

One Year Evening Course in Statistical Methods  
and Applications, 1983-84  
PART I

Probability  
PERIODICAL EXAMINATION

Date: ~~21-9-83~~ Maximum marks: 100 Duration: 2 hours

Note: The paper is set for 108 marks. You may attempt any part of any question. The maximum you can score is 100. The marks for each question are given in the brackets towards the end of that question.

1. Define each of the following and illustrate them with an example in each case:
  - (a) Random experiment; (b) Sample space; (c) Event.

(3 x 2 = 6)
2. Show that  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  where A and B are events. (10)
3. (a) When are two random variables X and Y, said to be independent ?  
 (b) Show that, if X and Y are independent, then the probability generating function of  $X + Y$  is the product of the probability generating functions of X and Y.  
 (c) Obtain the probability generating function of the negative binomial distribution with parameters r and p. (r is a positive integer and  $0 < p < 1$ ). (2+10+20 = 32)
4. Peter and Paul are to play a table tennis match to be decided by best of three games. Peter and Paul are equally good players but have different match temperaments. The probability that each of them wins the first game is  $\frac{1}{2}$ . For each of the subsequent games, the probability that Peter wins that game given that he won the previous game is  $\frac{2}{3}$  and the probability that Peter wins that game given that he lost the previous game is  $\frac{2}{5}$ . Find the probability that Paul wins the match. (20)
5. Describe a procedure to select a person at random from 12 persons using an unbiased coin. Prove that your procedure, indeed, ensures that the probability of selection for each person is  $\frac{1}{12}$ . (20)
6. In a lottery, t tickets are sold and prizes are given on k of these. If I buy n tickets (I have chosen them at random from the t tickets.) find the probability for each of the following events:
  - (a) I got at least one prize;
  - (b) I got exactly r prizes.

(20)

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and applications : 1983-84  
Part I

Economic Statistics(Theory and Pract.)  
Group A

PERIODICAL EXAMINATION

Date: 5.10.83.                      Maximum Marks: 50                      Time: 1 hour.

Note: Answer all questions. Maximum marks allotted are given at the margins.

1. State Fisher's tests of consistency of index numbers. Examine whether Laspeyres' index number formula satisfies these tests. (13+12)
2. The table below gives the wholesale prices and quantities produced of a number of commodities in India. Calculate Laspeyres', Paasche's and Fisher's 'ideal' index numbers for the year 1952 with 1951 as base. (10+10+5)

Commodity	1951		1952	
	p	q	p	q
Rice	16.9	210	17.5	225
Jowar	10.1	60	11.9	72
Bazra	10.1	23	13.3	331
Maizo	21.8	20	15.7	28
Ragi	9.5	13	9.3	13
Wheat	18.6	61	23.7	74
Barley	20.7	23	17.3	29
Gram	24.1	33	19.0	41

p: price in Rs. per maund

q: quantity produced in hundred thousand tons.

INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and Applications : 1983-84.

Part I

Economic Statistics(Theo. and Pract.)  
Group B

PERIODICAL EXAMINATION

Date: 5.10.83.                      Maximum Marks: 50                      Time: 1 Hour.

Note : Answer all questions.

- 1.(a) With what characteristic movement of time series would you associate
  - (i) a recession                      (ii) a Bangla Bandh
  - (iii) stock clearance sale before the commencement of the Bangali new year.                      (1+1+1)
- (b) Both seasonal and cyclical fluctuations are periodic movements. Where do they differ ?                      (2)
2. Discuss the preliminary adjustments to be made in time series data with respect to (i) calendar variation, (ii) population variation.                      (10)
3. Discuss the characteristics of the logistic curve with respect to its (i) asymptotes (ii) point of inflexion, (iii) any symmetry about the point of inflexion.
4. Fit a linear trend to the following data and estimate the imports for the year 1960.

Values of imports to India (in suitable units)

Year	1952	1953	1954	1955	1956	1957	1958
Value	179	184	194	200	210	219	208

(20)

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Applications, 1983-84  
Part-I

Descriptive Statistics(Theo.and Prac.)  
PERIODICAL EXAMINATION

Date: 19.10.83.      Maximum Marks: 100      Time: 2 hours.

Note: Attempt questions 1 and 2 and any two out of the rest. Marks allotted to questions are indicated in brackets at the end.

1. Answer either (A) or (B):

(A) The following table shows the age distribution of 450 males of a particular race. Represent the distribution by means of a histogram.

<u>Age<sup>+</sup>(year)</u>	<u>Frequency</u>	<u>Age<sup>+</sup>(years)</u>	<u>Frequency</u>
0-1	16	20-30	74
1-3	23	30-40	64
3-5	21	40-50	54
5-10	47	50-70	58
10-20	84	70-90	9

+ [The procedure of classification is illustrated by the following example.

Example: The class interval 1-3 includes all those who have completed 1 year of age but not completed 3 years. It includes e.g. the age 2 years  $11\frac{1}{2}$  months but excludes the age 3 years 1 month. This implies that the width of the class interval 1-3 is 2 years.]

[10]

(B) The marks obtained by 345 students in a certain examination were as follows.

<u>Marks</u>	<u>no. of students</u>	<u>marks</u>	<u>no. of students</u>
30-39	1	80-89	103
40-49	2	90-99	55
50-59	12	100-109	17
60-69	50	110-119	5
70-79	99	120-129	1

Calculate:

(i) the median mark

p.t.o.

- 1(B)(ii) the number of students who pass if the pass mark is 60 (i.e. those getting  $\geq 60$  pass and those getting  $\leq 59$  fail)
- (iii) the approximate number of students who pass if the pass mark is 62.
- (iv) what the pass mark should be in order that approximately 90% of the students pass. [5 + 2 + 4 + 5 = 16]

2. Answer either (A) or (B)

- (A)(i) For the frequency distribution of length  $X$  of earhead (in cm.) for 150 ears of wheat, you are given

$$\begin{aligned} \sum u_i f_i &= -8 \\ \sum u_i^2 f_i &= 406 & \text{where } u_i &= \frac{x_i - 10.05}{0.8} \\ \sum u_i^3 f_i &= -59 \end{aligned}$$

$x_i$  being the mid-point of the  $i$ th class interval and  $f_i$  its frequency.

Measure the coefficient of variation and the skewness of the distribution.

