INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[201]

PERIODICAL EXAMINATION

Calculus

Date: 14.9.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Draw the graphs of the following functions:

i) $y = \sin(x/2)$; (ii) $y = 2 + 3x^2$;

iii) $y = x^2$ for $x \neq 1$ (iv) $y = x \sin \frac{1}{x}$;

$= 2$ for $x = 1$ (v) $y = \frac{x^2}{x}$. [20]

2.a) A function $\phi(x)$ is defined as follows:-

$$\phi(x) = x^2 \quad \text{when} \quad x < 1$$
$$= 2.5 \quad \text{when} \quad x = 1$$
$$= x^2 + 2 \quad \text{when} \quad x > 1$$

Does $\underset{x \to 1}{\text{Lt}} \ \phi(x)$ exist ?

b) Do the following limits exist?

i) $\underset{x \to 2}{\text{Lt}} [x]$, where $[x]$ denotes the integral part of x.

ii) $\underset{x \to 1}{\text{Lt}} \left\{ x^2 + \sqrt{x-1} \right\}$

iii) $\underset{x \to \pi/2}{\text{Lt}} \frac{e^{\tan x} - 1}{e^{\tan x} + 1}$ [20]

3.a) A function $f(x)$ is defined as follows:

$$f(x) = 3 + 2x \quad \text{for} \quad -\tfrac{3}{2} \leq x < 0$$
$$= 3 - 2x \quad \text{for} \quad 0 \leq x < \tfrac{3}{2}$$
$$= -3 - 2x \quad \text{for} \quad x \geq \tfrac{3}{2}.$$

Find out whether $f(x)$ is continuous at $x = 0$ and $x = \frac{3}{2}$.

b)
$$f(x) = 1 \quad \text{for} \quad x < 0$$
$$= 1 + \sin x \quad \text{for} \quad 0 \leq x < \tfrac{\pi}{2}$$
$$= 2 + (x - \tfrac{\pi}{2})^2 \quad \text{for} \quad \tfrac{\pi}{2} \leq x.$$

Show that $f'(x)$ exists at $x = \frac{\pi}{2}$ but does not exist at $x = 0$. [15]

GO ON TO THE NEXT PAGE

4.a) Find the differential co-efficient of the following with respect to x.

i) $\frac{(1+x)^3}{x}$; ii) $2x^4 + 5x^2 - 4 - \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$;

iii) $x^n e^x$; iv) $x \tan x \log x$.

v) $x(1-x)(1-x^2)$.

b) Find $\frac{dy}{dx}$ when

i) $x = \sin^2 \theta$, $y = \tan \theta$.

ii) $\tan y = \frac{2t}{1-t^2}$; $\sin x = \frac{2t}{1+t^2}$.

c) The volume of a right circular cone remains constant. If the radius of the base is increasing at the rate of 3 inches per second how fast is the altitude changing when the altitude is 8 inches and radius 6 inches? [25

5.a) Verify Euler's Theorem for the following functions:

i) $u = (x^{1/4} + y^{1/4})/(x^{1/5} + y^{1/5})$; (ii) $u = \frac{x-y}{x+y}$.

b) Expand the following functions in the neighbourhood of $x = 0$ to three terms plus remainder (in Lagrange's form).

i) $\sin^2 x$; (ii) $\cos^3 x$. [20

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[202]

PERIODICAL EXAMINATION

Statistics-2: Data Processing (Theory and Practical)

Date: 21.9.70 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you like. Marks allotted for each question are given in brackets [ ].

1. Find the maximum absolute error and the maximum relative error for each of the following computations. (The figure in the bracket denotes the errors)

a) 2.001 (± 0.002) + 13.000325 (± 0.005)

b) 3932.51 (± 0.001) × 43.5 (± 0.25)

c) 15.235 (± 0.0025) × 15.0 (± 0.5) ÷ 19.29 (± 0.001)

d) 15.235 (± 0.0025) - 15.0 (± 0.5) × 19.29 (± 0.001). [10]

2.a) Find the maximum absolute error in Y, where Y = A.B, A and B are correct upto n significant digits and $1 \leq A \leq 2$ and $5 \leq B \leq 10$.

b) Let x be equal to 5.25 and it be correct upto second decimal place and let y be defined as

$$y = 2.35 + 5.0x + 3.9x^2.$$

Find the value of y and the maximum absolute error in it.

c) If y, defined in (b) above, is to be computed correct upto 2 places of decimal and if the approximate value of x is 5.25, what is the maximum allowable error in x? [7+8+10]=[25]

3. Find, among the following list of variables, which are fixed-point and which are floating-point variables.

(a) XV3 (b) 3I2 (c) IJX (d) FIX (e) FLOAT

(f) INTEGER (g) FLOATING (h) 123 (i) NO (j) YES [10]

4. Are the following FORTRAN statements valid? If not, why not?

(a) IF(X), 10, 11, 12

(b) GO TO I

(c) X = I + 2.0

(d) 4.0 = I + J

(e) I + 1 = J + K + L

(f) Y = (Y ** 2)(X ** I)/(X - 1.0)

(g) X = (9.0 * Y + 13.0 * Y * Y + (13.0 **, 0.5)/2.0 * Y

(h) IF (X - I + 2.0) 11, 12

(i) GO TO 10, 11

(j) 15X = 30Y [10 X 3]=[30]

GO ON TO THE NEXT PAGE

5. What will be the value of X when the following set of FORTRAN statements are executed?

(a)
```
      X = 0.0
      Y = 1.0
      I = 0
5     X = X + Y
      I = I + 1
      X = X + 1.0
      Y = Y - 1.0
      IF (I- 5) 5, 6, 6
6     STOP
```

(b)
```
      I = 3
5     J = I - 1
      X = X + 1
      I = I + 1
      IF (J - 3) 5, 6, 6
6     IF (I - 10) 5, 7, 7
7     STOP
```

(c)
```
      A = 1.0
      I = 1
1     X = A ** 2.0
      IF (I - 5) 2, 3, 3
2     I = I + 1
      A = A + 2.0
      GO TO 1
3     X = X - 5.0
      STOP
```

[3 X10]=[3

6. Convert the following decimal numbers into binary and octal numbers:

(a) 115, (b) 192, (c) 101, (d) 292 and (e) 512.

[5 X2]=[1

7. Convert the following octal numbers into decimal numbers:

(a) 777, (b) 170, (c) 111, (d) 321 and (e) 577.

[5 X 2]=[1

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [203]

PERIODICAL EXAMINATION

Statistics-2: Probability

Date: 28.9.70 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Marks allotted for each question are given in brackets [ ].

1. n distinguishable balls are placed in r boxes numbered $1, 2, \ldots, r$. Let X = number of balls in the first box.

   (a) Find $\Pr\{X = k\}$. [7]

   (b) Find the mode of X. [7]

2. Do problems 1(a), 1(b) assuming the balls are not distinguishable. [14]

3. When do you say that a random variable is without memory? Suppose X is a random variable which takes only the values $0, 1, 2, \ldots, k, \ldots$. Show that X has no memory iff X has a geometric distribution. [14]

4. A man wants to open his door. He has n keys of which only one fits the door. He tries the keys at random so that at each try each key has probability n^{-1} of being tried. What is the probability that the man will succeed exactly at the r-th trial ? [7]

5. Let a, b, c, d be integers with $a+b+c+d = 13$. Find the probability that a hand of 13 cards will contain a spades, b hearts, c diamonds and d clubs. [7]

6. If cards are drawn one by one at random from a pack of fifty two, find the probability that the first ace turns up at k-th draw. What is the probability that the second ace turns up at the k-th draw? [10]

7. n_1 H's and n_2 T's are arranged at random. Find the probability that there are exactly k runs of H's. [10]

8. Let $A_1, \ldots, A_n$ be n events. Show that

$$\Pr\{\text{exactly } m \text{ of } A_1, \ldots, A_n \text{ occur}\}$$
$$= S_m - {}^{(m+1)}c_m \, S_{m+1} + {}^{(m+2)}c_m \, S_{m+2} - \ldots + (-1)^{n-m} \, {}^{n}c_m \, S_n$$

where $S_1, S_2, \ldots$ have their usual meaning. [12]

9. Find the probability of exactly 2 quadruples in a hand of 13 cards. [7]

10. Calculate the m.g.f. of the negative binomial distribution. Hence or otherwise find its mean and variance. [14]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[204]

PERIODICAL EXAMINATION

Economics-2

Date: 5.10.70 Maximum Marks: 100 Time: 3 hours.

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Micro and Macro Economics

Marks: 70 Time: 2 hours

Attempt any three questions. One mark is reserved for neatness.

1. Explain the different forms of deposit creation by commercial banks. Why should bank deposits be regarded as money? [15+8]=[23]

2. Examine the view that central bank's discount policy is not as effective in the matter of controlling the lending potential of the commercial banks as its other two policies. [23]

3. Show how the factors determining the production decisions of firms and consumption decisions of households ultimately determine the equilibrium level of national income. Give an alternative demonstration for arriving at the same result by using the households' propensity to save. [15+8]=[23]

4. a) Analyse the different motives behind the demand for money.
b) Explain the interest effect of open market policy. [15+8]=[23]

5. Starting with a macro-economic equilibrium situation, explain what would be the new equilibrium corresponding to a change in net investment of a given amount. Also give a period analysis of the process of movement from the initial equilibrium situation to the final equilibrium situation. [23]

Group B: Indian Economic Structure

Marks: 30 Time: 1 hour

Answer any two questions.

6. Explain, with illustrations, the concepts of:
a) Balance of trade; and
b) Balance of payments. $[7\frac{1}{2}+7\frac{1}{2}]=[15]$

7. Indicate the chief features of foreign trade in India after Independence. [15]

8. Describe the present composition of India's foreign trade. [15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[205]

PERIODICAL EXAMINATION

Statistics-2: Statistics (Theory)

Date: 26.10.70 Maximum Marks: 100 Time: 4 hours

Note: Answer as many questions as you can. Marks allotted for each question are given in brackets [].

