $f : [0, \infty) \longrightarrow \mathbb{R}$ be Riemann-integrable on [0, a] for each a > 0 such that

$$\int_0^\infty |f(x)|\,dx \quad \text{converges,}$$

show that $\int_0^\infty f(x)\,dx$ exists.

(b) Find the values of p for which

$$\int_0^\infty e^{-x} x^p \,dx \quad \text{exists.}$$

(8+8) = [16]

3. Let $f(x) = \sum_{n=0}^{\infty} a_n x^n \qquad -R_1 < x < R_1$

and $g(x) = \sum_{n=0}^{\infty} b_n x^n \qquad -R_2 < x < R_2$.

Show that

$$f(x)\,g(x) = \sum_{n=0}^{\infty} c_n x^n \qquad -R_1 \quad R_2 < x < R_1 \quad R_2,$$

where $c_n = \sum_{m=0}^{n} a_m b_{n-m}$, $n = 1, 2, \ldots$ [8]

Find

$$\int_0^\infty \frac{\sin x}{x}\,dx .$$ [12]

p.t.o.

5.(a) Give an example to show that if $f_n \to f$ uniformly on $\mathbb{R}$ and $\int_{-\infty}^{\infty} f_n$, $n = 1, \ldots$ as well as $\int_{-\infty}^{\infty} f$ exist, it is not necessary that $\int_{-\infty}^{\infty} f_n(x)\,dx \to \int_{-\infty}^{\infty} f(x)\,dx$ as $n \to \infty$.

(b) Differentiate under the integral:

$$f_1(y) = \int_0^{\infty} e^{-x^2} \cos 2xy \, dx .$$

(6+6) = [12]

6.(a) Show that if $f$ is twice continuously differentiable on $[0, 2\pi]$ and

$$f(0) = f(2\pi)$$
$$f'(0) = f'(2\pi)$$

then the Fourier series of $f$ converges to $f(x)$ at each $x$.

(b) Using a suitable Fourier series, find

$$\sum_{n=1}^{\infty} \frac{1}{n^2} .$$

(8+6) = [14]

7.(a) Prove that if $f_n \to f$ uniformly on an interval $(a, b)$ and if each $f_n$ is continuous, then $f$ is continuous on $(a, b)$.

(b) Let $f(x) = \sum_{n=1}^{\infty} \frac{1}{n^2 + x^2}$, $x \in \mathbb{R}$.

Show that you can obtain $f'(x)$ for any $x$ by differentiating the series term-by-term.

(6+6) = [12]

8. Find the primitives

$$\int \frac{3x + 2}{x(x+1)^3}\,dx \qquad \int \frac{dx}{5 - 3\cos x} .$$

(6+6) = [12]

---

:bcc:

| 1992-93 | 128(b) |
|---|---|

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) I Year : 1992-93

BACKPAPER SEMESTRAL II EXAMINATION

Probability Theory and its Applications II

Date: 25.6.1993 Maximum Marks: 100 Time: 3 hours

1. The joint distribution of the random variables X and Y is uniform over the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$ i.e., the joint density is

$$f(x,y) = \begin{cases} \frac{1}{\pi ab} & \text{if } \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find (i) the marginal distributions of X and Y
(ii) the Cov.(X,Y).

Are X and Y independent ?

(5+4+7+4) = [20]

2. Let X, Y be two random variables each taking values 0 and 1. Suppose X and Y have a joint probability distribution. Show that X and Y are independent if they are uncorrelated.

[12]

3. Let X, Y have joint density function

$$f(x,y) = \begin{cases} \frac{1+xy}{4}, & |x| < 1, |y| < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Show that X, Y are not independent. Find the densities of $X^2$ and $Y^2$. Obtain the joint distribution of $X^2$ and $Y^2$ and show that $X^2$ and $Y^2$ are independent.

(10+8+8) = [26]

4. A random variable X is said to have double exponential distribution if its density function is

$$f(x) = \frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}}, \quad \mu \in \mathbb{R}, \sigma > 0, x \in \mathbb{R}.$$

Find E(X), Var(X). Also find E|X|.