- (A)(ii) If the fourth central moment of a variable  $Z$  is 10  $\text{cm}^4$ , can any of the central moments of  $Y=2Z+5$  be calculated? If so, calculate it. [19+5]=24

- (B)(i) Complete the following table which refers to a class of students divided into two sections.

	no. of students	mean age (years)	s.d. of age (years)
Section I	20	12.0	0.4
Section II	40	—	0.2
Whole class	60	12.5	0.5

- (B)(ii) Suppose you know the mean and s.d. of a variable  $X$  and nothing else. Which of the following will you be able to calculate and which will be impossible to calculate :

- (a) median of  $X$  .  
 (b) mean of  $X^2 - X$  .  
 (c) mean of  $\sqrt{X}$  .  
 (d) third central moment of  $X$   
 (e) variance of  $x+5$

[17+7=24]

3. Explain with examples the following basic concepts:  
(a) population, (b) sample, (c) parameter, (d) statistic,  
(e) sampling error (f) inductive inference. Define and  
explain the uses of partition values of a distribution.  
[4x6+6=30]
4. Explain what is meant by 'central tendency'. Suggest  
a 'good' measure of central tendency and discuss some  
of its important properties. In what sense do you call you  
measure 'good' ? [5+2+20+3=30]
- 5(a) Define central and raw moments of a distribution and  
express the former in terms of the latter.
- (b) On what features of a distribution the second and the  
third moments throw light ? Explain your answer with  
illustrations clarifying the concepts involved.  
[15+15=30]
6. (a) Define the normal distribution and explain its impor-  
tance in Statistics. State some of its important pro-  
perties.
- (b) In an examination it is laid down that a student passes  
if he secures 30% or more marks. He is placed in the  
first, second or third division according as he secures  
60% or more marks, between 45% and 60% marks, and  
marks between 30% and 45% respectively. He gets a  
distinction in case he secures 80% or more marks. It  
is noticed from the results that 10% of the students  
failed in the examination, whereas 5% of them obtained  
distinction. Calculate the percentage of the students  
placed in the second division, assuming the distribu-  
tion of marks to be normal. [8+7+15=30]
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INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Applications : 1983-84  
Part I

Probability  
PERIODICAL EXAMINATION

Date : 26.10.83.      Maximum Marks: 100      Time : 2 hours.

Note : Attempt all the questions. The marks for each question are indicated in the brackets on the right at the end of the question.

1. Let  $\rho_{X,Y}$  denote the correlation coefficient between the random variables X and Y.

(a) Show that  $-1 \leq \rho_{X,Y} \leq 1$ .

(b) Show that  $\rho_{X,Y} = \pm 1$  if and only if

$pX + qY + r = 0$  with probability 1 for some real numbers p, q and r.

(c) Show that  $\rho_{pX+q, rY+s} = \rho_{X,Y}$  ( Sign of pr )

whatever be the values of the constants p, q, r and s.

(d) Give an example of nondegenerate random variables X and Y such that  $\rho_{X,Y} = 0$ .

[8+6+4+7=25]

2. Let  $X \sim N(3, 16)$ . Find the following probabilities using the tables supplied to you.

(a)  $\text{Prob}\{X \geq -9\}$ , (b)  $\text{Prob}\{X \geq 0\}$ ,

(c)  $\text{Prob}\{-2 \leq X \leq 3.5\}$ , (d)  $\text{Prob}\{X \leq -5 \text{ or } X \geq 5\}$ .

[4x4=16]

p.t.o.

3. State and prove Chebyshev's inequality.

[4x4=16 ]

4. Compare the properties of discrete probability density function and the continuous density function. Illustrate with suitable examples. [12]

5. Define moment generating function of a random variable. Obtain the moment generating function of  $B(n,p)$ . Hence obtain the mean and variance of  $B(n,p)$ . [2+10+8=20]

6. Let  $X \sim N(\mu, \sigma^2)$  where  $\sigma^2 > 0$ . Derive the density of  $\frac{(X-\mu)^2}{\sigma^2}$ . Show that it has a Gamma distribution. What are the corresponding parameters ?

]12+5=17]

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INDIAN STATISTICAL INSTITUTE  
 One Year Evening Course in Statistical Methods  
 and Applications: 1983-84.  
 Part I

Economic Statistics (Theo. and Prac.)  
 PERIODICAL EXAMINATION

Date: 2.11.83.                      Maximum Marks: 100                      Time: 2 hours.

GROUP A

Note: Answer all questions. Answers should be  
 brief and to the point. Marks allotted  
 for each question are given in brackets.

1. What is a chain index ? Discuss its advantages and disadvantages over a fixed base index number. [15]
2. Write short notes on the following :
  - (a) Purchasing power
  - (b) Homogeneity error in index numbers
  - (c) Deflation. [15]
3. Explain the following terms :
  - (a) Balance of Payments
  - (b) National income. [10]
4. The following data relate to a given economy for a certain year :

	Amount in Rs. Crores
Gross Domestic Product	32514
Export	376
Import	734
Depreciation	241
Net income from foreign assets	683

- Find (i) Gross Retained Output (ii) Net Retained Output  
 (iii) Net Domestic Product (iv) National Income and  
 (v) Balance of Payments. [10]

GROUP B

Answer all questions

- 1.(a) Discuss the factors that are responsible for seasonal variations.  
(b) How do businessmen benefit from a study of seasonality, [5+5]
2. The following table gives the revenue expenditure of the Government of India for the four quarters of the year ending in June (J), September (S), December (D) and March (M). Compute the indices of seasonal variation.

Revenue expenditure of Government of India (Rs. Crores)

Year	Quarter			
	J	S	D	M
1971-72	78	62	56	100
1972-73	84	64	61	81
1973-74	92	70	63	72

If the total expenditure in the first quarter in a particular year is Rs. 200 crores, determine the amounts of expenditure for the other three quarters. [25+5]

3. Write short note on any one of the following:  
(i) Correlation  
(ii) Population Census Statistics. [10]

## INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and Applications, 1983-84.  
Part-I

Descriptive Statistics(Theo.and Pract.)  
PERIODICAL EXAMINATION

Date: 9.11.83.

Maximum Marks: 100

Time: 2 hours.

Note : Attempt two questions from group A, and two questions from group B. Marks allotted to questions are indicated in brackets at the end.

GROUP A

Note: Answer any two questions from group A. Each question carries 20 marks.

1. The bivariate frequency distribution of X (age of entry into service) and Y (retirement age) for 110 ex-employees of an organisation is shown below.

X (years) \ Y (years)	55	58	60
19.5-29.5	13	23	44
29.5-39.5	3	6	11
39.5-49.5	1	3	6

- (a) Draw up the three conditional relative frequency distributions of Y given  $19.5 < X < 29.5$ ,  $29.5 < X < 39.5$  and  $39.5 < X < 49.5$ . [6]
- (b) Graphically represent the three conditional relative frequency distributions obtained in (a), on the same graph paper. [5]
- (c) Using the graph, comment on the relationship between X and Y. [2]
- (d) Find the mean and variance of the conditional distribution of Y given  $19.5 < X < 29.5$ . [7]

p.t.o.

2. The heights of 174 Indian adult males have the frequency distribution shown below. Fit a normal distribution to the data.

height (inches)	frequency
30 - 62	3
62 - 64	24
64 - 66	58
66 - 68	60
68 - 70	27
70 - 72	2
Total	174

[20]

3. The mean, s.d. and third raw moment about 0 were calculated as follows for the distribution of weights of 100 men :

$$\text{Mean} = 58.7 \text{ kg.}$$

$$\text{S.d.} = 5.20 \text{ kg.}$$

$$m_3' = 203070.1 \text{ kg}^3.$$

It was then found that the weights of two men, which were actually 67.0 kg. and 57.0 kg., had been wrongly taken as 61.0 kg. and 51.0 kg. respectively. Find the correct mean, s.d. and third raw moment of the distribution.

