KARL PEARSON 1857—1936

WITH A NOTE ON HIS STATISTICAL AND BIOMETRIC WRITINGS.

By P. C. MAHALANOBIS

A Lecture delivered in the Statistical Laboratory, Calcutta, on the 27th November, 1936, and published in *Sankhyā*: The Indian Journal of Statistics, December, 1936.


Karl Pearson

KARL PEARSON, 1857-1936.

INTRODUCTION

With the passing away of Karl Pearson in April 1936, a great period in the history of modern statistics came to an end. He was the most eminent statistician of a new epoch which was largely his own creation. He laid the foundations of correlational analysis, constructed exact methods for drawing valid inferences from statistical material, founded the greatest statistical laboratory, and built up the most important centre of statistical research of the present time. When he started work, statistics and biometry were not recognised subjects for scientific study. During his own life time he succeeded however in establishing them as independent subjects for academic study and scientific research. This is not the place for a full memoir of his life and labours, and I am not competent to write it. I shall only try to give a general idea of his work for the advancement of statistical studies, dwelling on such aspects of it as are likely to be of especial interest to statistical workers in India.*

GENERAL CAREER

The external events of Karl Pearson's life are simple. He was born on the 27th March, 1857, and was educated at first in the University College School, London, and then at King's College, Cambridge, which he joined as a scholar in 1875. He was third Wrangler (Mathematical Tripos) in 1879, and was made a Fellow of King's College in 1880. He then spent some time in Germany, and after his return was called to the Bar in the Inner Temple in 1882, but he gave up law and was appointed professor of applied mathematics and mechanics in the University College, London, in 1884. In 1911 he was appointed the first Galton Professor of Eugenics in the University of London, and retired from this post in 1933. He was elected a Fellow of the Royal Society in 1896, and awarded the Darwin Medal in 1898. He received the Huxley Medal of the Royal Anthropological Society in 1902, and presided over the Anthropological Section of the British Association in 1920. He was an honorary Fellow of King's College, Cambridge, hon. D.Sc. of London, hon. LL.D. of St. Andrews, honorary Member of the Royal Society of Medicine, the Institute of Actuaries, and of Anthropological Societies of Paris, Washington and U. S. S. R.†

---

* Lecture delivered in the Statistical Laboratory, Calcutta, on the 27th November, 1936.

† I am indebted to Mr. G. U. Yule's obituary notice in the *Nature* of 23 May, 1936, and Dr. G. M. Morant's article in *Man*, 1936, 118, for biographical facts.

Reprinted from Sankhyā: The Indian Journal of Statistics, Vol. 2, Part 4, 1936.

These events do not however indicate in any way the real significance of his life in the history of statistics in recent years. To understand this we must know something of the personality of the man himself and the difficulties and struggles of his long life.

PARENTAGE AND EARLY LIFE

Karl Pearson believed that "from our stock we each one of us obtain our physical and mental faculties." His father William Pearson, who was a King's Counsel, came from old Yorkshire stock. After his father's death in 1907, Karl Pearson wrote in a letter to Galton :—

"My father was a man of immense will and endless power of work, with a wonderful physique. . . . . . He would be up at 4-30 to prepare his briefs, take a standing breakfast at 9, and rush into his brougham: back at 7 o'clock, dinner over at 8, he was in bed at 9, and so for month on month, we only saw him at these hurried meals, when speaking was scarce allowed. . . . . An iron man with boundless working power, who never asked a favour in his life, and never really got on because he forgot to respect any man's prejudices, and never knew when he was beaten. I learnt many things from him, and know that I owe much to him physically and mentally." (F. G. 3, 327-328).[1]

"On my Mother's side I am also descended from Yorkshire folk, formerly termed the 'mad' Bethels, and from another Yorkshire family, the roaming Whartons. Put into me a combination of those characteristics—a capacity for hard work and a capacity for roving into other people's preserves—and you have an explanation of my life." (*Speeches*, 1934, p. 20).[2]

It is necessary to add one other fact, the Quaker tradition of the family. To the zest of life and wide range of interests, capacity for hard work, a strong will, great intellectual power and forensic skill, the quaker blood in his veins added an uncompromising independence of spirit perhaps to the point of stubbornness, personal loyalty to a cause and to friends, and a sense of the mystery of life.

Karl Pearson has himself described in playful language

"the result of this germinal mixture. In Cambridge I studied Mathematics under Routh, Stokes, Cayley and Clerk Maxwell—but wrote papers on Spinoza. In Heidelberg I studied Physics under Quincke, but also Metaphysics under Kuno Fischer. In Berlin I studied Roman Law under Bruns and Mommsen, but attended the lectures of Du Bois Reymond on Darwinism. Back at Cambridge I worked in the engineering shops but drew up the schedule in Mittel and Althochdeutsch for the Medieval Languages Tripos. Coming to London, I read in Chambers in Lincoln's Inn, drawing up bills of sale, and was called to the Bar, but varied legal studies by lecturing on Heat at Barnes, on Martin Luther at Hampstead and on Lassalle and Marx on Sundays at revolutionary clubs round Soho. Indeed, I contributed to the Socialist Song Book hymns which I believe are still chanted." (*Speeches*, 20).

EARLY WRITINGS

In 1882 he published anonymously *Trinity, the Nineteenth Century Passion Play* in verse, lectured on pure mathematics at the University College as a *locum tenens* in 1883, wrote on Maimonides in the *Mind*, and a book on *The Ethic of Freethought* in 1883 which

---

[1] Karl Pearson: *The Life, Letters and Labours of Francis Galton*, Vol. IIIA, 1930, p. 327. This volume has been referred to everywhere as F. G. 3.

[2] *Speeches Delivered at a Dinner held in University College, London, in Honour of Professor Karl Pearson, 23 April, 1934*, referred to as *Speeches*.

attracted considerable notice. I believe it was about this time that he was offered the chair of mathematics at King's College, London ; unfortunately in an unofficial letter the College authorities expressed the hope that on accepting the chair he would concentrate his attention on mathematics, and give up his studies in linguistics in which he was engrossed at this time. This was sufficient reason for the young man of 26 to refuse the offer of the professorship. He had no great desire to leave the Bar, but was persuaded by Prof. Alexander Kennedy to return to mathematics as a successor of W. K. Clifford in the Goldsmid Chair of Applied Mathematics at University College, London, in 1884.

The great classic on the *History of the Theory of Elasticity* which was originally started by the late Isaac Todhunter, but for the major part of which Karl Pearson was responsible, was published in 1886-1893. To this period also belonged some of the technical series of memoirs in applied mathematics on stresses in cranes and masonry dams, torsional vibrations in axles and shaftings, and the theory of arches, etc. In the *Grammar of Science* (1892) he explored with penetrating logic the philosophical foundations of scientific knowledge, and in many ways anticipated the post-relativity revolution in scientific thought.[4] The *Chances of Death and other Studies in Evolution* which was published in two volumes in 1897 contained a collection of essays on an amazing variety of topics which included such subjects as the incidence of mortality at different ages, scientific aspects of Monte Carlo roulette, reproductive and natural selection, politics and science, women and labour, mother-right, mediæval witch-craft, group marriage, words for sex and kinship, and the German Passion Play, a study in the evolution of western Christianity.

Beginning of Interest in Statistics

Most of these belonged to the pre-statistical period. In the meantime he was beginning to get interested in other problems. As early as 1875 Francis Galton had been conducting experiments with sweet pea seeds to study the law of inheritance in size ; and he had reached the idea of regression in heredity (or 'reversion' as it was then called) in 1877. But the concept of correlation first occurred to Galton in 1888, and found expression in his book entitled *Natural Inheritance* published in 1889. Karl Pearson was of opinion that this marked a new epoch, and was the starting point of the Biometric School. He said :—

> "Here was a field for an adventurous roamer! I felt like a buccaneer of Drake's days—one of the order of men 'not quite pirates, but with decidedly piratical tendencies', as the dictionary has it! I interpreted Galton to mean that there was a category broader than causation, namely correlation, of which causation was only the limit, and that this new conception of correlation brought psychology, anthropology, medicine and sociology in large parts into the field of mathematical treatment. It was Galton who first freed me from the prejudice that sound mathematics could only be applied to natural phenomena under the category of causation. Here for the first time was a possibility—I will not say a certainty, of reaching knowledge—as valid as physical knowledge was then thought to be—in the field of living forms and above all in the field of human conduct." (*Speeches*,. 22-23).

---

[4] It is interesting to note that the President of the Mathematics and Physics Sections of the British Association in his presidential address of this year has expressed the opinion that Karl Pearson's views offer the most satisfactory basis for the reconstruction of the foundations of scientific knowledge in the light of modern researches.

At the age of 25, when he first started teaching, he had begun to examine critically the fundamental concepts of science. He challenged the crude materialism of orthodox scientists of that time, and suggested that the entities with which science worked—matter, force, ether, space or time—were all constructs of man's own mind, just so far useful as they described adequately his experience, and to be replaced by wider concepts as necessity arises. To him all science was "description and not explanation." A scientific law was a brief description in mental shorthand of a wide range of sequences of sense impressions. The category of cause and effect therefore did not have for him any absolute validity. It was very natural that he should have welcomed so enthusiastically the new concept of correlation, and it is easy to understand the importance attached to it in his philosophy of science. He has explained his ideas on many occasions:—

"This measure of partial causation was the germ of the broad category—that of correlation, which was to replace not only in the minds of many of us the old category of causation, but deeply to influence our outlook on the universe. The conception of causation—unlimitedly profitable to the physicist—began to crumble to pieces. In no case was B simply and wholly caused by A, nor indeed by C, D, E and F as well! The physicist was clearly picking out a few of the more important causes of A, and wisely concentrating on those. But no two physical experiments would—even if our instruments of measurement, men and machines, were perfect—ever lead to absolutely the same numerical result, because we could not include all the vast range of minor contributory causes. The physicist's method of describing phenomena was seen to be only fitting when a high degree of correlation existed. In other words he was assuming for his physical needs a purely theoretical limit—that of perfect correlation. Henceforth the philosophical view of the universe was to be that of a correlated system of variates, approaching but by no means reaching perfect correlation, *i.e.*, absolute causality, even in the group of phenomena termed physical. Biological phenomena in their numerous phases, economic and social, were seen to be only differentiated from the physical by the intensity of their correlations." (F. G. 3, 1-2).

Or again in a letter, dated April 22, 1907:—

". . . . . Not even two atoms are precisely identical. They form a class with variation about a mean character. Hence even in physics the ultimate basis of knowledge is statistical—the category is of course correlation not causation. The main difference is that in physics the correlation coefficients are nearly unity but in biology they diverge considerably from unity." (F. G. 3, 314).

