INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Backpaper Examination
Statistical Methods

Date: 7.7.89 | Maximum Marks: 100 | Time: 3 Hours.

Note: From Group A Answer any THREE questions.
From Group B Answer all the questions.

Group A

1. Let $x_1, --- x_{n_1}$ and $y_1, --- y_{n_2}$ be the random samples from two normal populations with the same mean m and variances σ^2 and $\lambda\sigma^2$ respectively, where λ is a known positive constant:
Obtain the maximum likelihood estimates of m and σ^2.
Are they unbiased? If not obtain the unbiased estimates. [21]

2.(a) State and prove Neyman-Pearson's theorem of obtaining most powerful test for testing a simple hypothesis against a simple alternative.

(b) Let $x_1, --- x_{10}$ be a random sample of size 10 from a normal distribution $N(0,\sigma^2)$. Find the best critical region of size $\alpha = .05$ for testing $H_0\ [\sigma^2=2]$ against $H_1[\sigma^2=3]$. Is this a best critical region of size .05 for testing (i) $H_0[\sigma^2=2]$ against $H_1[\sigma^2=1]$; (ii) $H_0[\sigma^2=2]$ against $H_1[\sigma^2>2]$.
[illegible] results in each case.
Indicate how will you compute the size and power of the test under $H_0[\sigma^2=2]$ against $H_1[\sigma^2=3]$. [7+14=21]

3.(a) Let $x_1, -- x_{n_2}$ and $y_1, -- y_{n_2}$ be random samples from $N(\mu_1,\sigma^2)$ and $N(\mu_2,\sigma^2)$ respectively, where all the parameters are unknown. Derive the likelihood ratio test for testing $H_0[\mu_1=\mu_2]$ against $H_1[\mu_1\neq\mu_2]$.

(b) What do you mean by stabilization of variance? Derive the fundamental lemma for arriving at this result. Hence obtain transformation on sample correlation coefficient. [11+10=21]

4.(a) Derive the sampling distribution of the sample regression coefficient b_{yx} on the basis of a random sample $(x_1,y_1),--(x_n,y_n)$ from $BN(\mu_1,\mu_2,\sigma_1,\sigma_2,\rho)$. Hence show that the likelihood ratio test for testing $H_0[\beta = \beta_0]$ against $H_1[\beta \neq \beta_0]$ is equivalent to t-test where t follows t-distribution with n-2 d.f.

(b) Discuss the important uses of frequency Chi-square in testing of hypothesis. [13+8=21]

p.t.o.

Group- B

5. To examine the growth of cosk of certain trees, observations of the thickness of cosk borings were obtained on 10 trees in a block of plantations from the Northern (X) and Eastern (Y) directions of t trunk. The weights of these borings (measured in centigrams) are given below:

Tree No.:	1	2	3	4	5	6	7	8	9	10
X :	72	60	56	41	32	30	39	42	37	33
Y :	66	53	57	29	32	35	37	43	40	29

Analyse the data to test

(i) whether there is a significant difference between the mean weights of the cork borings on the Northern and Eastern sides of the trees.

(ii) whether the variabilities of weights for the northern and eastern sides differ significantly.

use α = .05 in each case. [6+8=14]

6. The following records the operating life in hours of 10 electric bulbs.

980, 992, 1005, 998, 1010, 1020, 1028, 1042, 975, 1015.

Obtain 90% confidence interval for the population median life of electric bulbs. [7]

7. In a radiation experiment, 20 egg cells were treated with X-ray and 20 were untreated (control).

Examine the data given below to find out whether radiation does affect the growth of cells.

	X-ray	Control	
Affected :	12	4	16
Not affected:	8	16	24
	20	20	40.

[6]

8. Practical Note Book. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Backpaper Examination
Calculus-4

Date: 5.7.89 | Maximum Marks: 100 | Time: 3 Hours.

Note: Answer all questions.

1.(a) Give an example of a form $Pdx + Qdy$ on an open subset $\Omega \subseteq \mathbb{R}^2$ such that $\frac{Q}{x} = \frac{P}{y}$ everywhere on Ω but $Pdx + Qdy$ is not an exact differential.

(b) Using the form of Green's theorem proved in the class, prove that

$$_{R}(u \quad w - w \,\Delta\, u)\, dxdy = {}_{R}(u \frac{dw}{dn} - w \frac{du}{dn})\, d$$

with the usual assumptions on u, w, R and R. [8+8=16]

2.(a) For a vector field $\underline{f} = (f_1, f_2, f_3)$ on $\mathbb{R}^3$, prove that

$$\text{curl}\,(\text{curl}\,\underline{f}) = \text{grad}\,(\text{div}\,\underline{f}) - {}^2\underline{f}$$

(b) State Stokes' theorem for surfaces in $\mathbb{R}^3$.

(c) Obtain Green's theorem for a plane domain from Stokes' theorem in (b). [4+4+6=14]

3.(a) Find

$$_{S}\; z\, d\, x\, d\, y$$

where S is the surface $x^2+y^2+z^2 = R^2$.

(b) Find the arc-length of the curve

$$(t,\ t \sin \frac{1}{t}) \qquad t \quad (0,1]$$

if it exists. [8+6=14]

4.(a) State and prove the Lagrange theorem about maximisation of function under constraints.

(b) Let $f(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2$. Find the maximum value of f subject to the restrictions

$$x_1^2 + x_3^2 + x_4^2 = 4$$

$$x_2^2 + 2x_3^2 + 3x_4^2 = 9$$

[13+10=23]

p.t.o.

5. Find the tangent plane of the surface

$$1 + x \cos \pi z + y \sin \pi z - z^2 = 0 \text{ at } (0,0,1).$$ [8]

6.(a) If $\underline{a} = (a_1, \dots a_n)$ $\mathbb{R}^n$, define

$$\|\underline{a}\|_p = (\Sigma |a_i|^p)^{1/p}$$

where $\leq p < \infty$. Show that $\|a\|_p$ in a norm

(b) Find $\lim_{p \to \infty} \|a\|_p$ [12+3=15].

7. Prove that if f is a continuous periodic function on $\mathbb{R}$, and σ_n is the sequence of cesaro means of the Fourier partial sum; then $\sigma_n \longrightarrow f$ uniformly. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Backpaper Examination
Linear Estimation and ANOVA

Date: 3.7.89 Maximum Marks: 100 Time: 3 Hours

1.(a) Consider the model: $y_{ij} = \mu_{ij} + e_{ij}$

$\{e_{ij}\}$ are independent $N(0,\sigma^2)$

$i = 1,2,\ldots,m;\ j = 1,2,\ldots, n.$

Can you test the hypothesis $\mu_{ij} = \alpha_i+\beta_j \quad \begin{cases} i=1,2,\ldots, m \\ j=1,2,\ldots, n \end{cases}$

Justify your answer.

(b) Consider the model: $y_{ijk}=\mu_{ij} + e_{ijk}$

$\{e_{ijk}\}$ are independent $N(0, \sigma^2)$,

$i=1,2,\ldots,m;\ j=1,2,\ldots,n;\ k=1,2,\ldots,c.$

Test the hypothesis $\mu_{ij} = \alpha_i+\beta_j+\gamma_{ij}$ given that

$\sum_{i=1}^{m} \alpha_i=0=\sum_{j=1}^{n} \beta_j, \sum_{j=1}^{n} \gamma_{ij}=0 \quad i=1,2,\ldots,m$

$\sum_{i=1}^{m} \gamma_{ij}=0$ for $j=1,2,\ldots,n$ [5+13=18]

2. Clearly state and prove the S-method of multiple comparison in the general case. [20]

3. Consider the set-up

$$Y \sim N(X\beta,\sigma^2 I), \text{ rank } (X)=r, \sigma^2>0 \text{ unknown.}$$

Let $\Psi^T=\{\psi_1,\psi_2,\ldots,\psi_k\}$ be specified linear parametric functions. Define a confidence set for Ψ with confidence coefficient $1-\alpha\ (0<\alpha<1)$.

Given $0<\alpha<1$, derive a confidence set with confidence coefficient $1-\alpha$. (State the results you may make use of). [20]

4. Consider the observations $Y^T=(y_1,y_2,\ldots,y_n)$ such that

$E(y_i)= \sum_{j=1}^{m} x_{ij}\beta_j$, $V(y_i)=\sigma^2$ and $\text{cov}(y_i,y_k)=\rho\sigma^2$ for $i\neq k$ where $i=1,2,\ldots,n;\ j=1,2,\ldots,m$ and $\sigma^2>0$ is unknown. Make an orthogonal transformation from Y to Z such that $z_1=\frac{1}{\sqrt{n}}(y_1+y_2+\ldots+y_n)$ and

$Z_2^T=(z_2,z_3,\ldots,z_n)$. Show that z_i's are uncorrelated and $E(Z_2)=U\beta$,

p.t.o.

where U depends on $X=((x_{ij}))$ and the particular matrix of transformation. If ρ is unknown, show that $(Z_2, U\beta, \sigma^2(1-\rho)I)$ is a model under the Gauss-Markov set up. [15]

5. Suppose we wish to determine whether or not four different tips produce different readings on a hardness testing machine. The machi operates by pressing the tip into a metal specimen, and from the depth of the resulting depression, the hardness of the specimen can be determined. We have decided to obtain four observations for each tip. We test each tip once on each of four specimens. The data is given below.

Hardness Testing Experiment

Type of Tip	Specimen (Block)			
	1	2	3	4
1	9.3	9.4	9.6	10.0
2	9.4	9.3	9.8	9.9
3	9.2	9.4	9.5	9.7
4	9.7	9.6	10.0	10.2

Carry out the Analysis of Variance treating the above data as

(a) One-way layout (b) Two-way layout.

Comment on your findings. [10+12+5=27]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year:1988-89
Semestral-II Examination
Elective:3 Physical and Earth Sciences

Date: 10.5.89 Maximum Marks:100 Time: 3 Hours

Note: Answer four of the following questions taking at least one from any of the groups.

Group-A

1.(a) Define the term adsorption and writedown the forms of Fruendlick, Longmier and BET equations.

(b) State the basic assumptions of Longmuir theory and derive the equation. Explain the physical significance of the constants involved.

(c) The following data on the adsorption of CO_2 gas on one gram of charcoal at 0^oC. were found to satisfy Longmuir equation.

p (pressure)	25.1	137.4	416.4	858.6 mm
a (amt of CO_2 absorbed)	0.77	1.78	2.26	2.42 mg

By appropriate graphical plot of the data calculate the monolager capacity of the charcoal sample. Assuming the cross section over of CO_2 molecule as $26A^2$ evaluate the specific surface area of the Charcoal. [7+10+8]=25

2.(a) What is meant by the electric double layer surrounding a charged colloid dispersed in an electrolyte solution. Describe the Stern model and point out its superiority over both Helmlotz and Gouy-Chepman model.

(b) Explain the terms: Electrophonsis, electroosmosis, Dorn effect, zeta potential, straming potential. [12+3+3+3+2+2]=25

3.(a) Calculate the p^H of the following aqueons system

(i) 0.1 N Naett solution (ii) 0.1 N CH_3 COO H solution
(iii) 0.2 N solution of H COO Na (iv) 0.2 N solution of NH_4Cl
(v) 100 ml 0.1 N NaoH mixed with 110 ml of 0.2 N CH_3 CooH.
[dissociations constants of H CooH., CH_3 CooH and NH_3 are 1.774×10^{-4}, 1.752×10^{-5} and 1.74×10^{-5} respectively

(b) Explain the terms buffer capacity and indicator range. What indicator would you use to trace the end point in the titration of a nearly 0.1 N CH_3cooH solution by a 0.1 N NooH. solution and why ? [15+10]=25

p.t.o.

Group B

4.(a) What is meant by the equilibrium of a chemical reaction? Enumerate the factors that influence equilibrium. State and explain Le chatelier principle.

(b) Deduce the relationship between standard (Gibbs) free energy change and equilibrium constant.

(c) Derive on expression for the equilibrium constant (k_p) for the reaction: $N_2O_4 \quad 2NO_2$

(d) Write down vant' Hoff equation for the influence of temperature on equilibrium constant

(e) The value of k_p for the dissociation of iodine vapour to iodine atoms s.c $I_2 \quad 2I$ is 1.14×10^{-2} at $800^{\circ}C$ and 4.74×10^{-2} " at $900^{\circ}C$. Calculate the mean heat of dissociation in the temperature range mentioned. [7+6+4+3+5]=25

5.(a) State Raoultt's law and define osmosis. Derive a relationship between lowering of vapour pressure and osmotic pressure.

(b) Vapour pressure of a solution at 293 K is 0.98 times the vapour pressure.of the pure solvent at this temperature. What is the mole fraction of the solute in the solution ?

(c) When 4 g of a symthetic orgonic polymer is dissolved in an organic solvent the solution develops an osmotic pressure of 1.6×10^{-2} at mosphere at 300 K. What is the average molecular with of the polymer.

(d) What is meant by 'molal depression of freezing point'? Is it a property of the solvent, solute or solution?
An orgonic solvent freezer at $16.6^{\circ}C$ under standard atmoshperic pressure, its heat of fusion being 43.2 calonis perg. By how many degrees the freezing point will be lowered by dissolving 0.05 mole of a non-volatile, non-dissociating and non-interacting solute in 1000 g of the solvent ? [9+4+5+7]=25

6.(a) Explain the terms 'order of a reaction' and 'rate-constant'. What type of relation, if any, does exist between rate constants and equilibrium constant of a reversible reaction.

(b) Half-life of a first order reaction in 3600 seconds. Evaluate its rate constant.

(c) Thermal decomposition of HI to the elements follows a second order rate. In an experiment such a reaction is half-completed in 170 seconds. If the value of the rate constant is 7.5×10^{-3} litre $mole^{-1}$ see^{-1} calculate the initial concentration of HI.

contd. 3/-

(d) A first order reaction is apprcciably reversible and the reverse reaction is also of first order. Derive a relationship among the rate constant, time and reactional concentration taking cognisonce of the opposing reaction.

(e) Give a brief out-line of the collision the ory of reaction rate. Write down Avvhenius equation and explain the terms frequency factor and energy of activations. [5+3+5+5+7]=25.

