

## MODEL FERTILITY TABLES

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**SUMMARY.** In an attempt to estimate the age-specific fertility rates (a.s.f.r.) for countries lacking necessary data, it was found that reasonably accurate estimates could be obtained from the census age-distribution. Examinations of available data have shown fairly large values of correlation coefficients between the a.s.f.r. (for quinquennial age-groups), and the general fertility rate (g.f.r.), as well as between the a.s.f.r. themselves, specially for successive age-groups. The g.f.r. can be obtained more or less accurately from the census age-distribution. Table of a.s.f.r. for different values of g.f.r., as well as of the sex-age adjusted birth rate (s.a.b.r.) developed by the UN, computed by using linear regression model, are expected to serve as a ready reference to the users of such data.

### 1. INTRODUCTION

1.1. The model life tables (UN Population Studies No. 25) prepared by the United Nations furnish a plausible set of age-specific mortality rates and hence of other life table functions, for countries, lacking primary data necessary to construct such indices. The only *a priori* information required for the purpose is a fairly reliable estimate of the expectation of life at birth.

1.2. Apart from their usefulness in the derivation of the past and current sex-age specific mortality rates, the necessity of which needs no mention, the model life tables were constructed with a view to make population projections. Assumptions being made regarding expectations of life at birth for future years, the corresponding sets of mortality rates or survivorship functions can be calculated from the model life tables separately for the two sexes. Thus the derivation of one of the components required for population projections for countries without adequate data, has been simplified to a great extent.

1.3. Although the projection of population by the method of components requires the knowledge of mortality rates by sex and age-groups, detailed information on the fertility component, i.e., the age specific fertility rates, is not essential for the purpose. Estimates of the total number of births during the intervening period and of the sex-ratio at birth are just enough. The sex-age adjusted birth rate<sup>1</sup> developed by the UN furnishes a useful tool to deal with the fertility component of the population projection.

<sup>1</sup> The sex-age adjusted birth rate was constructed with a view to eliminate the differential of sex-age composition between populations. For this purpose, a weighting system was developed for the quinquennial distribution of women in the age-range 15-44. For details, see UN Population Studies No. 25.

## SANKHYĀ : THE INDIAN JOURNAL OF STATISTICS : SERIES B

1.4. The object of this study is to construct a set of model fertility tables which will provide probable sets of specific fertility rates when some non-age specific fertility index like the general fertility rate is known. No techniques are yet known for estimating the specific fertility rates from census age distributions (although mortality rates however approximate can be built from them), which are the only population statistics available in many parts of the world till now.

### 2. DATA AND METHODS OF PROCESSING

2.1. Our approach to the problem is as follows. The UN Demographic Year Book, 1959 provides information with respect to age-specific fertility rates for a number of countries and for different time periods. For some of those countries information is available for a number of years. In all, we have selected 80 sets of age-specific fertility rates relating to 57 countries (vide Table 3). For each set of tables we have taken into consideration women in the age-group 15-44 (although data are available beyond this age-span), since the contribution to the total number of births by the remaining women, is relatively small.

One may question the use of the age-specific fertility rates as given in the UN Demographic Year Book, because the figures for some of the countries are not sufficiently accurate and the data relate mostly to current years. The procedure adopted, therefore, is to select those countries for which the vital rates are known to be fairly accurate. Further, the use of current data may be justified by the fact that the range of general fertility rates or sex-age adjusted birth rates as observed for the selected countries is wide enough to include extreme values of low as well as of high fertility. Moreover, we may perhaps assume that countries which have low fertility at present, had at some time or other in the past, the same fertility pattern as that found in countries of currently high fertility.<sup>1</sup>

2.2. For each set of fertility rates calculations have been made of the following characteristics taking all (including non-legitimate) births into consideration :

(a) General fertility rate (g.f.r.) (births per thousand females aged 15-44<sup>2</sup> per year).

(b) Sex-age adjusted birth rate (s.a.b.r.) (ratio of total birth per year to a weighted sum of mothers aged 15-44, expressed as per 1,000, the weights for the six quinquennial age-groups being 1, 7, 7, 6, 4 and 1 respectively).

<sup>1</sup> Bourgeois-Pichat has, however, observed a different fertility pattern for many of the European countries in the nineteenth century. For example, the specific fertility rates at that period often reached a maximum for the age-group 30-34, instead of 25-30, a pattern usually observed at present. Such a pattern is currently observed in Ireland where age at marriage is very high. But unless the population has, during this period, undergone a significant biological change or has lowered its age at marriage, such findings cannot perhaps meet the challenge of reliability of data. Bourgeois-Pichat, however, feels that the selection of current data might be more appropriate for estimating fertility rates for the underdeveloped countries today.