With the replacement of 'causation' by 'correlation' the categoric separation of the organic and the inorganic also disappears. "The mystery of change in the inorganic world is just as great and just as omnipresent as in the organic world." (*Grammar of Science*, 2nd edition, 1899, preface). From this followed his "endeavour to see all phenomena, physical and social, as a connected growth, and describe them as such in the briefest formula." (*Chances of Death*, I, vi). This supplied on the theoretical side the fundamental note in his writings although they covered the most widely diverse subjects.

STATISTICAL PAPERS

G. U. Yule has remarked that 1893 was an epoch-making date in the history of statistics. Karl Pearson's first statistical memoir (the first in the classic series of contributions to the Mathematical Theory of Evolution) was communicated to the Royal Society on the 18th October of this memorable year. A collected edition of his statistical and biometrical writings will form a library by itself. This is not the place for a detailed exposition of this vast literature, and I shall mention here only a few leading themes to give some idea of the wide scope of this work.*

* A note with selected references is being added for students of statistics on pp. 411-422.

For the first five or six years Karl Pearson was engaged in building up the mathematical apparatus for the analysis of statistical data. Between 1893 and 1898 he developed the method of moments to describe the Pearsonian family of frequency curves, gave the product-moment formula for the coefficient of correlation, and worked out the theory of probable errors for large samples drawn from normal multi-variate correlated populations. In 1901 he formulated the Chi-square statistics which made a rational and precise comparison of actually observed results with theoretically expected values possible for the first time, and thus served to start a new epoch in the history of theoretical statistics. In the same year he gave a method for finding the correlation between characters not quantitatively measurable, and a little later in 1904 introduced the fruitful concept of contingency and gave the theory of non-linear regression in 1905. He continued to work on various methods of finding correlation, and on the theory of exact distributions and small samples up to the very end.

His approach to statistics was I think twofold. The possibility of replacing causality by the wider concept of correlation appealed to him strongly, and he had started work on correlation very early.

"I was immensely excited by Galton's book of 1889—*Natural Inheritance*—and I read a paper on it in the year of its appearance. In 1891-92 I lectured on probability at Gresham College, taking skew whist contours as illustrations of correlation. In 1892 I lectured on variation, in 1893 on correlation to research students at University College. . . . . The field was very wide and I was far too excited to stop to investigate properly what other people had done. I wanted to reach new results and apply them." (*Biom.* XIII, 1920, 29).

BIOMETRIC STUDIES

The other approach was definitely biological. He became interested in statistical methods as tools for the systematic study of evolution and the science of eugenics. His friend, W. F. R. Weldon, the biologist (later co-founder and co-editor of *Biometrika* with Pearson), had already started work on crabs, "On certain correlated Variations in *Carcinus Moenas*". (*R. S. Proc.* 51 (1892), 2-21). This paper gave rise to statistical problems which were turned over to Pearson for solution, and led to his first memoir in the Contributions to the Mathematical Theory of Evolution.

During the first few years he had concentrated his energies on developing the technique of correlational analysis. The next step was the systematic study of variation and selection in the mass. The general scheme was explained in the editorial note in the first issue of *Biometrika* in October 1901:—

"The first condition necessary, in order that any process of Natural Selection may begin among a race, or species, is the existence of differences among its members; and the first step in an enquiry into the possible effect of a selective process upon any character of a race must be an estimate of the frequency with which individuals, exhibiting any given degree of abnormality with respect to that character, occur. The unit, with which such enquiry must deal, is not an individual but a race, or a statistically representative sample of a race; and the result must take the form of a numerical statement, showing the relative frequency with which the various kinds of individuals composing the race occur."

A large number of laborious studies of variation and correlation in lower animals and in plants was published between 1900 and 1910. The question of inheritance was also taken up quite early. Karl Pearson was definitely of opinion that

"all Darwin's ideas fit themselves to algebraic definition. . . . . . exactly as in the case of the mathematical treatment of Faraday's conception of electromagnetism."

In several papers spread over nearly 30 years he discussed how far the laws of inheritance and evolution could be formulated in terms of mathematical theories.

Interest in Eugenics

Since the beginning of his interest in statistics Karl Pearson was also getting into more and more intimate touch with Francis Galton whom he came to regard as his acknowledged Master. Galton had introduced the term 'eugenics' in 1883 in his *Inquiry into Human Faculty*, and had defined it "as the science of improving stock, not only by judicious mating, but by all the influences which give the more suitable strains a better chance." (F. G. 3, 221). In 1891 Galton in his presidential address to the division of Demography of the Seventh International Congress of Hygiene and Demography came back to the subject, and after discussing the relative fertility of various classes within a nation, and the relative fertility of nations among themselves, pressed for a scientific study of the problem. Pearson started corresponding with Galton a little later, and gradually became interested in eugenics.

Study of Heredity in Man

Pearson had realized (and Galton fully agreed) that the first thing necessary was the study of man from the standpoints of heredity, anthropology and medicine. Concrete studies on heredity in man were therefore started early, and by 1903 or 1904 it had been shown that the resemblance between parents and offspring was given by a correlation of about $+0.5$ for both physical and mental characters. From that time the relative importance of heredity and environment was investigated in various ways by patient collection of an immense amount of data, and their laborious statistical analysis. The scope of the work was wide; health and disease, physical, physiological, mental characters and abnormalities, and environmental conditions of all kinds were studied in great detail.

As a result of these studies Karl Pearson became convinced of the overwhelming importance of heredity. Other equally painstaking studies showed that in England since about 1878 the poorer families were having a larger number of children. The association of greater fertility with lower economic status was definitely anti-social in character, and necessarily implied a progressive deterioration of the race. According to Karl Pearson, this constituted the greatest problem of the future. The only way of preventing race degeneration and of improving the stock was the practice of eugenics based on adequate scientific knowledge.

Eugenics and Anthropology

His work was not confined to scientific researches in eugenics. He gave numerous popular lectures and wrote a large number of pamphlets for the propagation of eugenic ideas. He emphasized the importance of heredity, the fact that nature was far more powerful than nurture. This involved him in many controversies with laymen and scientists who were ignorant of statistical methods. In the course of one such controversy he wrote:—

> "Every social problem belongs to a class embracing the hardest of all problems—it is vital not physical, it is biological, it is medical, it is statistical. It needs not less but far more investigation for its solution than any academic physical or biological problem. Yet every politician, every platform orator, who would hesitate to express even his opinion regarding a question in astronomical physics or cytology is ready with a decisive answer to each social problem that arises. The staff of the Galton Laboratory naturally lays no claim to any special infallibility in either conclusion or choice of method, but it does assert and will continue to assert that these social problems, with their intense complexity, cannot be solved by political and oratorial methods; they must be answered as all other scientific problems by inevstigation of an academic kind in university laboratories." (*Second Study on Parental Alcoholism*, p. 34).

Karl Pearson looked upon the study of man as the queen of all sciences, and was of opinion that researches in ethnology and sociology, psychology and medicine, or demography and evolution should be all combined and co-ordinated for its development. His biometrical and eugenic studies contain therefore much material of anthropometric interest. But he also made many notable contributions to comparative anthropmetry, especially in tracing the detailed evolutionary history of man, and in improving the technique of physical anthropometry by standardization of measurement and their adequate statistical treatment.

Miscellaneous Statistical Works

Besides the general theory of statistics and biometry, his two main spheres of work, he extended the application of the statistical method to other fields of study such as meteorology, psychology, historical chronology, medicine, public health, and epidemiology. He was also interested in the history of statistics about which he published a number of important papers.

Although the range of his interests was amazingly wide, Karl Pearson was above everything else an applied statistician and biometrician. He was never satisfied until he could put everything to the test of numbers. He realized the importance of numerical calculations, and statistical workers for many generations to come will remain indebted to him for the magnificent volumes of statistical tables computed and edited by him.

Finally mention must be made of the *Life, Letters, and Labours of Francis Galton* on which Pearson worked for 20 years and which was published in three (or actually four) volumes between 1914 and 1930. It has been truly called a monumental work, the greatest scientific biography, "never excelled in completeness, accuracy, insight, and keen judgment."

Lucid Exposition

On whatever subject he wrote, his exposition was clear and logical. He was careful about the minutest details, and explained everything in such a way that every serious student could follow the work. His scientific papers were written with great care and with a very high standard of accuracy in statements, references, or details of numerical calculations. This was most fortunate, for it enabled workers all over the world to learn his technique without difficulty. The more popular writings and works on general subjects were characterised by clarity and literary distinction.

Fifteen or twenty years ago good books on statistical theory were scarce, and many of us were obliged to learn our statistics from Karl Pearson's own writings. There was a spirit of adventure which was inspiring, and studying the papers in the original was a liberal education. They were full of side comments, frank discussion of difficulties, suggestions for further exploration. We could catch something of the personality of a great teacher. The younger workers who learn the theory from the formal lessons in a text-book miss a good deal, and I still think that statistical education cannot be considered to be complete until one has studied at least some of the classical papers of Pearson.

THE BIOMETRIC LABORATORY

I must now turn to the work of Karl Pearson as the founder and director of the greatest Statistical Laboratory in the world. I have already mentioned that Karl Pearson had been giving lectures on variation and correlations from 1891 to 1894. The lectures were mostly of a popular and general nature. It was in 1895 that a Statistical Laboratory (I think the word Biometry was coined by Karl Pearson four or five years later) was established in the University College, London, and the story of its early endeavours can be best told in Karl Pearson's own words written in 1922 in a small pamphlet on the history of the Biometric and Galton Laboratories:—

"From that year [1895] the statistical course became annual, and as the field of this form of investigation had been very little worked a school sprung up which has since been recognized as the "Biometric School", and the group of workers, occupying a small room at University College later termed the Biometric Laboratory, issued a long series of memoirs, which formed the basis of the English School of mathematical statistics. The object of this school was to make statistics a branch of applied mathematics with a technique and nomenclature of its own, to train statisticians as men of science, to extend, discard or justify the meagre processes of the older school of political and social statisticians, and in general to convert statistics in this country from being the playing field of dilettanti and controversialists into a serious branch of science which no man could use effectively without adequate training, any more than he could attempt to use the differential calculus, being ignorant of mathematics. This task was a very arduous one, for statistics in one form or another are fundamental in nearly every branch of science in precisely the same manner as mathematics are fundamental in astronomy and physics. Inadequate and even erroneous processes in medicine, in anthropology, in craniometry, in psychology, in criminology, in biology, in sociology, had to be criticised not for the pleasure of controversy, but with the aim of providing those sciences with a new and stronger technique." (*Pamphlet*, 1922)

The spirit of the Laboratory was explained by Karl Pearson in Galton's words:—

"Until the phenomena of any branch of knowledge have been submitted to measurement and number it cannot assume the status and dignity of science."

THE STARTING OF THE *Biometrika*

There was at first great difficulty in publishing the researches of the new laboratory. Karl Pearson had sounded a number of persons in 1899 with regard to the publication of a journal of pure and applied statistics but without much encouragement. In 1900 there was trouble with the Royal Society in connexion with Karl Pearson's paper on Homotyposis. After a good deal of opposition the paper was finally accepted for publication in the *Phil. Trans.* but Karl Pearson felt "it is a practical notice to quit." This was confirmed by subsequent events.