INDIAN STATISTICAL INSTITUTE

PRACTICAL EXAMINATION

Date: 9.5.89 Maximum Marks:20 Time: $3\frac{1}{2}$ Hours

Note: Use separate answerscripts for Group A and Group B.

Group A

1. Determine the amount of glucose present in the given blood sample. [6]

2. Viva-voce [4]

PLEASE TURN OVER

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Date: 9.5.89

Group B

1. Mention the purpose of the set up experiment. Write the working principle of the experiment. (2+4)=[6]

2. Viva-voce [2]

3. Practical note book [2]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Examination
Elective-3: Biological Sciences

: 10.5.89 Maximum Marks: 80+20(Practical) Time: $3\frac{1}{2}$ hrs.+$3\frac{1}{2}$ (Prac.)

Note: Use Separate answerscript for Group A and Group B.

Group-A

... r any two of the following: (5x2)=[10]

... Gluconeogenesis is not a reversal of glycolysis. ________ Justify.

. All 20 amino acids are synthesized from citric acid cycle and other major metabolic intermediates. _____ Justify with simple diagram no need for chemical formula or enzymatic details).

Glutathione is an extremely important biological molecule. What is its chemical nomenclature? How is it synthesized and what is its major function?

... up the blanks (answer any five) (1x5)=[5]

________ is the major donor of methyl group.

"Active acetate" is ______________.

(iii) The complete oxidation of palmitic acid (16: 0) yields ____________________ ATP.

(iv) Cystein is a member of the ________________ family.

(v) Lysine is a member of both glutamate as well as ______________ family.

(vi) Canavanine is a ________________ amino acid.

(vii) Amino acid biosynthesis is regulated by ___________________.

... Answer any four from the following: ($2\frac{1}{2}$ x4)=[10]

) Diagrammatically show the concept of negative feedback.

) Draw an amplification cascade in the stimulation of glycogen (in glycogenolysis) by epinephrine in the liver cell to yield blood glucose.

What are the major functions of the hormones secreted by the medulla of adrenal gland and the islets of Langerhans ?

What are the hormones that regulates calcium metabolisen? What would happen due to the pathophysiologic state of these hormones?

What are the functions of testicular and ovarian hormones?

) What are the major physiologic functions of LH and FSH ?

... a detailed account of the Oxidative Phosphorylation process with all enzyme sequences and energy production.

Or

... a detailed account of the Oxidative and non Oxidative branches of ... Phosphate Pathway with the enzyme sequences. [15]

p.t.o..

GROUP-B

Answer question no.1 and any Two from the rest

1. Fill up the blanks (answer any Five): (2x5) =[10]
   (a) ______________ have the unique ability among recognized plant hormones to promote extensive growth of many intact plants
   (b) ______________ and ____________ appear to be the main hormones that control leaf abscission.
   (c) Movements of a plant organ in response to directional fluxes or gradients in environmental stimuli are called ______________.
   (d) The most popular and widely used germination promoter is ______________.
   (e) The most exciting discovery in the search for specific cell division-inducing compounds was that of ______________.
   (f) The process by which cells become specialized is called ______________.
   (g) In the ______________ phase of a S-shaped growth curve, the size increases exponentially with time.
2. Justify the following statements (any three): (5x3)=[15]
   (a) Acetyl C_0A is the " connecting link" between glycolysis and Krebs cycle.
   (b) Indole-acetic acid has some inhibitory effect in lateral bud production.
   (c) Specific moisture level is essential for seed germination.
   (d) Mineral salt absorption is a ATP dependent process.
   (e) Leaf primordia do not develop randomly around the circumference of the shoot apex.
3. What is dormancy? Describe five mechanisims of seed dormancy particularly as they involve seed germination. (3+12)=[15]
4. Explain the current theories that account for Phototropism and geotropism. $(7\frac{1}{2}+7\frac{1}{2})$=[15]
5. From where Gibberellin was first isolated? How Gibberellins stimulate the mobilisation of food and mineral elements in seed storage cells? Write briefly about agricultural application of Gibbrellins. (2+7+6)=[15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Examination
Elective-3: Economics

.: 10.5.89 Maximum Marks: 100 Time: 3 Hours

Note: Answer any four questions.
All questions carry equal marks.

: A rise in real investment must be accompained by a rise in equilibrium saving. However, the latter could be brought about either through a rise in real output or through a rise in price". Discuss the truth of the above statement.

. Construct a macroeconomic model which recognizes the simultaneous existence of excess capacity and scarcity in different sectors of the economy. Discuss in terms of such a model the likely impact on employment of changes in autonomous expenditure. In particular show how a rise in aggregate employment might be accompained by a fall in employment in certain specific pockets.

What is meant by an IS-LM equilibrium? Trace out the possible impact of a cut in the exogenously specified money wage rate on this equilibrium.

What, according to Keynes, are the different motives for holding money? In this context, discuss Keynes' derivation of the schedule of speculative demand for money. Further, indicate clearly the nature of the Keynesian liquidity trap.

a) Define M_1 as the sum of currency and demand deposits (DD) and M_2 as the some of M_1 and time deposits (TD). Assume a currency-demand deposit ratio cu and a desired ratio of demand to time deposits of the public, $d = DD/TD$. Assume, too, that banks have reserve preferences described by r_D and r_T with respect to demand and time deposits.

(i) Use the definition of M_1 and the ratios d, r_T and r_D to derive an expression for the equilibrium stock of M_1.

(ii) Use the definition of M_2 and the different ratios to derive an expression for the equilibrium stock of M_2.

(iii) What would be the effect of a rise in d on equilibrium M_1 and M_2? Argue out the intuition underlying your answer.

Suppose the Central bank decides to pay the banks interest on their reserve holdings. What would be the likely effect of this decision on money supply?

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Examination
Statistical Methods

Date: 8.5.89 | Maximum Marks: 100 | Time: $3\frac{1}{2}$ Hours

Note: From Group A answer any THREE questions.
From Group B answer all the questions.

Group-A

1. Suppose a random sample of size n has been drawn from a population having p.d.f.

$$f(x) = \frac{\alpha^{\nu}}{\Gamma\nu} e^{-\alpha x} x^{\nu-1}, \quad x > 0$$

where ν is a known constant.

(a) Show that the method of moments and the method of maximum likelihood give the same estimate of α.

(b) Show that this estimate is biased. Hence find an unbiased estimate of α.

(c) Assuming $n\nu > 2$, show that the variance of the unbiased estimate is $\frac{\alpha^2}{\nu n-2}$.

(d) Verify whether this variance is the same as that obtained from Rao-Cramer lower bound of the variance of an unbiased estimate.

[5+6+6+4=21]

2. (a) What do you mean by a statistical test? Define type I and type II error and explain their role in testing of hypothesis.

(b) Define most powerful (MP) and uniformly most powerful (UMP) tests. Given a random sample of n observations from the distribution having p.d.f.

$$f(x) = \frac{1}{\theta} x^{\frac{1-\theta}{\theta}}, \quad 0 \le x \le 1, \ \theta > 0,$$

Obtain a sufficient statistic for θ. Using Neyman-Pearson's theorem, obtain MP test for testing H_0 $[\theta = \theta_0]$ against H_1 $[\theta = \theta_1]$. Show that the test statistic so obtained is a function of the sufficient statistic obtained above.
Hence derive the distribution of the test statistic.
State giving reasons, whether the MP test obtained above is UMP against the alternative (i) $\theta > \theta_0$, (ii) $\theta \neq \theta_0$. [5+16=21]

3. Define a generalized likelihood ratio test (LRT). On the basis of a random sample from $BN(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$, derive the LRT for testing $H_0[\rho = 0]$ against $H_1[\rho \neq 0]$. Hence derive the distribution of the test statistic under H_0.
Also show that this LRT reduces to t-test where t follows student's t distribution with n-2 d.f. Discuss what will be the distribution of this t-statistic under alternative hypothesis H_1. [21]

p.t.o.

4. (a) What is contingency table ?
Show that under the hypothesis of independence of the classification in a contingency table, the joint probability of the cell frequencies for fixed marginals is independent of the parameters of the population.

(b) Derive the distribution of the sample median for a sample of size n=2r+1, where r is a positive integer, when sample is drawn from a population with pdf,

$$f(x) = \theta e^{-\theta x}, \ x > 0, \ \theta > 0.$$

(c) Develop a method for estimating the correlation coefficient of a bivariate normal population by a confidence interval. [7+7+7=21]

Group-B

5. An engineer wishes to estimate the mean setting time of a new gypsum cement mix used in a highway spot repairs. Prior to sampling, the engineer wishes to determine the sample size (i.e. the number of observations of the setting time with the new mix) required to attain a desired precision in estimating the mean. From experience with other cement mixes, the engineer expects the standard deviations of the measurements to be approximately 5. How many observations of the setting times with the new mix should the engineer collect to be 95% certain that the error of estimation of the true mean does not exceed one minute ? [6]

6. The following are the 12 determinations (in ^{0}C) of the melting point of a chemical compound made by an analyst, the true melting point being $165^{o}C$.
165.5, 169.7, 163.9, 162.1, 160.9, 160.8, 161.4, 162.2, 168.5, 163.4, 162.9, 167.7,
Would you conclude from these data that his determinations are free from bias ? Use $\alpha = .01$.
Obtain 99% confidence interval for the true melting point. [6]

7. The following correlation coefficients r_i were obtained from independent samples of sizes n_i from bivariate normal populations.

r_i :	.34	.40	.81	.72
n_i :	100	120	90	95

If the first two values are from a common population P_1 and the other two from a common population P_2, then test whether P_1 and P_2 have the same correlation coefficient. [1...]

8. Of 64 offsprings of a certain cross between guinea pigs, 34 were r... 10 were black, 20 were white. According to genetic model these numbers should be in the ratio 9:3:4.
Are the data consistent with the model at the .05 level ? [5]

9. Practical Note Book. [1...]

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INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Examination
Stochastic Processes

Date: 5.5.89 Maximum Marks: 100 Time: 3 Hours

1. Consider a Markov chain with 8 states 1,2,....8 and the following transition probability Matrix:

$$\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

(a) Describe all the classes of states and explain which are transient
(b) For each recurrent class, find the period and if the period is greater than 1, describe the cyclically moving subclasses.
(c) Find all stationary initial distributions of this chain.

[10+10+20]

2. Consider Chandrasekhar's model for the number of partioles in a given volume of space.
(a) What are the basic assumptions of the model? Deduce transition probabilities using the assumptions.
(b) Show that the chain is irreducible.
(c) For $j \geq 0$, $n \geq 1$ calculate explicitly $p_{0j}^{(n)}$ and show that

$\underset{n}{Lt}\ p_{0j}^{(n)}$ exists. Call it μ_j.

(d) Verify that $(\mu_j, j \geq 0)$ is a stationary initial distribution.

(e) Calculate $m_{5,5}$ (the mean recurrence time starting from 5).

[8+2+10+5+5]

Consider the following statements:
(A) j is a transient state.
(B) There is a state k such that $j \to k$ but $k \not\to j$.
(a) Show that B implies A.
(b) In a finite state chain, show that A implies B.
(c) Give an example to show that in general A does not imply B.

[5+5+5]

p.t.o.

4. Consider a recurrent chain P

(a) State the definitions of the taboo probabilities ${}_k p_{ij}^{(n)}$ and ${}_k f_{ij}^{(n)}$

(b) Show that $f_{ij}^{(n)} = \sum_{k=0}^{n} {}_j p_{ii}^{(k)} \cdot {}_i f_{ij}^{(n-k)}$

and $${}_i p_{ij}^{(n)} = \sum_{k=0}^{n} {}_i f_{ij}^{(k)} \cdot {}_i p_{jj}^{(n-k)}$$

(c) Fix a state i.

For any state $j \neq i$, let $v(j)$ be the expected number of visits j starting from i before hitting i again. Show that $v(j) < \infty$.

[5+10+10]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Examination
Linear Estimation and ANOVA

Date: 2.5.89 | Maximum Marks: 100 | Time: $3\frac{1}{2}$ Hours

Note: The paper carries 106 marks. Answer as many you can. Maximum one can score is 100.

1.(a) Clearly describe the S-method of multiple comparison in the General case, stating all the underlying assumptions.

(b) Consider the One-way layout

$$y_{ij} = \alpha_i + e_{ij} \quad (i=1,2,\ldots,m;\ j=1,2,\ldots,n_i)$$

$\{e_{ij}\}$ are independent $N(0,\sigma^2)$, $\sigma^2 > 0$ unknown.

Define a contrast among the parameters $\alpha_1,\alpha_2,\ldots,\alpha_m$. State the theorem on the S-method of simultaneous comparison of all contrasts. Derive the proof from (a). [8+2+8=18]

2. Consider the following form of the one-way layout:

$$\left.\begin{array}{l} y_{ij} = \mu + \alpha_i + e_{ij} \\ \{e_{ij}\} \text{ are independent } N(0,\sigma^2) \end{array}\right\} \begin{array}{l} i = 1,2,\ldots,m \\ j = 1,2,\ldots,n_i \end{array}$$

Let $\tilde{\mu} = \mu+\alpha$ and $\tilde{\alpha}_i = \alpha_i - \alpha$ where $\alpha = \frac{1}{m}\sum_{i=1}^{m}\alpha_i$, so that

$$\mu+\alpha_i = \tilde{\mu}+\tilde{\alpha}_i \text{ for all } i.$$

Show that only a particular type of linear parametric functions of μ and $\{\alpha_i\}$ is estimable while any linear parametric function of $\tilde{\mu}$ and $\{\tilde{\alpha}_i\}$ is estimable. [5+7=12]

3. Consider the observations $Y^T=(y_1,y_2,\ldots,y_n)$ such that

$$E(y_i) = \sum_{j=1}^{m} x_{ij}\beta_j,\quad V(y_i) = \sigma^2 \text{ and } \operatorname{cov}(y_i,y_k) = \rho\sigma^2 \text{ for } i \neq k$$

where $i = 1,2,\ldots,n$; $j = 1,2,\ldots,m$ and $\sigma^2 > 0$ is unknown.

