INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year and III Year: 1995 - 96

SEMESTRAL - I EXAMINATION

Geology

Date: 24.11.1995 Maximum Marks: 100 Time: 3 hours

Note: Attempt Question No.1 and any FIVE from the rest.

1. Fill up the blanks (any 10). Only write one of the four choices for each blank. (2x10) = [20]

(a) The skeleton mount in the ISI geology museum is of a dinosaur whose bones have been obtained from the continental rocks of _____ (Palaeozoic/Triassic/Jurassic/Cenozoic) age.

(b) A horizontal sequence of rock layers lies over a tilted sequence of rock layers with a/an _____ (Conformable/Paraconformable/unconformable/disconformable) contact.

(c) The top and bottom surfaces of a sedimentary rock layer are determined from the _____ (colour/cross-bedding/dip/metamorphism) of it.

(d) Initial addition of free oxygen in the primeval atmosphere was by means of a process that was associated with _____ (volcanic activity/water vapour/photochemical dissociation/photosynthesis).

(e) A palaeontologist analyzes fossils to trace the _____ (morphology/stratigraphy/evolution/importance) of the organism(s).

(f) The Principle of Lateral Continuity of rock layers is due to _____ (Cuvier / Smith/ Steno/Hutton).

(g) The freshwater in the continents and the continental ice together make up around _____ (20%, 10%, 5%, 2%) of the entire water-budget of the hydrosphere.

(h) The chance for the presence of fossils in the igneous rocks is _____ (very rare/quite common/nil/none of these).

(i) Shale is a _____ (metamorphic rock/sedimentary rock/igneous rock/fossil).

contd..... 2/-

(j) The term texture involves _____ (grain size/structural/ cross-bedding/viscosity) study in a rock.

(k) A mineral assemblage which defines a particular P-T environment is called _____ (index/facies/retrogression/progression).

(l) The overall density of the earth is _____ (4.5/5.0/ 5.5/6.0) gm/cc.

(m) Water may be considered to be a _____ (rock/mineral/ organic substance/crystal).

2. Write what you know about the Principle of Uniformitarianism and the Principle of Superposition. Who are said to be the propounders of the two principles ?

How is the succession of rock units determined in course of the Geological field work ? What would the presence of an unconformity in the succession imply ?

(6+1+6+3) = [16]

3. Define erosion and weathering, and indicate their role in the production of sedimentary rock. Describe the different weathering processes. What is the role of weathering in the formation of different physiographic (geomorphological) feature on the surface of the earth.

(4+8+4) = [16]

4. What is the difference between a body and a trace fossil ? Describe the various ways by which the altered hard parts of an organism may be preserved as fossils.

Give one good example of the use of fossil as an economic tool.

(4+10+2) = [16]

5. What is meant by Archaean in the Geological Time Scale. How does it differ from Proterozoic ?

... Define Precambrians, and explain why it is so called. What is the process involved in the ^{14}C method of age-dating ? How far this method is useful for the dating of rocks, say, of 10 million years old ?

(2+2+5+4+3) = [16]

6. What is a sedimentary rock and what are its characteristics ? What do you understand by the terms 'bed', 'bedding', and lamination ? In what ways mudcracks and graded bedding are useful ?

(3+4+6+3) = [16]

7. Describe the major factors and their effects in metamorphism. What is the difference between a schistose structure and a foliation ? Name one typical metamorphic rock and describe its mineralogical composition.

(8+4+4) = [16]

8. In 'plate tectonics', what is meant by 'plate' and 'tectonics' ? Describe in short the usefulness of the plate tectonics to the earth scientists.

(6+10) = [16]

OR

"In understanding the evolution of hydrosphere, salinity is taken as a factor". Elucidate.

What is the logic given when geoscientists say that a considerable portion of all the water in the present day oceans has appeared since late Mesozoic ?

(8+8) = [16]

9. Write short notes on (any four): (4 x 4) = [16]

(a) Absolute Geochronology.

(b) Clastic sedimentary rock.

(c) Folding and Faulting.

(d) Crystalline substance.

(e) Magma and Lava.

(f) Role of volcanic activity in the evolution of atmosphere.

:bcc:

INDIAN STATISTICAL INSTITUTE

B.STAT.(HONS.) II,III YEAR: 1995-96

SEMESTRAL-I EXAMINATION

BIOLOGY I

Date:24.11.95 Maximum Marks:100 Time: 3 Hours

Note: Q.1 and Q.2 are compulsory. Attempt any three from the rest.