"Shortly afterwards a resolution of the Council was conveyed to me, requesting that in future papers on mathematics should be kept apart from biological applications. *βιος* was an admissible topic, *μετρον* also, but their combination was anathema, and that at a time when statistical theory had to be worked out step by step as the biological applications demanded." (F. G. 3, p. 100, footnote).

The need of a new journal was urgent, and the question was discussed with W. F. R. Weldon who drew up a prospectus, while Pearson invented a new word 'biometry', and suggested the name *Biometrika* for the new journal. By April 1901 only about 60 promises of subscription had been received which was not sufficient; a guarantee fund of £200 was essential before anything could be done. Fortunately, Francis Galton sent a cheque for the whole amount (for, as he put it, "I like forlorn hopes in a good cause"), and the *Biometrika* was started in October 1901 with Francis Galton as the Founder, and Weldon and Pearson as joint editors.

Galton Laboratory for National Eugenics

In 1902 a grant was received from the Company of Drapers which was earmarked for research and the publication of memoirs. In 1904 Galton gave a donation of £500 a year for three years to the University of London for the study of eugenics, and in 1905, the Eugenics Record Office was started "steadily to collect data bearing on the effect of environment, of heredity and of inter-caste marriage upon man", under the supervision of Galton himself assisted by Karl Pearson. National Eugenics was defined at this time "as the study of the agencies under social control that may improve or impair the racial qualities of future generations either physically or mentally." (F. G. 3, 305). Owing to Galton's failing health it was decided by the end of 1906 that Karl Pearson would take over charge of the office. The work was reorganized and the 'Galton Laboratory for National Eugenics'—the plan for which had been prepared by Pearson and approved by Galton—was started in February, 1907. It was stated in the scheme that "the Laboratory shall act (*i*) as a store-house for statistical material bearing on the mental and physical conditions in man and the relation of these conditions to inheritance and environment, (*ii*) as a centre for publication or other form of distribution of information concerning National Eugenics." The sudden death of Weldon in 1906 threw the entire responsibility for the editorial work of the *Biometrika* on Pearson at about the same time.

In eugenics Karl Pearson's activities were not confined only to scientific researches. Galton had always emphasized the need for propaganda. Karl Pearson agreed in this but he thought that the foundations must be securely laid on scientific research. In a letter to Galton (Feb. 7, 1909), he had explained his programme clearly:—

> "We have got to convince not only London University but the other universities (*i*) that Eugenics is a Science and that our research work is of the highest type and as reliable and sober as any piece of physiological or chemical work, (*ii*) that we are running no hobby and have no end in view but the truth. If these things can be carried out we shall have founded a science to which statesmen and social reformers can appeal for marshalled facts." (F. G. 3, 371-372).

But he always insisted "that the solutions to these problems are in the first place statistical, and in the second place statistical, and only in the third place biological." (F. G. 3, 127-28).

The Growth of the Joint Laboratories

The organization of the Biometric Laboratory was kept distinct, but the two laboratories worked in close association under the direction of Karl Pearson. Provision was made for giving training in statistical methods and for assisting research workers in eugenics. The material facilities were however meagre. The total staff of the two laboratories in the early days consisted of about four or five research assistants. The accommodation at the time the Statistical (later called the Biometric) Laboratory was started, in 1895, has been described by Mr. G. U. Yule:—

> "There was no working room except the Professor's private room and a long slip of a room behind the lecture room which was turned into an instrument room, and housed a Coradi co-ordinatograph, integraph, and pantograph—and not much else. The purchase of the first Brunsviga (in 1895) was great excitement." (*Speeches*, 12).

Two or three rooms were added when the Eugenics Record Office was transferred to University College. But even then:

> "Up to 1914 the joint laboratories occupied four small rooms in University College, which their staffs were terribly ashamed to state were all that they could show to the long series of foreigners who between 1904 and 1914 came to inspect what they supposed were laboratories in actuality as well as in name." (*Pamphlet*, 1922).

DIFFICULTIES IN EARLY YEARS

The difficulties in early years were indeed very great. Neither biometry nor eugenics had then been recognized as fit subjects for scientific research. In fact an offer made at that time "of the publications of the Eugenics Laboratory to the Royal Society Library met with a refusal, presumably on the ground that they were not scientific." (*Annals of Eugenics*, I, 1925, 2). The opposition from one section of the Royal Society continued unabated. In 1906, a biometric paper by Raymond Pearl was communicated by Karl Pearson and was refused. He then seriously considered whether he should not resign the Fellowship of the Royal Society as a mark of protest. He wrote to Galton (May 13, 1906):—

"My chief work and interests now lie in biometry. If the R. S. will have nothing to do with it, and publishes papers and reports of which the writers lack the most elementary knowledge of statistics, then the Society ceases to appeal to me in any way. I cannot see that I shall do any harm in raising my protest, however feeble it may be. It is not, I trust, a personal point, for I have sent nothing for three years to the Society." (F. G. 3, 283).

On the advice of Galton he however refrained from sending his resignation.

In the Biometric Laboratory he could work only in his spare time, for he was obliged to do his full work as the professor of applied mathematics. He wrote in a letter to Francis Galton (April 8, 1907):—

"College teaching—a good deal of an elementary kind, but of the bread and butter sort—has gone on increasing year by year, so that I get little time for research work during term. . . . . . . If one were in Germany, or had accepted one of the posts that have been offered in America, one would by fifty be able to do the work one is best fitted for. But this in England is only possible at Oxford and Cambridge; at all the newer Universities one has to undertake endless teaching work, which has no relation to the field of one's greatest efficiency." (F. G. 3, 312).

At this time Pearson "was giving 24 hours a week to teaching and demonstrating, apart from aiding research workers, supervising Galton's Eugenics Laboratory and much heavy editorial work." (*F. G. 3,* 345, footnote). In another letter of about the same time (Jan. 8, 1909), we find under what difficulties he had to carry on his researches:—

"I have asked for a holiday for six months on the grounds: (*i*) that I have been feeling inert and below par, and (*ii*) that the arrears of research work are so great, that nothing can get finished. I have not had a real holiday since my marriage tour, eighteen years ago. . . . . . If my request be granted I shall have no elementary teaching of any kind. I shall be a "half-timer" at College, spending my afternoons with the research students and with them only, while I devote my mornings to polishing off arrears of work, so that in the summer I can take a complete holiday." (F. G. 3, 368).

In a letter to Galton (December 15, 1908), we find Pearson referring to the discouragement from both outside and inside against which they had to work and toil forward:—

"Only last week a lecturer in the College read a paper 'On the Influence of Heredity on Conduct,' which consisted chiefly of abuse of the Eugenics Laboratory work and workers." (F. G. 3, 359).

In the beginning of the present century even in England prospects of statistics were not bright and abler students were still standing away from this subject.

"At present the training in statistics does not lead to paid positions. It is beginning to, but the posts available are few and the best men who want to get on in life won't enter this field. [But] . . . . . the time is coming when governmental and municipal work will demand men of the kind we are training. We are only a little bit (not very much) ahead of public needs. . . . . There must be sooner rather than later a government statistical bureau, and this will demand trained statisticians. Once we have a flow of such men who mean to make statistics their profession in life, there will be ample material to select from." (F. G. 3, 381).

But Pearson's Laboratory was gradually attracting able students :—

"I have nine biometric research students this term and my new Laboratory is *full*. It is the first time I have had more students than I want on this side. A man came this afternoon for admission wanting to do research work, and I took quite a lordly tone with him and told him to go away for a fortnight and write a paper and I would take him if it was good enough for publication. I have never been able before to pick and choose postgraduate workers—and this man was a Cambridge Wrangler!" (F. G. 3, 356).

In 1909, a grant was sanctioned by Government for Dr. Charles Goring's work on criminal anthropometry, and Pearson rejoiced at what he called the "first semi-official recognition of our statistical laboratory." (F. G. 3, 377).

Galton Chair of Eugenics

In 1911 Sir Francis Galton died and left the residue of his estate (worth considerably more than £30,000 I believe) to the University of London for the establishment of a full professorship in eugenics. Karl Pearson relinquished the chair of applied mathematics in the same year and became the first Galton Professor.

Things became easier from this time as he was relieved of all the elementary teaching in mathematics. The laboratory equipment and accommodation were still very limited. In response to an appeal by the University, Sir Herbert Bartlett provided a building for the combined laboratories in Gower Street in 1913. Work was however interrupted by the war. The new building was used as a military hospital and its occupation was postponed until October 1919. The staff including Karl Pearson himself undertook very heavy work for Government for the preparation of hundreds of diagrams and charts, investigations of propeller stresses, distribution of elastic constants, computing of sights, elaborate calculations of trajectories and ballistic tables, etc., which exhausted the resources of the laboratory. The new building was opened on the 4th June, 1920, but it took two or three years more to build up the organization again. Pearson was thus already over 65 years of age before he got the opportunity to apply himself uninterruptedly to statistical and biometric researches. For most men it would have been too late. But Karl Pearson's energy was titanic and he carried on his work for nearly fifteen years more with unabated vigour.

Annals of Eugenics

In October 1925 at the age of 68, Karl Pearson started a new journal, the *Annals of Eugenics*. Seventeen volumes of *Biometrika* had been already issued which had placed it in a secure position as the leading journal for mathematical statistics and biometry. But there was still need for a journal which would be more directly concerned with eugenics. Emphasis was laid on the 'national' character of eugenics, not in a narrow patriotic sense, but because the nation as a whole not the family nor the individual was the proper unit for study.

The editorial work was heavy because Karl Pearson did not merely attend to correct reproduction of the copy ; Dr. G. M. Morant, who was associated with him as the Assistant Editor of the *Biometrika* for many years, writes that Karl Pearson "considered *that* to be the lesser part of his task, and the greater part was the correction and improvement in other ways of the copy. . . . He *worked* on every paper submitted to him for publication. Many new contributors must have been surprised to receive detailed criticisms instead of unconditional acceptance. . . . There were few who did not appreciate in the end the advantage to their studies gained from his inspiring and unstinted help." (*Man*, 1936, 118). In a private letter (June 26, 1936) Dr. Morant tells me that before Pearson's death "for more than six months he had been quite seriously ill, but this had not prevented him from carrying on his editorial labours."

Work as a Teacher

Karl Pearson was a great teacher, and successive generations of students who came under the influence of his personality were inspired by it. As he himself had said:—

> "Science no less than theology or philosophy, is the field for personal influence, for the creation of enthusiasm, and for the establishment of ideals of self-discipline and self-development." (*Biom.* V, 1906, 1).

