INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1973-74

[301]

PERIODICAL EXAMINATION

Stat.-8: Linear Estimation

Date: 12.11.73 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [] .

1. Consider the model $E(\underline{Y}) = A'\underline{\theta}$ where $\underline{Y}^{n\times 1}$, $A^{m\times n}$ and $\underline{\theta}^{m\times 1}$ have their usual significance.

(a) Explain in this connection the terms 'linearly estimable linear parametric function' and 'best linear unbiased estimate'. [2×2]=[4]

(b) Prove that $\lambda'\theta$ is linearly estimable (l.e.) if and only if the system of equations $AA'\underline{q} = \underline{\lambda}$ in the elements of $\underline{q}$ is solvable. [10]

(c) State and prove a condition for the estimability of θ_m. [5]

(d) Assume $D(\underline{Y}) = \sigma^2 I_n$. Prove that the BLUE of l.e. $\underline{\lambda}'\underline{\theta}$ is $\underline{\lambda}'\hat{\underline{\theta}}$ where $\hat{\underline{\theta}}$ is any solution of $AA'\underline{\theta} = A\underline{Y}$. Also suggest an UE of σ^2 and prove its unbiasedness. [10+8]=[18]

2.a) Define V_{est} and V_e in connection with the theory of linear estimation. [1+1]=[2]

b). Prove that $\underline{a}'\underline{Y}$ is BLUE if and only if $Cov[\underline{a}'\underline{Y}, \underline{\xi}'\underline{Y}] \equiv 0$, for all linear functions $\underline{\xi}'\underline{Y}$ satisfying $E[\underline{\xi}'\underline{Y}] \equiv 0$. [10]

c) Y_1, Y_2, Y_3, Y_4 are four random observations having the same expectation θ and the dispersion matrix $D(\underline{Y}) = \sigma^2 G$ where

$$G = \begin{pmatrix} 1 & a & a^2 & a^3 \\ & 1 & a & a^2 \\ & & 1 & a \\ & & & 1 \end{pmatrix} \quad \text{with } 0 < a < 1, \text{ a known.}$$

Obtain the BLUE of θ. [Hint: Use (b)] [15]

3. Consider the model $E(\underline{Y}) = A'\underline{\theta}$, $D(\underline{Y}) = \sigma^2 I_n$. Define

$Q_i = \underline{\alpha}_i'\underline{Y}$, $i = 1,2,\ldots, m$ where $A' = (\underline{\alpha}_1, \underline{\alpha}_2, \ldots, \underline{\alpha}_m)$.

Prove that

(a) Any linear function of Q_i's with zero expectation must identically vanish. [7]

(b) The covariance between Q_i and any linear function of y_i's with zero expectation is zero. [6]

(c) If $\sum_1^m c_i Q_i$ estimates unbiasedly the parametric function $\sum_1^m \lambda_i \theta_i$, then the variance of the estimate is $(\sum_1^m c_i\lambda_i)\sigma^2$. [5]

p.t.o.

4. n independent random observations $y_1, y_2, \ldots, y_n$ satisfy the following model:

$E(y_i) = \alpha x_i + \beta x_i^{-1}$, var $(y_i) = \sigma^2 x_i^{-1}$, $i = 1, \ldots, n$, where $x_1, \ldots, x_n$ are known constants and α, β, σ^2 are unknown parameters. On the basis of the following data, obtain BLUE's of α and β and an UE of σ^2. (Here $n = 5$)

x:	1	2	3	4	5
y:	1	1.8	1.3	2.5	6.3

[5+5+5]=[15]

5. Practical record books. [5]

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PERIODICAL EXAMINATION

Statistics-10: Probability

Date: 19.11.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 109 marks. You may attempt any part of any question. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Define the generating function of a random variable taking non-negative integer values. State clearly the continuity theorem for generating functions. [7]

b) In a sequence of Bernoullian trials with probability of success p, let the trials upto the first failure be called the first turn etc. If S_r is the total number of successes in the first r turns, find the generating function of S_r and the distribution of S_r. From the generating function, find $E(S_r)$ and $V(S_r)$. [8+8]=[16]

c) Show that if $r \to \infty$ and $rp = \lambda$ then the distribution of S_r approaches a Poisson distribution. [8]

2.a) If X_n, $n \geq 1$ and X are random variables, define the convergence of X_n to X almost surely, in probability and in distribution. [6]

b) Show that $X_n \to X$ a.s. implies $X_n \to X$ in probability.
Show that $X_n \to X$ in prob. implies $X_n \to X$ in distribution. [8+8]=[16]

c) If the sample space is finite, show that convergence in probability implies convergence almost surely. Where does this argument fail for uncountable sample spaces? Does convergence in distribution imply convergence in probability? (Why ?) [10+6]=[16]

3.a) State and prove Borel-Cantelli lemmas. [12]

b) Show that with probability 1, any particular sequence of k S's and F's occurs infinitely many times in a sequence of Bernoullian trials when $0 < p < 1$. [8]

4.a) State and prove Kolmogorov's inequality. [12]

b) Prove that if $X_1, X_2, \ldots$ are independent and if $\Sigma \sigma_n^2$ converges where $\sigma_n^2 = V(X_n)$ then $S_n - ES_n$ converges almost surely. Here S_n denotes $X_1 + \ldots + X_n$. [8]

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PERIODICAL EXAMINATION

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Sample Surveys (Theory and Practical)

Date: 26.11.73 Maximum Marks: 100 Time: 3 hours

Note: Answer Q. 1 and any other three from the rest. Marks allotted for each question are given in brackets [ ].

1. The table given below shows the total cultivated area during 1960 as also the area under rice in 1962 for a sample of 20 villages in a certain state. The villages were selected by simple random sampling without replacement. The total cultivated area in 1960 for all the 170 villages of the state is known to be 78051 acres. Estimate

Table: Total cultivated area and area under rice for a sample of 20 villages.

Sl.No. of village	Total cultivated area in 1960	Area under rice in 1962	Sl.No. of village	Total cultivated area in 1960	Area under rice in 1962
1	401	75	11	470	109
2	634	163	12	1625	498
3	1194	326	13	827	125
4	1770	442	14	96	6
5	1060	254	15	1304	427
6	827	125	16	377	79
7	1737	559	17	259	105
8	1060	278	18	186	45
9	360	101	19	1767	564
10	946	356	20	604	238

the total area under rice in 1962 by using the three methods of estimation:- (i) mean per unit; (ii) ratio; and (iii) regression. Estimate the sampling variances of the estimates so obtained. [31]

2. A sample of size n is drawn with equal probability and without replacement from a population of size N. Let

$$\hat{\bar{y}}_N = \sum_{r=1}^{n} a_r y'_r \qquad \ldots \qquad (1)$$

be any linear estimate of the population mean $\bar{y}_N$, where the a_r's are some constants and y'_r denotes the value of the unit included in the sample at the r-th draw.

a) Show that $\hat{\bar{y}}_N$ is an unbiased estimate of $\bar{y}_N$ if and only if $\sum_{r=1}^{n} a_r = 1$. .. (2)

b) Show that under (2) the variance of $\hat{\bar{y}}_N$ is given by

$$V(\hat{\bar{y}}_N) = \frac{S^2}{N}\left[N \sum_{r=1}^{n} a_r^2 - 1\right] \qquad \ldots \qquad (3)$$

where S^2 is the mean square for the population.

p.t.o.

c) Show that $V(\hat{\bar{y}}_N)$ is minimised subject to (2) if $a_r = 1/n$ for $r = 1, 2, \ldots, n$. Hence prove that $\bar{y}_n = \sum^{n} y_i/n$ is the best unbiased estimate of $\bar{y}_N$ in the class of linear estimates given by (1). [25]

3. What is staratified sampling? Describe (i) optimum allocation, (ii) Neyman's allocation and (iii) proportional allocation. Obtain the variances of the estimates of population mean under (i) stratified sampling with Neyman's allocation; (ii) stratified sampling with proportional allocation; and (iii) simple random sampling. Compare the three variances and draw conclusion. [25]

4.a) Derive the expression for the estimate of the gain in precision due to stratification.

b) What is post-stratification? How much can one improve the precision of a simple random sample by resorting to post-stratification? [25]

5. Describe the ratio estimate of a population mean and state a situation where you think you would use it. Obtain the bias in it and examine its magnitude. Obtain also an approximate expression for the variance of the ratio estimate of population mean. [25]

6. What is cluster sampling and why is it resorted to? Assuming that the clusters are of equal sizes, suggest an estimate which is unbiased under simple random sampling. Obtain its variance in terms of the intra-class correlation coefficient and examine the efficiency of cluster sampling with respect to single unit sampling. [25]

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PERIODICAL EXAMINATION

Science-5: Geology

Date: 3.12.73 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions.
All questions carry equal marks.

