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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1989-90

BACKPAPER EXAMINATION

Economics II

Date: 6.7.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any THREE questions all of which carry equal marks.

1. How would you go about defining "value added in production" during any year ? Show that in the absence of government transactions, a closed economy consisting of capitalist production processes alone would be characterized by an equality between aggregate value added in production and aggregate of incomes (i.e., wages, salaries, profits etc.) and depreciation allowance for the year.

2. "A rise in equilibrium income must be accompanied by a rise in equilibrium saving. However, the latter could be brought about either through a rise in real output or through a rise in price."

   Discuss the truth of the above statement.

3. What is meant by an IS-LM equilibrium ? Trace out the possible impact of a cut in the exogenously specified money wage rate on this equilibrium.

4. Construct a macroeconomic model which recognizes the simultaneous existence of excess capacity and scarcity in different sectors of the economy. Discuss in terms of such a model the likely impact on employment of changes in autonomous expenditure. In particular, show how a rise in aggregate employment might be accompanied by a fall in employment in certain specific pockets.

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1989-90

SEMESTRAL-II EXAMINATION

Elements of Algebraic Structures

Date: 11.5.1990 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer as many questions as you can. The maximum possible score is 100 marks. Marks assigned to questions are given in parenthesis.

1.(a) If H and K are finite sub-groups of a group G, find the number of elements in HK. Is HK a sub-group of G ? Justify your answer.

(6+1+3) = [10]

(b) Either

(i) If a and b are elements of a group G such that the orders of a , b and a b are 2 , 2 and n respectively, show that the order of the subgroup H = < a , b > generated by a and b is 2n.
[Hint: Show that the cyclic subgroup of H generated by a b is normal in H and list all the elements of H.]

[8]

Or

(ii) If θ is a rational multiple of π, show that $e^{i\theta} = \cos\theta + i\sin\theta$ generates a finite cyclic sub-group of $S' = \{z \in \mathbb{C} : |z| = 1\}$, the multiplicative group. What is the order of this cyclic group ?

[8]

2.(a) In the alternating group A_4 on 4 symbols, write down a sylow 2-subgroup and a sylow 3-subgroup. Which of these is normal ? Justify.

(3+2+2) = [7]

(b) Either

(i) If N is a nontrivial normal subgroup of a finite group G of prime power order, show that N intersects the centre Z(G) of G nontrivialy. Use this to show that any group of order 16 is abelian.

(6+6) = [12]

Contd..... 2/-

(ii) If G is any abelian group of order 432, find precise the possibilities for the number of elements of G each of which is divisible by at most 2 primes.

[12]

3.(a) Prove that, if D is any integral domain containing the multiplicative identity, them D contains precisely one of the integral domains: $\mathbb{Z}$, $\mathbb{Z}_p$, p - a prime. [4]

(b) Define an Euclidean domain. Prove that the set $\mathbb{Z}[i]$ of all complex numbers of the form a + ib, a and b integers, is a Euclidean domain. (3+6) = [9]

(c) Show that no prime number of the form 4n + 3, n an integer, can be the sum of 2 integral squares.

[5]

(d) Decide which of the Polynomials

$$p(X) = X^3 - 9 \quad \text{and} \quad q(X) = X^2 + X + 4$$

is irreducible over $\mathbb{Z}_{11}$. Justify. (3+3) = [6]

4.(a) If F is an extension field of a field E, when is an element a of F called algebraic over E. Prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}+\sqrt{3}$ and $\sqrt{2}.\sqrt{3}$ are algebraic over $\mathbb{Q}$.

(3+1+1+3+2) = [10]

(b) If f(X) ε F[X], F a field, define the splitting field of f(X) over F. If p is a prime number, show that the degree of the splitting field of $X^p - 1$ over $\mathbb{Q}$ is (p-1).

(4+6) = [10]

(c) If f is an extension field of a field E, and $\bar{E}$ is the algebraic closure of E, then show that there is an embedding of F in $\bar{E}$.

[9]

5.(a) Given vector spaces V, W and Z over $\mathbb{R}$, define the concept of a bilinear form f : V x W -> Z, and the concept of $V \otimes W$. If $\dim_{\mathbb{R}} V = n$ and $\dim_{\mathbb{R}} W = m$, is $V \otimes_{\mathbb{R}} W$ isomorphic to the vector space of all n x m - matricies over $\mathbb{R}$? Justify.

(3+4+3) = [10]

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(b) If V, W, Z, are as in 5.(a), show that the vector space of all bilinear forms from $V \times W$ to Z is isomorphic to the vector space of all vector space homomorphisms from $V \otimes W$ to Z. [10]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year: 1989-90

BACKPAPER EXAMINATION

Elements of Algebraic Structures

Date: 28.6.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. The maximum possible score being 100 marks. Marks assigned to questions are given in parenthesis.

1.(a) If H and K are sub-groups of a group G of finite indices, show that $H \cap K$ is also a sub-group of G of finite index. Find an upper bound for the index of $H \cap K$ in G.

(7+3) = [10]

(b) Prove that if a finite group admits an automorphism of order 2, then it must necessarily be abelian. [6]

(c) Prove that a cyclic sub-group $\langle e^{i\theta} \rangle$ of the multiplicative group $S^1 = \{ z \in \mathbb{C} : |z| = 1 \}$ is finite if and only if θ is a rational multiple of π. [10]

2.(a) Using Sylow's theorem, determine the number of Sylow p-sub-groups of the alternating group A_5 for p = 2,3, and 5.