Mr. G. Udny Yule, his first pupil, speaking in 1934 of the early days, said:—"The luminous clarity of his exposition, his own keen interest in his subject and his students, stand out in my memory" (*Speeches*, 10). Another old pupil, Prof. M. Greenwood, said that Karl Pearson had

> "the power to fire the enthusiasm of a lad, to make him realise, once and for ever, that scientific research, undertaken not for some petty, material end, but because
>
> right is right, to follow right
> Were wisdom in the scorn of consequence.
>
> is the greatest of joys. Most of us have faltered in the quest, all of us have made mistakes, have fallen far below the ideal the Professor set before us. But none of us quite lost sight of the glorious vision he showed us in our youth or can ever think of him without gratitude and affection." (*Speeches*, 17).

I had the opportunity for a few months in 1927 to work in the Biometric Laboratory and to see him at work. It was his practice almost invariably to go at least once a day to each advanced worker and enquire how things were getting on. Sometimes he would stay over to discuss a difficult point, or suggest a new approach to the problem. Every worker was quite at liberty to go into the Professor's room and consult him. Although seventy years old, he was usually the first to come and the last to leave the Laboratory, so that he was almost always available. Guidance of research was therefore a whole time job with him.

He gave complete freedom to his students to find out the truth for themselves:—"My policy with my young people is to show them my own standpoint, but in no way to control their action." (F. G. 3, 372). Speaking of academic workers he had once said:—

> "Let us hope they will investigate and teach, not because to do so is to fulfil an academic duty, but because their natural impulse is to seek the truth and utter it." (*Annals of Eugenics*, I, 1925, 2).

It is no exaggeration to say that he himself fully lived up to his own ideals.

Controversies

He was a man of strong convictions. Like his father, I think, he also forgot to respect any man's prejudices. He had very strong loyalties, and was a born fighter. It was not surprising therefore that he was involved in many bitter controversies. This was to some extent inevitable, for he was the pioneer in a new scientific method which had not yet been accepted as orthodox. As he himself had said:—

> "The professors of other branches of knowledge—of which a new science is now-a-days almost certainly compelled to be an extension or a development—are usually disturbed, often contemptuous. They give solemn warnings to their brethren, like the priests of old, to avoid the unclean novelty; to beware of worshipping new gods or of playing with new-fangled tools." (*Annals of Eugenics* I, 1925, 2).

But this was not all. I think the vehemence with which he carried on some of the controversies can be explained only if we remember that he was fighting for a cause. With his temperament (and possibly partly due to his puritanic stock) he would naturally look upon any one who was not with him as being against him and against the cause he espoused. As an adventuring pioneer he felt himself surrounded by foes on every side, and he must have considered it his duty to fight every issue to a finish. (The name he gave to the series of popular lectures and addresses, "Questions of the Day and of the Fray" reflects this attitude of mind). When he himself discovered that he was in the wrong he was quick to acknowledge his mistakes. But when the adverse criticism came from what he considered to be a different camp, his subtle power of argument and skill in controversy would make him defend his own position with great tenacity. I think he had also inherited a little of his father's legal attitude of mind. He would often concentrate on whether his opponent's arguments were valid rather than on whether his opponent was in the right. This made him sometimes unappreciative of the strength of his adversary's position. Unfortunately this created a good deal of bitterness and isolated him from other workers.

Personal Loyalties

On the positive side the same loyalty made him the staunchest of friends. He would never spare himself to help a person who had been once admitted into the inner circle. He was ever watchful of the reputation and material welfare of his pupils. He has written to me more than once to find out whether suitable openings could be secured in India for Indian students who had completed their statistical studies under him.

For his colleagues and fellow-workers he had the warmest affection and love. They became integral parts of his own life. When W. F. R. Weldon died in 1906 he was overwhelmed with sorrow. Going through Weldon's papers he wrote to Galton:—

> "At times there seems so much to talk to you about and then again it all passes from me. It was possible to go on as long as I was attempting to put the papers at Oxford in order, but I seem now quite dazed, and for the first time in all my teaching experience the idea of facing my students and lecturing seems positively repellent,—at times impossible."(F. G. 3, 282).

And again:—

> "It is the fourth time I have had to throw all my energy into a dead man's papers and work, and three times the man has been so to speak a part of my own life. How can one tell the tale?" (F. G. 3, 285).

In his personal relations the highest peak was reached in his devotion to Francis Galton. It was hero-worship in the best sense of the word. Pearson had not only the greatest admiration and respect for the scientific genius of Galton, but entertained feelings of deepest affection and love for him. Pearson was a man of stubborn character. Once he had decided on a particular course of action it was practically impossible to make him change his mind. The only person to whom he would yield was Galton who always exercised a moderating influence on him. Speaking of Galton's death in 1911, Pearson wrote:—

> "I personally had lost the master in whose footsteps I had trod since I met his *Natural Inheritance* in 1889, and the man with whom my friendship had grown closer and closer year by year, even to his death."

For twenty years he worked on Galton's Life:—

"I have written my account because I loved my friend and had sufficient knowledge to understand his aims and meaning of his life for the science of the future. . . . . . Civilisation has gained nothing from rivalry in destructive warfare; it can gain enormously from the rivalry of nations in rearing their future generations from the most efficient of their citizens. Galton was the first to realise this great truth, to preach it as a moral code, and to lay the foundations of the new science which it demands of man. In the centuries to come, when the principles of Eugenics shall be commonplaces of social conduct and of politics, men, whatever their race, will desire to know all that is knowable about one of the greatest, perhaps the greatest scientist of the nineteenth century." (F. G. 3, v-vi).

Outlook on Life

Eugenics supplied Pearson with a complete philosophy of life. He realized that 'new modes of transport were rapidly making the world too small for mankind,' and that the human species itself might become extinct through increasingly destructive wars between the nations. The very survival and future progress of man required the emergence of a new type of mankind with a more scientific outlook.

"We know little of how it came about that Aurignacian man replaced the Mousterian man; but the ascent was a steep one, and man once more needs some such rapid elevating. With our present acquaintance with the laws of heredity, with our present knowledge of how customs and creeds have changed, can we not hasten the evolutionary process of fitting man to the needs of his present environments?" (F. G. 3, 218).

The need for international eugenics was urgent without which there was no hope of a satisfactory solution of existing struggles and conflicts.

"Those who fully realise the marvellous evolution of certain types of humanity at the expense of others will smile—sadly, it may—and wonder whether it is feasible for any League of Nations, however strong, to fix and maintain national and racial boundaries, unless it shall have first fixed the relative fertility of all the tribes of man and, what is more, internationalised all the world's resources! As inter-class struggle finds its hope of solution only in the socialism which teaches the nationalisation of the materials and means of production, so international struggle can only reach its conclusion by the universalism which demands internationalisation of world's wealth. In the first case, national eugenics is the only means left to provide any nation with men strong in mind and body; in the second case, international eugenics is the sole possibility of producing finer races of mankind." (F. G. 3, 219).

I think Pearson also accepted Galton's idea that Eugenics would take the place of religion and become the new Humanism of the future.

As regards accepted creeds and religions Pearson was a rationalist and a free-thinker:

"For me there is no absolute truth in scientific knowledge or in religious creed, the one provides conceptual models of more or less descriptive exactness of our sensations of phenomena, the other fits itself to the emotional needs of differing races, periods and individuals." (F. G. 3, 288).

He was of course definitely against the dogmatic view of life:—

"There always will be [war between science and dogma] so long as knowledge is opposed to ignorance. To know requires exertion, and it is intellectually easiest to shirk altogether by accepting phrases which cloak the unknown in the undefinable." (*Grammar of Science*, 2nd edition, 1899, preface).

But like Galton he also I think had that sense of the mystery of existence which is the basis of all spiritual life. Speaking of the well-known lines "Of sense and outward things" in Wordsworth's famous *Ode to Immortality* Pearson had said:—

"It seems to me that to every contemplative man, or at least to every contemplative child, such slipping away from their momentary environment, even in a crowded gathering, will not be unfamiliar, and that they can remember instances when they have experienced a distinct effort to recall themselves to their space and time relations; even if they do not need to shake a gatepost, they may require to shake themselves." (F. G. 3, 115).

Pearson was full of the zest of life:—

"The wonderful part of life is that the problems are so manifold and as long as we retain our mental curiosity, there is no cessation to our activity or to the pleasure of life. I have felt this even in moments of physical disablement. (F. G. 3, 381).

βιος and *μετρον*

For forty years he had worked to build up the foundations for the "study of man, which is destined in the future to be—what theology was in the Middle Ages—the supreme form of knowledge, the queen of all sciences", and had always insisted that

"in the future a new race of biologists will arise trained up in Galtonian method and able to criticise from that standpoint both Darwinism and Mendelism, for both now transcend any treatment which fails to approach them with adequate mathematical knowledge." (F. G. 3, vi).

In 1933, when he relinquished the Galton Chair of Eugenics and the editorship of the *Annals of Eugenics*, his successor in both the offices, Prof. R. A. Fisher, the most eminent statistician of to-day, wrote in his Foreword to the Journal:—

"During the past eight years, under the energetic guidance of its founder, Prof. Karl Pearson, the *Annals of Eugenics* has established its position as the leading scientific journal devoted to this complex subject. Many of us who had watched its progress, at first with undue diffidence, are now satisfied that . . . . the confidence of the founder [was] justified. The contents of the Journal will continue to be representative of the researches of the Laboratory, and of kindred work, contributing to the further study and elucidation of the genetic situation in man, which is attracting increasing attention from students elsewhere. The two primary disciplines which contribute to this study are Genetics and Mathematical Statistics."

Biology and mathematics, which had been ordered to be kept apart in 1901, had in the course of a single generation become indissolubly connected. On the applied side also on his retirement a separate department of statistics was created in the University of London with his son, Prof. Egon S. Pearson as Director. Karl Pearson lived long enough to see that his position was fully vindicated.

He had once quoted Lord Halifax with approval (*Annals of Eugenics,* I 1925, 4):—

"A Difficulty raiseth the Spirits of a Great Man. He hath a mind to wrestle with it and give it a Fall. A Man's Mind must be low, if the Difficulty doth not make a Part of his Pleasure."

If this be the test of a great man, we may be sure that Karl Pearson was a very great man indeed. He lived a full life and enjoyed its difficulties to the full. This perhaps is the greatest thing of all.

CONCLUSION

It is difficult for me to say anything regarding the personality of Karl Pearson as I did not have the privilege of knowing him intimately or of belonging to his inner group of workers. His mind was very alert, and one was deeply impressed by his intellectual power. It was also clear that he was entirely engrossed in his scientific work. But every day at the Biometric Tea in the Laboratory he would relax, and then one could see how wide was his intellectual and cultural interests. He could also be very entertaining to non-statisticians, and my wife who was present on one or two occasions cherishes his memory as a most charming host.

He had a good deal of friendly interest in India, and especially in the Indian peoples. He told me that in his younger days he used to know some of our earlier leaders. He was keenly interested in the Indian caste from an eugenic point of view. His earliest paper in the anthropometry of living persons was on Risley's Measurements of Indian castes and tribes (1903) in which he came to the conclusion that there was a great mixture of races in Bengal. More than once he asked me whether there was any possibility of re-organizing the caste system on scientific lines for serving eugenic purposes. Some of our customs had a special appeal for him. For example he considered cremation a much more fitting end than burial for what must one day perish.

He was interested in the progress of statistical studies in India, and when we requested him to accept the Honorary Fellowship of the Indian Statistical Institute in 1934 he readily agreed, and yet he told us that he thought what India needed most was the provision of advanced teaching in statistics in the Indian Universities. When I wrote to him in 1935 in connexion with our scheme of diploma and certificate examinations in statistics he immediately sent us his frank opinion. He thought the original scheme too ambitious, and again emphasized the need of providing sound teaching. We remember him to-day not only as an eminent scientist but as a great personality with whom we had the privilege of being associated as fellow workers in a common cause.

The time has not come to assess the value of Karl Pearson's work. But two things may be stated with confidence. He will come to be recognised in future as the great pioneer who first placed the science of statistics on a secure basis not only by his own researches but also by his life-long labours as a teacher, as an editor, and as the founder of a new school of research. Also when Eugenics is fully established, Karl Pearson will be associated with Francis Galton as the joint founders of the new science. If Galton's was the original idea, Pearson gave it the first concrete form.

I have a feeling that statistics will steadily grow in importance, and facts of nature in all branches of science will increasingly find their most convenient representation in statistical models. As statistics develops, the significance of Karl Pearson in the history of science will also grow in importance, and he will occupy a place with the greatest scientists of modern times.

P. C. MAHALANOBIS.

A NOTE ON THE STATISTICAL AND BIOMETRIC WRITINGS OF KARL PEARSON.

BY P. C. MAHALANOBIS.

INTRODUCTION.

This note was prepared for use by the workers in the Statistical Laboratory, Calcutta, and gives a selected list of Karl Pearson's statistical and biometric writings. It does not profess to cover all his writings or scientific papers. A full list of his works completed by Dr. G. M. Morant will be published in England shortly.

Karl Pearson's general views on the nature of scientific knowledge will be found in the *Grammar of Science* first published in 1892 and revised in 1900 and 1911. This is a book of outstanding importance, and is indispensable for a proper understanding of Pearson's point of view. A second collection of essays *The Chances of Death and other Studies in Evolution* was published in two volumes in 1897. The first volume contains a number of essays on statistical and biometric topics; and the second volume a number of studies in sociology and folk-lore.

FREQUENCY CURVES.

The papers on statistical theory started in 1893* with the classic series of Contributions to the Mathematical Theory of Evolution (or as the name was changed later, the Mathematical Contribution to the Theory of Evolution). In the first memoir of this series "On the Dissection of Asymmetrical Frequency Curves,[1] which was communicated to the Royal Society on 18 October, 1893, was first used the method of moments which was later developed into a powerful statistical tool. Next year, in the second memoir on Skew Variation[2] the early types of the Pearsonian frequency distributions were formulated in terms of the beta-coefficients. Pearson had noted in this paper the theoretical importance of Thiele's semi-invariants, but preferred to use the method of moments in order to avoid parameters of high order. Further developments and more detailed classification of the Pearsonian family of curves were completed in two supplements[3, 4] in the *Phil. Trans.* in 1901 and 1916. For practical convenience and flexibility the Pearsonian system still stands unrivalled for the empirical description of univariate samples, and the Pearsonian Type III remains the most fundamental form of frequency distribution of which the 'normal' distribution itself is only a special case.

The use of the method of moments for systematic graduation† of observations was considered quite early in 1902 in two papers in the *Biometrika*.[5, 6] Other topics treated by the method of moments or related to it are the dissection of frequency curves composed of different binomial distributions (1915)[7]; the corrections for grouping on more elaborate lines than Sheppard's adjustments (1919)[8]; and the use of the hypergeometric series in fitting observations.[9, 10, 11].

As early as 1895 he had felt the need of constructing skew frequency surfaces for the graduation of observations involving more than one variable, and a small group of papers in 1923,[12, 13] 1925,[14] and 1935[15] give a summary of his ideas on the subject.

* The first published statistical writing appears to be a letter to *Nature* dated 26th October, 1893 (p. 615-616) in which the term 'moment' is first used, and the familiar notation μ_n is first used. The use of the ratio μ_3 to $\mu_{..}$ is also suggested as a measure of the asymmetry of the curve.

† A recent letter in *Nature* (24 August, 1935, 296) should be referred to in this connexion for his views on the function of graduation curves in statistics.

CORRELATION AND CONTINGENCY.

The basic paper in the theory of correlation was the third memoir on 'Regression, Heredity and Panmizia'[16] in 1896 in which the product moment formula for the correlation coefficient was proved. The general theory of correlation for three varieties was given in the same memoir, and various biological terms like 'variation', 'correlation', 'natural, sexual and reproductive selection', 'heredity', etc., were also defined. The mathematical work was based on the assumption of a normal distribution, but the need for studying skew variations and skew correlations was pointed out.

The work on different methods of finding correlation and various aspects of the theory of correlation was continued during the next 30 years. In 1901 a method was worked out for determining the correlation (later known as the 'tetrachoric r') between characters not quantitatively measurable,[17] and its use was illustrated with a good deal of biological and medical data.[18] The next important step was the introduction of a new concept, namely, that of 'contingency' in a memoir in 1904.[19] The new concept is of great value in dealing with economic, social, biometric and other data of all kinds which can be classified, but which cannot be arranged according to any kind of a metric scale. The coefficient of contingency was on one hand connected with the very fundamental chi-square statistics which had been introduced in 1900, and on the other hand with association and normal correlation for quantitative variates. Next year (1905) saw the publication of the paper on skew correlation and non-linear regression[20] in which the method of fitting of polynomial curves of any degree was worked out; (the fact that the successive terms are fully orthogonal was however pointed out much later in another paper in 1921.[21]) The eta-square statistics for measuring the degree of correlation (when the correlation was non-linear) between two variates was introduced in the same paper,[20] and was the starting point of other far-reaching developments. This practically completed the basic structure of the theory of correlation and contingency.

Various methods were at the same time worked out for determining the linear and skew correlation[22] (such as biserial r,[23] biserial η,[24] rank and grade correlations,[25] polychoric r[26]), and corrections for broad groupings[27, 28, 29] when the variates were either not quantitatively measurable or were classified in broad categories. A critical review of theories of Association[30] was published in 1913, and among other papers on the theory of correlation may be mentioned those dealing with index correlation,[31, 32] the influence of selection on correlation,[33, 34] partial and multiple correlation,[35, 36] and first power methods of finding correlation.[37]

PROBABLE ERRORS AND THE THEORY OF DISTRIBUTION.

The problem of probable errors and sampling distribution had received early attention. The probable errors of means, standard deviations and higher moments of the normal distribution for a single variate had been determined much earlier, but scarcely anything else was known when Pearson took up the work. The general theory of the sampling distribution of the parameters of correlated normal populations and of univariate skew frequency curves was given for the first time in 1898 in the fourth memoir of the evolution series, 'On the Probable Errors of Frequency Constants and the Influence of Random Selection on Variation and Correlation'.[38] Many important results were given in this paper, but they were all worked out for the case of large samples. Pearson clearly recognized that "the distribution of errors of the frequency constants, if treated exactly, will generally be skew" (p. 234), and he emphasized that the normal distribution of probable errors was only an approximate result. The work on the general theory of probable errors was continued in three subsequent papers[39] published in the *Biometrika* in 1903, 1913 and 1920. The generalized probable error in multiple normal correlation was given in 1908,[40] and the product-moments for the bivariate correlated surface were given in 1918.[41] The probable errors of the coefficient of correlation calculated by various special methods,[42, 43] of various coefficients of contingency[44, 45, 46] and of a Mendelian class frequency[47] were discussed in a number of papers published mostly in the *Biometrika* between 1906 and 1919.

His first paper on the theory of small samples was published in 1915 as an appendix to the papers of "Student" and R. A. Fisher.[48] In subsequent papers in the *Biometrika*,[49, 50] the distributions of various statistics for small samples drawn from correlated normal populations were considered in great detail with a wealth of numerical examples. A paper on the sampling distribution of the product moment coefficient[51] in 1929 led to the development of certain special forms of the Bessel function which were shown in 1932[52] and in 1933[53] to have important applications in statistical problems. The variance of ranked individuals,[54] the use of probability integrals for symmetrical curves[55] for solving problems in small samples, and studies and comments on "Student's" *z*-statistics[56, 57, 58] were given recently between 1931 and 1935.

The Chi-square Statistics and Statistical Inference.

One of the most fundamental problems of statistics is to determine the probability of an observed sample having been drawn from a given population, or as it occurs more frequently in practice, to test whether two samples have been drawn from the same parent population. Karl Pearson constructed the Chi-square statistics to solve this problem, and found its exact distribution in a most elegant manner in 1900.[59] The Pearsonian Chi-square is the first and the most important of the modern tests of significance, and in the opinion of many statisticians, constitutes Pearson's greatest contribution to statistical theory. He considered different aspects of the same question and various applications of the Chi-square statistics in a number of important papers in the *Biometrika* between 1911 and 1934.[60-70, 85, 86]

How far the percentage of success in past experience can be made the basis of forecasting future expectation was studied in a paper in the *Phil. Mag.* in 1907.[71] This question, which is intimately connected with inverse probability and Bayes' theorem, was in Pearson's opinion the fundamental problem of statistics, and he returned to it in two important papers in 1920[72] and 1928[73] and in a number of shorter notes. An interesting paper on Tchebycheff's theorem[74] in 1919 may also be mentioned here.

Miscellaneous Statistical Papers.

Apart from the contributions to the general theory of statistics there are many ingeneous applications of the statistical method to special problems only a few of which can be mentioned here. In an early paper 'On the Distribution of Frequency (Variation and Correlation) of the Barometric Height at Divers Stations'[75] in 1897, correlation analysis was first used in meteorology. It was confidently stated in this paper that the future of meteorological forecasts would depend on an extended knowledge of correlations 'special attention being paid to the changes of the correlation with intervals of time' (p. 467). It is by the use of just these methods that the development of long-range forecasting of the monsoon has become possible in India in recent years.

Galton's problem of the most suitable proportion between values of first and second prizes was considered in 1902,[76] and again in 1925[77]. Errors of judgment in performing simple experiments and personal equations were studied in detail in 1902.[78] A memoir on the mathematical theory of random migration[79] in 1906 investigated how fast a species could spread in an unoccupied area. The method of variate difference correlation was used in 1915 to study the fluctuations of economic data.[80] A long paper on the 'Mathematics of Intelligence'[81] gave an elaborate and critical study of the statistical foundations of the Two-Factor theory of general intelligence. In the paper on 'Biometry and Chronology'

(1928)[82] it was shown how the statistical method could be used with advantage in calculating the average duration of the reign of sovereigns and hence in reconstructing historical chronology.

HISTORY OF MATHEMATICAL STATISTICS.

There is a group of about half a dozen studies in the history of statistics which deserve mention in passing. The originality of Galton's concept of correlation was discussed in detail against the background of the work done by Gauss and Auguste Bravais in 'Notes on the History of Correlation' in 1920,[83] and the work of Giovanni Plana in this connexion was considered in 1928.[84] In a second note in 1924 the discovery of the normal curve was traced to De Moivre in the second quarter of the eighteenth century.[85] In another note in 1925,[86] it was shown that the proof of James Bernoulli's theorem was actually given by De Moivre. A note on Laplace's solution of a special problem connected with inverse probability in 1928[87] and an interesting study in 1929 on Laplace[88] bear testimony to Pearson's great interest and ability in historical research. There are also historical notes on the theory of small samples of which one in 1931[89] is particularly important in which it is pointed out that the exact distribution of standard deviations was first given by Helmert in 1875-76.

The memoir on W. F. R. Weldon[90] gives a good deal of information about the beginnings of biometric studies. The monumental *Life of Francis Galton*[91] in three (or actually four) volumes is the greatest scientific biography of the present times, and is particularly valuable to students interested in the history of statistics and eugenics for a very detailed exposition of the events up to the time of Galton's death in 1911. There are also two short appreciations of a popular nature of Francis Galton[92] and Charles Darwin.[93]

BOOKS OF STATISTICAL TABLES.

Among books of Tables first and foremost are the *Tables for Statisticians and Biometricians*,[94] Part I of which was published in 1914 and Part II in 1931. The introductions (70 pages in Part I, and 238 pages in Part II) form probably the most lucid and most fascinating introduction to modern statistical practice.

Tables of the Beta-function (1920),[95] the Incomplete Gamma-function (1922),[96] and Elliptic Integrals' (1934)[97] with 53, 26 and 39 pages of Introduction respectively, are the three other magnificient volumes of Tables which we owe to the energy of Karl Pearson. He was also the general editor of more than a dozen 'Tracts for Computers'[98] of which he himself wrote the two parts on the 'Construction of Tables and on Interpolation', and contributed illuminating introductions to the other parts. Another great undertaking was the publication under his leadership of the Logarthmetica Britannica,[99] a standard table of logarithms to 20 decimal places, in 9 parts. That all these books of tables are indispensable for serious statistical workers is only a bare statement of fact. One important paper on the numerical evaluation of Eulerian Integrals[100] which was published shortly before his death may also be mentioned here.

BIOMETRICAL STUDIES.

Karl Pearson had become interested in theoretical statistics primarily as a tool for the systematic study of evolution and the new science of eugenics. During the first few years he had concentrated his energies on developing the technique of correlational analysis in the series of mathematical contributions already mentioned. The next step was a systematic study of variation and correlation in plants and lower animals, the study of laws of inheritance in general and also with special reference to man, followed by more directly anthropological and eugenic investigations.

The biometric studies started in 1900 with a comprehensive paper on "Homotyposis in the Vegetable Kingdom"[101] of which the chief objects were to compare the variability of the individual with the variability of the race, and to study the inheritance of variability in the individual. The general conclusion was that homotyposis and therefore heredity was a primary factor of living forms and a condition for the evolution of life by natural selection. A reply to certain biological criticisms of this paper was given in a paper 'On the Fundamental Conceptions of Biology' in 1901.[102] This was followed by a shorter study of 'Homotyposis in Homologous but Differentiated Organs' in 1903.[103] To this group also belong the laborious co-operative investigations on the Shirly Poppy (1903),[104] the Lesser Celandine (1903),[105] studies on the *Vespa Vulgaris* (1909-11),[106] and on the blood corpuscles of the Common Tadpole (1909)[107] during the next few years, and a later study in 1931 on the duration of pregnancy and the size of litter in bitches.[108]

LAWS OF HEREDITY.

In several papers spread over nearly 30 years he considered how far the laws of inheritance and evolution could be formulated in terms of general mathematical theories. In 1903 the 'Laws of Ancestral Heredity'[109] were developed mainly on the lines of Galton's earlier work on the subject. In 1904 in an attempt at developing a generalized theory of pure gametic inheritance it was clearly recognized "that the biometric or statistical theory of heredity does not involve a denial of any physiological theory but it serves in itself to confirm or refute such a theory."[110] In another paper on the determinantal theory of inheritance based on Weldon's ideas,[111] an attempt was made in 1908 to cover both the Mendelian and the biometric views in one unified theory of inheritance. It was suggested in this paper that "the Mendelians were merely working at one end of the scale, and the biometricians somewhat further down." There were occasional notes[112] on the subject, but a complete theory was worked out in detail in 1930 only a few years before his death in a long paper on 'Progressive Evolution'[113] in the *Annals of Eugenics* in which he sought to show that all organisms are in a constant state of evolution, and it is only inter-breeding and the selective action of environment which preserve a type. That is, natural selection controls evolution, but the progressive urge is provided by heredity itself.

INHERITANCE IN MAN.

Concrete biometric studies on man were started quite early. A very comprehensive paper on 'Inheritance in Man'[114] was published in two parts in the *Biometrika* in 1903 and 1904 in which it was shown that the correlation between parents and offspring was about +0.5 for a large number of both physical and mental characters.

Monographs on the inheritance of diathesis[115] and marital infection[116] in pulmonary tuberculosis were published in 1907 and 1908, and a study of the inheritance of vision[117] in 1909, in all of which heredity was found to be the more important factor. The systematic compilation of detailed pedigrees had been seriously taken up a little earlier in the form of a 'Treasury of Human Inheritance'[118] for "the publication of family histories—whether they concern physical abnormality, ability or achievement". This involved heavy editorial work by Pearson, and the fruits of his labours can be seen in the fifteen parts (in two completed and one incomplete volumes) of the book published during the next two decades. The monograph on 'Albinism in Man',[119] work on which had started in 1903, was published in three parts in 1911-1913, and contain an elaborate study of albionotic skin, eye, hair, and the geographical distribution of albinism in man, as well as studies of albinism in lower animals. Short papers on the inheritance of special defects were also published from time to time.[120]

EUGENIC STUDIES.

With the re-organization of the Eugenics Laboratory under the supervision of Karl Pearson in 1907, work was started on a number of problems of more direct eugenic interest. Two studies on Alcoholism[121] were published in 1910 in which "no marked relation [was] found between the intelligence, physique, or disease of the offspring and parental alcoholism." In a third study[122] in 1911, it was shown however that there was a close relation between mental defectiveness in children and extreme alcoholism in adults. The explanation suggested was that the mentally defective tended to become extreme alcoholists, and the remedy proposed was the segregation of mentally defectives (but not of alcoholics, as alcoholism was often due to other causes).

The question of differential fertility had been receiving attention at the same time.[123] In a co-operative study[124] published in 1913 it was shown that in England fertility had ceased to be correlated with social value since 1878, and that since then the size of the family varied inversely with wages. It was pointed out that this constituted "the greatest political problem of the near future", for "if fertility is correlated with any anti-social hereditary character, then a population will fairly rapidly degenerate."

The hereditary character of general health[125] was considered in 1913, and the evidence in favour of the view that natural selection is still weeding out the weaker individuals among human infants was discussed in some detail a little later.[126] In 1914 we have a very elaborate study of oral temperatures in school children[127] in which the individuality in temperature was clearly brought out, and its relation to various hereditary, environmental, and social factors were studied in great detail. Eight years later was published another elaborate study of data from a baby-clinic[128] in which also the relative effect of nature and nurture on baby's health was investigated very fully. The inheritance of psychical characters was considered in a note in 1919,[129] in which the correlation between siblings was shown to be just over +0.5. A little later in a study of the health of school children[130] published in 1923 it was shown that there was practically no connexion between health and psychical characters, and that general health and general intelligence of children remained practically unchanged throughout the school career. It was also pointed out that external anthropometric characters were "of the smallest importance from the standpoint of health," and the need of studying the dynamical aspects of the organism, that is the functioning of organs, was emphasized. The last great eugenic study was on alien immigration in Great Britain based on the detailed biometric studies of Jewish children of Russian and Polish extraction which was published in several parts in the *Annals of Eugenics* between 1925 and 1928.[131] The inheritance of mental disease was considered in a lecture in 1931.[132]

Medical Biometry and Public Health.

There is a group of papers on the application of the statistical method in medicine and public health on a wide variety of topics such as the opsonic index,[133] multiple cases of disease in the same house,[134] separation of different strains of Trypanosomes,[135] the existence of differential death-rates,[136] influence of isolation on the diphtheria attack and death-rate,[137] the fall in the phthisis death-rate,[138] study of dietaries,[139] and experiments on the influence of drinking milk on the growth of children.[140]

Popular Writings on Biometry and Eugenics.

Karl Pearson had started quite early writing popular lectures and addresses on statistical topics. A number of essays, written between 1894 and 1897 were collected in *The Chances of Death and other Studies in Evolution*,[141] which included the lecture on 'The Chances of Death' at various ages (I, 1-41); 'Reproductive Selection' (I, 63-102) in which the fertility curve for man is discussed in detail; and the first scientific and comprehensive study of 'Variation in Man and Woman' (I, 256-377) in which the coefficient of variation was used effectively for a large number of anthropometric characters.

In eugenics also Karl Pearson did not restrict his activities to scientific researches, but worked hard to spread eugenic ideas through the medium of popular lectures and tracts. Soon after he had taken charge of the reconstructed Eugenic Record office he chose the scope and importance of national eugenics as the subject for the Boyle Lecture at Oxford on May 17, 1907.[142] The exposition was non-technical, but it was emphasized that the problems "must be studied, not by verbal arguments, but be dissected under the statistical microscope, if we are to realize why nations rise and fall" (p. 15). In 1909, 1910 and 1911 he gave a number of lectures on the ground work,[143] practical problems[144] and academic aspects of eugenics,[145] and stressed the importance of heredity, the fact that nature was far more important than nurture.[146, 147] The memoirs on parental alcoholism already mentioned led to a fierce controversy in 1910-11 with both medical men and economists.[148, 149]

In the Cavendish lecture for 1912 given before the medical profession the subject selected was "Darwinism, Medical Progress, and Eugenics,[150] and an earnest appeal was made to the profession for its co-operation in eugenic work. Lectures on tuberculosis and public health,[151, 152] of course from the point of view of heredity and eugenics, were given about the same time. Subjects for other popular lectures were side-lights on the evolution of man,[153] the biological disadvantage of being the eldest child[154] and the right of the unborn child.[155] Another group of lectures dealt with the function of science in dealing with social problems,[156, 157, 158] and eugenics and public health.[159] The highly controversial subject of the segregation of the mentally defective was discussed in two pamphlets[160, 161] in 1914 in which it was shown that there was no evidence of a simple Mendelian segregation and that feeble-mindedness and intelligence varied continuously. The War interrupted the series of popular lectures and addresses which practically ceased in 1914, except for the appreciations of Francis Galton[92] and Charles Darwin,[93] and the presidential address to the anthropological section of the British Association in 1920.[120]

Anthropometry.

Karl Pearson looked upon anthropometry primarily as a foundation for eugenics, and many of his anthropological papers are tinged with eugenic colouring. The question of variability within and between living races was studied in 1903[162] on the basis of H. Risley's anthropometric measurements of the tribes and castes of north India. In another early study in 1906 it was shown that there was practically no relation between intelligence and the size and shape of head or other physical characters.[163] The question was considered again in greater detail about 20 years later in a paper on 'Mind and Body',[164] and it was concluded that there was practically no relationship between physical and mental characters. The stability of the cephalic index within the race[165] was considered in 1924, the dextrality and sinistrality of hand and eye in 1927,[166] and the correlation between various characters and their variation with age was discussed in an elaborate memoir in 1933.[167]

Karl Pearson always emphasized the need of standardization in anthropometric measurements, and insisted upon the use of adequate statistical methods for their analysis. He made many notable contributions for improving the technique of physical anthropometry. He devised an instrument for measuring the height of the crown of the head above the ear-hole on the living, and took the initiative in standardizing the measurements on the skull. In 1911 he described a method of preparing cranial type contours[168] for comparing long series of skull measurements. Discussion of the definition of alveolar point,[169] and the use of simometers[170] may also be mentioned here. Following a suggestion made by Galton in 1907, Pearson developed the method of type-silhouettes[171] which has proved fruitful in practice. In 1933 he suggested the use of three-dimensional analysis for the study of skulls[172] and said : "I unhesitatingly believe that there is a most promising field for the cranio-metricians who are the first to apply Cartesian geometry to the skull."

The reconstruction of cranial capacities,[173, 174] and of cranial measurements,[175, 176] and the use of the statistical technique for sexing osteometric data,[177] or for deciding the question of artificial deformation of heads[178] were considered in a number of papers.

On the analytic side a most important contribution was the formulation of the Coefficient of Racial Likeness which was first used in 1921[179] for measuring the degrees of resemblance in physical appearance between different groups or races of men. The usefulness of this coefficient and the question of standardizing its use were considered in a number of later papers.[180-182] He had noted however in 1926 that the fundamental weakness of the Coefficient of Racial Likeness lies in the fact that it neglects the correlations between the characters dealt with.[180]

Very careful work was also done in a detailed comparison of particular anatomical structures in man and other animals. In 1913 was published a comparative study of the nasal bridge in anthropoid apes and in man.[183] This was followed by the great monograph on long bones in 1919[184] in which the course of human evolution and the relation of recent man to his palæolithic ancestors and to anthropoid apes was worked out in considerable detail, and reasons were given for believing that in the proto-human stage there was more of Chimpanzee (rather than of Gibbon) features. In 1921 a long memoir was published on the sesamoids of the knee-joint in man and its evolution.[185] The morphometric characters of individual bones of the human skull were studied in another paper in 1935[186] shortly before his death. Finally there is a group of papers, chiefly of historical interest, on the skulls and portaits of George Buchanan,[187] Lord Darnley[188] and Oliver Cromwell[189] in which great ingenuity was shown in the comparative study of portraits in relation to skull measurements.

The presidential address to the anthropological section of the British Association in 1920[190] gives Karl Pearson's considered views on what should be the aims and objects of the science of man.

REFERENCES

1. I. On the Dissection of Asymmetrical Frequency Curves. *Phil. Trans.*, A 185 (1894), 71-110.
2. II. Skew Variation in Homogeneous Material. *Phil. Trans.*, A 186 (1895), 343-414.
3. X. Supplement to a Memoir on Skew Variation. *Phil. Trans.*, A 197 (1901), 443-459.
4. XIX. Second Supplement to a Memoir on Skew Variation. *Phil. Trans.*, A 216 (1916), 429-457.

5-6. On the Systematic Fitting of Curves to Observations and Measurements. Part I, *Biom.* I (1901), 265-303; Part II, *Biom.* II (1902), 1-23.

7. On Certain Types of Compound Frequency Distributions in which the Components can be individually described by Binomial Series. *Biom.* XI (1915), 139-144.
8. On Corrections for Moment-Coefficients of Limited Range Frequency Distributions. *Biom.* XII (1919), 231-258.
9. On Certain Properties of the Hyper-Geometrical Series and on the fitting of such Series to observation Polygons in the Theory of Chance. *Phil. Mag.* 49 (1899), 236-246.
10. On the Moments of the Hyper-geometrical Series. *Biom.* XVI (1924), 157-162.
11. On a Certain Double Hyper-geometrical Series and its Representation by Continuous Frequency Surfaces. *Biom.* XVI (1924), 172-188.
12. Notes on Skew Frequency Surfaces. *Biom.* XV (1923), 222-230.
13. On Non-Skew Frequency Surfaces. *Biom.* XV (1923), 231-244.
14. The Fifteen Constant Bivariate Frequency Surface. *Biom.* XVII (1925), 268-313.
15. On the Welch-Kolodziejczyk Frequency Surface. *Biom.* XXVII (1935), 228-259.
16. III. Regression, Heredity and Panmixia. *Phil. Trans.*, A 187 (1896), 253-318.
17. VII. On the Correlation of Characters not Quantitatively Measurable. *Phil. Trans.*, 195 (1901), 1-47.
18. VIII. On the Inheritance of Characters not capable of Exact Quantitative Measurement. *Phil. Trans.*, 195 (1901), 79-150.
19. XIII. On Contingency and its Relation to Association and Normal Correlation. *Biometric Memoirs*, I (1904).
20. XIV. On the General Theory of Skew Correlation and Non-Linear Regression. *Biometric Memoirs*, II (1905).

21. On a General Method of Determining Successive Terms in a Skew Regression Line. *Biom.* XIII (1921), 296-300.
22. XVI. On Further Methods of Measuring Correlation. *Biometric Memoirs*, IV (1907).
23. On a New Method of Determining Correlation between a measured Character A and a Character B, of which only the Percentage of cases etc. *Biom.* VII (1909), 96-105.
24. On a New Method of Determining Correlation, when the Variable is given by Alternative and the other by Multiple Categories. *Biom.* VII (1910), 248-257.
25. On an Extension of the Method of Correlation by Grades or Ranks. *Biom.* X (1914-15), 416-418. Also "Second Note" in *Biom.* XIII (1920-21), 302-305.
26. On Polychoric Coefficients of Correlation. *Biom.* XIV (1922), 127-156.
27. On the Measurement of the Influence of Broad Categories on Correlation. *Biom.* IX (1913), 116-139.
28. On a Correction to be made to the Correlation Ratio η. *Biom.* VII (1911), 254-256. Also *Biom.* XIV (1923), 412-417.
29. On the Correction for Broad Categories. *Biom.* XXVII (1935), 364-372.
30. On Theories of Association. *Biom.* IX (1913), 159-315.
31. On a Form of Spurious Correlation which may arise when Indices are used in the Measurement of Organs. *Proc. Roy. Soc.* LX (1897), 489-498.
32. On the Constant of Index-Distributions as deduced from the like constants for the Components of the Ratio with special reference to the Opsonic Index. *Biom.* VII (1910), 531-541.
33. On the Influence of Double Selection on the Variation and Correlation of two Characters. *Biom.* VI (1908), 111-112.
34. On the General Theory of the Influence of Selection on Correlation and Variation. *Biom.* VIII (1912), 437-445.
35. On Certain Errors with regard to Multiple Correlation. *Biom.* X (1914), 181-187.
36. On sòme Novel Properties of Partial and Multiple Correlation Coefficients in a Universe of Manifold Characteristics. *Biom.* XI (1916), 231-238.
37. On First Power Methods of finding Correlation. *Biom.* XVII (1925), 459-469.
38. IV. On the Probable Errors of Frequency Constants and on the Influence of Randon Selection on Variation and Correlation. *Phil. Trans.*, A 191 (1898), 229-311.
39. On the Probable Errors of Frequency Constants. Introductory Part I, *Biom.* II (1903), 273-381; Part II, *Biom.* IX (1913), 1-10; Part III, *Biom.* XIII (1921), 113-132.
40. On the Generalised Probable Error in Multiple Normal Correlation. *Biom.* VI (1908), 59-68.
41. On the Product Moments of Various Orders of the Normal Correlation Surface of two Variates. *Biom.* XII (1918), 86-92.
42. On the Probable Error of a Coefficient of Correlation as found from a Fourfold Table. *Biom.* IX (1913), 22-27.
43. On the Probable Error of Biserial η. *Biom.* XI (1917), 292-302.
44. On the Probable Error of a Mean-Square Contingency. *Biom.* V (1906), 191-197.
45. On the Probable Error of a Coefficient of Mean-Square Contingency. *Biom.* X (1915), 570-573.
46. On the Probable Error of à Coefficient of Contingency without Approximation. *Biom.* XI (1916), 215-239; with corrections in *"Peccavimus"*. *Biom.* XII (1919), 259-281.
47. The Probable Error of a Mendelian Class Frequency. *Biom.* XI (1917), 429-432.
48. On the Distribution of the Standard Deviations of Small Samples: Appendix I to Papers by "Student" and R. A. Fisher. *Biom.* X (1915), 522-529.
49. On the Distribution of the Correlation Coefficient in Small Samples: Appendix II to the Papers of "Student" and R. A. Fisher. A Co-operative Study. *Biom.* XI (1917), 328-413.
50. Further Contributions to the Theory of Small Samples. *Biom.* XVII (1925), 176-199, with correction note in *Biom.* XIX (1927), 441-442.
51. On the Distribution of the First Product Moment-Coefficient in Samples drawn from an indefinitely large Normal Population. *Biom.* XXI (1929), 164-193.
52. Further Applications in Statistics of the $T_m(x)$ Bessel Function. *Biom.* XXIV (1932), 293-350.
53. On the Application of the Double Bessel Function to Statistical Problems .*Biom.* XXV (1933), 158-178.
54. On the Mean Character and Variance of a Ranked Individual and on the Mean and Variance of the Intervals between Ranked Individuals, Part I, *Biom.* XXIII (1931), 364-397; Part II, Case of certain Skew Curves. *Biom.* XXIV (1932), 203-279.
55. Tables of the Probability Integrals of Symmetrical Frequency Curves in the case of Low Powers such as arise in the Theory of Small Samples. *Biom.* XXII (1931), 253-283.
56. On the Nature of the Relationship between two of "Students" Variates (z_1 and z_2) when Samples are taken from a Bivariate Normal Population. *Biom.* XXII (1931), 405-422.
57. Some Properties of "Student's" z: Correlation, Regression and Scedasticity of z with the Mean and Standard Deviation of the Sample. *Biom.* XXIII (1931), 1-9.
58. Further Remarks on the z-test. *Biom.* XXIII (1931), 408-415.
59. On the Criterion that a given System of Deviations from the Probable in the Case of a

Correlated System of Variables is such that it can be reasonably supposed to have arisen from Random Sampling. *Phil. Mag.*, 50 (1900), 157-175.