1. Define the following terms - Rock, Mineral.

What are the major groups in which rocks are classified? Give two examples from each group.

2. Name the important structures associated with the igneous rocks.

Indicate diagrammatically the sequence in which minerals crystallise out of a magma during the process of cooling.

3.a) What is the difference between weathering and erosion?

b) What are the different agents for weathering?

c) Name two natural agencies that transport sediment.

d) What is the difference between 'saltated load' and 'rolled load'?

4. Complete the following statements -

a) Point bar deposits occur in ... .

b) Foreshore is a part of the ... .

c) Large deposits of limestone occur in ... .

d) During transportation silt and clay move in ... .

e) In the isometric crystal system, three axes are always at ...

f) Asymmetrical ripple marks can help in ... .

5.a) What do you understand by 'dip' and 'strike' of a bed?

b) What is a 'fold'?

c) Draw sketches of any four of the following (label the different parts in your diagrams) -

i) Syncline
ii) Recumbent fold
iii) Fan fold
iv) Symmetrical fold
v) Strike-slip fault
vi) Reverse fault.

6.a) What is a fossil?

b) In what type of rocks do you expect fossils?

c) What are the different forms of preservation of fossils?

d) How do fossils help in stratigraphic correlation?

p.t.o.

7. Indicate the correct statement in each of the following:-

a) In a sedimentary sequence, the order of deposition is from
   (i) bottom upwards (ii) top downwards.

b) Very well rounded sands are found in
   (i) eolian environments (ii) lake environments.

c) There are four crystal axes in the crystals belonging to
   (i) hexagonal system (ii) monoclinic system

d) Very well sorted sands occur in
   (i) beach environment (ii) glacial environment.

e) In the monoclinic system:
   (i) three axes are at right angles to one another
   (ii) two axes are at right angles to each other.

f) Stratification is found in
   (i) igneous rocks (ii) sedimentary rocks.

8.a) What are the broad divisions of the geological time-scale?

b) Where are the sedimentary rocks of the Mesozoic age found in the Peninsular India?

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PERIODICAL EXAMINATION

Analysis

Date: 10.12.73 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all the questions.
Marks allotted for each question are given in brackets [ ].

1.a) $f(x)$ is a bounded function defined on $(-\infty, \infty)$. Let $g(x) = |x| \cdot f(x)$. Prove that $g(x)$ is continuous at $x = 0$. [8]

b) Give an example to show that even if $\{f(x)\}^2$ is continuous at $x = 0$, $f(x)$ may be discontinuous at $x = 0$. [8]

2.a) $\{f_n(x)\}$, $n = 1,2,3,\ldots$ is a sequence of functions defined on the closed interval $[0, 1]$ and $\lim_n f_n(x) = 0$ at each x of $[0, 1]$. [8]
Each $f_n(x)$ is a nondecreasing function. Prove that the convergence of $\{f_n(x)\}$ to the zero function is uniform

b) Precisely state a theorem which says that under uniform convergence, the limit of a sequence of continuous functions is a continuous function. Prove it. [8]

3.a) What is meant by the supremum of a set which is bounded above? [5]

b) $\sup A = \alpha$ and $\sup B = \beta$. What is $\sup (A \cup B)$? Give reasons (briefly). A and B are bounded above. [8]

c) λ is a limit point of A. $A \subset B$. Show that λ is a limit point of B. [5]

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PERIODICAL EXAMINATION

Economics-2

Date: 17.12.73 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

Group A: Maximum Marks: 50
Answer any two questions.

1. Starting from the equilibrium conditions of a utility maximizing two-commodity consumer, prove that if one of the two commodities is inferior, then substitution effect and income effect for it will have opposite signs. [25]

2. A firm is said to be efficient if it

i) maximizes output for a given cost

or ii) minimizes cost for a given output.

If the firm produces a single commodity with two inputs which it buys at given prices, show that the two definitions of efficiency given above are equivalent. [25]

3.a) Show that a Cobb-Douglas production function obeys (i) the law of constant return to scale and (ii) the law of diminishing marginal product of each input. [15]

b) If $Q = AK^{\frac{1}{2}}L^{\frac{1}{2}}$ [where Q = output, K and L are respectively capital and labour inputs] show that the marginal rate of input substitution (calculated per unit of labour) is equal to the capital labour ratio. [10]

Group B: Maximum Marks: 50
Attempt any two questions.

4. Show how the IS and LM functions can be graphically derived. Use them to arrive at the general equilibrium of the product and money markets. Examine carefully the situations in which shifts may occur in these functions and comment on the resulting changes in income and interest rate. [12+3+10]=[25]

5. What are Keynes' hypotheses about the consumption function? Examine any policy recommendations that may be based on his formulation of consumer behaviour. Describe how different types of statistical data give rise to differently shaped consumption functions. Briefly discuss some theories suggested to reconcile conflicting indications about the basic form of the relationship of consumption to income. [3+3+5+14]=[25]

6. Discuss the three different policies of the Central Bank. Examine the situations where they can be used. [25]

7.a) Explain how commercial banks create money.

b) Discuss the different motives behind the demand for money. [13+12]=[25]

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PERIODICAL EXAMINATION

Calculus

Date: 24.12.73 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all the questions.
All questions carry equal marks.

1. A function $u = f(x, y)$ is defined on a region R in xy-plane. P (x_0, y_0), is a point in R. Define continuity of f at P in two distinct ways and show that one implies the other.

   Give an example of a function $u = f(x, y)$ which has partial derivatives at a point in R but is not continuous. Under what conditions a function having partial derivatives will be continuous?

2. Define partial and directional derivatives of a function $u = f(x, y)$. Define differentiability of f also. Is continuity of partial derivatives necessary for f to be differentiable at a point? Define total differential of f having continuous partial derivatives. Evaluate the directional derivative of $f(x, y) = 2x^2 - y^2$ at (1, 2) in the direction of the line from (1, 2) to (3, 5).

3. Let $f(x, y, z)$ be a function depending only on

$$r = \sqrt{x^2 + y^2 + z^2}, \text{ i.e. let } f(x, y, z) = g(r)$$

   a) Calculate $f_{xx} + f_{yy} + f_{zz}$

   b) Prove that if $f_{xx} + f_{yy} + f_{zz} = 0$, it follows that

$$f = \frac{a}{r} + b \quad \text{(where a and b are constants).}$$

4. Find the Taylor series for the following functions and indicate their range of validity.

   a) $\frac{1}{1 - x - y}$, b) e^{x+y}

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MID-YEAR EXAMINATION

Statistics-10: Probability

Date: 7.1.74 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 106 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. Show that if $X_1, X_2, \ldots$ are i.i.d. random variables then $\frac{1}{n}(X_1+\ldots+X_n)$ converges almost surely to a constant iff $E|X_1| < \infty$.
(You may assume that if $X_1, X_2, \ldots$ are independent and $\Sigma\,\sigma_n^2$ is finite then S_n-ES_n converges almost surely). [20]

2.a) Define the characteristic function of a random variable. [4]

b) If φ is a c.f. (characteristic function), show that $\psi(t) = \varphi(-t)$ is also a c.f. [6]

c) If φ_1, φ_2 are c.f.'s and $0 \le \alpha \le 1$ then $\alpha\,\varphi_1 + (1-\alpha)\varphi_2$ is also a c.f. [8]

3.a) State and prove the inversion formula for the probability in an interval. (State clearly the results of calculus you assume). [15]

b) State and prove the inversion formula for the probability carried by a point. [10]

c) If X has c.f. $\frac{1}{3}(1 + 2\cos t)$, find $P(0 \le X \le 1)$ using (a) and (b). [7]

4.a) Define weak convergence of distribution functions. Show that if $F_n \xrightarrow{w} F$ then the c.f. of F_n converges to the c.f. of F. If you need any additional condition for this, state it clearly. [4+14]=[18]

b) Show that the c.f. of X_n which assumes the values $-n, -n+1, \ldots, -1, 0, 1, \ldots, n$ with equal probabilities is

$$\frac{\sin(n+\frac{1}{2})t}{(2n+1)\sin\frac{t}{2}}.$$

Find the weak limit F of F_n. Does the c.f. of of F_n converge to the c.f. of F? [10+8]=[18]

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MID-YEAR EXAMINATION

Mathematics-6: Analysis

Date: 9.1.74 Maximum Marks: 100 Time: 3 hours

Note: All questions may be attempted. Marks allotted for each question are given in brackets [].

