

INDIAN STATISTICAL INSTITUTES

Statistician's Diploma Examination - November 1974

Paper I : Official Statistics and Descriptive Statistics (Theoretical)

Time : 4 hours

Full marks : 100

Group A : Official Statistics

( Attempt any three questions from this group)

Describe the functions of a statistical system. What are the different types of statistical system? Discuss critically the system existing in India. (16)

What are the various types of agricultural statistics collected in India? Name one important publication for each type, mentioning the periodicity of publication. Describe critically the method of collection of yield statistics. (15)

- a) Name the different sources which give the statistics of industrial production in India. Describe briefly the differences in respect of the scope and coverage of the data collected from these sources.
- b) The capital-output ratio may be defined as the ratio of total capital employed to the value added. What data would you use for estimating capital-output ratios for individual manufacturing industries? Comment on the reliability of the data available for the purpose, and also on their drawbacks. (8+8)=16

Who compiles national income estimates of India? Explain the method of compilation used in India. What are the various problems encountered while compiling the estimates? (16)

- a) What was the basis of classification of activity status (workers and non-workers) for the 1961 population census. How was it different from the previous censuses? What was the main defect of this basis? Did it come to be removed in the 1971 census?
- b) We are interested in knowing the employment figures for a particular industry during any year. What are the three different sources which can be utilised for the purpose? Are the figures obtained from these sources likely to be the same? If not, what will be the differences? (Assume that there is no non-reporting from factories). (3+8)=16

Name the official publications of India, which present the following statistical series:

- (i) Statistics of live-stock, (ii) Statistics of birth and death rates, (iii) Statistics of foreign trade, (iv) Statistics of wholesale prices, (v) Statistics of consumer price indices for industrial workers, (vi) Statistics of factory employment, (vii) Number of sewing machines produced per month, (viii) Total number of passengers and total weight of goods carried by Indian Railway during any year.

Mention the names of the agencies issuing these publications and the periodicity of publication in each case. (16)

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## 33 PAPER I - Group B : Descriptive Statistics

(Attempt any three questions from this group.)

7. For any statistical survey with which you may be acquainted, give details about (i) the scope and objective of the survey, (ii) the schedule or questionnaire used, (iii) the method of data collection followed, (iv) the nature of scrutiny made. Also add sketches of important tables prepared for the report. (16)

8. (a) Why are deviations of observations from the mean invariably used for calculating the standard deviation, while deviations from the median are found to be appropriate for the calculation of the mean deviation? (Prove the underlying inequalities).  
 (b) Show that, for any discrete distribution, the standard deviation is greater than or equal to the mean deviation from the mean. (8+8) = 16.

9. (a) Write down the forms of the following distributions :  
 (i) binomial, (ii) Poisson, (iii) negative binomial and (iv) hypergeometric. Give an illustration from real life, in each case.  
 (b) Show that for a normal distribution with mean  $\mu$  and standard deviation  $\sigma$

$$\begin{aligned} \mu_{2r+1} &= 0 \\ \text{and } \mu_{2r+2} &= \sigma^2 \mu_{2r} + \sigma^3 \cdot \frac{d \mu_{2r}}{d\sigma} \end{aligned}$$

where  $\mu_r$  is the  $r$ th moment about the mean. (8+8) = 16

10. (a) Explain with illustrations, the meaning of the correlation coefficient ( $r$ ) between two variables  $X$  and  $Y$ . If  $\eta_{YX}$  denote the correlation ratio of  $Y$  on  $X$ , show that  $0 \leq r^2 \leq \eta_{YX}^2 \leq 1$   
 when will  $r^2$  be equal to  $\eta_{YX}^2$ ?

- (b) Show that the correlation between  $x_1 - b_{13}x_3$  and  $x_2 - b_{23}x_3$  is given by

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}}$$

where  $x_1, x_2, x_3$  are the deviations of the variables  $X_1, X_2, X_3$  from their means and  $b_{13}, b_{23}$  are the linear regression coefficients of  $X_1$  on  $X_3$  and of  $X_2$  on  $X_3$ . Explain clearly the meaning of  $r_{12.3}$

- (c) If the total correlation coefficients in a set of  $p$  variates are equal to  $\rho$  (rho), show that the partial correlations of the second order are all equal to  $\rho/(1+\rho^2)$ . (6+5+5) = 16
11. (a) How do you select the base for an index number? Discuss the suitability of the following in the calculation of index number.  
 (i) arithmetic mean of price relatives, (ii) geometric mean,  
 (iii) Laspeyres' formula and (iv) Paasche's formula
- (b) What is cyclical fluctuation in a time series? How does it differ from seasonal fluctuation? Describe, briefly, two methods for determining the cyclical component of a time series. (9+8) = 16

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INDIAN STATISTICAL INSTITUTE

Postgraduate Entrance Examination - November 1974

Paper II : Official Statistics and Probability Theory

Time : 4 hours

Full marks : 100

Group A : Probability Theory

(Attempt any three questions from this group)

- Explain the concepts of (i) random experiment, (ii) sample space, (iii) event, (iv) elementary event (v) union of two events and (vi) intersection of two events.
  - A coin is tossed repeatedly until there is a succession of two heads. Describe the sample space of this random experiment.
  - If an event A occurs, whenever the events  $A_1, A_2, A_3$  occur simultaneously prove that  $P(A) \geq P(A_1) + P(A_2) + P(A_3) - 2$ . (6+4+6)=16
- Explain, with illustrations, the concepts of (i) a random variable (ii) distribution function of a random variable and (iii) probability density function of a random variable.
  - A motorist has to leave his car in a garage for a minor repair, before proceeding to keep another engagement. The garage repair time X is assumed to be a continuous random variable with

$$f(x) = \frac{3}{4}(2-x), \text{ where } (0 \leq x \leq 2)$$

The engagement lasting time Y on the other hand is a continuous random variable independent of X with p.d.f.

$$(iii) \quad \beta(y) = \frac{1}{2}y \quad \text{where, } (0 \leq y \leq 2)$$

Find (i) the probability of the repair taking up more time than time t, and (ii) the probability that the car will not be ready when the motorist returns after keeping his engagement (6+4+6)=16

- Define the binomial, Poisson and negative binomial distributions. What are the mutual relations amongst these three distributions?
  - For the negative binomial distribution, obtain the generating function, a recurrent relation to get the successive probabilities and also the mean of the distribution. (6+10)=16
- Show that in sampling from a univariate normal distribution, the sample-mean and the sample-variance are independently distributed. Hence, or otherwise, derive Student's t-distribution.
  - If  $X_1, X_2, X_3$  and  $X_4$  are independent standard normal variates obtain the distribution of  $s = (3X_1 + X_2) / (4X_2 + 3X_3)$  (8+8)=16
- Derive the normal limit to the binomial distribution.
  - Derive the weak law of large numbers, and deduce Bernoulli's law of large numbers.
  - Obtain the distribution of the range in random samples of size n from a rectangular distribution with range R. (5+5+5)=16

Neatness

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(Attempt any 5 questions from this group)

6. (a) Explain the concepts of unbiasedness, consistency, sufficiency and efficiency of an estimator. Give an example of an estimator which is unbiased but not consistent and also an example of an estimator which is consistent but not unbiased.
- (b) Prove the consistency of the sample median.
- (c) Calculate the efficiency of the sample median in case of a random sample from a normal distribution. (10+3+3)=16

7. (a) A sample of size  $n$  is drawn at random (and with replacement) from a population with  $k$  categories, the population proportions for the categories being  $p_i$  ( $i = 1, 2, \dots, k$ ). If  $n_i$  ( $i = 1, 2, \dots, k$ ) be the corresponding sample frequencies, show that the statistic (called a frequency  $\chi^2$  (chisquare))

$$\sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}$$

has asymptotically the  $\chi^2$  (chisquare) distribution with  $k-1$  d.f.

- (b) Indicate how the frequency  $\chi^2$  (chisquare) statistic may be used (a) in testing for independence of two attributes and (b) in testing for homogeneity of a number of distributions. (8+8)=16
8. (a) What is likelihood function? Define the maximum-likelihood estimate (m.l.e) of a parameter. What are the optimum properties of a maximum-likelihood estimator?
- (b) Let  $X_1, \dots, X_n$  be a sequence of independent and identically distributed random variables, each with density

$$f_{\theta}(x) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

find the m.l.e of  $\theta$  based on the above observations.

- \*Ques 2X Consider  $Z_n^* = \sum_{i=1}^n (X_i + \dots + X_n)$ . How does it compare with the m.l.e, in respect of efficiency? (8+8)=16

9. (a) State and prove the Neyman-Pearson fundamental lemma to test a simple hypothesis against a simple alternative.
- (b) Show how the above method can be used to find a U.P test for testing  $H_0$  ( $\mu = \mu_0$ ) against  $H_1$  ( $\mu > \mu_0$ ) on the basis of a random sample of size  $n$ , drawn from  $N(\mu, \sigma^2)$ ,  $\sigma^2$  being known.
- (c) State how the procedure changes when  $\sigma^2$  is not known and is estimated from the sample. (8+8+4)=16
10. (a) What do you mean by a sequential probability ratio test? How does it differ from a fixed-sample size test?
- (b) Construct a sequential probability ratio test for testing  $H_0$  ( $p = p_0$ ) against  $H_1$  ( $p > p_0$ ) for a binomial population with probability of success  $p$ .
- (c) Find the A.S.N. of the test. (4+8+4)=16

Paper III : Sample Surveys and Design & Analysis of Experiments (Theoretical)

Time : 4 hours

Full marks : 100

Group A : Sample Surveys

(Attempt any three questions from this group)

1. (a) What do you understand by the bias of an estimator? Discuss why a large bias in an estimator is considered very undesirable?
- (b) For certain population, it is known that the observations  $y_1$  are all zero on a portion  $q^1$  of  $N$  units ( $0 < q < 1$ ). Sometimes (with varying expenditure of efforts) these units can be found and listed, so that they need not be sampled. If  $\sigma_0^2$  is the variance of  $y_1$  in the original population, and  $\sigma_0^2$  is the variance when all zeros are excluded, show that:

$$\sigma_0^2 = \frac{\sigma^2}{p} - \frac{q}{2} \bar{y}^2$$

where  $p = 1 - q$  and  $\bar{y}$  is the population mean.

If the population total is estimated from a simple random sample of size  $n$ , show that with the exclusion of 'zero' units, the fractional reduction in the variance of the estimate is

$$q(\sqrt{2} + 1)/\sqrt{2}$$

where  $\sqrt{2} = \sigma_0^2/\sigma^2$ . You may omit the f.o.c.

(3+3+5+5)=16

2. State the conditions under which the ratio estimator is more efficient than the simple sample mean, for estimating a finite population mean  $\bar{Y}$ . Obtain the bias and the variance of the ratio estimator. Also give an estimator of this variance and indicate whether your estimator is unbiased and consistent. (2+3+5+3+3)=16

3. (a) From a population of  $N$  units, a systematic sample of size  $n$  ( $N=kn$  where  $k$  is an integer) is selected. Assuming the presence of linear trend in the population show that the systematic sample mean is  $n$ -times more efficient than the mean of simple random sample/size  $n$  as an estimator of the population mean. The f.o.c. may be omitted.