1. (Only one answer is correct. Identify.) $[1\frac{1}{2} \times 12 = 18]$

A. One of the following is not a part of RNA

(a) guanine (b) adenine (c) cytosine (d) thymine

B. A molecule comprising of base, sugar and phosphate is a

(a) nucleotide (b) nucleoside (c) histone (d) none of the above

C. Germ cells are produced from somatic cells by a process known as

(a) mitosis (b) meiosis (c) oogenesis (d) embryogenesis

D. A cellular differentiation is

(a) regeneration (b) uncontrolled cell proliferation (c) successive mitotic division leading finally to matured specific cell (d) none of the above

E. The period of cell cycle devoted to DNA-synthesis is

(a) M-phase (b) S-phase (c) interphase (d) cytokinesis

F. The method of DNA-replication is

(a) semiconservative (b) conservative (c) random (d) none of the above

G. Human body cells usually have 46 chromosomes. During the anaphase stage of mitosis, a cell will have

(a) 92 chromosomes (b) 46 chromosomes (c) 23 chromosomes (d) none of the above

H. Colchicin arrest the cell cycle mostly at

(a) prophase (b) metaphase (c) interphase (d) telophase

I. In electron transport system electrons are transferred from NADH to 0_2 through a series of electron carriers, all of which are prosthetic groups of proteins except the

(a) flavins (b) iron-sulfur complexes (c) quinones (d) hemes

J. ADP-phosphorylation requires energy. What happens to most of the energy ? Is it

(a) lost as heat to the surroundings

(b) converted into electrical energy

(c) converted into electromagnetic energy

(d) stored as chemical-bond energy

K. Electrons donot usually flow through the electron transport chain to 0_2 unless

(a) the pH is maintained at 7.5

(b) temperature set to 32^0

(c) ADP is simultaneously phosphorylated to ATP

(d) NAD is completely exhausted

1. What do nicotinic acid, ascorbic acid and folic acid have in common ? They are

(a) fat soluble vitamins (b) lipid antioxidant (c) all B-vitamins (d) water soluble vitamins

2. [$1\frac{1}{2} \times 8 = 12$]

Column I is the deficiency symptoms of certain vitamins. Match them with the respective vitamins given in column II

Column I	Column II
A. Night blindness	(i) Vitamin D
B. Rickets	(ii) Ascorbic acid
C. Muscular dystrophy	(iii) Vitamin A
D. Beriberi	(iv) Thiamin
E. Megaloblastic anemia	(v) Niacin
F. Scurvy	(vi) Folic acid
G. Pellagra	(vii) α-Tocopherol
H. Sweet clover disease	(viii) Vitamin K

3. [10+7+6]

(a) What are the differences between the plant cells and animal cells?

(b) "Plant cells can withstand wider fluctuations of osmotic pressure than the animal cells" - justify

(c) Give a brief account on various types of animal tissues and their function.

4. [5+9+2+7]

(a) Hill reaction is considered as a 'land mark' in photosynthesis. Why ?

(b) How photosystem I is coupled to photosystem II in photosynthesis ?

(c) What do photosynthesis and oxidative phosphorylation have in common ?

(d) Give the carbon-cycle in dark reaction.

5. [5+10+8]

(a) What are the acceptable distinguishing features of a living object ?

(b) Design an experiment which can simulate the stages of chemical evolution of primitive earth.

(c) Give a brief account of the primitive forms of life which may have been the precursors of present day microbes.

6. [5+2+3+3+3+1+7]

(a) What is a coenzyme? How does it differ from a cofactor? Give examples of two coenzymes which are derived from vitamins. Also, name the corresponding vitamins.

(b) What are apoenzyme and holoenzyme ?

(c) What is an active site of an enzyme?

(d) What are the major classes of enzymes under the enzyme nomenclature?

(e) 'Enzymes can be inhibited by specific molecules' - justify

(f) Kinetically identify two types of inhibition.

7. Write short notes on [12+4+7]

(a) Meiosis I and Meiosis II

(b) Plant tissues (i) responsible for new growth (ii) resemples in function the skin of animals.

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) II Year : 1995-96

Statistical Methods III

Semestral-I Examination

Date : 17.11.1995 Maximum Marks : 100 Time : 4 Hours

GROUP A

All questions are compulsory.

1.(a) In a coin tossing experiment, examine the possibility of generating an event A for which $P(A) = 3.2\,pq$ where $p = P(\text{head}), q = P(\text{tail})$ and $\Omega = \{0<p<1,\ p+q = 1\}$.