1. Prove that $(\sqrt{2} + \sqrt{3})$ is irrational. You may assume that $\sqrt{n}$ is irrational if n is any positive integer which is not a perfect square. [8]

2. The infinite series $a_1 + a_2 + \ldots$ is convergent and has sum S. Prove rigorously that the series $a_2 + a_1 + a_4 + a_3 + a_6 + a_5 + a_8 + a_7 + \ldots$ is also convergent and has sum S. [10]

3. The sequence $\{b_n\} \longrightarrow 0$ and each $b_n \neq -1$. Prove from fundamentals that the sequence $\{c_n\} \longrightarrow 0$ where $c_n = \dfrac{b_n}{1 + b_n}$. [10]

4. A is a set of real numbers which is bounded above. B(=-A) is the set of negatives of the numbers occurring in A. For example, if A is the set consisting of 2 and -3, B consists of -2 and +3. [10]

   Show that B is bounded below and that inf B = -(sup A). [10]

5. $f(x)$ is continuous on $[a, b]$. $f(a) > 0$ and $f(b) < 0$. Prove that $f(x) = 0$ at least at one point x. [10]

6. What is a closed set? State the Heine-Borel theorem. Give an example where the covered set is bounded but not closed and the Heine-Borel property does not hold. [10]

7. $f(x)$ is defined and continuous on $[0, \infty)$. $f(x) \rightarrow 0$ as $x \longrightarrow \infty$. Prove that $f(x)$ is uniformly continuous on $[0, \infty)$. [12]

8. Give an example of a bounded continuous function on (0, 1) which is not uniformly continuous. [8]

9. Prove that the equation $x^3 + ax^2 + bx + c = 0$ has at least one real root. Here a, b and c are real numbers.

   Hint:- When x is very large, how does the function behave? So also when x is huge and negative? [12]

10. Give an example of a sequence $\{f_n(x)\}$ of functions defined on (0, 1) and converging pointwise to a limit function $f(x)$ and satisfying all of the following conditions.

    i) Each $f_n(x)$ is discontinuous at every point of (0,1),

    ii) The convergence is uniform on (0, 1), and

    iii) the limit function $f(x)$ is continuous on (0, 1).

    Briefly explain how your example satisfies all the conditions given. [10]

11. Consider the statement:

Let $f(x)$ be bounded on $[a, b]$. Also let $f(x)$ be continuous at all points of $[a, b]$ except at one point c where $a < c < b$. Then $f(x)$ must attain its supremum or its infimum (or both).

Give an example to show that this statement is false.

You may use the properties of $\sin x$, including the fact that $\sin x$ is continuous. [12]

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MID-YEAR EXAMINATION

Economics-2

Date: 11.1.74 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answer-scripts. Answer any two from Group A and Q.5 and any one from Group B.
Marks allotted for each question are given in brackets [].

Group A

1. A firm produces a homogeneous commodity under constant return to scale using only two inputs namely, labour and capital. Show that the firm can not increase the marginal products of both inputs at the same time by varying its capital-labour ratio. [25]

2. Show that for a first degree homogeneous production function involving one output and two inputs, namely labour and capital, the elasticity of input substitution is equal to the elasticity of the average product of labour with respect to its marginal product. [25]

3. Show that the production function of question 2 above is Cobb-Douglas if and only if the elasticity of input substitution is equal to unity. [25]

4. Taking the proposition of question 2 as proved, express the output as an explicit function of the inputs, assuming that the elasticity of input substitution is a constant different from unity. [25]

Group B

5. Give an analytical exposition of the interaction between the multiplier and the acceleration principle and derive the different time paths followed by income for different values of the multiplier and the accelerator. [30]

6. Develop the Keynesian theory of employment. Discuss briefly where it differs with the classical theory of employment. [20]

7. 'The way the budget affects the quantity of money and liquidity preference in the private sector is of considerable importance in determining the total effect of a given combination of government expenditure and revenue on national income'. Discuss the statement. [20]

8.a) Discuss the multiplier effect of an increase in government expenditure when net taxes are a rising function of national income at market prices.

b) Given that y_0 is the initial level of national income at market prices, b is the constant proportion of private disposable income which is spent on consumption, and t is the ratio of net tax receipts to national income at market prices, prove that an increase in government expenditure can lead to a doubling of national income at market prices if a budget deficit of the amount $y_0(1-b)(1-t)$ is incurred.

Assume the budget to be balanced initially, private investment expenditure to remain constant throughout and the economy to be a closed one. [10+10]=[20]

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MID-YEAR EXAMINATION

Statistics-8: Linear Estimation (Theory and Practical)

Date: 14.1.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Consider the model $E(\underline{Y}) = A'\underline{\theta}$ and suppose $\underline{a}'\underline{Y}$ is an UE of $\lambda'\underline{\theta}$. Prove that $\underline{a}'\underline{Y}$ is BLUE if and only if $Cov[\underline{a}'\underline{Y}, \underline{r}'\underline{Y}] \equiv 0$, for all linear functions $\underline{r}'\underline{Y}$ satisfying $E[\underline{r}'\underline{Y}] = 0$. [10]

2. Assume $\underline{Y}^{n\times 1} \sim N[A'\underline{\theta}, \sigma^2 I_n]$ with rank $(A) = r \le m < n$. Define $R_o^2 = \min_{\underline{\theta}} (\underline{Y} - A'\underline{\theta})'(\underline{Y} - A'\underline{\theta})$, $R_1^2 = \min_{\underline{\theta},\ H'\underline{\theta}=\underline{\xi}} (\underline{Y}-A'\underline{\theta})'(\underline{Y}-A'\underline{\theta})$ where rank $(H)^{m\times k} = k \le m$.

   a) Prove that $U = \frac{R_o^2}{\sigma^2} \sim \chi^2_{n-r}$, $V = \frac{R_1^2 - R_o^2}{\sigma^2} \sim \chi^2_k$ when $H'\underline{\theta} = \underline{\xi}$ holds, and that U and V are independent. [15]

   b) Hence develop a suitable test procedure for testing the hypothesis H_o: $H'\underline{\theta} = \underline{\xi}$. Clearly justify the procedure you propose. [5]

3. EITHER

   x_{ij} $(1 \le i \le m, 1 \le j \le n)$ are mn independent normal variables with common unknown variance σ^2 and $E(X_{ij}) = a_i + b_j$, $1 \le i \le m$, $1 \le j \le n$ where the a's and the b's are unknown parameters. Obtain a suitable te test for H_o: $b_1 = \dots = b_n$.

   (You need not derive any distribution result.) [15]

   OR

   Suppose there are two series of observations $\{(x_{1i}, y_{1i}), 1 \le i \le n_1\}$ and $\{(x_{2i}, y_{2i}), 1 \le i \le n_2\}$ where the x's are the values of a non-stochastic variable X and y's those of a stochastic variable Y. Assume that the regression of Y on X in both the cases is linear. Obtain a suitable test (stating clearly the assumptions you make) for H_o: the two regression lines coincide with the specified line $E(Y|X = x) = \alpha_o + \beta_o x$.

   (You need not derive any distribution result.) [1.]

p.t.o.