- (b) What is 'circular systematic sampling'? Discuss the advantages it has over 'simple systematic sampling'. (3+4+4)=16

4. A Survey is being repeated for the second time on a population  $\Omega$  of  $N$  units. On the first occasion a simple random sample ( $s_1$ ) of  $n$  units was selected without replacement from  $\Omega$ . The study variable  $y$  was observed on this sample. On the second occasion, a simple random sample of  $m$  ( $m < n$ ) units is selected without replacement from  $s_1$ , and a second sample of  $(n-m)$  units, is similarly selected from  $(\Omega - s_1)$ . Treating the information collected on  $y$  from the units in  $s_1$  on the first occasion as the auxiliary variable, obtain an estimate of population mean  $\bar{Y}$  on second occasion and find its variance. Also obtain the optimum value of  $m$ . (4+6+6)=16

5. Write short notes on any two of the following :

- (a) Double sampling, (b) Interpenetrating sub-samples, (3+3) = 16  
(c) Collapsed strata (d) Ordered and unordered estimators

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## Paper III - Group B : Design and Analysis of Experiments

(Attempt any three questions from this group)

6. (a) Explain the concepts - randomization, replication and local control. Discuss fully the role they play in the design of experiments.
- (b) Define analysis of variance, stating clearly the underlying assumptions.
- (c) When these assumptions are not valid, what methods are adopted to carry out the analysis? Discuss briefly. (5+4+6)=16.
7. Define balanced incomplete block design and state the relation among its parameters. Describe the method of its analysis. (4+4+9)=16
8. (a) Explain the concept of confounding in factorial experiments. When is confounding said to be balanced?
- (b) A factorial experiment with five factors A, B, C, D and E each at two levels, was laid out in blocks of 8 plots involving two replications. The treatment combinations of control blocks of replication I and II were:
- (i) 1, bd, abd., cde, acd, bce, abc, ae
- (ii) 1, ac, abd, ade, bcd, cde, be, abce.
- Find out the treatment effect confounded in each of the replications. Outline the procedure of analysis. (8+8)=16
9. (a) When do you use a split-plot design? What are its advantages and disadvantages? How does it differ from a strip-plot design?
- (b) Obtain the expressions for various treatment comparisons in a split-plot design.
- (c) Give the outline of the method of analysis of the split-plot design. (5+6+4)=16
10. (a) In three species of citrus trees the ratio of leaf area to dry weight was determined for three conditions of shading. The following ANOVA table was obtained:
- | Source  | df | S.S. |
|---------|----|------|
| Shading | 2  | 1884 |
| Species | 2  | 230  |
| Error   | 4  | 81   |
- The total of the ratio of leaf-area to dry weight under the 3 shading conditions stated below, are as follows:
- | Sun | Half-shade | Shade |
|-----|------------|-------|
| 325 | 248        | 223   |
- Split the treatment sum of squares in two contrasts:
- (i) Effect of shade  
(ii) Half-shade versus Rest.
- Calculate the 95% confidence limit for the above contrasts. (The value of 't' at 5% level is 2.78).
- (b) How does the use of concomitant variable help to reduce the experimental error? Describe the method of analysis by using a Completely Randomized Block Design. (8+8) = 16.
11. Write notes on any two of the following:
- (i) Youden squares, (ii) Fractional replication  
(iii) Orthogonal Latin squares (iv) Uniformity trial (8+8)=16
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INDIAN STATISTICAL INSTITUTE

Statistician's Diploma Examination - November 1974

Paper IV : Applied Statistics : (Theoretical)

(Two half-groups of subjects with 50 marks each)

Time : 4 hours

Full marks :100

- (a) Candidates will be required to answer questions from those two groups of subjects only for which they have registered their option.
- (b) Separate answer books are to be used for each of the two groups attempted.
- (c) Use of calculating machines is not permitted.

SD IV : GROUP (a) : ECONOMIC STATISTICS : Half-paper  
(50 marks)

(Attempt any three questions from this group)

1. Explain how you will estimate income elasticities of demand for specified items of consumption from family budget data. Indicate the usefulness of these estimates.  
(12+4) = 16
2. Give an account of the Pareto law of income distribution. How will you determine graphically whether a given body of economic data follows this distribution? Derive an expression for the Lorenz ratio, when income distribution obeys this law.  
(7+3+6) = 16
3. What purpose is served by an index number? Discuss the different steps involved in the construction of a wholesale price index number for India.  
(4+12) = 16
4. Discuss the problems of multicollinearity and least squares bias that arise in the estimation of demand functions from time-series data. Indicate broadly, how these can be solved.  
(16)
5. Explain any three of the following concepts :
- (a) national income at factor cost
  - (b) disposable income
  - (c) net domestic product
  - (d) c.o.s. production function
  - (e) the concept of the 'poverty line' in India.

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SD IV : GROUP (b) : STATISTICAL QUALITY CONTROL : Half-paper  
(50 marks)

(Attempt question 1 and any other two questions from this group)

1.

Either

An experiment involves three factors A, B and C, each at two levels. The 8 treatment combinations have to be replicated twice during the experimentation. It is possible to run only 4 treatment combinations in one day and thus the entire experiment extends over four days.

The three main effects and interactions AB, AC are to be determined with the same precision. Interactions BC and ABC may be determined with lesser but equal precision, as between them.

Give a lay out of the experiment and indicate how the results are to be analysed.

(12+8) = 20

Or

Discuss, in detail, the applicability, methodology and the utility of Group Control Charts for measurements.

(8+8+4) = 20

2. (a) Distinguish between control limits and confidence limits.  
(b) The carbon content of a batch of steel is estimated by testing a small number of specimens selected from the batch in a random manner. Obtain an expression for 100  $\alpha$ % confidence limits (two sided) for the carbon content when the value of standard deviation is given, where  $\alpha$  is a specified number such that  $0 < \alpha < 1$ .  
(c) How would you determine the number of specimens to be tested in (b), if the width of the confidence interval is specified as 'W' and the standard deviation is not known?

(5+5+5) = 15

3. (a) Derive expressions for acceptance and rejection lines in an item-by-item sequential sampling plan for attributes, when the values of AQL, LTPD, Producer's risk and Consumer's risk are specified.  
(b) Indicate a tabular procedure for the plan mentioned in (a).

(12+3) = 15

4. Write short notes on any three of the following

- (i) Type A and type B OC curves.  
(ii) Probability versus 3-sigma control limits.  
(iii) Accuracy, precision and reproducibility of test results.  
(iv) OC curve of a p-chart.  
(v) Process capability and specifications.

(5+5+5) = 15

Please turn over



SD IV : GROUP (c) : STATISTICAL METHODS IN GENETICS : Half-paper  
(50 marks)(Attempt any three questions from this group)

1. Given the frequencies of the four phenotypic classes AB, Ab, aB, ab — obtained by the back-cross method to study linkage between the factors A and B, explain how the  $\chi^2$  (chi-square) can be partitioned into 3  $\chi^2$  components with single degrees of freedom, to study the segregation of individual factors and linkage between factors.

Derive the expected frequencies of the four phenotypic classes for the offspring of the matings :

$$(i) \begin{array}{c} AB \\ ab \end{array} \times \begin{array}{c} ab \\ ab \end{array} \quad (ii) \begin{array}{c} Ab \\ ab \end{array} \times \begin{array}{c} Ab \\ ab \end{array}$$

Introducing linkage parameter. Give a method of estimation of linkage parameter for one of the above cases.

(8+4) = 12

2. What are the similarities and dissimilarities in the action of major genes and polygenes? How do they come to be reflected in the genetic analysis in the two cases?

What are the two criteria for judging the adequacy of a scale for the measurement of polygenic characters?

Explain the procedure of testing experimental data with respect to these criteria.

(6+4+3) = 13

3. Show how the regression of progeny means on parental values, may be taken as an estimate of the genetic fraction of the total variation in the parental population. Suggest a method of separation of genetic component of variation from the non-heritable part of heritable variation in the case where the parental generation, has been raised from a cross between pure strains.

(8+8) = 16

4. Write notes on any two of the following :

- (i) Hardy-Weinberg equilibrium frequencies.
- (ii) Effect of migration, mutation and selection in a large randomly mating population.
- (iii) Estimation of gene frequencies in the blood group system O-A-B, in a randomly mating population from observed phenotype frequencies.
- (iv) Use of discriminant function for plant selection.

(8+8) = 16

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SD IV : GROUP (d) : VITAL STATISTICS & DEMOGRAPHY : Half-paper  
(50 marks)  
(Attempt any three questions from this group)

1. (a) Describe King's method of constructing an abridged life-table. (C)  
(b) Define (i) curtate expectation of life  
(ii) complete expectation of life  
Deduce an approximate relation between the two. (2+2)=4  
(c) Comment on the statement :  
"The average age at death of bishops is found to be higher than that of teachers. Thus the profession of priesthood is healthier than that of teaching." (4)
2. (a) Deduce the equation to the logistic curve, stating clearly the conditions involved.  
Comment on the suitability of such a curve for representing Indian census data over the last 70 years. (5+3)=8  
(b) What is a 'stable population'? Define the symbols appearing in the equation  $R_0 = e^{rT}$ . Explain the implication of the equation.  
(2+6) = 8
3. (a) What is the significance of net reproduction rate in a non-stationary population? Are these rates 'indicators' of future population changes?  
Show that in a stationary population the net reproduction rate is unity.  
(4+2+2) = 8  
(b) (i) In India widow remarriages are comparatively negligible in number. In the light of this special knowledge, what modification would you suggest in the method of calculation of women's net reproduction rate?  
(ii) The age at marriage among girls has exhibited an upward trend in recent years. How is it likely to affect population growth?  
(5+3) = 8
4. Write short notes on any two of the following :  
(i) Standardisation of death rates.  
(ii) Morbidity rates and their uses.  
(iii) Technique of population projection.  
(8+8) = 16

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SD IV : GROUP (e) : EDUCATIONAL AND PSYCHOLOGICAL STATISTICS : (50 marks) Half-paper

( Attempt any three questions from this group )

1. State the main defects of the present system of ranking in scholastic tests. Discuss in this connection the role of normalised scores and linear derived scores.

In a college test, student A got the highest marks in Statistics Honours (in a group of 10) but he got the second highest marks in Mathematics Pass (in a group of 200 students). The second highest score in pass mathematics was however also obtained by one more student of the same class.  
Determine the percentile ranks of the student A in the two subjects and interpret these results.

$$(4+6+4) = 16$$

2. What are the main differences between a physical scale and a psychological scale ? Define the terms reliability of a test and validity of a test.

Derive the Kuder-Richardson formula for obtaining the reliability of a test of K parallel items in terms of K, the item and test variances.

$$(5+5+8) = 16$$

3. What are achievement and aptitude tests? Discuss the different stages of standardising an achievement test, indicating in detail the role of item analyses, reliability, validity and norms in this connection.

State what you mean by the statement :

'The mental age of a student A is 17 while his chronological age is 14'. Find also the I.Q. of the student A.

$$(7+10+2+1)=16$$

4. What is the purpose of factor analysis? Write down in this connection the fundamental equation for the scores of N individuals in n tests, explaining the different quantities occurring in the equation.

What are conveyed by the terms 'factor pattern' and 'factor structure' ? Describe the centroid method for obtaining factor loadings in a multiple-factor model.

$$(2+3+3+8)=16$$

5. Write short notes on any three of the following : (16)

(i) Parallel forms of a test and their use in determining reliability of a test.

(ii) Predictive validity and concurrent validity.

(iii) Likert's scale.

(iv) Intelligence and intelligence tests.

(v) Classificatory techniques in personnel selection programmes.

Neatness :

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INDIAN STATISTICAL INSTITUTE

Statistician's Diploma Examination - November 1974

Paper V : Methods of Numerical Computation; Descriptive Statistics  
and Official Statistics (Practical)

Time : 5 hours

Full Marks : 100

1) Figures in the margin indicate full marks

ii) Use of calculating machines is permitted.

GROUP A : Methods of Numerical Computation (25 Marks)

(Attempt any two questions)

1. Given  $A = \begin{pmatrix} 1.95 & 1.58 & -3.27 & 2.38 \\ -0.86 & -3.20 & 2.42 & 3.21 \\ 2.95 & 1.86 & -2.14 & 1.44 \\ 3.62 & -2.26 & -1.68 & 4.17 \end{pmatrix}$   $B = \begin{pmatrix} 3.19 & -1.12 \\ 0 & -2.17 \\ -1.17 & 0 \\ 2.00 & 0.85 \end{pmatrix}$

Obtain the values of i)  $AB$  and ii)  $A^{-1}$ .

$(4 + 6 \frac{1}{2}) = 12 \frac{1}{2}$

2. a) The following table gives the value of  $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x -t^2/2 dt$  for certain values of  $x$  :

$f(x)$ :	.9484493	.9494974	.9505285	.9515428	.9525403
$x$ :	1.63	1.64	1.65	1.66	1.67

By suitable interpolation, find the value of  $x$ , for which  $f(x) = 0.95$

b) By using a suitable quadrature formula for equidistant ordinates

evaluate  $I = \int_{10}^{13} \log_e x dx$ , correct to 6 places of decimal.

$(6 \frac{1}{2} + 6) = 12 \frac{1}{2}$

3. a) Find the positive root of the equation :  $a^x = 1 + 2x$

correct to 5 places of decimal .

b) Approximate the function  $f(x)$ , by an appropriate 3rd degree polynomial.

Using the following information on  $f(x)$  and  $f'(x)$  estimate the value of  $f(8.5)$  by making use of suitable interpolation formula.

$x$	$f(x)$	$f'(x)$
8	0.288	0.290
9	0.316	0.266

$(6 + 6 \frac{1}{2}) = 12 \frac{1}{2}$

$[f'(x) = \frac{d}{dx} f(x)]$

(13)

P.T.O.

## SD V - GROUP B : DESCRIPTIVE STATISTICS (50 Marks)

(Attempt all the questions from this group)

4. For a group of 1000 trees in an orchard the following  $2 \times 2 \times 2$  classification was made :
- High yielder vs. Low yielder
  - Shaded vs. non-shaded
  - Heavy-bruned vs. low-bruned

The summary classification was reported as under :-

i) High yielder trees	-	520
ii) Shaded trees	-	540
iii) Heavy bruned trees	-	600
iv) High yielder and shaded trees		290
v) High yielder and heavy bruned trees	-	230
vi) High yielder shaded and heavy bruned trees	-	140

Scrutinise the summary figures and give your comments.