1. Explain the importance of statistical data collection and analysis by giving at least three well-chosen examples. [6]

2. Explain the various stages in a statistical investigation. [6]

3. Give three measures of central tendency of data and give an example for each of the measures where that measure is the more appropriate than the other two in describing the central tendency of the data in question. [6]

4. The mean and standard deviation (s.d) of the heights of 10 boys are 150 cm and 4 cm respectively. If the heights of the tallest and the shortest boys are 165 cm and 136 cm respectively, find the mean and the s.d. of the heights of the remaining 8 boys. [6]

5. Prove that (a) the second moment is least when taken about the mean and
(b) the mean deviation is least when taken about the median

for a given univariate discrete data. [8]

6. If σ^2 is the variance of the observations $x_1, x_2, \ldots, x_n$, show that

$$\sigma^2 = \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_j)^2 .$$

Hence prove that

$$\sigma^2 \leq \frac{(b-a)^2}{4}$$

where $b = \max_i x_i$ and $a = \min_i x_i$. [8]

7. Describe clearly the physical models that give rise to each of the following distributions:

a) binomial distribution

b) negative binomial distribution and

c) Truncated Poisson distribution, truncated at left at $X = 0$.

Give at least two realistic examples for each case where you intuitively expect the data to conform to the respective models. [12]

8.a) From the binomial distribution $B(n,p)$ derive as an appropriate limiting form under suitable conditions to be stated by you, the Poisson distribution, Poi (λ).

b) Derive the first three central moments of $B(n,p)$ and

c) hence or otherwise write down the first three raw moments of Poi (λ). [6+6+4]=[16]

GO ON TO THE NEXT PAGE

9.a) Define the probability generating function (p.g.f.) $P(s)$ of a non-negative integer valued random variable X (n.n.i.v.r.v.) and explain its significance and use.

b) Prove that

$$E(X) = P'(1)$$

$$\text{and} \quad V(X) = P''(1) + P'(1) - [P'(1)]^2.$$

c) Hence find the mean and variance of a geometric distribution. [3+(3+5)+5]=[16]

10. Using the result that if X and Y are two independent rv's the p.g.f. of $X+Y$ is the product of the p.g.f.'s of X and Y prove that if $X \cap B(n_1, p_1)$ and $Y \cap B(n_2, p_2)$ then $X+Y$ is also a binomial variable under suitable conditions to be stated by you. [8]

11. A symmetric die is thrown 10 times. What is the probability that the total score is 15? [8]

12. Write an essay on 'fitting statistical models to data'.

[Hints: What is meant by 'fitting a model to a data? How do we select the model? What are the motives in fitting models? How do we fit a model when some of the 'parameters' of the model are unknown? How do we judge the 'goodness of fit? What are the practical rules in fitting a model?] [12]

13. Practical record. [10]

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[206]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

PERIODICAL EXAMINATION

General Science-3: Geology

Date: 2.11.70. Maximum Marks: 100 Time: 3 hours

Note: Answer question 1 and any five from the rest. Marks allotted for each question are given in brackets [ ].

1. What is a mineral? What are the chief rock-forming minerals? State the physical properties and approximate chemical compositions of three common minerals. [5+5+10]=[20]

2. How do you know that the crust of the earth is layered? Explain with the help of a diagram the essential difference between the continental and the oceanic crust. [16]

3. How were the major mountain belts of the earth formed? Briefly discuss if there is any relation between these mountains and the earthquake zones spread around the world. [16]

4. What is a sedimentary rock? Describe the processes involved in the formation of a typical sedimentary rock. [16]

5. What are fossils? Describe the various ways in which fossils can be preserved. Why fossils are useful to the geologists? [16]

6. Which parts of the geological time scale are represented by the rock groups found in the Peninsular India? Indicate their time relations. [16]

7. What are the different types of stratification? Briefly describe them with the help of sketches. [16]

8. Briefly discuss some geological problems where statistical methods of analysis are of help. [16]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[207]

PERIODICAL EXAMINATION

Mathematics-2: Matrix Algebra

Date: 9.11.70. Maximum Marks: 100 Time: 3 hours

Note: You can answer any part of any question without restriction. The maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

[The following notations are used throughout the paper V(F) denotes a vector space over the field F. $E_n(F)$ denotes the vector space of n-tuples over the field F. R denotes the field of real numbers].

1. Define the following terms:

   group, field, vector space over a field and subspace of a vector space. $[4 \times 2\frac{1}{2}]=[10]$

2. In each of the following cases check whether the given subset S of a vector space V(F) is a subspace of V(F).

   (a) $V(F) = E_n(R)$

   $S = \{ (x_1,\ldots,x_n):$ each $x_i \in R$ and $x_1=x_2=\ldots= x_n \}$

   (b) V(F) = vector space of all polynomials of degree ≤ 3 with real coefficients. $S = \{ p(x): p(x) \in V$ and constant term of $p(x) = 0 \}$.

   (c) $V(F) = E_n(R)$.

   $S = \{(x_1,\ldots,x_n):(x_1,\ldots,x_n) \in E_n(R)$ and $x_1 \geq 0\}$. $[3\times 4]=[12]$

3.a) When is a set of vectors $\alpha_1,\alpha_2,\ldots,\alpha_n$ in V(F) said to be linearly independent ?

b) Show that a set of non-null vectors $\alpha_1,\alpha_2,\ldots,\alpha_n$ $(n \geq 2)$ in V(F) is linearly dependent if and only if $\exists\, i : 2 \leq i \leq n$ such that α_i is a linear combination of $\alpha_1,\ldots,\alpha_{i-1}$. [2+8]=[10]

4. Prove that if ξ_1,ξ_2,ξ_3 are linearly independent in $E_3(R)$ then so are $\xi_1+\xi_2$, $\xi_2+\xi_3$ and $\xi_3+\xi_1$. Is this true for vectors in any vector space V(F)? [6+4]=[10]

5.a) Define a basis of a vector space.

b) Let $S = \{\alpha_1,\alpha_2,\ldots,\alpha_n\}$ be a generating set of vectors for a vector space V(F). Show that there exists a subset of S which is a basis of V.

c) Let $V(F) = E_3(R)$. Let $\alpha_1 = (1,2,1)$, $\alpha_2 = (2,4,2)$ and $\alpha_3 = (2,1,3)$. Obtain a basis of $\mathcal{M}(\alpha_1,\alpha_2,\alpha_3)$. [2+6+5]=[13]

GO ON TO THE NEXT PAGE

6. Let S and T be subspaces of a finite dimensional vector space $V(F)$. Let $S+T = \{\alpha : \alpha = \beta + \gamma$ where $\beta \in S$ and $\gamma \in T\}$.

Show that

(a) $S+T$ and $S \cap T$ are subspaces of $V(F)$

(b) $d(S+T) = d(S) + d(T) - d(S \cap T)$ and

(c) $d(S+T) = d(S) + d(T)$ if and only if $S \cap T = \{\emptyset\}$

where $d(S)$ denotes the dimension of S. [4+4+8+3]=[19]

7.a) Define complement of a subspace of a vector space.

b) Let $V(F)$ be a finite dimensional vector space and S be a subspace of V. Show that there exists a complement of S.

c) Let $V(F)$ and S be as defined in 7(b). Let T be a complement of S. Show that every vector $\alpha \in V$ can be expressed uniquely as $\alpha = \beta + \gamma$ where $\beta \in S$ and $\gamma \in T$.

d) Give an example to show that complement of a subspace is in general not unique. [2+6+5+4]=[17]

8. Consider the homogeneous linear system

$$x_1\alpha_1 + x_2\alpha_2 + \dots + x_n\alpha_n = \emptyset \quad \text{where } \alpha_1, \alpha_2, \dots, \alpha_n$$

are given vectors in $V(F)$.

Let $S = \{ Y : Y = (y_1, \dots, y_n) \in E_n(F)$ and

$$y_1\alpha_1 + \dots + y_n\alpha_n = \emptyset \}$$

(a) Show that

(i) S is a subspace of $E_n(F)$

and (ii) $d(S) = n - d(M(\alpha_1, \dots, \alpha_n))$.

(b) Let $V(F) = E_3(R)$ and $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ and

$\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. Obtain a basis of S where S is as defined above. [4+8+8]=[20]

9. Let F be a commutative field. Let a_{ij}, $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, n$ belong to F. Define

$$\alpha_i = \begin{pmatrix} a_{1i} \\ \vdots \\ a_{ni} \end{pmatrix}, \quad i = 1, 2, \dots, n \quad \text{and} \quad \beta_j = (a_{j1}, \dots, a_{jn}),$$

$j = 1, 2, \dots, n$. Consider the non-homogeneous linear system

$$x_1\alpha_1 + \dots + x_n\alpha_n = \gamma \qquad \dots *$$

where $\gamma = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$

where $Y \in E_n(F)$.

Show that * is consistent if and only if

$$c_1\beta_1 + c_2\beta_2 + \dots + c_n\beta_n = \emptyset \quad \Rightarrow \quad c_1b_1 + \dots + c_nb_n = 0$$

for scalars $c_1, \dots, c_n$ in F. [15]

10. When is a vector space $V(F)$ said to be isomorphic to a vector space $J(F)$? Show that a finite dimensional vector space $V(F)$ is isomorphic to a vector space $J(F)$ if and only if $d(V) = d(J)$.

[2+13]=[15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

PERIODICAL EXAMINATION

General Science-2: Physics

Date: 16.11.70 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. Marks allotted for each question are given in brackets [].

1. Define the terms -

i) Moment of inertia

ii) Radius of gyration

iii) What is the physical significance of moment of inertia?

iv) and (v) State and prove the parallel axis theorem as applied to moment of inertia.

vi) Derive an expression for the moment of inertia of a solid sphere about a diameter.

vii) Show that the acceleration of a body rolling down an incline is $f = g \sin\theta / (1 + k^2/r^2)$ with usual meaning of the symbols. [2+2+3+2+4+5+7]=[25]

2. Deduce the differential equation for the simple harmonic motion of a particle.

Solve the equation and show that the motion is isochronous.

A particle is moving with S.H.M. in a straight line. When the distance of the particle from the mean position has the values x_1 and x_2, the corresponding values of the velocity are u_1 and u_2. Show that the maximum velocity is

$$\left[\frac{u_1^2 x_2^2 - u_2^2 x_1^2}{x_2^2 - x_1^2}\right]^{1/2}$$

What is meant by Lissajous' figures? [4+5+2+12+2]=[25]

3. Obtain an expression for the time period of a compound pendulum and show that the centres of oscillation and suspension are interchangeable.

When does the time period become minimum and what is the minimum value ?

Discuss the Bessel's suggestion in connection with the determination of g by Kater's reversible pendulum.

