Essays on the Dynamics of Credit Contracts

Dyotona Dasgupta

September, 2017

Thesis submitted to the Indian Statistical Institute in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy

Thesis Advisor: Prabal Roy Chowdhury

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7 SJS Sansanwal Marg, New Delhi, India.

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Contents

Chapter 1

Introduction[1]

This thesis studies the applications of contract theory in credit market along two dimensions: the first part provides two explanations of one of the most widely used dynamic incentive schemes: "progressive lending" with and without the possibility of 'graduating' to a high productive activity and the second part examines the effects of competition among Microfinance Institutions (MFIs hereafter) on informal moneylenders and borrowers.

All the three chapters address the problem of *ex post* moral hazard in that the borrowers are strategic and do not repay whenever they have incentives to do so, in a context where repayment rate is very high. The following table shows high repayment rate in Microfinance industry:

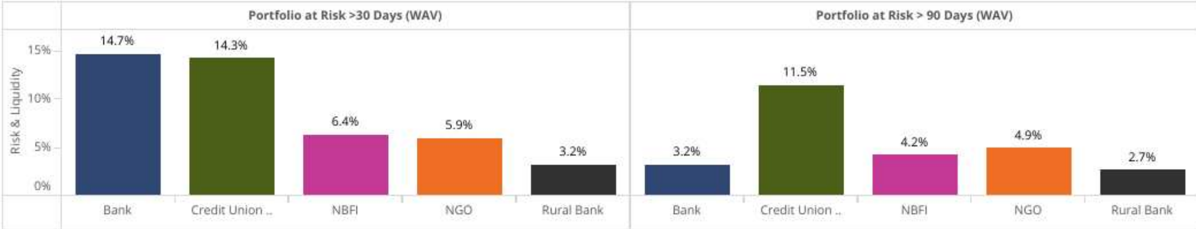
Regions	Percentage of Total Borrowers	Percentage of Gross Loan Portfolio (GLP)	Portfolio at Risk> 30 Days (PAR)
Africa	5%	9%	10.60%
East Asia and the Pacific (EAP)	1%	16%	3.40%
Eastern Europe and Central Asia (ECA)	3%	11%	10.00%
Latin America and the Carribean (LAC)	19%	42%	5.40%
Middle East and North Africa (MENA)	2%	1%	3.60%
South Asia	57%	20%	2.60%

"Portfolio at Risk (PAR)": is one of the indicators of repayment rate. Portfolio at Risk [xx] days is defined as the value of all loans outstanding that have one or more installments of principal past due more than [xx] days. This includes the entire unpaid principal balance, including both the past due and future installments, but not accrued interest. It also includes loans that have been restructured or rescheduled.

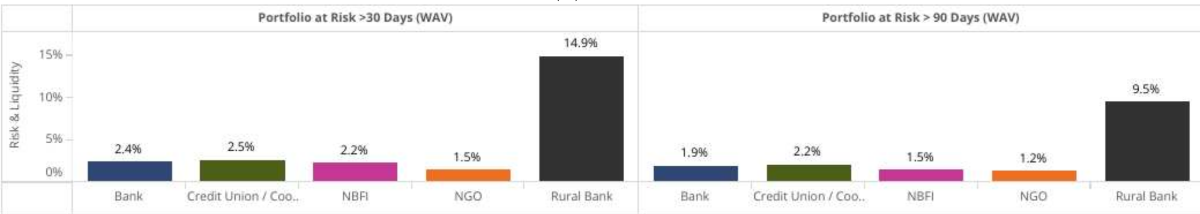
"Gross Loan Portfolio (GLP)": All outstanding principals due for all outstanding client loans. This includes current, delinquent, and renegotiated loans, but not loans that have been written off.

Source MIX (2017): Global Outreach and Financial Performance Benchmark Report 2015.

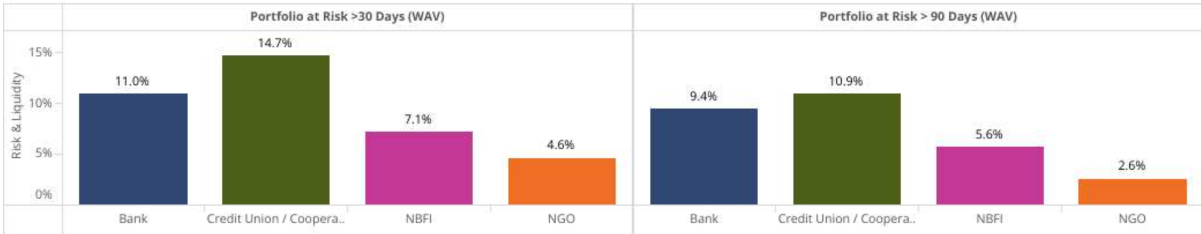
[1]The first chapter of my thesis is a part of a bigger project on which I am working with Dilip Mookherjee (Boston University). This project was started when I was visiting Boston University, I thank the World Bank for the fellowship and the Institute for Economic Development (IED), Boston University for hosting me. The second and third chapters are joint works with my thesis supervisor Prabal Roy Chowdhury.



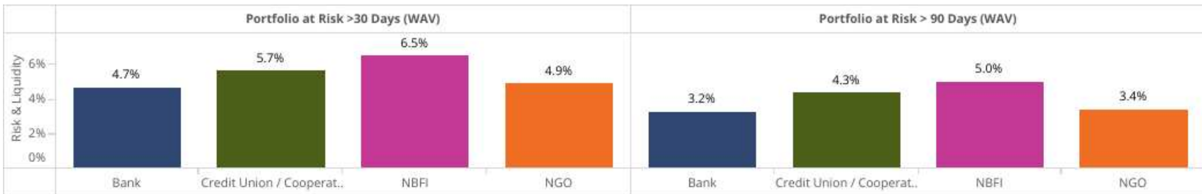
(a) Africa



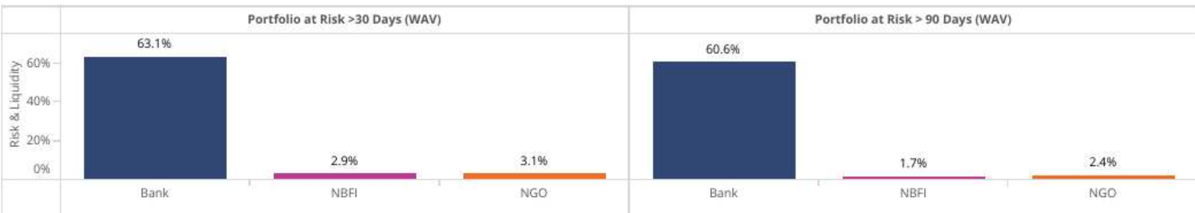
(b) East Asia and the Pacific (EAP)



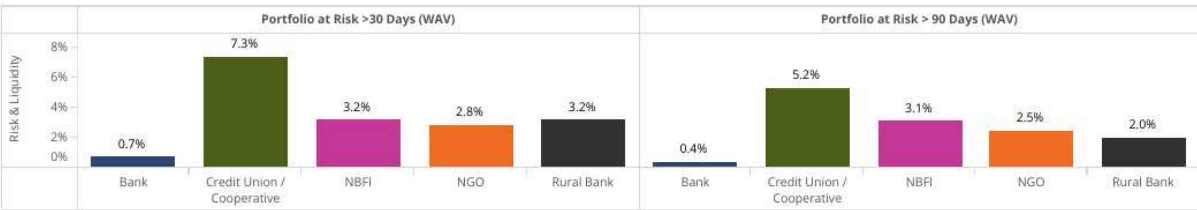
(c) Eastern Europe and Central Asia (ECA)



(d) Latin America and the Carribean (LAC)



(e) Middle East and North Africa (MENA)



(f) South Asia

Figure 1.1: Portfolio at Risk (PAR) Source MIX (2017)

All the three chapters of this thesis try to explain this incidence of high repayment where legal enforcement of contracts is not possible, so repayment solely depends on dynamic incentives. The objective of this thesis is to study the dynamics of credit contracts, in three different frameworks, which ensure that the strategic borrowers always repay.

1.1 Part I

As documented in Morduch (1999), Armendàriz and Morduch (2000, 2005) among others dynamic incentives especially that in the form of progressive lending: increasing loan size over time contingent on successful repayment is one of the most widely used instruments, by the MFIs, to ensure repayment. The following table shows the same:

Country	Number of Active Borrowers '000	Gross Loan Portfolio (GLP)(USD)m	Financial Service Provider (FSP)	Number of Active Borrowers '000	Gross Loan Portfolio (GLP)(USD)m	Progressive Lending?
India	38,097.6	11,640.8	Bandhan	-	2,352.66	Yes
			Janalakshmi	5,888.75	1,973.48	Yes
			Bharat Financial (SKS)	5,323.06	1,413.30	Yes
			Share	3,740.00	251.68	Yes
			SKDRDP	3,612.43	754.60	
Bangladesh	23,977.7	5,753.7	Grameen Bank	7,180.00	1,294.65	Yes
			ASA	6,207.69	1,533.97	Yes
			BRAC	5,356.52	1,768.61	
			BURO Bangladesh	917.46	311.61	Yes
			TMSS	736.98	188.95	Yes
Vietnam	7,533.9	7,351.9	VBSP	6,863.04	6,434.69	Yes
			CEP	288.49	108.28	Yes
			Coopbank	111.93	726.19	
			TYM	97.42	45.45	Yes
			MOM	40.17	7.26	

Top three countries and top five MFIs from each of them by the number of active borrowers, except Bandhan as number of active borrowers is not available in MIX Market data, however it is well known that this the largest MFI in India (Gross Loan Portfolio is the maximum).
Source MIX (2017): Global Outreach and Financial Performance Benchmark Report 2015 and respective websites.

Progressive lending can potentially incentivise a borrower to repay, as repayment of a small amount enables her to get a higher amount of loan subsequently. As a result, it allows the borrowers to increase the scale of production and consumption over time which conforms to the empirical findings. Banerjee et al. (2015a) find in their six country study - "each study finds at least some evidence, on some margin, that expanded access to credit increases business activity." However Bulow and Rogoff (1989b) and more generally Rosenthal (1991) have shown that only increasing loan amount is not sufficient to incentivise a strategic borrower to repay. A strategic borrower would default at the time when present discounted value of future debt is the maximum. As documented above, however default on MFI loan is rare.

The objective of this part is to provide theoretical explanations of two empirical observations: very high repayment rate and progressive lending, in the following two contexts

1. Infinite horizon, discrete time framework where an agent has strong preference for consumption smoothing (captured through concave utility function - a natural assumption in the context of MFI borrowers a large majority of which are poor) and also has access to a technology which enables her to transfer wealth from one period to another, even at autarky.

2. Infinite horizon, continuous time framework where a borrower graduates to a more productive sector/activity at an endogenous finite date. We assume that a poor agent (wealth is below a certain threshold) does not have access to any technology to transfer wealth from one period to another or in other words capital depriciates completely within a period. This is a plausible assumption since poor people have very limited access to any kind of savings technology (see for example Collins et al. (2009), Rutherford (2009) or Dupas and Robinson (2013a)).

In both the contexts we assume that the lender provides access to superior technologies (or marketing opportunities) which generate higher rates of returns and show that in such contexts welfare (and wealth) improving contracts do exist, and characterize the dynamics of optimal contracts.[2][3]

1.1.1 Chapter 2: Dynamics of 'Bundled' Aid-Debt Contracts: Progressive Lending

In this essay, in an infinite horizon discrete time framework, we consider an agent who has a strong preference for consumption smoothing (concave utility function) and has access to a neoclassical production technology along with savings technology. Her objective is to maximize present discounted value of lifetime utility and her future discount factor is assumed to be equal to the inverse of the rate of interest on savings. So, this is a standard Ramsey problem where the agent maintains her wealth if that is above a certain threshold otherwise her consumption and investment increase over time ultimately converging to the respective thresholds.