Calculate the distribution function F of X.

(3+5+4+5) = [17]

p.t.o.

5. Let $U_1$, $U_2$ be independent random variables each having uniform density on $[0,1]$. Let $X = \min(U_1, U_2)$ and $Y = \max(U_1, U_2)$.

   (i) Show that X and Y are random variables.

   (ii) Find the distribution functions of X and Y and the corresponding densities.

   (iii) Find the joint distribution of X and Y.

(3+4+5+5+8) = [25]

---

:bcc:

INDIAN STATISTICAL INSTITUTE
BSTAT – I : 1992 – 93
SEMESTRAL EXAMINATION II (BACK)

COMPUTATIONAL TECHNIQUES AND PROGRAMMING II

Date: 28.6.1993 Maximum Marks: 100 Time : 3 hrs.

*You can answer any part of any question.*
*Maximum you can score is 100.*

1. (a) What is meant by the term "cancellation" ? Will cancellation always occur in addition and subtraction operation ? Why is cancellation not a problem in multiplication and division ?

   (b) How does floating point arithmetic differ from conventional arithmetic.

   (d) Give an example which shows that

$$fl[fl(a+b)+fl(c)] \neq fl[fl(a)+fl(b+c)]$$

   where, $a, b, c$ are floating point numbers.
   State clearly any assumption you make in defining the floating point arithmetic.

[ 10 + 5 + 5 = 20 ]

2. (a) Given (n+1) values $x_0, x_1, \ldots, x_n$ and the corresponding functional values $f(x_0), f(x_1), \ldots, f(x_n)$, define $k^{th}$ order divided difference of f(x) relative to $x_i, x_{i+1}, \ldots, x_{i+k}$ for $i = 0, 1, \ldots, n-k$.

   (b) Given (n+1) equispaced arguments $x_i = x_0 + i.h$, for $i = 0, 1, \ldots, n$ and an integer $k$, such that $0 \leq k \leq n$. Show that

$$f[x_i, x_{i+1}, \ldots, x_{i+k}] = \frac{1}{k!h^k}\Delta^k f(x_i)$$

   where $i = 0, 1, \ldots, n-k$

   (c) Prove that the polynomial of degree $\leq n$ which interpolates $f(x)$ at $(n+1)$ distinct points is $f(x)$ itself, in case $f(x)$ is a polynomial of degree $\leq n$.

   (d) Prove or disprove the following statement

   Addition of a distinct point will always increase the degree of the interpolating polynomial.

[ 5 + 5 + 5 + 5 = 20 ]

3. (a) Given a vector $w' = (w_1, w_2, \ldots, w_n)$ and an integer $t < n$, obtain the Householder's matrix $H$ such that $Hw$ has zeros in positions $t+1, t+2, \ldots, n$. Verify your claim.

   (b) Given a real symmetric tridiagonal matrix $T$ of order n, show that if $T$ has an eigen value of multiplicity $k$ then atleast $k-1$ super-diagonal element of $T$ are zeros but the converse is not true.

P.T.O.

(c) Reduce the following matrix $A$ to tridiagonal form $T$, using Householder's transformation.

$$A = \begin{bmatrix} 1.00 & 0.42 & 0.54 & 0.66 \\ 0.42 & 1.00 & 0.32 & 0.44 \\ 0.54 & 0.32 & 1.00 & 0.22 \\ 0.66 & 0.44 & 0.22 & 1.00 \end{bmatrix}$$

(d) Find the characteristic polynomial of $T$.

(e) Find the largest eigenvalue of $T$ correct upto 2 significant digits and the corresponding eigen vector.

[ 12 + 5 + 10 + 8 + 10 + 5 = 50 ]

4. Find a fixed point iteration function to compute the smallest positive zero of the function $f(x) = \exp^{-x} - \sin x$.

Write a FORTRAN program to compute the smallest positve zero of the function $f(x) = \exp^{-x} - \sin x$ correct upto 4 places of decimal, using the fixed point iteration function obtained above.

[ 10 + 10 = 20 ]