[20]

4. X is a variable with the following frequency distribution.

X	0	1	2	3	4
frequency f	5	11	19	12	3

- (a) Draw up frequency distributions for the variables

$$Y_1 = X + 2$$

$$\text{and } Y_2 = 2X \quad [5]$$

- (b) Find the mean and s.d. of X. [6]

- (c) Find the mean and variance of  $Z = \frac{X+3}{2}$ . [4]

- (d) Calculate the mean deviation of X. [5]

(contd...3)

GROUP B

5. Explain what is meant by dispersion. Describe a good measure of dispersion and discuss some of its properties. [5+25=30]
6. Define the binomial distribution with parameters  $n$  and  $p$ . Obtain the mode of this distribution. If  $X$  has a binomial distribution with mean 4 and variance 3, what will be the modal value of  $X$  ? [3+20+7=30]
- 7.(a) Show that a Binomial distribution with parameters  $n$  and  $p$  tends to a Poisson distribution with parameter  $\lambda$ , when  $n \rightarrow \infty$ ,  $p \rightarrow 0$  such that  $np = \lambda$  is finite. [25]
- (b) Suppose  $X \sim \text{Binom.}(n,p)$ . Then what is the normal approximation of the probability  $P(a \leq X \leq b)$  ? [5]
8. Define Poisson distribution with parameter  $\lambda$ , and give its first four moments. Suppose  $X$  has a Poisson distribution such that
- $$P(X=2) = 9 P(X=4) + 90 P(X=6).$$
- Show that the variance of  $X$  is 1. [10+20=30]
-

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Applications : 1983-84  
Part I (Calcutta and Delhi)

Probability  
SEMESTRAL-I EXAMINATION

Date: 14.12.83. Maximum Marks: 100 Time:  $2\frac{1}{2}$  hours.

Note: The paper is set for 127 marks. You may attempt any part of any question. The maximum you can score is 100.

1. Prove or disprove each of the following statements.

(a)  $\text{Prob. } \{A|B\} + \text{Prob. } \{A|B^c\} = 1$  where A and B are any two events.

(b) If  $X \sim B(n_1, p)$  and  $Y \sim B(n_2, p)$ , then  $X+Y \sim B(n_1+n_2, p)$

(c) If X and Y are independently distributed, then  $\text{Cov}(X, Y) = 0$ .

(d) There exists a nondegenerate random variable X such that

$$\rho_{X, X^2} = 0.$$

(e) If  $f(x)$  is the density function of continuous real random variable, then  $0 \leq f(x) \leq 1$  for all real numbers x. [5x6 = 30]

2. I have a coin for which I do not know the probability of head.

I have to select one person from Peter, Paul and John at random with the help of this coin. Describe a procedure how I can make this selection. Show that your procedure selects one of them with probability 1 and that  $\text{Prob}\{\text{Peter is selected}\} = \text{Prob}\{\text{Paul is selected}\} = \text{Prob}\{\text{John is selected}\} = \frac{1}{3}$  if we use your procedure. [15]

3. A mathematician carries a match box in his right pocket and another in his left pocket. If he wants a match, he selects a pocket at random and takes a match from the match box in that pocket. Let initially each match box contain N matches. Consider the moment when for the first time he discovers that a box is empty. Let X denote the number of matches in the other pocket at that moment. Find the probability distribution of X. [20]

p.t.o.

- 4.(a) Let  $\phi_X(t)$  denote the moment generating function of  $X$ . Show that if  $X$  and  $Y$  are independent then  $\phi_{X+Y}(t) = \phi_X(t) \cdot \phi_Y(t)$ .
- (b) Find the moment generating function of Gamma distribution with parameters  $\alpha$  and  $n$ . Hence or otherwise obtain the mean and variance of this distribution.
- (c) Let  $X \sim N(0,1)$ . Obtain the distribution of  $X^2$ .
- (d) Hence or otherwise show that if  $X_1, \dots, X_n$  are iid  $N(0,1)$  random variables,  $X_1^2 + \dots + X_n^2$  is distributed as a Gamma distribution. What are the corresponding parameters?

[8+8+5+8+6=35]

- 5.(a) State and prove Tchebyshev's inequality.
- (b) Deduce the weak law of large numbers.
- (c) State clearly any version of the Central limit theorem.

[8+6+3 = 17]

6. In a bolt factory machines A, B, C manufacture, respectively, 25, 35 and 40 per cent of the total. Of their output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the produce and is found defective. What is the probability that it is manufactured by machine B ? [10]

## INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods and Its Applications:  
1983-84

PART I : FINAL EXAMINATION

PAPER II : DESCRIPTIVE STATISTICS (THEORY)

Date: 16.12.1983

Time: 2 hours

Maximum marks: 100

Note: Answer any FOUR questions.  
Marks are given in brackets.

1. (a) Describe briefly the various stages involved in processing statistical data and the steps you would take to control the error in processing work. [13]
- (b) Discuss the following statement:  
"The advantage proposed by presenting data with the help of graphs and charts, is not that of giving a more accurate statement than by figures, but it is to give a more simple and permanent idea of the gradual progress and comparative amounts, at different periods, by presenting to the eye a figure the proportions of which corresponds to the amounts of the sums intended to be expressed." [12]
2. (a) What is a frequency table? What important considerations are to be borne in mind for the choice of class intervals for such a table? [13]
- (b) What do you understand by Skewness and Kurtosis? Give some suitable coefficients for measuring skewness and kurtosis. [12]
3. (a) What is meant by dispersion of a frequency distribution? Compare the range, the mean deviation and the standard deviation as measures of dispersion. [11]
- (b) Show that
  - (i) Mean deviation is minimum when measured from median.
  - (ii) the standard deviation remains unchanged under a change of origin.
 [7+7]



4. (a) Show that the central moments of the binomial distribution satisfy the relation

$$\mu_{r+1} = pq \left( r\mu_{r-1} + \frac{d\mu_r}{dp} \right)$$

- (b) Let  $X$  be a random variable having binomial distribution with parameters  $n$  and  $p$ , the symbols having their usual significance. Give the normal approximation of the probability  $P(a \leq x \leq b)$ . By what name is this result known?
- (c) Derive the Poisson distribution as a limiting form of a binomial distribution. [10+5+10=25]

5. (a) Define the normal probability distribution and derive its moment generating function.
- (b) Show that a normal distribution with variance  $\sigma^2$ , the  $p$ th central moment  $\mu_p$  satisfy

$$\mu_{2r+1} = 0 \text{ and } \mu_{2r} = 1.3.5 \dots (2r-1)\sigma^{2r} \quad [12+13]$$

6. (a) Obtain the normal equations and estimate the constants  $a$  and  $b$  for fitting  $y = a+bx$  to a data  $(x_i, y_i)_{i=1,2,\dots,n}$ .
- (b) Define the multiple correlation coefficients of  $X_1$  on  $X_2$  and  $X_3$ .
- (c) Define the partial correlation coefficient  $r_{12.3}$  for the three variables  $X_1, X_2$  and  $X_3$ , and show that

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{1-r_{13}^2} \sqrt{1-r_{23}^2}}$$

where  $r_{ij}$  is the correlation between  $X_i$  and  $X_j$  for  $i, j = 1, 2, 3$ .

[10+5+10=25]

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Applications : 1983-84  
Part I (Calcutta and Delhi)

Descriptive Statistics (Practical)  
SEMESTRAL-I EXAMINATION

Date: 19.12.83.                      Maximum Marks: 100                      Time:  $2\frac{1}{2}$  hours.

Note : The paper carries 115 marks. Answer as much as you can. The maximum you can score is 100.