(b) What is the minimum number of tosses required for unbiased estimation of pq ? Of 3.2 pq ? Show that with 4 tosses, an unbiased estimator of 3.2 pq may exceed unity ! Comment on the result. [10+(1+1+5+3)=20]

2.(a) Discuss the nature of statistical inference problems in a parametric set-up with suitable examples.

(b) Explain the following terms in this context : estimator, error, bias, mean absolute error, mean squared error, confidence interval, level of significance and power. [9+11 = 20]

3. Suppose $X_i \sim N(m_i, \sigma_i^2)$, $1 \le i \le 2k$ and X_i's are independent.

Derive, indicating suitable transformations, sampling distributions of the following statistics when

$m_{2i-1} = m_{2i} + \delta$ and $\sigma_{2i-1}^2 + \sigma_{2i}^2 = \sigma^2$, $1 \le i \le k$, δ being nonzero :

(a) $T_1 = \sum_{i=1}^{K} (X_{2i-1} - X_{2i}) / k\sigma$

(b) $T_2 = \sum_{i=1}^{K} (X_{2i-1} - X_{2i})^2 / \sigma^2$

(c) $T_3 = T_1 / \sqrt{T_2 - kT_1^2}$.

[5+5+5 = 15]

p.t.o.

GROUP B

Attempt any three questions

4. On the basis of a random sample of size n from a normal population with unknown mean m and unknown variance σ^2 obtain explicit expressions for $\hat{m}$ and $\hat{\sigma}$, the mle's of m and σ respectively.

Derive the sampling distribution of $\hat{\sigma}^2$ and indicate its use in determining $100(1-\alpha)\%$ both-sided confidence interval for σ.

[(2+3)+(7+3) = 15]

5. In the set-up of Question ##3(Group A), using $\{(x_{2i-1} - x_{2i}),\ 1 \le i \le k\}$ values,

(i) determine mle's of δ and σ^2, assuming both unknown;

(ii) determine $100(1-\alpha)\%$ both-sided confidence interval for δ. Examine if it is possible to determine confidence interval for δ and σ^2 simultaneously.

[(2+3)+(5)+5 = 15]

6.(a) State and prove Neyman-Pearson Lemma.

Explain one use of the lemma with reference to a suitable inference problem.

(b) It is required to test $H_0 : m = m_0$ vs. $H_1 : m < m_0$ where m refers to the mean of a normal population with known σ.

Define the uniformly most powerful test for this proble and derive its form explicitly. Work out the expression for its power function.

[(2+3+2)+(1+3+1+3] =15]

7.(a) A coin tossed 10 times produced 3 heads while another toss 7 times also produced the same result.

Test if the two coins possess the same chance of showing a head.

(b) Three pages chosen at random from Hedayat - Sinha Book are found to contain altogether 7 misprints. On the other hand, two pages chosen at random from Cochran's Book are found t contain altogether 3 misprints.

Test if Cochran's Book may be claimed to be relatively more free of misprints.

write[7+8=15]

For both the problems, you are required to down clearly the theory and the working procedure before doing the calcu lations and drawing the conclusions.

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1995-96

SEMESTRAL - I EXAMINATION

Calculus III

Date: 22.11.1995 Maximum Marks: 60 Time: 3 hours

Note: 1. This paper is set for 65 marks.
2. Maximum you can score is 60.

1.(a) Consider the function $f(x,y) = \exp(-\frac{|x-y|}{x^2 - 2xy + y^2})$. What is its domain of definition in R^2 ?

Can you define f on R^2 so that it is a continuous function ? Justify your answer.

(b) Calculate the derivative of $f(x,y) = 2x^2 - y^2$ at the point (1,2) in the direction of the line joining (1,2) to (4,0).

(5+5) = [10]

2.(a) F is a C^2 function on R^3 to R^3. Show that Div(Curl F) = 0.

(b) Consider the curve x(t) in R^3 defined by

$x_1(t) = \rho \cos t$, $x_2(t) = \rho \sin t$, $x_3(t) = kt$; $0 \le t \le 2\pi$

Find the tangent, normal and binormal vectors at each point. Here $\rho > 0$, $K > 0$ are constants.

(5+5) = [10]

3. $f: R^2 \longrightarrow R^2$ is defined as

$$f(x,y) = (e^x \cos y,\ e^x \sin y)$$

(a) What is the exact range of f ?