<sup>2</sup> We have taken the age-bracket 15-44 as the fertile period although births are known to occur on either side of this range. In the calculation of g.f.r. we have preferred to exclude such births which are comparatively small in number. Inclusion of any age-group 45-49 might also have vitiated the comparability of g.f.r. and s.a.b.r.

## MODEL FERTILITY TABLES

Computations were then made of the following :

- (1) Correlation coefficients between the general fertility rate (g.f.r.) and age-specific fertility rates (a.s.f.r.).
- (2) Correlation coefficients between the sex-age adjusted birth rate (s.a.b.r.) and a.s.f.r.'s.
- (3) The correlation matrix of the a.s.f.r.'s.

2.3. The correlation matrix of the age-specific fertility rates is shown below. The predictability decreases with the increase in the gap separating the age-groups. The correlation coefficients between contiguous age-groups increase along the age-axis (with one exception), and the same feature is noticeable for the age-groups separated by a fixed interval. The explanation is provided by the possibility of stabilization of relationships among age-specific fertility rates at advanced ages and comparatively greater variability at the younger ages. For the latter, the age at marriage, child bearing practices, etc., are obviously destabilizing factors, which have not been controlled, because of limitations of data.

TABLE 1. CORRELATION MATRIX OF AGE-SPECIFIC  
FERTILITY RATES

age-group	15-19	20-24	25-29	30-34	35-39	40-44
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15-19	—	0.80	0.51	0.43	0.38	0.33
20-24		—	0.82	0.69	0.63	0.51
25-29			—	0.94	0.80	0.70
30-34				—	0.90	0.90
35-39					—	0.85
40-44						—

The correlation coefficients between successive age-groups seem to be quite high, so that plausible estimates of age-specific fertility rates could be made, if the fertility rate for one of the age-groups is known or can be estimated *a priori*. Alternatively, tables of rates can be constructed starting from several arbitrary values for any one specific age-group. These rates could then be applied to a given age-distribution and compared with an over-all rate or some such index available through other sources. Given the age distribution, the construction of such an index is usually possible if reliable estimates of children below 5 and of appropriate survivorship probabilities could be made.

The correlation coefficients (apart from a bit of spuriousness<sup>4</sup>) shown in Table 2, are quite large except for the first age-group. The coefficient for the age-group 40-44 also may not be regarded as sufficiently large. The similarity (or more properly the identity) between the two sets is not unexpected, as the different weighting systems of

<sup>4</sup> The spuriousness, perhaps, raises the value of the correlation coefficient but does not introduce any methodological problem.

## SANKHYĀ : THE INDIAN JOURNAL OF STATISTICS : SERIES II

TABLE 2. CORRELATION COEFFICIENTS BETWEEN AGE-SPECIFIC FERTILITY RATES (a.s.f.r.) AND GENERAL FERTILITY RATE (g.f.r.) AND SEX-AGE ADJUSTED BIRTH RATE (s.a.b.r.)

age-group	correlation coefficients	
	g.f.r.	s.a.b.r.
(1)	(2)	(3)
15-19	0.69	0.70
20-24	0.89	0.89
25-29	0.85	0.93
30-34	0.93	0.93
35-39	0.89	0.89
40-44	0.81	0.81

the female population used in the derivation of g.f.r. and s.a.b.r. should have little influence on the correlation coefficients. In spite of that, both of them are considered to be useful and have been treated separately. The g.f.r. is widely used, easily computed and has a large range of variation. The s.a.b.r., on the other hand is readily comparable with the crude birth rate and at the same time is practically independent of the age-sex composition of the population. With these facts in view, model fertility tables have been constructed which give sets of a.s.f.r.'s for suitable values of g.f.r. as well as of s.a.b.r. These are shown in Tables 4 and 5.

2.4. The lower and upper limits of g.f.r. have been taken as 40 and 225<sup>1</sup> respectively (expressed as per thousand female population) and sets of fertility rates have been computed at intervals of 5 points, i.e., for values of g.f.r. of 40, 45, 50, ..., 225. The corresponding limits of s.a.b.r. are 10 and 50 and rates are presented for values of s.a.b.r. at intervals of 2.5 points, i.e. for values of 10.0, 12.5, 15.0, 17.5, ..., 50.0.