1. The following data refer to 3 variables  $X_0$ ,  $X_1$  and  $X_2$ .

$$\bar{X}_0 = 14 \quad S_{00} = 98 \quad S_{01} = 57$$

$$\bar{X}_1 = 8 \quad S_{11} = 34 \quad S_{02} = 52$$

$$\bar{X}_2 = 4 \quad S_{22} = 30 \quad S_{12} = 31$$

where  $S_{ij} = \sum (X_i - \bar{X}_i)(X_j - \bar{X}_j)$ ,  $i, j = 0, 1, 2$ .

- (a) Obtain the linear regression of  $X_0$  on  $X_1$  and  $X_2$ . Calculate the multiple correlation coefficient  $\rho_{0.12}$  and interpret the value obtained.
- (b) Calculate the value of the regression coefficient ' $\beta$ ' obtained when  $X_0$  is regressed on  $X_2$  only. Hence, or otherwise, calculate the partial correlation coefficient of  $X_0$  and  $X_1$  eliminating the effect of  $X_2$ .

[23+12=35]

2. Consider the following data :

x	-4	-3	-2	-1	0	1	2	3	4
y	0.1	2.5	3.4	3.9	4.1	3.8	3.5	2.8	0.3

Calculate the correlation coefficient between x and y and comment on the value obtained.

[20+5=25]

3. The bivariate frequency distribution of X ( age of entry into service) and Y ( retirement age) for 110 ex-employees of an organisation is shown below.

X (years) \ Y (years)	55	58	60
19.5 -29.5	15	22	43
29.5 -39.5	4	6	13
39.5 -49.5	1	2	4

- (a) Obtain the three conditional relative frequency distributions of X given  $Y = 55$ ,  $Y = 58$  and  $Y = 60$  respectively.
- (b) Graphically represent the three distributions obtained in (a) in the form of frequency curves (by plotting conditional relative frequencies against class mid-points) on the same graph.
- (c) Using the graph, comment on the relationship between X and Y.
- (d) Calculate the mean and variance of the conditional distribution of X given  $Y = 60$ .

$$[6+7+4+8=25]$$

4. The life ( in hours) of electronic tubes of a certain type is supposed to be normally distributed with  $\mu=155$  hrs. and  $\sigma=19$  hrs. What is the probability that the life of a tube will be
- (a) greater than 200 hrs. ?
- (b) less than 140 hrs. ?
- (c) between 117 hrs. and 193 hrs. ?

If a sample of 200 tubes is taken, how many are expected to be in each of the above groups ?

$$[4+4+5+3=16]$$

(contd....3)

5. A part of the cumulative frequency table for the marks obtained by 60 students in a certain examination is reproduced below :

marks (x)	cumulative frequency (no. of students scoring $\leq x$ )
0	1
...	...
...	...
30	7
40	17
50	32
...	...
...	...
100	60

Calculate approximately:

- (a) the number of students who pass if the pass mark is 42,  
i.e. those getting  $\geq 42$  pass  
and those getting  $\leq 41$  fail
- (b) what the pass mark should be in order that 75% of the  
students pass. [6+8=14]

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## INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods and Its Applications:  
1983-84

## PART-I FINAL EXAMINATION

Paper III : Economic Statistics  
(Index Number and National Income Accounting)

Date: 21.12.1983

Time: 2 hours

Maximum marks: 100

Note: Answer any THREE questions from Group-A and any ONE question from Group-B. Marks allotted to each question are given in brackets.

G R O U P - A  
INDEX NUMBER

1. (a) Explain briefly what you understand by an 'index number.'
- (b) Discuss the problems involved in the construction of a price index number explaining clearly the considerations underlying the choice of items and the base period.  
[8+12]
2. (a) What are 'time reversal' and 'factor reversal' tests as applied to index number formulas.
- (b) Show that Laspeyres' and Paasche's price index number formulas do not satisfy the two tests.
- (c) Write down the formula for Fisher's Ideal Index Number and show that it satisfies both tests.  
[4+8+8]
3. (a) You are given prices ( $p_i$ ) and quantities ( $q_i$ ) for each period  $i = 1, 2, 3$  and 4 for each of  $n$  commodities to be included in an index number. Construct a chain base index number for period 4 with period 1 as base using base period quantities as weights.
- (b) Linked price indices using current period quantities as weights for periods 2, 3 and 4 are respectively  $a, b$  and  $c$ . Derive an expression in terms of  $a, b$  and  $c$  for the chain base index for period 1 with period 4 as base using base period quantities as weights.

P.T.O.

(c) Prove that weighted aggregative price index numbers using fixed quantity weights yield algebraically the same result whether fixed base or chain base method is used. [8+8+4]

- 4. (a) Explain what is meant by cost of living index number.
- (b) Distinguish true cost of living index number from consumer price index number.
- (c) Name three important price index numbers issued by the Government of India and write a brief note on any one of them. [6+6+8]

G R O U P - B  
 NATIONAL INCOME ACCOUNTING  
 (Answer briefly in about 10 sentences  
 for each question)

- 5. (a) (i) Define the term national income.
- (ii) Explain the distinction between the terms factor cost and market prices; gross national product and net national product, in the context of national income accounts.
- (iii) What do you understand by the term capital formation. Give two examples where this takes place in the Indian economy.
- (b) The following data relate to the Indian Economy for the year 1976-77.

	<u>Figures in Rs. Crores</u>
i) gross national product at factor cost	71231
ii) corporation tax	984
iii) consumption of fixed capital	4477
iv) net factor income from abroad	-233
v) private income	67442
vi) direct taxes paid by households	1792
vii) indirect taxes	9926
viii) subsidies	1393
ix) savings of private corporate sector	259
x) miscellaneous receipts of government administration department	246

From the above data find:

- i) national income
- ii) gross national product at market prices
- iii) net national product at market prices
- iv) net domestic product at market prices
- v) personal income
- vi) personal disposable income

[16+24=40]

6. (a) i) Define the term value added.  
 ii) What are the methods used for estimating the national product?  
 What are the purposes for which national accounts are used?  
 How can we make international comparisons using the national product?
- (b) i) Fill up the blanks in the following table.  
 ii) Find also the national income for the year 1977-78. (Explain briefly your steps).

Sl.No. No.	National product and related aggregates for 1977-78 (Rs. crores)	
1.	gross national product at factor cost	---
2.	indirect taxes	10689
3.	subsidies	1772
4.	gross national product at market prices	89410
5.	consumption of fixed capital	4957
6.	net national product at market prices	---
7.	net domestic product at market prices	84686
8.	net domestic product at factor cost	---
9.	income from entrepreneurship and property accruing to government administrative department	1790
10.	saving of non departmental enterprises	327
11.	income from domestic product accruing to private sector	---
12.	national debt interest	697
13.	net factor income from abroad	---
14.	current transfers from government and administration departments	1762
15.	other current transfers from the rest of the world	1022
16.	private income	---
17.	savings of private corporate sector net of retained earnings of foreign companies	359
18.	corporation tax	1221
19.	personal income	---
20.	direct taxes paid by households	1657
21.	miscellaneous receipts of government administration departments	239
22.	personal disposable income	---

[16+24=40]

## INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and Applications : 1983-84

Part II

Sample Surveys ( Theory and Practical)

PERIODICAL EXAMINATION

Date: 22.2.84. Maximum Marks: 100 Time: 2 hours.