60. On the Probability that two Independent Distributions of Frequency are really Samples from the same Population. *Biom.* VIII (1911), 250-254.

61. On the General Theory of Multiple Contingency with Special Reference to Partial Contingency. *Biom.* XI (1916), 145-158.

62. On the Application of "Goodness of Fit" Tables to test Regression Curves and Theoretical Curves used to describe Observational or Experimental Data. *Biom.* XI (1916), 239-261.

63. On the χ^2 test of Goodness of Fit. *Biom.* XIV (1922), 186-191.

64. Further Note on the χ^2 test of Goodness of Fit. *Biom.* XIV (1923), 418.

65. Note on the relation of (P, χ^2) Goodness of Fit Test to the Distribution of Standard Deviations in samples from a Normal Population. *Biom.* XIX (1927), 215.

66. Experimental Discussion of the (χ^2, P) Test for Goodness of Fit. *Biom.* XXIV (1932), 351-381.

67. On the Probability that two Independent Distributions of Frequency are really Samples from the same Parent Population. *Biom.* XXIV (1932), 457-470.

68. On the Parent Population with Independent Variates which gives the Minimum Value of ϕ^2 for a Given Sample. *Biom.* XXV (1933), 134-146.

69. On a Method of determining whether a sample of size *n* supposed ot have been drawn from a Parent Puoplation having a known Probability Integral has probably been drawn at random. *Biom.* XXV (1933), 379-410.

70. On a New Method of Determining Goodness of Fit. *Biom.* XXVI (1934), 425-442.

71. On the Influence of Past Experience on Future Expectation. *Phil. Mag.* (1907), 365-378.

72. The Fundamental Problem of Practical Statistics. *Biom.* XIII (1921), 1-16. Also "Addendum" to above. *Biom.* XIII (1920), 300-301.

73. On a Method of Ascertaining Limits to the Actual Number of Marked Members in a Population of given size from a Sample. *Biom.* XXA (1928), 149-174.

74. On generalised Tchebycheff Theorems in Mathematical Statistics. *Biom.* XII (1919), 284-296.

75. On the Distribution of Frequency (Variation and Correlation) of the Barometric Height at Divers Stations. *Phil. Trans.*, 190 (1897), 423-469.

76. On Francis Galton's Problem. (The Most suitable Proportion between the Values of First and Second Prizes. *Biom.* I (1902), 390-399.

77. On the Multiple Correlation of Brothers etc. *Biom.* XVII (1925), 129-141.

78. On the Mathematical Theory of Errors of Judgment with Special Reference to the Personal Equation. *Phil. Trans.*, 198 (1902), 235-292.

79. On the Mathematical Theory of Random Migration. *Biom. Memoirs*, III (1906).

80. Numerical Illustrations of the Variate Difference Correlation Method. *Biom.* X (1914), 340-355.

81. The Sampling Errors in the Theory of a Generalized Factor. *Biom.* XIX (1927), 246-291.

82. Biometry and Chronology. *Biom.* XXA (1928), 241-262.

83. Notes on the History of Correlation. *Biom.* XIII (1920), 25-45.

84. The Contribution of Giovanni Plana to the Normal Bivariate Frequency Surface. *Biom.* XXA (1928), 295-298.

85. Historical Note on the Normal Curve of Errors. *Biom.* XVI (1924), 402-404.

86. James Bernoulli's Theorem. *Biom.* XVII (1925), 201-210.

87. Note on a Solution due to Laplace. *Biom.* XXA (1928), 166-174.

88. Laplace. *Biom.* XXI (1929), 202-217.

89. Historical Note on the Distribution of the Standard Deviation of Samples of any size drawn from an indefinitely large Normal Parent Population. *Biom.* XXIII (1931), 416-418.

90. Walter Frank Raphael Weldon, (1860-1906). *Biom.* V (1906), 1-52.

91. *The Life, Labours and Letters of Francis Galton*, Vols. I, II, IIIA and IIB (*Camb. Univ. Press*, 1914-1930).

92. *Francis Galton, A Centenary Appreciation.*

93. *Charles Darwin, 1809-1882.*

94. Tables for Statisticians and Biometricians. (*Biometric Laboratory*). Part I. 1914, 1924, 1930 (Pp. lxxxiii+143). Part II. 1931 (Pp. ccl+262).

95. Tables of the Complete and Incomplete Beta-Function. (*Cambridge Univ. Press*), 1934 (Pp. lix 494).

96. Tables of the Incomplete Γ-Function (*H.M.S.O.*) 1922 (Pp. xxxi+164).

97. Tables of the Complete and Incomplete Elliptic Integrals. (*Biometric Laboratory*) 1934. (Pp. xliii+93).

98. Tracts for Computers. (*Cambridge Univ. Press*). Nos. I-XX (1919-1934).

99. Logarithmetica Britannica (Tracts for Computers Nos. XI, XIV, XVI-XIX). 1924-1934.

100. On the Numerical Evaluation of High Order Incomplete Eulerian Integrals. *Biom.* XXVII (1935), 409-423.

101. On the Principle of Homotyposis and its relation to Heredity, to the Variability of the Individual, and to that of Race. Part I Homotyposis in the Vegetable Kingdom, *Phil. Trans.*, 197 (1902), 285-379.

102. On the Fundamental Conceptions of Biology. *Biom.* I (1901), 320-344.
103. On Homotyposis in Homologous but Differentiated Organs, *Proc. Roy. Soc.*, 71 (1903), 288-313.
104. On Inheritance in the Shirley Poppy. *Biom.* II (1902), 56-100.
105. Variation and Correlation in Lesser Celandine from Divers Localities. *Biom.* II (1903), 145-164.
106. Co-operative Study of *Vespa Vulgaris*. *Biom.* VII (1909), 48-63; and *Biom.* VIII (1911), 1-12.
107. A Biometric Study of the Blood Corpuscles of the Common Tadpole. *Biom.* VI (1909), 402-419.
108. On the Relation of the Duration of Pregnancy to Size of Litter and other Characters in Bitches. *Biom.* XXII (1931), 309-323.
109. The Laws of Ancestral Heredity. *Biom.* II (1903), 211-236.
110. On the Generalized Theory of Alternative Inheritance with Special Reference to Mendel's Law. *Phil. Trans.*, 203 (1904), 53-86.
111. On a Mathematical Theory of Determinantal Inheritance from Suggestions and Notes of the Late W. F. R. Weldon. *Biom.* VI (1908), 80-93.
112. Further Remarks on the Law of Ancestral Heredity. *Biom.* VIII (1911), 239-243.
113. On a New Theory of Progressive Evolution. *Annals of Eugenics* IV (1930), 1-40.
114. On the Laws of Inheritance in Man: Part I Physical Characters. *Biom.* II (1903), 357-462.
Part II Mental and Moral Characters. *Biom.* III (1904), 131-190.
115. A First Study of the Statistics of Pulmonary Tuberculosis (Inheritance). (1907).
116. A Second Study of the Statistics of Pulmonary Tuberculosis. Marital Infection (1908).
117. A First Study of the Inheritance of Vision and of the Relative Influence of Heredity and Environment on Sight. (1909).
118. *The Treasure of Human Inheritance*: Vol. I, Parts I and II—Diabetes Insipidus; Split-Foot, Polydactylism, Brachydactylism, Tuberculosis, Deaf-Mutism. Part III—Angioneurotic Oedema, Hermaphroditism, Deaf-Mutism, Insanity. Part IV—Cleft Palate, Hare-Lip, Deaf-Mutism. Part V, VI—Hæmophilia. Part VII--Dwarfism. Part VIII—Prefatory Matter, and Indices.
Vol. II, Part I--Retinitis pigmentosa. Part II -Colour Blindness. Part III—Blue Sclerotics and Fragility of Bone. Part IV—Hereditary Optic Atrophy. Part V—Structural Anomalies of the Eye and Inheritance of Glaucoma. Part VI—Index.
Vol. III, Part I—Hereditary Disorderas of Bone Development.
119. *Albinism in Man.* Part I (1911); Parts II and IV (1913).
120. Split-foot or Lobster-claw. *Biom.* VI (1908), 69-79.
Internal Albinism. *Biom.* VII (1910), 244-247. Muscular Dystrophy. *Annals of Eugenics* V (1933), 179-191.
121. *The Inflence of Parental Alcoholism on the Physique and Intelligence of the Offspring.* 1910. *A Second Study.* 1910.
122. *A Preliminary Study of Extreme Alcoholism in Adults.* 1911.
123. On the Effect of a Differential Fertility on Degeneracy. *Biom.* VII (1909-10), 258-275.
124. *On the Correlation of Fertility with Social Value.* 1913.
125. On the Hereditary Character of General Health. *Biom.* IX (1913), 320-329.
126. Further Evidence of Natural Selection in Man. *Biom.* X (1915), 488-506.
127. *A Statistical Study of Oral Temperatures in School Children with special reference to Parental, Environmental and Class Differences.* 1914.
128. *Study of Data provided by a Baby-Clinic in a large Manufacturing Town.* 1922.
129. Inheritance of Psychical Characters. *Biom.* XII (1919), 367-372.
130. On the Relationship of Health to the Psychical and Physical Characters in School Children. 1923.
131. The Problem of Alien Immigration into Great Britain. *Annals of Eugenics* I (1925), 5-127; II (1927), 111-244, 290-317; III (1928), 1-76, 201-262.
132. Inheritance of Mental Disease. *Annals of Eugenics* IV (1931), 362-380.
133. Opsonic Index: Mathematical Error and Functional Error. *Biom.* VIII (1911), 203-256.
134. On the Appearance of Multiple Cases of Disease in the same House. *Biom.* VIII (1912), 404-412.
135. On the Probability that two Independent Distributions of Frequency are really Samples of the same Population, with special Reference to Recent Work on the Identity of Trypanosome Strains. *Biom.* X (1914), 85-143.
136. On Criteria for the existence of Differential Death-rates. *Biom.* XI (1916), 159-184.
137. The Influence of Isolation on the Diptheria Attack- and Death-rates. *Biom.* X (1915), 549-569.
138. The Check to the Fall in Phthisis Death-rate since the Discovery of the Tubercle Bacillus and the Adoption of Modern Treatment. *Biom.* XII (1919), 374-376.
139. Statistical study of Dietaries. *Biom.* X (1914), 172-174.
140. Lanarkshire Milk Experiment. *Annals of Eugenics* V (1933), 337-338.
141. *The Chances of Death and other Studies in Evolution.* Vols. I and II (1897).
142. *The Scope and Importance to the State of the Science of National Eugenics.* 1907; and 1909.
143. *The Groundwork of Eugenics.* 1909; and 1912.
144. *The Problem of Practical Eugenics.* 1912.
145. *The Academic Aspects of the Science of National Eugenics.* 1911.