(b) What are the images under f of the lines parallel to the coordinate axes ?

(c) Show that f' is nonsingular at all points.

(d) Show that f is not one to one on R^2.

(e) Given a point (x_0, y_0), prescribe a neighbour hood of (x_0, y_0) where f is one to one.

(3x5) = [15]

4. Calculate volume of the solid bounded by

$$y = x^2,\quad x = y^2,\quad z = 0,\quad z = 12 + y - x^2.$$

[10]

p.t.o.

5.(a) Find a Potential function if any, for the vector field

$$x^2 i + y^4 j + z^6 k \quad \text{on} \quad R^3.$$

(b) Calculate the work done by the force field

$f(x, y, z) = yi + zj + yzk$ in moving a particle along the curve

$$a(t) = \text{Cost}\, i + \text{Sint}\, j + e^t k \qquad 0 \leq t \leq \pi .$$

(5+10) = [15]

6. $f(x,y)$ is a real valued continuous function on $[0,1] :: [0,1]$. Show that $\varphi(y) = \int_0^1 f(x,y)\, dx$ is a continuous function on $[0,1]$.

[5]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1995 - 96

SEMESTRAL - I EXAMINATION

Economics I

Date: 20.11.1995 Maximum Marks: 65 Time: $2\frac{1}{2}$ hours

Note: The paper carries 75 marks. You may attempt any part of any question. Your maximum score will be 65.

1. A consumer's utility function is of the form $U(x_1, x_2, x_3) = f_1(x_1) + f_2(x_2) + f_3(x_3)$, where $f_i' > 0$ and $f_i'' < 0$ for $i = 1, 2, 3$. (a) How many of the three goods can be inferior? (b) When p_i rises, what are the possible kinds of behaviour for x_i ?

(5+5) = [10]

2. Clearly distinguish between

(a) Compensated and uncompensated demand curves,

(b) Cardinal utility and ordinal utility.

(2.5 x 2) = [5]

3. Employ the weak axiom of cost minimization to demonstrate non-negativity of the slope of a factor demand function.

[7]

4. A firm uses labour L, and capital K, to produce a single good x. Capital is fixed, but labour is variable. The firm's production function is $x = -.2L^3 + 18L^2 + 1620L$. If the firm is currently producing so as to maximize the average product of labour, by what proportion would it have to increase employment to maximize total product ?

[10]

5. Consider a two-commodity consumer. Show that of two taxes, one a lump-sum tax on general purchasing power and the other a tax on a specific commodity - collecting the equal tax revenue, the lump-sum tax decreases the consumer's utility least. Give the intuitive reasoning behind this result.

[8]

6. In the short-run the demand for cigarettes is totally inelastic. In the long run suppose that it is perfectly elastic. What is the impact of a cigarette tax on the price that consumers pay in the short-run and in the long-run ?

[5]

p.t.o.

INDIAN STATISTICAL INSTITUTE

B.STAT. (HONS.) II YEAR : 1995-96

SEMESTRAL-I (BACK-PAPER) EXAMINATION

PROBABILITY III

Date: 27.12.95 Maximum Marks:100 Time: 3 Hours

Note: Answer all questions.

1. (a) Let X_1, X_2 be i.i.d. exponential (1) random variables. Let $U = X_1, V = X_1 + X_2$. Find the conditional distribution of U given $V = v$.

(b) Let X and Y be random variables, where we assume that Y has a finite variance. Show that $Y - E(Y \mid X)$ is uncorrelated with every $Z = f(X)$ which has a finite variance. [10+10=20]

2. (a) Let X, Y be a independent standard normal random variables. Find the joint density of (U, V) where

$$U = \frac{1}{X}$$

$$V = \frac{Y^2}{X}$$

(b) Let $(X_1, \ldots, X_n)$ have the multivariate normal density with parameter $\mu = 0$ and $\Sigma = (\sigma_{ij})$. Show that there exist independent standard normal random variables $Y_1, \ldots Y_n$ such that every X_i is a linear combination of $Y_1, \ldots Y_n$. [10+10=20]

3. (a) Let ϕ be a characteristic function. Show that $\frac{1}{2}\phi(t) + \frac{1}{2}\phi(2t)$ is a characteristic function.

(b) Let $X_1, \ldots, X_n$ be independent random variables each having Cauchy distribution with the parameter equal to 1. Show that $\frac{X_1 + \ldots + X_n}{n}$ has the same Cauchy distribution. [10+10=20]

4. (a) Let X be a random variable such that $E(|X|) < \infty$. Prove that $\phi_X(t)$ is differentiable at any $t \in R$.