(5+5+5) = [15]

(b) If P is a group of order p^n, p-a prime and n : an integer, then show that the centre Z(P) of P is nontrivial. Use that to show that any group of order 49 is either isomorphic to $\mathbb{Z}_{49}$ or to $\mathbb{Z}_7 \times \mathbb{Z}_7$.

(6+4) = [10]

(c) If P is a finite abelian group of order p^m, p a prime and n an integer, such that there are at most p elements x of P such that $x^p = 1$, then show that P is cyclic. [6]

3.(a) If R is a finite ring without zero divisors, prove that R is a division ring. [6]

(b) Find all the units of the ring $\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$, what is the greatest common divisor of 11 + 7i and 18 - i in $\mathbb{Z}[i]$?

(5+5) = [10]

p.t.o.

(c) If a is a rational number and $X - a$ divides an integral monic polynomial, show that a must be an integer.

[8]

4.(a) If K is an extension field of F and $a \in K$, show that a is algebraic over F if and only if $F(a)$ is a finite extension of F.

[10]

(b) If F is a field containing p^n elements, p a prime and n an integer, find the automorphism group of F.

[12]

(c) Determine the splithing field of $X^5 - 3$ over $\mathbb{Q}$. [7]

5.(a) Given vector spaces V, W and Z over $\mathbb{R}$, define the concept of a bilinear form $f : V \times W \longrightarrow Z$ and the concept of $V \otimes W$. If $\dim_{\mathbb{R}} V = n$ and $\dim_{\mathbb{R}} W = m$, is $V \otimes W$ isomorphic to the vector space of all $n \times m$ - matrices over $\mathbb{R}$? Justify.

(3+4+3) = [10]

(b) If V, W, Z are as in 5.(a), show that the vector space of all bilinear forms from $V \times W$ to Z is isomorphic to the vector space of all vector space homomorphisms from $V \otimes W$ to Z.

[10]

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INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) II Year:1989-90

STATISTICAL METHODS 4

SEMESTRAL EXAMINATION

Date:7 May 1990 Maximum Marks:100 Time:$3\frac{1}{2}$ Hours

Figures in brackets [ ] indicate marks allotted to the questions.

1. In a multiple-choice test for an item candidates were classified by their responses being correct and wrong and the mean scores of the two groups so obtained in the test as well as in their M.Sc. examinations were computed as follows:

	response to item	
	correct	wrong
No. of candidates	136	20
Mean test score	11.13	9.06
Mean M.Sc. score	66.57	63.81

Compute appropriate correlation coefficients between item response and test score and between item response and M.Sc. score; interpret the results. [10]

2. (a) Let $X = (X_1, X_2, \ldots, X_p) \sim N_p(\mu, \Sigma)$. Derive the distribution of the sample correlation coefficient r_{12} between X_1 and X_2 based on a sample of n from X, under the condition that the correlation coeffcient ρ_{12} between X_1 and X_2 is zero. [15]

   (b) Let $X = (X_1, X_2) \sim N_2(\mu, \Sigma)$, where $\Sigma = ((\sigma_{ij}))$. Develop a test for $H_0 : \sigma_{11} = \sigma_{22}$ against $H_1 : \sigma_{11} \neq \sigma_{22}$ based on n independent observations on X. [10]

PLEASE TURN OVER

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3. Let $S \sim \mathcal{W}_p(k, \Sigma)$, with a *positive* definite Σ.

   (a) Let C be a nonsigular matrix. Show that $CSC' \sim \mathcal{W}_p(k, C\Sigma C')$. [5]

   (b) Let Σ be partitioned as

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}.$$

   where Σ_{11} and Σ_{22} are of orders $r \times r$ and $(p-r) \times (p-r)$ respectively. Let S be similarly partitioned with similar notations S_{11}, S_{12}, etc. and S^{-1} with S^{11}, S^{12} etc. Show that

$$(S^{22})^{-1} \sim \mathcal{W}_{(p-r)}((\Sigma^{22})^{-1}, k-r),$$

   and is distributed independently of (S_{11}, S_{12}). [10]

   (c) Show that if A is an $r \times p$ matrix of rank r, then

$$(AS^{-1}A')^{-1} \sim \mathcal{W}_r(A\Sigma^{-1}A')^{-1}, k-p+r).$$

   [Hint: Extend A to a nonsingular matrix.] [10]

   (d) Show that the distribution of $\mid S \mid$ involves Σ only through $\mid \Sigma \mid$. [5]

4. Let $X = (X_1, X_2, X_3) \sim \mathcal{N}_3(\mu, \Sigma)$ where $\mu = (\mu_1, \mu_2, \mu_3)$ and Σ are unknown. Based on a sample of size 50, the following statistics are available:
Mean vector: (2.13, 2.45, 0.203)

Corrected sum of squares and products matrix

$$\begin{pmatrix} 3.41 & 1.23 & -1.35 \\ & 4.37 & -2.10 \\ & & 4.53 \end{pmatrix}.$$

Test the hypothesis H_0: $\mu_1 = \mu_2$, $\mu_3 = 0$ against the most general alternative, at 5% level of significance. (*You have to clearly explain your procedure.*) [15]

CONTINUED

5. The following data represent the failure times (X) (in hundreds of hours) of a certain type of light bulb:

5.1, 5.9, 6.1, 4.2, 8.2, 6.6, 8.3, 8.0, 7.8, 7.1

Assuming that the failure-time distribution F has a density,

(a) suggest a suitable test statistic for the hypothesis H_0: $\mu = 6$ against H_1: $\mu > 6$, where μ is the median failure time. Carry out the test and comment on your findings. [10]

(b) suggest a suitable test for H_0: $\nu = 5.5$ against H_1: $\nu > 5.5$, where $Pr_F(X < \nu) = 0.25$. Carry out the test and comment on your findings. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1989-90

SEMESTRAL-II EXAMINATION

Demography

Date: 4.5.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any FIVE questions.