4. The following calculations are based on sets of observations on 15 individuals regarding four characters y, x_1, x_2 and x_3.

$$\bar{y} = 3.1685, \quad \bar{x}_1 = 2.2752, \quad \bar{x}_2 = 2.1523, \quad \bar{x}_3 = 2.1128,$$

$$\Sigma y^2 = 150.7178$$

Corrected sum of products

	x_1	x_2	x_3
y	0.0303	0.0441	0.0363

Inverse of the matrix of corrected sum of squares and sum of products.

	x_1	x_2	x_3
x_1	64.21	-15.57	-10.49
x_2		41.71	- 9.00
x_3			39.88

a) Obtain a linear forecasting formula for y in terms of x_1, x_2 and x_3.

b) Calculate the sample multiple correlation coefficient $R_{y.123}$ and test if the population multiple correlation coefficient $\rho_{y.123}$ is zero.

c) Test if all of x_1, x_2 and x_3 (which are similar in nature) are equally important for predicting y.

d) Test H_o: $(\beta_{y2.13} = 1, \ \beta_{y3.12} = 0)$.

e) Obtain 95 % confidence interval for $\beta_{y1.23}$.

The symbol $\beta_{yi.jk}$ stands for the population partial regression coefficient of y on x_i. State clearly the assumptions you make.

[5 +10+15+10+5]=[45

5. Practical Record Book. [10

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MID-YEAR EXAMINATION

Mathematics-5: Calculus

Date: 16.1.74 Maximum Marks: 100 Time: 3 hours

1. State and prove the implicit function theorem for functions of several variables. [20]

2. Let $u = f(x_1, x_2, x_3)$ be any differentiable function defined in a region of x_1, x_2, x_3 — space. Define the directional derivative $D_\alpha f$ at a point (x_1, x_2, x_3) in the direction making angles α_1, α_2 and α_3 with the coordinate axes. If $\bar{e}$ is a unit vector in this direction show that

$$D_\alpha f = \bar{e} \ \text{grad} \ f.$$

Prove that in the direction of gradient vector, the function increases most rapidly. [20]

3.a) Prove that $xy + \log xy = 1$ has unique solution for y in a neighbourhood of $(1, 1)$. [10]

b) Prove that for a homogeneous function f of first degree,

$$x^2 f_{xx} + y^2 f_{yy} + z^2 f_{zz} + \ldots + 2xy f_{xy} + \ldots = 0.$$

4.a) Find the expression for Laplacian in polar coordinate system (r, θ) by changing independent variables from rectangular coordinates (x, y) to (r, θ). [10]

b) Find the expression for curvature for the function $F(x, y) = 0$. For the ellipse

$$x^2 / a^2 + y^2 / b^2 = 1, \quad (a > b)$$

find the points where the curvature has maximum and minimum values. [10]

5.a) Define divergence and curl of a vector field. Find curl $\bar{u}$ where $\bar{u}$ has components

$$u_1 = x_3^2, \quad u_2 = x_1^2, \quad u_3 = x_2^2.$$ $\bar{u}$ is a vector in

$x_1 x_2 x_3$-space. Prove that curl grad $f = \bar{0}$ where f is a function of x_1, x_2, x_3 variables. [12]

b) Find the tangent plane of the surface

$$x^3 + 2xy^2 - 7z^3 + 3y + 1 = 0$$

at $(1, 1, 1)$. [8]

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MID-YEAR EXAMINATION

Statistics-7: Sample Surveys

Date: 18.1.74 Maximum Marks: 100 Time: 3 hours

Note: Answer Q. No.1 and any other three out of the rest. Marks allotted for each question are given in brackets [].

1. A yield survey on rice was carried out in West Godavari district of Andhra Pradesh in 1946-'47. Five villages were selected in each of the seven strata into which the district was divided, three fields were selected in each village and one plot of 1/100 acre was harvested in each field. The data are reproduced below. Assuming homogeneity of S^2_{ib}'s and $\bar{S}^2_{iw}$'s over i, obtain pooled values of s^2_b and $\bar{s}^2_w$ and get the estimates of S^2_b and $\bar{S}^2_w$. Finite multipliers at the sub-sampling stage may be ignored.

Stratum Number (i)	No. of villages in the population (N_i)	sample (n_i)	Sample Mean (oz./plot)	s^2_{ib}	$\bar{s}^2_{iw}$
1	88	5	347.5	1452.1	2791.5
2	142	5	297.8	1937.0	27422.5
3	119	5	201.1	7107.2	1864.0
4	90	5	438.9	9603.9	11824.0
5	114	5	282.9	20702.0	13628.2
6	102	5	301.9	2510.8	2007.5
7	146	5	186.7	3342.2	7441.8
Total	801	35	-	46655.2	66979.5

Give the estimate of the district mean yield and calculate its sampling variance. Assuming that the sample of villages is to be allocated in proportion to the number of villages in the several strata and that the cost in rupees of the survey is represented by $C = 7n + 2nm$, calculate the values of n and m that may be recommended for a subsequent survey in order that the district mean yield may be estimated with a standard error of 10 ozs. per plot for the minimum cost. [6+9+15]=[30]

2. Describe Harvitz-Thompson estimate of population mean and give an unbiased estimate of its variance. Obtain also the Yates' and Grundy's estimate of this variance and state a necessary condition under which the estimate always assumes positive values. [4+6+10+2]=[22]

3. Suggest an unbiased estimate of population mean under sampling with varying probabilities with replacement. Give also its variance and an unbiased estimate of the variance. Describe Des Raj's estimate and obtain an unbiased estimate of its variance. [(3+4+5)+(5+5)]=[22]

4. What is two-stage sampling? Assuming that first-stage units are of equal size, give an unbiased estimate of the population mean, and obtain its variance and an unbiased estimate of the variance (assume simple random sampling in both the stages). Hence show that under this sampling scheme,

$$\hat{p} = \frac{1}{n} \sum_{1}^{n} \hat{p}_i$$

p.t.o.

4. (contd.)

is an unbiased estimate of p, the population proportion of units possessing a given attribute, where n is the number of primary units selected and $\hat{p}_i$ denotes the observed proportion of units which possess the attribute under study in the i-th selected primary unit. [2+3+6+8+3]=[22]

5. Describe cluster sampling with an illustration. Assuming unequal clusters and simple random sampling of clusters, suggest <u>four</u> estimates of the population mean and examine their unbiasedness or otherwise. Give also their variances and the estimates of the variances. [2+4x5]=[22]

6. Describe systematic sampling with an illustration. Give unbiased estimate of population mean and obtain its variance under systematic sampling scheme. What is circular systematic sampling? Under this scheme, suggest an unbiased estimate of mean and give its variance. Compare systematic sampling with simple random sampling from efficiency point of view. [(3+4+4)+5+6]=[22]

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MID-YEAR EXAMINATION

General Science-5; Physics

Date: 19.1.74 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any three questions. Marks allotted for each question are given in brackets [ ].

1. Describe the essential constructional features of a triode. What are the constants of a triode? Explain the meaning of each of them.

The following data are given for a triode:

Plate voltage (volts)	Grid voltage (volts)	Plate current (mA)
320	- 4.0	1.2
320	- 3.0	1.6
280	- 3.0	1.2

Calculate the constants of the triode. [5+3+5+3]=[16]

2. Draw the circuit diagram of an R-C coupled amplifier employing a triode, and explain how it amplifies an alternating single voltage.

What is the function of the coupling capacitance? How does it affect the performance of the amplifier? What is the function of the grid-leak resistance? [3+6+2+3+2]=[16]

3. Explain, with suitable diagrams, the function of a triode as an oscillator.

A triode amplifier has an a.c. resistance of 20,000 Ω and an amplification factor of 20. Calculate the voltage obtainable across the anode load if it is a resistance of 30,000 Ω. [10+6]=[16]

4. Discuss the principle of modulation. How is this effected in practice?

The maximum and the minimum amplitudes of a modulated voltage wave are 18 V and 2V. Calculate the percentage of modulation. [5+6+5]=[16]

5. What is the ionosphere? What are the effects of the ionosphere on wave propagation? What are 'fading' and 'noises'? [6+5+2+3]=[16]

Neatness [2]

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PERIODICAL EXAMINATION

Application of Calculus

Date: 25.3.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Solve any four of the following differential equations.

i) $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.

ii) $x^2 y dy - (x^3 + y^3)dy = 0$.

iii) $(y - 3x + 3)\frac{dy}{dx} = 2y - x - 4$

iv) $(a^2 - 2xy - y^2)dx - (x+y)^2 dy = 0$.

v) $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$. [20]

2. Find the solution of the following differential equations.

i) $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

ii) $\frac{dy}{dx} + \frac{xy}{1-x^2} = xy^{1/2}$. [10]

3. Answer any one of the following:-

i) Determine the curve whose sub-tangent is n times the abscissa of the point of contact, find the particular curve which passes through the point (2, 3).

ii) A body falls from rest, assuming that the resistance of the air is proportional to the square of the velocity, find

a) its velocity at any instant.

b) the distance through which it has fallen. [10]

4. Solve any three of the following differential equations.

i) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

ii) $\frac{d^2y}{dx^2} - 4y = 2\sin\frac{x}{2}$

iii) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}\sin x$.

iv) $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. [20]

p.t.o.