In these tables, fertility rates for the terminal age-groups, i.e., for 15-19 and 40-44 have been obtained by linear regression on the estimated rates for the adjoining groups, i.e., 20-24 and 35-39, rather than from regression of g.f.r. or s.a.b.r. on the specific rates, in view of relatively higher values of the correlation coefficients between the age-specific fertility rates for these groups. For the intermediate age-groups, however, regressions of g.f.r. or s.a.b.r. on the specific rates have been used. The estimated values for the age-groups 20-24 and 35-39 thus obtained were used to derive the rates for the terminal groups.

\* Computation can easily be extended for values beyond this range.

## MODEL FERTILITY TABLES

### 3. ANALYSIS OF THE RESULTS

3.1. The pattern of the model fertility tables is briefly as follows. The specific fertility rate for the age-groups 25-29 is uniformly the maximum, followed very closely by that for the preceding age-group, irrespective of the level of fertility as measured by g.f.r. or s.a.b.r. This behaviour is not in absolute conformity with the general experience as the fertility rate for some of the countries has sometimes been observed to attain its maximum value in the age-group 20-24 and to fall to a lower level at ages 25-29. The factors involved in such irregularities are the ages at marriage, child bearing practices, etc., which have not explicitly been taken into consideration. As a result, the derived specific fertility rates regarded only as a function of g.f.r. or s.a.b.r. cannot be expected to reveal features other than monotonicity and differentials in rates of changes for different age-groups. Fortunately however, the dispute over the maxima can, for all practical purposes, be ignored since the absolute difference in rates between the two age-groups is relatively small in almost all cases. In fact, the effect on the over-all fertility index, of an interchange of rates between the two groups, is negligible.

The pattern for the remaining age-groups is simple. They may be ranked in decreasing order with successively higher age-groups starting with 30-34, 15-19 being intermediate between the last two age-groups.

3.2. The specific rates, expressed as ratios of the over-all index show the following trends. The ratio is virtually constant for the age-group 30-34. The 20-24 and 25-29 age-groups contribute relatively less and less as the g.f.r. or s.a.b.r. goes higher up; the trend is seen to be in the reverse direction for the remaining age-groups.

3.3. It may be seen that none of the sets of specific rates conforms to the pattern consistent with the set of weights recommended by the UN for the construction of s.a.b.r. A reconciliation was sought by examining the deviations produced by the same weighting system under widely differing situations. Operationally, eight countries were selected with different s.a.b.r.'s, and on the age-distribution of each of them, s.a.f.r.'s as estimated from assumed values of s.a.b.r. were applied to recalculate the s.a.b.r. Table 6 shows the results, where the recalculated s.a.b.r.'s compare favourably with the assumed s.a.b.r.'s almost everywhere. This finding either justifies that the weights recommended by the UN are uniformly applicable, or at least suggests, that a different weighting system might not be an improvement.

A few general comments regarding the use of model fertility tables may not be out of place. Just as the model life tables provide a set of average values of life table functions corresponding to a given level of mortality, the model fertility rates similarly will, in all probability, differ from the rates observed. Conversely, the levels of actual rates need not be the same for different age-groups. The straightforward application of the model fertility rates derived from a given g.f.r. might, therefore, lead to errors to which estimates of any kind are subjected. The model rates are therefore recommended in situations when necessary data are either non-existent or unreliable.