1. Define population census. Describe briefly methods of census enumeration. Narrate a brief history of the Indian census.

(4+8+8) = [20]

2. What are the basic sources of demographic data ? What types of error in data are generally encountered in a demographic survey or census ? Discuss net difference rate and gross difference rate in measuring errors of content.

(5+5+10) = [20]

3. Define rate of natural increase. Derive the exponential growth rate of the population. Prove that the population will quadruple in size at time $1.386/r$, where r is the growth rate of the population. What is the balancing equation of the population ? Interpret it.

(4+8+4+2+2) = [20]

4. Define and derive logistic curve of population growth. Find out the initial and final population with the logistic curve. Why do we prefer the logistic curve to other curves for population growth ?

(12+4+4) = [20]

5. Define crude death rate and why this death rate is called 'crude' ? What is the difference between infant mortality rate and infant death rate ? Can we say that the perinatal mortality rate is always higher than the neo-natal mortality rate ?

(8+6+6) = [20]

6. What are the assumptions involved in the construction of a life table ? What is the difference between a generation life table and a periodic life table ? If between age x and

Contd..... 2/-

$x + t$, ${}_tp_x$ is the probability of surviving and μ_{x+t} is the force of mortality, then prove that

$$\int_0^{\infty} {}_tp_x \, \mu_{x+t} \, dt = 1 .$$

Show that for five consecutive values of l_x, μ_x can be estimated by

$$\mu_x = \frac{8(l_{x-1} - l_{x+1}) - (l_{x-2} - l_{x+2})}{12 \, l_x}$$

where l_x and μ_x have their usual meanings.

(4+4+6+6) = [20]

7. What do you mean by median age at first marriage ? The median age at first marriage for females in Malaysia is 21.6 years and in India is 15.4 years: comment on the status of fertility in India in comparison with Malaysia. Estimate singulate mean age at marriage in a given population from census data.

(5+5+10) = [20]

8. Define total fertility rate. Find the mathematical relationship between crude birth rate and total fiertility rate. Interpret the situations: (i) NRR = 1, (ii) NRR > 1 and (iii) NRR < 1.

(4+10+6) = [20]

9. What do you mean by standardisation of vital rates ? Discuss methods for direct as well as indirect standardisation of death rates. The CDR for country A is 37 per thousand population and that for country B is 40 per thousand population : Can we say that the mortality in country B is higher than country A ? If not, then what have we to do for comparison of mortality between country A and country B ?

(6+8+6) = [20]

10. Explain " Demographic Transition Theory ". Is this theory relavant to developing countries now a days ?

(12+8) = [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1989-90

SEMESTRAL-II EXAMINATION

Economic and Official Statistics

Date: 2.5.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any THREE questions from Qs. 1-5 and also Qs. 6 and 7.
Marks allotted to the questions are shown within brackets.

1. Explain the two most important tests of consistency of index number formulas and examine Laspeyres' and Fisher's formulas in the light of these tests.

   Briefly mention what is meant by formula error in the measurement of price changes over time.

   [18]

2. Describe the ratio - to - moving average method of determining constant seasonal indices from a series of quarterly figures. You may assume the multiplicative model for the time series. Justify the different steps briefly.

   How would you modify this procedure if the seasonal pattern changes gradually over time ? State briefly.

   [18]

3. Prove the moment distribution property of the lognormal distribution and use this property to derive the equation of the Lorenz curve of the distribution.

   Find the expression for the Lorenz ratio of the lognormal distribution.

   How does the Lorenz curve of a lognormal distribution $\Lambda(\mu, \sigma^2)$ depend on the parameter σ of the distribution ?

   [18]

4. Discuss the various methodological problems that arise in the estimation of demand functions from time series data.

   [18]

p.t.o.

5. Write short notes on any two of the following:

(i) The law of proportionate effect.

(ii) Economies of scale in household consumption.

(iii) Properties of the Cobb - Douglas production function.

[18]

Either

6. The following shows the average per capita consumption of cereals for households in different levels of per capita income, according to a household budget enquiry:

per capita monthly income (Rs.)	percentage of population	average monthly per capita income (Rs.)	average monthly per capita consumption of cereals (Rs.)
0 - 50	8	39	19
51 -100	12	72	30
101 -150	25	123	41
151 -200	18	171	48
201 -250	16	221	57
251 -300	11	270	64
301 -	10	380	71

Estimate the constants of a semi-logarithmic Engel curve for cereals by the weighted least squares method, using the percentages of population as weights. Then estimate the Engel elasticity for the item at per capita monthly income (Rs.) = Rs.100/- and Rs.200/-.

[26]

OR

6. Fit a logistic curve to the following data on the number of unemployed persons in Japan (in ten thousand) by any suitable method. Compute the expected values and plot them along with the observed values in a graph.