5.a) What are the necessary conditions for a function of a complex variable $f(z)$ to be differentiable at a point $z = \zeta$?

b) Show that if the Cauchy-Riemann Equations are satisfied throughout the neighbourhood of the point ζ, however small that neighbourhood may be and if the four partial derivatives

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

are continuous at the point ζ, then the function $f(z) = u + iv$ of a complex variable $z = (x + iy)$ has a derivative with respect to z at the point ζ. [10]

6.a) What is an analytic function?

b) What is a singularity of an analytic function.

c) If $f(z)$ is analytic within and on a closed curve C then show that

$$\int_C f(z)\,dz = 0$$

assuming continuity of $f'(z)$ within and on C. [10]

7.a) If $f(z)$ be analytic within and on a closed curve C, then show that

$$f(z) = \frac{1}{2\pi i}\int_C \frac{f(t)}{t-z}\,dz.$$

b) If $f(t)$ be a continuous function of t on C and

$$F(z) = \frac{1}{2\pi i}\int_C \frac{f(t)\,dt}{t-z}$$

then show that $F(z)$ is analytic within C. [20]

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PERIODICAL EXAMINATION

Physics

Dated: 8.4.74 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer Q. No.5 and any two of the rest. Marks allotted for each question are given in brackets [ ].

1. Describe Thomson's method of determining e/m for cathode rays. What is the present accepted value of e/m for cathode rays? [15+4]=[19]

2. Describe Millikan's oil-drop method of measuring the electronic charge, giving a sketch of the apparatus used. [13+6]=[19]

3. Describe the Banibridge mass spectrograph and explain how it is used to identify positive ions of different masses. [14+5]=[19]

4. Give an elementary account of Bohr's theory of the hydrogen spectrum. Write the expression for the Balmer series. [14+5]=[19]

5. A radioactive sample has its half-life equal to 60 days. Calculate (i) its decay constant (ii) average or mean life (iii) the time required for 2/3 of the original number of atoms to disintegrate. [4 x 3]=[12]

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PERIODICAL EXAMINATION

Economics

Date: 15.4.74 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

Group A

Answer any two questions

1. A profit-maximizing firm can sell its product in two markets in which the prices (p_i, $i = 1,2$) are respectively connected with the quantities sold (q_i, $i = 1,2$) by the relations,
$p_1 = a_1q_1 + b_1$ and $p_2 = a_2q_2 + b_2$ where a_1, a_2, b_1, b_2 are given constants. Will the firm practise price-discrimination if

   i) $a_1 = a_2$ and $b_1 \neq b_2$,
   ii) $b_1 = b_2$ and $a_1 \neq a_2$?

   Justify your answer. [25]

2. Two firms are selling a homogeneous product in a market of competing buyers. If each firm in trying to maximize its profit treats its rival's output as a parameter, show how the problem of market equilibrium can be solved. [25]

3. Is there any valid ground for assuming, the existence of a solution of the problem in question 2? Justify your answer. [25]

4. In a simple duopoly, the profit-maximizing firms can act either as leader or as follower. Find the conditions under which the market will have (i) a determinate equilibrium and (ii) no determinate equilibrium. [25]

Group B

Answer any two questions.

5. Discuss the main features of foreign trade of India during the first three five year plans. [25]

6. Analyse the changing pattern of India's imports during the period of planning. Do you think that the changes are consistent with the requirements of a developing economy? Give reasons for your answer. [15+10]=[25]

7. Discuss the main directions of India's foreign trade indicating the balance of trade position with the major trade partners of India. [15+10]=[25]

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PERIODICAL EXAMINATION

Sampling Distributions and Tests of Hypotheses

Date: 22.4.74 Maximum Marks: 100 Time: 3 hours

1. Define χ^2-statistic with n.d.f. and derive its distribution. State and prove the additive-property of χ^2-distribution. Indicate some uses of the χ^2-statistic in tests of hypotheses.

[2+5+1+5+3] = [16]

2. Define Student's t-statistic with n.d.f. and derive its distribution. Indicate some uses of the t-statistic in tests of hypotheses.

OR

Define F-statistic with n_1, n_2 d.f. and derive its distribution. Indicate some uses of the F-statistic in tests of hypotheses.

[2+8+4] = [14]

3. Obtain the distribution of the sample median in odd samples of size $(2n + 1)$ from the uniform distribution over $[0, \theta]$.

[5]

4. $X_1, \ldots, X_k$ are independent normal with common mean m and unequal variances $\sigma_1^2, \ldots, \sigma_k^2$. Prove that $\sum_1^k \frac{1}{\sigma_i^2}(X_i - \bar{X}_w)^2$ has a central χ^2-dist. with $(k-1)$ d.f. where

$$\bar{X}_w = \left(\sum_1^k X_i/\sigma_i^2\right) \Big/ \sum_1^k \frac{1}{\sigma_i^2}.$$

[12]

OR

$Y_1, \ldots Y_n$ are independent normal with common variance σ^2 and $E(Y_i) = \alpha + \beta u_i$, $i = 1, \ldots, n\ (>2)$, $\sum_1^n u_i = 0$, $\sum_1^n u_i^2 > 0$.

Prove that $\dfrac{(b - \beta)\sqrt{\Sigma u_i^2}}{\sqrt{\left\{\sum_1^n (Y_i - \bar{Y})^2 - b^2 \sum_1^n u_i^2\right\}/n-2}}$ is a central t-variable with $(n-2)$ d.f.

[12]

5. Indicate the distributions of (i) $\dfrac{X_1 - X_2}{|X_1 + X_2|}$ where X_1, X_2 are independent normal with mean 0 and variance $\sigma^2 > 0$.

[5]

(ii) $\dfrac{(X_1 - X_2)^2/3}{2(X_1 + X_2 + X_3)^2 + (X_1 + X_2 - 2X_3)^2}$ where X_1, X_2, X_3 are independent normal with mean zero and variance $\sigma^2 > 0$.

[8]

GO ON TO THE NEXT PAGE

6. X_1, X_2, X_3 are independent random variables having absolutely continuous distributions. If F_i is the c.d.f. of X_i $(i = 1,2,3)$ prove that $-2 \sum_{1}^{3} \log F_i(X_i)$ is a χ^2_6 variable.

[10]

7. The mean and standard deviation (s.d.) of scores of 16 students in some test are 45.5 and 9.51 respectively. Test at 5°/. level whether

   a) the population mean is 42.7

   b) the population s.d. is 8.0

State your assumptions clearly. [6 + 6] = [12]

8. A six-faced die was thrown 9000 times, and a 5 or a 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate that the die is unbiased ? (level 5°/.)

[8]

9. A correlation coefficient of 0.6 is obtained in a sample of 27 pairs from a bivariate normal population. Test whether the population correlation coefficient differs significantly from zero at 5°/. level.

[5]

10. Practical records. [5]

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PERIODICAL EXAMINATION

Complex Variable

Date: 29.4.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1.a) What is meant by a function f of a complex variable z.

i) being continuous at a point z_o

ii) having a derivative at z_o

iii) being analytic at z_o?