SANKHYĀ : THE INDIAN JOURNAL OF STATISTICS : SERIES B

TABLE 3. NAME OF COUNTRIES AND REFERENCE PERIOD OF FERTILITY DATA

sl. no.	country	reference period	sl. no.	country	reference period
1.	Africa (coloured)	1951	29.	Luxembourg	1958
2.	Algeria (white)	1954	30.	Malta	1958
3.	American Samoa	1956	31.	Martinique	1956
4.	Australia	1940, 1958	32.	Morocco (S 2)	1951
5.	Austria	1951, 1958	33.	Morocco (RU)	1956
6.	Belgium	1949, 1957	34.	Netherlands	1949
7.	Berlin	1950, 1958	35.	Norway	1949, 1958
8.	British Guiana	1950	36.	New Guinea	1954
9.	Bulgaria	1958	37.	Norfolk Island	1954
10.	Canada	1949, 1958	38.	Norway	1949, 1957
11.	Chile	1952	39.	Panama	1950
12.	China (Taiwan)	1950, 1958	40.	Papua	1954
13.	Cyprus	1949, 1958	41.	Portugal	1949, 1958
14.	Denmark	1957	42.	Puerto Rico	1950
15.	England and Wales	1949, 1957	43.	Rhodesia (w)	1951
16.	Finland	1940, 1958	44.	S. Africa (union)	1952
17.	France	1949, 1958	45.	S. Africa (white)	1956
18.	Germany	1958	46.	Scotland	1949, 1958
19.	Ghana	1950	47.	Singapore	1958
20.	Guadeloupe	1956	48.	Spain	1950
21.	Guan	1950	49.	Sweden	1949, 1958
22.	Hungary	1949, 1958	50.	Switzerland	1949, 1958
23.	Iceland	1955	51.	St. Pierre	1951
24.	Israel	1950, 1957	52.	S. W. Africa	1951
25.	Israel (Jewish)	1949	53.	Trinidad	1954, 1958
26.	Italy	1951	54.	U.S.A.	1949, 1957
27.	Jamaica	1949, 1950	55.	Virgin Islands	1950
28.	Japan	1950, 1957	56.	West Indies	1958
			57.	Yugoslavia	1950, 1957

TABLE 4. MODEL FERTILITY TABLE BASED ON SEX-AGE ADJUSTED BIRTHRATE

sex-age adjusted birth rate	fertility rate (per thousand females) for age-groups					
	15-19	20-24	25-29	30-34	35-39	40-44
(1)	(2)*	(3)	(4)	(5)	(6)	(7)**
10.0	3.6	93.8	99.2	50.2	15.2	(9)
12.5	10.7	107.8	113.5	63.0	26.2	3.5
15.0	17.8	121.7	127.7	76.9	37.2	8.6
17.5	24.9	135.7	142.0	90.2	48.3	13.8
20.0	32.0	149.6	156.3	103.5	59.3	19.0
22.5	39.1	163.6	170.6	116.9	70.3	24.1
25.0	46.2	177.5	184.8	130.2	81.3	29.3
27.5	53.3	191.5	199.1	143.5	92.4	34.5
30.0	60.4	205.4	213.4	156.8	103.4	39.6
32.5	67.5	219.4	227.7	170.2	114.4	44.8
35.0	74.6	233.3	241.9	183.5	125.4	50.0
37.5	81.7	247.3	256.2	196.9	136.5	55.1
40.0	88.8	261.2	270.5	210.2	147.5	60.3
42.5	95.9	275.2	284.8	223.5	158.5	65.5
45.0	103.0	289.1	299.0	236.8	169.5	70.6
47.5	110.1	303.1	313.3	250.1	180.0	75.8
50.0	117.2	317.1	327.6	263.4	191.6	81.0

\*Estimated values are based upon the equation  $f_{15-19} = -44.10 + 0.51 f_{10-11}$

\*\* " " " " " " " " " " " "  $f_{10-11} = -8.82 + 0.47 f_{15-19}$

n) Figures in negative.

MODEL FERTILITY TABLES

TABLE 5. MODEL FERTILITY TABLE BASED ON GENERAL FERTILITY RATE

general fertility rate	fertility rate (per thousand females) for age-groups					
	15-19	20-24	25-29	30-34	35-39	40-44
(1)	(2)*	(3)	(4)	(5)	(6)	(7)**
40	-(a)	90.0	95.0	45.0	11.4	(a)
45	4.8	96.3	101.4	51.0	10.4	(a)
50	8.0	102.5	107.8	57.9	21.4	1.2
55	11.2	108.8	114.2	63.0	26.4	5.5
60	14.4	115.0	120.6	69.9	31.4	5.9
65	17.5	121.3	127.0	75.0	36.4	8.2
70	20.7	127.5	133.4	81.0	41.4	10.6
75	23.9	133.8	139.8	87.0	46.4	12.0
80	27.1	140.0	146.2	93.0	51.4	15.3
85	30.3	146.3	152.6	99.0	56.4	17.8
90	33.4	152.5	159.0	105.0	61.4	20.0
95	36.6	158.8	165.4	111.0	66.4	22.3
100	39.8	165.0	171.8	117.0	71.4	24.6
105	43.0	171.3	178.2	123.0	76.4	27.0
110	46.2	177.5	184.6	129.0	81.4	29.3
115	49.3	183.8	191.0	135.0	86.4	31.7
120	52.6	190.1	197.4	141.0	91.4	34.0
125	55.7	196.3	203.8	147.0	96.4	36.4
130	58.9	202.5	210.2	153.0	101.4	38.7
135	62.1	208.8	216.6	159.0	106.4	41.0
140	65.2	215.0	223.0	165.0	111.4	43.4
145	68.4	221.3	229.4	171.0	116.4	45.7
150	71.6	227.5	235.8	177.0	121.4	48.1
155	74.8	233.8	242.2	183.0	126.4	50.4
160	78.0	240.0	248.6	189.0	131.4	52.8
165	81.2	246.3	255.0	195.0	136.4	55.1
170	84.3	252.5	261.4	201.0	141.4	57.4
175	87.5	258.8	267.8	207.0	146.4	59.8
180	90.7	265.0	274.2	213.0	151.4	62.1
185	93.9	271.3	280.6	219.0	156.4	64.5
190	97.1	277.5	287.0	225.0	161.4	66.8
195	100.2	283.8	293.4	231.0	166.4	69.2
200	103.4	290.0	299.8	237.0	171.4	71.5
205	106.6	296.3	306.2	243.0	176.4	73.8
210	109.8	302.5	312.6	249.0	181.4	76.2
215	113.0	308.8	319.0	255.0	186.4	78.5
220	116.2	315.0	325.4	261.0	191.4	80.9
225	119.3	321.3	331.8	267.0	196.4	83.2

\*  $f_{15-19} = -44.10 + 0.61 f_{10-14}$

\*\*  $f_{10-14} = -8.82 + 0.47 f_{15-19}$

(a) Figure is negative.

## SANKHYĀ : THE INDIAN JOURNAL OF STATISTICS : SERIES B

TABLE 6. ADJUSTMENT FOR AGE AND SEX DISTRIBUTION BY THE WEIGHTS  
RECOMMENDED BY THE UN FOR THE CALCULATION OF SEX-AGE  
ADJUSTED BIRTH RATE, REVERIFIED

sl. no.	country	year	s.a.b.r.	derived s.a.b.r., corresponding to assumed s.a.b.r. as							
				10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1.	Japan	1957	16.0	10.61	12.08	15.45	17.91	20.38	22.85	25.31	27.78
2.	Sweden	1949	18.2	9.09	12.46	14.92	17.39	19.86	22.33	24.80	27.27
3.	Scotland	1949	20.0	10.09	13.50	15.10	17.61	18.03	22.63	25.14	27.65
4.	U.S.	1949	23.3	10.22	12.68	15.13	17.58	20.03	22.48	24.94	27.39
5.	Norfolk Isl.	1954	26.8	9.58	12.07	14.67	17.06	19.56	22.05	24.64	27.04
6.	Canada	1958	20.8	10.03	12.51	15.00	17.49	19.97	22.46	24.93	27.43
7.	S. Africa (union)	1952	39.1	10.75	13.24	15.73	18.22	20.71	23.20	25.69	28.18
8.	Trinidad	1954	45.2	10.27	12.77	15.26	17.75	20.24	22.73	25.22	27.71

sl. no.	country	year	s.a.b.r.	derived s.a.b.r., corresponding to assumed s.a.b.r. as								
				30.0	32.5	35.0	37.5	40.0	42.5	45.0	47.5	50.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1.	Japan	1957	16.0	30.25	32.71	35.18	37.65	40.12	42.58	45.05	47.52	49.99
2.	Sweden	1949	18.2	29.73	32.20	34.67	37.14	39.61	42.08	44.54	47.01	49.48
3.	Scotland	1949	20.0	30.15	32.66	35.17	37.68	40.19	42.70	45.21	47.72	50.22
4.	U.S.	1949	23.3	29.84	32.20	34.73	37.20	39.63	42.10	44.56	47.01	49.46
5.	Norfolk Isl.	1954	26.8	29.53	32.03	34.52	37.02	39.51	42.00	44.50	46.99	49.49
6.	Canada	1958	29.8	29.92	32.40	34.89	37.38	39.87	42.35	44.84	47.32	49.81
7.	S. Africa (union)	1952	39.1	30.67	33.16	35.65	38.15	40.64	43.13	45.62	48.11	50.59
8.	Trinidad	1954	45.2	30.20	32.69	35.19	37.68	40.17	42.66	45.15	47.64	50.14

## 5. ACKNOWLEDGEMENT

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