Year	1970	'71	'72	'73	'74	'75	'76	'77	'78
no. of unemployed persons	59	64	73	68	73	100	108	110	124

[26]

7. Practical Record. [20]

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B.Stat. (Hons.) II Year : 1989-90

SEMESTRAL-II EXAMINATION

Economics II

Date: 30.4.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer question no.4 and any TWO of the remaining. Marks are indicated in the margin.

Marks will be deducted for purely mechanical calculations. As far as practicable the calculations should be backed by economic reasoning.

1.(a) Show that in any economy aggregate saving and investment must be identically the same when the government's budget is balanced, aggregate exports equal aggregate imports and there are no transfers to foreigners by private individuals. [15]

(b) How would the identity in (a) change when the simplifying assumptions are given up ? [5]

(c) Distinguish the identity in (a) from saving-investment equality as a condition for macroeconomic equilibrium. [10]

2.(a) Consider a simplified macroeconomic system with the following characteristics:

the marginal propensity to consume out of aggregate income = 0.8; the marginal income tax rate = 0.25; the slope of the investment demand function = -0.02; the income sensitivity of money demand = 0.1 and its interest rate sensitivity = -0.05.

Calculate the size of the multiplier for a change in the size of real government expenditure. [10]

(b) Derive a general expression for the change in equilibrium real income in response to a change in the proportional tax rate. Interpret the expression by separating out the multiplier from the multiplicand. [10]

(c) Discuss in terms of a simple model the balanced budget multiplier theorem. [10]

p.t.o.

3.(a) Discuss the nature of the classical dichotomy. Why is it the case that in the classical model, output and employment are determined independently of the aggregate demand for output ? [10]

(b) What is the mechanism that guarantees an equality between investment and full employment saving in the classical world ? [5]

(c) What are the effects of a rise in the aggregate demand in the classical model ? [5]

(d) Why does the classical model face an inconsistency in the presence of a liquidity trap ? [10]

4. Consider a dual economy model with the following features: There are two sectors, industry (y) and food (x). The industrial sector is characterized by a capitalist mode of production, the price (p_y) being determined on the basis of a mark-up (m) over wage cost. There is excess capacity in the y-sector. Industrial profits are denoted π_y. The mode of production in agriculture is noncapitalistic, with the price p_x of x being determined by the interaction of demand and supply. The landlord's income is denoted R. The nominal expenditure on y consists of an autonomous component A; industrial profit earner's consumption expenditure $(1-s_\pi)(\pi_y-X_\pi)$, where, $0 < s_\pi < 1$ and X_π is the industrial capitalists' nominal expenditure on food; the agricultural landlord's consumption expenditure R; and industrial workers' consumption expenditure $(1-z)W_y$, where z is the share of industrial wages W_y spent on food. Suppose further that the marketed surplus of food (x^S) is the following function of p_x :

$$x^S = p_x + 2 .$$

(a) Determine the equilibrium food price and the corresponding marketed surplus of food when A = Rs.0.75 crores, $s_\pi = 0.75$, m = 0.1 and the π_y earners demand 1 unit of food irrespective of the level of p_x. [20]

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(b) Determine the level of π_y, R and W_y in equilibrium. [5]

(c) What is the equilibrium value of industrial output ? Can you determine the equilibrium level of real output ? [5]

(d) How do the equilibrium magnitudes in (a), (b) and (c) change when A rises to Rs.7.5 crores ? [10]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1989-90

SEMESTRAL-II EXAMINATION

Anthropology

Date: 30.4.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions. The figures in the right-hand margin indicate marks.

1. Discuss the biological and cultural uniqueness of Homo sapiens sapiens. [16]

2. Discuss the changing concept of "race" from earlier to recent times. [14]

   Can the concept of "race" in Anthropology be equated with the concept of subspecies in Zoology or Botany ? [2]

3. (i) Homeostasis essentially involves maintenance of a stable internal environment of an organism. True or false ? [2]

   (ii) The basic design for detecting the effect of high altitude stress on human biological traits involves the following comparisons : HAN - HAN(D); LAN - LAN(v); HAN(D) - LAN ; LAN(U) - HAN

   (HAN : High altitude native
   HAND(D): High altitude native, downward migrant
   LAN : Low altitude native
   LAN(U): Low altitude native, upward migrant).

   Can the principle of this design be used for detecting the effects of other environmental stresses ? [2]

   (iii) The following diagram represent a concept in demography. What is the term used for this concept ?

   High Birth rate

   Medium Death rate

   Low

   Stage 1 Stage 2 Stage 3 [2]

(iv) Does the uniqueness of Anthropology as a discipline lie in its holistic approach or subdisciplinary specialisation ? [2]

(v) The concepts of Caste and Class are entirely different. True or false ? [2]

(vi) Preferential choice of mate in the South Indian Hindu Society is generally made on the basis of (A) Homogamy (B) Hypergamy (C) Uncle-niece or consin relationship (D) Social Status.

Which one of the above alternatives is correct ? [2]

4. Write short notes on any <u>four</u> of the following:

(a) Crossing over [5]

(b) Linkage [5]

(c) Genetic variation [5]

(d) Polymorphism [5]

(e) Random genetic drift [5]

5. (i) (a) Define Mendel's laws of inheritance. [3]

(b) A brown-eyed man marries a blue-eyed woman. The first child is blue eyed. What is man's genotype ? (Assume that brown eye colour is dominant over blue eye colour). [7]

(ii) (a) What is the Hardy-Weinberg principle ? [3]

(b) In a population's gene pool, the alleles 'A' and 'a' occur at initial frequencies p and q, respectively. Prove that the gene frequencies and the Zygote frequencies do not change from generation to generation, after the first generation of random mating. [7]

6. What are the types of mutation and the magnitudes of their phenotypic effects ? [14]

What is the evolutionary significance of mutation ? [2]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.)II Year:1989-90

PROBABILITY THEORY AND ITS APPLICATIONS III
SEMESTRAL-I BACKPAPER EXAMINATION

Date : 2.1.1990 Maximum Marks : 100 Time : 3 Hours.