A circular disc of radius 20 cms. oscillates as a pendulum about a point on its circumference. Find the position of the centre of oscillation. [5+4+2+2+6+6]=[25]

4. Deduce the bending equation:- $M/I = f/z = Y/R$ with usual meaning of the symbols.

Apply it to deduce the expression for the depression of the free end of a cantilever.

How does the expression change in the case of a double cantilever?

Indicate in outlines how a double cantilever could be used to find the Young's modulus of a specimen. [8+8+4+5]=[25]

GO ON TO THE NEXT PAGE

5. State and explain Kirchoff's laws for an electrical circuit and apply them to calculate the current through the galvanometer in the Wheatstone's network.

Twelve equal resistances each 1 ohm form a cube. A 2 volt accumulator with 0.09 ohm internal resistance is put directly across an arm of the cube. Calculate the distribution of current in the network. [6+4+6+9].

6. Derive Helmholtz equations for the growth and decay of currents in a circuit containing inductance, resistance and a constant E.M.F. Interpret the equations.

What is meant by the time constant of the circuit?

A resistance of 5 ohms and self-inductance 4 henries is connected in series to a battery of e.m.f. 10 volts and negligible resistance. After how long will the current in it rise to 1 amp? [6+4+6+2+7].

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [209]

Mid-year Examination

Mathematics-2: Calculus

Date: 21.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. An open tank of a given volume consists of a square base with vertical sides. Show that the expense of lining the tank with lead will be least if the height of the tank is half the width. [15]

2. In enclosing a rectangular lawn that has one side along a neighbour's plot, a person has to pay for the fence for the three sides on his own ground and for half of that along the dividing line. What dimensions would give him the least cost if the lawn is to contain 4800 sq. ft.? [15]

3. Show that the tangents to the curve

$$3x^2 + 4xy + 5y^2 - 4 = 0$$

at the points in which it is intersected by the lines

$$3x + 2y = 0 \quad \text{and} \quad 2x + 5y = 0$$

are parallel to the axes of co-ordinates. [15]

4. Show that at any point of the hyperbola $xy = c^2$ the subtangent varies as the abscissa and the subnormal varies as the cube of the ordinate of the point of contact. [10]

5. Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at an extremity of the major axis is equal to half the latus rectum. [10]

6. Determine the centres of curvature of the following curves at the points indicated

   i) $xy = 12$ at $(3,4)$

   ii) $y = \sin^2 x$ at $(0,0)$. [10]

7. Find the asymptotes of the following curves

   i) $3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$

   ii) $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0.$ [15]

8. Show that the asymptotes of the curve

$$x^2y^2 - a^2(x^2 + y^2) - a^3(x + y) + a^4 = 0$$

form a square, two of whose angular points lie on the curve. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School

B. Stat. Part II: 1970-71

MID-YEAR EXAMINATION

Mathematics-2: Matrix Algebra

Date: 22.12.70 Maximum Marks 100 Time: 3 hours

Note: Answer question 1 and any four of the rest. Marks allotted for each question are given in brackets [ ].

Instruction: In questions 2,3 and 4,R denotes the field of real numbers and $E_n(R)$ denotes the vector space n-tuples over the field of real numbers.

1. Let V and J be finite dimensional vector spaces over a field F. Show that V is isomorphic to J if and only if $d(V) = d(J)$. [16]

2.a) Define a function f on $E_n(R) \times E_n(R)$ into R as follows:

$$f(X, Y) = \sum_{i=1}^{n} \lambda_i x_i y_i \qquad \forall X,Y \in E_n(R)$$

where $X = (x_1, x_2, \ldots, x_n)$, $Y = (y_1, y_2, \ldots, y_n)$, and $\lambda_1, \lambda_2, \ldots, \lambda_n$ are real constants. Show that f is a valid inner product on $E_n(R)$ if and only if $\lambda_i > 0$ for $i = 1, 2, \ldots, n$. [9]

b) Check whether each of the following functions on $E_n(R)$ is a norm on $E_n(R)$. (In the following, X denotes the vector $(x_1, x_2, \ldots, x_n)$.)

i) $f_1(X) = \sum_{i=1}^{n} |x_i|$ for all $X \in E_n(R)$ [3]

ii) $f_2(X) = \underset{i}{\text{Min}} |x_i|$ for all $X \in E_n(R)$ [3]

3. Let V be a finite dimensional (nontrivial) vector space in which an inner product $(\cdot,\cdot)$ is defined.

a) Show that there exists an orthogonal basis for V. [12]

b) Let $\alpha_1, \alpha_2, \ldots, \alpha_k$ be a set of orthogonal vectors in V. Obtain the minimum value of (α, α) for choices of α in V subject to conditions $(\alpha, \alpha_i) = p_i$, $i = 1, 2, \ldots, k$ where $p_1, p_2, \ldots, p_k$ are given scalars. [9]

4.a) Consider the following basis of $E_3(R)$:

$\alpha_1 = (3,1,1)$, $\alpha_2 = (1,3,1)$ and $\alpha_3 = (1,1,3)$.

Using Gram-Schmidt's orthonormalisation process on the above basis, obtain an orthonormal basis of $E_3(R)$. [10]

b) Let S and T be subspaces of a finite dimensional inner product space V. Show that

$$(S+T)^{\perp} = S^{\perp} \cap T^{\perp}.$$ [11]

PLEASE TURN OVER

5.a) State and prove the Frobenius inequality on ranks of matrices. [8]

b) Show that if A and B are matrices of orders $m \times n$ and $n \times p$ respectively, then

$$R(A) + R(B) - n \leq R(AB) \leq \text{Min} \{ R(A), R(B) \}.$$ [7]

c) Let A be a square matrix. Show that if $R(A^m) = R(A^{m+n})$ for some positive integers m and n then $R(A^m) = R(A^s)$ for all positive integers $s \geq m$. [8]

6.a) Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be a nonsingular matrix of order $n \times n$ where A is a nonsingular matrix of order $r \times r$ and B, C and D are matrices of orders $r \times (n-r)$, $(n-r) \times r$, $(n-r) \times (n-r)$ respectively. Obtain the inverse of M in terms of

$$A, B, C, D \text{ and } A^{-1}.$$ [12]

b) Let $M = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 0 & 3 \end{pmatrix}$ be a nonsingular matrix of order 3×3. You are given that

$$\begin{pmatrix} \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{3}{8} \end{pmatrix} \text{ is the inverse of } \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

Use this information and compute the inverse of M. [8]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

MID-YEAR EXAMINATION

Statistics-2: Probability

Date: 23.12.70. Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions and parts of questions as you can. The paper carries 110 marks. Marks allotted for each question are given in brackets [ ].

1. Consider an electric fixture containing 5 electric light bulbs which are connected so that none will operate if any one of them is defective. If the light bulbs in the fixture are selected randomly from a batch of 1000 bulbs, 100 of which are known to be defective, find the probability that all the bulbs in the fixture will operate. [5]

2. An urn contains M balls numbered 1 to M. Let N numbers be designated lucky, where $N < M$. Let a sample of size n be drawn with replacement. Find the probability that the sample will contain exactly k balls with lucky numbers. [10]

3. A sample of size 4 is drawn with replacement from an urn containing twelve balls of which eight are white. Find the conditional probability that the ball drawn on the third draw was white given that the sample contains exactly three white balls. [5]

4. An urn contains M balls of which M_W are white. n balls are drawn and laid aside (not put back in the urn) their colour unnoted. Another ball is drawn (it is assumed $n < M$). What is the probability that it will be white? [5]

5. Prove the following statements for any events A, B, C such that $P(C) > 0$.

   i) $P(S|C) = 1$ if $P(S) = 1$.

   ii) $P(A|C) = 1$ if $C \subset A$.

   iii) $P(A|C) = 0$ if $P(A) = 0$.

   iv) $P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$.

   v) $P(A^c|C) = 1 - P(A|C)$. [10]

6. A ball is drawn at random from an urn containing four balls numbered 1 to 4. Let A be the event 'ball no. 1 or 2 is drawn', B the event 'ball no. 1 or 3 is drawn' and C the event 'ball no. 1 or 4 is drawn'. Show that A, B and C are pairwise independent but not independent. [10]

7. $A_1, \ldots, A_n$ are independent events and have same probability $P(A_i) = p$. Find the probability that exactly k of the events occur. [5]

8. Suppose one makes $m+n$ independent tosses of a coin whose probability of head at each toss is p. Let $q = 1-p$.

   i) For any $k = 0, 1, \ldots, n$ find the conditional probability that exactly $m+k$ tosses will result in head given that the first m tosses result in head. [illegible]

   ii) Find the conditional probability that exactly [illegible] tosses will result in head given that at least [illegible] tosses result in head. [illegible]

9. An urn contains 5 white and 7 black balls. A ball is drawn and its colour noted. It is then replaced; in addition 3 balls of the same colour are added to the urn. This procedure is repeated n times, i.e., n balls are drawn and each time 3 balls of the same colour are added. Find the probability

i) that the first two balls are black,

ii) that the (n-1)th ball drawn is white and n-th ball drawn is black. [10]

10. State and prove Bayes theorem. [10]

11. a) Let $p_n = ap_{n-1} + b, \quad n = 2,3,\ldots$

and suppose $a \neq 1$. Show that

$$p_n = (p_1 - \frac{b}{1-a})a^{n-1} + \frac{b}{1-a}.$$ [10]

b) Consider a communication system which transmits the digits 0 and 1. Each digit transmitted passes through several stages. At each of these there is a probability p_{ij} that the digit i which enters it comes out as the digit j. What is the probability p_{ii}^n that if the digit i is transmitted it will come out of the n-th stage as the digit i? [10]

c) A certain young lady tries not to be late for office too often. If she is late on one day she is 90 per cent sure to be on time next day. If she is on time there is a chance of 60 per cent of her being late on the next day. In the long run how often is she late? [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[212]

MID-YEAR EXAMINATION

Economics-2: Economic Theory

Date: 24.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions.
All questions carry equal marks.

1. Give an analysis of excess demand inflation. Will the inflationary process come to a halt or will it continue indefinitely if the monetary authority allowed the supply of money to increase in proportion to the rise in prices? Give reasons for your answer, stating clearly the assumptions you may have to make in this connection.

2. Derive graphically the IS and LM functions and use them to arrive at the general equilibrium of the product and money markets. Examine carefully the situations in which shifts may occur in these two functions and comment on the resulting changes in income and interest rate.