Note : Answer as much as you can.

1. Suppose that you are required to conduct a survey to estimate mainly the number of colour T.V. sets in your locality, besides obtaining an estimate of the amount spent on the T.V. and the number of hours devoted to watching the T.V. and other relevant information. Explain the principle steps you would take to conduct such a survey. Also draw up a brief schedule for this survey.

[20+10]=[30]

- 2.(a) How do you draw a random sample of size 4 from a population of size 123 using random number tables. Explain what you would do if no random number tables are available.
- (b) For Simple Random Sampling With Replacement (SRSWR) show that  $\bar{y}$  the sample mean is an unbiased estimate of the population mean  $\bar{Y}$ .

(c) Show that  $\hat{V}(\bar{y}) = \frac{s^2}{n}$  where  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2$ .

What modification is needed when sampling is by without replacement and with equal probability.

- (d) In a population of 4826 plots it is found that only 329 plots grow jute. Estimate the area under jute and obtain an estimate of the sampling error assuming that the sample is drawn by srswor. [10+4+13+13]=[40]
- 3.(a) What is 'Probability Proportional to Size' sampling ?
- (b) Explain Lahiri's method of selection with an illustrative example. What are the advantages and disadvantages of this method over the other methods.
- (c) How do you use split method ? [6+12+7]=[25]



4. From a population of 47 shops, 5 shops were selected with probability of selection proportional to their turnover(x). The profits during one season (y) are observed for this sample. It is also known that the total turnover for all the 47 shops is 8493.66.

<u>Shop No.</u>	<u>x</u>	<u>y</u>
1	141.37	70.12
2	214.64	101.43
3	99.14	49.31
4	312.46	164.14
5	200.11	99.33

- (a) Estimate the total profits for all the 47 shops .  
(b) Obtain an estimate of the sampling error.

[13+12]=[25]

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1983-84/E242

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Application: 1983-84

Statistical Inference (Theory and Practical)

Part-II

PERIODICAL EXAMINATION

Date: 9.3, 1984

Maximum Marks: 100

Time: 2 hours

Note: Answer all the questions.

1. State and prove Neyman Pearson lemma. Find the most powerful test of size  $\alpha$  for testing  $H_0: \lambda = \lambda_0$  vs  $H_1: \lambda > \lambda_0$  where  $x_1 \dots x_n$  iid Poisson ( $\lambda$ ).

[25]

2. (a) Show that  $\bar{x}$  is sufficient for  $\theta$  where  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ ,  $x_1 \dots x_n$  iid with  $P(X = x) = \theta^x (1-\theta)^{1-x}$ ,  $x = 0, 1$ .

[10]

- (b) Explain the following i) efficiency of an estimator.  
ii) Minimum variance estimation.

[15]

3. Let  $(x_i, y_i)$   $i = 1 \dots n$  be  $n$  independent observations from  $N_2(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$  population where all the parameters are unknown. Describe test procedure for testing

$$H_0: \frac{\sigma_x}{\sigma_y} = \frac{\sigma_x}{\sigma_y} \quad \text{vs} \quad H_1: \frac{\sigma_x}{\sigma_y} > \frac{\sigma_x}{\sigma_y}$$

[15]

4. The following data relates to the height of 10 males and 7 females in ft.

Male : 5.4, 6.2, 4.8, 5.1, 5.7, 5.2, 4.9

6.0, 5.6, 5.9

Female : 4.8, 4.9, 5.0, 5.6, 5.5, 5.7, 4.9

under the assumption of normality and the equality of unknown variances of two population, test whether mean height differs significantly in two groups (take  $\alpha = .05$ ).

[35]

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Application  
1983-84

SUPPLEMENTARY EXAMINATION

PART-I

Paper III : Economic Statistics (Theory and Practical)

Date: 14.3.1984

Maximum Marks: 50

Time: 1 hour

GROUP-A

Note: Answer either question 1 or question 2.  
Marks for each question are given in  
brackets.

1.(a) Describe the different problems faced in constructing index numbers.

(b) Write few lines on gross National Product and Net National Product explaining clearly the difference between the terms 'Gross' and 'Net'.

(40 + 10) = [50]

2.(a) What is a chain index? When can you treat a fixed-base index as a chain base index? What are the advantages and disadvantages of a chain index number over a fixed-base index number?

(b) Write short notes on any three of the following:

(i) Capital formation

(ii) Value added approach

(iii) Depreciation

(iv) Cost of Living index number

(v) Uses of index numbers.

(25 + 25) = [50]

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P.T.O.

INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and Applications, 1983-84  
Part I

Paper III : Economic Statistics(Theory and Practical)  
SUPPLEMENTARY EXAMINATION

Date: 14.3.84. Maximum Marks : 50 Time : 1 hour.

GROUP-B

Note : Attempt question 1 and any two of the rest.

1. Fit a straight line trend to the following figures and estimate the imports for the year 1964.

Value of imports into India ( in suitable units)

Year	1951	1952	1953	1954	1955	1956	1957	1958	1959
Value	159	179	184	194	200	210	219	208	241

(20+2)=[22]

2. (a) What are the factors that give rise to seasonal fluctuations ?  
(b) Briefly explain the major uses of seasonal indices.  
[7+7=14]
3. Describe the use of the method of moving averages in finding the trend values of a time series. [14]
4. Discuss the salient features of the logistic curve. [14]
5. Write a note on any one of the following :  
(a) National sample survey organization  
(b) Central Statistical Organisation(C .S.O.) [14]
6. Give an account of the trend and cyclical components of a time series. [14]
-

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Applications : 1983-84

SAMPLE SURVEYS ( THEORY AND PRACTICAL )  
PERIODICAL EXAMINATION

Date: 21.3.84

Maximum Marks: 100

Time: 2 hours.

Note : The paper carries 110 marks. Answer as much as you can. The maximum you can score is 100. Marks are given in brackets [ ] .

- 1.(a) What are the advantages of stratification ? [6]
- (b) Write down ( no proofs needed) the proportional and Neyman's optimum allocation for a sample of size  $n$  when Simple Random Sampling (SRS) With Replacement (WR) is used in all strata. [4+6]=[10]
- (c) Obtain the values of  $V(\hat{Y}_{st})$  under proportional and optimum allocations given above, where  $\hat{Y}_{st}$  is the estimate of the population mean of  $y$ -values of a study variate when the population is divided into  $k$  strata. [6+8]=[14]
- (d) Compare the above variances and compare them with  $V(\hat{Y}_D)$  where  $\hat{Y}_D$  is the estimate of the population mean obtained by a Direct sampling of  $n$  units by SRSWR from the whole population of size  $N$ . [6+6+6]=[18]
- 2.(a) What are the differences between 'Linear' and 'circular systematic' sampling schemes. [5]
- (b) Explain why the variance cannot be estimated based on a single systematic sample. [7]
- (c) From a population of size 105 a sample of size 8 is required. The random start chosen was 1. The values obtained for a  $y$ -value for this sample which turns out to be of size '9' are given by 201, 211, 196, 144, 216, 178, 144, 201, 203. Estimate the population mean unbiasedly. Under certain conditions ( to be stated by you) estimate its error. Is an unbiased estimate for the error ? [7+8+5]=[20]

p.t.o.