(b) Let ϕ be the characteristic function of the standard normal random variable. Prove that

$$\phi'(t) = -t\phi(t) \quad t \in R. \qquad [12+8=20]$$

5. (a) Let $X_n, n = 1, 2, \ldots$ be i.i.d with $E(X_1) = \mu$ and $\mathrm{Var}(X_1) = \sigma^2$. Show that

$$\frac{X_1 + \ldots + X_n}{n} \to \mu \text{ in probability.}$$

(b) Let X_n be the random variable : $P(X_n = 2^n) = \frac{1}{n}$ and $P(X_n = 0) = 1 - \frac{1}{n}, n = 1, 2, \ldots$ If the X_n's are independent, prove that $\frac{X_1 + \ldots + X_n}{n} \to 0$. [10+10=20]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) II Year : 1995-96

Statistical Methods III

Semestral-I Backpaper Examination

Date : 29.12.1995 Maximum Marks : 100 Time : 3 Hours

Attempt all questions.

1.(a) In a coin tossing experiment, generate an event A for which $P(A) = 3.5pq$ where $p=P(\text{head}), q=P(\text{tail}), 0<p<1$.

(b) Show that the usual unbiased estimator of pq may exceed 0.25 if a coin is tossed n times, $n>2$. Comment on the result. [10+8+2=20]

2. Define the non-central Chi-square statistic with ndf. What do you mean by the non-centrality parameter ? Derive the sampling distribution of the above statistic. [4+2+14=20]

3.(a) What is a likelihood function ? What is the principle of maximum likelihood estimation ?

(b) Find the mle of the relevant parameter in the following cases :

(i) $X \sim U(0,\theta), \Omega = \{0<\theta\}$, $(x_1, x_2, .., x_n)$ = Random sample

(ii) $f(x|\theta) = e^{-(x-\theta)}$, $x>0$, $\Omega = \{0<\theta\}$, $(x_1 .. x_n)$ = random sample. [2+3+5+5 = 15]

4.(a) Obtain $100(1-\alpha)\%$ lower sided confidence interval for the standard deviation of a normal population with unknown mean, based on a random sample of size $n(>1)$.

(b) In the above set-up with known mean, find uniformly most powerful test for $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma<\sigma_0$. [10+10 = 20]

5. A breeder claims that his variety of cotton contains, at the most, 40 per cent lint in seed cotton. Fourteen samples of 100 gms each were taken, and after cleaning, the following quantity of lint was found in each sample.

5. Contd.....

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Quantity of lint in 100 gm samples	36	37	37	38	38	38	37	37	36	38	38	39	37	37

Examine if the breeder's claim is justified.

[15]

6. The yield of two strains of crop was found to be as given below :

Strain 1 : 15.4 20.5 15.9 38.3 18.7 37.8 26.4 39.2

Strain 2 : 33.3 16.7 6.5 24.0 42.5 22.9

Determine, under suitable assumptions to be clearly stated by you, 95% confidence interval for the difference in mean yields of the two strains.

[10]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1995-96

Statistical Methods IV

Semestral-II Examination

Date : 22.4.1996 Maximum Marks : 100 Time : $3\frac{1}{2}$ Hrs.

Answer any six questions.
All questions carry equal marks.

1.(a) Explain the concept of an array distribution with a concrete example and thereby explain the meaning of "regression".

(b) Under suitable assumptions (to be clearly stated by you); suggest tests for
(i) no regression, (ii) linear regression,
mentioning about the distributions of the test statistics under the null hypotheses.

2. Consider polynomial regression of Y on a non-stochastic regressor x.
(a) For a p^{th} degree polynomial regression equation, obtain the least squares estimates of the parameters and an expression for the minimum error sum of squares, to be denoted by S_p^2.
(b) Show that S_p^2 is a decreasing function of p in general.

3. (a) In a large family gathering it was observed that the sons of tall fathers, though tall, were generally shorter than their respective fathers while those of short fathers, though short, were generally taller than their respective fathers.

Using the principle of linear regression, or otherwise, provide a satisfactory explanation for the above phenomena.

p.t.o.

3.(b) The objective is to recommend the fixed sitting height of chairs to be used in a computer room for all the students of the Institute. A survey revealed the following information regarding the height distribution of student users :

Mean Height	158.3 cm
Modal Height	157.8 cm
Median Height	158.0 cm
s.d.	4.5 cm

As a thumbrule, the sitting height is about 70% of the actual height.