3. What are Keynes's hypotheses about the consumption function? Examine any policy recommendations that may be based on his formulation of the consumer behaviour. How far do statistical studies corroborate his views? What are the differently shaped consumption functions based on different types of statistical data? Briefly discuss some of the theories suggested to reconcile conflicting indications about the basic form of the relationship of consumption to income.

4. Discuss the main features of the classical and Keynesian theories of employment, demonstrating carefully how the respective demand and supply functions in the two cases are derived.

5. Write short notes on:

a) the liquidity trap and its implications

b) the marginal efficiency of capital

c) the money-earning assets of commercial banks.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[213]

MID-YEAR EXAMINATION

Economics-2: Indian Economic Problems

Date: 25.12.70 | Maximum Marks: 100 | Time: 3 hours

Note: Answer any four questions. All questions carry equal marks.

1. Explain the problems and requirements of a developing economy like India in the field of foreign trade.

2. Describe, with statistical data, the present direction of India's foreign trade.

3. 'A review of land reforms reveals much that has been achieved as well as a great deal that requires urgent attention. There are many gaps between objectives and legislation and between the laws and their implementation'. - Fully examine the statement.

4. Do you agree with the view that land reform measures are absolutely essential for the economic development of the country? Give reasons for your answer.

5. Explain fully the consequences of land-tenure systems introduced by the Britishers on our rural economy.

6. Indicate the main arguments in favour of 'co-operative farming' in India. Do you agree with the view that the expansion of co-operative farming would create more unemployment?

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

214

MID-YEAR EXAMINATION

Statistics-2: Data Processing Theory and Practical

Date: 26.12.70 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 120 marks. You may attempt any part of any question.
Marks allotted for each question are given in brackets [ ].

1. Are the following statements valid? If not, why not?

```
(a)      READ INPUT TAPE 1, 2, 3, A(I), I = 1, 10
(b)      WRITE INPUT TAPE 1, 2, (A(I), I = 1, N)
(c)      X + 1 = SQRTF (Y + 2)
(d)      - Y = Z + 1.0
(e)   1  FORMAT (1HO, 5X, 9HABSOLUTE, 6HVALUE =, I2,
                          20(5X, 32))
(f)      DIMENSION A(N), B(M), C(I)
(g)      PRINT, 2, A, B
(h)      ARRAY (I-J) = B(I) - C(J) + B(I+J)*C(I-J)
(i)      DO 1  I = 1, 10,   J = 1,20
(j)      END FILE 1, 2.
```

[10×2]=[20]

2. Are the following set of FORTRAN statements valid? If not, state the reason and correct them, if possible.

```
(a)   1  READ 2, A
      2  FORMAT (I2)
         IF END OF FILE 4,3
      3  I = FIXF(A)
         I1 = I1 + I
         GO TO 1
      4  PRINT 2, I1
         STOP

(b)      READ 1, X
      1  FORMAT (F 10.5)
         PRINT 1, X
         STOP

(c)      S = 0.0
         DO 1 I = 1, 10
         I = I + 1
      1  S = S + FLOATF (I)
         STOP
```

GO ON TO THE NEXT PAGE

```
(d)       DIMENSION  A(5,5), B(5)
          READ 1,((A(I,J), J = 1,5), B(I), I = 1,5)
    1     FORMAT (6F 10.2)
          DO 3 I = 1,5
          B(I) = B(I) + 1.0
          DO 4 J = 1,5
    3     A(I, J) = A(I,J) - B(I)
    4     CONTINUE
          STOP
```

[4×5]=[2(

3. What will be the value of X after the following set of instructions are executed?

```
(a)       S0 = 0.0
          S1 = 0.0
          S2 = 1.0
          N  = 0
    1     READ  2, X
    2     FORMAT (5X, F8.3)
          IF END OF FILE 4.3
    3     S0 = S0 + X
          S1 = S1 + X*X
          N  = N + 1
          S2 = S2 * X
          GO TO 1
    4     PRINT 5, S0, S1, S2, N, X
    5     FORMAT (1H1, 5X, 3F15.5, I5, 5X, F10.4)
          X = 1.0/FLOATF(N)
          S1 = S1 - S0 * S0 * X
          X = S2 ** X /(S0 * X )
          STOP
```

The following 4 data cards are placed:

Card No.	Content of cols. 1 to 13
1	bbbbbbb 01.200
2	bbbbbbb 02.000
3	bbbbbbb 00.500
4	bFINIS bbbbbbb

'b' denotes blank.

(b)

```
      DIMENSION A(5,5), B(5,5)
      X = 0.0
      A (1, 1) = 0.0
      B (1, 1) = A(1, 1)
      DO 3 I = 1,5
      DO 3 J = 1,5
      IF (I - J) 1, 2, 2
1     B(I,J) = - 1.0
      GO TO 3
2     B(I,J) = 1.0
3     B(I,I) = 0.0
      DO 4 I = 1,5
      L = 6- I
      DO 4 J = 1,5
      A(I, J) = B(J, I)
      DO 4 K = 1,5
4     X = X + A(J,K) + B(K, J)
      STOP
```

[2 ×15]=[30]

4. The following bits of information are punched in cards according to the card-design given below:

Card-columns	Content	Remarks
1 - 4	Class	Integer
5 - 8	Roll No.	Integer
9 - 10	Percentage score in Mathematics	Integer
11 - 12	Percentage score in Statistics	Integer
13 - 14	Percentage score in other subjects	Integer

You have to write a program to print the scores of different subjects and the total score for each student along with the roll number and the class. The program should also compute the mean and standard deviation of the total score for a class and print them. You should also draw the flow-chart for the program. [80]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [215]

MID-YEAR EXAMINATION

General Science-2: Physics Theory

Date: 28.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1\. a) What do you mean by isothermal and adiabatic changes of a gaseous system? [6]

b) Establish that for an adiabatic change pV^{γ} = constant. [7]

1 lb of dry air (volume = 12.39 cu.ft.) at atmospheric pressure (14.7 lbs./in.2) requires compressing to a pressure of 200 lbs./in. 2. What will be the saving if the air is compressed isothermally instead of adiabatically? (Assume γ for air as 1.4.) [12]

2\. a) What is a Carnot's cycle? [4]

b) Calculate the work done in each operation of the cycle, when the working substance is a perfect gas. [8]

c) Find an expression for the thermal efficiency of a Carnot's engine. [6]

d) To increase the efficiency of a Carnot's engine would you increase the higher temperature T_1 or decrease the lower temperature T_2? [2]

A Carnot's engine whose low-temperature reservoir is at 7°C has an efficiency of 40 per cent. It is desired to increase the efficiency by 10 per cent to 50. By how many degrees should the temperature of the high-temperature reservoir be increased? [8]

3\. a) State the second law of thermodynamics and show on the basis of it that no engine can be more efficient than a reversible engine operating between the same two temperatures. [4 +7]=[11]

b) An inventor claims to have developed an engine which takes in 100,000 calories from its fuel supply, rejects 25,000 calories in the exhaust, and delivers 25 kilowatt-hours of mechanical work. Do you advise investing money to put this engine on the market? [8]

4\. a) State the laws of electromagnetic induction. [6]

b) Hence define the self inductance of a circuit. [4]

c) A condenser is charged by means of a constant e.m.f. E, the connecting wires having a resistance R. At t = 0, the e.m.f. is switched on. Show that at time t = t, the charge on the condenser is

$$q = CE\,(1 - e^{-t/CR}).$$ [8]

d) A 2 volt battery of negligible internal resistance is applied to a coil of inductance 1 henry and of resistance 1 ohm. Find the time required by the current to attain a value half that in the steady state. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

216

MID-YEAR EXAMINATION

General Science-2: Physics Practical

Date: 29.12.70 Maximum Marks: 100 Time: $2\frac{1}{2}$ hours

Note: Answer all the questions. Marks allotted for each are given in brackets [ ].

1. Perform the experiment as indicated in Card A. [60]

2. Class work [20]

3. Laboratory Note Book [10]

4. Oral Test [10]

Distribution of marks of Q.1.

Theory and working formula	[8]
Tabulation	[40]
Calculation, graphs etc.	[7]
Accuracy	[5]
	[60]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [217]
MID-YEAR EXAMINATION

Statistics-2: Statistics Theory

Date: 30.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Let X be a Poisson distribution with mean μ, and let Y be the truncated Poisson, truncated at left at $X = 1$. i.e. Y is the variable X subject to the condition $X \geq 2$.
   a) Find the probability distribution of X. [6]
   b) What is $E(Y)$? [4]
   c) What is $V(Y)$? [8]

2. If $X_1 \sim B(3, 1/4)$
   $X_2 \sim Poi\ (2)$
   and X_1 is independent of X_2.
   a) Write down the distribution of $X_2 - X_1$. [10]
   b) Find the mean and variance of $X_2 - X_1$ without using the explicit distribution of $X_2 - X_1$. [6]

3. Show that for any discrete distribution, the standard deviation is not less than the mean deviation about the median. [6]

4. Let $A_1, A_2, \ldots, A_k$ be the classes into which any member of a population can be classified so as to belong to one and only one of the A_i's. Let p_i be the probability that a random individual falls in the class $A_i (1 \leq i \leq k)$. Let $n_1, n_2, \ldots, n_k$ be the observed frequencies of the classes $A_1, \ldots, A_k$, in a sample of size $n (= \Sigma n_i)$.
   a) Find the joint probability distribution of the variables $n_1, \ldots, n_k$. [10]
   b) Without making use of the distribution of (a) above find $E(n_i)$ and $V(n_i)$. [6]
   c) Find $cov\ (n_i, n_j)$. [8]

5. Let X take the values -1, 0 and 2 with probabilities $\frac{1}{2}p$, $1 - p$, and $\frac{1}{2}p$ respectively. Let $x_1, x_2, \ldots, x_n$ be n independent realisations of X.
   a) Find the probability generating function of
   $$Y = \sum_{i=1}^{n} x_i^2 .$$ [10]
   b) Hence find $E(y)$ and $V(y)$. [10]

Let r be the product moment correlation between X and Y.
   a) Prove that the correlation between $aX + b$ and cY [illegible] is also equal to r.
   b) Find the correlation between $fX + gY + h$ and $lX + mY + n$.