3. A sample of 6 plots were selected from 27 plots in a region and the area(x) and yield (y) are observed as follows :

<u>S.No.</u>	<u>area (x)</u>	<u>yield (y)</u>
1	21.11	104.32
2	24.21	124.61
3	46.46	218.24
4	15.17	69.83
5	22.34	101.43
6	9.97	43.21

---

It is also known that the total area of the 27 plots is 764.11 acres. Estimate the yield in the region (a) by ratio method of estimation and (b) by regression method of estimation. Explain briefly which of the methods you would use ( theoretical justification).

[10+13+7]=[3

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INDIAN STATISTICAL INSTITUTE  
 One Year Evening Course in Statistical Methods  
 and Applications: 1983-84  
 Part II

Design of Experiments  
 PERIODICAL EXAMINATION

Date: 11.4.1984.      Maximum Marks : 100      Time : 2 hours.

Note : The paper carries 110 marks. You can answer any part of any question. Maximum marks you can score is 100.

1. What is meant by 'Randomisation' in the context of planning of an experiment ? Describe its role with suitable examples, at least one each from the field of agriculture and the field of industry. [30]
2. What is a Latin Square Design ? How do you use it for the elimination of heterogeneity in two directions ? Describe in brief the method of analysis of a Latin Square Design. [30]
3. An experiment was conducted to determine the optimum dose of ammonium sulphate for a variety of wheat. Details of the experiment are given below :
  - (i) Design - Randomised Block Design.
  - (ii) Treatment
 

A	-	0	Kg	N	per	hectore
B	-	40	"	"	"	"
C	-	80	"	"	"	"
D	-	120	"	"	"	"
E	-	160	"	"	"	"
F	-	200	"	"	"	"
  - (iii) Plot size 8 m x 2 m

Layout plan and wheat yield in Kg per plot

Block I	F	2.19	E	2.50	D	2.27	C	1.62	B	1.82	A	0.91
Block II	E	2.27	C	1.41	A	0.91	D	1.91	F	2.13	B	1.95
Block III	B	2.04	A	0.91	F	2.29	E	2.29	D	2.50	C	2.07

Analyze the data.

[50]

INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and Applications, 1983-84.

Industrial Statistics (Theory and Practical)  
PERIODICAL EXAMINATION

Date: 18.4.84      Maximum Marks : 100      Time: 2 hours.

Note : Question 1 is compulsory. Attempt any two  
more questions from the remaining.

- 1.(a) Give example of different kinds of quality characteristics of a product and explain the meaning of 'Quality of a product.'  
(b) Explain the meaning of the 'state of statistical control' and give five practical examples where Poisson model holds. (20+20)=[40]
2. When would you say that a process is capable to produce a product according to specified quality? Discuss various cases in detail comparing process capability with the permissible tolerance. [30]
3. Twenty successive samples each of size 5 were taken from a process. Measurement of length of each item in the sample was taken and following results in the form of sample mean and sample range was obtained

<u>S.No.</u>	<u>Average</u>	<u>Range</u>	<u>S.No.</u>	<u>Average</u>	<u>Range</u>
1.	40	11	11.	41	17
2.	42	14	12.	42	10
3.	42	9	13.	45	11
4.	46	10	14.	48	9
5.	50	8	15.	51	12
6.	39	9	16.	40	9
7.	41	12	17.	42	14
8.	43	11	18.	39	12
9.	47	7	19.	43	7
10.	49	15	20.	49	9

- (a) Compute control limit(s) for ranges (b) compute control limits for averages (c) Is the process in statistical control? (10+12+8)=[30]



- 4.(a) Distinguish between chance and assignable causes of variation.
- (b) Number of surface defects observed on galvanised sheet of the same area were recorded for 20 sheets separately as follows :

<u>Sheet No.</u>	<u>Number of surface defects</u>	<u>Sheet No.</u>	<u>Number of surface defects</u>
1.	15	11.	24
2.	9	12.	16
3.	13	13.	8
4.	20	14.	10
5.	11	15.	15
6.	15	16.	12
7.	7	17.	10
8.	11	18.	7
9.	22	19.	8
10.	12	20.	16

---

Compute control limits for c-chart. Comment whether process is in a state of statistical control.

$$(5+20+5) = [30]$$

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INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods and Its Applications:  
1983-84

STATISTICAL INFERENCE (THEORY)

Date: 28.6.1984  
Time: 2 hours  
Max.marks: 100

ANNUAL EXAMINATION

Note: Answer as many questions as you can.  
The maximum you can score is 100.

1. (a) When do you say that a statistic  $T$  is sufficient for a population parameter  $\theta$ ?
- (b) State (without proof) the factorization theorem for sufficient statistic.
- (c) Show that the sample mean is a sufficient statistic for  $\theta$  in the case of a sample from  $N(\theta, 1)$ . (7+8+10)
2. (a) What is a maximum likelihood estimate?
- (b) Is the maximum likelihood estimator always unbiased?
- (c) Obtain the maximum likelihood estimator of the parameter  $\sigma^2$  in  $N(0, \sigma^2)$ . Is the estimator unbiased? (8+7+10)
3. (a) State and prove Rao Blackwell Theorem.
- (b) Explain the method of moments for estimating the unknown parameters of a population. (15+10)
4. (a) Write down the expression for the Cramer-Rao lower bound for the variance of an unbiased estimator of a parameter stating the condition under which the lower bound is attained.
- (b) Show that if  $X_1, X_2, \dots, X_n$  is a sample of  $n$  independent observations from a Poisson distribution with parameter  $\lambda$ , then the sample mean is a minimum variance unbiased estimator of  $\lambda$ . (13+12)

P.T.O.

5. (a) State the Neyman Pearson Lemma.
- (b) Let  $X_1, X_2, \dots, X_n$  be a sample of  $n$  independent observations from  $N(0, \sigma^2)$  ( $\sigma^2$  unknown).
- (i) Derive the most powerful test for testing  $H_0 : \sigma = \sigma_0$  against  $H_1 : \sigma = \sigma_1$
- (ii) Derive the uniformly most powerful test for testing  $H_0 : \sigma = \sigma_0$  against  $H_1 : \sigma > \sigma_1$ .  
(9+8+8)
6. Write short notes on the following
- (i) Chisquare test of goodness of fit.
- (ii) One way analysis of variance
- (iii) Uniformly most powerful test
- (iv) Level of significance and power function of a test.  
(6+7+6+6)

INDIAN STATISTICAL INSTITUTE  
One Year Evening Course in Statistical Methods  
and Applications : 1983-84  
Part - II

Statistical Inference ( Practical )  
ANNUAL EXAMINATION

Date: 30.5.84.                      Maximum Marks: 100                      Time: 2 hours.

Note : Answer all the questions.

1. A coin was tossed 150 times, and the no. of head and tail was 87 and 63 respectively. Test whether the coin was unbiased.

[Note: Take  $\alpha = .05$ ]

[15]

2. Sitting height (SH) and stature (S) of 7 persons are given below. Do you think SH and S of a person are positively correlated. Examine it.

SH (in cm.)	83	83	81	85	83	81	84
S (in cm.)	172	166	164	164	168	165	170

[Note: You can assume, SH and S follows Bivariate Normal.

Take  $\alpha = .05$ ]

[30]

3. The data in the table below relate to the degree of speech defects and physical defects of a no. of handicapped school children. Examine whether there is any association between the degree of two types of defects.