Now a lender can costlessly provide access to a neoclassical production technology which is more productive than the autarky technology, along with the savings technology. The lender is assumed to be benevolent and wants to maximize the agent's present discounted value of lifetime utility without making any loss.

Technology access in the absence of any loans would result in growing investment financed by own savings, ultimately converging to the efficient level (as represented by the standard Ramsey model). Technology assistance by itself would also help raise the borrower's welfare and generate the same long run outcome. Bundling it with credit enables borrower's financing costs to be lowered and thereby generates additional welfare improvements.

We assume that the lender has unlimited access to external finance at a constant cost equal to the borrower's discount rate. In the absence of any moral hazard, efficient investment can be sustained at every date, combined with perfect consumption smoothing. With moral hazard, the first-best allocation cannot be sustained if the borrower's initial wealth falls below a threshold.

We characterize the optimal contracts in this case. We show that the dynamics are qualitatively similar to the case where investments are self-financed: there is under-investment, but investments grow and eventually converge to the efficient level. In parallel, the agent's net

[2]See Thomas and Worrall (1994) for a similar approach. Examples of such bundling abound — e.g. in sovereign debt where default is punished by termination of access to lender markets and technology, or MFIs that provide borrowers access to technology and marketing channels conditional on not defaulting.

[3]A paper in similar spirit but in a different context is by Anderson et al. (2009). They show that in the absence of external (social) sanctioning ROSCAs are never sustainable, even if the defecting member is excluded from all future roscas.

wealth (value of production less inherited debt) and consumption grow and converge to stationary levels which correspond to the threshold where the moral hazard problem disappears. Loan sizes also grow; hence the optimal policy is characterized by 'progressive lending' policies of the sort commonly observed in lender-borrower relationship.

1.1.2 Chapter 3: Progressive Lending with Graduation

In the next essay, we provide an explanation of progressive lending where there is a possibility of graduation. We assume that when an agent's wealth becomes more than a certain threshold (say $\bar{S}$), her economic opportunity increases, in particular she gets access to a formal sector technology. We assume that it is not possible to bundle technology beyond this threshold.

For example, $\bar{S}$ is the set up cost for a shop or a formal enterprise. This huge set up cost creates a nonconvexity in the technology. An MFI can improve it's client's payoff by providing credit along with access to informal sector technology and savings technology. When an agent starts her own shop which is more productive than the informal sector technology, bundling of access to technology becomes redundant.

Given Bulow and Rogoff (1989b), Rosenthal (1991), this implies providing credit to an agent whose wealth is more than $\bar{S}$ is not incentive compatible. We consider an agent who is poor, in particular her endowment is assumed to be zero. The job of the benevolent MFI is to help the borrower to accumulate $\bar{S}$ so that she can start investing in the formal sector technology. When an agent moves to the formal sector, we say that she has graduated. Suppose present discounted value of lifetime utility from graduation is V (gross of $\bar{S}$).

In the first best scenario, the lender would have given $\bar{S}$ at the very first period so that the borrower can graduate immediately, again due to *ex post* moral hazard problem that is not incentive compatible. So, the lender provides small amounts of loans and access to the low productive informal sector technology and the borrower invests that loan amount in that technology.

At each period, at the time of repayment, the borrower has to save a part of her net return with the lender. It is this savings with the lender which allows the borrower to graduate. On successful repayment of all the loans at a prespecified date, the lender gives back the borrower her entire savings with him. The lender designs the contract in such a way that by that prespecified date, the borrower's savings becomes at least $\bar{S}$ so that after termination of the contract, she can use that savings to graduate. In case the borrower defaults, the lender terminates the contract, withdraws her access to the low productive technology and confiscates her savings. So, in case of default she not only loses her savings with the lender but also the opportunity to graduate using that savings.

As their relationship matures, her savings with the lender increases and the remaining time till the prespecified date at which she graduates decreases, hence her present discounted value from repayment increases. This increases the maximum loan amount which is incentive compatible, so optimum loan size increases over time (more formally, optimum loan size weakly increases over time, it remains constant when the loan reaches the effiecient amount). Hence, the second essay of this thesis shows how savings and the provision of graduation using that savings give rise to progressive lending at optimum.

1.2 Part II

1.2.1 Chapter 4: How Does Competition among MFIs affect Moneylenders and Borrowers?

The objective of this chapter is to understand the effects of MFI competition on MFI and informal moneylender loan size, moneylender interest rate, moneylender's profit and borrowers' welfare. We assume that the MFIs are more benevolent than the moneylender, and take an extreme case where the MFIs are benevolent and the moneylender is profit maximizer. The moneylender has informational advantage, due to thick interaction with the borrowers. We capture this in our model in the following way: the project which is funded by the MFIs loans becomes successful only with probability p, which is strictly less than one. While the moneylender can perfectly observe the return of the project, the MFI cannot.

We address the problem of *ex post* moral hazard in that the borrower is strategic and does not repay whenever she has incetive to do so. We moreover assume that the MFIs must break even *ex post* in every period. So, the MFIs design contracts such that the borrower has to always repay their loans, in case of success on her own and in case of failure by taking bridge-loans from the moneylender. If the borrower does not repay a particular MFI loan, that MFI terminates the credit contract. The MFIs do not share information among themselves, so default on one MFI loan does not affect relationships with the other MFIs.

As it turns out, the incentive of the moneylender who has all the information and provides the bridge-loans in case of failure, plays an important role. Though an MFI does not provide any contract to the moneylender directly but it knows the moneylender's strategic reponses to its contracts, so the MFI has to incorporate those while designing the contract.

We solve for symmetric, stationary equilibria when there is one MFI in a village and also when there are $n(> 1)$ MFIs in a village. Then we compare the optimum outcomes under the two scenarios. We find that the aggregate loan amount received by a particular borrower is higher when there is only one MFI vis-a-vis when there are n MFIs, in fact aggregate loan amount decreases with the number of MFIs in a village. Similarly, moneylender's loan amount decreases with the number of MFIs, however moneylender's interest rate is increasing in the number of MFIs. This is quite intuitive in that as the number of MFIs increases, loss from default on one MFI loan decreases, as there are $n-1$ more MFIs from which the borrower can get loans (and hence the moneylender can provide bridge-loans for those). So, the maximum loan amount which the moneylender allows the borrower to repay decreases with the number of MFIs, since the MFIs know this strategy of the moneylender, they in turn provide lower amount of loan. Finally, we compare the borrower's welfare in both the scenarios and find that the borrower's welfare decreases as the number of MFIs in a particular village increases whereas moneylender's profit increases.

1.3 Plan of the Thesis

The thesis is organized as follows: Next I provide a brief survey of related literature. Then, two essays of Part I providing two explanations or progressive lending are presented in chapter 2 and chapter 3. The third essay studying the effects of competition among MFIs is presented in the second part of the thesis which consists of one chapter - chapter 4. Finally chapter 5 concludes.

1.4 Related Literature

1.4.1 Part I

Recall, Part I provides two explanations of progressive lending in a framework where legal enforcement is not possible. Also all loans are individual liability. In this common framework, we study the dynamics of credit contracts in two different contexts:

> In Chapter 2, the borrower has access to a technology to transfer wealth from one period to another and has a strong preference for consumption (concave utility function),
>
> In Chapter 3, poor borrowers do not have access to any technology on her own and there is a possibility of graduation. We study the dynamics of credit contracts in a continuous time, infinite horizon framework with an endogenous finite end date where the borrower's utility function is assumed to be linear.

This part is related to different strands of literature. Like in sovereign debt models contractual enforcement of repayment is weak and hence default can only be prevented through dynamic incentives and threat of termination of relationships.

One of the very first papers is by Eaton and Gersovitz (1981). In their model the country loses access to future loans in case of default. This threat ensures repayment. The key assumption being the sovereign cannot even save in case of default. Bulow and Rogoff (1989b) show that this threat of losing access to future loan is not enough to ensure repayment, if the country can save at the same interest rate as that on the loan. The country, in that case, can simply save up the amount to be repaid and enjoy higher welfare.

Rosenthal (1991) generalizes this result with a simple one sector growth model where the borrower has access to a deterministic, neoclassical production technology return to which exceeds the rate of return on the world credit market, at least initially and it is assumed that the sovereign's wealth cannot be seized. He shows that when the sovereign's future discount factor is not more than the inverse of the rate of return in the credit market, threat of losing future access to credit market[4] cannot deter the sovereign from defaulting. More precisely, this impossibility result holds irrespective of its endowment when the future discount factor is less than the inverse of the rate of return in the credit market and when the discount factor is exactly equal to that, the impossibility result holds when the agent's endowment is below the return from the steady state investment in the neoclassical production technology. The intuition is very simple - when future discount factor is less than the inverse of the rate of return from savings, at steady state the sovereign invests only in the production technology and does not save, even if it can. So, the threat of losing access to savings cannot deter the sovereign from defaulting. On the other hand, when future discount factor is exactly equal to the inverse of the rate of return from savings, the sovereign saves only when its endowment is higher than the return from the steady state investment in the production technology.

[4]That is it can neither save nor borrow.

In both the essays of Part I, borrower's future discount factor is assumed to be equal to the inverse of the rate of return from savings. We show that bundling access to superior technology with credit, with or without the possibility of graduation, ensures repayment, even when the borrowers are poor.[5]

A paper in similar spirit is by Cole and Kehoe (1997). They show that *reputation spillovers* support debt. They argue that countries are involved in many different types of relationships and assume that when a country breaks trust of one of those relationships that adversely affects other trust relationships. So even when the country does not get directly affected from default may get adversely affected from that spillover effect. In that case the country would repay if the loss in lifetime utility is higher than the short term gain from default. In particular they added a labor relationship to the debt model where country's project return is dependent on workers it hires. He assumes that the workers stop working not only when the country breaks its contracts with them, but even if it does not repay. Due to this assumption they find that when the country's future discount factor is not very low, the country repays to avoid this loss in project return via workers' *spillover strategy*. However, unlike us they do not characterize the optimum contract.

Thomas and Worrall (1994) study self-enforcing credit contracts between a risk neutral host country and a transnational corporation where none of them can commit. They assume that "transnational corporation is providing not only capital but also technology and expertise not otherwise available to the host country". Albuquerque and Hopenhayn (2004) build on their framework with a firm and a bank where the bank can committ but the firm cannot. They allow for a more general outside option than Thomas and Worrall (1994) and do not assume that this investment opportunity is specific of those two parties but allow for strict bankruptcy law in that the lender can liquidate the firm whenever he wants to do so. Optimum investments in both the models increase over time and since the borrower is risk neutral she starts getting any dividends or tranfers only when the investment reaches the steady state amount. In Albuquerque and Hopenhayn (2004) there is an initial setup cost which is funded by a long term debt and short term loans are provided as working capital. So in this case, the borrower starts getting dividends only after the investment reaches efficient amount and also the long term debt has been repaid.

In chapter 2, the borrower's utility function is concave and contrary to Thomas and Worrall (1994) assumption that capital deprciates completely within a period, the borrower has access to a neoclassical production technology which is inferior to the technology provided by the lender along with savings technology. So outside option takes a very different form.

In chapter 3 the borrower is risk neutral and like Albuquerque and Hopenhayn (2004) an initial setup cost is required for a high productive technology to which the borrower has access on her own and liquidation is not possible, so if the borrower at all starts investing in this technology she would never repay. So the benevolent lender provides access to another less productive technology to which the borrower does not have access on her own and which does not require any initial investment. The job of the benevolent lender is to help the borrower to accumulate the initial set up cost.

[5] A paper in similar spirit but in a different context is by Anderson et al. (2009). They show that in the absence of external (social) sanctioning ROSCAs are never sustainable, even if the defecting member is excluded from all future ROSCAs.

In a very general setup Ray (2002) studies the optimum time path of “efficient-self-enforcing” (ESE) aggreements between a principal (lender in our case) and an agent (borrower), where principal can commit and the agent cannot. He shows that irrespective of bargaining power of the individuals, all ESE aggreements takes the same form in the long run: “there exists a finite date such that the agent’s best self-enforcing sequence is followed thereafter”. He considers the question of allocation of surplus between lender and borrower over time, an issue we abstract from by assuming that the surplus goes to the borrower.

Ghosh and Ray (2016) generate increasing loans over time owing to adverse selection reasons where default on early loans reveal the types of borrowers, so by the structure of the framework defaults arise on equilibrium path.

Though dynamic incentives especially that in the form of progressive lending is one of the most widely used institutions to ensure repayment in the microfinance industry (Morduch (1999), Armendàriz and Morduch (2005)), there are very few papers which address this. One of the very first paper to address this is by Armendàriz and Morduch (2000). They consider a simple two period model and show that how repayment incentive of the first period increases as loan amount in the second period increases. However, since this is a two period model, the borrower defaults at the second period with certainty and knowing that the lender has to set interest rate of the first period accordingly. Egli (2004) and Shapiro (2015) both unearth some limitations of the dynamic incentives involved in progressive lending. Egli (2004) develops a two period model with two types of borrowers - honest, and strategic. He shows that progressive lending may be counter-productive, in that strategic borrowers may be tempted to repay in the first period, so as to access larger loans later on. Shapiro (2015) examines a framework with uncertainty over borrower discount rates. He shows that even in the efficient equilibrium almost all borrowers default. Note that both the papers involve default along the equilibrium path. However, in reality microfinance schemes appear to involve little or no default.

Other kinds of dynamic incentives in microfinance, among others, have been studied by Aniket (2014), Roy Chowdhury (2005, 2007) (sequential lending), Jain and Mansuri (2003), Fischer and Ghatak (2016) Chowdhury et al. (2014) (immediate and frequent repayment). Tedeschi (2006) examines environments where default may be both strategic as well as non-strategic, and argues that in such cases borrower utility would be higher if default was not punished by cutting off access to loans for ever.

The most popular institution used by the MFIs to incentivise the borrowers to repay are group lending with and without joint liability. For a survey see Ghatak and Guinnane (1999) and Armendàriz and Morduch (2005). However, de Quidt et al. (2016) empirically show that there is a decline in joint liability. All the three essays of this thesis are in the context of individual lending.

Finally in chapter 3 the borrower saves her entire net return with the MFI which helps us her to graduate. While there are some empirical papers which address saving among poor people: Dupas and Robinson (2013b), Dupas and Robinson (2013a), Collins et al. (2009), Baland et al. (2011), Atkinson et al. (2013), Ashraf et al. (2010), Roodman (2009), Rutherford (2009)

There are a very few theoretical papers to explain this. Aniket (2011) analyzes endogenous group formation with two members, where one individual borrows from the MFI and the other one saves with the MFI. In contrast, this paper addresses individual lending, with the focus being on explaining progressive lending, as well simultaneous borrowing and saving. Basu (2016) analyzes a model with present biased preferences which addresses the puzzle of simultaneous borrowing and saving. Bernheim et al. (2015) and Banerjee and Mullainathan (2010) explain the empirical finding that poor people act as if they are myopic and save much less. In contrast, we explain simultaneous borrowing and saving in a framework with time consistent borrowers.

1.4.2 Part II

As stated above this part studies the effects of competition among MFIs on moneylender and borrowers - along the loan amount and interest rate as well as welfare of the borrowers and the moneylender.

To start with we briefly review the empirical literature. McIntosh et al. (2005), Vogelgesang (2003), Marconi and Mosley (2005) find evidence for double-dipping. Using Ugandan data McIntosh et al. (2005) examined the effect of competition on loan size and found no effect. Zeller et al. (2001) and McKernan et al. (2005) suggest that households in Bangladesh that borrow from MFIs often borrow from informal lenders as well. Furthermore, Mallick (2012) finds that moneylender interest rates are higher in the villages where higher percentage of households borrows from MFIs.

The theoretical literature studying competition among lenders includes among others, Hoff and Stiglitz (1997), Conning et al. (2003), McIntosh and Wydick (2005), de Janvry et al. (2010), de Quidt et al. (2016), Ghosh and Ray (2016), Ghosh et al. (2000), Guha and Roy Chowdhury (2013), Shapiro (2015).

In three different frameworks Hoff and Stiglitz (1997) study the effect of subsidised formal credit on moneylender interest rate where they assume "exclusivity": "each borrower is a customer of only one moneylender". Unlike their model we do not study strategic interaction between the MFIs as we study only symmetric contracts, but we study strategic interaction between the MFIs and the moneylender.

Conning et al. (2003) address adverse selection and moral hazard problem where client maximizing lenders offer either "One For All contract" or "Screening Contract". They do not address the issue of double dipping which is central to our framework. Also unlike our model, there is no strategic interaction among the lenders as they assume that one lender (BancoSol) was traditionally there in the market and the other lender (Caja Los Andes) enters as a Stackelberg follower. Further they predict that competition lowers welfare of the poorest borrowers since increase in competition decreases the capacity to cross-subsidize. In our model also borrower's welfare decreases with increase in competition among MFIs, however that happens due to decrease in the optimum loan size.

This possibility of decrease in poor borrowers' welfare due to decrease in cross-subsidization is also predicted by McIntosh and Wydick (2005). They also predict that competition increases average debt of some (most impatient) borrowers and decreases expected equilibrium repayment rate among all borrowers. Our model predicts that the average debt of a representative borrower decreases which is supported by the empirical finding of Chowdhury et al. (2017) and as shown above repayment rate in microfinance industry is universally very high.

The objective of de Quidt et al. (2016) paper is to capture the effect of competition among

MFIs on individual and joint liability. In their main model, they capture competition among MFIs by an increase in the borrower's outside option. They do not allow for double dipping.

Ghosh and Ray (2016) address competition among lenders, however, they assume "exclusivity" - a borrower, at a time, takes loan from only one lender. The production function, in their framework, is assumed to be deterministic, so no bridge-loan is required. Due to their assumption a certain proportion of borrowers are myopic and hence, always default, there is default along the equilibrium path. In our model, all borrowers are strategic and repay whenever they have incentive to do so, so the contracts are designed in such a way that there is no default along the equilibrium path.

Like us Guha and Roy Chowdhury (2013) allow for double dipping in a model where lenders are motivated and borrowers are subject to *ex ante* moral hazard. One of their main findings is that default rate necessarily increases with increase in competition. Double dipping, in their model, is inefficient in that taking loan only for productive investment is efficient. They assume that an agent can take two units of loans and she can invest only one unit in the project and also maximum amount she can consume using loan money is one. So in their framework double dipping gives rise to inefficiency and default along the equilibrium path. However, as we have documented default rate in microfinance industry is low.

Shapiro (2015) also allow for double dipping and find that initial loan amount in double dipping is lower than that in single dipping. Unlike us, they do not study the interaction between the moneylender and MFIs though.[6]

Another important aspect of our model is that we explicitly study the interaction between the informal moneylender and the MFIs. Perhaps the first paper to study this is by Jain and Mansuri (2003). Similar to their model, at the heart of the symbiotic relationship between the MFIs and the moneylender is the latter's informational advantages about the borrower's project return. However unlike their model, there is no problem of project choice as by assumption there is only one risky project in which the borrower can invest using MFI loan. Further they assume that moneylenders are perfectly competitive whereas in our framework there is one strategic moneylender, or if there are more than one moneylenders in a village following Mookherjee and Motta (2016) we assume they collude and behave strategically, given the informational advantage of the moneylender which is a plausible assumption.

In Mookherjee and Motta (2016) borrowers differ in two aspects: risk type and landholding, however in our model all borrowers are similar. Also they do not study the effects of competition among MFIs on the moneylender. One of the important findings of their model is that the moneylender's interest rate increases with the entry of MFI, we also find that as the number of MFIs increases moneylender's interest rate increases. Gine (2011), Madestam (2014), Bose (1998) among others address interaction between informal and formal moneylenders.

[6] This issue of multiple lending is related to a broader literature on non-exclusive contracts viz. Kahn and Mookherjee (1998).

Part I

Two Theoretical Explanations of Progressive Lending

Chapter 2

Dynamics of 'Bundled' Aid-Debt Contracts: Progressive Lending

2.1 Introduction

The objective of this chapter is to provide a theoretical explanation of these two empirical observations: progressive lending and very high repayment rate, in a context where borrower's utility function is concave (a natural assumption in the context of MFI borrowers a large majority of which are poor).

To the best of our knowledge, this is the first paper to explain progressive lending and negligible default rates in a context with concave utility function.

We address the Bulow-Rogoff-Rosenthal problem by assuming the lender bundles the loan with access to a superior technology (or marketing opportunity) which generates higher rates of return.[1] Examples of such bundling abound — eg in sovereign debt where default is punished by termination of access to lender markets and technology, or MFIs that provide borrowers access to technology and marketing channels conditional on not defaulting. In such a context, we show it is possible for benevolent lenders to design lending policies that raise borrower welfares without inducing any defaults on the equilibrium path. We characterize the dynamics of optimal contracts, and show they involve progressive lending for borrowers whose assets fall below an endogenous threshold and stationary loans for those above the threshold. For poor borrowers below the threshold, loans, production and consumption grow over time. In the long run these converge to the stationary levels associated with borrowers starting at the threshold. Hence long run outcomes are not affected by progressive lending, i.e., are the same as under self-financing (coupled with access to the superior technology).

In particular, we consider a setting, without any uncertainty, where in autarky an agent, potential borrower, has access to a technology represented by a neoclassical production function and savings technology; marginal return from the production function is higher than that from the savings technology, initially. Also, to abstract from time trends created only from time preference, we assume that the agent's future discount factor is equal to the inverse of the rate of interest on savings. So, this a standrad Ramsey problem where a rich agent maintains

[1]See Thomas and Worrall (1994) for a similar approach but where borrowers have linear utility and there is no capital accumulation.

her wealth and poor agent's consumption and investment increase over time and ultimately converge to the steady state levels.

The lender provides access to a superior technology, represented by a neoclassical production function, which is more productive than the autarky technology. So even if the lender provides only access to this technology, the agent's welfare would increase as that would decrease investment cost. But providing loan over and above that access to the technology would further increase the borrower's welfare. In case of default the lender withdraws access to this superior technology and it is due to this loss the strategic borrower chooses to repay. We characterize optimal contracts in this case and as stated above, find that equilibrium outcomes depend on the borrower's endowment. For a poor borrower, in the short run loan size increases over time before converging to the stationary level. Also, there is zero default along the equilibrium path.

So in a nutshell, the contribution of this chapter is to provide a theoretical explanation of the most widely used dynamic incentive viz. progressive lending in a framework where default is rare and the borrowers have strong preferences for consumption smoothing.

2.2 Framework

2.2.1 Payoffs and Technology

Consider an agent with endowment w, in an infinite horizon discrete time framework. Her current payoff is given by $u(c)$ where c denotes consumption. We assume $u(\cdot)$ to be time-stationary, continuous, strictly increasing, concave and satisfy Inada conditions: $u^{'}(0) = \infty$ and $u^{'}(\infty) = 0$. The agent's objective is to maximize present discounted value of her lifetime utility: $\sum_{t=0}^{\infty} \delta^t u(c_t)$; where δ is her future discount factor.

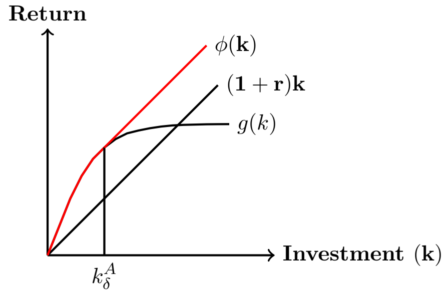
We assume that an agent has access to (i) a neoclassical production technology $g(\cdot)$ which is strictly increasing, strictly concave and satisfies the Inada conditions: $g'(0) = \infty$ and $g'(\infty) = 0$ and (ii) a savings technology, net interest being r. To abstract from time trends created only from time preference, we make the following assumption

Assumption 2.1. $\delta = \dfrac{1}{1+r}$.

Given this, the agent has to decide how much to invest in which technology, accordingly we define k_δ^A: the maximum amount of investment such that the marginal return from investing in $g(\cdot)$ is higher than that from investing in the savings technology. Formally.

Definition 2.1. k_δ^A *solves* $\max\limits_{k} \delta g(k) - k$.

So, autarky technology $\phi(\cdot)$ takes the following form:



$$\phi(k) = \begin{cases} g(k) & \textit{if} \quad k \leq k_\delta^A \\ g(k_\delta^A) + (1+r)(k - k_\delta^A) & \text{otherwise.} \end{cases}$$

Hence at autarky, the problem of the agent with endowment w is to

$$\underset{\{k_{\tau+1}\}_{\tau=0}^{\infty}}{\text{Maximize}} \; u\big(w - k_1\big) + \sum_{\tau=1}^{\infty} \delta^\tau u\big(\phi(k_\tau) - k_{\tau+1}\big)$$

Subject to,

$k_1 \leq w$ and $\forall \tau \geq 1 \;\; k_{\tau+1} \leq \phi(k_\tau)$.

We denote the soultion to this problem by $V_A(w)$.

A lender seeks to raise welfare of the agent by providing loans bundled with access to a superior technology represented by a neoclassical production function $(f(\cdot))$ which satisfies the following assumptions $f(\cdot)$ is strictly increasing, strictly concave and satisfies the Inada conditions: $f^{'}(0) = \infty$ and $f^{'}(\infty) = 0$. and $f(k) > g(k)\ \forall k$, $f^{'}(k) \geq g^{'}(k)\ \forall k$ and $\forall k \leq \kappa$ $f^{'}(k) > g^{'}(k)$, where κ is very large.

Similar to autarky, given this $f(\cdot)$, $g(\cdot)$ and savings technology, it has to be decided how much to invest in which technology. First, there should be any investment in the savings technology only when the marginal return to the savings technology is higher than that to $f(\cdot)$ technology as well as to $g(\cdot)$ technology. Second, given Inada conditions, in particiular $\lim_{k\to 0} f^{'}(k) = 0 = \lim_{k\to 0} g^{'}(k)$, for any amount of investment, it is efficient to invest in both the technologies.

So, first we consider given any investment k, how to optimally allocate that between $f(\cdot)$ and $g(\cdot)$ technologies. This requires investing $x(k)$ in $f(\cdot)$ and $k - x(k)$ in $g(\cdot)$, so that $f^{'}(x(k)) = g^{'}(k - x(k))$. The resulting production is $y(k) = f(x(k)) + g(k - x(k))$. It has a marginal product $y^{'}(k) = g^{'}(k - x(k))$, which is decreasing in k since $1 - x^{'}(k) > 0$. So, this function is strictly increasing, strictly concave, satisfies Inada conditions and dominates both $f(\cdot)$ and $g(\cdot)$. Further, is decreasing

Given this, we define k_δ, which is the maximum amount of investment such that the marginal return from investing in $y(\cdot)$ is higher than that from investing in the savings technology. Formally.

Definition 2.2. k_δ *solves argmax* $\delta y(k) - k$.

So the effective"transformation" technology ($\psi(k)$) is given by

$$\psi(k) = \begin{cases} y(k) & \text{when} \quad k \leq k_\delta \\ y(k_\delta) + (1+r)(k - k_\delta) & \text{otherwise.} \end{cases}$$

Finally, observe $y^{'}(k_\delta) = 1$

$$\Rightarrow g^{'}(k_\delta - x(k_\delta)) = g^{'}(k_\delta^A)$$

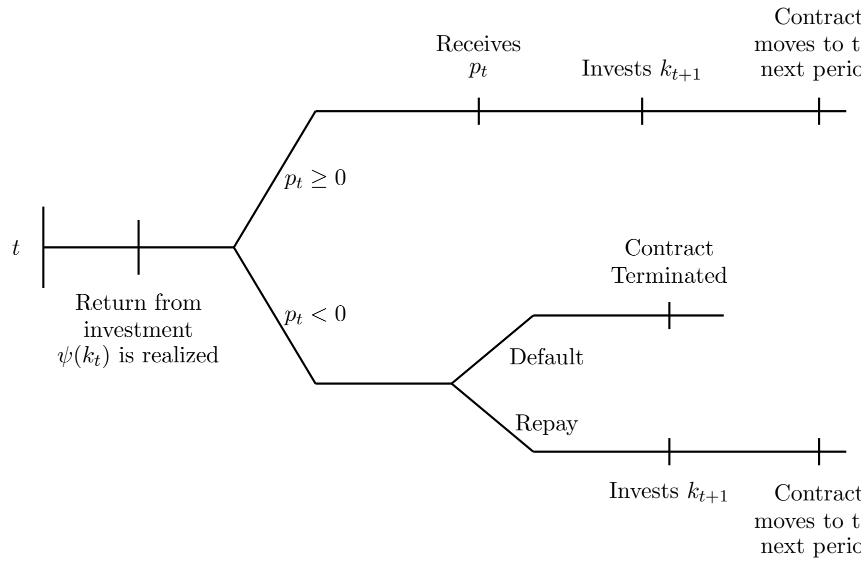
$$\Rightarrow k_\delta = k_\delta^A + x(k_\delta); \quad \text{where } x(k_\delta) \text{ is given by } f^{'}(x(k_\delta)) = g^{'}(k_\delta - x(k_\delta)) = 1.$$

2.2.2 Contracts and Timeline

We assume that the lender can commit and the borrower cannot, in that she defaults on debt whenever she has incentive to do so. The lender provides a contract $\xi \equiv \langle \{p\}, \{k\} \rangle$ where $p \equiv \{p_0, p_1, p_2, ...\}$ and $k \equiv \{k_1, k_2, ...\}$ denote net transfer and investment of each period respectively. Specifically, p_t denotes net transfer at t from the lender to the borrower and k_t denotes investment of $t - 1^{th}$ period. Net interest rate on loan is assumed to be r.

When $p_t < 0$ the borrower has to decide whether to repay or not, if she does not repay[2] the lender terminates the contract and withdraws her access to $f(\cdot)$ technology. The borrower, then optimally chooses the amount to be consumed and invested. We do not allow for any renegotiation. The timeline of this game is as follows.

Timeline: If the agent accepts the contract, having there been no default till date, at any $t \geq 1$ the following things happen:



2.2.3 Analysis of the Optimum Contract

First, given our observation that $k_\delta^A < k_\delta$ note that when $y(k_t) \geq g(k_\delta^A)$ the agent would maintain her wealth after deviation with $y(k_t)$. So in this case, she would invest k_δ^A in $g(\cdot)$

[2] Observe, partial repayment is not allowed in that even if the agent repays partially, the lender terminates the contract, so the borrower either repays the full amount or does not repay at all.

technology and $\delta[y(k_t) - g(k_\delta^A)]$ in the savings technology, so that her wealth in the next period becomes $g(k_\delta^A) + (1+r)\delta[y(k_t) - g(k_\delta^A)] = y(k_t)$. Hence,

$$\text{When } y(k_t) \geq g(k_\delta^A) \qquad V_A(y(k_t)) \equiv \frac{u\big((1-\delta)y(k_t) + \delta g(k_\delta^A) - k_\delta^A\big)}{1-\delta}.$$

Finally observe that $V_A(w)$ is increasing in w.

Given this deviation payoff, the problem of the lender is to

$$\underset{\langle \{p_t\}_{t=0}^{\infty}, \{k_{t+1}\}_{t=0}^{\infty}\rangle}{\text{Maximize}} \left[u(w + p_0 - k_1) + \sum_{t=1}^{\infty} \delta^t u\big(y(k_t) + p_t - k_{t+1}\big)\right]$$

Subject to

Feasibility: $w + p_0 - k_1 \geq 0$ and $\forall t \geq 1, \quad y(k_t) - p_t - k_{t+1} \geq 0$

Sustainability: $p_0 + \sum_{t=1}^{\infty} \delta^t p_t \leq 0$

$$\text{DIC: } V_t \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u\big(y(k_\tau) + p_\tau - k_{\tau+1}\big) \geq V_A\big(y(k_t)\big) \quad \forall t \geq 1. \tag{2.2.1}$$

Sustainability or break even condition ensures that the lender does not make any loss. DIC or dynamic incentive compatibility condition ensures that at any period t, the borrower does not have any incentive to default. Or in other words, the borrower's present discounted value of lifetime utility from repayment is (weakly) higher than that from default at any $t \geq 1$. Feasibility condition ensures that the borrower's consumption at any period is non negative. We ignore this condition as it is implied by the Inada condition in particular $u^{'}(0) = \infty$. Given sustainability and DIC constraints the lender chooses net transfer and amount of investment at each period in order to maximize the borrower's present discounted value of lifetime utility.

We denote the maximum value of the above problem by $V(w)$. It is evident that V is strictly increasing, since there is always the option of consuming the incremental wealth immediately which does not disturb any incentive constraints.

Note also at the outset that since p_0 does not enter any incentive constraint, sustainability condition always binds.

2.2.4 First-best Contracts

Consider the optimal contract when all the incentive constraints are dropped. It involves full consumption smoothing (via choice of transfers p_t) and efficient investment $k_t = k_\delta$. The constant consumption $c^*(w)$ is obtained by the requirement that the present value of consumption $\frac{c^*}{1-\delta}$ equals the present value of production minus investment, plus enodowment $w - k_\delta + \frac{\delta}{1-\delta}[y(k_\delta) - k_\delta]$, so

$$c^*(w) = (1-\delta)w + \delta y(k_\delta) - k_\delta \qquad (1^{st} \text{ Best Consumption})$$

The borrower then attains welfare $V^*(w) = \dfrac{u(c^*(w))}{1-\delta}$.

As the first best contract is stationary, all the incentive constraints collapse to a single constraint $c^*(w) \geq (1-\delta)y(k_\delta) + \delta g(k_\delta^A) - k_\delta^A$, which reduces to the borrowers endowment exceeding a threshold w^*:

$$w^* \equiv y(k_\delta) - \frac{\delta\big(y(k_\delta) - g(k_\delta^A)\big) - \big(k_\delta - k_\delta^A\big)}{1-\delta} \qquad \text{(Wealth Threshold)}$$

Note that the incentive problem arises only for intermediate ranges of the discount factor. If δ approaches 1, the first best can be sustained for any initial w, as the threshold w^* goes to minus infinity. While if δ approaches zero, the threshold approaches zero (as in this case, the efficient investment approaches zero), and the demand for loans vanishes.

When $w \geq w^*$, it follows the first-best contract is incentive compatible from the very first period, and $V(w) = V^*(w)$. And $V(w) < V^*(w)$ whenever $w < w^*$.

2.2.5 Second-best Contracts for Poor Borrowers

First-best is implementable whenever the agent's wealth is at least w^*, so now we focus on poor borrowers, for whom $w < w^*$ and characterize features of the optimal contract. Let $c_t = y(k_t) + p_t - k_{t+1}$ and $c_0 = w + p_0 - k_1$ denote cosumption at date $t \geq 0$.

Lemma 2.1. *$c_t \geq c_{t-1}$ for all t.*

Proof. Suppose otherwise, and $c_t < c_{t-1}$ for some t. Lower p_{t-1} slightly, and raise p_t correspondingly to keep $p_{t-1} + \delta p_t$ unchanged. This smooths consumption, raising V_l for every $l \leq t$, while leaving it unchanged for every $l > t$. Hence all incentive constraints are preserved, while raising $V(w) = V_0$. ■

To make further progress we use the recursive formulation of the problem: $l_0 = p_0, p_t = l_t - \dfrac{l_{t-1}}{\delta}, t \geq 1$. In this notation, the problem of the lender becomes

$$\underset{\langle \{l_t\}_{t=0}^\infty, \{k_{t+1}\}_{t=0}^\infty}{\text{Maximize}} \Big[u(w + l_0 - k_1) + \sum_{t=1}^{\infty} \delta^t u\big(y(k_t) - \frac{l_{t-1}}{\delta} + l_t - k_{t+1}\big)\Big]$$

Subject to

$$\text{DIC: } V_t \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u\big(y(k_\tau) - \frac{l_{\tau-1}}{\delta} + l_\tau - k_{\tau+1}\big) \geq V_A\big(y(k_t)\big) \quad \forall t \geq 1. \tag{2.2.2}$$

Now observe that starting from any date t, the effect of past history is summarized in the single state variable $w_t \equiv y(k_t) - \dfrac{l_{t-1}}{\delta}$, the borrower's net wealth which is the value of current production less inherited debt. So the contracting problem has the following recursive representation.

Lemma 2.2. *The maximum attainable welfare $V(w)$ for a borrower with initial wealth w must satisfy*

$$V(w) = \max_{l,k}[u(w + l - k) + \delta V(y(k) - \frac{l}{\delta})] \quad \textit{subject to:} \quad V(y(k) - \frac{l}{\delta}) \geq V_A\big(y(k)\big) \tag{2.2.3}$$

Proof. Let $\{l_t, k_{t+1}\}_{t\geq 0}$ be an optimal contract. The following conditions are necessary for optimality: (a) given l_0, k_1, the continuation contract $\{l_t, k_{t+1}\}_{t\geq 1}$ must be optimal for the borrower starting with $w_1 = y(k_1) - \dfrac{l_0}{\delta}$ (a problem in which constraints $IC_t, t \geq 2$ are incorporated), and (b) the initial loan and investment size l_0, k_1 satisfy condition (2.2.3) (a problem in which IC_1 is incorporated). ■

We denote target wealth by $\Omega(l, k) \equiv y(k) - \dfrac{l}{\delta}$. Then condition (2.2.3) can be restated as:

$$V(w) = \max_{l,k}[u(w + l - k) + \delta V(\Omega(l, k)] \quad \text{subject to:} \quad V(\Omega(l, k)) \geq V_A(y(k)) \tag{2.2.4}$$

This problem can be broken into two stages.

At the first stage, given any 'target' wealth Ω for the next date, select (l, k) to minimize the net investment cost, i.e, the sacrifice of current consumption $k - l$, subject to the incentive constraint $V(\Omega(l, k)) \geq V_A(g(k))$. Let the resulting minimized cost be denoted by $C(\Omega)$. Formally,

$$C(\Omega) = \min_{l,k}(k - l) \quad \text{subject to:} \quad y(k) - \frac{l}{\delta} = \Omega \quad \text{and} \quad V(\Omega) \geq V_A(y(k)) \tag{2.2.5}$$

Then at the second stage, select the optimal target wealth $\Omega(w)$ for the next date, given current wealth w. We summarize this as follows.

Lemma 2.3. *The maximum attainable welfare $V(w)$ for a borrower with initial wealth w must satisfy*

$$V(w) = \max_{\Omega}[u(w - C(\Omega)) + \delta V(\Omega)] \tag{2.2.6}$$

Since V is strictly increasing, a higher target wealth is always valuable. The borrower must trade off a higher target wealth against the current cost. Nevertheless, the concavity of u implies the following.

Lemma 2.4. *$\Omega(w)$ is nondecreasing.*

Proof. If this is false, there exist $w_1 < w_2$ with $\Omega_1 \equiv \Omega(w_1) > \Omega(w_2) \equiv \Omega_2$. Then $V(\Omega_1) > V(\Omega_2)$ and

$$u(w_2 - C(\Omega_2)) - u(w_2 - C(\Omega_1)) \geq \delta[V(\Omega_1) - V(\Omega_2)] > 0 \tag{2.2.7}$$

which implies $C(\Omega_1) > C(\Omega_2)$. On the other hand,

$$\delta[V(\Omega_1) - V(\Omega_2)] \geq u(w_1 - C(\Omega_2)) - u(w_1 - C(\Omega_1)) \tag{2.2.8}$$

which implies

$$u(w_2 - C(\Omega_2)) - u(w_2 - C(\Omega_1)) \geq u(w_1 - C(\Omega_2)) - u(w_1 - C(\Omega_1)) \tag{2.2.9}$$

This contradicts the concavity of u. ■

Let us now focus on the first stage cost minimization problem. Given target wealth Ω and capital choice k, the associated current loan must be $l(\Omega, k) = \delta y(k) - \delta\Omega$. Hence we can simplify (2.2.5) and reduce it to choice of investment alone as follows:

$$C(\Omega) = \delta\Omega + \min_k (k - \delta y(k)) \quad \text{subject to:} \quad V(\Omega) \geq V_A(y(k)) \tag{2.2.10}$$

So when $\Omega \geq w^*$, the agent invests k_δ from the very first period and hence $C(\Omega)$ in that case becomes $\delta\Omega - [\delta y(k_\delta) - k_\delta]$.

While if $\Omega < w^*$, the DIC constraint binds and in particular

$$V(\Omega) = V_A\big(y(k)\big) \tag{2.2.11}$$

so the resulting investment size is $k(\Omega) = y^{-1}\big(V_A^{-1}(V(\Omega))\big)$. Since $V(\Omega) < V(w^*) = V_A\big(y(k_\delta)\big)$, and V_A, V and y are increasing, hence when $\Omega < w^*$ there will be underinvestment: $k(\Omega) < k_\delta$. It is summarized in the following lemma.

Lemma 2.5. *For target wealths Ω smaller than w^*, investment $k(\Omega)$ is smaller than the efficient level k_δ, and equal to the efficient level otherwise.*

So given target wealth optimum investment is uniquely determined:

$$k \equiv \begin{cases} k_\delta & \text{if } \Omega \geq w^* \\ y^{-1}\big(V_A^{-1}(V(\Omega))\big) & \text{otherwise} \end{cases}$$

Further whenever the agent's current wealth w is at least w^*, every period she gets $\delta[y(k_\delta) - w]$ as loan and repays $y(k_\delta) - w$, observe that this means when the borrower's wealth is higher than $y(k_\delta)$ she saves $\delta[y(k_\delta) - w]$ with the lender. Finally at this parametric condition target wealth and the optimum consumption are constant at w and $c^*(w)$ (determined according to (1^{st} Best Consumption)) respectively.

Hence now we turn to the case where initial wealth is below the threshold w^*. In the next two lemmas we show that the wealth is monotonically increasing and converges to w^*, but the limit wealth w^* cannot be reached in finite time from any initial wealth $w < w^*$.

Lemma 2.6. *If $w < w^*$, the sequence of net wealths w_t is nondecreasing and converges to the first best threshold w^*; the corresponding investment sequence k_t is nondecreasing and converges to k_δ, and consumption c_t is nondecreasing and converging to $c^*(w^*)$.*

Proof. Suppose not and w_t is decreasing: there is some $w < w^*$ such that $\Omega(w) < w$. Then combining with Lemma 2.4, it follows that wealth is nonincreasing over time, so at every date it is smaller than w, and converges to some limit $w_\infty < w$. The corresponding sequence of investments will then be nonincreasing, all strictly smaller than k_δ and converging to some $k_\infty < k_\delta$. At the same time, we have already established that consumption is nondecreasing, hence converging to some c_∞. Observe that there can be two cases: $k_\infty \leq k_\delta^A$ and $k_\infty > k_\delta^A$. Observe, IC will be satisfied only if $c_\infty \geq c_\infty^A$ where $c_\infty^A = g(k_\delta^A) - k_\delta^A$ in the first case and $c_\infty^A = (1-\delta)y(k_\infty) + \delta g(k_\delta^A) - k_\delta^A$ in the second case.

Now consider a small increase in investment k_{T+1} at some date $T \geq 1$. Let S_t denote the surplus on IC_t, i.e., $S_t \equiv V_t - V_A(y(k_t))$. The effect on continuation payoffs and incentive surpluses at different dates are as follows. There is no effect on any of these at any date after $T+1$, so we focus on effects for $t \leq T+1$.

$$\frac{\partial S_{T+1}}{\partial k_{T+1}} = [u'(c_{T+1}) - u'(c^A_{T+1})]y^{'}(k_{T+1}) \geq 0 \tag{2.2.12}$$

IC$_{T+1}$ will not be jeopardized if the inequality is true. We argue that it is true in both the cases, that is when $k_\infty \leq k^A_\delta$ as well as when $k_\infty > k^A_\delta$. First, if IC$_{T+1}$ does not bind that is true trivially. Now, if IC$_{T+1}$ binds, observe $c_{T+1} \leq c^A_{T+1}$ because both c_{T+1} and c^A_{T+1} are non-decreasing and $c_\infty \geq c^A_\infty$. Given, this observation $u^{'}(c_{T+1}) \geq u^{'}(c^A_{T+1})$ follows from the concavity of $u(\cdot)$.

Next

$$\frac{\partial V_T}{\partial k_{T+1}} = \frac{\partial S_T}{\partial k_{T+1}} = u'(c_{T+1})\delta y^{'}(k_{T+1}) - u'(c_T) \tag{2.2.13}$$

As $T \to \infty$, this converges to $u'(c_\infty)[\delta f'(k_\infty) - 1]$ which is strictly positive. Hence for all T sufficiently large, expression (2.2.13) is strictly positive. The same is true at all earlier dates. Hence the variation is feasible and welfare improving, so the contract cannot be optimal.

Now we show that w_t convergers to the first-best Wealth Threshold w^*, k_t is nondecreasing and converges to k_δ and c_t converges to $c^*(w^*)$.

Since the wealth sequence is nondecreasing the investment sequence is also nondecreasing (since it is an increasing function of the target wealth). So both converge to limits w_∞, k_∞ respectively.

If $k_\infty < k_\delta$ we will get a contradiction (using the same argument as above). Hence k_t converges to k_δ.

If $w_\infty < w^*$, the corresponding limit of investment $k_\infty < k_\delta$, which has been ruled out above. Hence $w_\infty \geq w^*$. Since $\Omega(w^*) = w^*$, Lemma 2.4 implies that $\Omega(w) \leq w^*$ for any $w < w^*$. Hence $w_\infty \leq w^*$, and it follows that $w_\infty = w^*$. The corresponding sequence of consumption must therefore converge to $c^*(w^*)$. ■

Next we show that when $w < w^*$, target wealth is strictly increasing in current wealth, however it can never reach the first-best Wealth Threshold w^* within a finite time.

Lemma 2.7. *$w < w^*$ implies $w < \Omega(w) < w^*$.*

Proof. First we show that $\Omega(w) < w^*$. We have already seen that since $\Omega(\cdot)$ is nondecreasing, we have $\Omega(w) \leq \Omega(w^*) = w^*$. Suppose the claim is false and there exists $w < w^*$ such that $\Omega(w) = w^*$. In an optimal contract starting from $w_0 = w$, we must then achieve the first-best allocation from date 1 onwards, with $c_t = c^*(w^*) = (1-\delta)y(k_\delta) + \delta g(k^A_\delta) - k^A_\delta, k_t = k_\delta$. The borrower's welfare is then $u(w + p_0 - k_\delta) + \dfrac{\delta u((1-\delta)y(k_\delta) + \delta g(k^A_\delta) - k^A_\delta)}{1-\delta}$. Now budget balance implies $p_0 = \dfrac{\delta[\delta y(k_\delta) - \delta g(k^A_\delta) + k^A_\delta - k_\delta]}{1-\delta}$ (since the borrower repays $\delta y(k_\delta) - \delta g(k^A_\delta) + k^A_\delta - k_\delta$ at every date $t \geq 1$ onwards). Hence date 0 consumption $c_0 = w + \dfrac{\delta^2(y(k_\delta) - g(k^A_\delta)) - (k_\delta - \delta k^A_\delta)}{1-\delta} < (1-\delta)y(k_\delta) + \delta g(k^A_\delta) - k^A_\delta$, as $w < w^*$.

Now consider a variation on this contract where the production level continues to be stationary (denoted k such that $k \in (k_\delta^A, k_\delta)$). The loan at date 0 is modified to $\frac{\delta[\delta y(k) - k - \delta g(k_\delta^A) + k_\delta^A]}{1-\delta}$, while future per period repayments are modified to $\delta y(k) - k - \delta g(k_\delta^A) + k_\delta^A$. Welfare in this contract is $u(w + \frac{\delta^2(y(k_\delta) - g(k_\delta^A)) - (k_\delta - \delta k_\delta^A)}{1-\delta}) + \frac{\delta u((1-\delta)y(k) + \delta g(k_\delta^A) - k_\delta^A)}{1-\delta}$. Incentive compatibility are maintained and so is budget balance. The first order effect of a slight reduction in k below k_δ equals $u'(c_0) - u'(c^*(w^*))$ which is strictly positive since $c_0 < c^*(w^*)$. Hence a small reduction in k will improve welfare, and we obtain a contradiction.

Next, to show that $\Omega(w) > w$ for any $w < w^*$, note that otherwise there is some $w < w^*$ for which $\Omega(w) = w$. Then starting with initial wealth of w, wealth will remain stationary. This contradicts Lemma 2.6 since w_t must converge to w^*. ■

Finally we show that when $w < w^*$ optimum loan size increases over time.

Lemma 2.8. *Starting with any $w < w^*$, the borrower obtains a loan $l(w)$ which is strictly positive and strictly increasing. Hence loan size $l_t = l(w_t)$ rises over time, eventually converging to* $\frac{\delta\left[\delta\big(y(k_\delta) - g(k_\delta^A)\big) - \big(k_\delta - k_\delta^A\big)\right]}{1-\delta}$.

Proof. Let $n(w)$ denote date 1 output $y(k(w))$, where $k(w)$ is the investment at date 0 with wealth w. As we have observed $l(w) = \delta[n(w) - \Omega(w)]$ and since $w < w^*$ $V(\Omega(w)) = V_A(n(w))$. We establish the lemma using the following two properties

i) $V(w) > V_A(w)$

Observe, $y(k) > g(k)$ $\forall k$, so even if the lender does not provide any loan borrower's welfare would be higher than that from autarky where she has access to $f(\cdot)$, $(g(\cdot))$ and the savings technology: $V(w)|_{\text{without loan}} > V_A(w)$. Now, the lender is benevolent so $V(w) \geq V(w)|_{\text{without loan}}$ (since the lender can simply not provide any loan). Hence $V(w) \geq V(w)|_{\text{without loan}} > V_A(w)$.

ii) $V'(\Omega(w)) > V_A'(n(w))$ since $w < w^*$ $k < k_\delta$. So we consider two cases

a) $k \in [k_\delta^A, k_\delta)$

Observe here $V_A(n(w)) \equiv \frac{u\big((1-\delta)n(w) + \delta g(k_\delta^A) - k_\delta^A\big)}{1-\delta}$ and $V_A'(n(w)) = u'\big((1-\delta)n(w) + \delta g(k_\delta^A) - k_\delta^A\big)$.

Now, $V'(\Omega(w)) \geq u'(c_0(\Omega(w))$ where $c_t(w)$ denotes date t consumption starting with wealth w (as the borrower always has the option of immediately consuming any incremental wealth). Next, note that $c_0(\Omega(w)) < (1-\delta)n(w) + \delta g(k_\delta^A) - k_\delta^A$, since $V(\Omega(w)) \equiv u(c_0(\Omega(w))) + \sum_{t=1}^{\infty} \delta^t u(c_t(\Omega(w))) = V_A(n(w))$, $c_0(\Omega(w)) \leq c_t(\Omega(w))$ for all $t \geq 1$ with strict inequality for some t as the optimal consumption sequence starting with wealth $\Omega(w)$ is not stationary (this in turn follows from noting that if consumption were stationary then V_t would be stationary, while we have shown above that k_t is strictly increasing, so IC_t could not bind at every date t). Hence $u'(c_0(\Omega(w))) > u'((1-\delta)n(w) + \delta g(k_\delta^A) - k_\delta^A) = V_A'(n(w))$, so $V'(\Omega(w)) > V_A'(n(w))$.

b) $k < k_\delta^A$

Recall $V(\Omega(w)) = V_A(n(w))$, we also know that on-contract consumption converges to $c^*(w^*)$ and post-deviation consumption converges to $g(k_\delta^A) - k_\delta^A$ and in both the cases consumption is (weakly) increasing over time, which implies that after a finite point of time on-contract consumption becomes higher than that from post-deviation. So to have $V(\Omega(w)) = V_A(n(w))$, $c_0(\Omega(w)) < c_0^A(n(w))$ where $c_0(\Omega(w))$ and $c_0^A(n(w))$ are defined as above: $c_t(w)$ and $c_0^A(n(w))$ denote on-contract and post-deviation consumption starting with wealth w at date t, respectively. So we have

$$V^{'}(\Omega(w)) \geq u'(c_0(\Omega(w)) > u'(c_0^A(n(w))) = V_A^{'}(n(w))$$

where the first inequality is coming from the fact we discussed above, second inequality is coming from the discussion above: $c_0(\Omega(w)) < c_0^A(n(w))$, finally, the last equality is coming from Envelope theorem.

Claim (i) now implies that $l(w) = \delta[n(w) - \Omega(w)] > 0$ since $V_A(n(w)) = V(\Omega(w)) > V_A(\Omega(w))$. Moreover, $V_A^{'}(n(w))x'(w) = V^{'}(\Omega(w))\Omega^{'}(w)$ so (ii) implies $x'(w) > \Omega^{'}(w)$. Therefore $l^{'}(w) = \delta[x'(w) - \Omega^{'}(w)] > 0$. l_t must converge to $l(w^*) \equiv \dfrac{\delta\Big[\delta\big(y(k_\delta) - g(k_\delta^A)\big) - \big(k_\delta - k_\delta^A\big)\Big]}{1-\delta}$. ■

Proposition 2.1. *When the agent's endowment $w \geq w^*$*

i) Net wealth remains constant at w

ii) The agent invests k_δ at every $t \geq 0$

iii) $\forall t \geq 0$ she receives $\delta[y(k_\delta) - w]$ from the lender and $\forall t \geq 1$ she repays $y(k_\delta) - w$

iv) at each $t \geq 0$ she consumes $c^*(w) \equiv (1-\delta)w + \delta y(k_\delta) - k_\delta$.

When the agent's endowment $w < w^*$

i) Net wealth increases over time and converges to w^* but the limit amount w^* is not reached within a finite time

ii) Investment is uniquely determined as a function of next period's target wealth: $k(w) \equiv y^{-1}\big(V_A^{L^{-1}}(V(\Omega))\big)$. Hence investment is increasing over time and converges to $k(w^*) \equiv k_\delta$

iii) The borrower obtains a loan $l(w)$ which is strictly positive and strictly increasing, that is optimal contract features progressive lending. Hence loan size $l_t = l(w_t)$ rises over time, eventually converging to

$$\frac{\delta\Big[\delta\big(y(k_\delta) - g(k_\delta^A)\big) - \big(k_\delta - k_\delta^A\big)\Big]}{1-\delta}$$

iv) Consumption is nondecreasing and converges to $c^*(w^*)$.

2.3 Conclusion

We address Bulow-Rogoff-Rosenthal problem and show that bundling access to superior technology with credit ensures repayment. We, then, characterize the optimum loan scheme where the borrower has concave utility function. We find that when endowment of the borrower falls below a certain threshold loan size, consumption, investment increase over time. So, this chapter provides an explanation of progressive lending commonly observed in dynamic credit relationships.

Chapter 3

Progressive Lending with Graduation

3.1 Introduction

This chapter provides another explanation of progressive lending in a framework where graduation to a high productive activity is possible. For simplicity we assume that the poor agents do not have access to any technology to transfer wealth from one period to another or capital depriciates completely within a period. Their utility functions are assumed to be linear.

We study the problem of a benevolent lender in an infinite horizon, continuous time framework with an endogenously determined finite date at which the borrower graduates. The lender provides credit along with access to a deterministic, informal production technology and savings technology. The borrower invests the loan and return is realized instantaneously, then she decides whether to repay or default. In case of default the lender terminates the contract, withdraws access to the technology and confiscates her entires savings with him till date. In case the borrower repays always, the contract gets terminated at an endogenously determined finite date, at that instance the borrower gets back her entire savings (along with interest) from the lender which enables her to graduate. So in case of default the borrower loses her savings as well her opportunity to graduate. Hence, her loss from default increases over time which gives us our central result - loan size (weakly) increases over time.

Later we consider a general case where the borrower has access to a savings technology (return to which is equal to that of the savings technology provided by the lender) and also, in case of default, the lender can confiscate only a part of the borrower's savings with him. We find that the qualitatively all the results hold.

The rest of the chapter is organized as follows: Next we analyse the benchmark case, then we consider a more general case where the borrower has access to a savings technology on her own and also the lender cannot confiscate her entire savings in case of default. In the section following that, we compare the optimum outcomes of these two cases. Finally we conclude.

3.2 Benchmark Case

3.2.1 Payoff, Technologies and Graduation

We study a dynamic relationship between a poor borrower who is subject to an *ex post* moral hazard problem and a benevolent MFI that operates under a zero profit condition. The borrower has access to a nonconvex technology $\langle V, \bar{S}\rangle$ where $\bar{S}(>0)$ is the required fixed initial investment, and V is the present discounted value of lifetime utility from investing in that technology (gross of $\bar{S}$). Her endowment is zero, so she cannot start the project on her own. The MFI could provide adequate credit to start the project, but once the borrower starts investing in $\langle V, \bar{S}\rangle$ technology, it is not possible to incentivise her to repay.[1]

Apart from zero endowment, the borrower also has limited access to technology: In this benchmark case, when her wealth is less than $\bar{S}$ she does not have access to any technology. The MFI can provide access to a savings and a deterministic neoclassical technology $f(\cdot)$. This technology $f(\cdot)$ does not require any minimum initial investment and satisfies the usual assumptions:

Assumption 3.1. $f(0) = 0,\ f'(\cdot) > 0,\ f''(\cdot) < 0,\ \lim_{k\to 0} f'(k) = \infty$ *and* $\lim_{k\to\infty} f'(k) = 0.$

Now let the efficient scale of investment k^e solve $\underset{k}{\operatorname{argmax}}\ [f(k) - k]$. Turning to the MFI-savings technology, given the zero profit condition of the MFI, the interest rate on savings is equal to the real rate of interest. Let r denote the real rate of interest and both the agents (the borrower and the MFI) discount the future at the rate r.

We also assume that the borrower's technology $\langle V, \bar{S}\rangle$ is more productive – the net gain from investing in that technology exceeds the present discounted net payoff from running the $f(\cdot)$ technology at its efficient level.

Assumption 3.2. $V - \bar{S} > \dfrac{1}{r}[f(k^e) - k^e]$.[2]

When a borrower starts investing in this nonconvex, more productive technology $\langle V, \bar{S}\rangle$ we say that she has *graduated*. The first-best thus involves providing $\bar{S}$ amount of loan at the very beginning so that she can graduate immediately. But as discussed above, that is not achievable – the borrower would default and the MFI would make a loss with certainty. The problem of the MFI thus is to design a dynamic self-enforcing scheme such that the borrower's lifetime utility is maximised and she chooses to repay always.

3.2.2 Contracts and Timeline

We consider an infinite horizon, continuous time framework, where $t \in [0, \infty)$. At $t = 0$, the MFI announces a contract $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M\rangle$, where T_M is the "successful" termination date of this contract[3] and k_t and α_t, respectively, denote the loan amount and the part of net

[1] First, the MFI does not have any control over $\langle V, \bar{S}\rangle$ technology. Second, once the borrower's wealth becomes no less than $\bar{S}$ she gets access to a storage technology as well. Hence it is not possible to incentivise her to repay. This is a direct consequence of Bulow and Rogoff (1989b) and Rosenthal (1991).

[2] Observe, $\dfrac{1}{r}[f(k^e) - k^e] = \displaystyle\int_0^\infty e^{-rt}[f(k^e) - k^e]dt.$

[3] A contract gets terminated in two ways – in case of default and in case the borrower always repays at this prespecified date T_M. To differentiate the latter from the former, when the contract gets terminated in the latter way, we say that it has been terminated "successfully".

return which the borrower saves[4] with the MFI, at any instance t, where $0 \leq t \leq T_M$. So, at any instance t, the borrower not only repays the amount k_t, but also saves a part α_t of her net return, i.e. $\alpha_t(f(k_t) - k_t)$, with the MFI. We shall consider α_t such that $0 \leq \alpha_t \leq 1$. Given limited liability α_t cannot be higher than 1, this condition actually implies that dissaving is not allowed.[5]

The borrower either accepts or rejects this contract, with the game ending in case she rejects. If she accepts then the continuation game at any instance t, where $0 \leq t \leq T_M$, is as follows:

Stage 1: The MFI lends k_t, the borrower invests that amount in the $f(\cdot)$ technology which yields an instantaneous output of $f(k_t)$.

Stage 2: The borrower then decides whether to repay, or not:

(i) In case of repayment, she repays k_t, deposits a part α_t of her net return $\alpha_t[f(k_t) - k_t]$ with the MFI, consumes the rest instantaneously, and the game continues.

(ii) In case of default, she obtains her current gross income $f(k_t)$, that is we assume that the amount yielded cannot be seized. The MFI, however, terminates the contract, so that the borrower does not get any more loan from that instance onwards and withdraws her access to both $f(\cdot)$ and the savings technology. Moreover, the MFI confiscates her entire savings with it till date.

Finally, at the successful termination of the contract T_M, the borrower gets back her entire savings with the MFI till date, along with interest. If that amount is no less than $\bar{S}$ then she graduates immediately. Otherwise, given that she does not have access to any technology to transfer wealth from one instance to another she has to consume the entire amount immediately.

We solve for the subgame perfect Nash equilibrium (SPNE) of this game.

3.2.3 Analysis: Optimal Loan Scheme

Let us start with a brief overview of the results and the intuition behind them. Since graduation is welfare improving (Assumption 3.2), the benevolent MFI enables the borrower to graduate. In fact, due to the same reason its objective is to minimise the time required to graduate. In order to do that it (a) successfully terminates the contract and returns the borrower's savings as soon as that becomes $\bar{S}$ and (b) designs the contract such that the time required to accumulate $\bar{S}$ is the minimum, given that the borrower has incentive to repay. This in turn implies that the objective of the MFI is to maximise the instantaneous savings. Therefore, the MFI lends the efficient amount k^e whenever that is incentive compatible otherwise the maximum amount which is that. The borrower on the other hand saves the entire net return. The last result is due to our assumptions of linear utility function and that discount factor is equal to the rate of interest on savings. Due to these assumptions the borrower is indifferent between consuming

[4]Observe, the alternative interpretation of savings could be "advance repayment", that is the MFI towards the end is in debt. So like in Ray (2002), there is frontloading pain (small loan, high interest) and backloading rewards (large loans, low interest). Rutherford (2009) and Roodman (2009) indeed point out "as a matter of money flow, the service differ only in when the large sums are paid out." From the notational standpoint we follow Rutherford (2009) "saving up ahead of a pay-out, and borrowing ahead of repayment".

[5]The assumption that $\alpha_t \geq 0$ is without loss of generality as at the optimum α_t takes the maximum value possible which is 1 (Lemma 3.2).

an amount now, and saving and consuming that amount (along with interest) later, but she is strictly better off as the time required to graduate decreases.[6]

For ease of exposition, we make the following assumption which ensures that the borrower cannot graduate in case of default.[7]

Assumption 3.3. $\bar{S} > f(k^e)$.

Now, there can be two cases – at the optimum the borrower may or may not graduate. In the former case, the problem of the MFI is to select a scheme $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle$ which maximises the borrower's lifetime utility, subject to (i) the graduation condition (GC hereafter) which ensures that, by the end of the scheme, the accumulated savings exceeds $\bar{S}$ and (ii) the dynamic incentive compatibility constraints (DICs hereafter) which ensure that the borrower does not have any incentive to default at any instance t, where $0 \leq t \leq T_M$.

$$\underset{\langle \alpha_t, \{k_t\}_{t=0}^{T_M}, T_M \rangle}{\text{Maximise}} \int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t) - k_t]dt + e^{-rT_M}\left[\int_0^{T_M} e^{r(T_M - t)}\alpha_t[f(k_t) - k_t]dt - \bar{S} + V\right]$$

$$\text{Subject to: GC: } \int_0^{T_M} e^{r(T_M - t)}\alpha_t[f(k_t) - k_t]dt \geq \bar{S},$$

$$\text{DIC: } \int_t^{T_M} e^{-r(t'-t)}(1-\alpha_{t'})[f(k_{t'}) - k_{t'}]dt' + e^{-r(T_M - t)}\left[\int_0^{T_M} e^{r(T_M - t')}\alpha_{t'}[f(k_{t'}) - k_{t'}]dt' - \bar{S} + V\right]$$

$$\geq f(k_t), \qquad \forall t \leq T_M.$$

Let us briefly explain these two constraints. The left hand side of the GC is the total savings till T_M, which needs to be at least $\bar{S}$, so that the borrower can graduate at T_M. On the other hand, the DIC states that at any $t \leq T_M$ the borrower's present discounted value of lifetime utility from repayment is higher than that from default. The borrower's present discounted value of lifetime utility from repayment, i.e. the left hand side of the DIC has two components: the first term denotes the present discounted value of her utility from consumption till the T_M^{th} instant (evaluated at t) and the second term denotes the present discounted (again evaluated at t) value of her utility at the T_M^{th} instant when she gets back her entire savings (along with interest) and graduates, $V - \bar{S}$ being the net gain from graduation. Finally, the right hand side of the DIC is the borrower's utility at t from default, i.e. her instantaneous return $f(k_t)$. Observe, due to our assumption 3.3 and the fact that the borrower does not have access to any storage technology, she cannot graduate in case of default. Thus her present discounted value of lifetime utility from default at t is her utility from consumption.

[6] We conjecture that qualitatively all the results go through even if we assume strictly concave utility function, we discuss that briefly in Remark 3.2.

[7] We relax this assumption in the general case (section 3.3) and in Remark 3.1 we briefly discuss what happens if we relax this assumption in this benchmark case.

Similarly, in case the borrower does not graduate, the problem of the MFI is to

$$\underset{\langle \alpha_t, \{k_t\}_{t=0}^{T_M}, T_M \rangle}{\text{Maximise}} \quad \int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t)-k_t]dt + e^{-rT_M}\int_0^{T_M} e^{r(T_M-t)}\alpha_t[f(k_t)-k_t]dt$$

Subject to:

$$\text{DIC: } \int_t^{T_M} e^{-r(t^{'}-t)}(1-\alpha_{t^{'}})[f(k_{t^{'}})-k_{t^{'}}]dt^{'} + e^{-r(T_M-t)}\int_0^{T_M} e^{r(T_M-t^{'})}\alpha_{t^{'}}[f(k_{t^{'}})-k_{t^{'}}]dt^{'} \geq f(k_t),$$
$$\forall t \leq T_M.$$

In our first proposition we show that at the optimum the borrower always graduates. This is a direct consequence of our assumption 3.2 – graduation is welfare improving. The borrower's lifetime utility increases as she graduates, so the MFI, being benevolent, designs the optimum contract in such a way that the borrower can graduate. We prove the result by contradiction, we start with an optimum contract where the borrower does not graduate and show that given Assumptions 3.1, 3.2 and 3.3, it is possible to construct another DIC contract where the borrower graduates and her present discounted value of lifetime utility is higher. So, the original contract cannot have been optimum. For the rest of the paper, we restrict ourselves to the class of contracts where k_t and α_t are continuous in t. All the proofs can be found in Appendix A.

Proposition 3.1. *Let Assumptions 3.1, 3.2 and 3.3 hold. At the optimum the borrower always graduates.*

In fact in the next lemma we show that at the optimum the borrower graduates as soon as possible, i.e. she graduates as soon as her savings becomes $\bar{S}$. This is because graduation is welfare improving (Assumption 3.2) and beyond $\bar{S}$, V is independent of the wealth with which the borrower graduates.

Lemma 3.1. *Let Assumptions 3.1, 3.2 and 3.3 hold. Optimally, the contract is terminated as soon as the borrower accumulates enough savings to graduate i.e. start the $\langle V, \bar{S} \rangle$ technology: so the graduation constraint GC binds at the optimum.*

Given Proposition 3.1 and Lemma 3.1, the problem of the MFI can be expressed as follows:

$$\underset{\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle}{\text{Maximise}} \quad \int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t)-k_t]dt + e^{-rT_M}V$$

$$\text{Subject to GC: } \int_0^{T_M} e^{r(T_M-t)}\alpha_t[f(k_t)-k_t]dt = \bar{S}, \tag{3.2.1}$$

$$\text{DIC: } \int_t^{T_M} e^{-r(t^{'}-t)}(1-\alpha_{t^{'}})[f(k_{t^{'}})-k_{t^{'}}]dt^{'} + e^{-r(T_M-t)}V \geq f(k_t), \quad \forall t \leq T_M. \tag{3.2.2}$$

Let the optimum scheme be denoted by $\langle \{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^* \rangle$, where T_M^* is the time required to save $\bar{S}$ under this scheme, i.e.

$$\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*)-k_t^*]dt = \bar{S}.$$

Before proceeding further let us introduce the following technical definition.

Definition 3.1. *Given a scheme* $\langle\{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M\rangle$, *let* $k_{It}(\langle\{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M\rangle)$ *denote the maximum loan amount at* t, *such that DIC at* t *holds.*[8]

In the next lemma we identify the optimum loan amount and the part of net return to be saved at any instance t, where $0 \le t \le T_M^*$. We find that the MFI optimally chooses a contract such that the borrower's instantaneous savings is the maximum. It involves maximizing the instantaneous net return $f(k_t) - k_t$, given DIC, and setting α_t as high as possible.

Lemma 3.2. *Let Assumptions 3.1, 3.2 and 3.3 hold. The MFI chooses the optimal scheme* $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$ *such that the instantaneous savings* $\alpha_t^*[f(k_t^*) - k_t^*]$ *is the maximum:*

(a) It lends the efficient amount whenever that is DIC, otherwise it lends the maximum amount which is DIC; formally $k_t^* = \min\{k_{It}, k^e\}$ *for all* t,

(b) The borrower saves her entire net return with the MFI, formally $\alpha_t^* = 1$ *for all* t.

This is quite intuitive. First, given our assumptions that the discounting rate of the borrower and the interest rate on savings both equal r, and that her utility function is linear, she is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later. Second, given Assumption 3.2 and Lemma 3.1, her utility increases as the time required to save $\bar{S}$ decreases. These imply that the objective of the MFI is to maximise the instantaneous savings $\alpha_t[f(k_t) - k_t]$ which involves $k_t = k^e$ and $\alpha_t = 1$. So at the optimum, the MFI lends k^e whenever that is incentive compatible, otherwise, it lends k_{It} and sets $\alpha_t = 1$.