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INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[218]

MID-YEAR EXAMINATION

Statistics-2: Statistics Practical

Date: 31.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions.
Marks allotted for each question are given in brackets [ ].

1. Two candidates A and B stand for an election. In the population of voters A is favoured by 57 % of the people while the rest of the voters favour B.

A pre-election opinion poll is conducted by taking a random sample of 100 voters who are asked for their preference.

What is the probability that the opinion poll indicates the winner (i.e. A) correctly? [25]

(Use suitable approximations.)

2. A distributor of elective switches knows from past experience that about 2 % of his switches are defective. A buyer buys 120 switches and makes 12 circuits utilising 10 switches in each. A circuit breaks down if at least one switch is defective. What is the probability that not more than 2 circuits fail? [30]

(Use suitable approximations.)

3. 1200 people comprising 400 each of English, French, and Italian nationality were asked for their preferences for English, French and Italian music. The report is as follows:

132 among the English, 267 among the French and 243 among the Italians prefered their own music. In all 273 people liked English music and 361 liked French music. Only 45 Englishmen liked French music.

a) Rewrite the report in an easily understandable form. [5]

b) Is there any association between nationality of the person and the type of music he prefers. [30]

4. Practical records and neatness [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[218]

PERIODICAL EXAMINATION

Mathematics-2: Matrix Algebra

Date: 29.3.71 Maximum Marks: 100 Time: 3 hours

Note: The whole paper consists of 110 marks. You may attempt any part of any question. The maximum you can score is 100.
Marks allotted for each question are given in brackets [ ].

1. Define the following terms:

Linear transformation, rank of a linear transformation, nullity of a linear transformation, linear functional, trace of a square matrix. [5 x 2]=[10]

2. Examine whether each of the following functions T from V into V is a linear transformation. In case T is a linear transformation find the matrix of T with respect to any two ordered bases $\mathcal{B}$ and $\mathcal{B}'$ (to be chosen by you) of V.

(a) $V = E_2(R)$. $T\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}x_1^2\\x_2^2\end{pmatrix}$ where x_1 and x_2 are real numbers.

(b) V = Vector space of all polynomials of degree ≤ 3 over the field of real numbers. T(p) = derivative of p, for all $p \in V$. [11]

3. Let V be a vector space of dimension n over a field F. Let S_1 and S_2 be two subspaces of V such that

(i) $S_1 + S_2 = V$ and (ii) $S_1 \cap S_2 = \{\emptyset\}$.

For $\alpha \in V$, define $T(\alpha) = \alpha_1$ where $\alpha = \alpha_1 + \alpha_2$, $\alpha_1 \in S_1$ and $\alpha_2 \in S_2$.

(a) Show that T is a linear operator on V.

(b) Identify the range and null spaces of T.

(c) Show that T is idempotent (i.e. $T^2 = T$).

(d) Let $\beta_1, \beta_2, \ldots, \beta_r$ form a basis of S_1 and $\beta_{r+1}, \ldots, \beta_n$ form a basis of S_2. Show that $\beta_1, \ldots, \beta_n$ forms a basis of V. Consider the ordered basis $\mathcal{B} = \{\beta_1, \ldots, \beta_n\}$ of V and find the matrix of T with respect to $\mathcal{B}$. [5 x 5]=[25]

4. Let V be a vector space of dimension n over a field F and W be a vector space over F. Let T be a linear transformation from V into W.

(a) Show that $R(T) + \eta(T) = n$.

(b) Let W = V. Show that the following statements are equivalent.

GO ON TO THE NEXT PAGE

4\. (contd.)

i) Range of $T \cap$ Null space of $T = \{\emptyset\}$

ii) $T^2(\alpha) = \emptyset \implies T(\alpha) = \emptyset$

iii) $R(T^2) = R(T)$. [7+20]=[27]

5\. Let V and W be vector spaces over a field F. Let T be a linear transformation from V into W. Show that if $T(\alpha_1), \ldots, T(\alpha_n)$ are linearly independent then so are $\alpha_1, \ldots, \alpha_n$. Is the converse true? If yes, prove it. If no, give a counterexample. [5+4]=[9]

6\. a) Show that the trace function is a linear functional on the vector space of $n \times n$ matrices over a field F.

b) Show that $tr(AB) = tr(BA)$ whenever AB and BA are both defined.

c) Hence show that similar matrices have the same trace.

d) Let f be a linear functional on the vector space V of all $n \times n$ matrices over a field F such that $f(AB) = f(BA)$ for all $A, B \in V$. Show that f is a scalar multiple of the trace function. Further show that if $f(I) = n$ then f is the trace function.

e) Let V be defined as in 6(c). Do there exist A and B belonging to V such that $AB - BA = I$ (the identity matrix)? Give reasons for your answer. [5+6+3+10+4]=[28]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[220]

PERIODICAL EXAMINATIONS

Economics - 2

Date: 5.4.71 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A : Economic Theory

Attempt any three questions. One mark is reserved for neatness.

1. Give an analytical proof of the proposition that under perfect competition in both the output and factor markets, if Q is the maximum output which can be obtained at the cost of C rupees, then C rupees is the minimum cost at which the output Q can be produced.

   Give a geometrical interpretation of your analysis. [18+5]=[23]

2. a) Work out the multiplier effect of a change in government expenditure when net taxes are a rising function of income.

   b) State and prove the Haavelmo theorem on balanced budget. [13+10]=[23]

3. Do you think that fiscal policy has to be supplemented by monetary policy in order to cope up with a depression or an inflationary process? Give reasons for your answer. [23]

4. a) What are the conditions to be satisfied by a perfectly competitive commodity market? Bring out the implications of these conditions.

   b) Derive the market demand curve for a commodity. How will the demand curve for the output of an individual enterpreneur look in a perfectly competitive market? Give reasons for your answer. [15+8]=[23]

Group B: Indian Economic Problems

(Answer any two questions)

5. Discuss the main arguments against the formation of co-operative farms in India. [15]

6. Critically examine the 'scheme of integrated rural credit' as recommended by the All India Rural Credit Survey Committee. [15]

7. Explain how Government can act effectively to provide the basic facilities needed for agricultural marketing. [15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [221]

PERIODICAL EXAMINATIONS

Statistics-2: Statistics Theory and Practical

Date 12.4.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Determine the line that best fits the following points in the least squares sense, minimizing the sum of squares of vertical deviations: (0,2), (1,1), (4,3), (5,2). [10]

2.a) Explain the terms 'partial correlation' and 'multiple correlation.' [8]

b) Show that

$$\rho_{1(23\ldots n)} = \sqrt{1 - \frac{P}{P_{11}}} .$$ [12]

3. Given the following joint distribution of X and Y examine whether (i) X and Y are independent and (ii) X and Y are uncorrelated.

X =	-1	0	1
Y =			
0	.1	.1	.1
2	.1	.2	.1
4	.1	.1	.1

[10]

4. A man holds 5 tickets in a lottery in which 1000 tickets are sold. Ten tickets are to be drawn for prizes. What is the probability that the man wins at least one prize? What is the probability that he does not win any prize? (Approximate numerical answer is required). [15]

5. If (X, Y) has the joint density $f(x,y) = 2$ for $0 \leq x \leq y \leq 1$ and $f(x,y) = 0$ otherwise, compute the marginal distributions of X and Y and $E(X|Y)$. [15]

6. Given the following data, examine whether the distribution involved is normal.

Class interval	Frequency
28 - 30	2
30 - 32	15
32 - 34	30
34 - 36	20
36 - 38	5

[30]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[202]

PERIODICAL EXAMINATION

General Science-2

Date: 19.4.71. Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Physics

Answer all the questions. Maximum Marks: 50

1. Define the terms- reactance, impedance and power factor of a single phase a.c. circuit.

   An electric lamp which runs at 40 volts D.C. and consumes 10 amperes current is connected to A.C. mains at 100 volts 50 cycles. Calculate the inductance of the choke. [4×3+12]=[24]

2. Explain elaborately the term 'Entropy' and show that it increases in the process of heat conduction between two bodies.

   Show, from the consideration of the properties of the entropy, that the efficiency η of a Carnot engine is given by $\eta = (1 - T_2/T_1)$ where T_1 and T_2 are the temperatures of the source and the sink respectively. [10+4+4]=[18]

3. Write down (do not deduce) the Maxwell's four thermodynamic relations and hence show that the boiling point of a liquid changes with the change of pressure. [4+4]=[8]

Group B: Chemistry

Answer Q.1 and any other two questions. Max. Marks: 50

4.a) The apparently octa-molecular reaction represented by the equation

$$2Fe\,Cl_3 + 6KI \rightarrow 2Fe\,I_2 + 6K\,Cl + I_2$$

was experimentally found to be of the third order.

How can you account for the fact?

b) Heat of combustion of carbon is found to be 97,650 calories per gram atom. What is the intrinsic energy of CO_2?

c) t_1 and t_2 are the half-life periods of a second order reaction corresponding to the initial concentration of 0.1 Molar and 0.025 Molar respectively.

Find out the ratio of t_1 to t_2.

d) State the law of mass action and hence define the equilibrium constant for a chemical reaction.

What are the factors on which the equilibrium constant depends? [20]

5. What is Le Chatelier principle? With its help or otherwise explain the effect of increasing the pressure on the system

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$ [7]

GO ON TO THE NEXT PAGE

5.b) The density of gaseous nitrogen peroxide (N_2O_4) at 80°C is 25.6. What inferences regarding its state of dissociation can you draw from the fact?

6. Utilise the following data on the decomposition of H_2O_2 to specify the order of the reaction and calculate its half-life period.

Time in Seconds	0	10	20
N	22.8	13.8	8.25

where N is the number of c.c of $KMnO_4$ required to decompose a definite volume of H_2O_2 solution.

7.a) State Hess's law. Explain its thermodynamic basis.

b) The vapour density of phosphorus penta chloride (PCl_5) was found to be 57.92 at 250°C. Calculate its degree of dissotiation at this temperature. [P = 31, Cl = 35.5].