Make an orthogonal transformation from Y to Z such that $z_1 = \frac{1}{\sqrt{n}}(y_1+y_2+\ldots+y_n)$ and $Z_2^T = (z_2,z_3,\ldots,z_n)$. Show that z_i's are uncorrelated and $E(Z_2) = U\beta$, where U depends on $X = ((x_{ij}))$ and the particular matrix of transformation. If ρ is unknown, show that $(Z_2, U\beta, \sigma^2(1-\rho)I)$ is a model under the Gauss-Markov set up. [15]

p.t.o.

4. The following data relate to head breadth of 142 skulls belonging to three series. Can the true mean head breadth be considered the same in the three series ?

Series	Sample Size	Head Breadth Total	Head Breadth Mean
1	83	11277	135.87
2	51	7049	138.22
3	8	1102	137.75
Total	142	19428	136.817

The total corrected sum of squares is found to be 4616.64. Set out the ANOVA Table and write down your conclusion. Also perform the S-method of inference on the contrasts $\alpha_2-\alpha_1$, $\alpha_3-\alpha_1$ and $\alpha_3-\alpha_2$ at 5 % level of significance where α_i stands for the true mean head breadth of the i-th series (i=1,2,3). [10+8=18]

5. Suppose $E(Y) = X\beta = \sum_{j=1}^{m} \beta_j X_j$ and $D(Y) = \sigma^2 I$. Let $X_1 = X_1^* + X_1^{\perp}$, where X_1^* is the projection of X_1 on the space spanned by $\{X_2, X_3, \ldots, X_m\}$. Show that, if β_1 is estimable, $X_1^{\perp} \neq 0$ and the variance of the least square estimate of β_1 is $\|X_1^{\perp}\|^{-2}\sigma^2$. [7]

6. An industrial engineer is conducting an expriment on eye focus time. He is interested in the effect of the distance of the object from the eye on the focus time. Four different distances are of interest. He has five subjects available for the experiment. Because there may be differences among individuals, he decides to conduct the experiment in a randomised block design. The data obtained follow. Carry out a Two-way Analysis of variance and draw appropriate conclusions.

Distance (ft)	Subject 1	2	3	4	5
4	10	6	6	6	6
6	7	6	6	1	6
8	5	3	3	2	5
10	6	4	4	2	3

[16]

7. Assignments and Practicals. [20]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Semestral-II Examination
Calculus - 4

Date: 28.4.89 | Maximum Marks: 100 | Time: 3 Hours

Note: This paper carries a total of 114 marks. Answer as many questions or parts thereof as you can. Your score is subject to a maximum of 100.

1. Let F be a real valued function on $\mathbb{R}$ such that F' exists and is continuous everywhere on $\mathbb{R}$. Given a fixed vector $\underline{a} \neq 0$ in $\mathbb{R}^3$, define

$$\underline{g}(\underline{x}) = F(\underline{x} \cdot \underline{a})\, \underline{f}(\underline{x}),$$

where $\underline{f}(\underline{x}) = (2x_1x_2x_3,\ x_1^2 x_3,\ x_1^2 x_2)$ if $\underline{x} \in \mathbb{R}^3$.
Show that curl $\underline{g}(x)$ is orthogonal to $\underline{a}$. [6]

2. Find the surface area of the paraboloid $z = x^2+y^2$ intercepted between the cylinders $x^2+y^2 = a$ and $x^2+Y^2 = b$ where $a = \frac{1}{4}(2m-1)^2-1$ and $b = \frac{1}{4}(2n-1)^2-1$, m and n being natural numbers with $n > m > 1$. [10]

3.(a) State the divergence theorem in $\mathbb{R}^3$.

(b) Let p denote the distance from the centre of the ellipsoid S :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

to the tangent plane at $P(x,y,z)$ and dS the element of surface area at this point. Prove

$$\int_S p\,dS = 4\pi abc$$

[4+10=14]

4.(a) Show that a plane curve $C(t) = (C_1(t), C_2(t))$, $t \in I$ is recifiable if and only if C_1 and C_2 are functions of bounded variation.

(b) Prove that if $(C_1(t), C_2(t))$ is continuously differentiable, then the arc-length of C is equal to

$$\int_a^b \sqrt{C_1'^2(t) + C_2'^2(t)}\,dt$$

when $I = [a,b]$. [6+10=16]

p.t.o.

5. Prove that if a continuously differentiable function $f : \Omega \to \mathbb{R}^n$, where Ω is an open subset of $\mathbb{R}^n$, is one-one on Ω and if f has a non-vanishing Jacobian at $\underline{x}_0 \in \mathbb{R}^n$, then $f(\Omega)$ contains a neighbourho of $f(x_0)$.

6. Find the taugent plane of the surface

$$x^3+2xy^2-7z^3+3y+1 = 0$$

at the point (1,1,1)

7.(a) Maximise the function $z = \cos \pi (x+y)$ subject to the condition $x^2+y^2=1$.

(b) Calculate the maximum value of the expression $\frac{x^4+2x^3y}{x^4+y^4}$. [8+8=

8.(a) Show that if $u_1, \ldots u_n, v_1, \ldots v_n$ are arbitrary non-negative numbers, then

$$\sum_{i=1}^{n} u_i v_i \leq \left(\sum_{i=1}^{n} u_i^{\alpha}\right)^{1/\alpha} \cdot \left(\sum_{i=1}^{n} v_i^{\beta}\right)^{1/\beta}$$

where $\alpha, \beta > 1$ and $\frac{1}{\alpha} + \frac{1}{\beta} = 1$.

(b) Show that if f and g are Riemann integrable functions on $[a,b]$, then

$$\left| \int_a^b f(x)g(x)\, dx \right| \leq \left(\int_a^b f(x)^{3/2} \right)^{2/3} \left(\int_a^b g(x)^3 \right)^{1/3}.$$

9.(a) Prove that

$$D_n(x) = \frac{1}{2} + \cos x + \ldots\ldots + \cos n x = \frac{\sin (n+\frac{1}{2})x}{2 \sin \frac{x}{2}}$$

(b) Show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

[4+6=10

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Periodical Examination
Elective-3: Biological Sciences

Date: 1.3.89 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Use separate answerscript for Group A and Group B.

GROUP-A

Answer question no.1 and any Two from the rest.

1. Fill up the blanks (answer any Five): (2x5)=[10]

(a) Photosynthesis is the reduction of __________ and the oxidation of __________ resulting in the accumulation of energy in organic molecule (sugar).

(b) The end products formed in Photosynthesis are __________, __________ and __________.

(c) A part of the light energy absorbed by green cells is transformed into the chemical energy known as __________.

(d) The source of oxygen evolved during Photosynthesis is __________.

(e) The intermediate steps which are the same in both aerobic and anaerobic respiration are called __________.

(f) Chlorophyel molecule contains a metal atom of __________.

(g) Maximum rate of Photosynthesis occurs in __________ and __________ lights.

(h) Ethylene evolution is highest when a fruit is __________.

2. Discuss the Krab's cycle and its significance. (10+5)=[15]
3. Describe the various factors affecting Photosynthesis. [15]
4. Discuss the biosynthetic pathway of auxins. Mention the physiological activity of auxins in plant. (5+10)=[15]
5. Write short notes on (any THREE): (5x3)=[15]

(a) Glycolysis.

(b) Respiratory quotient (RQ).

(c) Calvin cycle.

(d) Fermentation.

(e) Apical dominance.

(f) β-Oxidation path way

p.t.o.

GROUP-B

Attempt all questions.

1. You have isolated a octapeptide from a rare fungus that prevents baldness. Analysis gives the composition [Lys_2/Asp_1/Tyr_1/Phe_1/Gly_1/Ser_1/Ala_1]. Reaction of the intact peptide with FDNB followed by acid hydrolysis liberates FDNB-Ala. Cleavage with trypsin gives peptides with compositions [Lys_1/Ala_1/Ser_1] and [Gly_1/Phe_1/Lys_1], plus a dipeptide. Reaction with chymotrypsin liberates free aspartic acid, a tetrapeptide with composition [Lys_1/Ser_1/Phe_1/Ala_1] and a tripeptide which liberates FDNB - Gly on reaction with FDNB followed by acid hydrolysis.

What is the sequence of the Octapeptide ?

Or,

You are given the following Peptide

```
Ala - Tyr - Cys - Phe - Glu - Asn - Arg - Gly - Ala - Lys
             |
             S
             |
             S                 S ———————— S
             |                 |          |
Gly - Ser - Cys - Trp - Met - Val - Cys - Ala - Cys - Leu
```

(a) If this were subjected to performic acid treatment, how many peptides would be formed ?

(b) If it were subjected to Edman's degradation, which amino acid would be present as the thiohydantoin derivative in the first cycle ?

(c) What are the peptides/amino acids that you would expect to see after treatment with the following :

(i) CNB_r ; (ii) Trypsin ; (iii) Papain ; (iv) Carboxypeptidase

(15x1)=[

2. Determine whether the following statements are True or Flase. If false explain why. (3x5)=[15]

(Answer any five)

(a) All 20 standard amino acids found in proteins have at lease One asymmetric carbon atom.

(b) An equimolecular mixture of D- and L- alanine does not rotate the plane of polarized light.

(c) Under physiologic conditions, the polypeptide chain of a protein has only one (or a few) conformations. This is known as the NATIVE STRUCTURE.

contd. 3/-

2.(d) The α-helix and β-structure are held together by interchain and intrachain H-bonding, respectively.

(e) The α-helical conformation and β-structure of polypeptides are a direct result of their amino acid composition and sequence.

(f) Secondary structure of a protein results from the interaction between distantly separated amino acid residues in the sequence, whereas tertiary structure results from the interaction of closely spaced amino acid residues in the sequence.

(g) Enzymes affect the equilibria of the reactions they catalyze.

(h) The initial rate of an enzyme - catalyzed reaction is increased if the amount of ES complex is increased.

(i) The Km of an enzyme - catalyzed reaction is independent of the substrate used.

(j) The Km value for a substrate is altered by a non-competitive inhibitor.

3. Distinguish between the members of each pair. (5x4)=[20]
(Answer any four)

(a) Sub-unit/Domain
(b) Secondary structure/Tertiary structure of a Protein
(c) Enzyme/Coenzyme
(d) Cofactor/Prosthetic Group
(e) Competitive/Non-competitive Inhibitor
(f) Oxido reductase/Isomerase
(g) Transferase/Lyases
(h) Hydrolases/Ligases

4. (Answer any two) (5x2)=[10]

(a) Name three extensive and three intensive properties which relate to thermodynamic quantities. What is the basic difference between the two types of properties?

(b) Amino acids are frequently separated by either paper electrophoresis or ion-exchange chromatography. What is the basis for these separation techbiques? Can these be used to separate peptides? Explain.

(c) Fill up the blanks:

(i) The pH at which a protein does not migrate in an electrical filled is called ____________.

(ii) A protein can be cleaved by treatment with the enzyme ____________(specific for Lys and Arg)Or ____________ (specific for Phe, Trp, Tyr).

(d) The Michaelis-Menten Equation is written as follows:

$$V_0 = \frac{V_{max}\,[S]}{K_m + [S]}$$

. Define each term of this equation.

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Periodical Examination
Elective-3: Economics

Date: 1.3.89 Maximum Marks: 50 Time: 2 hours

Note: Answer any 3 questions, all of which carry equal marks. Answers should be brief and analytical. All algebraic manipulations must be supplemented by a clarification of the underlying economic intuition.

1. Discuss the meaning of the term effective demand. What are its determinants? What is the relationship between effective demand and Keynes' conception of macroeconomic equilibrium?

2. Discuss, in terms of a simple two sector model, how a change in the parameters of income distribution might affect the equilibrium of a macro system. Clarify your answer with reference to the Keynesian multiplier.

3. Illustrate by means of a simple model the concept of voluntary unemployment. What sort of measures, in your opinion, should constitute ways out of such forms of unemployment?

4. What is meant by the balanced budget multiplier? Discuss the nature of the balanced budget multiplier for the cases where income taxes take the (a) lump sum form and (b) proportional form.

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Periodical Examination
Elective-3: Physical and Earth Sciences

Date: 1.3.89 Maximum Marks: 100 Time: 3 hours

Answer the following questions

1.(a) Derive the rate equation of a second order reaction involving two reactants with unequal initial concentrations. [10]

(b) In a first order reaction 75% of the reactant is converted to the product in 120 minutes. Evaluate the rate constant of the reaction in appropriate unit? What is the half-life of the reaction?

(c) Derive an expression for the rate constant of a reversible reaction of which both the forward and the reverse reactions are of first order.

(d) 18.23 centi mole of a reactant X undergoes reversible isomeric transformation to the product Y at 160°C. The reverse reaction is also of first order. Amounts of the product formed after 6000 seconds and at equilibrium are 7.29 and 13.29 centimoles respectively. Evaluate the rate constants for both the forward and the backward reaction. What is the value of the equilibrium constant for the reaction.

(e) Write down Arrchinius equation on the reaction rate. How it is developed from collision theory of reaction. Explain the terms: Activation energy and frequency factor.

(10+10+10+10)=[40]

2.(a) Develop the expression of the equilibrium constant of homogeneous chemical reaction from free energy consideration.

(b) Write down Van't Hoff equation correlationg equilibrium constant and temperature. Ratio of the equilibrium constants of a homogeneous reaction at temperature 17°C and 27°C is 12.5. What is the heat of reaction? Is the reaction exothermic or endothermic?

(c) Ratio of K_p to K_c for a chemical reaction at 17°C is 620. Utise the information to show whether the reaction is favourably influenced by increase of pressure.

(12+10+8)=[30]

p.t.o.

3.(a) State Raoult's law.

0.4 mole of a non-volatile solute is dissolved in 75 gm of a solvent of molecular weight 58 at 25°C. If the vapour pressure of the pure solvent at this temperature is 229 mm what is the vapour pressure of the solution.

(b) Deduce an expression connecting lowering of freezing point and molal concentration of a non-volatile solute in solution.

(c) 1.75 gm. of a non-volatile substance of unknown molecular weight is dissolved in 50 gm of carbon tetrachloride (CCl_4). Boiling point of this solution is found to be 77.105°C. What is the molecular weight of the substance? The boiling point of the pure solvent is 76.75°C and its heat of vaporisation at this temperature is 46.4 calories per gram.