If $f(z)$ is analytic at z_o, does it have a derivative at z_o? If it has a derivative at z_o, is it analytic there? Give reasons for your answer. [10]

b) Find the regions, if any, in which the following functions are analytic and find their derivatives there:

i) $f(z) = e^{\frac{1}{z^2}}$ if $z \neq 0$
$= 0$ if $z = 0$

ii) $f(z) = \frac{1}{z}$ if $z \neq 0$
$= 0$ if $z = 0$

iii) $f(z)$ = Real part of z. [15]

2.a) If $u+iv$ is analytic in some domain, u, v being real valued functions of a complex variable z, show that $-v+iu$ is analytic in the same domain. [15]

b) If $u+iv$ and $u-iv$ are analytic in some domain, show that $u+iv$ is constant in that domain. [10]

3. Suppose f is analytic within and on a closed contour C, show that the value of f as well as that of its derivative at any point interior to C is completely determined by the values f takes on C. [25]

4. Let C be the circle $|z| = 2$. Find the values of the following:

i) $\int_C \frac{2z}{z-1}\,dz$; (ii) $\int_C e^z\,dz$; (iii) $\int_C \frac{1}{(z+4)^2}\,dz$;

iv) $\int_C \frac{e^{2z}}{z^2-2iz-1}\,dz$; (v) $\int_C \frac{1}{(z-1)^2}\,dz$. [25]

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PERIODICAL EXAMINATION

Design of Experiments

Date: 27.5.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Two brands of sleeping pills A and B are to be compared for their effectiveness in producing extra hours of sleep. Analyse the following data to test the hypothesis that A and B do not differ, under the two following alternative designs of the experiment.

a) For each serial number in the table below, the figures in the columns A and B represent extra hours of sleep for the same patient to whom pills A and B have been administered on two different occasions;

b) The figures in the columns A and B represent two independent samples of 10 individuals each, members of one sample having been given the pill A and members of the other the pill B.

sl. no.	A	B
1	+ 0.7	+ 1.9
2	- 1.6	+ 0.8
3	- 0.2	+ 1.1
4	- 1.2	+ 0.1
5	- 0.1	- 0.1
6	+ 3.4	+ 4.4
7	+ 3.7	+ 5.5
8	+ 0.8	+ 1.6
9	0.0	+ 4.6
10	+ 2.0	+ 3.4

On the basis of your results comment on the relative sensitivity of the two designs. If possible, state the reasons for any difference in sensitivity observed. [25]

2. In a randomised block design and with the additive form of the equations of estimation,

a) Show that the estimation space can be divided up into three mutually orthogonal sub-spaces, generated respectively by contrasts among block totals, contrasts among treatment totals and the grand total; indicate the rank of each sub-space. [6]

b) Show that the best linear estimate of any contrast among treatment effects is the corresponding contrast among the treatment means [5]

c) If there are 3 blocks with block effects $\beta_1, \beta_2, \beta_3$ and 4 treatments with treatment effects $\tau_1, \tau_2, \tau_3, \tau_4$, examine if the following parametric functions are estimable and, for those which are estimable, write down the best estimates:

i) $\beta_1 - 2\beta_2 + \beta_3$; (ii) $\beta_1 + \beta_2$;

iii) $(\beta_1 - \beta_2) - (\tau_1 - \tau_2)$. [9]

pt
p.t.o.

3. An experiment was conducted in a 4x4 Latin Square design, to compare the selling potential of 4 types of apples.

4 shops were each observed on 4 days of the week; 4 types of apples (A, B, C, D) were each displayed once in each shop and once on each day. The data consist of weight of apples sold per 100 customers.

Day of week	Shop number 1	2	3	4	Day total
MONDAY	A 14	B 8	C 40	D 48	110
TUESDAY	B 20	A 22	D 48	C 25	115
WEDNESDAY	D 24	C 12	B 12	A 27	75
THURSDAY	C 31	D 16	A 32	B 22	101
Shop total	89	58	132	122	Grand total
Totals for types	A 95	B 62	C 108	D 136	401

a) Test for the equality of sales of the 4 types of apples. [[illegible]]

b) Test if the sales potentials of the 4 types obey the relation

$$A + C = B + D .$$ [7]

4.a) Denoting by A, B, C, D the treatments in a balanced incomplete block design with the parameters

$$b = 4, \quad k = 3, \quad v = 4, \quad r = 3, \quad \lambda = 2,$$

write down the full layout of such a design. [12]

b) Show that in a b.i.b. design the best estimate of a linear function of only the treatment effects must be in the form of a linear function of the 'adjusted yields' Q_j . [8]

c) Show that the coefficient of correlation been Q_j and $Q_{j'}$ is $-1/(v-1)$. [8]

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PERIODICAL EXAMINATION

Probability

Date: 20.5.74 Maximum Marks: 100 Time: 3 hours

Note: The whole paper carries 120 marks. Answer as much as you can. Maximum marks one can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Write down the characterizing properties of the distribution function of an n-dimensional random variable. [8]

b) If X and Y have absolutely continuous distributions, is the joint distribution of X and Y absolutely continuous? Is the converse true? Give complete proofs. [14]

2. Let X_1, X_2 be random variables such that $X_1 \sim U[0, 1]$ and the conditional distribution of X_2 given $X_1 = x_1$ is $U[0, x_1]$. Find

a) the joint density function of X_1 and X_2. [6]

b) the distribution function of X_1 and X_2. [6]

c) the density function of $X_1 - X_2$. [10]

3. Let $X = (X_1, \ldots, X_p)'$ be such that $t'X$ is univariate normal for all vectors t.

a) Find the characteristic function of X and show that if $X_1, \ldots, X_p$ are pairwise uncorrelated, they are independent. [7]

b) If X is partitioned into two parts $\underset{\sim}{X}_1$ and $\underset{\sim}{X}_2$ with k and p-k components respectively find the conditional distribution of $\underset{\sim}{X}_2$ given $\underset{\sim}{X}_1$. (You may assume that $\underset{\sim}{X}_1$ is non-singular.) Show that the regression of X_p on $X_1, \ldots, X_{p-1}$ is linear. [13]

4.a) Let $X_1, X_2, \ldots, X_n$ be independent standard Normal variates and $Q_i = X'A_iX$ be a quadratic form in $X = (X_1, \ldots, X_n)$ fo for $i = 1, \ldots, k$, such that $X'X = Q_1 + \ldots + Q_k$. Then show that each Q_i has a chi-square distribution and $Q_1, \ldots, Q_n$ are independent iff $n = n_1 + \ldots + n_k$ where n_i = rank of A_i. [15]

b) Deduce from the result in (a) that if $X_1, \ldots, X_n$ is a sam sample from N(0, 1), then $\bar{X}^2$ and $\Sigma (X_i - \bar{X})^2$ are independently distributed. How can you conclude from this that $\bar{X}$ and $\Sigma (X_i - \bar{X})^2$ are independent? [10]

p.t.o.

5.a) If $X_1, X_2, \ldots$ is a sequence of i.i.d. random variables with common mean μ and common variance σ^2, show that $\frac{S_n - n\mu}{\sqrt{n}\sigma}$ converges to $N(0, 1)$ in distribution. [10]

b) State the Lindeberg-Feller central limit theorem and deduce the result in (a) from that. [6]

6. Two points are selected at random from [0, 1]. Find the probability that the three segments can be the sides of a triangle. [15]

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ANNUAL EXAMINATION

Mathematics-5: Calculus

Date: 24.6.74 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. All questions carry equal marks.

1. Find by contour integration the values of any two of the following integrals

i) $\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx$ (ii) $\int_0^\infty \frac{\sin x}{x} dx$, (iii) $\int_0^\infty \frac{\cos x}{\sqrt{x}} dx$

iv) $\int_0^\infty \frac{\sin ax}{\sinh \pi x} dx \ (a > 0)$.

2. Find a series solution in the neighbourhood of an ordinary point of a homogeneous ordinary differential equation of second order such that the solution together with its first derivative acquire arbitrarily assigned values at that point. Find also the radius of convergence of the series.

3. What is a regular singularity of an ordinary differential equation ? When is an ordinary homogeneous equation of second order of Fuch's type ? When there are γ singularities in the finite part of the plane with assigned exponents, find out the forms of the equations of Fuch's type, the point at infinity being either an ordinary point of the differential equation or a regular singularity.

4. Show that the hypergeometric equation is an equation of the Fuch's type. What are the singularities of this differential equation ? Find the solutions in the neighbourhood of the origin and at infinity.

5. Show that the differential equation

$$(1 - z^2) \frac{d^2 w}{dz^2} - 2z \frac{dw}{dz} + n(n + 1) w = 0$$

is an equation of the Fuch's type.