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[223]

PERIODICAL EXAMINATION

Statistics-2: Probability

Date: 26.4.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Let $Pr\{X = r\} = n_{c_r} \cdot p^r q^{n-r}$. Find the probability generating function of $X+1$ and $2X$. [5]

2. Y and Z are two independent random variables (whose values are non-negative integers) such that $Y+Z$ has the same distribution as X of Q.1. Show that there exists integers n', n'' such that $n' + n'' = n$ and $P\{Y = r\} = n'_{c_r} p^r q^{n'-r}$, $P\{Z = r\} = n''_{c_r} p^r q^{n''-r}$. [10]

3. Let $X_1, \ldots, X_n, \ldots$ be independent random variables with $P\{X_i = 1\} = p$, $P\{X_i = -1\} = q$. Let $S_n = \sum_1^n X_i$. For a positive integer x denote by $\lambda_n^{(x)}$, the probability that $S_1 < x, \ldots S_{n-1} < x, S_n = x$.

   a) Show that if $\phi^{(x)}(t)$ is the generating function of $\lambda_n^{(x)}$ then $\phi^{(x)}(t) = \{\phi^{(1)}(t)\}^x$. [7]

   b) Find $\phi^{(1)}(t)$ and hence write down $\lambda_n^{(1)}$. What is the probability that all S_n's are ≤ 0 ? [7]

   c) Let f_n be the probability of the first return to zero at time n (i.e. of the event $S_1 \neq 0, \ldots, S_{n-1} \neq 0, S_n = 0$) Find f_n. [8]

4.a) Find the probability generating function for a negative binomial distribution. [5]

   b) Show that for positive values of the parameters $p_0^a \{1 - p_1 t - p_2 t_2\}^{-a}$ is the probability generating function of a pair of random variables (X,Y) such that the marginal distributions of X, Y and X+Y are negative binomial. [8]

5.a) What is meant by a compound Poisson distribution? Briefly explain its uses. [6]

   b) Show that the negative binomial distribution is a compound Poisson distribution. [5]

   c) There are N Bernoulli trials, where N is a random variable with a Poisson distribution; let the numbers of successes and failures be X and Y. Are X and Y independent? [8]

The number of living offsprings an insect can have is a random variable X_1 having the binomial distribution $n_{c_r} p^r q^{n-r}$. Each offspring of the insect gives birth to more insects. The number of insects born to an offspring follows the same probability distribution. Also these numbers corresponding to different offsprings are independent random variables. Let Y = number of grand children of the insect. Find the p.g.f. of Y and hence or otherwise find E(Y). [10]

GO ON TO THE NEXT PAGE

7. Let $X_1, \ldots, X_n$ be mutually independent random variables each having the uniform distribution $\Pr\{X_i = k\} = \frac{1}{N}$ for $k = 1, 2, \ldots, N$. Let U be the maximum among $X_1, \ldots, X_n$. Find the probability distribution of U. Find the conditional probability distribution of X_1 given U. [1

8. For a group of n people find the expected number of days of the year which are birth days of exactly K people. (Assume 365 days and that all arrangements are equally likely.) [2

9. Let $X_1, \ldots, X_n$ be independent identically distributed random variables with Poisson distribution. Find

$$\Pr\left\{X_1 = K_1, \ldots, Y_n = K_n \;\middle|\; \sum_{i=1}^{n} X_i = K\right\}$$ [3

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [224]

PERIODICAL EXAMINATION

Mathematics-2: Calculus

Date: 3.5.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Integrate the following:

i) $\int x^2 \sqrt{a^3 + x^3}\, dx$, (ii) $\int \frac{dx}{e^x + e^{-x}}$

iii) $\int \frac{7x - 9}{x^2 - 2x + 35}\, dx$, (iv) $\int \frac{dx}{\sqrt{1 - x - x^2}}$

v) $\int e^x \sin x\, dx$, (vi) $\int \sin^4 x \cos^2 x\, dx$

vii) $\int \frac{dx}{13 + 3\cos x + 4\sin x}$, (viii) $\int \frac{dx}{1 - x^3}$. [32]

2.a) Find the area bounded by the hyperbola $xy = c^2$, the x-axis and the ordinates $x = a$, $x = b$.

b) Find the area bounded by the x-axis and one arch of the sine curve $y = \sin x$.

c) Find by integration the area of the triangle bounded by the line $y = 3x$, the x-axis and the ordinate $x = 2$. Verify your result by finding the area as half the product of the base and the altitude. [18]

3. Evaluate the following integrals:

i) $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$, (ii) $\int_0^{\pi/2} \sin^6 \theta \cos^3 \theta\, d\theta$

iii) $\int_0^{\pi/2} \sin^n x\, dx$, ($n$ = even integer),

iv) $\int_0^{\pi} \cos^n x\, dx$, ($n$ = integer),

v) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$. [20]

4. Evaluate when possible, the following integrals:

i) $\int_{-1}^{+1} \frac{dx}{x^3}$, (ii) $\int_0^2 \frac{dx}{2 - x}$, (iii) $\int_0^{\infty} \frac{dx}{(x+1)(x+2)}$,

iv) $\int_0^{\infty} \frac{x\, dx}{(x^2 + a^2)(x^2 + b^2)}$ ($a, b > 0$). [20]

5. Find the volume of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71
ANNUAL EXAMINATION

[225]

Mathematics-2: Matrix Algebra

Date: 7.6.71 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 140 marks. You may attempt any number of questions. The maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. Let A be a $m \times n$ matrix.
   a) When is a matrix B said to be a right inverse of A?
   b) Show that A has a right inverse if and only if $R(A) = m$.
   c) Show that if A has a right inverse C and a left inverse B, then A is non-singular and $B = C = A^{-1}$.

[3+5+3]=[11]

2. a) When is a square matrix A (over the field of real numbers) said to be strictly diagonally dominant ?
   b). Show that if A is strictly diagonally dominant then A is non-singular.
   c) Let A be a square matrix such that $A^2 + 2A + I = 0$. Show that A is non-singular. Obtain the inverse of A.
   d) Show that if $(I+A)^{-1} A = D$ where D is a nonsingular diagonal matrix, then A is also a non-singular diagonal matrix.

[2+7+5+6]=[20]

3. a) Let V be a finite-dimensional vector space over the field of real numbers and $\alpha_1, \alpha_2, \ldots, \alpha_n$ be non-null vectors belonging to V. Show that there exists a linear functional f on V such that $f(\alpha_i) \neq 0$ for $i = 1, \ldots, n$.
   b) Let T be a linear operator on V. Show that the following statements are equivalent:
      i) T is a projection operator projecting arbitrary vectors in V onto range of T along the null space of T.
      ii) T is idempotent.

[10+10]=[20]

4. a) Let $B = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ be a square matrix where A_{11} is non-singular. Show that $|B| = |A_{11}|\,|A_{22} - A_{21} A_{11}^{-1} A_{12}|$.
   b) Let A be a non-singular matrix of order $n \times n$ and u a $n \times 1$ vector. Explain how you can use the above result to compute $u' A^{-1} u$.

[8+3]=[11]

5. a) Show that $R(AB - I) \leq R(A - I) + R(B - I)$.
   b) Show that $R(A'A) = R(A)$ if A is a real matrix. Is this true for a matrix over any field? If yes, prove it. If no, give a counter example.
   Show that if $R(A+B) = R(A) + R(B)$ then $\mathcal{M}(A) \cap \mathcal{M}(B) = \{0\}$.

[5+7+5]=[17]

GO ON TO THE NEXT PAGE

6.a) When is a square matrix H said to be in Hermite Canonical Form (H.C.F.)?

b) Show that if H is in H.C.F. then H is idempotent.

c) Show that if H is idempotent then $R(H) = tr(H)$.

d) Let H be a square matrix of order $n \times n$. Show that H is idempotent if and only if $R(H) + R(I - H) = n$.

[2+6+5+6]=[19]

7. Describe a method to reduce a square matrix A to H.C.F. using only elementary row operations. Prove that a square matrix A is indeed reduced to H.C.F. if one uses the above method.

[7+7]=[14]

8. (In this question we consider matrices and vectors over the field of real numbers) Consider the system of linear equations $Ax=y$ where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 5 & 6 \end{pmatrix} \quad \text{and} \quad y = \begin{pmatrix} 6 \\ 12 \\ 19 \end{pmatrix}.$$

a) Check whether the above system is consistent. If it is consistent obtain a solution.

b) Obtain a g-inverse of A.

c) What is the rank of A?

[15]

9. Let A be a $m \times n$ matrix. Then a $n \times m$ matrix G is said to be a g-inverse of A if $AGA = A$.

a) Show that G is a g-inverse of A if and only if (i) AG is idempotent and (ii) $R(AG) = R(A)$.

b) Let $R(A) = R(AH)$. Show that if $GAH = H$ then G is a g-inverse of A.

[8+5]=[

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[226]

ANNUAL EXAMINATION

Statistics-2: Statistics Theory

Date: 8.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Suppose that the probability of no flat tyres in a time period of $t = 5$ hours is 0.9. What is the probability of less than 25 flat tyres in a time period of 100 hours? [Hint: Assume that the number of flat tyres in a time interval of length t follows a Poisson law with parameter λt, where λ is a fixed positive constant.] [10]

2. Two random variables x and y have the least squares regression lines $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find the mean values and the correlation coefficient. [15]

3. A large business firm wants a quick estimate of total rupee sales in a one-day period. To save time the amount of each sale is rounded to the nearest rupee. It is assumed that costs from sale to sale are independent, and for simplicity that the rounding off error has a uniform density on [-.5, .5]. Get a reasonable upper bound for the probability that the error in totalling 10,000 sales exceeds Rs.100/-. [10]

4. Let X_i, for $i = 1, 2, \ldots, n$ be independent random variables following $N(0, \sigma^2)$, where σ^2 is unknown. Write down the likelihood function. Get an unbiased estimate for σ^2 by the maximum likelihood method. Examine whether the estimate that you propose is efficient. [20]

5. Suppose $X_1, \ldots, X_n$ are independent random variables, each distributed uniformly on $[0, \theta]$. Find a 95% confidence interval for θ whose length goes to zero as $n \to \infty$. [10]

6. Explain briefly the following terms:
   a) Type I error and Type II error.
   b) Critical region.
   c) Level of significance. [15]

7. Examine whether the following statements are true or false (if any is false, give reasons):
   a) The maximum likelihood method always yields unbiased estimates for the parameters.
   b) Let X_1 and X_2 be independent binomial random variables each taking two values only, 0 and 1, and let $P[X_1 = 1] = p$ and $P[X_2 = 1] = 2p$, where $0 \le p \le 1/2$. Then $X_1 + X_2$ is sufficient for p.
   c) The hypothesis that X is normally distributed is simple.
   d) $\bar{X}$ is not a consistent estimate for θ when the X_i's are independent observations from a Cauchy distribution with parameter θ. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [227]

ANNUAL EXAMINATION

Statistics-2: Statistics Practical

Date: 9.6.71 Maximum Marks: 100 Time: 3 hours

1. The following table gives the means and the variance-covariance matrix of 4 characters X_0, X_1, X_2 and X_3:

	X_1	X_2	X_3	X_0	Means
X_1	15.129	23.860	1.793	0.998	14.9
X_2		54.756	3.633	3.511	30.5
X_3			18.225	21.122	7.8
X_0				60.516	18.3

a) Obtain the multiple linear regression equation of X_0 on X_1, X_2 and X_3.

b) Compute the multiple correlation coefficient of X_0 on X_1, X_2 and X_3.

c) Compute the partial correlation coefficient between X_0 and X_3 eliminating the effects of X_1 and X_2. [35]

2. Fit a suitable frequency distribution to the following data:

Number of heads in throwing 12 coins.