(8+12+10)=[30]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year 1988-89
Periodical Examination
Stochastic Processes

Date: 24.2.89 Maximum Marks: 100 Time: 3 hours

1.(a) Let $(X_n\ n \geq 0)$ be a sequence of random variables taking values in a countable set S. Complete the following sentence: The sequence is said to be a Markov chain if........ . Define the one step transition matrix of the Markov chain.

(b) Let $(X_n\ n \geq 0)$ be a Markov chain with state space S. Let $A \subset S$. Show that $P(X_n \in A$ for infinitely many values of

$n \mid X_0=i_0,\ X_1=i_1, \ldots,\ X_6=i_6,\ X_7=j)$

$= P(X_n \in A$ for infinitely many values of $n \mid X_0=j)$, (5+5+10)=[20]

2. Let $Z_1, Z_2, \ldots$ be i.i.d r.v $Z_1 = +1$ with probability p

$= -1$ with probability q.

Define $X_0=1$ and for $n \geq 1$, $X_n = X_{n-1} \cdot Z_n$. Show that (X_n) is a Markov chain. What is the state space. What is the [illegible] Evaluate the π matrix. (10+1+3+6)=[20]

3. Consider a Markov chain with S = {0,1,2,3,4,5,6} and transition matrix P as given below: (a) Explain which states are essential and which are inessential. (b) Describe all the classes.

(c) Calculate $p^{(89)}_{4,4}$, $p^{(90)}_{4,4}$.

	0	1	2	3	4	5	6
0	1/7	1/7	1/7	1/7	1/7	1/7	1/7
1	1/2	0	0	0	1/4	1/4	0
2	0	0	1	0	0	0	0
3	0	0	0	1/3	0	0	2/3
4	0	0	0	0	0	1	0
5	0	0	0	0	1	0	0
6	0	0	0	2/3	0	0	1/3

(8+8+4)=[20]

p.t.o.

4. Let $S = \{0, 1, 2, \ldots\}$. Fix numbers a_i $i \geq 1$ strictly positive and $\sum_1^\infty a_i = 1$. Here is the transition matrix of a markov chain with state space S

$$P_{0j} = a_j \quad j = 1, 2, \ldots\ldots$$

$$P_{i, i-1} = 1 \quad i = 1, 2, \ldots\ldots$$

(a) Show that the chain is irreducible.

(b) Calculate $f_{00}^{(n)}$ $n = 1, 2, 3, \ldots\ldots$

(c) Show that the chain is recurrent.

(d) Show that the chain is positive recurrent

iff $\sum_{n=1}^{\infty} n a_n < \infty$. (5+10+5+5)=[25]

5.(a) State the mean ergodic theorem.

(b) When do you say that a state i is positive recurrent.

(c) Assume that recurrence is a class property. Show that positive recurrence is also a class property. (3+2+10)=[15]

6. Consider an irreducible chain with one step transition matrix P. Suppose $\underset{\sim}{u}$ is a solution of $\underset{\sim}{u} P = \underset{\sim}{u}$ with $\sum_{i \in S} |\underset{\sim}{u}(i)| < \infty$.

Show that for some number c, $\underset{\sim}{u} = c \underset{\sim}{\mu}$ where $\underset{\sim}{\mu}(i) = \pi_{ii}$ with usual notation. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year: 1988-89
Periodical Examination
Calculus-4

Date: 20.2.89 Maximum Marks: 100 Time: 3 Hours

Note: Answer all questions.

1.(a) Give an example of functions P and Q continuously differentiable on an open subset $\Omega \subseteq \mathbb{R}^2$ such that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ on Ω but there is no function φ on Ω with $P = \frac{\partial \varphi}{\partial x}$ and $Q = \frac{\partial \varphi}{\partial y}$. [10]

(b) Let P and Q be continuously differentiable functions on Ω such that

$$\int_{\gamma} P dx + Q dy = \int_{\gamma'} P dx + Q dy$$

whenever γ, γ' are C' curves : $[0,1] \longrightarrow \Omega$ with $\gamma(0) = \gamma'(0)$ and $\gamma(1) = \gamma'(1)$. Show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ on Ω. [8]

(c) Find a function φ on $\mathbb{R}^3$ (explicitly) whose partial derivatives are

$y \cos(xy)$, $x \cos(xy) + 2yz^3$, $3y^2z^2$ [8]

2.(a) State Green's theorem for a planar region.

(b) Calculate the integral

$$\int_C y^n dx + x^n dy$$

where C is the positively oriented unit circle.

(c) Prove the following form of Green's theorem in polar co-ordinates

$$\int_{\partial R} f(r,\theta)\, dr + g(r,\theta)\, d\theta = \iint_R \left(\frac{\partial g}{\partial r} - \frac{\partial f}{\partial \theta}\right) dr\, d\theta,$$

stating the assumptions on f, g, R. (4+10+8)=[22]

3.(a) Show that a function f of bounded variation on a closed bounded interval [a,b] is the sum of two monotone functions φ_1 and φ_2.

(b) If the function f in a) is continuous, can we choose φ_1 and φ_2 to be continuous ? Give reasons for your answer.

(c) Give an example of a function of bounded variation which is not absolutely continuous. (10+10+15)=[35]

p.t.o.

4.(a) Find the surface area of the parametric surface

$$x = (a+b\cos\rho)\cos\theta$$

$$y = (a+b\cos\rho)\sin\theta$$

$$z = b\sin\rho$$

$$0 \le \theta,\ 0 \le 2\pi.$$

(b) Let $R \xrightarrow{x} S \subseteq \mathbb{R}^3$ be the parametrisation of a surface. If $\alpha : [0,1] \longrightarrow R$ is a C^1-curve and $\gamma = x(\alpha)$, show that

$$\ell(\gamma) = \int_0^1 \left(\dot\alpha \begin{pmatrix} E & F \\ F & G \end{pmatrix} \dot\alpha'\right)^{1/2} dt$$

when $\ell(\gamma)$ is the length of γ and E, F, G have the usual meaning. (8+9)=[17].

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Periodical Examination
Linear Estimation and ANOVA

Date: 22.2.89 Maximum Marks: 50 Time: $2\frac{1}{2}$ hours

In the sequel we assume the Model $(Y, X\beta, \sigma^2 I)$ where Y_{nx1}, X_{nxm}, β_{mx1}, I_{nxn} carry the usual connotations.

1. Show that under this model, every estimable function $\psi = c^T\beta$ has a unique unbiased linear estimate $\hat{\psi}$ which has minimum variance in the class of all unbiased linear estimates. Also deduce that the estimate $\hat{\psi}$ may be obtained from $\psi = c^T\beta$ by replacing β by any solution $\hat{\beta}$ of the normal equations. Also deduce that $E(X\hat{\beta}) = X\beta$ where $\hat{\beta}$ is a solution of the normal equations. (9+6+3)=[18]

2. Denoting the columns of X by $X_1, X_2, \ldots, X_m$ let us write

   $X_1 = X_1^* + X_1^{\perp}$, where X_1^* is the projection of X_1 on the space spanned by $X_2, X_3, \ldots, X_m$. Show that β_1 is estimable if and only if $X_1^{\perp} \neq 0$. [10]

3. Show that a linear function a^TY has minimum variance as an unbiased estimate of $E(a^TY)$ if and only if $cov(a^TY, d^TY) = 0$ for all d such that $E(d^TY) = 0$ i.e., $d^TX = 0$. [12]

4. Estimate the weights of 4 objects from the following observational equations (ignoring bias).

β_1	β_2	β_3	β_4	Weight
1	1	1	1	20.2
1	-1	1	-1	8.1
1	1	-1	-1	9.7
1	-1	-1	1	1.9
1	1	1	1	19.9
1	-1	1	-1	8.3
1	1	-1	-1	10.2
1	-1	-1	1	1.8

Find the dispersion matrix of the estimates and also the least square estimate of σ^2 (σ^2 bing the unknown variance of each measurement). [15]

ISS:

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year: 1988-89
Periodical Examination
Statistical Methods

Date: 27.2.89 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can.
Total marks in the margin is 108 but
maximum that you can score is 100.

1.(a) Define the standard error (S.E.) of a statistic. Find the S.E. of the sample mean when sample is drawn without replacement from a finite population with mean μ and variance σ^2. Interprete the resulted S.E. when the population size is large.

(b) Let Y be distributed as $F(\nu_1, \nu_2)$. Show that under certain condition, $\sqrt{Y}$ follows t-distribution. (10+6)=[16]

2. What do you mean by an unbiased and consistent estimator? Let $X_1, \ldots X_n$ be a random sample from a $N(\mu, \sigma^2)$, where n is an even number. Determine the constant 'C' such that

$$C \sum_{i=1}^{n/2} (X_{2i} - X_{2i-1})^2$$

is an unbiased estimator of σ^2. Hence verify whether this is a consistent estimator of σ^2. [16]

3.(a) Define a sufficient estimator. State and prove Rao-Cramer inequality for the variance of an unbiased estimator of a parameter. State under what condition the equality holds. Show that this condition leads to the conclusion that the estimator whose variance attains the lower bound must be a sufficient estimator.

(b) Let $X_1, \ldots X_n$ be a random sample of size $n > 2$ from a distribution having p.d.f.

$$f(x,\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \ \theta > 0.$$
$$= 0 \quad , \text{elsewhere.}$$

Show that $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, where $Y_i = -\log_e X_i$, is a sufficient estimator of θ. Is it an unbiased estimator of θ? If not, obtain an unbiased estimator and compute its variance.

Further compute the Rao-Cramer lower bound of the variance of an unbiased estimator of θ and hence compute the efficiency of the unbiased estimator of θ obtained above.

(15+25)=[40]

p.t.o.

4.(a) What do you mean by maximum likelihood estimate of a parameter?
Obtain the m l e s of the parameters of a lognormal distribution.

(b) Let $X_1, \ldots X_{n_1}$ and $Y_1, \ldots Y_{n_2}$ be two samples drawn from two independent normal populations, $N(\mu, \sigma^2)$ and $N(\theta\mu, \sigma^2)$ respectively. Obtain $100(1-\alpha)\%$ confidence limits for the parameter θ. (8+8)=[16]

5.(a) Assume that the amount of rainfall recorded at a certain station on a given date is uniformly distributed on the interval (0,b). If a sample of 10 years' records show that the following amounts were recorded on that date :

0, 0, .7, 1, .1, 0, .2, .5, 0, .6

(measurements are recorded in inches).
Compute the estimate of b by (i) using method of moments and (ii) using maximum likelihood method.

(b) Assume that the top speed attainable by a certain design racing car is a normal random variable X with mean μ and standard deviation σ.
A random sample of 10 cars, built according to this design, was selected and each car was tested. The sum and the sum of squares of the top speeds attained (in miles per hour) were

$$\sum_{i=1}^{10} x_i = 1652 \quad , \quad \sum_{i=1}^{n} x_i^2 = 273,765$$

Compute the observed 95% confidence limits for μ when (i) σ = 10 miles per hour and (ii) σ is unknown.

Hence compute the length of the confidence interval in case (i) and (ii) and give your comments. (10+10)=[20]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year . 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Elementary Algebraic Structures

Date: 2.1.1989 . Maximum Marks: 100 Time: 3 hours

Note: Answer as many as you can.

1. State Sylow's Theorem.

   Show that if p is a prime number then

   $$(p-1)! \equiv -1 \pmod p.$$ [10]

2. What is a Euclidean ring ? Give an example.

   Show that a Euclidean ring has an identity. In a Euclidean ring R show that the ideal $(a) = \{ra : r \in R\}$ is maximal iff $\underline{a}$ is a prime element. [15]

3. Show that the characteristic of a field of finite characteristic is a prime p and hence that the field contains Z_p as a subfield. [10]

4. Show that if p(x) is a polynomial irreducible over a field F, then there exists a field $K \cong F[x]/(p(x))$ which is a simple algebraic extension of F generated by a root u of p(x).

   Prove that $x^3 + x - 1$ is irreducible over the field Z_5. If a root of this polynomial is adjoined to Z_5, how many elements has the resulting field ? (10+8) = [18]

5. State Eisenstein's Criterion for irreducibility. Show that for any prime p, $x^n - p$ is irreducible over the rationals. [7]

6. Prove that the set of all algebraic numbers is a field which is algebraically complete. [10]

p.t.o.

7. Define a root field of a polynomial over a field F.

Show that any finite field F of characteristic p is the root field of $x^q - x$ over Z_p, where $|F| = q$.

Hence conclude that any two finite fields with the same number of elements are isomorphic.

Find the root field of $x^3 - 5$ over $\mathbb{Q}$. [20]

8. Define the Galois group of a polynomial. Find the Galois group of the polynomial $x^4 - 3$ over $\mathbb{Q}$, the field of rationals.

[15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year: 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Statistical Models

Date: 29.12.1988 Maximum Marks: 100 Time: 4 hrs.

Note: Answer as many questions as you can. Marks in the margin total 110 but the maximum mark that you can score is 100.

1.(a) Show that under certain conditions, a binomial distribution can be approximated by a normal distribution.

(b) Define a compound distribution. Hence derive Neyman's type A distribution and find its mean and variance.

Show that, under certain assumptions, this distribution can be approximated by a modified Poisson distribution.

(7+10) = [17]

2.(a) Show that the differential equation

$$\frac{df(x)}{dx} = \frac{(x-a)\, f(x)}{b_0 + b_1 x + b_2 x^2}$$

for a family of probability distributions can be obtained as the limiting case of a hypergeometric distribution with parameters (N, n, p). From this equation obtain Pearson's type III curve and prove that for this curve $2\beta_2 - 3\beta_1 - 6 = 0$.

(b) Derive Gram-Charlier type A series to represent the p.d.f. of a random variable.

[(8+10) + 8] = [26]

3.(a) Let X be distributed as $N(\theta, \sigma^2)$. Work out the compound distribution when θ is a random variable distributed as $N(\mu, \sigma_1^2)$.

(b) Let X_1, X_2, X_3 be three independent lognormally distributed random variables. Find the distribution of $X_1^{a_1} X_2^{a_2} X_3^{a_3}$, where a_1, a_2, a_3 are some positive constants.

(7+7) = [14]

p.t.o.