Find out the integrals in the neighbourhood of the origin. What is Legendres polynomial ? Deduce Rodrigue's formula for Legendres polynomial.

6. Is Bessel's equation an equation of the Fuch's type? Find out the integrals in the neighbourhood of the origin.

7.a) Find out the solution of Laplace's equation in polar co-ordinates assuming that the solution can be expressed as a product of three functions which are functions if r, θ and Ø alone respectively.

b) Find out the solutions of the wave equation in polar co-ordinates assuming that it can be expressed as a product of four functions which are functions of t, r, θ and Ø alone respectively.

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ANNUAL EXAMINATION

Mathematics-6: Complex Variables

Date: 25.6.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Let C be the contour given by $z = 2e^{i\theta}$, $0 \leq \theta \leq \frac{\pi}{2}$.
Show that $\left| \int_C \frac{dz}{z^2+1} \right| \leq \frac{\pi}{3}$. [10]

2. Let $f = u + iv$ be entire, u, v being real. Let m be a real constant such that $u \leq m$ at all points.
Show that f is constant. [15]

3. State and prove the formula for calculating the radius of convergence of a power series

$$\sum_{n=0}^{\infty} a_n z^n.$$

What can you say about the behaviour of a power series on its circle of convergence? Give reasons for your answer. [25]

4. Define a pole of order m of a function. If ϕ is analytic at z_0, $\phi(z_0) \neq 0$ and [illegible]

$$f(z) = \frac{\phi(z)}{(z - z_0)^m}$$

show that f has a pole of order m at z_0.
Find the residue of f at z_0. Show that $|f(z)| \to \infty$ as $z \to z_0$. [20]

5. Calculate (i) $\int_{-\infty}^{\infty} \frac{\cos x}{(x+a)^2 + b^2}\, dx$,

(ii) $\int_0^{\infty} \frac{x^2\, dx}{(x^2+1)^2}$. [30]

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Statistics-10: Probability

Date: 27.6.74 Maximum Marks: 100 Time: 4 hours

Note: The whole paper carries 110 marks. Answer as much as you can. Maximum marks one can score is 100. Marks allotted for each question are given in brackets [ ].

1. State the weak compactness lemma for distribution functions. Assuming this (and the values of any standard integrals) prove that if the characteristic functions ϕ_n of a sequence of distribution functions F_n converge to a function ϕ continuous at 0, then ϕ is the characteristic function of a distribution function F and $F_n \xrightarrow{w} F$. [20]

2.a) If X is a p-dimensional random variable with dispersion matrix Σ and mean 0, show that X belongs to the column space of Σ with probability 1. [8]

b) Show that X has (possibly singular) $N_p(\mu, \Sigma)$ distribution iff $X = \mu + BY$ a.s. where $Y \cap N_p(0, I)$ and $\Sigma = BB'$. Hence obtain the density function of X when Σ is nonsingular. [13]

3. Show that if $X_1, X_2, \ldots, X_n$ is a sample from $N(0, 1)$ then $X'AX$ has a chi-square distribution iff A is idempotent. What is the number of degrees of freedom? [12]

4. In the gambler's ruin problem, find the probability of ultimate ruin of the gambler and also the expected duration of play if the initial capitals of the gambler and his opponent are z, $a - z$ and p is the probability of the gambler's winning a game. Show that if the stake is doubled (to 2 units of money), the probability of ruin decreases or increases according as $p < 1/2$ or $p > 1/2$. [22]

5.a) Define a Markov Chain with stationary transition probabilities. State and prove the Chapman-Kolmogorov equations. Show that the matrix of n-step transition probabilities is the nth power of the matrix of 1-step transition probabilities. [12]

b) Define the classes in a M.C Define when a state is essential and show that this is a class property. [7]

c) Prove that in a M.C. with a finite number of states, all the states cannot be inessential. [6]

6. Viva Voce [10]

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Economics-2

Date: 29.6.74 Maximum Marks: 100 Time: 4 hours

Note: Answer Groups A and B in separate answerscripts Marks allotted for each question are given in brackets [ ]. Answer any two questions from Group A and any four questions from Group B.

Group A:

1.a) Taking as given the conditions for equilibrium of a two commodity consumer who maximizes utility subject to his budget restraint, derive the Slutsky equation. [15]

b) Show that if this consumer's demand for a commodity is perfectly inelastic, then he must regard it as an inferior good. [5]

2. A firm has a first degree homogeneous production function with the following properties:

i) it involves a single output and only two inputs, namely labour and capital ;

ii) the quantities of output and inputs are strictly positive;

iii) the marginal product of each input is strictly positive;

iv) the elasticity of input substitution is equal to unity.

Prove that the firm's production function is Cobb-Douglas. [20]

3. Assuming that a duopoly problem can be regarded as a two-person constant sum game, construct a model to illustrate the proposition that there is a solution of the problem in mixed strategies if the pay-off matrix of neither player has a saddle point. [20]

4. Derive (a) the Hicks-Walrasian and (b) the Marshallian conditions for the stability of equilibrium in a competitive market for a given commodity. Show that the conditions (a) and (b) are mutually inconsistent if both demand and supply curves are negatively sloped. [20]

p.t.o.

Group D:

5. Discuss the changes that have taken place in the nature of India's imports during the period of planning. Show how these changes reflect the changing pattern of the Indian economy. [10+5]=[15]

6. Examine the main types of co-operative farming and examine their usefulness under Indian conditions. [10+5]=[15]

7. Describe the present system of marketing of agricultural produce in India. Do you agree with the view that 'State take-over of foodgrains trade' would be an effective step to improve the present system ? Give reasons for your answer. [10+5]=[15]

8. Critically examine the main provisions of the Industrial Policy Resolution of 1956. [15]

9. Give an account of the functions of the Industrial Development Bank of India indicating the main objectives of its establishment. [15]

10. Discuss the salient features of foreign collaboration arrangements in Indian industry. [15]

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Sampling Distributions and Tests of Hypotheses

Date: 1.7.74 Maximum Marks: 100 Time: 4 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

Group A: Maximum Marks : 60

1. Let $v \sim \chi_r^2$ and n be a positive integral function of v. If, given v, $a_1, a_2, \ldots, a_n$ are constants so determined that $\sum_1^n a_i^2 = 1/v$, and also, given v, the random variables $X_1, \ldots, X_n$ are independently distributed as N(0, 1), prove that $\sqrt{r} \sum_1^n a_i X_i$ is unconditionally distributed as a Student's t with r d.f. [15]

2. $X_1, X_2, \ldots, X_p$ follow a joint multinormal distribution with zero means and dispersion matrix which is idempotent and of rank $r(< p)$. Show that

$$\sum_1^p X_i^2 \sim \chi_r^2$$ [5]

3. Let $Y, X_1, \ldots, X_p$ follow a (p+1)-variate normal distribution with certain mean vector and certain non-singular dispersion matrix. Let $(y_\lambda, x_{1\lambda}, \ldots, x_{p\lambda})$, $\lambda = 1, \ldots, n$ be a random sample of size n from this (p+1)-variate distribution. Define

$$S_{yy} = \sum_1^n (y_\lambda - \bar{y})^2, \quad S_{yi} = \sum_{\lambda=1}^n y_\lambda (x_{i\lambda} - \bar{x}_i),$$

$$i = 1, \ldots, p, \quad S_{ij} = \sum_{\lambda=1}^n (x_{i\lambda} - \bar{x}_i)(x_{j\lambda} - \bar{x}_j), \quad i, j = 1, \ldots, p.$$

Obtain the sampling distribution of

$$Z = \begin{vmatrix} S_{yy} & S_{y1} & \cdots & S_{yp} \\ S_{y1} & S_{11} & \cdots & S_{1p} \\ \vdots & \vdots & & \vdots \\ S_{yp} & S_{p1} & \cdots & S_{pp} \end{vmatrix} \div \begin{vmatrix} S_{11} & \cdots & S_{1p} \\ \vdots & & \vdots \\ S_{p1} & \cdots & S_{pp} \end{vmatrix}$$ [15]

4. Assume the same formulation as in Q.3.

a) Define the sample multiple correlation coefficient $R_{y.12..p}$. [2]

b) Obtain the distribution of R^2 regarding $(x_{1\lambda}, \ldots, x_{p\lambda})$; $\lambda = 1, \ldots, n$ as fixed and then obtain the unconditional distribution of R^2. [8]

c) Hence, or otherwise, prove that $\frac{R^2/p}{(1-R^2)/n-p-1} \sim F_{p,n-p-1}$ when $\rho_{y.12\ldots p} = 0$. [5]

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p.t.o.