(10)

5. Draw the Ogive in a neat form for the following frequency distribution:

Mid-value of class:	2	7	12	17	22	27	32	37	42	47
Frequency:	7	18	30	45	61	72	82	90	96	100

(10)

6. A telephone switch-board, on an average, handles 720 calls during a rush hour. If the board can make 15 connections per minute, estimate the probability that the board will be overtaxed during any minute in the rush hour. (10)
7. In the course of an experiment, 15 mosquitoes were placed in each of 120 jars and were next subjected to a dose of D.D.T. After 4 hours the number alive in each jar was counted and the following frequency distribution was obtained :

No. of mosquitoes (alive after treatment)	0	1	2	3	4	5	6	7	8
Number of jars	2	12	14	22	28	17	13	10	2

Find the frequencies that one would expect, on the assumption that each mosquito has a common probability of survival.

(10)

8. Either

- 1) Eight students made the following scores on Test-1, Test-2 and in the final examination in a certain mathematics course.

Test 1	Test 2	Final examination	Test 1	Test 2	Final examination
43	22	66	35	20	25
39	29	38	21	8	17
27	23	55	19	17	33
29	33	63	13	19	19

Calculate the multiple correlation coefficient of final examination score, on test 1 and test 2 scores. Comment on the value obtained. (10)

(7)

(20 contd. in next page)  
Please turn over

Qr (contd. from page No.2)

8. i) A and B are two commodities judged to be of equal importance and their prices are as follows :-

	<u>Year I</u>	<u>Year II</u>
A (per kilogram)	10 rupees	15 rupees
B (per yard)	90 paise	60 paise

How will you combine the two commodities so that the price index taking year I as base and the same with year II as base, may be comparable ?

- ii) Two seasonal indices of the sales of garments of a particular type in a certain shop are given below :-

<u>Quarter</u>	<u>Seasonal Index</u>	<u>Quarter</u>	<u>Seasonal Index</u>
Jan - Mar.	97	July - September	83
Apr. - June	95	Oct. - December	135

If the total sales in the first quarter of a year be worth Rs. 15,000/- determine how much worth of garments of this type, should be kept in stock by the shop owner to meet the demand for each of the other three quarters of the year.

(10)

SD V - GROUP C : OFFICIAL STATISTICS ( 25 marks)

(Attempt both the questions)

- Notes: i) The information furnished by the candidates should be methodically arranged and, wherever possible, shown on neat tabular form.  
ii) Full references to sources are to be given along with the answers. Any other information considered relevant is also to be given.
9. From the official publications supplied, collect information on any three of the following items :
- Area under Wheat and Sugarcane in India for any three latest consecutive years as may be available.
  - Total number of bicycles produced in (i) West Bengal, (ii) Uttar Pradesh and (iii) Punjab, during any two recent years.
  - Number of British and Japanese tourists visiting India during three consecutive latest available months.
  - Total number of applicants on live-registers of employment exchanges in (i) West Bengal, (ii) Maharashtra and (iii) Tamil Nadu separately for any two recent years.
  - Total strength of Indian Merchant Shipping Fleet in terms of number of vessels and gross registered tonnage for any three recent years.
- ( 5 + 5 + 5 ) = 15
10. From the official publications supplied, compile data on consumer's price index numbers for industrial workers in Calcutta, Bombay and Madras for 12 available recent consecutive months. Present the data graphically to provide a comparison among the 3 cities.

( 3 + 4 )

INDIAN STATISTICAL INSTITUTE  
Statistician's Diploma Examination - November 1974

Time : 5 hours

Full marks : 100

- (a) All questions carry equal marks.  
(b) Use of Calculating Machines is permitted.

SD VI : GROUP (a) : STATISTICAL METHODS (40 marks)

(Attempt any two questions from this group)

1. Under the assumption that  $x$  and  $y$  are two log normally distributed variates, independently distributed with  $E(\log x) = 1$  and  $E(\log y) = \log 2-1$ , test the hypotheses :

- (a)  $H_0 : E(x) = 1$  on the basis of the following 9 observations on  $x$  : .53, 2.58, 3.21, .87, 1.09, .29, 4.02, 2.93 and 1.53.  
(b)  $H_0 : E(x)/E(y) = 0.5$ , on the basis of the above 9 observations of  $x$  and the following 9 observational values of  $y$  : 1.54, 1.69, 3.02, 2.38, 2.05, 3.78, .56, 2.82 and 3.01

(7+13) = 20

- 2.(a) Assume that  $x$  follows the n.d.f.

$f(x) = (\alpha+1)x^\alpha, 0 < x < 1, \alpha > 0$ ,  
work out the maximum likelihood estimate (m.l.e.) of  $\alpha$  on the basis of the following observations of  $x$  :  
.47, .38, .54, .79, .82, .53, .75 .

- (b) Also work out the m.l.e. of  $p$  and the standard error of the estimate, for the following data on hypothetical probabilities and observed frequencies :

Classes	probabilities	frequencies
1	$\frac{1}{P}$	97
2	$\frac{1-2\sqrt{2}}{P}$	59
3	$\frac{1}{P}$	102

(13+7) = 20

- 3.(a) A filling machine is expected to put 16 oz. of sugar in each bag. From the following weights of sugar put in 20 bags, test whether the machine is fair or biased.

16.1, 15.8, 15.8, 15.9, 16.1, 16.2, 16.0, 15.9, 16.0, 16.2, 16.1, 15.7, 15.9, 16.3, 16.1, 16.3, 15.7, 16.0, 15.8, 15.7

State the assumption underlying your analysis. What inference can be drawn about the fairness of the machine if no assumption is made regarding the population?

- (b) The following table gives the number of trials to learning required by 12 post operative rats (E-rats) and the number of trials to learning by 15 normal rats (C-rats).

Test the homogeneity of the two samples by

- (i) Kolmogorov - Smirnov test  
(ii) Run-test

E-rats : 20, 55, 29, 27, 75, 56, 31, 45, 25, 65, 72, 49  
C-rats : 23, 8, 24, 15, 6, 21, 9, 16, 22, 18, 14, 28, 13, 30, 11.

(You may use large sample approximations).

(8+12) = 20

(17)

P.T.O.

## SD VI : GROUP (b) : DESIGN AND ANALYSIS OF EXPERIMENTS (30 marks)

(Attempt all questions from this group)4.  
EITHER

each Draw up a layout for a factorial experiment, with three factors, A, B and C at three levels, in nine blocks each of nine plots, so that the losses of information on the three-factor interactions  $ABC^2$ ,  $AB^2C$  and  $AB^2C^2$  are the same.

OR Give a design for a varietal trial with nine varieties in six-plot blocks, so that differences of all pairs of varietal effects are estimated with same variance. The design is required to be complete completed in twelve blocks.

(10)

5.  
EITHER

number The following data relate to the initial weights (in pounds) and the growth rate (in pounds per week) of 15 pigs, classified according to the sex and the type of food given. Examine the differences between the three types of food A, B, C in their effect on the growth of pigs, correcting for the difference in the initial weights of the animals.

pan no.	type of food	initial weight (lbs)	growth rate (lbs/week)	pan no.	type of food	initial weight (lbs)	growth rate (lbs/week)
I	A	48	9.91	IV	C	50	10.37
	B	48	10.00		A	48	10.56
	C	48	9.75	B	46	9.68	
II	B	32	9.24	V	B	37	9.67
	C	28	8.66		A	32	8.82
	A	32	9.48		C	30	8.57
III	C	33	7.63				
	A	35	9.32				
	B	41	9.34				

OR Below are given the plan (in usual notations) and the yields (in certain units) of an experiment involving three fertilizers N, P and K, each at two levels in eight blocks of four plots each. Analyse the data and give its salient features.

Plan and yields of a  $2^3$  experiment in 4-plot blocks

Replication	Block 1		Block 2	
	Treatment	Yield	Treatment	Yield
1	(1)	145	k	199
	pk	191	p	272
	nk	300	n	180
	np	210	npk	305
2	(1)	223	p	266
	k	153	nk	300
	npk	210	pk	233
	np	182	n	278
3	p	186	n	209
	npk	173	k	93
	(1)	170	pk	224
	nk	213	np	245
4	pk	182	k	233
	(1)	175	nk	226
	npk	156	np	219
	n	183	n	220

(11c)

(20)

P.T.O.



## SD VI : GROUP (e) : SAMPLE SURVEYS : (30 marks)

(Attempt one question only from this group)

2. Draw a sample of size 6 with replacement with probabilities proportional to area (x), from the following population of 20 units :

County	Area (acres)	Yield of paddy (00 lbs)	County	Area (acres)	Yield of paddy (00 lbs)
	x	y		x	y
1	870	8521	11	1093	9562
2	883	8554	12	1111	10512
3	894	8783	13	1123	10974
4	901	8962	14	1177	11560
5	914	7025	15	1203	11631
6	973	8987	16	1224	11913
7	995	9569	17	1229	11517
8	1031	9598	18	1290	11955
9	1043	10316	19	1290	11639
10	1054	8963	20	1302	12428

Obtain an unbiased estimate of the total yield of paddy (y) and an unbiased estimate of the variance of this estimator. Also give an unbiased estimate of variance of the corresponding estimator from a simple random sample of size 6 from this population. Hence obtain estimate of percent gain in efficiency of the sampling with probability proportional to area, over the simple random sampling.

$$(6+6+8+8+2) = 30$$

2. (a) Below are given the number of inhabitants (in thousands) for the two years, 1920 (x) and 1930 (y), in each of a random sample of 32 cities drawn from a list of 196 cities, with equal probabilities but without replacement.

x	y	x	y	x	y
76	80	172	183	77	83
158	143	78	106	61	63
67	67	66	86	54	77
23	50	80	57	56	142
391	464	44	87	40	60
93	48	32	50	40	64
37	63	507	634	38	72
120	115	179	200	136	169
61	89	121	113	118	130
357	459	50	61	46	53
93	104	41	59		

The total number of inhabitants in all the 196 cities in 1920 was 22,919 thousands.

Estimate the total number of inhabitants in all the 196 cities in 1930, using each of the following two methods :

- (a) ignoring the x values, and  
(b) using the x values in the ratio method of estimation.

Also obtain the standard errors of the estimates given by you.

$$(7+8) = 15$$

P.T.O.

(b) The results obtained from a census of all the 34 villages of a district are as follows :

Stratum No.	Size class of village (acres)	$N_i$	$\mu_i$	$\sigma_i$
1	0 - 500	63	112.1	58.1
2	501 - 1500	109	276.7	118.4
3	1501 - 2500	53	858.1	186.0
4	2501 -	25	980.1	361.3

The villages were divided into four strata by size. For the  $i$ th stratum,  $N_i$  denotes the number of villages,  $\mu_i$  the mean area under wheat per village, and  $\sigma_i$  the standard deviation of the area under wheat in a village. A 10% sample of the villages is to be taken to estimate the (current) average area under wheat per village.

Using the above data, compare the efficiency of the following three schemes of sampling:

- (1) simple random sampling without stratification.
- (2) stratified sampling with simple random sampling within each stratum, and samples allocated to the strata in proportion to

$$(i) N_i \quad \text{and} \quad (ii) N_i \sigma_i^2$$

Comment on your findings.

$$(CV\%) = 15$$

INDIAN STATISTICAL INSTITUTE  
 Statistician's Diploma Examination - November 1974  
 Paper VII : Applied Statistics (Practical)  
 (covering two half-paper groups of 50  
 marks each)

Time : 5 hours

Full Marks : 100

- a) Candidate will be required to answer questions only from those two groups for which they have already registered their options.
- b) Separate answer books are to be used for each of the two groups.
- c) Use of calculating machines is permitted.

SD VII : GROUP (a) : ECONOMIC STATISTICS Half paper  
(50 marks)  
 (Attempt any two questions from this group)

1. The following table gives the index numbers of demand for agricultural products ( $Y$ ), prices of agricultural products ( $X_1$ ) and national income ( $X_2$ ), for the years 1952 to 1959. Obtain the elasticities of demand for agricultural products with respect to price and income, by making use of a suitable regression equation.

Year	Y	$X_1$	$X_2$	Year	Y	$X_1$	$X_2$
1952	100	100	100	1956	120	75	190
1953	105	91	98	1957	127	91	113
1954	117	93	105	1958	118	91	114
1955	120	72	96	1959	134	96	125

= 25

2. The following table gives the average weekly earnings (in pounds) in Australia, for the quarters ending in September (S), December (D) March (M) and June (J).