If the measure of discomfort of a student (while using the chair) is the difference (in absolute terms) between his/her sitting height and that of the chair and if the purpose is to minimize the average discomfort, what would be your recommendation ? Give reasons for your answer.

4.(a) Consider multiple linear regression of Y on k non-stochastic regressors $X_1, X_2, \ldots, X_k$.

(i) Define multiple correlation coefficient $R_{Y.12\ldots k}$ of Y or $X_1, \ldots, X_k$.

(ii) Show that $R^2_{Y.12\ldots k}$ is the maximum possible correlation between Y and its linear predictor.

(b) If $(X,Y) \sim BN(0,0,1,1,\rho)$, compute the value of Prob. $(Y<X<2Y)$ as a function of ρ and hence that of Prob. $(0<Y<X<2Y)$ as well.

5.(a) Obtain the joint distribution of the r^{th} and s^{th} order statistics in a random sample of size n from a continuous population with density function f.

(b) Obtain an approximate expression for the variance of the r^{th} order statistic in the set-up of (a) above.

(c) In normal samples, show that the sample median is approximately 64% efficient compared to the sample mean.

Contd.....

6. Let $\underset{\sim}{X} = (X_1,\ldots,X_p)'$ follow a p-variate normal distribution with a null mean vector and a dispersion matrix

$$\Sigma = \begin{pmatrix} 1 & \theta & \theta^2 & \ldots & \theta^{p-1} \\ \theta & 1 & \theta & \ldots & \theta^{p-2} \\ \theta^2 & \theta & 1 & \ldots & \theta^{p-3} \\ & & \vdots & & \\ \theta^{p-1} & \theta^{p-2} & \theta^{p-3} & \ldots & 1 \end{pmatrix}$$

where $p \geq 3$ and $|\theta| < 1$. What will be the correlation coefficient between X_1 and X_2 in their conditional distribution given $X_3,\ldots,X_p$? Simplify your answer as far as practicable.

7.(a) Let $X_1,\ldots,X_p$ be jointly distributed random variables such that marginally each of $X_1,\ldots,X_p$ is univariate normal. Then is the joint distribution of $X_1,\ldots,X_p$ necessarily multivariate normal ? Justify your answer and give an example, if necessary.

(b) Show that if $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, where Σ is positive definite, then $(\underset{\sim}{X}-\underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{X}-\underset{\sim}{\mu})$ follows the Chi-square distribution with p degrees of freedom.

8. On the basis of a random sample from a multivariate normal population with unknown mean vector $\underset{\sim}{\mu}$ and unknown positive definite dispersion matrix Σ, derive the maximum likelihood estimators of $\underset{\sim}{\mu}$ and Σ.

9. Under random sampling from a multivariate normal population with a positive definite dispersion matrix, show that

$$\frac{r_{12.34\ldots p}\sqrt{N-p}}{\sqrt{1-r^2_{12.34\ldots p}}}$$

follows the t-distribution with N-p degrees of freedom, provided $\rho_{12.34\ldots p} = 0$ (the notation is as usual). Briefly indicate a use of this result in statistical inference.

INDIAN STATISTICAL INSTITUTE
B.STAT (HONS.) II YEAR:1995-96
ALGEBRAIC STRUCTURES
SEMESTRAL-II EXAMINATION

Date:24.4.96 Maximum Marks:60 Time: 3 Hours

Note: The paper carries 72 marks. The maximum you can score is 60.

1. (a) Let σ, τ, π be elements of the permutation group S_n such that $\tau = \pi\sigma\pi^{-1}$. Show that for every m, number of m-cycles of τ = number of m cycles of σ. [8]

   (b) If for σ, τ in S_n, $\sigma\tau = \tau\sigma$ and $(k_1, \ldots, k_m)$ is a cycle of τ, show that $(\sigma k_1, \ldots \sigma k_m)$ is also a cycle of τ. [7]

2. (a) Let H be a subgroup of a group G. Show that

   $N(H) = \{g : gHg^{-1} \subseteq H\}$ is a subgroup of G. [5]

   (b) A subgroup of the form gHg^{-1} is called a conjugate of H. Suppose G is finite. Show that the no. of distinct conjugates of H = order of $G/N(H)$. [10]