Note : Attempt any five of the following questions.

1.(a) Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the order statistics in a random sample of size n from an absolutely continuous c.d.f. F(.). Show that the random variables

$$Y_i = \left[\frac{F(X_{(i)})}{F(X_{(i+1)})} \right]^i , \quad i = 1,2,\ldots,n-1$$

are independently distributed. Find the distribution of each Y_i.

(b) Consider an urn containing N tickets marked 1,2,..,N. A simple random sample of size n is drawn from the urn without replacement. Let X be the largest number in the sample. Obtain the p.m.f. of X and calculate E(X) and V(X). Simplify your expressions as far as practicable. [8+12=20]

2.(a) Let $X_1, X_2, \ldots, X_n$ be iid each following $N(\mu,\sigma^2)$ distribution. Define $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$. Derive the conditional distribution of X_1 given $\bar{X}$.

(b) Let $(X,Y)'$ be a bivariate random vector with a concentration ellipsoid given by

$$7x^2 + 6xy + 8y^2 - 14x - 6y + 7 \leq 3.$$

Obtain the mean vector and the dispersion matrix of $(X,Y)'$.

(c) If $\underset{\sim}{X}$ be a px1 random vector having the multivariate normal distribution with mean vector $\underset{\sim}{\mu}$ and a positive definite dispersion matrix then find a quadratic function of $\underset{\sim}{X}$ which has the chi-square distribution with p degrees of freedom. [10+5+5=20]

3.(a) Define the multivariate Cauchy distribution. Show that if $\underset{\sim}{X} = (X_1,\ldots,X_p)'$ has the multivariate Cauchy distribution then marginally each of $X_1,\ldots,X_p$ has a Cauchy distribution.

p.t.o.

3.(b) If X_1, X_2 are independent random variables each uniformly distributed over (0,1) then show that

$$U = \sqrt{-2\log X_1}\ \cos(2\pi X_2)$$

$$\text{and}\quad V = \sqrt{-2\log X_1}\ \sin(2\pi X_2)$$

are independent. Find the distribution of each of U,V. What can be said about the distribution of $W = U/|V|$? Justify your answer. [10+10=20]

4.(a) Let X,Y be independent each having the normal (0,1) distribution. Show that

$$P[(X,Y)\in C] \geq P[(X,Y) \in S]$$

where S is a square with vertices (-k,-k), (-k,k), (k,-k) and (k,k) and C is a circle with centre at origin, the areas of C and S being equal. Hence or otherwise, show that

$$\frac{1}{\sqrt{2\pi}} \int_0^k e^{-\frac{1}{2}t^2}\, dt \leq \frac{1}{2} \sqrt{1-\exp(-2k^2/\pi)}.$$

(b) Let (X,Y) be jointly distributed with pdf

$$\frac{1}{\Gamma m\ \Gamma n}\, x^{m-1}\, (y-x)^{n-1}\, e^{-y},\ 0 < x < y < \infty.$$

Obtain the correlation coefficient between X and Y in the simplest possible form. [10+10=20]

5.(a) Derive a formula for the <u>exact</u> variance of the sample variance. State your assumptions and notations clearly. Simplify your answer as far as possible.

(b) Let $\{X_n\}$ be a sequence of independent random variables such that

$$P[X_n = n^{\alpha}] = P[X_n = -n^{\alpha}] = \tfrac{1}{2}.$$

Find a sufficient condition, if any, on α under which the weak law of large numbers (WLLN) holds. Also find a sufficient condition, if any, on α under which the WLLN does not hold

[10+10 = 20]

6. Prove the following statements if you consider them to be true. Otherwise disprove them by counterexamples.

(a) Let $\{X_n\}$ be a sequence of random-variables. If $X_n \xrightarrow{P} 0$ then $\mathrm{Var}(X_n) \to 0$ as $n \to \infty$.

(b) Convergence in distribution implies convergence in probability.

(c) If marginally each of X,Y is univariate normal then the joint distribution of X,Y is bivariate normal.

(d) Let $\{\phi_n(t)\}$ be a sequence of characteristic functions converging (pointwise) to a function $\phi(t)$. Then $\phi(t)$ is continuous at t=0.

[5x4 = 20]

Contd.....

7.(a) Using probabilistic arguments, show that

$$\lim_{n\to\infty} \sum_{j=0}^{n} e^{-n} \frac{n^j}{j!} = \frac{1}{2}.$$

(b) Let $X_{1n}, \ldots, X_{6n}$ be the frequencies of the faces 1,...,6 in n independent throws of a fair die. Show that for a suitable choice of the constant a,

$$\frac{1}{\sqrt{n}} \{(X_{1n} + X_{2n} + 2X_{3n} - 4X_{4n})/a\}$$

is asymptotically normal (0,1). Find a. State all auxiliary results clearly.

(c) In the set-up of (b), what can be said about the limiting distribution of the random vector

$$\frac{1}{\sqrt{n}} \begin{pmatrix} X_{1n} - X_{2n} \\ X_{2n} - X_{3n} \end{pmatrix} \quad ?$$

Justify your answer. [6+7+7=20]

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ECONOMICS - I

SEMESTRAL-I EXAMINATION

Date : 1.12.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Answer ANY FOUR questions. Marks allotted to each question are shown in brackets [ ].