4.(a) Define multinomial probability distribution. For this distribution obtain the multiple correlation coefficient of one variable on the rest of the variables.

(b) Let X and Y jointly follow a bivariate normal distribution with parameters $(0,0,1,1,\rho)$.

Derive the moment generating function $M(t_1, t_2)$ of this distribution.

Show that

$$\frac{\partial^2 M}{\partial t_1 \partial t_2} = \rho\left[M + t_1 \frac{\partial M}{\partial t_1} + t_2 \frac{\partial M}{\partial t_2}\right] + (1-\rho^2) t_1 t_2 M.$$

Hence obtain the following recurrence relation among bivariate central moments,

$$\mu_{r+1,s+1} = (r+s+1)\rho\mu_{r,s} + rs(1-\rho^2)\mu_{r-1,s-1}.$$

(8+10) = [18]

5.(a) Let X be the number of successes through n independent repitations of a random experiment having the probability of success $p = \frac{1}{4}$.

Determine the smallest value of n so that $Prob[X \geq 1] \geq 0.70$.

(b) The following statistics were obtained from a bivariate frequency distribution on yield of plants (X) and that of offsprings (Y) in suitable units:

	mean	variance	correlation
X	55	15	
Y	42	12	0.75

If from the parent plants only those with the top 10% of yield are selected and allowed to propagate, what will be the expected yield in their offsprings ?

(Assume that the joint distribution is bivariate normal).

(c) Following data relate to the number of outbreaks of War for the year between 1500 and 1931 :

Contd..... Q.No.5.(c)

No. of outbreaks of War	Frequency
0	223
1	142
2	48
3	15
4	4
5	0
	432

Fit a Poisson distribution to the data and test for the goodness of fit.

(7+8+10) = [25]

6. Practical Note Book. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89
BACKPAPER (SEMESTRAL-I) EXAMINATION
Calculus - 3

Date: 26.12.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL the questions.

1.(a) Let $S \subseteq \mathbb{R}^2$ and let (x,y) be a boundary point of S that is not in S. Show that there exists a sequence $(x_n, y_n) \in S$ such that $\lim_{n \to \infty} (x_n, y_n) = (x,y)$.

(b) Prove that $S \cup \partial S$ is a closed set.

(c) State and prove the Bolzano-Weierstrass theorem in $\mathbb{R}^n$.

(5+5+10) = [20]

2.(a) Let f be the function on $\mathbb{R}^2$

$$f(x,y) = e^{-\frac{1}{x^2+y^2}} \quad \text{if } x^2 + y^2 > 0$$
$$= 0 \quad \text{otherwise}$$

Show that f is continuous at $(0,0)$.

(b) Decide if the function f in (a) is differentiable at $(0,0)$.

(c) Let $f : \Omega \longrightarrow \mathbb{R}$ be a function on an open subset $\Omega \subseteq \mathbb{R}^2$ s.t. at all $(x,y) \in \Omega$

$$| f_x(x,y) | < K$$
$$| f_y(x,y) | < K$$

for some $K > 0$. Show that f is continuous on Ω.

(4+8+8) = [20]

3.(a) Let f be a function on $\mathbb{R}^3 - \{(0,0,0)\}$ such that

$$f(x,y,z) = g(\sqrt{x^2 + y^2 + z^2})$$

for some function g. Prove that if $f_{xx} + f_{yy} + f_{zz} = 0$ at all (x,y,z), then $f(x,y,z) = \frac{a}{\sqrt{x^2+y^2+z^2}} + b$ where a and f are constants.

Contd..... 2/-

Contd..... Q.No.3

(b) Find the direction u, $|u| = 1$ in which the function

$$f(x,y) = x^2 + y^2 + 3xy^3 - 2y^4$$

has the maximum directional derivative at $(1,1)$.

(12+8) = [20]

4.(a) Let f be a nonnegative bounded function on the interval $[0,1]$ and let

$$S = \{(x,y) . \ x < [0,1] \text{ and } 0 \leq y \leq f(x)\}.$$

Prove that S has Jordan content if and only if f is Riemann-integrable on $[0,1]$.

(b) Show that a continuous function f on the bounded closed rectangle $[0,1] \times [0,1]$ is integrable.

(10+8) = [18]

5.(a) Find the area in the positive quadrant enclosed between the curves $a^2y = x^3$, $b^2y = x^3$, $p^2x = y^3$ and $q^2x = y^3$ where $0 < b < a$ and $0 < q < p$.

(b) Find the position of the centroid of the homogeneous mass distribution with mass density 1 over the region

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \ ; \quad z \geq 0.$$

(c) Decide for which values of α, the improper integral

$$\int_D \frac{1}{\sqrt{(x^2 + y^2)^{\alpha}}}$$

exists, where $D = \{(x,y) : x^2 + y^2 < 1\}$.

(7+7+8) = [22]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89
BACKPAPER (SEMESTRAL-I) EXAMINATION
Probability - 3

Date: 28.12.1988 Maximum Marks: 100 Time: 3 hrs.

Note: Answer ALL questions

1. Let $(X_1, \ldots, X_n)$ be multivariate normal with mean vector $\underset{\sim}{\mu}$ and covariance matrix Σ. Fix $1 < k < n$. Evaluate the conditional distribution of $(X_1, \ldots X_k)$ given $X_{k+1}, \ldots, X_n$. [20]

2. Let $1 \leq k \leq n$. Let U_k be the kth orderstatistic of a sample of size n from uniform (0,1). Evaluate the distribution of U_k. Calculate its mean and variance. [20]

3. Let U_1, U_2 be orderstatisti of size 2 from Exp(λ). Show that U_1 and $U_2 - U_1$ are independent and have exponential distributions. Evaluate their parameters. [20]

4.(a) Let (X_n) be a sequence of i.i.d. r v s with mean μ and finite variance. Let f be a bounded continuous function on the real line. Stating precisely the theorems you need, prove that

$$\underset{n}{\text{Lt}}\ E[f(\frac{X_1 + \ldots + X_n}{n})] = f(\mu).$$

(b) Show $$\underset{n}{\text{Lt}} \underbrace{\int_0^2 \ldots \int_0^2}_{(n \text{ times})} (\frac{x_1 + \ldots + x_n}{n})^{99} dx_1 \ldots dx_n = 1.$$

[20]

5.(a) Define the following:

(i) X_n converges to X in probability.

(ii) X_n converges to X in distribution.

(b) Let X_n be a sequence of i.i.d. N(0,1) random variables. Show that X_n converges in distribution but does not converge in probability. [10]

p.t.o.

. Let $\varphi = u + iv$ be a characteristic function,

(a) Using

$$1 - \cos 2\theta \leq 2(1 - \cos^2 \theta) \leq 4(1 - \cos \theta)$$

show $0 \leq 1 - u(2t) \leq 4(1 - u(t))$.

(b) Show that for any characteristic function $\varphi(t)$

$$|\varphi(t)|^2 \leq 1 - \frac{1 - |\varphi(2t)|}{4} \leq e^{-\frac{1}{4}(1 - |\varphi(2t)|)}.$$

[10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year: 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Elective-2: Physical and Earth Sciences

Date: 30.12.1988 Maximum Marks: 100 Time: 3 hrs.

Note: Answer Q.No.10 and any FIVE of the rest.

1.(a) Derive Maxwell's thermodynamic relations and show that for a Van der waal gas

$$C_P - C_V = \frac{R(p + \frac{a}{V^2})}{P - \frac{a}{V^2} + \frac{2ab}{V^2}}.$$

(b) Deduce, using Maxwell's relation, the clausius-clapeyron equation

$$\frac{dp}{dT} = \frac{L}{T(V_2 - V_1)}$$

symbols having usual significance.

(9+5+4) = [18]

2.(a) State the essential difference between the First and the Second law of thermodynamics.

(b) Define entropy and state briefly its significance. Show that entropy increases in natural processes.

(6+6+6) = [18]

3.(a) Assuming that the critical state corresponds to a point of inflexion of an isothermal, find the critical constants for a gas obeying the equation of state

$$p = \frac{RT}{V-b} e^{-a/RTV}$$

where e is the exponential function and a,b,R are constants.

(b) Calculate the Boyle temperature for such a gas.

(12+6) = [18]

p.t.o.

4.(a) What is photo-electric effect ? Give Einstein's theory of this effect.

(b) It is found that the maximum wavelength of radiation that can cause the emission of photoelectrons from a metallic plate is 3500 A. Calculate

(i) the work function of metal in eV.

(ii) the velocity of photoelectrons when the same plate is irradiated by radiation of wavelength 2500 A.

Given $h = 6.62 \times 10^{-34}$ J.s, $c = 3 \times 10^{8}$ m/s.

(6+6+6) = [18]

5.(a) X-rays of wavelength 0.3A undergo Compton scattering by electrons. Find the wavelength of the scattered radiation observed at a scattering angle of 45°.

(b) Deduce the necessary formula.

(6+12) = [18]

6.(a) Define Q-value of a nuclear reaction. How do you express Q-value in terms of masses of the reacting nuclei ? What is the threshold energy for a reaction ?

(b) State de Broglie's expression for the wavelength of matter waves. Show that in a Bohr atom if the electron is considered as a wave travelling along the circular path, the nth orbit will contain n complete de Broglie wavelength.

(5+3+3+7) = [18]

7.(a) State and explain the postulates of Special Relativity.

(b) What do you understand by relativistic 'time dilatation' and 'length contraction' ?

(c) Establish from relativistic considerations the equivalence of mass and energy.

(6+3+3+6) = [18]

8.(a) Explain the terms: phase space, micro-state, macro-state and thermodynamic probability.

(b) Assuming that the atmosphere consists of monatomic ideal gas, deduce the law of atmosphere (pressure - height relation) by the application of Maxwell-Boltzmann Statistics.

(8+10) = [18]

p.t.o.

9.(a) What are the characteristics of Fermi-Dirac Statistics ? Explain Fermi energy ?

(b) Apply Fermi-Dirac Statistics to obtain the Richardson-Dushman equation.

(4+4+10) = [18]

10. Pick out the correct answer from those given in each of the following questions:

(a) C^{14} is radioactive. The activity and the daughter product are

(i) β - active, N^{14} (ii) β - active, B^{14}

(iii) γ - active, C^{14} (iv) α - active, Be^{10}.

(b) For pair production the γ - ray must have a minimum frequency of

(i) 2.5×10^{20} Hz (ii) 3×10^{10} Hz

(iii) 2.5×10^{21} Hz (iv) 6×10^{20} Hz

(c) An e^- of mass M_e and a proton of mass M_p are accelerated through the same potential difference. The ratio of the wavelength associated with an e^- to the associated with a proton is

(i) 1 (ii) $\sqrt{M_e/M_p}$ (iii) $\sqrt{M_p/M_e}$

(ii) M_p/M_e (v) none of these.

(d) An e^- moves around a proton in a circle of radius r. Assuming that the uncertainty in the momentum of the electron is of the same order as the momentum itself, the momentum itself, the momentum of e^- is of the order of

(i) $h/2\pi r$ (ii) $h/2\pi$ (iii) $2\pi r/h$

(iv) 2π (v) none of these.

(e) In nuclear fission 0.1% mass is converted into energy. The energy released by fission of 1 kg mass is

(i) 2.5×10^{5} KWH (ii) 2.5×10^{7} KWH

(iii) 2.5×10^{9} KWH (iv) 2.5×10^{-7} KWH.

(5 x 2) = [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Elective 2 : Biological Sciences

Practical

Date: 25.11.1988 Maximum Marks: 20 Time: 3 hours

Note: Use separate answerscript for Group A and Group B.

GROUP - A

1. Cut transverse section of specimen A, mount and observe; draw labelled sketches of the section. Identify its anatomical structures with reasons. (1+2+3) = [6]

2. Viva-Voce ... ... [2]

3. Practical note books ... ... [2]

GROUP - B

1. Identify the following slides with reasons. [6]

2. Viva-Voce ... ... [4]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Probability-3

Date: 26.11.1988 Maximum Marks: 100 Time: 3 hours

1.(a) A sample of size $(2n+1)$ is taken from uniform $(0,1)$. Show that the median of the sample has Beta distribution. What are the parameters ?

(b) Six soldiers takeup random positions on a road 2 miles long. Show that the probability that the distance between any two soldiers will be more than $\frac{1}{3}$ mile equals $(\frac{1}{6})^6$.

(10+10) = [20]

2. X, Y are independent $Exp(\lambda)$ random variables. Show that $Max(X,Y)$ and $X + \frac{1}{2}Y$ have the same distribution.

[20]

3. Let $(X_1, X_2, X_3, X_4) \sim D(\nu_1, \nu_2, \nu_3, \nu_4; \nu_5)$. Show that $(X_1 + X_2, X_3) \sim D(\nu_1 + \nu_2, \nu_3; \nu_4 + \nu_5)$.

[20]

4.(a) Let $X_n \xrightarrow{P} X$. Let f be a uniformly continuous function on the real line. Show that $Y_n \xrightarrow{P} Y$ where $Y_n = f(X_n)$ and $Y = f(X)$.

(b) Let F and $F_n (n \geq 1)$ be univariate c.d.f's. Assume that for every rational number r, $F_n(r) \to F(r)$ as $n \to \infty$. Show that $F_n \xrightarrow{w} F$.

(10+10) = [20]

5.(a) Let X be a.r.v. having density

$$f(x) = \frac{1}{2} e^{-|x|} \qquad -\infty < x < \infty$$

show that its characteristic function $\varphi_X(t) = \frac{1}{1+t^2}$.

Contd..... 2/-

Contd..... Q.No.5

(b) If Y is a Cauchy variable with density

$$g(y) = \frac{1}{\pi(1+y^2)} \qquad -\infty < y < \infty$$

show that its characteristic function $\varphi_Y(t) = e^{-|t|}$

[Hint: Use (a) and inversion formula].

(c) If $Y_1, \ldots Y_n$ are independent Cauchy variables as above, show that $\frac{Y_1 + \ldots + Y_n}{n}$ is again Cauchy.

(5+5+5) = [15]

6. Select a number at random from (0,1). Call it X. Take a coin whose chance of heads is X and toss it 100 times. Let Y be the number of heads obtained.

(a) Write the joint distribution of (X,Y).