Number of heads	Frequency	Number of heads	Frequency
0	185	7	1331
1	1149	8	403
2	3265	9	105
3	5475	10	14
4	6114	11	4
5	5194	12	0
6	3067		
		Total	26306

Test for the goodness of fit. [30]

3. The following table gives the frequencies of eggs laid by gall-flies on flower heads. The count of flower heads with 'no eggs' is not available. Find the maximum likelihood estimate of the average number of eggs laid, assuming that the number of eggs laid follows a Poisson distribution.

GO ON TO THE NEXT PAGE

Number of eggs laid	Number of flower heads
(1)	(2)
1	22
2	18
3	18
4	11
5	9
6	6
7	3
8	0
9	1
Total	88

[15]

4. Practical Record [10]

5. Viva Voce. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[228]

ANNUAL EXAMINATION

Economics-2: Economic Theory

Date: 10.6.71 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [].

Group A: Attempt any two questions.

1.a) Prove that in a monopoly market the imposition of a sales tax will induce the monopolist to reduce his output and to raise the price, but that of a lump sum tax will not affect his output and price.

b) A monopolist can sell his product in two economically isolated markets in each of which the demand curve is known to be a negatively sloped straight line. Will price discrimination pay,

i) if the two demand curves are parallel?

ii) if they intersect at some point on the price axis?

Give reasons for your answer. [15+10]=[25]

2. 'There are many different solutions to the duopoly problem. Each solution is based upon a different set of behaviour assumptions'. Discuss the statement with particular reference to the Cournot and Stackelberg solutions. [25]

3.a) Briefly explain how factor prices are determined under imperfect competition in factors markets and pure competition in product markets.

b) A monopsonist's production and labour supply functions are:

$$q = 15x^2 - 0.2x^3$$

and

$$w = 144 + 23.4x$$

(where q = units of output

x = number of labour units used

w = wage rate).

He sells his output in a competitive market at a price of 3 rupees per unit. Determine his maximum profit and the corresponding output and wage rate. [15+10]=[25]

4.a) Show that the scale elasticity of the production process when the cost of inputs is at a minimum, becomes equal to the ratio of average cost to marginal cost under suitable conditions.

b) A firm is operating with a given plant at a fixed cost of Rs.98 and a variable cost of V rupees given by

$$V = q^3 - 12q^2 + 40q,$$

q being the number of units produced. How much will it supply in a competitive market if the price is Rs.19 per unit and what will be its profit or loss? [13+12]=[25]

GO ON TO THE NEXT PAGE

Group B.

The maximum that you can score from this group is 50.

5. Give an analytical exposition of the interaction between the multiplier and the acceleration principle. [30]

6. In a closed economy, the community's planned consumption expenditures are four-fifths of its private disposable income and the government's net tax receipts are one-fourth of the national income. Initially, the government expenditures on goods and services are just balanced by its net tax receipts. Show that if the government doubles the national income by increasing its expenditures (private investment and prices remaining unchanged), the total budget deficits will amount to Rs.1500 crores, if initially the level of national income was at Rs.10000 crores.

[National income = national income at market prices. Assume that there are no productive enterprises owned by the government]. [15

7. In a two-country model how will incomes in both countries change on account of an increase in autonomous exports in one of them? Give an analysis of the corresponding change in income if, alternatively, autonomous investment were to rise in the same country. [20]

8. Build up the Keynesian theory of employment. [15]

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(229)

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

ANNUAL EXAMINATION

Economics-2: Indian Economic Problems

Date: 11.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. All questions carry equal marks.

1. 'A review of land reforms reveals much that has been achieved as well as a great deal that requires urgent attention. There are many gaps between objectives and legislation and between the laws and their implementation'. - Fully examine the statement.

2. Do you agree with the view that in the present context, expansion of co-operative farms can solve some of the urgent problems of the agricultural sector? Give reasons for your answer.

3. Indicate the existing sources of rural credit in India and the role played by the Reserve Bank of India in this field.

4. Elucidate the main features of the Industrial Policy of the Government of India as embodied in their resolution of 1956.

5. Examine the need for expansion of the public sector in the context of economic planning in India. Do you agree with the view that the expansion of the public sector becomes necessary since the pattern of investments in the plans is such that it has to be undertaken by the Government?

6. Examine the objectives and functions of the Industrial Development Bank of India.

7. With special reference to the 'locational pattern' of Indian industries, comment on the view that since the industrial activity has been concentrated in a few selected areas, regional planning of industries should be given due place in our programme for development.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[230]

ANNUAL EXAMINATION

Mathematics-2: Calculus

Date: 14.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each questions are given in brackets [].

1.a) If $f(x) = 2 + 2x$ for $-\frac{3}{2} < x \leq 0$

$= 3 - 2x$ for $0 < x < 3/2$

Show that $f(x)$ is continuous at $x = 0$ but $f'(0)$ does not exist.

b) Find the differential coefficient of the following with respect to x:

i) $2^x \sin x$ (ii) $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ (iii) x^{x^x}

c) Find $\frac{dy}{dx}$ when $x = a\cos^3\theta$, $y = a\sin^3\theta$.

d) Differentiate $\tan^{-1} x$ with respect to x^2.

e) A man 5 ft. tall walks away from a lamp post $12\frac{1}{2}$ ft. high at the rate of 3 miles per hour.

i) How fast is the farther end of the shadow moving on the pavement?

ii) How fast is his shadow lengthening? [5+6+2+2+10]=[25]

2. If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. [5]

3. A conical tent of a given capacity has to be constructed. Find the ratio of the height to the radius of the base for the minimum amount of the canvas required for the tent. [10]

4. Find the point on the curve $y = x^2 + 3x + 4$, the tangent at which passes through the origin. [5]

5. Find the centre of curvature at any point (x,y) on the parabola $y^2 = 4ax$. [10]

6. If the asymptotes of the curve

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad (h^2 > ab)$$

pass through the origin, prove that

$$af^2 + bg^2 = 2fgh.$$ [10]

GO ON TO THE NEXT PAGE

7. Find the envelopes of the straight lines

$$\frac{x}{a} + \frac{y}{b} = 1$$

where the parameters a and b are connected by the relation.

i) $a^2 + b^2 = c^2$ (ii) $ab = c^2$, c being a constant. [10]

8.a) Find the centre of gravity of a solid hemisphere of uniform density.

b) Show that the C.G. if a thin hemispherical shell is at the middle point of the radius perpendicular to its bounding plane. [10]

9. Solve the following differential equations:-

i) $\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ (ii) $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$.

iii) $(2x - y + 1)dx + (2y - x - 1)dy = 0$. [15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part II:1970-71 [231]

ANNUAL EXAMINATION

Statistics-2: Probability

Date:15.6.71 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 111 marks.
Answer as many questions as you can.
The maximum you can score is 100.
Marks allotted for each question are given in brackets [ ].

1. Consider two independent events A and B such that $P(A) = .25$, $P(B) = .5$. Let the random variables X and Y be defined as follows:

$$X = 1 \text{ if } A \text{ has occurred}, \quad = 0 \text{ otherwise}$$

$$Y = 1 \text{ if } B \text{ has occurred}, \quad = 0 \text{ otherwise.}$$

Let $X^2 + Y^2 = Z$ and $XY - X^2Y^2 = V$.
State, with reasons, whether the following statements are true or false.

i) The random variables X and Y are independent. [3]

ii) $\Pr\{Z = 1\} = 1/4$ [3]

iii) $\Pr\{V = 0\} = 1$ [3]

2. Three dice are to be rolled. The first one has the numbers 1 to 6 on its faces, the 2nd one has each of the numbers 2,4,6 on a pair of faces; the third one has the numbers 1 to 4 on four faces and 5 on the other two faces. Let X = sum of the three numbers seen when the dice are rolled at random. Find the probability generating function of X. [7]

3. For a group of n people find the expected number of multiple birthdays (A multiple birthday is a day which is the birthday of more than one person. Assume 365 days in a year and all arrangements are equally likely.) [5]

4. Let S_n be the number of successes in n Bernoulli trials. Find $E|S_n - np|$. [8]

5.a) Let $\phi(x) > 0$ for $x > 0$ and be monotonically increasing and suppose that $E(\phi(|X|)) = M$ is finite. Prove that

$$P\{|X| \geq t\} \leq \frac{M}{\phi(t)}.$$ [5]

b) Suppose $X_1, \ldots, X_n$ are i.i.d. with mean μ, and fourth moment μ_4 about mean. Show that $P\{|\bar{X}_n - \mu| > a\} \leq \frac{A}{n^2}$ where A is a constant. [5]

6. In a sequence of Bernoulli trials with $p > q$, let S_n = number of successes in n trials - number of failures, in n trials and let a_n be the probability that there exists an integer $j > n$ such that $S_j = 0$. Find the generating function of a_n. [10]

GO ON TO THE NEXT PAGE

7. Let N have a Poisson distribution with mean λ and let N balls be placed randomly into n cells. Show that the probability of finding m cells empty is

$$ {}^{n}C_{m} \cdot e^{-\lambda m/n} \left[(1-e)^{-\lambda/n}\right]^{n-m}. $$ [8]

8. a) Suppose that in a ballot, candidate P scores p and candidate Q scores q votes where $p > q$. Find the probability that there are always more votes for P than for Q. What is the probability that votes polled for P remain greater than or equal to those polled for Q? [9+6]=[15]

b) $n+m$ people are waiting in line at a box office; n of them have five rupee notes and the other m have ten rupee notes. The tickets cost five rupees each. When the box office opens there is no money at the counter. If each customer buys just one ticket, what is the probability that none of them will have to wait for changes? [6]

9. a) Let $\Pr\{X_i = 1\} = \Pr\{X_i = -1\} = 1/2$, $S_n = X_1 + \ldots + X_n$. Find the following probabilities

i) $P(S_1 \neq 0, \ldots, S_{2n} \neq 0)$ [7]

ii) $P(S_1 \neq 0, \ldots, S_{2n-1} \neq 0, S_{2n} = 0)$ [7]

b) Prove that

$$ u_{2n} = \sum_{r=1}^{n} f_{2r}\, u_{2n-2r} $$

where $u_{2n} = {}^{2n}C_{n}\, 2^{-2n}$,

and $f_{2n} = \frac{1}{2n} u_{2n-2}$ if $n \geq 1$

$= 0$ if $n = 0$.

c) Let X be a random variable such that

$\Pr\{X = 2k\} = u_{2k} \cdot u_{2n-2k}$

where u_{2n} is defined in (b). Find the modes of X. [

d) State the arc sine law for the time spent on the positive side. [

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[232]

ANNUAL EXAMINATION

General Science-2: Physics Theory

Date: 16.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer five questions, taking at least two from each group. Marks allotted for each question are given in brackets [ ].