3. Let R be a commutative ring. Let $N = \{x \in R : x^n = 0$ for some positive integer $n\}$.

   (a) Show that N is an ideal in R. [8]

   (b) If $[y] \in R/N$ and $[y] \neq [0]$, show that $[y]^n \neq [0]$ for any positive integer n. [4]

4. Let p,q be prime. Show that there is an irreducible polynomial of degree q over the field J_p. [15]

5. Let K be the splitting field of x^3-1 over the field of rationals. Show that x^3+9x^2+3x+3 is irreducible over K. [15]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year:1995-96
Economic and Official Statistics
Semestral-II Examination

Date:26.4.96 Maximum Marks:100 Time: 3 Hours

Note: Answer any four questions.

1.(a) What is a chain base index number? What are the advantages of the chain base index number over the corresponding fixed base index number?

(b) Describe the test that an index number formula must satisfy in order that the chain base index number is equal to the corresponding fixed base index number.

(c) What are 'Time reversal test' and 'Factor reversal test'? Examine the Laspeyres' and Paasche's indices in the light of these tests.

(d) Name some important index numbers used in India.

(e) The following data show the price indices for industrial workers in an Indian city in March 1972.(Base: 1960=100).

Item group	Index	Weight
Food	221.6	44.65
Clothing	164.8	5.47
Fuel and light	202.9	4.26
House rent	187.4	9.16
Miscellaneous	164.5	36.46

X's basic pay was Rs.460/- in March, 1972. According to an agreement with the workers in 1969, cost of living index (CLI) upto 100 was merged with basic pay, and for every 4-point rise in CLI there would be a rise of 3% of basic pay as Dearness Allowance (DA). How much was the DA of X, an industrial worker?

[5+3+8+3+6=25]

2.(a) Consider the polynomial trend equation

$$Y_t = a+bt+ct^2.$$

Using Gram-Schmidt orthogonalisation procedure write down
(i) the orthogonal polynomial and
(ii) the estimators of its coefficients.

(b) Describe the ratio-to-moving average method of determining constant seasonal indices from a series of quarterly figures. You may assume that the time series follows the multiplicative model. Justify the different steps briefly. How would you modify the procedure for determining changing seasonal patterns?

[13+12=25]

contd. ....2.

3. (a) Define Lorenz curve (LC) and discuss its properties.
   (b) Define Lorenz Ratio (LR) and show that LR is equivalent to the Gini coefficient.
   (c) Obtain the LC and LR for Lognormal distribution.
   (d) What are the desirable properties that a measure of income inequality must satisfy? [6+6+7+6 = 25]

4. Write short notes on any two of the following.
   (a) The Indian Official statistical system;
   (b) Problems in estimating demand function from time series dat
   (c) The Pareto distribution. [2

5. Consider a production function with two inputs K and L.
   (a) Define the 'Marginal Rate of Technical Substitution' (MRTS) between K and L. Show that in equilibirum the MRTS is equal to the slope of the isoquant and also equal to the ratio of input prices.
   (b) Define the elasticity of substitution for a production funct Draw the isoquants corresponding to the cases where the ela ticity of substitution between inputs is (i) zero and (ii) infinity.
   (c) Derive the elasticity of substitution for the CES production function.
   (d) Obtain the constant returns to scale Cobb-Douglas production function as a special case of the CES production function. [7+6+4+8=25]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) II Year : 1995-96

Demography and Statistics

Semestral-I Examination

Date : 29.4.1996 Maximum Marks : 50 Time : $1\frac{1}{2}$ Hrs.

Attempt any FIVE questions.

1. Describe various systems of census-taking indicating their merits and demerits. Which system was adopted in the 1991 Indian census ? [8+2=10]

2. Describe a method to estimate the total number of births in a country during a year under the dual recording system. How dan you indicate this system ? [8+2=10]

3. For a logistic population model :

$$P(t) = [A+B\ e^{-Ut}]^{-1}\ ,\ t \geq 0;$$

show that the growth rate : r(t) satisfies

$$r(t) = U[1-AP(t)].$$ [10]

4. Describe various methods of estimating infant mortality rate. How does infant mortality rate differ from infant death rate ? [6+4=10]

5. Show that Reed-Merrel model is a particular case of Greville's model. [10]

6. Find out a mathematical relationship between crude birth rate and total fertility rate. [10]

7. Describe measures of fertility based on census data and comment on their merits and demerits. [10]

8. Describe various columns of a net nuptiality table. Also give the formula for net nuptiality. [10]

INDIAN STATISTICAL INSTITUTE
B.STAT (HONS.) II and III YEAR: 1995-96
SOCIOLOGY
SEMESTRAL-II EXAMINATION

Date : 2.5.96 Maximum marks : 100 Time : 3 Hrs.