1.(a) Suppose we have a duopoly industry. Each duopolist's cost of production is zero (at any level of output), and the market (inverse) demand curve is given by the following linear function :

$$p = a-b(y_1+y_2)$$

where p is the price of the good and y_i is the output level of the ith duopolist. Solve for the equilibrium values of price, outputs and profits of the duopolists under the Cournot behaviouristic assumptions.

(b) Suppose now duopolist 1 acts as a follower and the duopolist 2 acts as a leader. Obtain the Stockelberg solutions for price, outputs and profits of the duopolists.

[14+11 = 25]

2. Examine the following statement in detail :

" The long-run average cost function is drawn with much the same sort of shape as the short-run average cost function. However, the factors responsible for this shape are not the same in two cases". [25]

3. A monopolist with cost function

$$c(y) = y^2+1$$

faces a market (inverse) demand function :

$$p = Rs.200-y$$

where p is the price (in Rs.) of the good and y, the amount of the output of the monopolist.

(a) What levels of price and output will result ? What will the monopolist's profit be ?

(b) If the monopolist for some reason behaved as a competitor and, in fact, were one of 98 such identical competitors, what would be the equilibrium price of the good and the equilibrium quantity of output and profit of each competitor ?

[10+15=25]

p.t.o.

4. Indicate whether the following statements are TRUE or FALSE, giving sufficient arguments in support of your answer, [Attempt ANY THREE] :

(a) In a n-good world, if the income elasticity of the demand for each good is same and is independent of the level of income, then this elasticity must be equal to 1.

(b) A monopolist may operate at an inelastic portion of his demand curve.

(c) If a firm has a production function :
$y = f(x_1, x_2)$ which shows constant returns to scale and
$f_i > 0$, $f_{ii} < 0$ (for $i = 1,2$), then the cost function of the firm (when the amount of fixed cost is zero and the prices of the inputs are given) is proportional to the level of output (b).

(d) If an excise tax of t rupees per unit of output is imposed on the suppliers of a good operating under perfect competition, they will be able to " pass on " to the consumers the full amount of this tax. [25]

5.(a) What are the conditions necessary for successful price discrimination by a discriminating monopolist ?

(b) What is meant by " price discrimination of the third degree"? Show that in this case a discriminating monopolist will charge a higher price in the market with lower price-elasticity of demand ?

[7+18=25]

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GEOLOGY

SEMESTRAL-I EXAMINATION

Date : 29.11.1989. Maximum Marks : 100 Time : 3 Hours.

Note : Answer any four questions.

1. Write short notes on (an three) :
   (a) Rock cleavage and mineral cleavage
   (b) 'Low velocity laye'
   (c) Fold and fault
   (d) 'P', 'S' and 'L' waves in earthquakes
   (e) 'Room problem' of large granitic bodies.

[(3x8)=24+1=25]

2. 'A rock can be dated by its radiometric content' - elucidate. [25]

3. Write what you know about evolution of the atmosphere through geological time [25]

4. What causes an earthquake ? How would you determine the epicentre and focus of an earthquake ? What makes us suggest that the earth has a liquid core. [25]

5. Write what you know about the 'plate tectonics' model of the earth. [25]

6. What is isostasy ? How would you determine isostatic anomaly at a point on the surface of the earth ? How would you explain presence of isostatic anomalies over young mountain chains. [25]

7. What is Bowen's reaction series ? How can diverse igneous rocks originate from a parent basic magma. [25]

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SOCIOLOGY

SEMESTRAL-I EXAMINATION

Date : 29.11.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Attempt FIVE questions at least TWO from each group. Questions are of equal marks.

GROUP A

1. For critically understanding social reality it is necessary for the students of statistics to learn sociology. Do you agree ? Illustrate your answer.
2. Briefly compare Durkheim's and Marx's major ideas on studying society.
3. Define marriage. What are its different forms ? Briefly discuss the ways of acquiring brides in tribal India.
4. How did the social movement of the English educated middle class differ from that of the peasantry in Bengal during the early colonial phase ?
5. In your opinion what is the future of Joint Family in India ? Do you feel that it is on the decline ?

GROUP B

6. Suppose you are required to do a research project on "Drug-addiction" among college students. Or, if you may so desire, you can choose any problem other than drug-addiction also. In this project you are required to find out the extent and seriousness of the problem; why this is so, etc. Discuss your possible hypotheses (at least two) and the design of your enquiry.

7.(a) In the literature on sociological research two types of methodology are often referred to : qualitative methodology and quantitative methodology. What do they mean ?

(b) State the name of one published book, paper, report etc., where qualitative methodology was followed and also of one where quantitative methodology was the design of enquiry. Comment on whether the adopted methodologies were appropriate for these two studies. State how would you like to design your enquiry ?

8. Choose any two of the following concepts. State what do you mean by them and discuss how would you collect data and measure them ?

(a) occupational mobility (b) reciprocity (c) caste status
(d) social values and attitudes.

p.t.o.

9. Briefly describe the settlement pattern and socio-economic conditions of the village Chatro in Giridih, Bihar where you did your recent field work. Mention two major problems of the village that need immediate attention. State reasons of your choice of the problems.