(b) Calculate the marginal distribution of Y.

(c) Calculate the conditional distribution of X given Y = y for y = 0, 1, 2, ... 100.

(5+5+5) = [15]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Calculus - 3

Date: 16.11.1988 Maximum Marks: 100 Time: 3 hours

Note: This paper carries a total of 110 marks. Answer as many questions or parts thereof as you can. The maximum you can score is 100.

1.(a) For each of the following sets S, find the boundary S:

(i) $S = \left\{(x,y) \in \mathbb{R}^2 : 0 < x^2 + y^2 < 1\right\}$

(ii) $S = \left\{(x,y) \in \mathbb{R}^2 : x > 0,\ y = \sin\frac{1}{x}\right\}$

(b) Let f be a continuous real valued function on a closed and bounded subset $S \subseteq \mathbb{R}^2$. Show that f(S) is closed and bounded in $\mathbb{R}$.

(5+5+10) = [20]

2.(a) Let $f : \Omega \to \mathbb{R}$ have continuous partial derivatives on an open subset $\Omega \subseteq \mathbb{R}^2$. Let $S \subseteq \Omega$ be closed and bounded. Show that there exists K > 0 such that for all $\underline{x}, \underline{y} \in S$,

$$|f(\underline{x}) - f(\underline{y})| \le K|\underline{x} - \underline{y}|.$$

(b) State necessary results to conclude that there is a differentiable function f in a neighbourhood of 1 such that f(1) = 1 and

$$x^3 + f^3(x) - 2x f(x) = 0.$$

Also find f'(1).

(12+10) = [22]

3.(a) Show that if u, v, $f : \mathbb{R}^2 \to \mathbb{R}$ are functions satisfying the conditions

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

and if $\varphi(x,y) = f(u(x,y), v(x,y))$,

then $$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0.$$

Contd..... 2/-

Contd..... Q.No.3

(b) Express $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in terms of polar co-ordinates on $\mathbb{R}^2$.

(12+8) = [20]

4.(a) Define Jordan content of a bounded set $S \subseteq \mathbb{R}^2$. Show that $A(S)$ exists if and only if $A(\partial S) = 0$.

(b) State a set of conditions which ensure

$$\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy .$$

(12+4) = [16]

5.(a) Find (i) $\int_D x^2y^2$ where $D = \{(x,y) : 0 \leq x^2+y^2 \leq 1\}$

(ii) $\int_D x^{1/2}\, y^{1/3}\, (1-x-y)^{2/3}$ where

$D = \{(x,y) : x \geq 0,\ y \geq 0,\ x+y \leq 1\}$.

(b) Prove the identity

$$B(m,n) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}, \quad m > 0,\ n > 0.$$

($\overline{6+6}$+12) = [24]

6. Give an example of a function f on $[0,\infty) \times [0,\infty)$ such that the repeated (improper) integrals

$$\int_0^\infty \int_0^\infty f(x,y)\,dx\,dy$$

and

$$\int^\infty \int^\infty f(x,y)\,dy\,dx$$

exist but are unequal.

[8]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Statistical Models

Date: 18.11.1988 Maximum Marks: 100 Time: 4 hours

Note: Answer as many questions as you can. Marks in the margin total 110 but the maximum mark that you can score is 100.

1.(a) In a life-testing problem, the waiting time until "death" is a random variable. Let X denote the waiting time until K deaths occur, where K is a random variable having a Poisson distribution with parameter λ. Obtain the probability distribution of the random variable X.

(b) Let K(t) be the cumulant generating function about origin of a binomial distribution with parameters (n,p). Show that

$$\frac{dK(t)}{dt} = n[1 + e^{-(z+t)}]^{-1}, \text{ where } Z = \log_e \frac{p}{1-p} .$$

Using Taylor's expansion of the R.H.S. or otherwise, obtain the recurrence relation

$$k_r = p(1-p) \frac{dk_{r-1}}{dp}$$

where k_r is the rth cumulant of the distribution.

(6+10) = [16]

2. State the assumptions under which a basic differential equation is formulated to derive the system of Pearsonian frequency curves.

From this differential equation, obtain Pearson's type II curve and prove that for this curve $\beta_2 < 3$.

(5+8) = [13]

3. Develop Edgeworth's type A Series to represent the p.d.f. of a random variable.

Hence find the first four terms of the type A representation of the distribution having p.d.f.

$$f(x) = \frac{\gamma^{p+1}}{\Gamma(p+1)} e^{-\gamma x} x^p, \quad x > 0, \gamma > 0, p > 0.$$

in terms of the standardized variable.

(8+6) = [14]

p.t.o.

4.(a) A continuous random variable X, $(0 \leq X < \infty)$, having mean α_1 and variance α_2, is lognormally distributed with parameters m and σ^2.

Show that

$$m = \log_e\left(\frac{\alpha_1^2}{\sqrt{\alpha_1^2 + \alpha_2}}\right), \qquad \sigma^2 = \log_e\left(\frac{\alpha_1^2 + \alpha_2}{\alpha_1^2}\right).$$

Also find the mode and the maximum ordinate of the distribution of X.

(b) Define a compound distribution. Let X have a Paisson distribution with parameter θ. Work out the compound distribution when θ is a random variable having a gamma distribution with parameters (α, β).

(8+6) = [14]

5.(a) Define the multinomial probability distribution.

(i) Obtain the m.g.f. of this distribution and hence compute the correlation coefficients between pairs of variables.

(ii) Prove that the conditional distribution of a subset of random variables given the rest is multinomial.

Hence find the expectation and variance of the conditional distribution.

Give your comments on the results of (i) and (ii).

(b) After an extensive anthropometric survey of men a clothing manufacturer has come to the conclusion that the joint distribution of trouser length X and the waist measurement Y is bivariate normal with parameters $(m_1, m_2, \sigma_1^2, \sigma_2^2, \rho)$.

In the manufacture of trousers he decides that he will effectively produce trousers of all lengths, but that all trousers of length x will be made with an adjustable waist-band providing for waist measurements in the range

$$m_2 + \beta(x - m_1) - a\,\sigma_2\sqrt{1-\rho^2}, \quad m_2 + \beta(x - m_1) + a\,\sigma_2\sqrt{1-\rho^2},$$

where $\beta = \rho\frac{\sigma_2}{\sigma_2}$ and a is a constant.

Contd..... 3/-

Contd..... Q.5.(b)

Show that by this provision he will satisfactorily accommodate a proportion $\{2\,\Phi(a) - 1\}$ of men.

An alternative plan suggested is to provide the same waist-band adjustment from $m_2 - a\,\sigma_2\sqrt{1-\rho^2}$ to $m_2 + a\,\sigma_2\sqrt{1-\rho^2}$ for all trousers regardless of length. What proportion of men will be suited by this plan ?

[Here $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{x} e^{-x^2/2}\,dx$].

[(5+5)+8] = [18]

6.(a) The frequency distribution of height (in cm) for 177 Indian Adult males yields the following moments:

$m'_1 = 164.734$ cm

$m_2 = 29.945\ (cm)^2$

$m_3 = 10.071\ (cm)^3$

$m_4 = 3203.594\ (cm)^4$.

Determine the appropriate type of Pearsonian curve to be fitted to the above data.

(b) Let the number of chocolate drops in a certain type of cookie have a Poisson distribution. Find the smallest value of the mean of the distribution such that the probability that a cookie of this type contains at least two chocolate drops is greater than 0.99.

(c) The following gives the haemocytometer distribution count of yeast cells:

No. of cells (per square)	Frequency
0	213
1	128
2	37
3	18
4	3
5	1
	400

Fit a negative binomial distribution to the above data and test for the goodness of fit.

(8+7+10) = [25]

7. Practical Note Book. [10]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Elementary Algebraic Structures

Date: 21.11.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer any FIVE questions.

1.(a) Let p be a prime number which divides the order of a finite group G. Show that there exists an element in G of order p.

(b) Let G be the group of non-zero complex numbers under multiplication and let $N = \{a + ib \in G : a^2 + b^2 = 1\}$. Show that the quotient group G/N is isomorphic to the group of all positive real numbers.

(c) Show that the polynomial $1 + x + x^2 + \ldots + x^{p-1}$ is irreducible over the field of rationals iff p is prime.

(6+6+8) = [20]

2.(a) Define a Euclidean ring.

Show that the Gaussian integers $Z[i] = \{a + ib : a, b \in Z\}$ form a Euclidean ring.

(b) Let p be a prime integer such that for some integer c relatively prime to p there exist integers a and b such that $cp = a^2 + b^2$. Show that p can be written as sum of squares of two integers.

(c) Show that there are no non-trivial ideals in a field.

(8+8+4) = [20]

3.(a) Let $2^h + 1$ be a prime p.

Prove that in the multiplicative group mod p, the order of 2 is 2h.

Hence show that h must be a power of 2.

(b) Show that the multiplicative group of all non-zero elements of a finite field is cyclic.

(10+10) = [20]

p.t.o.

4. Let K be an extension of a field F and u an element of K algebraic over F. Let p(x) be the monic irreducible polynomial over F of which u is a root. Show that the simple algebraic extension F(u) is isomorphic to the quotient $F[x]/(p(x))$.

Show also that $[F(u) : F] = n$, where n is the degree of p(x).

Hence show that every element of F(u) is algebraic over F.

[20]

5.(a) Suppose $\mathcal{H}$ is a finite group of automorphisms of a field N. Let K be the subfield of all elements invariant under $\mathcal{H}$ i.e.

$$K = \{ a \in N : T(a) = a \quad \text{for all} \quad T \in \mathcal{H} \}.$$

Show that the degree $[N : K]$ is atmost the order of $\mathcal{H}$.

(b) Let $\mathcal{G}$ be the Galois group for the root field N of a seperable polynomial f(x) over F. Show that there is a one-to-one correspondence between the subgroups of $\mathcal{G}$ and there subfields of N that contain F such that

(i) given $K \subseteq N$, the corresponding subgroup consists of all automorphisms in $\mathcal{G}$ which leave each element of K fixed;

(ii) given $\mathcal{H} \subseteq \mathcal{G}$, the corresponding subfield consists of all elements in N left invariant by every automorphism of the subgroup $\mathcal{H}$.

(10+10) = [20]

6.(a) In any finite field of characteristic p, show that every element has a pth root.

(b) Using degrees, show that any subfield of $GF(p^n)$ has p^m elements, where m/n.

Also show that if m/n then $GF(p^n)$ has a subfield with p^m elements.

[Hint: $m/n \Rightarrow (p^m - 1) | (p^n - 1)$.]

(10+10) = [20]

7.(a) Let K denote the root field of $x^4 - 3$ over $\mathbb{Q}$, the field of rationals. Compute $[K : \mathbb{Q}]$ and find a basis of K over $\mathbb{Q}$.

(b) Let F be a finite field of order $q = p^n$ and let $S = \{a \in F : \text{ for some } b \in F,\ a = b^2\}$ be the set of perfect squares. Show that the cardinality of S is at least $(q+1)/2$.

Conclude that every element of F is a sum of two squares.

[Hints: (i) In F, $a^2 = b^2 \Rightarrow a = b$ or $a = -b$,

(ii) $S \cap (a - S) \neq \phi$].

(7+13) = [20]

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B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Elective - 2 : Economics

Date: 24.11.1988 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted to each question are shown in brackets [ ]. The question paper is set for a total marks of 105. You may attempt all questions. But the maximum that you can score is only 100.

1. "The long-run average cost function is drawn with much the same sort of shape as the short-run average cost function ... However, the factors responsible for this shape are not the same in two cases." Examine the above statement in detail. [25]

2.(a) What are the conditions necessary for successful price discrimination ?

(b) What is meant by 'price discrimination of the third degree' ? Show that in this case a discriminating monopolist will charge a higher price in the market with lower price-elasticity of demand. (5+15) = [20]

3. Let x_{ij} be the amount of the ith good consumed by the jth consumer and u_j be the utility level of the jth consumer. Consider a two-consumer two-good economy, jth consumer having a utility function given by

$$u_j = \log x_{1j} + \log x_{2j} \quad (j = 1,2)$$

there is one unit of each good available. Calculate the set of Pareto-optimal distributions of the goods over the two consumers and illustrate it in an Edgeworth box. [13]

4. What are the major characteristics of a market under monopolistic competition ? Suppose all firms under such a market have identical demand and cost functions.

Contd..... 2/-

Contd..... Q.No.4

Show graphically how each firm and hence the industry will attain equilibrium in the short-run.

[20]

5.(a) Suppose we have a duopoly industry. Each duopolist's cost of production is zero (at any level of output) and the market (inverse) demand curve is linear :

$$p = a - b(y_1 + y_2)$$

where p is the price of the good and y_i is the output level of the ith duopolist. Solve for the Cournot equilibrium values of price and outputs.

(b) Suppose now duopolist 1 acts as a follower and duopolist 2 acts as a leader. Solve for the stackelberg solution.

(8+7) = [15]

6. Suppose that we could perform an experiment in which an industry was first operating under conditions of perfect competition (with a large number of firms) and then under conditions of monopoly (say, with an amalgamation of these firms). Assuming that the demand curve for the industry's product and the industry's cost curves would be the same in either case, what would be the difference in the long-run equilibrium ?

[12]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Elective-2 : Physical and Earth Sciences

Date: 24.11.1988   Maximum Marks: 100   Time: 3 hours

Note: Answer Q.No.10 and any FIVE of the rest.

1.(a) Write down (do not deduce) Van der Waal's equation of state for an imperfect gas and explain, in brief, the significance of the correction terms.

(b) Obtain the expressions for (i) critical temperature, (ii) critical pressure and (iii) critical volume in terms of a, b, R.

(c) What is critical coefficient ? What is its value for a Van der Waal gas ? Show that the Boyle temperature, $T_B = (27/8)T_c$, where T_c = critical temperature.

(5+6+1+2+4) = [18]

2.(a) Obtain the equation (involving p.V.) which governs reversible adiabatic changes in an ideal gas.

(b) An adiabatic curve and an isothermal curve pass through a given point in the p-V diagram. Which curve is steeper ? Prove your statement.

(c) The radius of a ball of fire, immediately after explosion of a nuclear bomb, is 100 m and its temperature at the time is $100{,}000^\circ$ A. Find the temperature when the radius of the ball of fire increases to 1000 m. ($\gamma = 1.666$).