Distribution of Handicapped children by Degree of Defect

Physical defect	Speech defect		
	Serious	Intermediate	Mild
Serious	45	26	12
Intermediate	32	50	21
Mild	4	10	17

[ Note : Take  $\alpha = .05$ ]

[25]

p.t.o.

4. In one evening, three persons suspected to be driving under the influence of liquor were stopped and blood samples taken from each were sent to the laboratory. Percentage of alcohol in the blood were determined for each sample. They determined 3 times for person 1, 5 times for person 2 and 4 times for person 3.

Under normality assumption, test whether they were equally intoxicated.

	Person		
	1	2	3
Alcohol(%) in blood sample	0.08	0.14	0.00
	0.06	0.18	0.02
	0.06	0.16	0.01
		0.15	0.02
		0.12	

[ Note : Take  $\alpha = .05$  ]

[30]

5. Two experimenters A and B, take repeated measurements on the length of a copper wire. On the basis of the data obtained by them, which are given below, test whether B's measurements are more accurate than A's ( It may be supposed that the readings taken by both are unbiased)

A's measurements  
(in mm)

12.47  
11.90  
12.77  
11.96  
12.78  
12.13

B's measurements  
( in mm )

12.06  
12.23  
12.46  
11.98  
12.22

[Hints: Since the readings of both the experimenters are unbiased, B's measurements may be considered more accurate if they have a smaller population variance than A's measurements. So, under normality assumption, test this.

Take  $\alpha = .05$  ]

[15]

## INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and applications : 1983-84.

## Sample Surveys ( Theory )

## ANNUAL EXAMINATION

Date: 1.6.84. Maximum Marks : 100 Time : 2 hours.

Note : Attempt any four questions. Estimated time for each question is  $\frac{1}{2}$  hour.

1. It is desired to conduct a survey of those persons in a locality who take up a course of study or training during the evening hours in addition to their regular job or study during the day time. Draw up a brief schedule for this survey and state clearly your objectives, sampling design and the possible sources of nonsampling errors. [12+13]=[25]
- 2.(a) From a population of size 123 explain how you would draw a random sample of size 2 without replacement using an unbiased coin when random number tables are not available. [7]
- (b) For simple random sampling with replacement ( srswr ) show that  $s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{(n-1)}$  is an unbiased estimator of the population variance  $\sigma^2$ . Is  $s^2$  still an unbiased estimator for  $\sigma^2$  if sampling is done using srswo scheme. Explain what modification is needed. [6+5]=[11]
- (c) Explain how you would obtain 95% confidence interval for the proportion of Colour Television sets in your area. State your assumptions clearly. [7]
- 3.(a) Explain Lahiri's method of selection with a small illustrative example. [6]

p.t.o.

3.(b) A population of size  $N$  is divided into 2 strata of sizes  $N_1$  and  $N_2$  respectively. A sample of size  $n_1$  is drawn from the first stratum using SRSWOR scheme and a sample of size  $n_2$  is drawn from the second stratum using PPSWR scheme where the probabilities of selection are proportional to a given size measure.

- (i) Write down an unbiased estimate for the stratum total  $Y_1$  of stratum 1 and for the stratum total  $Y_2$  of stratum 2. Using these obtain an unbiased estimate for the population total. [10]
- (ii) How do you estimate the sampling error of the estimate of the population total. [9]

4.(a) Obtain expressions for the Bias and Mean Squared Error of the Ratio estimator  $\hat{R} = \hat{Y} / \hat{X}$ , where  $\hat{Y}$  and  $\hat{X}$  are unbiased estimators of the population totals  $Y$  and  $X$  of two characteristics  $y$  and  $x$  respectively based on any sampling design. [5+6]=[11]

(b) Compare the method of estimating the population total  $Y$  using (a) above with the conventional method which does not use information on the characteristic  $x$ . [8]

(c) If  $Y$  is estimated by  $\hat{Y} = \hat{Y} + (\hat{X} - \hat{X})$  would you accept this estimator? Give reasons. [6]

5. Write short notes on :

- (a) Optimal allocation of sample size to strata
- (b) Circular systematic sampling
- (c) Two stage sampling

[9+8+8]=[25]

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Year 1983-84	No. E 254
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INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods and Its Applications:  
1983-84

SAMPLE SURVEYS (PRACTICAL)

Date: 4.6.1984

Time: 2 hours

Max.marks: 100

ANNUAL EXAMINATION

Note:- Answer as many questions as you can.  
The maximum you can score is 100.

I A simple random sample (without replacement) of 10 students was drawn from a class of strength 60. The data on their weights are as follows:

Sl. No.	Weight (in kgs)	Sl. No.	Weight (in kgs.)
1	38.5	6	44.5
2	60.5	7	48.0
3	42.0	8	56.5
4	36.5	9	63.0
5	47.5	10	41.0

- (i) Find the estimate of the average weight of the class and its standard error.
- (ii) From the above sample, obtain an estimate of the percentage of students whose weight is not less than 47 kgs. and find its standard error. [15+10 = 25]

II The following data gives the stratification of tyre dealers. The dealers were assigned to strata according to the number of new tyres held at a previous census. The data is as follows, where

$N_i$  = Number of tyre dealers in the i-th stratum.

$\bar{Y}_i$  = mean of the i-th stratum

$S_i^2$  = variance of the i-th stratum

$$S_i^2 = \frac{1}{N_i - 1} \sum_{j=1}^{N_i} (Y_{ij} - \bar{Y}_i)^2$$

Stratum boundaries	$N_i$	$\bar{Y}_i$	$S_i^2$
1-9	19800	4.1	32.0
10-19	3250	13.2	91.0
20-29	900	25.0	170.0
30-39	600	35.0	310.0

A stratified sample of total size 400 is to be drawn using SRSWOR in each stratum.

P.T.O.



- (i) Obtain the sample sizes for each stratum in the case of (a) Proportional allocation and (b) Neyman allocation.
- (ii) Obtain the variance for the unbiased estimate of the total tyres held in case of both the allocations.
- (iii) Obtain the relative efficiency of the Neyman allocation over the proportional allocation.

$$[10+20+5 = 35]$$

III In order to estimate the number of milk cows in a state it was decided to use two-stage sampling. The following is the data for the population.

In each district, the total number of villages is given and in each village, the number of milk cows is given.

Districts	Number of villages.	Number of milk cows in the village
1	7	24, 18, 27, 16, 13, 0, 7
2	9	41, 37, 18, 27, 14, 10, 18, 11, 9
3	11	31, 32, 11, 17, 15, 16, 21, 23, 44, 32, 21
4	14	16, 8, 11, 21, 13, 9, 0, 14, 8, 3, 0, 14, 17, 11.
5	10	54, 39, 18, 26, 39, 44, 33, 18, 9, 24
6	8	17, 26, 0, 8, 19, 14, 21, 24
7	12	23, 13, 18, 21, 27, 26, 41, 31, 27, 22, 12, 21.