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PROBABILITY THEORY AND ITS APPLICATIONS III
SEMESTRAL-I EXAMINATION

Date : 27.11.1989   Maximum Marks : 100   Time : 3 Hours.

Note : Attempt any five of the following questions.

1.(a) Show that the distribution of sample range in a random sample of size 3 from a normal (0,1) population has the p.d.f.

$$f(w) = \frac{6\, e^{-w^2/4}}{\pi \sqrt{2}} \int_0^{w/\sqrt{6}} \exp(-\tfrac{1}{2} u^2)\, du.$$

(b) Consider a random sample of size n from the uniform $(0,\theta)$ distribution, where $\theta > 0$. Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the order statistics. For $i=1,2,\ldots,n$, find a constant c_i, free from θ, such that

$$E[c_i X_{(i)}] = \theta,$$

whatever be the true value of θ. With the c_i's so chosen, let $Y_i = c_i X_{(i)}$. Show that $Var(Y_i)$ is minimum over i $(1 \leq i \leq n)$ when $i = n$.

[12+8=20]

2.(a) Let $(X_1, X_2, \ldots, X_p)'$ be p-variate normal with mean vector μ and a positive definite dispersion matrix Σ. Show that the correlation coefficient between X_1 and X_2 in their conditional distribution for given $X_3 = x_3, \ldots, X_p = x_p$ is the same as the partial correlation coefficient between X_1 and X_2 eliminating $X_3, \ldots, X_p$.

(b) In the set-up of (a), let μ be the null vector and Σ be the pxp identity matrix. Show that

$$P[X \in S] \geq P[X \in C],$$

where S is a p-dimensional sphere with centre at origin and C is a p-dimensional cube with centre at origin, the volume of C being the same as that of S.

[13+7=20]

p.t.o.

3. Let the joint distribution of X, Y and Z be Dirichlet with parameters 3, 3, 3 and 4.
   (a) Derive regression of X on Y and Z. Hence calculate the multiple correlation coefficient between X and Y, Z.
   (b) Write down the correlation matrix of X, Y, Z. Hence verify your answer in (a) by direct computation.
   (c) Write down the expressions for the conditional expectation and variance of X given Y. Hence calculate the unconditional expectation and variance of X.

[11+3+6 = 20]

4. (a) Let (X,Y) follow bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. Let

$$\xi = \frac{1}{\sqrt{1-\rho^2}} \left\{ \frac{X-\mu_1}{\sigma_1} - \rho \left(\frac{Y-\mu_2}{\sigma_2} \right) \right\}, \quad \eta = \frac{Y-\mu_2}{\sigma_2}.$$

Show that ξ and η are iid each N(0,1).

   (b) Hence derive the distribution of $Z = a_1 X + a_2 Y$, where a_1, a_2 are scalars such that $(a_1, a_2) \neq (0,0)$. (use of m.g.f. is not allowed but a suitable orthogonal transformation may be helpful).
   (c) Use your result in (a) to find the distribution of

$$T = \frac{1}{(1-\rho^2)} \left\{ \left(\frac{X-\mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{X-\mu_1}{\sigma_1} \right) \left(\frac{Y-\mu_2}{\sigma_2} \right) + \left(\frac{Y-\mu_2}{\sigma_2} \right)^2 \right\}.$$

Also find a function of T which is uniformly distributed over (0,1).

[6+8+6 = 20]

5. (a) Using probabilistic arguments show that

$$\lim_{n \to \infty} \int_0^n \frac{1}{\Gamma(n)} e^{-y} y^{n-1} \, dy = \frac{1}{2}.$$

   (b) Derive the large sample variance of the sample coefficient of variation stating your assumptions carefully. Consider in particular the case of a normal population and indicate the simplifications, if any, in your results.

[8+12=20]

6. Derive the asymptotic distribution of Pearsonian chi-square. State all auxiliary results carefully.

[20]

Contd.....

7.(a) Let $\{X_n\}$ be a sequence of independent random variables with

$$P[X_n = 2^n] = P[X_n = -2^n] = \frac{1}{2},$$

for each n. Examine whether or not the sequence satisfies the weak law of large numbers.

(b) Let $\{X_n\}$ be a sequence of random variables and c_n be a sequence of scalars such that

$$X_n \xrightarrow{P} 0 \qquad \text{as } n \to \infty.$$
$$\text{and } c_n \longrightarrow C$$

Without using any auxiliary result, prove from first principles that $X_n - c_n \xrightarrow{P} 0$.

(c) Suppose a sequence X_n of random variables converges to a random variable X in distribution. Prove that $X_n = O_p(1)$.

[6+7+7 = 20]

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STATISTICAL METHODS 3

SEMESTRAL EXAMINATION

Date:23 November 1989 Maximum Marks:100 Time:3 Hours

Figures in brackets [ ] indicate marks allotted to the questions.

1. Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Let $c > 0$ such that

$$\Phi\left(\frac{1}{c}\right) - \Phi\left(-\frac{1}{c}\right) = \alpha.$$

Then show that $(0, c\,|X|)$ is a confidence interval for σ with confidence coefficient <u>*at least*</u> $1 - \alpha$.

[Hint: For what value of b is $\Phi(a - b) - \Phi(-a - b)$ largest?] [10]

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the *uniform* distribution on (θ_1, θ_2).