(7+2+4+5) = [18]

3.(a) Find, from entropy consideration, the thermal efficiency of a Carnot's engine.

(b) State and prove Carnot's theorem. What is its significance?

(c) Show, thermodynamically, that absolute zero is the lowest temperature conceivable.

Contd..... 2/-

Contd..... Q.No.3

(d) A Carnot engine has an efficiency of 30%, when the temperature of the sink is 27°C. What must be the change in temperature of the source to make its efficiency 50% ?

(3+7+3+2+3) = [18]

4.(a) Write down (do not deduce) Maxwell's thermodynamic relations and hence show that

(a) $C_p/C_V = E_S/E_T$

(b) $C_p - C_V = T(\partial p/\partial V)_T \, (\partial V/\partial T)_p^2$.

(b) Equal quantities of water, each of mass m and at temperatures T_1 and T_2 (degree Kelvin) are mixed up adiabatically and isobarically. If C_p be the specific heat of water at constant pressure, show that the net change in entropy of the universe in the process is

$$2m C_p \ln \frac{(T_1+T_2)/2}{\sqrt{T_1 T_2}} .$$

and is positive. (4+4+4+6) = [18]

5.(a) The expression for the Compton shift is given by $d\lambda = \frac{h}{m_o c}(1 - \cos\alpha)$, with usual meanings of the symbols.

Discuss the result for $\alpha = 0$, $\pi/2$ and π. If instead of a free electron, the X-radiation collides with a bound electron, what would happen to the Compton shift ?

(b) The masses of different nuclei taking part in the nuclear reaction

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H + Q$$

are $M(^{14}N) = 14.0075$ amu, $M(^{4}He) = 4.0039$ amu, $M(^{17}O) = 17.0045$ amu and $M(^{1}H) = 1.0082$ amu. Calculate the Q-value of the reaction in MeV. Is it exoergic or endoergic ?

(12+6) = [18]

6.(a) Describe, with a neat diagram, a modern cloud chamber with Blackett's modification and the basic principle of its operation.

Contd..... 3/-

Contd..... Q.No.6

(b) Deuterons in a cyclotron describe a circle of radius 0.32 m just before emerging from the dees. The a.c. voltage applied to the dees is 2×10^4 volts at 10 MHz. Find

(i) the velocity of deuterons (ii) the magnetic field and (iii) energy of deuterons.

(6+6+6) = [18]

7.(a) Define the terms 'disintegration constant', half-life and mean life of a radioactive substance. How are they related?

(b) If N_1, N_2 be the number of atoms of parent and daughter (of decay constants λ_1 and λ_2 respectively) present in a radioactive preparation at any time t, show that the activity A_2 of the doughter atoms, when $\lambda_2 >> \lambda_1$, is given

$$A_2 \simeq N_1(0)\,\lambda_1\,[1 - e^{-\lambda_2 t}]$$

where $N_1(0)$ is the number of parent atoms at t = 0 and $N_2 = 0$ at t = 0.

(6+5+7) = [18]

8. Either

Deduce Planck's radiation law by applying Bose-Einstein statistics. Hence show the T^4 law of radiation of Stefan.

(12+6) = [18]

Or

Derive Fermi-Dirac distribution function and apply FD - statistics to obtain the Richardson - Dushman equation for thermionic emisson.

[18]

9.(a) Write down Lorentz transformation formulae. Show that they are equivalent to a simple rotation in Minkowski's four-dimensional space-time world.

(b) Write down the relativistic expression for the Kinetic energy and obtain from it the classical expression when the velocity is small.

(c) When two electrons leave a radioactive sample in opposite directions, each having a speed of 0.67 c with respect to the sample, the speed of one electron relative to the other is 1.34 c according to classical physics. What is the relativistic result ?

[c = velocity of light in free space].

(4+6+4+4) = [18]

p.t.o.

10. Pick out the correct answer from the ones given in the following:

(a) The de Broglie wavelength of an electron is 1.66×10^{-10} m. Its energy will be

(i) 1.08 MeV (ii) 0.5 MeV (iii) 55 MeV (iv) 55 eV

(b) After 1 hour, one-eighth of the initial mass of a certain radioactive isotope remains undecayed. The half-life of the isotope is

(i) 20 Min (ii) 30 Min (iii) 45 Min (iv) 8 Min.

(c) Neutron binds a proton to form a deuteron due to

(i) weak interaction (ii) strong interaction
(iii) gravitational force (iv) electromagnetic force.

(d) The radius of the nucleus is of the order of

(i) 10^{-15} m (ii) 10^{-14} m (iii) 10^{-18} m (iv) 10^{-16} m.

(e) The energy of the electron in the nth Bohr orbit is $E_n = -13.6/n^2$ eV. The energy required to ionise hydrogen atom is

(i) 10.2 eV (ii) 1.36 eV (iii) 27.2 eV
(iv) 13.6 eV (v) None of these.

(5 x 2) = [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Elective-2 : Biological Sciences

Date: 24.11.1988 Maximum Marks: 80 Time: $3\frac{1}{2}$ hours

Note: Use separate answerscript for Group A and Group B.

GROUP - A

Note: Answer question no.1 and any TWO from the rest.

1. Fill up the blanks (answer any FIVE): (2x5) = [10]

(a) The endarch type of xylem is present in all ________.

(b) The roots possess the vascular bundles which are ________.

(c) All roots which possess xylem which is characterised as ________.

(d) The phellogen and the tissues produced by it are together known as ________.

(e) Multiple epidermis is found in the leaves of ________.

(f) The meristems which have been delayed in its differentiation is called ________.

(g) The calcium carbonate crystals that occur in the Ficus leaf hanging from the epidermal cell are called ________.

(h) The function of sclerenchyma is ________.

2. Describe with illustrations how secondary growth takes place in the stem of sunflower until it becomes two years old. [15]

3. Discuss any three theories with reference to organisation of the shoot apical meristems. [15]

4. What is cambium ? What is its function ? Explain with illustrations the abnormal behaviour and function of the cambium in the stem of Aristolochia triangularis. (3+4+8) = [15]

p.t.o.

5. Describe the secondary growth in Boerhaavia difusa stem and comment on abnormal features.

(8+7) = [15]

6. Write short notes on the following (any FIVE):

(5 x 3) = [15]

(a) Intercalary meristem,
(b) Hydathodes,
(c) Periderm,
(d) Casparian strips,
(e) Laticifers,
(f) Heart wood and sap wood,
(g) Growth rings,
(h) Anomocytic stomata.

GROUP - B

Note: Answer question no.1 and any TWO from the rest.

1. Answer the following questions (any FIVE):

(5 x 2) = [10]

(i) What do you understand by the term organ and organ system ?

(ii) What are the functions of ciliary movement ? Define the term metachronal waves.

(iii) What do you mean by the term synapsis ?

(iv) What are the functions of squamous and ciliated epithelial tissue ?

(v) What is the difference between axon and dendron ?

(vi) Explain how blood regulate the body temparature ?

(vii) What is erythrogenesis ?

(viii) How many types of neurones are found according to their functions ? Show with labelled diagram.

2. Define animal tissue. Describe compound epithelial tissue with diagram.

(4+11) = [15]

3. What is the source of tissue fluid ? Explain with labelled sketches how the intercellular tissue fluid is formed. State the functions of tissue fluid.

(2+4+9) = [15]

4. Describe with the help of diagram ____ the different types of neuroglia cells. Explain the characteristics of irritability and conductivity of nerve tissue.

(5+10) = [15]

5. What is bone marrow ? Draw a diagram and describe red and yellow bone marrow. Write down the general functions of periosteum.

(2+9+4) = [15]

6. Write short notes on (any THREE):

(5 x 3) = [15]

(a) Coblef cells, Pigment cells.

(b) Nutrophils, Bursae.

(c) Osseous tissue.

(d) Adipose tissue with functions.

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

PERIODICAL EXAMINATION

Elective-2: Physical and Earth Sciences

Date: 2.9.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can.
All questions carry equal marks.

1.(a) Write down (do not deduce) the equation for pressure of a gas from the kinetic theory and hence prove the Avogadro's law.

(b) Write (no deduction) the Van der waal's equation of state. Define critical constants for a gas. Starting from the Van der waal equation, derive the reduced equation of state.

(c) Compute the values of the critical coeeficient for (i) an ideal gas and (ii) Van der waal gas.

(2+4+2+4+4+4) = [20]

2.(a) What do you mean by adiabatic and isothermal transformations? Show that at a given point in a P-V diagram, an adiabatic curve is steeper than the isothermal curve.

(b) Show that for a gas obeying Van der waal's equation and having C_v independent of temperature,

$$T(V - b)^{R/C_v} = \text{constant} .$$

where the symbols have their usual significance.

(c) After detonation of an atom bomb, the ball of fire consisting of a spherical mass of gas was found to be 15 m radius at 3×10^5 K. Assuming adiabatic condition to exist, find the radius of the ball after 100 ms when its temperature is 3×10^3 K. Given $\gamma = 1.666$.

(4+3+8+5) = [20]

3. Prove the following thermodynamic relations :

(a) $E_s/E_T = C_p/C_v$

(b) $C_p/- C_v = T(\delta P/\delta T)_v(\delta V/\delta T)_p = -T(\delta P/\delta T)_v^2(\delta V/\delta P)_T$

(c) $C_p - C_v = R[p + a/V^2]/(p - a/V^2 + 2ab/V^3)$ for a gas obeying Van der waal equation.

(d) $\delta C_v/\delta V = T(\delta^2 P/\delta T^2)$.

(4+6+6+4) = [20]

p.t.o.

4.(a) Give the expression for the partition function and show that once the partition function of an ensemble are made known.

(b) What is an equation of a state ? Obtain the equation of state for a monatomic ideal gas using MB-statistics.

(c) In what respects does the FD-statistics resemble and differ from the BE-statistics ?

(1+6+2+6+5) = [20]

5.(a) What are bosons ? Name two boson particles.

(b) Write down (no deduction) Bose's distribution function and hence obtain Planck's law of radiation.

(c) Obtain from Planck's radiation law, the T^4 law of Stefan-Boltzmann.

(2+2+2+10+4) = [20]

6.(a) Write down the postulates of special relativity.

(b) Show that if the Newton's second law is valid for one inertial frame, it remains valid in all inertial frames moving relative to each other with a constant translational velocity.

(c) Write down Lorentz transformation formulae and hence prove 'length contraction' and 'time dilation'.

(d) Derive the relation that describes mass-variation with velocity.

(e) Prove the relation $K^2 + 2Km_oc^2 = p^2c^2$ with usual notational significance.

(4+3+6+5+2) = [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

PERIODICAL EXAMINATION

Elective-2 : Economics

Date: 2.9.1988 Maximum Marks: 50 Time: $2\frac{1}{2}$ hours

Note: (1) Marks allotted to each question are shown in brackets [ ]. The question paper is set for a total marks of 65 only. You may attempt all questions. But the maximum that you can score is only 50 marks.

(2) Some general notations are explained below:

G_i : good i
p_i : price of G_i
y : (money) income of a consumer
x_i : quantity of G_i purchased/consumed by a consumer.

QUESTIONS

1. Discuss briefly the nature of the major economic problems which every society has to solve regularly. [15]

2. Suppose in a two-good economy a consumer has a given income and faces given prices of the goods.

(a) Graphically show how the consumer's purchase of G_1 will change when p_1 falls.

(b) Explain what are meant by "income effect" and "substitution effect" of a price change and show these effects graphically for case (a). [12]

3. A consumer has a utility function of the form:

$$u = -\frac{1}{x_1} - \frac{1}{x_2}$$

Suppose his income and prices of the two goods are given.

(a) Derive the consumer's demand functions for two goods.

(b) Show that the indirect utility function is

$$-(\sqrt{p_1} + \sqrt{p_2})^2/y.$$

(6+4) = [10]

[Note: Indirect utility function is defined to be the function showing the maximum level of the utility attained by the consumer, given prices and his income.]

p.t.o.

4. Assume that there are n goods. Define the income elasticity of a consumer's demand for G_i. Show that if the income elasticity of the demand for G_i is the same for all goods and is constant, then this elasticity must be equal to one. [6]

[Hint: Use the budget constraint for the consumer.]

5. Assume that a consumer's daily income (y) equals the given hourly wage rate (r) times the number of hours of work performed by him daily (W).

Assume further that the consumer's utility (U) depends on his income and the number of hours of leisure enjoyed by him daily (L):

$$U = g(L, y), \text{ with } g_i > 0, \quad g_{ii} \leq 0, \quad (i = L, y).$$

(a) Derive (either graphically or algebraically) the condition for the optimum number of hours that he will work daily;

(Note: that the total number of hours available per day is 24 only).

(b) If the above utility function has the following specific form:

$$U = Ly,$$

show that whatever be the wage rate, the consumer will always work 12 hours per day.

(6+4) = [10]

6. Assume that in a three-good economy a consumer has a separable utility function, i.e., a utility function of the following form:

$$U = U(x_1, x_2, x_3) = U_1(x_1) + U_2(x_2) + U_3(x_3)$$

where $\dfrac{\partial u_i}{\partial x_i} > 0, \quad \dfrac{\partial^2 u_i}{\partial x_i^2} < 0 \qquad$ for $i = 1, 2, 3.$

(a) How many of the three goods can be inferior goods ?

(b) When p_1 rises what are the possible signs of $\dfrac{dx_1}{dp_1}$?

[12]

[Hint: Use the budget constraint and the condition for utility maximisation.]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1988-89

PERIODICAL EXAMINATION

Elective - 2: Biological Sciences

Date: 2.9.1988 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer all the questions.

1. Draw a diagram of a typical plant cell as you see it through an electron microscope and label different parts of the cell. (8+2) = [10]

2. Compare mitotic prophase with meiotic prophase 1 with the help of diagrams. What is the significance of meiosis ? (8+2) = [10]

3. What part of chromosome is likely to carry the source of genetic information ? Discuss briefly the structure of that part with special reference to the formation of nucleotide and nucleoside. Ellucidate briefly the major evidence for genetic role of that part.