Answer any Five questions but not more than Two from each group

GROUP -A

1. What are 'social facts'? Illustrate by using one of Durkheim's studies.

5 + 15=20

2. Describe Weber's concept of Legitimate authority.

20

3. Can you discuss Marx, Weber and Durkheim in a comparative manner? What do we learn from them ?

10 + 10=20

GROUP -B

4. What is a social network ? Illustrate your answer with the help of a diagram. Describe how can you identify a social network in a multi-caste village.

5 + 5 + 10=20

5. In order to study whether the use of chemical fertiliser has changed among the farmers of a village, state how will you draw a sample of your respondents, prepare a list of items of information and collect your data.

5 + 10 + 5=20

6. What are the differences between quantitative and qualitative data ? State one method of collection of data of each type. Illustrate their uses.

10 + 5 + 5=20

GROUP -C

7. Define kinship. What are its types ? Explain the significance of kinship terms.

5 + 5 + 10=20

8. What are exogamy and endogamy ? How do you account for these practices ? Give your answer with examples.

5 + 10 + 5=20

9. Briefly explain the process of developement of Sahajiya philosophy in the religious tradition of Bengal.

20

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II year: 1995-96
ECONOMICS II
SEMESTRAL-II EXAMINATION

Date : 2.5.96 Maximum marks: 60 Time : 3 Hrs.

Answer any four questions. All questions carry equal marks.

Maximum marks is 60

1. a) Consider a simple keynesian model with autonomous investment and exports. Suppose all investment goods are imported from abroad and assume that there are no other imports. Calculate the equilibrium output.

b) In a simple keynesian model with autonomous investment and no foreign trade, suppose there are two classes of consumers with marginal propensities to consume b_1, b_2 where $b_1 > b_2$. Calculate the effect on income if there is a transfer from the second group of consumers to the first.

c) In a simple keynesian model with autonomous investment and no foreign trade, what would happen to total savings if there is a rise in the saving ratio ?

(5 + 5 + 5)

2. Given the existence of capital flows, what mix of monetary and fiscal policy should be pursued to offset the following disturbances under fixed exchange rates.

a) A rise in exports ;
b) A fall in the foreign rate of interest ;
c) A decline in autonomous spending ;

(5 + 5 + 5)

3. Consider an aggregate demand-supply framework. Discuss the short run and long run effects of a rise in the productivity of labour on income and the price level.
(Hint : Assume that the production function takes the form $Y = aN$ and consider a rise in a)

(15)

p.t.o.

4. Discuss a Solow growth model with population growth and without technical progress. Derive the golden rule condition. Show how per capita income can grow over time once technical progress is introduced.

(15)

5. Consider an underdeveloped economy where the industrial sector is demand constrained and the agricultural sector is supply constrained. Discuss the effects of the following changes on industrial employment and the price of food :

a) A rise in autonomous expenditure.

b) A fall in industrial prices due to a reduction of mark-up.

c) A land reform redistributing land from large to small farmers.

(5 + 5 + 5)

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) II YEAR:1995-96
ANTHROPOLOGY
SEMESTRAL-II EXAMINATION

Date : 2.5.96 Maximum marks : 100 Time : 3 Hrs.

USE SEPERATE ANSWER SCRIPT FOR GROUP-A AND GROUP-B
ANSWER ANY THREE QUESTIONS FROM GROUP-A
ANSWER ANY THREE QUESTIONS FROM GROUP-B

GROUP - A

1. Enumerate briefly the major changes that took place in anatomical characteristics of man due to assumption of erect posture.

(16)

2. What are the major stresses on man at high altitude ? What are their major effects on human biological traits.

(16)

3. Define human biology. Indicate the relationship between Human Biology and Physical Anthropology. (16)

4. Discuss briefly the theory of demographic transition.

(16)

GROUP - B

1. Describe the structure and function of an animal cell.
2. Describe Mendel's laws of inheritance with suitable example.
3. Give an account of normal l.c. Chromosomes with special reference to their number, structure and classification.
4. Write short notes on any two of the following :
   a) Expressivity and penetrance.
   b) Mendelian population.
   c) Hardy-Weinerg principle.
   d) Albirism.

(8 + 8=16)