- (i) Draw a SRSWR sample of three districts.
- (ii) Draw three villages from each of the selected districts with SRSWOR.
- (iii) Estimate unbiasedly the total number of milk cows in the population based on your sample.
- (iv) Obtain the variance of the above estimate.

$$[5+5+10+20] \\ =40$$

-----/-

IV A simple random sample (SRSWOR) of 15 villages was selected from a population of 300 villages. The number of mango trees (y) together with the corresponding census figure (x) relating to a previous survey for each of the sampled village was noted. The data on the same is given below.

$$\sum_{i=1}^{15} y_i = 473 \quad \sum_{i=1}^{15} x_i = 551$$

$$\sum_{i=1}^{15} y_i^2 = 16791 \quad \sum_{i=1}^{15} x_i^2 = 22243$$

$$\sum_{i=1}^{15} x_i y_i = 18692$$

It is given that the total number of mango trees for the previous survey in the 300 villages was 13560.

- (i) Estimate 'Y' the total number of mango trees by using ratio estimate and estimate its standard error.
- (ii) What would be the estimate and its standard error if the information for the previous period was not used  
[23+12 = 35]

V A sample of 7 villages was selected with PPSWR, size being the village population, from a Tehsil. The relevant data is given below.

Sl.No.	$p_i$	$y_i$ (number of bufallows in the villages)
1	.08	147
2	.002	38
3	.07	263
4	.0094	77
5	.007	24
6	.005	-17
7	.034	160

where  $p_i$  is the proportion of the village population in the Tehsil.

- (i) Estimate the total bufallows population (the Tehsil, using
- (ii) Estimate the standard error of the estimate given in (i).

[5+10 = 15]

## INDIAN STATISTICAL INSTITUTE

One Year Evening Course in Statistical Methods  
and Applications! 1983-84.

## Part II

Design of Experiments (Theory and Practical )

ANNUAL EXAMINATION

Date: 6.6,84                      Maximum Marks : 100                      Time : 2 hours.

Note : Answer question number 5 and any three from the remaining.

1. Describe a general method of analysis of two-way classified data with  $m$  ( $> 1$ ) observations in each cell. Indicate the assumptions underlying the method you describe. [20]
2. Define a Latin Square Design.  
Indicate the randomisation procedure for these designs.  
For an experiment with 6 treatments, construct a suitable Latin Square Design and give the break up of the total degrees of freedom for the design. [20]
3. What is a factorial experiment? What are its advantages in comparison with a series of experiments each with one factor at a time. Explain the terms (i) main effects and (ii) interactions, and describe a method of analysis of  $2^3$  factorial experiment.
4. Write short notes on any two of the following :-
  - (i) Role of randomisation in the planning of an experiment.
  - (ii) A  $2^4$  design laid out in 2 blocks of 8 plots each.
  - (iii) Pairwise comparison of treatments, when the hypothesis that the treatment effects are equal is rejected. [20]

5. An experiment was conducted on cotton at a dry farming research station with the object of finding out a suitable level of farm yard manure. The treatments were 6 levels of farm yard manure A,B,C,D,E and F. The experiment was laid out in a Randomised Block Design. The layout plan along with the plot yields in kg. are given below.

Blocks

1	D(4.8)	F(3.9)	C(7.5)	E(4.6)	B(8.7)	A (2.5)
2	E(2.2)	D(5.4)	F(8.9)	B(5.8)	A(3.1)	C(5.0)
3	C(6.7)	B(8.4)	A(4.5)	F(3.9)	E(6.4)	D(3.6)
4	F(5.4)	C(3.2)	B(3.4)	A(4.4)	D(8.0)	E(6.7)

Analyse the data and given your conclusions. Test if the performance of C differs from the performance of B.

[40]

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## INDIAN STATISTICAL INSTITUTE

Year	No.
83-84	B 264

One Year Evening Course in Statistical Methods and Its Applications  
1983-84  
PART - II

## INDUSTRIAL STATISTICS (THEORY AND PRACTICAL)

ANNUAL EXAMINATION

Date: 8.6.1984

Time: 2 hours

Max.marks: 100

Note:- Question 1 is compulsory.  
Answer any other three questions from the  
remaining questions.

1. The following data relate to the average and range of bore diameter for 20 samples each of 4 components after fine boring operation.
- (a) Compute control limits for  $\bar{X}$  and R control charts and examine whether the process is in a state of statistical control by roughly drawing the charts.
- (b) Estimate the process capability of the process from the results in (a) and hence comment whether the process is capable to meet the specification of 23.99 and 24.01 on the bore diameter.

Sample No.	$\bar{X}$	R
1	25.5	8
2	22.5	5
3	23.0	17
4	22.5	5
5	18.5	7
6	28.0	16
7	24.5	8
8	24.0	12
9	21.0	2
10	27.5	15

Sample No.	$\bar{X}$	R
11	29.5	2
12	28.0	8
13	29.0	12
14	26.0	8
15	30.0	4
16	16.0	4
17	25.5	8
18	23.0	6
19	21.5	7
20	24.5	10

(30+10)=[40]

2. The probability that the average weight ( $\bar{x}$ ) of 25 tea packets is atleast 100 gms is 0.99. The weight of individual tea packet ( $x$ ) lies between 95 gms and 105 gms with a probability of 0.95. Find the mean ( $\mu$ ) and S.D. ( $\sigma$ ) of the weight of tea packets.

[20]

P.T.O.

3. (a) The Centre line of Number defective Chart (np-chart) is 12. The 3-sigma Control limits are given by 20 and 4. Find the value of the sample size (n) being used and average fraction of defective ( $\bar{p}$ ).
- (b) For a process which is in a state of statistical control how would you proceed to take advantages of the situations (i)  $6\sigma < T$  and (ii)  $6\sigma > T$  where  $6\sigma$  denotes the process capability and T denotes the permissible tolerance i.e., USL-LSL.

$$(8+12)=[20]$$

4. (a) Point out some of the draw backs of 100 percent inspection.
- (b) "Sampling inspection of a fixed proportion i.e., the inspection in which sample size is directly proportional to lot size and depends on the lot size alone, provides constant protection against acceptance of bad lots". Comment.
- (c) Explain the concept of operating characteristic function of a single sampling plan by attributes and hence state the steps you would take to find out whether a given plan is good, i.e., whether the given sampling plan has adequate power of discrimination between good and bad lots.

$$(6+4+10)=[20]$$

5. (a) What do you understand by AOQL? In what manner AOQL value of a given plan provides assurance to the consumer regarding the quality of the products accepted under the plan?

$$(4+2)=[6]$$

- (b) Obtain a single sampling plan by attributes for the following

Lot size = 20,000

AQL = 2.5 percent

using I.S.2500 Part I Tables. Draw the O.C. curve for the chosen plan

OR

Obtain single sampling plan by attributes using Dodge and Romig's sampling inspection table when lot size = 10,000,  $\bar{p} = 0.05$ , AOQL = 2 percent. Sketch the O.C. curve of the plan.

$$(4+10)=[14]$$

6. (a) Explain the procedure under double sampling plan by attributes.
- (b) Write down the mathematical expression for O.C. function of double sampling plan by attributes.
- (c) Explain the steps to construct a fraction defective control chart (p-chart) from a given data when the sample size varies. (4+8+8)=[20]

7. Write short notes on any two.

- (a) State of statistical control
- (b) Consumer risk and producer risk
- (c) O.C. Curve of  $\bar{X}$ -Chart and its role.
- (d) Control chart for count of defect data
- (e) Rational sub-grouping for control chart

(10x2)=[20]