Group A

1. Show that the law of conservation of energy holds good for a body executing simple harmonic motion.

Show that the velocity v and the acceleration f for a simple harmonic motion satisfy

$$\omega^2 v^2 + f^2 = a^2 \omega^4$$

where 'a' is the amplitude and ω the frequency of the motion.

A body falls from the earth's surface through a frictionless tunnel passing through its centre. Assuming the earth to be a homogeneous sphere of radius 6400 km. and g = 980 cm./sec^2, calculate the time taken by the body to reach the earth's centre. [7+5+8]=[20]

2. Establish the bending equation for an isotropic elastic solid. Apply the above equation to calculate the displacement at the free-end of a loaded cantilever of rectangular cross-section. Neglect the weight of the cantilever. Explain the term 'flexural rigidity'. [6+10+4]=[20]

3. Investigate the growth and decay of current in a circuit composed of an inductance L, resistance R and a battery of e.m.f. E all connected in series.

Explain the term 'electrical resonance' in reference to an a.c. circuit consisting of a capacity, an inductance and a resistance in series.

A telephone operates at a current of 120 milliampere and has an inductance 10 henries and resistance 100 Ohms. If a 24 volt battery having negligible internal resistance is suddenly applied calculate the operating time. [6+3+4+7]=[20]

4. Give the theory of the Helmholtz double coil galvanometer. In what respect it is superior to the single coil tangent galvanometer? Explain the term reduction factor of a galvanometer and give the expression for it in the case of a Helmholtz galvanometer. What do you mean by the current sensitivity of a suspended coil galvanometer? [10+4+3+3]=[20]

Group B

5. What do you mean by the term 'coherent sources'? Draw two neat diagrams to show how the coherent sources are produced in the cases of Fresnel's bi-prism and Lloyd's mirror and also how fringe patterns are generated in those arrangements. (No description necessary). Compare and contrast the bi-prism fringes with those formed in Lloyd's mirror. Explain the meanings of the terms 'fringes of equal inclination' and 'fringes of equal chromatic order'. [3+5+4+4+2+2]=[20]

GO ON TO THE NEXT PAGE

6. Explain the Fresnel's concept of half-period zones. Show how the principle of half period zones has been applied to explain the rectilinear propagation of light. [6+14]=[20]

7. Describe with a neat diagram (properly labelled) the construction and the principle of a Michelson Interferometer. Enumerate its different uses. [5+2+5+5+3]=[20]

8.a) Prove the following from thermodynamical considerations, the symbols having their usual significance.

a) $C_p/C_v = E_s/E_T$; (b) $C_p - C_v = TE\,\alpha^2 V$

c) $\partial C_v/\partial v = T\partial^2 p/\partial T^2$ [4+5+3]=[12]

d) A mass, m, of water at T_1 is isobarically and adiabatically mixed with an equal mass of water at T_2. Show that the entropy change of the universe is $2m \log_e\{[(T_1 + T_2)/2]/ (T_1 T_2)^{1/2}\}$. Assume that the specific heat of water at constant pressure is unity. [4]

b) Select the correct answer from among those supplied.

A. A frictionless heat engine can be 100 per cent efficient only if its exhaust temperature is

a) equal to its input temperature b) 0'K

c) less than its input temperature d) 0'K. [2]

B. Longitudinal waves do not exhibit

a) refraction b) reflection

c) interference d) polarisation. [2]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[233]

ANNUAL EXAMINATION

General Science-2: Chemistry Theory

Date: 16.6.71   Maximum Marks: 100   Time: 3 hours

Note: Answer any five questions.
Marks allotted for each question are given in brackets [ ].

1.a) State the first law of thermodynamics and show that Hess's law can be derived from the first law. [8]

b) What are exothermic and endothermic compounds? Give examples. [4]

c) The heat of combustion of acetylene gas (C_2H_2) is 312,700 calories and that of liquid benzene (C_6H_6) is 782,000 calories. Calculate the heat of polymerisation of acetylene to benzene. [8]

2.a) What do you understand by a reversible chemical reaction? How does the equilibrium constant of a reaction relate to the temperature? [8]

b) The equilibrium constant for the reaction between acetic acid and alcohol, forming ethyl acetate and water at 25° is 4. If 2.5 gram molecules of acetic acid react with 1 gram molecule of alcohol as completely as possible, what will be the composition of the equilibrium mixture? [12]

3.a) Derive the relationship between the osmotic pressure of a solution and its lowering of vapour pressure. [12]

b) What is the osmotic pressure in atmospheres at 24.2°C of a solution of glucose ($C_6H_{12}O_6$) containing 90 grams per litre? [8]

4.a Is the elevation of boiling point of a solution related to its concentration? If so give reasons in favour of your arguments. Deduce the relationship and explain the term 'molal elevation of boiling point'. [12]

4. The freezing point of pure benzene is 5.440°C and that of a solution containing 2.093 grams of benzaldehyde in 100 grams benzene is 4.440°C. Calculate the molecular weight of benzaldehyde. Cryoscopic constant for benzene = 5. [8]

.a) The following apparently octamolecular reaction

$$2FeCl_3 + 6KI \rightleftharpoons 2FeI_2 + 6KCl + I_2$$

was actually found to be of the third order. How do you account for this fact? [5]

b) The isomeric transformation of N-chloroacetanilide to p-chloroacetanilide was followed by iodometric estimation of the former in the reaction mixture from time to time and the following data were obtained.:

GO ON TO THE NEXT PAGE

5.b) (contd.)

Time minutes	0	60	120	180	240	360
Titration (a - x)	49.3	35.6	25.75	18.5	13.5	7.3

Utilise the data to prove that the reaction is of the first order. Also calculate the half life of the reaction. [15]

6. At 200°C and atmospheric pressure the vapour density of PCl_5 is 70 (H = 1). Calculate.

a) the degree of dissociation of the PCl_5 vapour. [6]

b) partial pressures of PCl_5, PCl_3 and Cl_2. [7]

c) concentrations of PCl_5, PCl_3 and Cl_2. [7]

when one gram molecule is heated under atmospheric pressures to 260°C.

7. Write what you know about the following:

a) Redox potential (b) External indicator

c) Le Chatelier principle (d) Raoult's law. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[234]

ANNUAL EXAMINATION

General Science-2: Physics Practical

Date: 17.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions.
Marks allotted for each question are given in brackets [ ].

1. Perform the experiment as indicated in Card A. [60]

2. Class work. [20]

3. Practical Note Book [10]

4. Oral Test [10]

Distribution of marks of Q.1

1.	Theory	5
2.	Working formula	5
3.	Tabulations	40
4.	Calculation, graph etc.	5
5.	Accuracy	5

N.B. No procedure of the experiment need be written. Special credit would be given for intelligent multiplication of data. All calculations should be neatly shown. Entry of data must be made in ink and not with lead pencil. No dot-pen should be used.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71 [235]

ANNUAL EXAMINATION

General Science-2: Chemistry Practical

Date: 17.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions.
Marks allotted for each question are given in brackets [ ].

1. Standardise the supplied sodium thiosulphate solution with the help of a standard $K_2 Cr_2 O_7$ solution and estimate the total quantity of Cu^{++} present in the supplied solution marked (c). [70]

2. Practical Record [10]

3. Viva voce [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

[256]

ANNUAL EXAMINATION

General Science-3: Biology Theory

Date: 18.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.1 and any three from the rest.
Give drawings wherever desirable.
Marks allotted for each question are given brackets [ ].

1. Give a detailed morphological description of the family Palmaceae with suitable illustrations. Give the names of ten plants belonging to Palmaceae. [15+10]=[25]

2. Mention the characteristic features of Leguminosaceae. Give a comparative morphological description of the sub-families of Leguminosaceae. [25]

3. Write brief accounts of any two of the following:

   a) Aestivation of corolla in Malvaceae;
   b) Androecium in Euphorbiaceae;
   c) Uses of palms. [25]

4. Write short notes on any five:

   a) Sungenesious stamens;
   b). Spikelet of rice;
   c) Androecium in Malvaceae;
   d) Spadix inflorescence;
   e) Fruit of Umbelliferae;
   f) Leaves in Rutaceae. [25]

5. What is a cereal crop? Give the morphology of any cereal crop. Write the names of ten plants belonging to Gramineae. [25]

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[237]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part II: 1970-71

ANNUAL EXAMINATION

General Science-B: Biology Practical

Date: 19.6.71 Maximum Marks:100 Time: 3 hours

Note: Answer all the questions.
Give drawings wherever desirable.
Marks allotted for each question are given in brackets [ ].

1. Give a botanical description of specimen A. Assign the plant to its family giving reasons. [20]

2. Describe the morphology of specimens B and C. Give the floral formula and floral diagram of a flower each from specimens B and C. [10+10]=[20]

3. Comment on specimens D and E making labelled drawings. [10+10]=[20]

4. Identify specimens F to O. [10×2]=[20]

5. Submit your practical record books. [20]