OR

Let $Y, Z, X_1, \ldots, X_p$ follow a (p+2)-variate normal distribution with certain mean vector and certain non-singular dispersion matrix. Let $(y_\lambda, z_\lambda, x_{1\lambda}, \ldots, x_{p\lambda})$, $n = 1, \ldots, n$ be a random sample of size n from this (p+2)-variate distribution.

a) Define the sample partial correlation coefficient $r_{yz.12..p}$ of order p. [2]

b) Prove that $r_{yz.12..p}$ can be recognised as the ordinary product moment correlation coefficient based on a sample of size n-p-1 from a bivariate normal population with means zero. [3]

c) Using (b), prove that $\frac{r_{yz.12..p}\sqrt{n-p-2}}{\sqrt{1-r^2_{yz.12..p}}} \sim t_{n-p-2}$ when $\rho_{yz.12..p} = 0$. [5]

5. Attempt any two of the following:

a) X and Y are independent $N(0, \sigma^2)$ variables. Derive the distribution of $Z = \frac{X}{Y}$. [5]

b) $X_1, X_2, \ldots, X_n$ are independent random variables and $X_i \sim N(m_i, 1)$, $i = 1, \ldots, n$. Obtain the distribution of $\sum_1^n (X_i - \bar{X})^2$ where $\bar{X} = n^{-1} \sum_1^n X_i$. [5]

c) If $\chi_1^2 \sim \chi^2_{n_1}(\lambda_1)$, $\chi_2^2 \sim \chi^2_{n_2}(\lambda_2)$ and χ_1^2, χ_2^2 are independent, prove that $\chi_1^2 + \chi_2^2 \sim \chi^2_{n_1+n_2}(\lambda_1 + \lambda_2)$. [5]

Group B: Maximum Marks: 40

Answer all the questions. State clearly your assumption if any. For question 9, answer either 9(a) or 9(b) and 9(c) both. Marks for each question are given within brackets. Carry out all the tests at 5% level.

6.a) From a normal population with mean μ and variance σ^2 (both unknown), a random sample of size 10 yielded mean as 1.79. Test the hypothesis

$$H_0 : e^{\mu + \frac{\sigma^2}{2}} = \sqrt{e}, \quad \sigma = 2$$

against the alternative

$$H_1 : e^{\mu + \frac{\sigma^2}{2}} \neq \sqrt{e}, \quad \sigma = 2.$$ [4]

OR

b) Random samples of sizes 21 and 11 were drawn from two independent normal populations (means not specified) with variances σ_1^2 and σ_2^2 respectively having the sum of squares of the deviation from the sample means 26.94 and 11.59. Test the hypothesis $H_0: \sigma_1 = 2\sigma_2$ against the alternative $H_1 : \sigma_1 \neq 2\sigma_2$. [4]

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ANNUAL EXAMINATION

General Science-5: Physics

Date: 3.7.74 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. Marks allotted for each question are given in brackets [ ].

1. Deduce an expression for the change in wavelength in Compton scattering. An x-ray photon of wavelength 0.1 Å ($1 Å = 10^{-8}$ cm.) is scattered at an angle of 90' with its original direction after collision with an electron at rest. Find the energy it will loss on collision. [12+8]=[20]

2. How did de Broglie infer that an electron of mass m moving with a velocity v would correspond to a wave of wavelength h/mv ? Give an account of G.P. Thomson's experiment lending support to the above inference. Calculate the wavelength associated with a proton moving with a velocity of 10^7 cm./sec. [5+9+6]=[20]

3. Give an elementary account of Bhor's theory of the hydrogen spectrum. Show that the radius of the first Bohr orbit for hydrogen is about 0.5 Å. Use the values of the constants e, h and m. [14+6]=[20]

4. What is Raman effect ? Give an elementary theory for the phenomenon. Give an experimental arrangement to obtain Raman lines with liquids. Explain why Stokes' lines are more intense than anti-Stokes' lines. [4+7+6+3]=[20]

5. Give a general account of the different quantum numbers. What is Pauli's exclusion principle ? Show how this principle can explain the electron configuration of an atom, illustrating your answer with the shell for which n = 2. [8+4+8]=[20]

6. Deduce relations between the disintegration constant, the half-life and the mean life of a radioactive substance. A radioactive substance disintegrates for a time equal to its mean life. Calculate the fraction of the original substance left behind. [12+8]=[20]

7. Describe the essential features of an ordinary cyclotron and the principle of its operation. Explain the difference between a cyclotron and a linear accelerator. [8+3+4]=[20]

8. What are the constants of a triode ? Calculate the constants for a triode from the following data :

Plate voltage (volts)	Grid voltage (volts)	Plate current (mA)
320	- 4.0	1.2
320	- 3.0	1.6
280	- 3.0	1.2

Draw (do not describe, but label the diagram) the circuit diagram of an RC-coupled amplifier employing a triode. What are the functions of the coupling capacitance and the grid-leak resistance? [6+4+4+3+3]=[20]

9. Write explanatory notes on any two of the following:

a) Cosmic radiation
b) Heisenberg's uncertainty relation
c) Geiger Muller counter
d) Bainbridge mass spectrograph. [10+10]=[20]

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ANNUAL EXAMINATION

Design of Experiments (Theory and Practical)

Date: 5.7.74 Maximum Marks: 100 Time: 4 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Write briefly to bring about the utility of the following in the conduct of experiments:

a) Randomisation

b) Replication

c) Control through i) choice of experimental units
ii) designed allocation of treatments to units. [18]

2. In an experiment to find the effect of a particular diet on the weight of animals, 15 animals were first weighed after feeding them on normal diet and with a lapse of time, weighed again after feeding them with the new diet. It was then discovered that the values of weights were not too reliable, although the direction of change of weight (i.e., increase or decrease) from the first to the second feeding could be relied upon. It was noticed that 9 animals had increases in weight and the rest had decreases. Test whether (a) the new diet has no advantage over the normal diet ;

(b) the new diet gives at least a 2/3 chance of weight increase. [14]

3. Five treatments have been submitted for comparison. Write down one layout each of a (i) completely randomised, (ii) randomised block, (iii) Latin square and (iv) balanced incomplete block design, for carrying out the experiment,

(a) taking care that in each design, every treatment should be replicated at least 4 times;

(b) in each case indicating the situation when the corresponding design is appropriate. [10]

4.a) For a balanced incomplete block design with parameters b, v, r, k, λ,

i) Show that

$$E(Q_j) = \sum_{j'=1}^{v} c_{jj'} \tau_{j'} ,$$

where Q_j is the j^{th} 'adjusted yield',

$c_{jj} = r(1 - \frac{1}{k})$; $c_{jj'} = -\frac{\lambda}{k}$, $j \neq j'$;

and the τ_j's are the treatment effects. [7]

ii) Prove that a linear function of only the τ_j's is estimable if and only if it is a contrast. [7]

p.t.o.

4.b) For a b.i.b. design with the above parameters, write down the analysis of variance table, and derive the anova table for a randomised block design as a special case. [6]

5. To test the hypothesis that the vegetative growth of 'Ranvolfia serpentina' becomes more when 'deflorated', six strips of land (to be called blocks) were taken and each strip divided into two equal plots. Six plants were planted in each plot. One of the plots in each block was chosen at random and the plants on it 'deflorated'. Twelve months later, the total moisture-free weight of the six plants on each plot was recorded. The data are given in the table below:

Block	Treatment	
	Deflorated	Control
1	290.56*	267.41
2	327.81	229.08
3	293.77	229.95
4	304.58	246.00
5	293.33	268.81
6	303.92	240.92

* estimated figure

One of the plants of block 1 in the deflorated category having died, its weight was estimated from the remaining five, and then the total made.

Test the relevant hypothesis regarding defloration

a) taking the estimated figure as an observation;

b) rejecting the estimated figure and considering it to be missing. [5]