Year	Quarter			
	S	D	M	J
1951-52	13.52	14.52	13.79	14.95
1952-53	15.14	16.19	14.70	15.77
1953-54	16.03	16.93	15.53	16.55
1954-55	16.79	17.63	16.27	17.91
1955-56	17.94	18.83	17.54	18.80
1956-57	18.86	19.92	18.35	19.53

Assuming the seasonal pattern to be constant, calculate the indices of seasonal variation; comment on them.

(22 + 3) = 25

3. Given below are the indices (base = 1939) of volume of production ( $Q$ ), of average number of wage earners ( $L$ ) and of physical volume of capital ( $K$ ), in manufacturing industries in the U.S.A. for some selected years. Estimate  $\alpha$  and  $\beta$  of the production function  $Q = AL^\alpha K^\beta$

Year	Q	L	K
1900	101	105	107
1903	121	123	131
1906	152	133	163
1909	155	140	199
1912	177	152	226
1915	180	154	266
1918	223	200	356

Examine the nature of returns to scale. Is it necessary to assume competitive market conditions?

(22 + 3) = 25

SD VII : GROUP (b) : STATISTICAL QUALITY CONTROL : Half paper  
(50 marks)

(Attempt question 1 and any two others from this group)

1. The percentage impurities in a standard chemical used in a laboratory, is known to be 1.59. Ten analysts performed duplicate analyses on samples from this chemical, yielding the following results.

Analyst	Percentage	Impurity	Analyst	Percentage	Impurity
1	1.59	1.37	6	1.58	1.56
2	1.73	1.65	7	1.53	1.69
3	1.70	2.28	8	1.52	1.66
4	1.79	1.72	9	1.69	1.68
5	1.64	1.67	10	1.76	1.69

Analyse the above data by drawing an  $\bar{X}$  and R chart, or otherwise. Comment on the accuracy, precision and reproducibility of the test results.

(8)

2. Obtain, without making use of any published sampling plans, a single sampling attributes plan which approximately satisfies the following criteria :

AQL = 1.5%

Producer's Risk = 0.05

LTPD = 6%

Consumer's Risk = 0.10

(15)

3. A component is machined to obtain a slot whose depth is to meet the specifications :

$$0.465 \text{ in} \pm 0.010 \text{ in.}$$

The standard deviation of depth for this operation is known as 0.002 in. It is also known that for every 60 pieces machined, there is a progressive linear decrease in average depth (to the extent of 0.001 in) attributable to tool-wear.

- (a) Set up the control chart with slowing limits as applicable to such a situation.  
(b) Hence estimate the number of pieces expected to be machined, before the tool has to be re-sharpened.

(9+6) = 15

4. Draw the AOQ curve for the following single sampling attributes plan :

Lot size	Sample size	Acceptance number
2000	150	2

From the curve drawn, read the approximate value of AOQL.

(15)

## SD VII - GROUP (c): STATISTICAL METHODS IN GENETICS (theoretical): Half paper (50 marks)

(Attempt all the questions from this group)

1. The following data relate to 3 backcross tests for the genes A and B.

test	frequencies			
	AB	Ab	aB	ab
I	58	92	85	65
II	53	78	72	47
III	58	77	75	50

Examine whether the four tests provide mutually consistent estimates of the linkage parameter.

(12)

2. In a cross, segregating for petal colour (A - a) and stigma colour (B - b) in linseed, the following observations are obtained in F<sub>2</sub> generation :

phenotype	AB	Ab	aB	ab
frequencies	357	37	33	94

Test for the segregation ratios and linkage.

If the linkage is present, find out the estimate of the cross-over value along with its standard error.

(12)

3. Estimate the O-A-B gene frequencies from the observed frequency distribution of O, A, B and AB blood group classes.

blood groups	O	A	B	AB
frequencies	70	45	68	15

Test also for the goodness of fit.

(12)

4. The frequencies of blood groups MN, MN and NN in two samples from two populations are as follows :

genotype } frequencies	MN	MN	NN
Sample I	36	52	16
Sample II	49	42	15

Examine whether the gene frequencies of m and n are the same, in both the populations.

(12)

Shortness :

(125)

(2)

P.T.O.

SD VII - GROUP (d) : VITAL STATISTICS AND DEMOGRAPHY : Half paper  
(50 marks)  
(Attempt any two questions from this group)

1. From the data furnished below, calculate the projected female population in 1956.

Age group	Female population in 1931 (000)	Survival factor for 5 years	Age group	Female population in 1931 (000)	Survival factor for 5 yrs
0-4	1436.2	.9810	35-39	1510.0	.9739
5-9	1614.8	.9950	40-44	1454.2	.9620
10-14	1595.8	.9934	45-49	1347.4	.9495
15-19	1735.0	.9987	50-54	1260.3	.9268
20-24	1735.3	.9971	55-59		
25-29	1728.1	.9950			
30-34	1632.0	.9921			

  

Age of mother	Fertility rate	No. of women in 1931 (000)	Age of mother	Fertility rate	No. of women in 1931 (000)
15-19	0.2962	1725.0	35-39	0.0667	1510.0
20-24	0.2110	1795.3	40-44	0.0206	1454.2
25-29	0.1956	1728.1	45-49	0.0033	1347.4
30-34	0.1158	1632.0			

You are given that : (a) the sex-ratio (females to total) at birth = 0.485  
(b) Survivors aged 0-4, in 1931  
Total No. of births during quinquennium 1931-36 = 0.945  
(25)

2. Construct an abridged life-table for a female population, from the data given below :

Age group	Number of females enumerated in 1961 ('000)	Number of female deaths registered during 1960-62
15-20	751	2978
20-25	767	4070
25-30	711	4455
30-35	627	4972
35-40	580	6591
40-45	572	9532
45-50	587	15340

Take  $l_{15} = 100,000$  and  $e_{50} = 19.7$

(25)

3. The following information is obtained from a rural sample survey :

Age group	No. of women	Proportion married	No. of births	Proportion surviving from birth to mid-point of age-group among married women
15-19	16,952	.82	2,612	.902
20-24	14,137	.83	4,272	.891
25-29	10,800	.84	2,179	.878
30-34	7,900	.85	790	.865
35-39	2,463	.74	203	.849
40-44	925	.61	47	.830

- (a) Calculate net reproduction rate, assuming that :  
(i) there was no illegitimate birth,  
(ii) ratio of female to total birth was 0.485 .  
(b) Determine the probability of a female child dying (after marriage) at age 31½.

(12+)

(20+5) = 25  
P.T.O.

SD VII - GROUP (c); EDUCATIONAL AND PSYCHOLOGICAL STATISTICS: Half paper  
(50 marks)

(Attempt any four questions from this group)

1. In a certain contest, five judges have been asked to give a simple rank order of the 10 contestants. The results are given below :

Judges	Contestants									
	1	2	3	4	5	6	7	8	9	10
J1	5	4	7	6	3	2	1	8	10	9
J2	8	6	7	4	1	3	5	2	10	9
J3	5	4	3	2	6	1	9	10	7	8
J4	4	5	6	3	2	1	9	10	8	7
J5	8	6	7	5	1	2	3	4	10	9

Test whether the rankings by the five judges may be regarded as concordant.

Assuming they are concordant, combine the five rankings suitably and assign final ranks to the contestants.

(8+4) = 12

2. (a) A test containing 10 items with a reliability of 0.61 is to be expanded so as to give a reliability of 0.90. How many new items of the same type as the original ones, must be added?
- (b) A certain intelligence test of 75 items has a validity coefficient of 0.68 and a reliability coefficient of 0.95. Estimate what the validity would be if the test was doubled in length.
- (c) A test has a reliability coefficient of 0.90 and  $\sigma = 20$ . What is the standard error of an obtained score on this test? What is the index of reliability?

(4+4) = 12

3. A large group of students was examined in a subject by three examiners,  $E_1$ ,  $E_2$  and  $E_3$ , independently. The standard of marking of the examiners are reflected in the percentage frequency distributions of scores given below :

score	percentage frequency distribution of			score	percentage frequency distribution of		
	$E_1$	$E_2$	$E_3$		$E_1$	$E_2$	$E_3$
0-10	5	5	10	50-70	10	21	10
10-30	25	15	20	70-90	8	5	2
30-50	50	50	58	90-100	2	1	-

P.T.O.

3. Determine the relative positions of three students A, B and C, (contd.) who scored the following marks with the three examiners :

Student	Score given by		
	$E_1$	$E_2$	$E_3$
A	25	60	70
B	47	55	38
C	76	23	52

(12)

- 4.(a) Obtain the correlation matrix for the tests included in the factor matrix appearing below:

Test	Factor			
	A	B	C	D
1	0.10	0.70	0.10	0.50
2	0.60	0.10	0.10	0.45
3	0.45	0.00	0.55	0.52
4	0.00	0.00	0.30	0.32
5	0.30	0.50	0.62	0.11

- (b) The following values are taken from the first row of a Factor matrix :

Test	Factor			
	A	B	C	D
1	0.45	0.00	0.55	0.52

The specific variance of the test is known to be 0.1175.

- (i) What is the communality of the test?  
 (ii) What is the reliability of the test?  
 (iii) What is the unique variance?
5. With the following data for 75 individuals, obtain a linear prediction formula for predicting  $X_1$  from a knowledge of  $X_2$  and  $X_3$ .

$X_1$  = criterion : average grade in undergraduate college.

$X_2$  = average grade over 4 years of high school.

$X_3$  = score on group intelligence test.

Means :  $\bar{X}_1 = 70.00$ ,  $\bar{X}_2 = 87.20$ ,  $\bar{X}_3 = 32.80$

SDs :  $s_1 = 10.21$ ,  $s_2 = 6.02$ ,  $s_3 = 10.35$

Correlation coefficients:  $r_{12} = 0.67$ ,  $r_{13} = 0.75$ ,  $r_{23} = 0.63$ .

What average grade in college would you expect of a candidate who earned a grade of 80 in high school and a score of 40 on intelligence test?  
 (10\*2)=12



Statistician's Diploam Examination - November 1974

Paper VIII (Theoretical) : Subjects of Specialization I

Time: 4 hours

Full marks : 100

- (a) Candidates will be required to answer from that group only for which they have already registered their option.  
 (b) Figures in the margin indicate full marks.

GROUP A : ECONOMIC STATISTICS

(Special Paper I : Econometrics)

(Attempt any five questions from this group)

1. "Econometrics is defined as a branch of economics in which economic theory and statistical methods are fused in the analysis of numerical and institutional data".  
Elucidate, giving appropriate examples. (20)
2. Give exact definitions of the concepts of 'bias' and 'efficiency' in statistical estimation. In the standard linear model  $Y = X\beta + u$ , show that  $b = (X'X)^{-1} X'Y$ , provides the best linear unbiased estimate of  $\beta$ . State all your assumptions very clearly. (20)
3. What do you understand by the term 'heteroscedasticity'? Explain two common types of heteroscedasticity and suggest suitable amendments to the ordinary least squares method of estimation in each case. (20)
4. In the complete linear simultaneous equations model :  
$$Ay + Bx = z$$
where the letters have their usual meaning, obtain the necessary and sufficient conditions for the parameters in A and B to be identified. (20)
5. Define the economic concepts of income and price elasticities of demand. How would you estimate them empirically? Suggest one or two practical uses for your estimates. (20)
6. Discuss the main criteria for choosing the algebraic specification of an Engel curve. Describe a method of estimating Engel elasticities from grouped family budget data. (20)
7. Give economic interpretation of the parameters in the production function as formulated by Arrow, Chenery, Kinhas and Solow. How can these parameters be estimated? (20)
8. Describe the main elements of an inter-industry flow matrix, with Indian examples. How is it used in economic planning in India? (20)
9. Write short notes on any two of the following :  
 i) Haavelmo bias                      iii) Neo-classical production function  
 ii) Two stage least squares        iv) Fractile graphical analysis (20)

Please turn over

## GROUP B : TECHNO-COMMERCIAL STATISTICS

## (Special Paper I : Statistical Quality Control)

(Attempt any five questions from this group)

1. (a) Explain the theoretical basis for fixing the control limits for  $\bar{X}$  chart. Derive an expression for the control limits when sample size is less than 10.
- (b) What is meant by rational sub-groups? Explain its role in operating a control chart.
- (c) Discuss the setting up of a control chart for the number of defects per unit, when the number of units inspected at a time varies? (10+4+6)=20
2. (a) What is meant by 'Natural Tolerance limits'?
- (b) Distinguish between specification limits, natural tolerance limits and control limits.
- (c)  $\bar{U}$  and  $\bar{L}$  are specification limits for a certain characteristic of a product known to be distributed as normal. If the process capability is less than 1.33, explain how you will modify the control limits for the chart for averages.
- (d) If  $\sigma$  is the standard deviation of a measured characteristic, find out at what level the process should be maintained in order to ensure that not more than 10% of the items produced will be having a value less than or equal to  $L$ . (2+6+6+6)=20. Define
3. (a) What is the standard cusum scheme?
- (b) Explain under what circumstances cusum charts will be preferable to  $\bar{X}$  chart. (12+6)=20
4. (a) Briefly state the problem of acceptance sampling.
- (b) Define the terms AQL and LTPD. What are the risks involved in adopting sampling inspection for lot acceptance?
- (c) Mention the salient features of Dodge and Romig single sampling plans, for inspection by attributes. (2+5+5+10)=20
5. (a) What is a sequential sampling plan?
- (b) Derive expressions for the acceptance and rejection lines of a sequential probability ratio test for proportion with AQL, LTPD, Producer's and Consumer's risk specified as  $P_1, P_2, \lambda$  and  $\beta$  respectively.
- (c) Derive the expression for ASN for the above plan. (4+6+10)=20
6. (a) What is meant by 'Interaction Effect'? Explain by an industrial example.
- (b) Explain Yates's method for computing factorial effects from the data of a  $2^3$  factorial experiment.
- (c) Write a short note on WOP. (6+10+6)=20
7. Write short notes on any three of the following:
- Use of components of variance analysis.
  - Work sampling vs. Time study.
  - Disk sampling.
  - Use of OC curves in the selection of sampling plans. (20)

Special Paper I - Statistical Aspects

(Attempt any six questions)

1. (a) Mention the various basic principles underlying a good experimental design. Discuss briefly the constructive roles of each of them.
- (b) In a randomized block experiment with  $v$  treatments and  $v$  blocks, the observations pertaining to the  $i$ -th treatment in the  $i$ -th block were reported missing ( $i = 1, \dots, v$ ). Indicate how you would carry out the analysis of the data. (8 + 8) = 16
2. Discuss the rationale behind analysis of covariance? Describe the analysis of covariance in a randomized block design with  $t$  treatments and  $b$  blocks, and derive an expression for the standard error of the difference of any two adjusted treatment means. (16)
3. (a) Why are Factorial experiments superior to single factor experiments?
- (b) Explain, with suitable illustrations, the concept of 'confounding'. What do you understand by Balanced Confounding?
- (c) How do you analyse a  $3^3$  experiment in 9 plot blocks and two replications. (4+4+2+5) = 15
4. (a) Define a Balanced Incomplete Block (BIB) design with parameters  $v, b, r, k, \lambda$ . When is a BIB design called resolvable? Show that for any resolvable BIB design,  $b \geq v + r - 1$ .
- (b) How do you recover inter-block information in resolvable BIB design. (2+2+5+6) = 15
5. (a) Define partially balanced incomplete block design with two associate classes.
- (b) Give an example of a 2-associate partially balanced incomplete block design and write down its parameters.
- (c) Discuss the intra-block analysis of a 2-associate partially balanced incomplete block design. (5 + 5 + 8) = 16
6. It is desired to test the efficacy of three feeds on dairy cattle. The feeds are to be applied over three seasons in a year, namely, summer, rainy and winter. It is known that the effect of any feed persists to the immediate next season in which it is applied. Assuming that you have twelve animals at your disposal, suggest a suitable design for the experiment and also give its analysis. (8 + 8) = 16
7. What are Second order rotatable designs? To what use can such designs be put? How do you carry on the analysis of such designs? (5 + 3 + 8) = 16
8. (a) Distinguish between fixed -, mixed -, and random effects models. Illustrate your answer with suitable examples.
- (b) Discuss the analysis of a two way classified data with equal number of observations per cell under a random effects model. (8 + 9) = 16
9. Write notes on any two of the following :-
  - (i) Split plot design.
  - (ii) Fractional replication.
  - (iii) Uniformity trials.
  - (iv) Youden Squares (9 + 9) = 16

NEATNESS

Please turn over <sup>4</sup>

## SD VIII - GROUP B : SAMPLE SURVEYS

Special Paper I - Theoretical Aspects

(Attempt any five questions)

1. For estimating the area under paddy in a district all the 3000 villages of the district are grouped into 4 strata, taking geographical continuity into account. Keeping in view the staff available for collection of data and also for scrutiny and analysis, it is decided to draw a sample of 300 villages from the district, for enumerating the area under paddy. If you are required to use a stratified random sampling design, for selecting the sample and building up the estimate of area in question, explain how you allocate the 300 villages to the different strata. Assuming that the cost of survey per village in each stratum is available, estimate the area and also obtain its variance. If the cost of survey per village is the same for all strata, compare this estimate with that which you would get, had a simple random sample of 300 villages, been drawn from the entire district.

(5+5+5+5) = 20

2. The values of  $y$  and  $x$  are measured for each unit of a simple random sample from a population. If  $\bar{y}$ , the population mean of  $y$  is known which of the following procedures you recommend for estimating  $\bar{y}/\bar{x}$  ?
- Always use  $\bar{y}/\bar{x}$
  - Sometimes  $\bar{y}/\bar{x}$  and some times  $\bar{y}/\bar{x}$
  - Always use  $\bar{y}/\bar{x}$

Give reasons for your answer, outlining brief proofs where necessary. (20)

3. In a population consisting of linear trend, show that a systematic sample of  $n$  units, (where the population size  $N = nk$ ) gives rise to an estimate which is less precise than that of a stratified random sample of two units per stratum from strata of size  $2k$ , if  $n > (4k+2)/(k+1)$ .

4. (a) A sample of  $n$  units is drawn from a population with probability  $p_i$  for the  $i$ -th unit ( $i=1, \dots, N$ ) and with replacement. If  $y_i$  is the value of the study variate on the  $i$ -th unit, obtain a suitable estimate of the population total  $Y = \sum_{i=1}^N y_i$  and also derive its variance. Find an unbiased estimate of this variance.
- (b) For the above sampling scheme, obtain the probability of inclusion of  $i$ -th and  $j$ -th units in the sample. Hence write down the Horvitz-Thompson estimator for the population total and its variance estimator due to Yates and Grundy.

5. A population consists of  $N$  primary stage units ('psu's) each containing  $M$  secondary stage units ('ssu's). A sample of 'n' psu's is drawn from the population by simple random sampling and from each selected psu a simple random sample of 'm' ssu's is chosen without replacement. Write down an unbiased estimate of the population total. Work out its variance and also the variance estimate. (5+4+7) = 20

6. In a survey for the estimation of area under a specific crop, a simple random sample of villages is chosen from a district and in each selected village the number of fields under a given crop and the number of fields growing a specific high yielding variety of the crop, are noted. Explain how you go about to obtain an unbiased estimate of the proportion of fields growing the given high yielding variety of the crop in question. Derive its variance and also the variance estimate. (5+9+7) = 20

7. What is the importance of double-sampling technique in sample surveys? Describe its use relating to Dinkins two variate ratio estimator. (5+15) = 20
8. What are non-sampling errors? Describe the Hansen & Hurvitz technique of assessing non-response, in detail. (5+15) = 20

(20)

Please turn over

## SD VIII - GROUP F : TECHNIQUES OF COMPUTATION

## Special Paper I - Numerical Analysis

(Answer any five questions)

1. (a) What are the uses of punch card machines? In what way digital computers differ from desk calculators? Give a functional description of a digital computer. What kinds of error are committed, in a digital computer?
- (b) Let  $x$  and  $y$  be two real numbers and  $x^*$  and  $y^*$  be approximations to  $x$  and  $y$ , with absolute errors  $\xi$  and  $\eta$  respectively. Show that the relative error in the quotient  $x/y$  is :

$$\left[ \xi/x^* - \eta/y^* \right] / \left[ \xi + \eta/y^* \right]$$

Under what conditions can you assert that the relative error in a quotient does not exceed in magnitude, the sum of the magnitudes of the relative errors in the individual terms.

- (c) If all coefficients in the definition,
- $$f(x) = (5.03241x + 0.11095)/(0.75995x + 0.611015)$$
- are rounded numbers, to how many significant figures can you determine  $f(x)$ , when  $x$  is known to lie between 3.25 and 3.25?  $(2+2+3+1)+(4+2)+8 = 20$ .
2. (a) Define divided difference of order 'r' and show that it is independent of the order of the arguments. Derive Newton's divided difference formula for interpolation and obtain its error term.
- (b) Show that the truncation error associated with linear interpolation of  $f(x)$ , using ordinates at  $x_0$  and  $x_1$  with  $x_0 \leq x \leq x_1$ , is not larger in magnitude than  $\frac{1}{2} (x_2 - x_0)^2$  where  $M_2$  is the maximum value of  $|f''(x)|$  in the interval  $(x_0, x_1)$ . Does this result hold when  $x \notin (x_0, x_1)$ ?  $(2+3+10)+(4+1) = 20$
3. (a) Derive Simpson's formula with the error term, for numerical integration. Interpret it geometrically.
- (b) Given three evenly spaced pivotal values  $y_0, y_1$  and  $y_2$ . Obtain  $y_1^2, y_0^2$  and  $y_2^2$  with errors of order  $h^2$ ,  $h$  being the difference between two consecutive pivotal points.  $(10 + 10) = 20$
4. Derive Euler-Maclaurin sum formula and hence or otherwise, deduce Stirling's asymptotic formula for the factorial, in the form  $(10+10) = 20$
- $$n! = \sqrt{2\pi n} \cdot n^n e^{-n} \left( 1 + \frac{1}{12n} + \frac{1}{288n^2} + \dots \right)$$
5. Give an iteration scheme to compute the value of a polynomial and its derivative at a given point. How do you obtain all the roots of a polynomial equation? Illustrate your answer with examples.  $(5 + 15) = 20$
6. (a) Define a linear difference equation. Describe a method for its solution. Give examples to illustrate your answer.
- (b) Describe an iterative method for solving a differential equation and discuss its convergence.  $(10 + 10) = 20$
7. Define a triangular matrix. Show how a matrix can be decomposed into the product of two triangular matrices. Prove that the inverse of a triangular matrix is triangular. Hence, suggest a method for computing inverse of a real non-singular square matrix.  $(2+6+6+6) = 20$

## DIVERSITY IN STATISTICAL INFERENCE

## (Special Paper I - General Theory)

(Attempt any five questions from this group)

1. (a) Define the terms uniformly most powerful test, unbiased test, similar test, test having Neyman structure, Baye's test and minimax test.
- (b) State conditions under which the problem of finding a most powerful unbiased test can be reduced to the problem of finding a most powerful similar test. Prove your statement.
- (c) State conditions under which a Baye's test is a minimax test. Prove your statement.
2. (a) Discuss the role of least favourable distributions in hypotheses testing.
- (b) Let  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$  be  $n$  independent random variables each distributed normally with unknown mean  $\mu$  and variance 1. Obtain a most powerful test of level of significance  $\alpha$  for testing the hypotheses  $H: \mu \geq \mu_0$  against the alternative  $H_1: \mu < \mu_0$  where  $\mu_0 > 0$ .
3. (a) Let  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$  be  $n$  independent random variables each distributed uniformly on  $(0, \theta)$ . Show that for testing the hypothesis  $H: \theta = \theta_0$  against the alternative  $H_1: \theta \neq \theta_0$ , a unique uniformly most powerful test  $\phi(x)$  exists and is given by  $\phi(x) = 1$  when  $\max(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) > \theta_0$  or  $\max(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \leq \theta_0 \sqrt{\alpha}$  and  $\phi(x) = 0$ . Otherwise, where  $\alpha$  is the level of significance.
- (b) Let  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$  be independent normally distributed random variables with means  $\mu_1\theta, \mu_2\theta, \dots, \mu_n\theta$  and standard deviations  $\sigma_1, \sigma_2, \dots, \sigma_n$  where  $\mu_i$  and  $\sigma_i$  are known numbers and  $\theta$  is unknown.
- Find the maximum likelihood estimator  $\hat{\theta}$  of  $\theta$ .
  - Show that  $\hat{\theta}$  is the minimum variance unbiased estimator of  $\theta$ .
  - Under what condition on the numbers  $\mu_1, \mu_2, \dots$  and  $\sigma_1, \sigma_2, \dots$  is  $\hat{\theta}$  a consistent estimator of  $\theta$  as  $n$  tends to infinity?
4. (a) Explain the concepts of sufficiency and completeness of a statistic.
- (b) Show that if a complete sufficient statistic exists, then every function of it is a uniformly minimum variance unbiased estimator of its expected value.
- (c) Let  $X$  be a random variable which is distributed in the Poisson form with parameter  $\lambda$ . Find the minimum variance unbiased estimator of  $P_r (X=r)$  on the basis of  $n$  independent observations  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$  from this population.
5. (a) Let  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$  be  $n$  independent observations on a random variable with a probability density function  $f(x; \theta)$ , where  $\theta$  is an unknown parameter. Prove any theorem which establishes that under certain conditions the corresponding most unbiased likelihood equation has a solution  $\hat{\theta}_n$  which is consistent as  $n$  tends to infinity.
- (b) Give an example of a statistic that is not sufficient.
- (c) Let  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$  be  $n$  independent random variables such that  $P_r \{X_i = 1\} = p$  and  $P_r \{X_i = 0\} = 1 - p$ , for some  $0 < p < 1$ . Obtain a consistent unbiased estimator of  $p^2$ . Does there exist an unbiased estimator of  $p^{-2}$ ?
6. Write notes on: (i) Confidence intervals and tests; (ii) Zero sum two person game and decision problems; (iii) Admissible decision rules and minimal complete class of decision rules.

INDIAN STATISTICAL INSTITUTE

Statistician's Diploma Examination - November 1974

Paper IX (Theoretical) - Subjects of Specialisation II  
Time : 4 hours Full Marks 100

- (a) Candidates are required to answer questions from that group only for which they have registered their options.  
(b) Figures in the margin indicate full marks.

GD IX GROUP A : ECONOMIC STATISTICS  
(Special Paper II)

Section I : Indian Economics (50 marks)

(Attempt any three questions from this section)

1. How important is 'Agriculture and the Allied Sectors' in India's Gross Domestic Product? Do you think that the importance of this sector should be reduced in the coming years? Give reasons for your answer. (4 + 12) = 16
2. What criterion is currently adopted to distinguish between (a) small-scale and large-scale industries, and (b) factory and non-factory industries? What are the special merits of small-scale, non-factory industries for which they deserve special areas of assistance? (4+4) = 8
3. Give arguments favouring restrictions on the sphere of operation of Large Business Houses in India. How are these L.B.H.s to be identified? (12 + 4) = 16
4. Comment on the following :-  
a) Indian Industry cannot stand up to the competition of foreign industries; so Import control is absolutely necessary for India.  
b) Monetary measures to check the inflationary situation in India are bound to bring about a recession. (8 + 8) = 16
5. Give 'yes' or 'no' answers to the following queries :  
a) Is national income in India smaller than the net domestic product at factor cost?  
b) Is the entire irrigated area in India sown more than once during the year?  
c) Do Employment Exchange registration figures convey any idea about the extent of unemployment in the country?  
d) Do people having a taxable income of Rs.12000 per year come under the Compulsory Deposit Scheme of 1974?  
e) Does the money supply in India vary with the amount of gold and silver in the Reserve Bank?  
f) Do railways contribute to India's general finances on the basis of the profit which they earn?  
g) Does the amount of land revenue payable by a farmer vary progressively with the amount of land which he holds?  
h) Have all Indian commercial banks been nationalised? (2 x 8) = 16

Section II : Economics of Planning (20 Marks)(Attempt any three questions from this Section)

6. 'Self-reliance should be the key-note in our economic planning.'  
Examine this statement in the context of India's Fifth Five Year Plan. (16)
7. In what way did the Harrod - Domar model of economic growth influence Indian planning? Did this model, in your opinion, provide planners with suitable policy instruments? Justify your answer. (8 + 8) = 16
8. What are the deficiencies of inter-industry analysis as applied to the construction of economic plans? Illustrate your answer with reference to the Draft of the Fifth Five Year Plan. (16)
9. Explain the economic significance of the 'dual' of a linear programme. If the market price of a scarce input is lower than its value (as obtained from the 'dual'), what policy conclusions will you derive? (10+6) = 16
10. In the context of the Indian Plan, distinguish between  
(a) Public Investment and Current Public Development Outlay,  
(b) Gross Foreign Aid and Net Foreign Aid,  
(c) Major and Minor Irrigation Projects, and  
(d) 'core' and 'non-core' industries. (4 x 4) = 16
- Markings (Sections I and II) (4)

## SD IX GROUP B : TECHNICAL COMMERCIAL STATISTICS

(Special Paper II)

## Section I : Operations Research (70 Marks)

Section I : Elements of Book-keeping & Accountancy (70 Marks)  
(Alternative)

## Section II : Statistical Methods in Business (50 Marks)

Please turn over for questions on Sections I, I (alt.) and II



## SD IX - GROUP B: TECHNICAL COMMERCIAL STATISTICS

## (Special Paper - II)

Section I : Operations Research (70 Marks)

- i) Use a separate answer book for this Section.  
 ii) Attempt any four questions from this Section.

1. Solve the following linear programming problem by Simplex method:

$$\text{Minimize } Z = x_1 + x_2$$

subject to the following constraints

$$(i) 2x_1 + x_2 \geq 4 \quad (ii) x_1 + 7x_2 \geq 7$$

$$(iii) x_1, x_2 \geq 0$$

2. There are three collieries a, b and c which can supply coal in bulk as specified against the colliery names.

(i) Colliery a	.....	14 tons
(ii) Colliery b	.....	12 tons
(iii) Colliery c	.....	5 tons

There are three consumers A, B and C who require quantities of coal as specified against each of them:

(i) Consumer A	.....	8 tons
(ii) Consumer B	.....	10 tons
(iii) Consumer C	.....	15 tons

The costs (in rupees) of transportation of 1 ton of coal from any one colliery a, b or c, to any one consumer A, B or C, are as shown below :-

	From Colliery		
	a	b	c
To consumer : A	8	8	4
B	4	9	5
C	1	2	6

Obtain the quantities of coal to be sent from the collieries a, b, c to each of the consumers A, B, C so that the total cost of transportation turns out to be the minimum.

3. A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amounts to Rs.0.60 per unit per year. The set-up cost per run is Rs.80/- only. Find the optimum run size and the minimum average yearly cost. Establish the formulae you use.
4. A supermarket has two girls ringing up sales at the counters. If the service-time for each customer, is exponential with mean value 4 minutes and if people arrive in a poisson fashion at the counter at the rate of 10 people an hour:-
- What is the probability of having to wait for service?
  - What is the expected percentage of idle-time for each girl?

Please turn over

Section I : Operation Research (Contd.)

5. (a) A firm buys Rs.10,000/- for its automobiles. Their operating and maintenance costs are about Rs.2,500/- per year for the first two years and then go up by approximately Rs.1,500/- per year. When should such cars be replaced? The discount rate is 0.9.
- (b) Discuss a machine repair problem.
6. Write notes on any two of the following :
- Dynamic programming
  - Ware-housing problem
  - Monte Carlo method.

## SD IX - GROUP B : TECHNICAL STATISTICS

(Special Paper II)

Section II : Statistical Methods in Business (30 Marks)

- (i) Use a separate answer book for this Section.
- (ii) Attempt any two questions from this Section.
- Discuss the estimation of economic relationships and identification problem. (15)
  - Write an essay on the role of Statistics in Market Surveys (15)
  - Write notes on any two of the following :-
    - Business Forecasting
    - Incentive Schemes
    - Selection of Salesman. (15)

SD IX GROUP C: BIO-METRY (Statistical Methods in Genetics & Bio-Assays) No Candidates available
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## SD IX - Group B : Techno-Commercial Statistics (Contd.)

Section I : Elements of Book-keeping & Accountancy  
(Alternative)

- 1) Use a separate answer book for this Section.  
ii) Attempt question no.1 and any three others from this Section.

1. From the following Trial Balance data of H.K. Singh as at 31st December, 1973, you are required to prepare a Trading and Profit & Loss Account for the year ended 31st December, 1973 and also a Balance Sheet as at this date, after making the necessary adjustments.

<u>TRIAL BALANCE</u>			
1. Drawing A/c (H.K.S.)	Rs. 6000	1. Capital A/c. (H.K.S)	Rs. 80000
2. Plant & Machinery :		2. Sundry Creditors	10000
a) Balance as on		3. Sales	120000
1.1.73	20000	4. Returns Outwards	1000
b) Addition as on		5. Provision for Doubtful Debts	400
1.7.73	5000	6. Discounts	800
3. Stock on Jan 1, 1973	15000	7. Rent of Premises sublet for a year (upto 30th June, 1974).	1200
4. Purchases	82000		
5. Returns Inward	2000		
6. Sundry Debtors	20000		
7. Furniture & Fixture	5000		
8. Freight & Duty	2000		
9. Carriage Outward	500		
10. Rents, Rates & Taxes	4000		
11. Printing & Stationery	800		
12. Trade Expenses	400		
13. Postage & Telegram	800		
14. Insurance charges	700		
15. Salaries & Wages	21300		
16. Cash in hand	6200		
17. Cash at Bank	20500		
	Rs. 213400		Rs. 213400

Adjustments required:

- a) Stock on 31st December 1973 was valued at Rs.14600 (b) Write off Rs.600 as Bad Debts (c) The provision for Doubtful Debts is to be maintained at 5 percent on Sundry Debtors. (d) Provide depreciation on furniture & fixture at 5 percent per annum and on plant and machinery at 20 percent per annum. (e) Insurance provided was Rs.100 (f) A fire had occurred on 25th December, 1973 in the godown and stock of the value of Rs.5000 was destroyed. It was fully insured, and the Insurance Company admitted the claim in full. (7+9+9) = 25
2. Enter the following transactions in a three column Cash Book of M/s. XY & Co.
- 1.4.73 Cash in hand Rs.237; balance at Bank Rs.659;
  - 2.4.73 Received from K.Son Rs.590; allowed him discount Rs.10.
  - 4.4.73 Paid salaries for March 1973 by cash Rs.200/-; Cash Sales Rs.134.
  - 5.4.73 Paid B. Bose by cheque Rs.300; Cash purchases Rs.60.
  - 6.4.73 Paid Q. Ahmad by cheque Rs.584; discount received Rs.26.
  - 8.4.73 Cash Sales Rs.112; paid cartage Rs.6/-
  - 10.4.73 Withdrew from Bank for office use Rs.200/-; paid rent in cash Rs.50/-.
  - 14.4.73 Cash Sales Rs.212/-; Received from G.C. Dhar Rs.194/- by cheque, discount Rs.6/-.
  - 16.4.73 Deposited into Bank Rs.1,000/-; Purchased a Motor Car for Rs.5,500/- and drew a cheque for the amount.

(15)

3. From the following particulars, prepare a Bank Reconciliation Statement for M/s. Scientific Equipment Co. Ltd., as on 30th June, 1974.

- (a) Balance at Bank as per Cash Book Rs.2,000/-  
 (b) They had sent cheques amounting to Rs.10,000/- to the bank before 30th June but it appears from the Pass Book that cheques worth Rs.4000/- only had been credited before that date  
 (c) Cheques for Rs.5,000/- issued during the month of June but cheques for Rs.2,500/- were presented and paid in July

The Pass Book showed the following payments:-

- (a) Rs.320/- Insurance Premium, as per standing instructions  
 (b) Rs.2,000/- against promissory notes as per instructions

The Pass Book also showed that the Bank had collected Rs.600/- as interest on Government securities. The Bank charged interest Rs.50/- and Rs.20/- as Bank charges. There was no entry in the Cash Book for payments, receipts, interest etc. (15)

4. (a) What is a Bill of Exchange?  
 (b) A, wanting to raise some funds, draws a bill on B for Rs.1,000/-, on 1st April, 1974 for three months. B accepts the bill and sends it to A, who gets it discounted with his Bank at 6%. On the due date A remits the amount to B, who meets the bill. Give the journal entries in the books of both A and B. (4+1)=15

5. Write short notes on any two of the following:

- (a) Provision and Reserve (b) Secret Reserve and how created  
 (c) Depreciation stating any two methods (d) Sinking Fund (15)

6. Point out what is wrong with the following journal entries; pass correct entries where necessary.

(a)	Purchase A/c.	Dr.	500	
	To M/s. Duggal & Co.			500
	(being the purchase of furniture from M/s. Duggal & Co.)			
(b)	G. Mitra	Dr.	450	
	To Bank A/c.			450
	(being the payment to G. Mitra his salary for the month)			
(c)	Cash account	Dr.	320	
	To Cash Sales Account			320
	(being the Cash Sales for the day as per cash memo (Bill to 350))			
(d)	S. Guha & Co.	Dr.	350	
	To Purchases A/c.			350
	(being the purchase of goods from S. Guha & Co. on credit)			
(e)	Bank Account	Dr.	500	
	To S. Ghose			500
	(being the amount received from the Official Receiver of the estate of S. Ghosh, whose debt was written off last year)			

(15)

## SD EX GROUP D: DESIGN AND METHODS OF EXPERIMENTS

Special Paper II: Combinatorial Aspects(Attempt any five questions from this group)

1. (a) Show that the polynomial  $f(x) = x^2 + x + 2$ , is irreducible over  $GF(3)$ .  
 (b) Use this polynomial to construct  $GF(3^2)$ . Verify that  $x$  is a primitive element of the field. Write down the sum and product tables.  
 (c) Show that the number of mutually orthogonal latin squares of order 's' is at most  $s-1$ .  
 (d) Write down a complete set of mutually orthogonal latin squares of order 9 (nine). (5+5+5+5)=20
  
2. (a) Define a projective plane. Show that the existence of a complete set of mutually orthogonal latin squares of order 'n', implies the existence of a projective plane of order 'n'. Illustrate this with  $n = 3$ .  
 (b) State the Bruck-Ryser-Chowla Theorem on the non existence of projective planes of certain orders. Illustrate with examples. (10+10)=20
  
3. (a) State and prove Fisher's inequality for a Balanced Incomplete Block Design (BIBD).  
 (b) Show that any two blocks of a symmetric BIBD, have the same number of treatments in common.  
 (c) Derive a necessary condition for the existence of a symmetric BIBD, with even number of treatments. Illustrate this with two examples. (7+6+7)=20
  
4. (a) Define a Partially Balanced Incomplete Block Design (PBIBD) and derive the relations between the different parameters of the design.  
 (b) Define the dual of an Incomplete Block Design.  
 (c) Show that the dual of a nonsymmetric balanced incomplete block design, with parameters  $v, b, r, k, \lambda = 1$  is a PBIBD with parameters  $b, v, k, r; n_1 = k(r-1); \lambda_1 = 1; \lambda_2 = 0; p_{11}^1 = (r-2) + (k-1)^2$   
 and  $p_{11}^2 = k^2$ . (5+5+10)=20
  
5. Consider the Galois Field of residue classes mod  $p$ , where  $p$  is an odd prime.  
 (a) Define Quadratic Residues.  
 (b) Show that there are exactly  $(p-1)/2$  quadratic residues and they are given by :  
 $(i^2)$  where  $i = 1, 2, \dots, (p-1)/2$ .

5. (c) Show that if  $p \equiv 3 \pmod{4}$ , the quadratic residues form a difference set with respect to addition. Write down the parameters of this difference set.
- (d) Write down the quadratic residues of GF(11) and construct a symmetric balanced incomplete block design with parameters  $v = b = 11$ ,  $r = k = 5$  and  $\lambda = 2$ . (5+5+5)=20
6. (a) Describe the concept of fractional replication in factorial designs. Construct a  $\frac{1}{2^2}$  replicate of a  $2^6$  experiment, with the identity relationship  $I = ABCD = ABC^2 = CDEF$  and find 'alias sets'.
- (b) Describe the concept of confounding in factorial designs. Construct a  $3^3$  experiment in blocks of 9, confounding  $ABD$ ,  $ABD^2$ ,  $AC^2C^2$  and  $BC^2D^2$  where  $A, B, C, D$  are four factors, each at three levels. (10+10)=20
7. When is a design involving  $k$  quantitative factors called rotatable of order  $d$ ? State the conditions on the moments of a design which is rotatable of order 2. Discuss any method for the construction of three dimensional rotatable designs of order 2. (4+4+12)=20
8. Write notes on any two of the following:
- (a) MacLeod matrices and their use in the construction of fractional factorials.
- (b) Orthogonal Arrays and their relation to mutually orthogonal latin squares.
- (c) Finite geometries and their applications in the construction of designs. (10+10)=20

Please turn over

Special Paper II : Organizational aspects

(Attempt any five questions from this group)

1. The agriculture department wants to find out the annual productivity of mango trees in the Malda District of West Bengal. The results of the survey are required to be submitted in the form of a report within a year.
- Work out a suitable plan for the sample survey and indicate the organisational details and the administrative set-up, you would suggest, for the purpose.
  - If the said work is required to be carried out through committee constitution what basic changes would be necessary in the above plan of work? Discuss with justifications. Enumerate the advantages and disadvantages clearly, of the alternative procedures. (14 + 6) = 20
2. a) Enumerate the different stages of work for annual compilation involved in the construction of Weekly Consumer Price Index Number from price data collected from shops and stalls in 35 markets covering 123 items and involving 30 to 40 quotations for each item, if the weights concerned are known from previous survey results.
- If machine-time be available for the purpose, what modifications would you suggest in the above mentioned stages of work?
  - Discuss the safeguards and controls that you would like to have for such jobs in order to ensure accuracy, speed and the optimum utilization of the available resources. (12+4) = 16
3. a) What are the essential/divisions, in the preparation of a report on a socio-economic survey? What are the important points to be touched or covered under each division? If you think that some promotion should be maintained in the matter of discussion for the sake of a balanced presentation of materials, what would be your prescription? ((procedural)
- If the report is to be printed, through an outside press, prepare suitable guidelines for the staff to follow, indicating therein the precautions to be taken.
  - Mention the points on which special instructions are required to be given to the staff of the press for transforming the manuscripts into printed matter.
4. a) Describe in detail the various components of the budget of a large-scale sample survey.
- Give an outline of the organisational set-up that should be followed covering both field work and compilation work.
  - Enumerate the items to be printed or duplicated in bulk for facility of field work and compilation work and give an indication of their basic contents. (7+5+7) = 20
5. On receipt of a report about extensive damage to standing crops owing to heavy rain and flood, in a certain district, it is decided to carry out a quick survey for an assessment of the extent of damage (or benefit, if any). It is necessary to have the report on the survey within 8 weeks, by employing the monthly paid regular staff of an existing organisation.

5. (a) Indicate how you would approach the job, if you are given the charge. Discuss the organizational aspect of the work indicating the arrangements you would consider essential for ensuring error assessment, and enumerating the safeguards that should be sought in such quick surveys.

(b) If the same work is required to be carried out through piece-rate workers, discuss the difficulties and hazards involved. Under what circumstances, should such piece-rate workers be preferred?

(14 + 6) = 20

6. Offer your critical comments on the following :

(a) The detailed instructions for field workers are not usually read carefully and thoroughly by the Investigators. It is therefore unnecessary to issue detailed instructions for field workers.

(b) In the drawing up of instructions for field workers, it is hardly necessary to enunciate the object of survey.

(c) "Concepts and definitions" of commonly used terms and ideas need not be discussed in the Instructions for field workers in order to save time, labour and money.

(d) In respect of a survey of establishments of handloom industry, the Investigators often find the units closed when they call the selected establishments for survey. The Inspector in-charge wants to save time on account of such fruitless calls. He, therefore, suggests the survey of a substitute establishment in such cases, the substitute being any unit in the neighbourhood found open, when the Investigator calls there.

(e) For survey of a sampled household in connection with a socio-economic enquiry, the head of the household is not often available for interrogation. The field instruction should therefore be to obtain the household particulars from any member available in the household when the Investigator calls on the family.

(5+5+5+5) = 20

7. (a) For ensuring satisfactory field work, a survey organisation has 6 Inspectors one for each of a group of 5 Investigators. An indication of the precise nature of the duties and functions of these Inspectors is required to be chalked out, so that the best possible utilisation of their services is made. Give your suggestions for an outline of their duties and functions.

(b) What do you understand by the terms "spot check" and "back check" of field work? Discuss their advantages and disadvantages.

(c) Is there any other method or mode of checking of field work? If so, give an example and indicate its advantages and disadvantages.

(10 + 5 + 5) = 20

SD IX Group F : Techniques of Computation -(Practical with Desk Calculators) Group G : Statistical Inference -(Special Topics) Group H : Probability Theory -(Limit Distributions) <u>Special Paper II</u> No candidates available.
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INDIAN STATISTICAL INSTITUTE  
Statistician's Diploma Examination November 1974

Paper X: Subjects of Specialisation - Paper III (Practical)

Time : 5 hours

Full marks : 100

- i) Candidates are required to answer questions from that group only for which they have registered their options.
- ii) Figures in the margin indicate full marks.
- iii) Use of calculating machine is permitted.

Paper X : GROUP A: ECONOMIC STATISTICS  
Special Paper III- Practical  
No candidates available

GROUP B : ECONOMIC STATISTICS  
(Special Paper III - Practical)

- Section I : Statistical Quality Control (50 marks)  
Section II : Operations Research/Elements of Book keeping and Accountancy (30 marks)  
Section III : Statistical Methods in Business (20 marks)

Section I : Statistical Quality Control (50 marks)

- i) Use separate answer book for this Section
- ii) Marks in the margin indicate full marks

(Attempt Question No.1 and any other two questions)

1. The following, suitably coded, data were obtained over a week to initiate  $\bar{X}$  and R control charts on tensile strength of insulators. Five insulators each, were tested each day and tensile strength was determined.

Sub group no.	$\bar{X}$	R	Sub group no.	$\bar{X}$	R
1	57.0	23	11	50.8	19
2	50.0	5	12	56.4	18
3	53.1	22	13	50.4	17
4	53.0	12	14	50.2	14
5	57.0	17	15	60.6	16
6	50.4	8	16	59.0	16
7	50.4	15	17	57.0	12
8	50.6	10	18	50.4	13
9	58.0	17	19	51.4	17
10	50.2	12	20	50.5	14

- (a) Determine control limits for  $\bar{X}$  and R control charts.
- (b) What preliminary conclusions can be drawn from the control charts? Plot the 20 points on the charts using a graph sheet and hence comment whether introduction of control charts may have had some influence on the process. (Assume that the limits so obtained in (a) were used to initiate the control charts).

1. (c) Specifications on tensile strength are given to be  $31 \pm 11$ .  
 contd.) An insulator with tensile strength below the lower specification has to be scrapped while if the strength is more than upper specification, certain costly rework is needed.  
 At what average level the process should be set so that rework may be almost eliminated? (Assume that it is possible to make simple adjustment to change process average).
- (c) Obtain  $\bar{1}$  in 200 control limit for  $\bar{X}$  chart in (a).  $(3 \times 5 + 1 \times 3) = 20$
2. (a) Consider an assembly with dimension  $AD$ , which is the sum of dimensions of parts  $AB$ ,  $BC$  and  $CD$ . Dimension of each of the three parts is statistically controlled, with averages and standard deviations as given below :

Part	Average (inch)	Standard deviation (inch)
AB	1.450	0.0010
BC	0.385	0.0008
CD	1.170	0.0007

Determine the tolerance range for each of the three parts  $AB$ ,  $BC$  and  $CD$  and also for  $AD$ , assuming that

- i) upper and lower specification limits are at 4 sigma distance from the average,
  - ii) assembly is random,
  - iii) each dimension has normal distribution, and
  - iv) dimensions are independently distributed.
- (b) A manufacturer purchases small bolts in cartons that usually contain several thousand bolts. Each shipment consists of a number of cartons. As part of acceptance procedure for bolts, 400 bolts are selected at random from each carton and are subjected to visual inspection for certain defects. In a shipment of 10 cartons, the respective percentages of defectives in the samples from each carton are 0, 0, 0.5, 0.75, 0, 2.0, 0.25, 0, 0.25 and 1.25. Does this shipment of bolts appear to exhibit statistical control?  $(8 \times 7) = 15$
3. (a) A certain product is submitted for inspection in lots of ten articles. The inspection procedure requires taking a random sample of one article from each lot. If the chosen article is good the lot is accepted. If the article is bad, the lot is screened - all defectives are sorted out and are replaced by good ones. Compute AQL of this plan.
- (b) A double sampling plan is as follows: (a) select a sample of size 2 from a lot of 20. If both the articles are found good on inspection, accept the lot. If both are defective, reject the lot. If 1 is good and 1 defective take a second sample of one article. (b) If the article in second sample is good, accept the lot. If it is defective, reject the lot.
- If a lot 25% defective is submitted, what is the probability of acceptance? Compute the probability by using theoretically exact formula.  $(8 \times 7) = 15$

4. With a view to studying the effect of storage-time and packing on moisture content of salt, two types of packing A and B were chosen and periods selected were 1, 3 and 6 months. Six different samples of salt from same lot were analyzed in duplicate after storing for 1, 3 and 6 months in two types of packing. The following are the data:

Storage period (months)	Packing type					
	A			B		
	Sample			Sample		
	1	2	3	1	2	3
1	5.03	4.43	4.10	5.10	5.07	5.50
	6.13	4.25	4.33	5.00	5.21	5.49
3	6.10	6.17	5.00	4.31	5.09	4.91
	6.11	6.13	5.00	4.25	5.73	4.93
6	5.00	6.55	5.03	5.12	4.16	6.74
	7.00	6.35	5.27	4.27	4.07	6.59

Analyse the data and interpret the results. (15)

GROUP B : Section II : Operations Research (30 marks)

- i) Use separate answer book for this section.  
ii) Marks in margin indicate full marks.

(Attempt any two questions from this section)

1. (a) Apply the simplex method to solve

$$\begin{aligned}
 &\text{Minimise} && 15x_1 + 0x_2 + 0x_3 + 2x_4 \\
 &\text{subject to} && 2x_1 + x_2 + 5x_3 + 0.6x_4 \leq 10 \\
 &&& 3x_1 + x_2 + 3x_3 + 0.25x_4 \leq 12 \\
 &&& 7x_1 + x_4 \leq 35 \\
 &&& \text{every } x_j \geq 0.
 \end{aligned}$$

- (b). Is the solution obtained in (a) unique? (10\*3)=15
2. (a) A manufacturer is offered two machines  $M_1$  and  $M_2$ .  $M_1$  is priced at Rs.5000/= and running costs are expected to be Rs.800/= for each of first five years, increasing by Rs.200/= per year in sixth and subsequent years.  $M_2$  has same capacity as  $M_1$  and costs Rs.2500/= but will have running costs of Rs.1200/= per year for six years, increasing by Rs.200/= per year thereafter. If money is worth 10% per year, which machine is economic to be purchased? (Assume that the machines may be eventually sold at negligible price.)
- (b) Solve the following assignment problem minimizing the total cost of assignment.

	1	2	3	4
A	0	12	6	10
B	7	3	7	5
C	6	5	11	11
D	6	0	2	10

(3\*7)=15

- 3) Relevant data for a transportation problem are given in the following table :

		Destinations				
		$D_1$	$D_2$	$D_3$	$D_4$	Availability
Store	$S_1$	7	9	1	0	0
	$S_2$	22	25	16	25	3
	$S_3$	28	29	21	23	7
	$S_4$	12	14	5	15	7
Requirement	5	5	7	0		

- (a) Solve this transportation problem and obtain optimal solution.  
 (b) ^ sensitivity analysis to show how the solution in (a) changes if availability at  $S_2$  changes to 4 and requirement at  $D_2$  goes to 9. What is the value of new objective function? (10\*3)=15

GROUP B : Section 'II' : Statistical Methods in business (20 marks)

- i) Use separate answer book for this section.  
 ii) Marks in margin indicate full marks.  
 (Attempt both questions from this section)

1. In preparing statement of yearly profit of a firm for the period 1965-1973 it has found that the profit figure for the year 1973 was omitted by oversight. Estimate the profit for 1973 along with the standard error. The data for 1965-72 are :

Year	'65	'66	'67	'68	'69	'70	'71	'72
Profit (Rs. lakhs)	3.7	2.3	3.9	4.5	4.8	4.0	4.7	5.1

2. In a market survey conducted by a radio manufacturing company two informants were shown ten radio models. The informants ranked the radios according to their own preferences as follows :

Model	1	2	3	4	5	6	7	8	9	10
Informant I	10	3	7	6	4	2	8	1	5	9
Informant II	8	7	4	3	5	2	9	6	0	10

Is there enough evidence to conclude that the informants are different with respect to their preferences? (9)

GROUP B : Section II (alt.) : Elements of Book Keeping and Accountancy (30 marks)

- i) Use separate answer book for this section.  
 ii) Figures in the margin indicate full marks.

(Attempt any three from this section)

1. What is meant by cost of sales? State briefly what are the main items of expenditure included in cost of sales. (10)  
 2. Define Budget. What is the utility in the preparation of Budget and how it helps the management? (10)  
 3. Distinguish between Capital and Revenue with three examples of each.  
 4. Write short notes on any two of the following :  
 (a) Contingent Liability (c) Asset Cover  
 (b) Window Dressing (d) Sinking Fund  
 5. What is the advantage of maintaining Petty Cash Book. Write up a Petty Cash book in suitable form giving three heads of expenditures with imaginary figures. (6)

GROUP C: BIOGENERIC METHODS  
 (Special Paper III: Practical)

No candidates available

Please turn over

(146)

## STATISTICS: DESIGN &amp; ANALYSIS OF EXPERIMENTS

Special Paper III - Practical

(Attempt any three questions from this group)

1. (a) Identify the following design, write down the values of its parameters, and give the partitioning of the degrees of freedom in the analysis of variance. The numbers in the body of the table indicate treatments.

<u>Block</u>	<u>1st Plot</u>	<u>2nd Plot</u>	<u>3rd Plot</u>
1	10	7	16
2	11	8	13
3	12	5	14
4	9	6	15
5	13	4	6
6	14	1	7
7	15	2	8
8	16	3	5
9	7	9	4
10	8	10	1
11	5	11	2
12	6	12	3
13	2	16	9
14	3	13	10
15	4	14	11
16	1	15	12

- (b) Construct a balanced confounded  $3^3$  design in blocks of 3 plots each in 4 replications.  
 (c) Construct the resolvable semi-regular group divisible design having parameters.

$$v = b = 16, \quad r = k = 4, \quad m = n = 4, \quad \lambda_1 = 0, \quad \lambda_2 = 1 \quad (12 \cdot 10 \cdot 10) = 32$$

2. Five treatments, of which the first one is a control, were tested in the following layout - the figures in brackets representing yields corresponding to the treatments.

1 (75)	2 (66)	3 (59)
1 (87)	4 (43)	5 (95)
1 (65)	3 (55)	4 (38)
1 (85)	5 (120)	2 (72)

- (a) Test if treatments 2 to 5, are different from 1, the control.  
 (b) If the yield figure for treatment 2 in the fourth row is found missing, will the conclusion drawn in (a) need to be modified?  $(15 + 16) = 32$

(147)

Please turn overP.T.O.

5. An experiment in 1/8th replicates of  $2^8$  factorial experiment, with factors A, B, C, D, E, F, G and K - each at two levels, and arranged in 4 blocks of 8 plots each, was carried out to study the effect of these factors on the yield of paddy. In the table below are given the lay-out plan and figures of yield of paddy, the treatments within a block being assigned at random to the 8 plots in the block :

<u>Lay-out plan and yield of paddy</u>							
<u>Block I</u>		<u>Block II</u>		<u>Block III</u>		<u>Block IV</u>	
<u>Treat-</u>	<u>Yield</u>	<u>Treat-</u>	<u>Yield</u>	<u>Treat-</u>	<u>Yield</u>	<u>Treat-</u>	<u>Yield</u>
<u>ment</u>	<u>of Paddy</u>	<u>ment</u>	<u>of Paddy</u>	<u>ment</u>	<u>of Paddy</u>	<u>ment</u>	<u>of Paddy</u>
	<u>(lbs./plot)</u>		<u>(lbs./plot)</u>		<u>(lbs./plot)</u>		<u>(lbs./plot)</u>
a d g	26.99	a c g	31.34	b d o f k	26.22	boefh	36.99
c d e f	32.81	a b c d f	20.85	a c f k	29.90	a b g k	27.77
a e o h	33.26	c d f g h	32.29	a d o f g	30.32	a b e d o	32.24
b d h	33.13	o f	31.02	o g h	32.95	a d f h	34.29
a b f	31.30	a d o h	30.45	c d	31.73	b d f g	31.73
abcdofgh	35.43	b c h	38.13	abcdgh	29.81	a e o f g	33.59
f g k	31.15	b d e g	30.45	a b o	28.48	o d o g h	31.60
b c e g	35.57	abofgh	33.61	b e f g	28.91	(1)	35.92

- (a) Identify the defining contrasts and the confounded interactions.  
 (b) Analyse the data and interpret the results of your analysis.  $(9 + 24) = 33$
4. Three objects A, B and C are weighed ten times in a Chemical Balance, by putting some objects on one pan and balancing against standard weights placed on the other pan. The weighing results are given below :

Weight (in gms) of Objects A, B, C as obtained on a Chemical Balance

<u>Sl No.</u>	<u>Object</u>		<u>Standard Weights</u>	
	<u>Right Pan</u>	<u>Left Pan</u>	<u>Right Pan</u>	<u>Left Pan</u>
1.	A, C	-	-	14.2
2.	A	C	4.6	-
3.	A, B	C	-	3.2
4.	A, C	B	-	8.5
5.	B, C	A	-	11.9
6.	-	A, B, C	15.4	-
7.	B, C	A	-	12.1
8.	-	A, B	8.3	-
9.	B	C	1.0	-
10.	C	B	-	3.9

In this experiment, it is not known whether the chemical balance had been adjusted for zero point or not.

- 1) Estimate the true weights taking into account any correction needed for non-adjustment of the balance at the start.  
 11) Find the standard errors of these estimates.

$(20 + 12) = 32$

## SD X - GROUP E : SAMPLE SURVEYS

(Special Paper III - Practical)Section I (relative to Theoretical Concepts)(Attempt three questions in all with at least one from each Section)

From a Tehsil of Gorakhpur district of U.P. consisting of 2000 villages (and having a total population of 2 lakhs in 1971) a simple random sample of 100 villages, was drawn without replacement. The data on the population and the annual rice consumption of the villages as collected, are given in the following grouped frequency table.

Distribution of annual consumption of rice in villages of different population levels in 1971.

Population for 1971	50 to 100 (quintals)	100 to 150 (quintals)	150-200 (quintals)	200 to 250 (quintals)	250 to 300 (quintals)
100 - 500	2	1	1	0	0
500 - 1000	2	1	1	0	0
1000 - 1500	0	5	8	2	1
1500 - 2000	1	6	10	2	2
2000 - 2500	0	10	10	7	1
2500 - 3000	0	7	15	5	0

- i) Obtain from the data in the table above, the conventional estimate, ratio estimate and regression estimate of the total consumption of rice by the Tehsil, for the year under consideration.
- ii) Obtain estimates of the variances of these three types of estimates.
- iii) Compare the relative efficiencies of the various estimates in (i).

$$(11+18+3) = 32$$

The 1050 companies, operating in India, with foreign capital participation were stratified according to the total capital employed in 1968. Out of this total a stratified simple random sample of 55 companies only, was selected using proportional allocation. The amount of foreign capital in these companies was observed for the year 1968. The sample data in consolidated form is given in the following table:

Stratum	Total foreign capital employed (crores of rupees)	Sample size	Sample mean for the foreign capital (crores of rupees)	Standard deviation
(1)	(2)	(3)	(4)	(5)
1.	0.01 - 0.25	10	0.15	8.7
2.	0.25 - 1.0	14	0.60	7.1
3.	1.0 - 5.0	15	5.10	15.1
4.	5.0 - 10.0	11	8.20	10.7
5.	10.0 -	7	15.90	12.9

- i) Obtain the usual weighted stratified unbiased estimate for Y, the total foreign capital employed in all the 1050 companies.
- ii) Obtain an unbiased estimate of the variance of the above estimate.
- iii) If an unstratified simple random sample of 55 companies were drawn and Y is estimated by the usual unbiased estimator, what would be an unbiased estimate of the variance of this estimate, based on the above data?
- iv) Obtain the percentage gain in efficiency, due to stratification.

$$(8+5+13+8) = 32$$

- 3) A block was suitably divided into 60 geographical areas containing 5 villages each. A sample of 4 areas was selected with equal probability and without replacement, and the yield of wheat was noted for each village. The data is given in the following table :

Area No.	Yield of wheat for the villages (in quintals)				
1	100,	125,	300,	200,	50
2	40,	80,	110,	200,	70
3	30,	80,	200,	400,	120
4	400,	500,	150,	200,	350

- i) Give an unbiased estimate of the average yield of wheat per village in all 60 areas and an unbiased estimate of the variance of this estimate.
- ii) If an equivalent sample of 20 villages were selected from all the 300 villages using simple random sampling without replacement, find an unbiased estimate of the variance of the estimate of mean per village.
- iii) Obtain the estimated relative efficiency of an area as the unit of sampling, compared with that of a village.  $(14 + 14 + 4) = 32$

#### SECTION II (relative to Organisational aspects)

4. 1) Data are required to be collected on (a) deaths due to starvation (b) deaths due to scarcity of life saving/(c) availability of meals in a year, in a particular district of a State in India. Suggest a suitable sampling design and draw up a brief schedule for the same.
  - ii) Draw up a scrutiny programme for checking the above filled-in schedules before processing.  $(20 + 12) = 32$
5. Information is collected for individual cycle rickshaw drivers and rickshaw pullers, in three areas of Calcutta city and its suburbs.
  - (i) Name of rickshaw puller/driver
  - (ii) The State to which he belongs
  - (iii) Living with the family or alone or with others.
  - (iv) Number of family members with their age and occupation
  - (v) Whether the rickshaw is owned by him or obtained on hire basis
  - (vi) Monthly earnings due to rickshaw pulling/driving
  - (vii) Other side work if any; monthly earnings from such side work
  - (viii) Monthly payments to the rickshaw owner, police department, municipality.
  - (ix) Details of expenditure on meals; clothing, housing, medicines, recreation etc.
  - (x) Nature of diseases from which he and/or his family suffer.
  - (xi) Loans taken and/or monthly savings
  - a) Suggest a suitable tabulation programme covering the above items.
  - b) Give a table containing the percentage of rickshaw drivers (pullers) owning their rickshaw, suffering from disease, earning above Rs.150/- per month, belonging to West Bengal, doing side work, owing police department/municipality more than Rs.5/- a month, separately for the above mentioned three areas.  $(22 + 1^*) = 32$

NEATNESS (Section I and II)

4