   (a) Derive the maximum likelihood estimators of θ_1, θ_2. [15]

   (b) Suppose $\theta_1 = -\theta$, and $\theta_2 = \theta$, $\theta > 0$. What is the maximum likelihood estimator of θ? Give reasons. [5]

3. It is desired to estimate the average daily number of traffic accidents (λ) in the city of Calcutta. It is known that the average daily number of traffic accidents in DUNLOP BRIDGE crossing is 10λ while that in a little-known intersection in BEHALA is 0.1λ. Consider the following alternative procedures for this:

   Procedure I: Randomly select n calendar days and observe $X_1, X_2, \ldots, X_n$, where X_i is the number of traffic accidents on the i^{th} calendar day in the DUNLOP BRIDGE crossing.

   Procedure II: Randomly select n calendar days and observe $Y_1, Y_2, \ldots, Y_n$, where Y_i is the number of traffic accidents on the i^{th} calendar day in that BEHALA intersection.

PLEASE TURN OVER

[10+8=18]

p.t.o.

The number n is the same in the two procedures. Explain, formulating the problem suitably, stating your assumptions and deriving the formulae used, which of the two procedures fares better. [25]

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from $\mathcal{N}(\mu, 1)$. Consider the hypothesis $H_0 : \mu = 0$ against $H_1 : \mu > 0$. Compute the power of a *Uniformly Most Powerful* test when $n = 25$ for $\mu = 1$. [15]

5. An experiment was conducted on pigs to examine the differences in the effects of *Limestone* and *Bonemeal* as feeds. For this, 8 pairs of pigs were used, each pair chosen from the same litter with approximately equal body weights. One member of the pair (chosen at random) was fed *Limestone* and another *Bonemeal*. The ash content in percentage of scapulas was the response measured. Data are given below.

Ash Content in % of scapulas of pairs of pigs fed on *Limestone* and *Bonemeal*

Pair	*Limestone*	*Bonemeal*
1	49.2	51.5
2	53.3	54.9
3	50.6	50.2
4	52.0	53.3
5	46.8	51.6
6	50.5	54.1
7	52.1	54.2
8	53.0	53.3

Examine the significance of the difference between mean effects of *Limestone* and *Bonemeal*. State clearly your assumptions. [20]

6. Write a short note (*not more than a page*) on any *one* of the following: Scoring Method; Likelihood Ratio Test; Goodness of Fit Tests. [10]

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CALCULUS - III

SEMESTRAL-I EXAMINATION

Date : 20.11.1989   Maximum Marks : 100   Time : 3 Hours.

Note : This paper carries 110 marks. You may answer all the questions. But the maximum you can score is 100.

1.(a) Let f be a real-valued function defined on $\mathbb{R}^n$, and let $\underset{\sim}{a} \in \mathbb{R}^n$. When do you say that f is differentiable at $\underset{\sim}{a}$?

(b) Let $f(x,y) = \sqrt{|xy|}$, $(x,y) \in \mathbb{R}^2$.
Show that f is not differentiable at $(0,0)$.

(c) If $f(x,y) = \int_{xy}^{x^2+y} \frac{\sin(t+y)}{t^2+y^2}\, dt$, show that f is differentiable everywhere and find $df(x,y)$. [5+7+8 = 20]

2. Prove that any three surfaces of the family of surfaces

$$\frac{xy}{z} = c_1 , \quad \sqrt{x^2+z^2} + \sqrt{y^2+z^2} = c_2 ,$$

$$\sqrt{x^2+z^2} - \sqrt{y^2+z^2} = c_3$$

that pass through a single point are orthogonal to one another. [10]

3.(a) Find the maximum volume of a rectangular parcel with side lengths x,y,z, $x \le y \le z$ such that $2(x+y)+z \le 100$.

(b) Find the maximum of $x_1^2 x_2^2 \ldots x_n^2$ under the restriction $x_1^2 + \ldots + x_n^2 = 1$. Hence show that

$$(x_1 x_2 \ldots x_n)^{1/n} \le \frac{x_1+\ldots+x_n}{n} \quad \text{if } x_i > 0,\ i=1,\ldots,n.$$

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[10+10=20]

4.(a) By transforming to polar co-ordinates evaluate the integral

$$\int_0^{a \sin\beta} \left(\int_{y\cot\beta}^{\sqrt{a^2-y^2}} \log(x^2+y^2)\,dx \right) dy , \quad 0 < \beta < \pi/2.$$

(b) Change the order of integration in the above integral.

[10+8=18]

p.t.o.

5.(a) Use Green's theorem to evaluate the line integral

$$\oint_C (x^2-y^2)dx + 2xy\ dy$$

where C is the boundary of the square $\{(x,y): |x|\leq 1, |y|\leq 1\}$

[10]

(b) Check whether the function

$$\underset{\sim}{f}(x,y) = (2xe^y+y,\ x^2e^y + x - 2y)$$

is a gradient. If $\underset{\sim}{f}$ is a gradient find a potential function of $\underset{\sim}{f}$. [10]

6.(a) Find the area of the region cut from the plane $x+y+z=a$ by the cylinder $x^2+y^2=a^2$.

(b) A real-valued function ϕ defined on $\mathbb{R}^3$, which is never zero, has the properties

$$\|\nabla\phi\|^2 = 4\phi,\ \text{div}\ (\phi\ \nabla\phi) = 10\phi.$$

Evaluate the surface integral

$$\iint_S \frac{\partial\phi}{\partial n}\, dS$$

where S is the surface of the unit sphere with centre at the origin and $\frac{\partial\phi}{\partial n} = \nabla\phi \cdot \underset{\sim}{n}$ where $\underset{\sim}{n}$ is the unit outer normal to S. [10+12=22]