Or

State the cellular site of Krebs cycle reactions. Diagramatically represent the Krebs cycle reactions. Make a list of enzymes and coenzymes involved in this process.. (2+12+6) = [20]

4. How grana can be differentiated with stroma ? State the structure and function of stroma and grana.

Or

Show diagramatically how the cell organelles are involved in photorespiration. [10]

5. Answer any two of the following :

(i) What are the different enzymes of DNA metabolism ?

(ii) What are the cases studied so far where Mendel's law of inheritance do not hold good in true sense ?

(iii) What do you mean by autocatalytic properties of DNA ? How does it differ in Prokaryotes and Eukaryotes ?

(5x2) = [10]

p.t.o.

6. Answer the following questions (any five):

(i) What are the general function of cellwall and plasmodesmata ?

(ii) What are the various methods by which the modification or changes of chemical nature of cellwall generally takes place ?

(iii) Give the modern definition of gene.

(iv) What are the different types of RNA ?

(v) Make a list of chemical mutagen.

(vi) The man made cereal Triticale.

(vii) What is centromere, plasmid and satellite DNA ?

(viii) Who got Noble prize for deciphering the genetic code ?

(ix) What is spontaneous mutation and gamma garden ?

(x) What is Test Cross, Nullisomic and Trisomic ?

(5 x 5) = [25]

7. Write notes on the following (any three) :

(i) Cellulose

(ii) Endoplasmic reticulum

(iii) Peptide linkage

(iv) β - oxidation

(v) Hexaploid cultivated wheat

(vi) Crop varieties developed by mulation breeding in India.

(vii) Tripalmitin.

(5 x 3) = [15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

PERIODICAL EXAMINATION

Elementary Algebraic Structures

Date: 29.8.1988 Maximum Marks: 100 Time: 3 hours.

Note: Answer as many as you can. The maximum you can score is 100.

1.(a) Define a cyclic group.
Show that a subgroup of a cyclic group is cyclic.
Show that the elements of finite order in any commutative group form a subgroup.

(b) Prove that any permutative ϕ can be written as

(i) a product of disjoint cycles;

(ii) a product of transpositions;

(iii) a product of 3-cycles, if ϕ is even.

(10+10) = [20]

2. When is a subgroup of a group said to be normal ?

Let N be a normal subgroup of a group G.

Define the quotient group G/N.

Show that the homomorphic images of a group G are (upto isomorphism) the quotient groups G/N by its different normal subgroups. Show that if M and N are normal subgroups of G such that $M \cap N = \{e\}$, then

$$ab = ba \quad \text{for all} \quad a \in M \quad \text{and} \quad b \in N.$$

(Hint: Show that $aba^{-1}b^{-1} \in M \cap N$)

(2+5+10+3) = [20]

3.(i) Let S be a subgroup of a group G. Show that any two right cosets of S are either identical or disjoint.

(ii) In (i) let G be a finite group of order n and suppose S is of order m. Show that the number of distinct right cosets of S is n/m.

(iii) Show that a group consisting of 23 elements is commutative.

Contd..... 2/-

Contd..... Q.No.3

(iv) Let G and H be two groups. Define multiplication in $G \times H = \{(g, h) : g \in G, h \in H\}$ by the formula

$$(g,h).(g', h') = (gg', hh').$$

Show that $G \times H$ with the above operation is a group.

(5+6+4+5) = [20]

4.(a) Suppose a finite set G is closed under an associative product and that both cancellation laws hold in G. Prove that G must be a group.

(b) Use the result in (a) to show that

(i) the non-zero integers relatively prime to n under multiplication mod n from a group ;

(ii) a finite integral domain is a field.

(c) Find all the units of $Z[\sqrt{2}] = \{a + b\sqrt{2} : a \in Z, b + Z\}$.

(7+8+5) = [20]

5.(a) Prove that if D is an infinite integral domain, then two polynomials over D which define the same function on D have identical coefficients.

Give an example to show that the result is not true if D is finite.

(b) Let $p(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$ be a polynomial with integral coefficients. Show that any rational root of the equation $p(x) = 0$ must have the form r/S, where $r | a_n$ and $S | a_0$. Hence, show that $x^4 + x^2 - 28$ is irreducible over the rational field.

(10+10) = [20]

6.(a) Show that any non-zero ideal C of F[x], where F is a field, consists of the set of multiples q(x) a(x) of any non-zero member a(x) of least degree.

Deduce that in F[x], any two polynomials a(x) and b(x) have a greatest common divisor d(x) satisfying

(*) $d(x) = s(x)\, a(x) + t(x)\, b(x)$, for some

polynomials s(x) and t(x).

Give an example to show that (*) need not true for polynomials in Z[x].

Contd..... 3/-

Contd..... Q.6

(b) Show that if $p(x)$ is irreducible over a field F and $p(x) \mid a(x)\, b(x)$, then $p(x) \mid a(x)$ or $p(x) \mid b(x)$.

(c) Let D be a finite integral domain with n elements $a_1, a_2, \ldots, a_n$. Let $m(x)$ be the polynomial form $(x - a_1)(x - a_2) \ldots (x - a_n)$. Show that $m(x) = x^p - x$ in case $D = Z_p$, where p is prime.

(Hint: Use Fermat's Theorem).

(10+5+5) = [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89
PERIODICAL EXAMINATION
Probability - 3

Date: 25.8.1988 Maximum Marks: 100 Time: 3 hours

Note: Question No.2 should be worked out directly without appealing to the general formulae on order statistics derived in the class.

1.(a) Write down the n dimensional normal density with mean vector μ and variance - covariance matrix Σ.

(b) For a random sample of size n from $N(\mu, \sigma^2)$ show that the sample mean and sample variance are independent.

(5+10) = [15]

2. Let $U_1, \ldots, U_n$ be the order statistics of size $n \geq 3$ from uniform (0,1). Calculate the joint density of U_1 and U_n. Calculate the cumulative distribution function, density and expected value of $R = U_n - U_1$.

(10+15) = [25]

3. X_1, X_2, X_3 be independent Exp(1) random variables. Let $S = X_1 + X_2 + X_3$.

(a) Find the joint density of (S, X_2, X_3).

(b) Find the conditional density of S given $X_2 = x_2 . (0 < x_2 < \infty)$.

(c) Find the conditional density of S given $X_2 = x_2$ and $X_3 = x_3$ $(0 < x_2 < \infty, 0 < x_3 < \infty)$.

(d) Find the conditional density of (X_2, X_3) given $S = s$ $(0 < s < \infty)$.

(6+8+8+8) = [30]

4. There are two random variables X and Y. Density of Y is given by

$$f_2(y) = cy^4, \quad 0 < y < 1$$
$$= 0 \quad \text{otherwise.}$$

Further, for every y, $0 < y < 1$, the conditional density of X given $Y = y$ is,

Contd..... 2/-

Contd..... Q.No.4

$$f_{X|Y=y}(x) = d\frac{x}{y^2} \quad 0 < x < y.$$

(a) Determine the constants c and d.

(b) Carefully write down the joint density of (X,Y).

(c) Evaluate $P(\frac{1}{4} < X < \frac{1}{2} \mid Y = \frac{5}{8})$.

(d) Evaluate $P(\frac{1}{4} < X < \frac{1}{2})$.

(6+6+6+7) = [25]

5.(a) Write down the conditions for a function F(x,y) of two variables to be a cumulative distribution function.

(b) Write the density function of F distribution with (m,n) degrees of freedom.

(c) When do you say that n random variables $X_1, \ldots, X_n$ are independent ?

(5+5+5) = [15]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1988 - 89

PERIODICAL EXAMINATION

Statistical Models

Date: 31.8.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Total marks in the margin is 110, but the maximum marks that you can score is 100.

1.(a) A random variable X has a probability distribution with p.d.f.

$$f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x, & 0 \le x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the moment generating function (m g f.) of the distribution and hence find its mean and variance.

(b) Explain why cumulants are called semi-invariants. Find the cumulant generating function (cgf) of a negative binomial distribution with parameters (N , p). Hence obtain the first four cumulants of this distribution.

Further, using cgf or otherwise, show that this distribution can be approximated by a Paisson distribution under the conditions that $N \to \infty$, $q = 1-p \to 0$ but $Nq = \lambda$, a finite quantity.

(10+15) = [25]

2.(a) Let the probability that a patient suffering from a certain disease will react favourably be p. The treatment is given to n_1 patients in hospital A and to n_2 patients in hospital B. If the total number of patients who react favourably to the treatment is K, find the probability that r of the n_1 patients in hospital A react favourably. (You may assume that both n_1 and n_2 are finite numbers).

(b) Let X be distributed as b(x, n, p). If Kth moment about origin be μ'_k, show that for this distribution

$$\mu'_{k+1} = np\mu'_k + pq\frac{d\mu'_k}{dp}, \quad K \ge 1 .$$

Contd..... 2/-

Contd..... Q.No.2

(c) In life-testing problem, the waiting time until "death" is a random variable. Let X denote the waiting time until K number of deaths occur, where K is a random variable having a Poisson distribution with parameter λ. Obtain the probability model of the distribution of the random variable X.

(7+6+7) = [20]

3.(a) Let X be distributed as $N(\mu, \sigma^2)$. Find the p.d.f. of X when the distribution is truncated below. Hence find the mean and variance of the truncated distribution.

(b) Let U and V be two i.i.d. N(0, 1) variables. Find the mgf of Z = UV.

(c) Let X_1, X_2, X_3 be three independent log-normally distributed random variables. Obtain the distribution of $X_1^p \; X_2^q \; X_3^r$, where p, q, r are some constants.

(7+6+7) = [20]

4.(a) Steel rods are manufactured to be 3 inches in diameter but they are acceptable if they are inside the limits 2.99 inches and 3.01 inches. It is observed that about 5% are rejected oversize and 5% are rejected undersize.

Assuming that the diameters are normally distributed, find the s.d. of the distribution. Hence Calculate what proportion of rejects would be if the permissible limits were widened to 2.985 inches and 3.015 inches.

(b) A newsboy is selling papers on a busy street. The paper he sells are events in a Poisson process with parameter λ = 50 per hour. If we have just purchased a paper from him, what is the probability that it will be at least 2 minutes until he sells another ?

(9+6) = [15]

5.(a) Let X be distributed as b(x, n, .55). Find the smallest value of n so that $P(\frac{X}{n} > \frac{1}{2}) \geq .95$.

(b) Following data give the distribution of the number of spontaneous ignitions per day in an explosive factory for a period of 200 days where at least one ignition occured:

No. of ignitions :	1	2	3	4	5	6 or more
Observed number of days:	95	64	30	7	3	1

Fit a truncated Poisson distribution to this data and test the goodness of fit at 5% level.

(8+12) = [20]

6. Practical Note Book. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1988-89

PERIODICAL EXAMINATION

Calculus-3

Date: 22.8.1988 Maximum Marks: 100 Time: 3 hours

Note: This paper carries a total of 110 marks.
Answer as many questions as you can.
You will be awarded the minimum of your actual score and 100.

1.(a) Given that g is a function of one variable, $\lim_{t \to 0} g(t) = \lambda$, prove that $\lim_{(x,y) \to (0,0)} f(x,y) = \lambda$ where $f(x,y) = g(x^2 - y^2)$.

(b) Show that if $\lim_{(x,y) \to (x_0,y_0)} g(x,y) = \lambda$ and if $\lim_{y \to y_0} g(x,y) = f(x)$ exists for all x in a neighbourhood of x_0, then $\lim_{x \to x_0} f(x) = \lambda$.

(c) Prove that a Cauchy sequence in $\mathbb{R}^3$ converges.

(4+6+6) = [16]

2.(a) Show that if $\mathcal{U}$ is an open cover of $[a,b] \times [c,d]$, then $\mathcal{U}$ admits a finite subcover.

(b) Let f be a continuous function on the open set $U \subseteq \mathbb{R}^2$. If $S \subseteq U$ is a closed and bounded set, prove that f is bounded on S.
[Hint. Consider the sets $f^{-1}((-n,n))$]

(10+8) = [18]

3. Classify the following subsets of $\mathbb{R}^3$ as 'open', 'closed' or 'neither open nor closed':

(a) $\{(x,y,z) : z = e^{x+y}\}$

(b) $\{(x,y,z) : \min(\sqrt{(x-1)^2 + (y-2)^2 + (z-1)^2}, \sqrt{x^2+y^2+z^2}) < \frac{1}{2}\}$

(c) $\{(x,y,z) : ax + by + cz > 0\}$; a, b and c given constants.

(4x3) = [12]

p.t.o.

4.(a) Find the tangent plane and the normal direction to the function surface $z = f(x,y)$ at $(a,b,2)$ where

$$f(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$$

(b) Find an approximate value of $\log[(1.02)^{1/4} + (0.96)^{1/6} - 1]$.

(c) Find $\frac{\partial^2 f}{\partial x \partial y}$ of the function $f(x,y) = e^{\cos(y^2 + x)}$.

(5+5+4) = [14]

5.(a) Give an example of a function having all directional derivatives at (0,0) which is not differentiable there.

(b) Prove that if f has continuous pertial derivatives at (x_o,y_o) then f is differentiable at (x_o,y_o).

(8+8) = [16]

6.(a) Decide if the following functions are differentiable at (0,0):

(i) $f(x,y) = x^2y^3$ if $x \geq 0$
$= -x^2y^3$ if $x < 0$.

(ii) $f(x,y) = \frac{2xy}{\sqrt{x^2+y^2}}$ $(x,y) \neq (0,0)$
$= 0$ $(x,y) = (0,0)$.

(b) If $u(x,y) = f(x)\, g(y)$ where f and g are differentiable functions of one variable, show that

$$u \cdot u_{xy} - u_x u_y = 0.$$

(4+6+2) = [12]

7.(a) Find $g''(t)$ where

$$g(t) = f(at, bt)$$

and f has continuous second order partial derivatives.

(b) Give an example to show that $\frac{\partial^2 f}{\partial x \partial y}(x,y)$ and $\frac{\partial^2 f}{\partial y \partial x}(x,y)$ may both exist but be different.

(c) Show that the function

$$x^2 - 3xy + y^2 + \cos 2(x+y)$$

has a strict local maximum at the point (0,0).

(8+8+8) = [24]

:bcc: