INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) II Year : 1990-91

Economics II

Semestral-II Examination

Date : 8.5.1991   Maximum Marks : 100 Time: 3 Hours.

Note : Answer any four questions. Marks allotted
to each question are shown in brackets.

1.(a) In a simple Keynesian model where investment is
autonomously given and consumption is propor-
tional to income, derive the balanced budget multi-
plier and interpret the result in terms of the
multiplier process.

(b) Instead of assuming a uniform saving ratio for the
economy as a whole, assume that there are two classes
of income earners, workers and capitalists, with saving
ratios s_w and s_c respectively where $s_w < s_c$. Suppose
there is an increase in government expenditure financed
by additional taxes, but the additional taxes are
imposed only on the capitalists. What would be the
value of the multiplier ? Explain your result intui-
tively.

(For simplicity, you may assume that workers are not
taxed at all and taxes on profits are lumpsum).

(12+13)

2. Suppose an economy imports oil from abroad. Analyse the
effect of an increase in the world price of oil on
output and employment of the domestic economy under fixed
and flexible exchange rates and capital mobility.

(25)

3. Consider a macroeconomic model of aggregate demand and
aggregate supply where the money wage rate adjusts
according to the lag between the actual level of
employment and the full employment level. Assuming that
prices are determined by a mark-up over costs, analyse
the effects of

(a) an increase in the price of an imported input;

(b) an increase in the rate of mark-up.

(13+12)

4. (a) Following Domar, derive the rate of growth of inves ment which guarantees full-capacity utilization. If the actual rate of growth is less than this equilib rate, find out an expression for the rate of capaci utilization in the long run.

   (b) In a one sector neo-classical growth model, analyse the path of adjustment if starting from an initial steady state equilibrium, half of the capital stock in the economy is suddenly destroyed due to an earthquake. (Assume that the labour force remains the same and continues to grow at a constant rate).

   (13+12)

5. Consider a simple multiplier-accelerator model. Assume values of various parameters are such as to produce either explosive cycles or steady divergence away from equilibrium income, (the latter is growing over time a the same rate, say g, at which autonomous expenditure is growing).

   Demonstrate how Hicksian trade cycles can emerge and persist in this economy. Mention some major limit of such a trade cycle model.

6. Consider an economy described by the following system linear equations :

   (i) $y_t = \alpha x_t - \beta p_t$ (aggregate demand)

   (ii) $y_t = Y + \lambda(p_t - p_t^*) + u_t$ (aggregate supply)

   (iii) $x_t = a p_{t-1} + b y_{t-1} + v_t$ (policy rule)

   where y_t = real income or output in period t, p_t = (actu price level in period t, p_t^* = expected price in period (i.e., prices expected to prevail in period t) Y = nat rate of output (assumed to be constant over time), x_t the level of government policy instrument (e.g., money supply government expenditures etc.), u_t and v_t are each a random disturbance in period t, (each of the series $\{u_t\}$ and $\{v_t\}$ are identically and independently distributed with $E(u_t) = 0$ and $E(v_t) = 0$ for all t).

   (a) What is meant by 'rational expectation' of prices

   (b) Show that in this economy no policy rule can have systematic effect on the deviation of output from 'natural' level, if expectations are formed 'ratio ally'.

6.(c) Instead of 'rational expectations' consider "extrapolative"
expectations given below :

$$p_t^* = p_{t-1} + \varepsilon \, (p_{t-1} - p_{t-2}),$$

where ε is a positive constant.

Show that with such expectations government
policy can systematically alter the level of
output.

[5+13+7]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year : 1990-91
Elements of Algebraic Structure
Semestral-II Backpaper Examination

Date : 26.6.1991   Maximum Marks : 100 Time: 3 Hours.

Note : Answer all questions. The paper carries
100 marks.

1.(a) Prove that every group of order n is isomorphic
to a subgroup of S_n.   [12]

 (b) Show that the converse of Lagrange's theorem is
false for the alternating group A_4.
                                                [7]

2.(a) Define an ideal I in a ring R and the quotient
ring R/I giving necessary proof.   [10]

 (b) Show that every ideal is isomorphic to the
kernel of some homomorphism and conversely.
                                                [12]

3.(a) Prove the division algorithm in F[x].
                                                [10]

 (b) Starting from (a), prove that any two non-zero
polynomials $f(x)$ and $g(x)$ in F[x] have a g.c.d. and
that the g.c.d. can be written as $f(x)u(x)+g(x)v(x)$
for some $u(x)$ and $v(x)$ in F[x]. Do not assume
any result on Euclidean rings.   [15]

4. Prove that an element a in an extension K of F is
algebraic over F iff F(a) is a finite extension of F.
                                                [22]

5. Find an irreducible polynomial of degree 3 (give proof)
over GF(2) and construct a field of order 8.
                                                [12]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year : 1990-91
SQC and OR
Semestral-II Examination

Date : 10.5.1991   Maximum Marks : 100 Time: 3 Hours.
Group A   Max. marks : 50
Note :   Answer both the questions.

1.(a) Define (any three)
Producer's Risk, LTPD, AOQ, ATI
  (b) Draw an O.C curve for the single sampling plan (200,2).
                              [(5x3)+10=25]

2.(a) Define and derive the ARL of an $\bar{X}$-chart.
  (b) The following data were obtained over a period to

initiate $\bar{X}$-R control charts for a quality characteris-
tic of a certain manufacturing product. All the figures
apply to products made on a single machine by a single
operation. The subgroup size was 5.

Subgroup number	$\bar{X}$	R	Subgroup number	$\bar{X}$	R
1	177.6	23	11	179.8	9
2	176.6	8	12	176.4	8
3	178.4	22	13	178.4	7
4	176.6	12	14	178.2	4
5	177.0	7	15	180.6	6
6	179.4	8	16	179.6	6
7	178.6	15	17	177.8	10
8	179.6	6	18	178.4	9
9	178.8	7	19	181.6	7
10	178.2	12	20	177.6	10

What will be the ARL of the $\bar{X}$-chart to detect a
shift of 1.5σ in the process average.

Group B   Max. marks : 50

Note : Answer as many questions as you can. You can
answer part questions also.

1.(a) State the general linear programming problem.
  (b) A T.V. company produces two models A and B. The sale
   volume of A is at least 60% of the total sales of the
   two models. Both use a special component where availa-
   bility is 100 per day. A uses 2 per unit, and B uses

p.t.o.

4 per unit of this special component. The sale
prices are Rs.10,000 and Rs.15,000 for A and B
respectively. Formulate the problem and deter-
mine graphically the optimal allocation of the
component to the two products.   [2+10]

2.(a) Define the following terms : Basic solution,
Feasible solution, Basic feasible solution and
optimal solution in respect of an LP problem.

(b) Given a set of m simultaneous linear equations
in 'n unknown (n$\geq$m), $Ax=b$ with $r(A)=m$, if there is a
feasible solution $x^2 \geq 0$ then prove that there
exists basic feasible solution.

(c) Reduce $2a_1+4a_2+a_3=b$; $a_1 = [2,1]$, $a_2= [-1,4]$

$a_3=[2,0]$ to a basic feasible solution.
     [2+8+8]

3. Smaller paper rolls are to be cut from the parent roll in
four sizes. Size I covers 20%, size II 45%,
size III 15% and size IV 30% of the parent roll width.
There are several cutting procedures. For example, 5 rolls
of size I can be cut from the parent roll with no waste.
Similarly 3 rolls of size IV with 10% left over which is
a waste.

It is desired to get 420 rolls of size I, 318 rolls
of size II, 510 rolls of size III and 48 rolls of size IV.
Formulate the problem as a Linear Programming problem if
the objective lists minimise waste.

4. Solve the following LP problem using simplex method.
     Maximise $2x_1+x_2+3x_3+5x_4$

     Subject to $x_1+7x_2+3x_3+7x_4 \leq 46$

     $3x_1-x_2 +x_3 +2x_4 \leq 8$
     $2x_1+3x_2-x_3 +x_4 \leq 10$ [10]

5.(a) Explain the terms : Lead time, reorder level,
Inventory carrying cost and Order cost.

(b) A retail shop stores an item whose annual demand
is 600 items. Demand is uniform. His order cost and
inventory carrying costs are Rs.10 per order and
20% per annum. The price per unit is Rs.5. Find
the EOQ for this item .   [12+8]

INDIAN STATISTICAL INSTITUTE

3.Stat.(Hons.) II Year : 1990-91

Differential Equation

Semestral-II Examination

Date : 8.5.1991   Maximum Marks : 75 Time: 3 Hours.

Note : 1. Numbers in the brackets on the right
margin indicate maximum credits for the
corresponding problems.

2. Arguments must be precise and complete

3. The prime operations ', ", etc. have
usual meanings as in the theory of
ordinary differential equations.

1. For the Chebyshev equation $(1-x^2)y" - xy' + \alpha^2 y = 0$

where α is a constant,

(a) Compute two linearly independent series solutions for
$|x| < 1$ (8)

(b) Show that for every non-negative integer $\alpha = n$, there
is a polynomial solution of degree n.   (4)

2. J_0 is the Bessel function of zero order

(a) Show that there exists an interval $0 < x < a$ for some
$a > 0$ such that for any x in this interval, $J_0(x) \neq 0$.
(2)

(b) Let $0 < x_0 < a$ where a is as obtained in part (a).
Show that there is a second solution ϕ_2 of the
Bessel equation of order zero which has the form

$$\phi_2(x) = J_0(x) \int_{x_0}^{x} \left[\frac{1}{t J_0^2(t)} \right] dt \quad (0 < x < a).$$
(5)

(c) Show that J_0 and ϕ_2 are linearly independent
on $0 < x < a$.   (3)

3. Let J_α be the Bessel function of order α of the
first kind $(R\alpha \geq 0)$. Use the formula for $J_\alpha(x)$ to prove
that

(a) $(x^\alpha J_\alpha)'(x) = x_\alpha J_{\alpha-1}(x)$ (3)

(b) $(x^{-\alpha} J_\alpha)'(x) = x^{-\alpha} J_{\alpha+1}(x)$ (3)

(c) $J_{\alpha-1}(x) - J_{\alpha+1}(x) = 2J_\alpha'(x)$ (2)

(d) $J_{\alpha-1}(x) + J_{\alpha+1}(x) = 2\alpha x^{-1} J_\alpha(x)$ (2)

4. Consider the equation $M(x,y)dx + N(x,y)dy = 0 \ldots(i)$
where M,N have continuous first partial derivatives
on some rectangle R.

p.t.o.

1.(a) Prove that a function u on R having continuous
first partial derivatives, is an integrating factor
of (i) if and only if $u(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}) = N \frac{\partial u}{\partial x} - M \frac{\partial u}{\partial y}$
on R. (4)

   (b) Show that if (i) has an integrating factor u which is a
function of y alone, then $q = \frac{1}{M} (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})$ is a

   continuous function of Y alone. You may assume
that the derivative of u with respect to y is
continuous. (3)

   (c) If q is continuous and independent of x, show that
an integrating factor u is given by $u(y) = e^{Q(y)}$ where
Q is any function such that $Q' = q$. (3)

5. Let f be a real-valued continuous function defined
on the strip S : $|x| \leq a$, $|y| < \infty$, (a>0) and let I
denote the interval $|x| \leq a$. Suppose ϕ is a real-
valued function on I.

   (a) Define what it would mean to say that ϕ is a solution
on I of the initial value problem
$y'' + \lambda^2 y = f(x,y)$, $y(0)=0$, $y'(0) = 1$ $(\lambda>0)\ldots.(*)$. (2)

   (b) Show that ψ is a solution of (*) on I if and
only if ϕ is a solution of the integral
equation
$$y = \frac{\sin \lambda x}{\lambda} + \int_0^x \frac{\sin \lambda(x-t)}{\lambda} f(t,y)dt \text{ on I.} \quad (8)$$

6. On the square R : $|x| \leq 1$, $|y| \leq 1$; let f be
defined by
$f(x,y) = 0$ if $x=0$, $|y| \leq 1$
$\qquad = 2x$, if $0 < |x| \leq 1$, $-1 \leq y < 0$
$\qquad = 2x - \frac{4y}{x}$ if $0 < |x| \leq 1$, $0 \leq y \leq x^2$
$\qquad = -2x$ if $0 < |x| \leq 1$, $x^2 \leq y \leq 1$
Note that f is continuous on R. Now
   (a) Show that $|f(x,y)| \leq 2$ on R. (2)
   (b) Show that f does not satisfy a Lipschitz condition on R. (2)

6.(c) Show that the successive approximations

ϕ_0, ϕ_1, ϕ_2, ... for the problem $y' = f(x,y)$, $y(0)=0$, satisfy

$$\psi_0(x) = 0, \quad \phi_{2m-1}(x) = x^2, \quad \psi_{2m}(x) = x^2 \quad (m=1,2, ...).$$

(3)

(d) Prove that neither of the convergent subsequences in (c) converges to a solution of the initial value problem.

(2)

(e) Does the initial value problem of (c) have a solution ? If yes, find such a solution.

(4)

7. Let f be a continuous function for (x,y,λ) in

R : $|x-x_0| \leq a$, $|y-y_0| \leq b$, $|\lambda-\lambda_0| \leq c$

where $a,b,c > 0$ and suppose there is a constant $K > 0$ such that $|f(x,y_1,\lambda) - f(x,y_2,\lambda)| \leq K |y_1-y_2|$ for all (x,y_1,λ), (x,y_2,λ) in R. Further suppose that $\frac{\partial f}{\partial \lambda}$ exists and there is a constant $L > 0$ such that $|\frac{\partial f}{\partial \lambda} (x,y,\lambda)| \leq L$ for all (x,y,λ) in R.

If ϕ_λ represents the solution of $Y' = f(x,y,\lambda)$, $y(x_0)=Y_0$ show that

$$|\phi_\lambda(x) - \phi_\mu(x)| \leq \frac{L|\lambda-\mu|}{K} (e^{K|x-x_0|} -1)$$

for all x for which ϕ_λ, ϕ_μ exist.

(10)

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II and III Year : 1990-91

Anthropology

Semestral-II Examination

Date : 8.5.1991   Maximum Marks : 100 Time: 3 Hours.

Note : Answer any five Questions. The questions
carry equal marks.

1. How do you define human biology ? How does human
   biology differ from classical physical anthropo-
   logy ? What are their areas of overlapping ?

   [20]

2. Bring out the differences between the Darwinian
   and Lamarckian theories of organic evolution. What is
   Neo-Lamarckism ?

   [20]

3. What are the major stresses on man at high altitude ?
   What are their major effects on human biological traits ?

   [20]

4. How do you distinguish between Neandertal man and Modern
   man ? Give a brief account of the cultural life of
   Neandertal man.

   [20]

5. If an XXY individual were to produce sperm cells,
   what kinds would he produce with regard to sex chromosome
   content ? If he married a normal woman, and no non-
   disjunction were involved in the production of her
   ova, what types of offspring would be expected ?

   [20]

6. What blood types will the children belong to when the
   father is type M and the mother is type N ? When the
   father is type MN, the mother type N ? When the father
   is type M, mother type MN ?

   [20]

7. A certain group of 300 people is constituted as follows :
   150 AA, 120 Aa, 30 aa. Do these people form a panmictic
   population at equilibrium ? If not, what number of indi-
   viduals would have been expected to have had each of the
   3 genotypes if the population had been in equilibrium ?

   [20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year : 1990-91
Economics and Official Statistics
Semestral-II Examination

Date : 6.5.1991   Maximum Marks : 100 Time: 3 Hours.

Note : Answer Question 1 and any two from the rest.

1. Consider a production function with two inputs K and L.
   (a) Define the 'Marginal Rate of Technical Substitution'
       (MRTS) between K and L. Show that the MRTS is equal
       to the slope of the isoquant and also equal to the
       ratio of input prices in equilibrium.
                                                    (3+4)

   (b) Define the elasticity of substitution for the produc-
       tion function. Draw the isoquants corresponding to the
       cases where the elasticity of substitution between
       inputs is (i) zero and (ii) infinity.
                                                    (6)

   (c) Obtain the elasticity of substitution for the CES
       production function.   (5)

   (d) Prove or disprove the following :
       The Cobb-Douglas production function must show
       increasing returns to scale for a determinate
       solution of the problem of profit maximisation
       under perfect competition.   (7)
                                                    [25]

2. (a) Assuming a multiplicative model for the time series,
       describe the ratio-to-moving average method for deter-
       mining moving seasonal indices from a series of monthly
       figures when the seasonal patterns are changing
       over the years.   (18)

   (b) The table below shows wholesale prices and quantities
       produced of selected foodgrain items in India. Here
       p = price in Rs. per maund, and q = quantity produced
       in thousand tons. Compute price and quantity index
       numbers for the year 1952 taking 1951 as base (=100),
       and also for the year 1951 taking 1952 as base (=100),
       using Laspeyres' and Fisher's formulae.

                                                p.t.o.

2.(b) contd.

Item	1951		1952	
	P.	Q.	P.	Q.
Rice	16.87	20,964	17.50	22,537
Wheat	18.60	6,085	23.67	7,382
Jowar	10.09	5,981	11.93	7,243
Bajra	10.07	2,309	13.33	3,142

(12)
[30]

3.(a) Define Lorenz curve and discuss its properties.
(10)

(b) Obtain the Lorenz ratio for Lognormal distribution.
(10)

(c) Describe the usual method of fitting a Pareto
distribution to an empirical income distribution
available in a grouped form. (10)
[30]

4.(a) Show that in a regression model

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon$$

with near-exact multicollinearity, variance of the
ordinary least squares estimator β_j tends to ∞ as
$R^2_{j.12\ldots j-1,j+1,\ldots k}$ approaches 1, where

$R^2_{j.12\ldots j-1,j+1,\ldots k}$ is the coefficient of determination
of the linear regression of X_j on the other regressors.
(10)

(b) The following table shows the average percapita consump-
tion of cereals for households in different levels of
percapita income according to a household budget enquiry.

Per capita monthly income (Rs.)	Percentage of population	Average income (Rs.)	monthly percapita consumption of cereals (Rs.)
0 – 50	8	39	19
51 –100	12	72	30
101 –150	25	123	41
151 –200	18	171	49
201 –250	16	221	57
251 –300	11	270	64
301 –	10	380	71

Estimate the constants of a semi-logarithmic Engel curve
for cereals by the weighted least squares method, using
the percentage of population as weights. (20) [30]

5. Practical Exercise
[15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year : 1990-91
Elements of Algebraic Structures
Semestral-II Examination

Date : 3.5.1991   Maximum Marks : 100 Time: 3 Hours

Note : Answer all questions. The whole paper carries
111 marks but the maximum you can score is
100. Marks allotted to questions are shown
in square brackets.

1. Determine all the groups of order $5^2 \times 7^2$. You may use any
standard theorem after quoting it but give the proof of all
other steps.   [15]

2. In a Euclidean ring R, prove that the ideal generated by an
element a is maximal iff a is a prime element of R.
                                                         [15]

3. Consider the element $a = \sqrt{2} + \sqrt{3}$ of the extension $\mathbb{R}$ of
the field Q.
   (i) Find a monic polynomial $f(x)$ of degree 4 over Q such
       that $f(a) = 0$.   [10]
   (ii) Prove that the polynomial f obtained in (i) is the
        minimal polynomial of a. (Hint: find the roots of f.)
                                                         [10]

4.(a) How would you prove that given any polynomial $f(x)$ over
      a field F with degree ≥ 1, there exists an extension of
      F containing a root of $f(x)$. Give the method omitting
      the proof.   [6]

   (b) If $f(x) \in F[x]$ is such that $(f, f') = 1$, prove that f
       cannot have a repeated root in any extension of F.
                                                         [10]

   (c) Assuming the result that every polynomial over F has a
       splitting field over F, prove that for every prime
       number p and every positive integer n, there exists a
       field of order p^n. (You may use (b) but prove all
       intermediate results you want to use.).   [20]

5.(a) Prove that the multiplicative group of any finite field
      is cyclic.
                                                         [15]

   (b) If c is an integer not divisible by the odd prime p,
       prove that there exists an integer x such that
       $x^2 \equiv c \pmod{p}$ iff $c^{(p-1)/2} \equiv 1 \pmod{p}$.   [10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year : 1990-91
Demography
Semestral-II Examination

Date : 2.5.1991   Maximum Marks : 100 Time: 3 Hours.

Note : Attempt question 10 and any FIVE questions
from the rest.

1. Describe the method of enumeration of individuals in the
   population census of India 1991 with special reference to
   the procedures adopted for enumerating (a) normally resi-
   dent members of the house hold absent on the day of enu-
   meration and (b) visitors.   [16]

2. What are the main sources of demographic data ? Discuss the
   pitfalls of birth and death registration system in India.
                                                        [16]

3. Explain briefly the need for evaluation and adjustment of
   basic demographic data. Discuss any one method for evalua-
   ting age data given by single years of age. [16]

4. Which is the best measure to forecast population growth ?
   Discuss.   [16]

5. What do you mean by infant mortality rate ? How does it
   differ from infant death rate ? Describe the different
   methods of computing the infact mortality rate. Comment on
   their merits and demerits.   [16]

6. What do you mean by a life table ? For a certain life table
   $$\ell_x = 20900 - 80x - x^2$$
   (i) What is the ultimate age in the table ?
   (ii) Find μ_x, q_x and $_{10}p_{20}$.   [16]

7. Define total fertility rate. Find the mathematical rela-
   tionship between crude birth rate and total fertility rate.
   Interpret the situations : (i) NRR=1, (ii) NRR>1 and
   (iii) NRR < 1.   [16]

8.(a) What is the difference between 'mean age at marriage'
   and 'singulate mean age at marriage' ? Discuss a pro-
   cedure for estimating the singulate mean age at marriage,
   if it is known that 5 per cent women never marry at all.
   (b) Describe a method of construction of the net nuptiality
   tables.   [16]

9. What do you mean by standardisation of vital rates ?
Discuss methods for direct as well as indirect standardi-
sation of death rates. The CDR for country A is 37 per
thousand population and that for country B is 40 per
thousand population :
Can we say that mortality in country B is higher than in
country A ? If not, then what have we to do for comparison
of mortality between country A and country B ?

[16]

10. Write short notes (any four)
    (a) Chandrasekaran-Deming Formula
    (b) UNSABR
    (c) Nuptiality rates
    (d) Comparative Mortality Index
    (e) Cohort and current life tables.

[20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) II Year : 1990-91
Statistical Methods IV
Semestral-II Examination

Date : 29.4.1991   Maximum Marks : 100 Time: 3 Hours.

Note : Clearly state results/assumptions/regulatory
conditions that you use to answer questions.
The notations have their usual meanings. .

Answer one question from each group.

Group A : Answer one question

1.(a) Let (X_1, Y_1), $i = 1,2,\ldots$, n be iid bivariate normal with
(population) correlation co-efficient ρ. Find the
distribution of the sample correlation coefficient in a
form suitable for testing the hypothesis that $\rho = 0$.

(b) The following correlation matrix was reported in 1991 in
connection with a Differential Impact Study of rural
households in some West Bengal villages.

   n = number of households = 801
   X_1= % area under High Yielding Variety
   X_2= % area irrigated
   X_3= % of rice area suitable for High Yielding
      Variety

   r_{12}= 0.5516, r_{13}= 0.1230, r_{23}= 0.1627.

Test, at 5% level, the significance of the correlation
co-efficient between X_1 and X_3 given X_2.

$[15+10 = 25]$

2.(a) Let (X_1, Y_1), $i = 1,2,\ldots$,n be iid bivariate normal.
Find a $(1-\alpha)$ 100% confidence interval for the regre-
ssion coefficient β of the regression of Y on X.

(b) The following statistics were computed, on the basis
of a sample of size 34, for two variables X and Y.

$\bar{X} = 58.3$, $\bar{Y} = 63.4$, $S_{XX} = 34(7.2)^2$
$S_{YY} = 34(5.25)^2$, $r = 0.243$

Find a 95% confidence interval for β.

$[15+10 = 25]$

p.t.o.

3. State and prove Fisher-Cochran theorem on the distribution of quadratic forms. [25]

4. Let $Y' = (Y_1 \ Y_2 \ ... \ Y_n)$ be with iid $N(0,1)$ components, and $Q_1 = Y'A_1Y$, $Q_2 = Y'A_2Y$, and $Q = Y'AY$.

   (a) If $Q = Q_1 + Q_2$, Q is $\chi^2_{(a)}$, Q_1 is $\chi^2_{(a_1)}$, and Q_2 is non-negative, then show that Q_2 also has a Chi-square distribution.

   (b) If Q_1 is $\chi^2_{(a_1)}$ and Q_2 is $\chi^2_{(a_2)}$, then show that a necessary and sufficient condition that they are independent is $A_1A_2 = 0$. . [18+ 7=25]

5. Let $U_1, U_2, ..., U_n$ be iid $N_p(\mu, \Sigma)$, $\Sigma > 0$, Partition U_i as $U_i' = (X_i' \ Y_i')$.

   (a) Show that, when the population covariance matrix between X_i and Y_i is null, $S_{YY} - S_{YX}S_{XX}^{-1}S_{XY}$ and $S_{YX}S_{XX}^{-1}S_{XY}$ have independent Wishart distributions.

   (b) Hence or otherwise find the distribution of $\dfrac{\ell'\Sigma^{-1}\ell}{\ell'S_{UU}^{-1}\ell}$ for any fixed non-null vector ℓ. [15+10= 25]

6. Let each of r populations be classified into s groups with π_{ij} being the probability of classification into jth group of an observation from the ith population.

   (a) Derive the conditional (exact) distribution of cell frequencies given the marginal totals under the hypothesis of homogeneity, $\pi_{ij} = \pi_j$, $j = 1,2 \ ... \ s$, for all i.

   (b) Suggest a test for homogeneity when π_{ij}'s are known parametric functions $\pi_{ij}(\Theta)$, Θ may be vector valued. [12+13 = 25]

7. (a) Give the essential steps for finding the asymptotic distribution of sample pth fractile.

7.(b) Following is a frequency distribution of % of family income spent on food.

Class	Frequency
28.5 – 30.5	8
30.5 – 32.5	10
32-5 – 34.5	28
34.5 – 36.5	32
36.5 – 38.5	28
38.5 – 40.5	28
40.5 – 42.5	22
42.5 – 44.5	19
44.5 – 46.5	13
46.5 – 48.5	6
48.5 – 50.5	6

Find a 95% confidence interval for the first quartile. (You may use a consistent estimator of the population variance.)

[10+15=25]

8. Let the sample size be large.

(a) What are variance stabilizing transformations ?
(b) Find a suitable variance stabilizing transformation for testing $\rho = \rho_0$ in a bivariate normal set-up.
(c) Find a suitable variance stabilizing transformation for testing about the Bernoulli parameter.
(d) Find a 95% confidence interval for the population correlation coefficient between X_1 and X_2 of problem 1(b) of group A.

[2+6+6+11 = 25]

Date:2.1.91   Maximum Marks:88(Theo.)   Time:$3\frac{1}{2}$ Hours.

Note: Answer all questions.

1. Give a comparative account of striated, cardiac and non-striated
   muscles.   [20]

                              OR

   What are the main characteristics of different types of
   Epithelial tissue ?   [20]

2.(a) Define a hormone-receptor. How does it affect a macromolecule
      after getting attached to a hormone ?   [3+5=8]

  (b) Distinguish (diagrammatically) between the receptor-mediated
      function of a peptide hormone vs. a steroid hormone. Give
      two examples of each kind of hormones: [10+2=12]

                              OR

  (c) Write short notes on any THREE

       (i)   Hormones and growth factors;
       (ii) Agonist and antagonist;
       (iii) Cytokinin;
       (iv) Gibberellins and Ethylene,
       (v)   Neurotransmitters and Pheromones.   [4x3=12]

3. Give the botanical names and the families of the following
   plants (any FIVE):
   Rice, Barley, Coconut, Jack-fruit tree, Gram, Banana, Wheat. [2x5=10]

4. Write short notes on the following (any THREE):
   a) Radioactive pollution;
   b) Ecology and ecosystem;
   c) Kreb cycle reactions;
   d) Calvin-Bassham cycle;
   e) Decomposers and ectocrine substances;
   f) Food chain and ecological pyramid.   [6x3=18]

5. What is plant secretion? Describe briefly the various secretory
   structures found in plants.   [3+17=20]

                              OR

   What do you understand by the term "primary anomalous structures"?
   Describe briefly the common types of primary anomalies found in
   the stems of angiosperms.   [3+17=20]

Date:2.1.91   Maximum Marks:100   Time:3 Hours.

Note: Answer all questions. Marks assigned to
questions are given in parentheses.

1.(a) Let $f = (f_1, \ldots, f_n)$ be a continuously differentiable vector field defined on an open convex set S in $\mathbb{R}^n$. Prove that f is a __gradient__ on S if and only if

$$D_k f_j(x) = D_j f_k(x)$$

for each x in S and all $k, j = 1, 2, \ldots, n$.   [15]

(b) Find a potential function for the vector field defined on $\mathbb{R}^3$ by

$$f(x,y,z) = (2xyz + z^2 - 2y^2 + 1)\ i + (x^2 z - 4xy)\ j + (x^2 y + 2xz - 2)k.   [5]$$

2.(a) Consider a plane region Q bying between the graphs of two continuous functions f and g over an interval $[a,b]$, where $0 \leq g \leq f$. Let S be the solid of revolution generated by rotating Q about the x-axis. If $A(Q)$ denotes the area of Q, $V(S)$ the volume of S and $\bar{y}$ the y-coordinate of the centroid of Q, prove that

$$V(S) = 2\pi\ \bar{y}\ A(Q).   [12]$$

(b) Use (a) to find the volume of a torus and the __centroid__ of a semicircular disc.   [4+4]

3.(a) Evaluate the double integral

$$I(p,r) = \iint_R \frac{dxdy}{(p^2 + x^2 + y^2)^p}$$

over the circular disc $R = \left\{(x,y): x^2 + y^2 \leq r^2\right\}$.
Determine those values of p for which $I(p,r)$ tends to a limit as $r \to \infty$.   [8+4]

(b) Establish the following equation by introducing a suitable change of variables:

$$\iint_S f(xy)\ dxdy = \log 2 \int_1^2 f(u)\ d_u$$

where S is the region in the first quadrant bounded by the curves $xy=1$, $xy=2$, $y=x$, $y=4x$.   [3]

4.(a) Find the volume of the solid bounded by the xy-plane, the
 cylinder $x^2+y^2=2x$ and the cone $z = \sqrt{x^2+y^2}.$ [10]

(b) Compute the area of that portion of the conical surface
 $x^2+y^2=z^2$ which his above the xy-plane and is cut off by the
 sphere $x^2+y^2+z^2=2ax.$ [10]

5. Let S be a closed surface with n the unit outer normal. S bounds
 a solid V of the type discussed in the divergence theorem. If
 f,g are scalar fields, denote by $\frac{\partial f}{\partial n}$, $\frac{\partial g}{\partial n}$ their directional
 derivatives in the direction n. Prove, assuming continuity of
 of all the derivatives involved, that

(a) $\iint\limits_{S} \frac{\partial f}{\partial n} \, dS = \iiint\limits_{V} \nabla^2 f \, dxdydz$

(b) $\iint\limits_{S} f \frac{\partial g}{\partial n} \, dS = \iiint\limits_{V} (f \, \nabla^2 g + \nabla f \cdot \nabla g) \, dxdydz$

(c) $\iint\limits_{S} f \frac{\partial g}{\partial n} \, dS = \iint\limits_{S} g \frac{\partial f}{\partial n} \, dS$ if both f,g are harmonic in V

(d) $\iint\limits_{S} f \frac{\partial f}{\partial n} \, dS = \iiint\limits_{V} \| \nabla f \|^2 \, dxdydz$ if f is harmonic in V. [20

....

:ss.

Date:2.1.91   Maximum Marks:12(Pract.)   Time: 2 Hours.

1. Identify the following FOUR slides with reasons.   [1x4=4]

2. Identify WBC in the given stained and focused field mounted on the
   microscope.   [4]

3. Practical note backs.   [2+2=4]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1990-91
SEMESTRAL-I EXAMINATION
BIOLOGY-I

Date: 28.11.90   Maximum Marks:The.88+(Prac.12)   Time:3½ Hrs.(The.)

Note: Answer all questions.

1. Define Pollution. Name a few common pollutants of human society.
   What is radioactive pollution? How does it pollute the Environment?
                                                          [2+2+4+2=10]
                                OR
   Define the following (any TWO):   [5+5=10]
   (a) Ecology and Ecosystem;
   (b) Food chain and Ecological Pyramid; ·
   (c) Decomposers and ectocrine substances.

2. Give the botanical names and the families of the following plants:
   (any FIVE)   [2x5=10]
   (a) Paddy ; (b) Cotton; (c) Wheat; (d) Jute;
   (e) Mustard; (f) Onion;   (g) Lentil; (h) Mango;
   (i) Castor.

3. Describe diagrammatically the reactions involved in Kreb's
   cycle. State the cellular site of such reactions. How many ATP
   molecules are produced during the whole process of aerobic
   respiration?   [8+1+1=10]
                                OR

   Discuss Calvin - Bassham cycle in the elucidation of chemical
   conversion of carbondioxide and water to carbohydrates.   [10]

4. Write short notes on the following (any FOUR):   [5x4=20]
   with diagrams
   (a) Adipose tissue;   (b) Reticular tissue;
   (c) Striated muscle fibers;   (d) Different types of Neurone,
   (e) Neuroglia cells;   (f) Schematic representation of the
                                      Haversian system in compact bone.

5.(a) Describe briefly a RBC-counting chamber. Write steps for
      counting total RBC per mm^3.   [2+4=6]

  (b) Write short notes on (any THREE)   [4x3=12]
      (i) Receptor;   (ii) Auxins; (iii) Gibberellins;
      (iv) Steroid hormone receptor-function;
      (v) Agonist and antagonist.

                                OR

Explain (any THREE):   [4x3=12]
   (i) Testosterone acts both as a hormone as well as a
        prohormone.
   (ii) Vitamin D is a hormone
   (iii) Cyclic AMP is a 'second messenger'
   (iv) Ethylene is a hormone
   (v) Epinephrine action is receptor - mediated.

6. What is secondary growth? What are the meristematic tissues
   involved in it? Describe briefly the ontogeny of secondary
   vascular tissues of the stem.   [2+3+15=20]

                              OR

   Write short notes on (any FOUR):   [5x4=20]
   (a) Laticifers;
   (b) Nectaries and hydathodes;
   (c) Tylosis;
   (d) Growth rings;
   (e) Trichomes and glands;
   (f) Polystelic condition of stem.

Date: 27.11.90   Maximum Marks:100   Time: 3 Hours

Note: Attempt any Five questions-at least two
from each Group. Questions carry equal
marks.

GROUP A

1. Discuss the place of sociology in social science and its relationships with social anthropology and social psychology.

2. Consider the following terms: " Community solidarity", " national integration", "social disintegration" and "country's unity". Choose any one of those terms and state how you may define it. Also discuss a method of how you may measure it.

3. What are polygyny and polyandry? How do you account for their occurrence in society? Give your answer with Indian examples.

4. Discuss briefly the substance of culturological approach in academic sociology.

GROUP B

1. A Village Level Worker (VLW) began to work in a Community Development Block (CDB). He obtained data on adoption of a new variety of High-Yielding rice in 5 villages by the farmers in the year of 1989 He observed that rates of adoption varied largely as follows:

Villages (No.of farmers interviewed shown in bracket)				
A.	B.	C.	D.	E.
Percent farmers				
40%	80%	60%	20%	17%
Adopting(160)	(200)	(100)	(75)	(48)
in 1989				

The VLW also noted that the first two villages A and B were situated near the CDB headquarters on the road. He interpreted that proximity to headquarters and road connection leads to adoption. Comment on the interpretation made by VLW.

2. Design a sample survey of a multi-ethnic village or a workers' colony for studying socio-economic condition of its inmates.

3. Choose any two methods of data collection in sociology and discuss their merits and demerits with brief illustration.

4. Write short notes on (any two)
(a) Hypothesis, (b) Sociometric star, (c) Quantitative approach.

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II AND III YEAR:1990-91
SEMESTERAL-I EXAMINATION
GEOLOGY

Date:27.11.90   Maximum Marks: 100   Time: 3 Hours

Note: Attempt Question no. 9 and any five from the
rest. Answers should be brief and to the point.
Draw sketches wherever necessary. Maximum score
for question no. 9 is 20 and for the rest of
each is 16.

1.(a) What are the major divisions of sedimentary rocks? Give examples,
   one from each division.   (4)

 (b) Indicate the parameter for primary classification of mechanically
   deposited sedimentary rocks.   (1)

 (c) What is sandstone? Which mineral generally occurs as the most
   dominant constituent of sandstones? Justify your answer.   (3)

 (d) Indicate the important parameters used for describing the
   texture of a mechanically deposited sedimentary rock.   (4)

 (e) What major differences you would expect between a feldspathic
   sandstone and a granite?   (4)

2.(a) What is a mineral? Is sugar cube a mineral? Name four important
   groups of rock forming silicate minerals.   (2+1+2)=(5)

 (b) What is polymorphism? How are calcite and aragonite related
   to each other?   (4+2)=(6)

 (c) What is a rock? Name a monomineralic rock type from each of the
   major groups.   (2+3)=(5)

3.(a) Briefly state the Bowen's reaction series.   (4)

 (b) (i) Separate the following rock types into two columns, one
        for the volcanic varieties and the other for their plutonic
        equivalents. (ii) Arrange the volcanic varieties in order
        of decreasing SiO_2 content and (iii) Match each of them
        against their plutonic equivalents:
        Granite, Basalt, Diorite, Rhyolite, Gabbro, Andesite,
        Granodiorite, Diorite.   (4+4+4)=(12)

4. What is a fossil? Describe the various processes involved when the
   hard parts of an organism is undergoing fossilization. Briefly
   discuss the utilities of studying fossils.   (2+7+7)=(16)

5.(a) What is Geological Time Scale? What is meant by Palaeozoic time?
   What is the most important fuel, found extensively in the
   Carboniferous rocks of Indian Gondwana? When did the first
   flowering plants appear in the earth? Name the first bird to
   appear in the sky of the earth.   (5+2+1+1+1)=(10)

5.(b) What major changes took place in the organic community on the earth (as evident from the fossil record) during (i) early Cambrian (ii) at the end of Cretaceous.   (3+3)=(6)

6.(a) Distinguish between brittle and plastic deformation.   (4)

   (b) 'Draw a folded layer and label the following features: anticline, syncline, hinge, inflection point, limb.   (5)

   (c) If the interlimb angle of a fold is 50°, what will it be called?   (1)

   (d) Draw a recumbent fold and label the axial plane.   (2)

   (e) On the scarp face along an E-W trending road stretched at 200 meter contour level, lower boundary of a sedimentary rock layer occurs at a fixed elevation all along the road. What would you conclude about the structural attitude of the bed?   (4)

7.(a) Draw an inclined fault plane and label the following features: Hangingwall, Footwall and netslip.   [6]

   (b) Draw a sketch to illustrate the horst and graben structures. (2)

   (c) Define with sketches (any two): Oblique-slip fault, Strike fault, Longitudinal fault, Parallel fault.   (4+4)=(8)

8. What is metamorphism? What are the three basic controlling factors for metamorphic changes? Briefly discuss about them. What is a metamorphic grade? Arrange the following index minerals in order of increasing metamorphic grade: biotite, chlorite, garnet, sillimanite [indicate the increasing grade with an arrow]. What type of metamorphic rock was used as the building material of Konarak Sun Temple?   (3+3+5+2+2+1)=(16)

9. Indicate the correct answer (Attempt any ten).
   (a) The age of the oldest fossil algae is (i)) 1.0 b.y. (ii)) 1.5 b.y (iii)) 2.8 b.y (iv)) 4.5 b.y.
   (b) The fold that closes upward with the youngest rock at its core is a/an (i) antiformal anticline (ii) synformal syncline (iii) synformal anticline (iv) antiformal syncline.
   (c) The rapid establishment of vascular plants by (i) Silurian (ii) Middle Devonian (iii) Triassic (iv) Jurassic, led to the establishment of the first forest of the world.
   (d) Early Homo sapiens appeared in the earth during (i) Jurassic (ii) Pleistocene (iii) Pliocene (iv) Recent time.
   (e) When the crest of a fold is flat and broad and there are two hinges, one on either side of the flat crest: the fold is a (i) chevron fold (ii) box fold (iii) fan fold (iv) isoclinal fold

(f) One of the common mineral produced by shock metamorphism is
(i) stishovite (ii) zeolite (iii) haematite (iv) fluorite.

(g) Aluminium is an important element in (i) magnetite
(ii) chalcopyrite (iii) dolomite (iv) andalusite.

(h) The overall density (in gm./c.c.) of the earth is in the range:
(i) 3.5 - 4.0 (ii) 4.0 - 4.5 (iii) 5.5 - 6.0 (iv) 6.0 - 6.5.

(i) The most dominant rock type in the stratigraphic record is·
(i) sandstone (ii) shale (iii) limestone (iv) conglomerate.

(j) During the last 2 b.y. of the earth history the main reason
for oxygen enrichment in the atmosphere is (i) photosynthesis
(ii) photochemical dissociation of water (iii) volcanic eruption
(iv) chemical disintegration of oxide minerals.

(k) The metamorphic equivalent of limestone is (i) slate (ii) marble
(iii) quartzite (iv) mica-schist.

(l) The internal structure of muscovite consists of (i) 3-D network
of silicate tetrahedra (ii) single-chain of silicate tetrahedra
(iii) double-chain of silicate tetrahedra (iv) sheet-like arrange-
ment of silicate tetrahedra.

(m) The dark colour of an igneous rock is due to the presence of ·
(i) Fe-Mg (ii) SiO_2 (iii) K-Cl (iv) Ca-Na.   (2x10)=(20)

:ss:

Date: 23.11.90   Maximum Marks:100   Time: 3 Hours

Note: Answer all questions. You may consult your
notes or books if necessary.

1. Consider a utility function

$$U = (x-a)^{\alpha} (y-b)^{1-\alpha}$$

where x and y are quantities of the two goods consumed, a and b
are positive constants and α is a given positive fraction. A
consumer maximises the above utility function subject to his
budget constraint

$$p_x x + p_y y = I$$

where p_x, p_y are the prices of the two goods and I is the consumer's
money income.

(i) Interpret the constants a and b in the utility function.
(ii) Derive the demand functions for good x and good y and inter-
pret the demand functions.
(iii) Calculate the price and income elasticities of demand. (5+12+8)

2.(a) At a given wage rate an individual would choose to work six
hours per day, but institutional constraints force that person
to work eight hours or not at all. Show that the unemployment
benefit necessary to induce the person to quit his job is more
if he were allowed to work six hours.

(b) A consumer consumes two goods: food and petrol. Determine if
the consumer is better off, worse off or indifferent when the
government, in order to restrict the consumption of petrol,
puts a tax on petrol as compared to the situation when a direct
quantity rationing is imposed on the consumption of petrol.
Assume that both goods are normal and assume that the consump-
tion of petrol is the same under the two alternative policies.
(13+12)

3. A firm faces the following demand function for its output:

$$D(p) = \begin{cases} 12 - p & \text{for } 0 \le p \le 8 \\ 20 - 2p & \text{for } 8 \le p \le 10 \\ 0 & \text{for } p \ge 10 \end{cases}$$

(a) Graph this demand function. For what value of output y does this
function have a kink ?

(b) What is the firm's total revenue function ?

3.(c) What is the firm's marginal revenue function? Graph this function.

(d) Can you think of a plausible reason why a firm's demand function might look like this? (Hint: Think of the firm's competitors' behaviour)

(e) Assume that the firm has a production function $y = q^{1/2}$ where q is the amount of labour input and w is the wage rate. For what values of w will the firm not produce?   (2+2+6+7+8)

4. Consider an economy producing two goods X_1 and X_2 with two factors of production V_1 and V_2. Each technology exhibits constant returns to scale. To produce one unit of X_1, 10 units of V_1 and 10 units of V_2 are required and to produce one unit of X_2, 20 units of V_1 and 10 units of V_2 are required. There is perfect competition in each sector and free mobility of factors. The economy is endowed with 150 units of V_1 and 100 units V_2. Finally, consumers spend $3/4$ of their income on X_1 and $1/4$ on X_2.

Determine the equilibrium levels of output, commodity prices and factor prices.

[Hint: First show that both factors cannot be fully employed in equilibrium].   (25)

:ss:

Date:21.11.90   Maximum Marks:100   Time: 3 Hours

Note: Answer any five questions.

1.(a) Define the terms quantal response, probit, and probit
regression line.

(b) Estimate the probit regression line by graphical method for
the following data and mark off LD75.

dose metameter	number of insects	percent kill
1.01	50	88
0.89	49	86
0.71	46	52
0.58	48	33
0.41	50	12

[1x3+15+2=20]

2. Let X be a discrete random variable taking finite number of
values with probability function $f(.,\theta)$.

(a) Define Fisher information about θ in X.

(b) Let X be Binomial (n,θ), n known. Compute Fisher information
about θ in X.

(c) Let $T = t(X)$ be a function of X. Show that Fisher information
in T about θ can at most be equal to that of in X. When are
these same?   [2+6+10+2=20]

3.(a) Discuss the method of scoring for a multinomial distribution
will cell probabilities depending on a single parameter.

(b) Following table gives class probabilities and observed
frequencies in a repulsion intercross of two linked traits
with a recombination fraction θ.

Class	Probability	Observed frequency
AB	$(2+\theta^2)/4$	125
Ab	$(1-\theta^2)/4$	18
aB	$(1-\theta^2)/4$	20
ab	$\theta^2/4$	34

Compute m.l.e. of θ after one cycle of interation.   [6+14=20]

4. Let X_1, X_2, X_3 be a random sample from a distribution which has p.d.f

$$f(x,\theta) = \begin{cases} \theta x^{\theta-1} & , \quad 0 < x < 1 \\ 0 & , \quad 0.w. \end{cases}$$

where $0 < \theta < \infty$. In order to test $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$, the following test is proposed.

Reject H_0 if and only if $\prod_{i=1}^{3} X_i > \exp(-\chi^2_{1-\alpha,6}/2\theta_0)$,

where $\chi^2_{1-\alpha,n}$ satisfies $P(Y_n > \chi^2_{1-\alpha,n})=1-\alpha$, $Y_n \sim \chi^2_n$.

(a) Find $\nu(\theta)$, the power function of this test.

[Hint: Use the distribution of $-\log X_1$.]

(b) Show that $\beta(\theta)$ is a monotone increasing function.

(c) Compute the size of the test. $[7+9+4=20]$

5. (i) Let $X_1, \ldots, X_n$ be iid $N(\theta,1)$, θ unknown. In order to test $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$, the following test is proposed.
Reject H_0 if and only if $\sum_{i=1}^{n} X_i > k$.

(a) Determine k so that the test will have size α, where $0 < \alpha < 1$.

(b) Denote the critical region of the test obtained in (a) by S. Suppose $A \subseteq R^n$ satisfies

$$\sup_{\theta \leq \theta_0} P_\theta(X \in A) \leq \alpha, \quad X = (X_1, \ldots, X_n).$$

Show that

$$P_\theta(X \in S) \geq P_\theta(X \in A), \text{ for all } \theta > \theta_0.$$

(c) Comment on the result proved in (b).

(ii) Let $X \sim U(\theta, \theta+1)$, $\theta = 0\&4$. Suggest a suitable test for the problem $H_0 : \theta = 0$ versus $H_1 : \theta = 4$. Offer your comment about the test you have suggested. $[2+10+3+5=2C]$

6. Following table gives the mileage (km/lit) of two fuel efficient makes of automobiles at a road test with a number of cars from each make.

make A test car number	1	2	3	4	5	6	7
mileage	6.9	6.6	6.8	6.8	6.7	6.9	6.8

make B test car number	1	2	3	4	5
mileage	7.0	6.4	6.8	6.8	6.6

Assume normality of mileage distribution. Test, at 5% level, if the two makes give same mileage.   [20]

7.(a) Based on 27 independent observations from a normal distribution m.l. estimates of the mean and the variance have been computed as 48.2 and 6.94 respectively. Obtain a 95% confidence interval for the mean and a 90% confidence interval for the variance.

  (b) A player scored, in the past, 116 times out of 150 penalty kicks. Construct a 95% confidence interval for his probability of success in the next kick.   [6+6+8=20]

8.(a) In two villages, 67 out of 310 and 133 out of 564 people were literate. Test, at 1% level, the equality of literacy rates in the two villages.

  (b) Assume Poisson distribution for the number of words ending with 'al' among 1000 words in texts of different authors. From two different texts, the number of words ending with 'al' was counted to be 6 and 4 respectively among 1000 words each from each text. Test (conditional), at 5% level, if the two texts are written by the same author.   [10+10=20]

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ss:

Date: 19.11.90   Maximum Marks:100   Time: 3 Hours.

Note: Attempt any five of the following questions.

1.(a) State the central limit theorem (iid case) and outline a proof using characteristic functions.

(b). Using the central limit theorem show that

$$\lim_{n \to \infty} \sum_{i=0}^{n} \bar{e}^n \frac{n^i}{i!} = \frac{1}{2} \qquad (14+6=20)$$

2.(a) State the weak law of large numbers.

(b) Let $\{X_i\}$ be a sequence of independent random variables such that $E(X_i)=m$, $V(X_i) = e^{\alpha_i}$ for each i, where $\alpha_i \to 0$ as $i \to \infty$. Suppose $m < 0$ and let $S_n = X_1 + \dots + X_n$. Show that

$$\lim_{n \to \infty} P(\frac{S_n}{n} < \frac{1}{2}m) = 1.$$

(c) Consider the event that in an even number of tosses of a fair coin exactly half the tosses lead to heads. What can you say about the probability of this event when the number of tosses is large? Justify your answer.   (2+9+9=20)

3.(a) Let X be a random variable. For $\epsilon > 0$, show that there exist a,b (a < b) such that $P[X \notin [a,b]] < \epsilon$.

(b) Let $\{X_n\}$ be a sequence of normally distributed random variables, each with mean zero, such that the sequence $\{Var(X_n)\}$ is unbounded. Show that the following is false. If $\epsilon > 0$ then there exist a,b (a < b) such that $P[X_n \notin [a,b]] < \epsilon$, for every n.

(c) Let $\{X_n\}$ be a sequence of iid random variables such that X_1 has the exponential distribution with density $\alpha e^{-\alpha x}(\alpha > 0)$. Let $M_n = \max(X_1,\dots, X_n)$. Find the distribution function F_n of M_n. Show that

$$\lim_{n \to \infty} F_n(x) = 0.$$

Let $T_n = M_n - \alpha^{-1} \log n$, and G_n be the distribution function of T_n. Find $\lim_{n \to \infty} G_n(x)$.   (3+7+10=20)

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4.(a) Obtain the expectation of the sample range based on n independent observations from the exponential distribution with pdf.

$$f(x) = \bar{e}^x, \quad x > 0.$$

(b) Let X_1, X_2 be iid each having the uniform distribution over $(0,1)$. Find the density of $W = |X_1 - X_2|$. Hence or otherwise find the density of the standard deviation of X_1, X_2 given by

$$s = \sqrt{\frac{1}{2}\{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2\}},$$

where $\bar{X} = \frac{1}{2}(X_1 + X_2)$ (10+10=20)

...

5.(a) Let $X_1, X_2, \ldots, X_6$ be the frequencies of the faces $1, 2, \ldots, 6$ as obtained in n independent throws of a fair die. Obtain the partial correlation coefficient $\rho_{12 \cdot 345}$.

(b) In the set-up of (a), what can be said about the multiple correlation coefficient $\rho_{1 \cdot 23456}$? Justify your answer. (15+5=

6.(a) Let $X = (X_1, \ldots, X_p)'$ follow a multivariate normal distribution with a mean vector μ and a dispersion matrix Σ which is positi definite. Show that the correlation coefficient between X_1 and in their conditional distribution given $X_3, \ldots, X_p$ is the same as the partial correlation coefficient between X_1 and X_2 eliminating $X_3, \ldots, X_p$.

(b) If X and Y follow the bivariate normal distribution with zero means, unit variances and correlation coefficient ρ then does there exist a unique choice of two linear functions of X and Y such that the linear functions are independent? Justify your answer with examples if necessary. (16+4=20)

...

7.(a) Let $Y = (Y_1, \ldots, Y_p)'$ and $Z = (Z_1, \ldots, Z_p)'$ be two p-variate random vectors such that the mean vectors of Y and Z are identical and the dispersion matrices of Y and Z are positive definite. If the concentration ellipsoid of Y is fully contained in the concentration ellipsoid of Z then show that

$$V(Y_1) \leqslant V(Z_1), \quad i = 1, 2, \ldots, p.$$

(b) Stating your assumptions clearly, find an approximate formula for the large sample variance of m_r, the rth sample (central) moment. (8+12=20)

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INDIAN STATISTICAL INSTITUTE
B. Stat. (Hons.) II and III Year : 1990-91
Biology II
Semestral-II Examination

Date : 8.5.1991   Maximum Marks : 100 Time: 3 Hours.
Group A

Note :   Answer all questions.

1. Give the characteristic differences between mass
   selection and pure line selection in plant breeding.

   Briefly narrate the application of plant breeding
   in agriculture.   (8+4=12)

   What is mutation ? What are mutagens ? How do you
   classify them ? Name the methods for detection of
   mutation in nature.
                                      (2+2+6+2=12)

2. Name four important Cattle breeds of India. Give an idea
   of cross breeding of Indian cattle with imported bulls in
   relation to average lactation yield.   (4+6=10)

3. Write notes on the followings (any three) :
   (a) Importance of keeping herd books.
   (b) Different methods of the determination of sex
       in animals of plants.
   (c) Major discoveries in molecular biology in
       twentieth century.
   (d) Feed sources for Indian Cattle.   (3x6=18)

Group B

Note : All questions carry equal marks. Answer any four.

1. How water is held in the soil ? Define field capacity,
   wilting point and available soil moisture with regard to
   crop production. How do you calculate the water requirement
   of crop plant ?   (2+6+2)

2. What is the primary purpose of tillage ? What are the
   important parameters in a tilled soil and how these affect
   soil condition ?   (2+6+2)

p.t.o.

3. What is the role of nitrogen in crop production ?
Give a diagramatic presentation of the nitrogen
cycle in nature.   (4+6)

4. Write short notes on :
(a) Nutrient absorption mechanism.
(b) Acid Soil.   (5x2)

5. What proportion of soil and fertilizer phosphorus
is usually available to the plants ? Write down
the P. contents of four important phosphorus
fertilizers. What should be managoment practices
for efficient use of fertilizer P.   (2+4+4)

6. Describe briefly the different vegetative growth
and developmental stages of a 120 days paddy cultivar
grown in Kharif season. Write the names of early, medium
and late cultivars (two each) generally grown in eastern
region of India during Kharif season.   (7+3)

Group C

Note : Answer any two questions.

1.(a) Define a dynamical system.   (2)

(b) Find by graphical method the trajectory of the

two dimensional system
$$\frac{dx}{dt} = f_1(x,y)$$
$$\frac{dy}{dt} = f_2(x,y)$$
in the XY-plane.   (8)

2.(a) Write down the Vander Pol equation of harmonic
oscillator.   (2)

(b) Discuss the nature of the trajectories described by

the Vander Pol equation. Show when the above system
exhibits limit cycle behaviour.   (8)

3.(a) Define Hamiltonian of a conservative system. (2)

(b) Derive Volterra equations and discuss the qualitative
properties of the system represented by these equations