The only potential problem we need to address here is that this increase in instantaneous savings, especially setting $\alpha_t = 1$ may affect the DICs adversely. If it does so, then this lemma is not so obvious, but fortunately, that is not the case. This is again because of the fact that borrower is indifferent between consuming an amount now, and saving and consuming that amount later. In fact, observe an increase in the instantaneous savings relaxes the DICs.

We summarise the above discussion in the following proposition which gives us the MFI's optimum contract in this benchmark case.

Proposition 3.2. *Let Assumptions 3.1, 3.2 and 3.3 hold. The optimal scheme* $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$ *satisfies the following:*

(a) The borrower graduates and moreover she graduates as soon as the minimum required amount $\bar{S}$ *is accumulated,*

(b) The MFI lends the efficient amount k^e, *unless constrained by the incentive condition,*

(c) The borrower saves the maximum possible amount with the MFI at all t.

3.2.4 The Time Path of The Optimal Loan Scheme

We next characterise the time path of the optimal loan scheme. We demonstrate that the optimal loan scheme is (weakly) increasing over time. Further, we find that when the increase

[8]Observe, k_{It} is continuous in t.

in utility from graduation is not too large (defined formally later), the optimal loan amount initially increases and then remains constant (at the efficient level k^e) over time. For ease of reference, we call such a scheme *progressive with a cap*. This is of interest given that in reality (a) almost all the MFIs practise lending that is "progressive with a cap" and (b) Banerjee et al. (2015a) suggest that the increase in utility from any microfinance scheme is "modestly positive, but not transformative". So the prediction of this model conforms with the empirical findings. We also find that when this increase in utility from graduation is "transformative", the optimal loan scheme remains "constant" at the efficient level k^e. Again for ease of reference, we call such a scheme *constant*. Similarly, a loan scheme which keeps on increasing over time is termed *strictly progressive*.

Finally, we say that increase in utility from graduation is "modestly positive" when

$$\frac{f(k^e) - k^e}{rf(k^e)}V - \bar{S} < \frac{f(k^e) - k^e}{r}.$$

Otherwise, we say that increase in utility from graduation is "transformative". Now we are in a position to state the main result of this section:

Proposition 3.3. *(The Dynamics of the Optimal Loan Scheme): Let Assumptions 3.1, 3.2 and 3.3 hold.*

A. The optimal loan scheme is weakly progressive.

B. The optimal loan scheme is either progressive with a cap or constant:

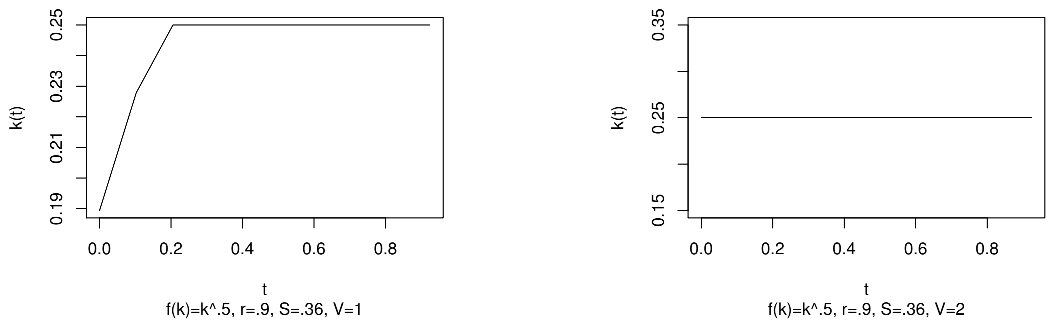
(i) The optimal loan scheme is "progressive with a cap" if and only if increase in utility from graduation is "modestly positive".

(ii) The optimal loan scheme is "constant" if and only if increase in utility from graduation is "transformative".

Intuitively, with the passage of time, on the one hand the borrower's savings increases, so that the loss from default increases, whereas on the other hand, the graduation date gets closer, so the present discounted value of lifetime utility from repayment increases. This ensures that the DICs get relaxed over time. Given Proposition 3.2(b), this implies that the optimal loan scheme is weakly progressive. So, there can be three cases – the optimal loan scheme is strictly progressive, progressive with a cap or constant.

When the increase in utility from graduation is *transformative*, then the present discounted value of lifetime utility from repayment is very high. This makes the efficient level of investment k^e incentive compatible from the very beginning. Thus in this case the optimum loan scheme is a constant. Correspondingly, when the increase in utility from graduation is *modestly positive*, it is not that attractive. This makes incentive for repayment weak – the efficient amount k^e is not incentive compatible, at least initially. Whether that would at all become incentive compatible or not depends on the parametric condition. It turns out that when $\bar{S}$ is not small, formally when assumption 3.3 is satisfied, the efficient amount k^e becomes incentive compatible towards the end.[9] Hence, this gives us the interesting result – when increase in utility from graduation is *modestly positive* the optimal loan scheme is "progressive with a cap".

[9]In other words, Assumption 3.3 precludes the possibility of optimal loan scheme to be strictly progressive.



(a) **Moderate V: Progressive with a cap** (b) **Large V: Constant Loan Scheme**

Figure 3.1: **The Optimal Loan Schemes under Different Parametric Values**

Remark 3.1. (Relaxation of Assumption 3.3) Suppose Assumption 3.3 is relaxed and consider the case where $f(k^e) \geq \bar{S}$. Recall at the optimum $k_t \leq k^e$. Therefore, at any $t \in [0, T_M]$ there can be two contingencies – $f(k^e) \geq f(k_t) \geq \bar{S}$ and $f(k_t) < \bar{S}$. This changes the DIC (utility from default is $f(k_t) - \bar{S} + V$ if $f(k_t) \geq \bar{S}$ and $f(k_t)$ otherwise.) Now observe in the former case, if the borrower defaults, her present discounted value of lifetime utility would be higher than that from repayment (recall, in case of repayment she graduates as soon as her savings becomes $\bar{S}$). So, she would never repay such a k_t. Thus, when $\bar{S} \leq f(k^e)$ the efficient amount k^e never becomes DIC which implies strict progressivity in the optimal loan scheme – it keeps on increasing over time without reaching the efficient amount. For ease of exposition, in the benchmark case we make this assumption and relax it in the next section.

Remark 3.2. (Concave utility function of the borrower) How robust is the analysis if the utility function of the borrower is strictly concave? Given there is no uncertainty, concavity in utility function implies that the borrower has a preference for consumption smoothing over time. We conjecture that the optimal loan scheme would continue to be progressive. We do not allow for dissaving[10] which implies that the borrower saves only when she graduates. In both the cases an amount which is incentive compatible at some instance remains incentive compatible in the future as well. Hence the conjecture.

Remark 3.3. (Profit-maximizing MFI) As discussed in the **??** section, Liu and Roth (2017) show that in their framework a profit-maximizing MFI optimally designs a contract such that a poor borrower can never graduate. In our framework, whether a borrower would be able to graduate or not is an open question. Intuitively graduation may or may not happen because there are two opposing forces in play. On the one hand, as a borrower graduates, the MFI loses a client. So depending on its outside option, availability of a new borrower for example, the MFI may choose to not give up a potential source of revenue.[11] On the other hand, when a borrower graduates a higher amount of loan becomes DIC towards the end in comparison to the case where she never graduates (as present discounted value of lifetime utility from repayment is higher in the former case). Since the profit of the MFI is increasing in loan amount, the MFI

[10]It is without loss of generality only when the borrower graduates.

[11]Also note that even if new borrowers are available, profit from a new borrower can be substantially lower than that from an old borrower.

may want her to graduate. If the latter effect dominates then at the optimum the borrower does graduate, otherwise she does not. Nevertheless, we conjecture that the optimum loan scheme continues to be progressive. While a complete analysis is beyond the scope of this paper, it can be shown that this is indeed true. Intuitively, since we do not allow for dissaving[12] at the optimum the MFI takes savings only if the borrower graduates. In both the cases an amount which is incentive compatible at some instance remains incentive compatible in the future as well. Hence the conjecture.[13]

3.3 General Framework: Allowing Another Savings Institution and A General Confiscation Rule

In this section, we introduce two changes in our basic framework which improve the borrower's utility from default: First, the borrower has access to a savings technology on her own – she can save with a Savings Institution (SI hereafter), at the same instantaneous interest rate provided by the MFI.[14] Second, the MFI cannot confiscate her entire savings with it even in case of default – it has to return at least $\underline{\gamma}$ part of that (along with interest), where $\underline{\gamma}$ is exogenously given and $0 < \underline{\gamma} < 1$.[15] These enable a borrower to graduate even in case of default – she can save a part of the amount with which she defaults, and graduate using that. Thus incentive for repayment is harder to satisfy here. The objective is to check whether our central result – progressivity in loan size, survives or not. Also, we relax Assumption 3.3, that is we allow for the case where $\bar{S} \leq f(k^e)$.

3.3.1 Contracts and Timeline

At $t = 0$, the MFI announces a contract $\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM} \rangle$,[16] where like before k_{st}, α_{st} and T_{sM} denote the loan amount, the part of the net return to be saved with the MFI at the instance t and the 'successful' termination date of the contract, respectively. γ denotes the part of savings, the borrower gets back from the MFI, in case of default; $\gamma \in [\underline{\gamma}, 1]$. Hence, in case of default, like before, the MFI terminates the credit contract and withdraws access to both the production and savings technologies provided by the MFI, but unlike the benchmark case it returns a part of the her savings with it till date $S_t^D \equiv \gamma \int_0^t e^{r(t-t')} \alpha_{st'} [f(k_{st'}) - k_{st'}] dt'$.

The individual either accepts or rejects the MFI contract, with the game ending in case she rejects. If she accepts, then in the next stage at $t = 0$, she chooses $\langle \{ \{\sigma_t^R\}_{t=0}^{T_{sM}}, \sigma_t^D \}, \{T_B^R, T_B^D(t)\} \rangle$, where σ_t^R denotes the part she wants to save with the SI at any arbitrary t, after repaying and saving with the MFI (or getting back her savings from the MFI which happens at T_{sM}), and σ_t^D denotes the part of $f(k_{st}) + S_t^D$ she wants to save with the SI after defaulting at t; where

[12] This assumption is *not* without loss of generality here.

[13] In the section marked For the Referee, we provide an argument for progressivity.

[14] Perhaps, it is more natural to assume that the interest rate provided by the SI is lower than that provided by the MFI, but we assume them to be equal as this is a robustness check exercise, and the borrower's outside option (weakly) increases with the interest rate provided by the SI.

[15] In the benchmark case $\underline{\gamma} = 0$.

[16] To distinguish the notations from the benchmark case, we subscript the variables of this general framework with 's', where s denotes the fact that in this framework we are allowing for a savings technology other than MFI.

$0 \leq t \leq T_{sM}$ and $0 \leq \sigma_t^R, \sigma_t^D \leq 1$. Similarly, T_B^R denotes the date at which she withdraws her savings from the SI in case she always repays (and saves), and $T_B^D(t)$ denotes the date of withdrawal of savings from the SI, in case she defaults at t.[17]

Given the MFI-contract, and the borrower's strategy, the continuation game at any instance t, where $0 \leq t \leq T_{sM}$, is as follows:

Stage 1: The MFI lends k_{st} which the borrower invests in the $f(\cdot)$ technology and gets $f(k_{st})$ instantaneously.

Stage 2: She then decides whether to repay, or not:

(i) In case she decides to repay, she returns k_{st} to the MFI, deposits $\alpha_{st}[f(k_{st}) - k_{st}]$ with the MFI, and $\sigma_t^R(1-\alpha_{st})[f(k_{st}) - k_{st}]$ with the SI, with all deposits attracting interest at the instantaneous rate r. She consumes the rest instantaneously and the game continues.

(ii) In case of default the MFI terminates the contract, and withdraws the borrower's access to both $f(\cdot)$ and the MFI-savings technology. She obtains her current gross income $f(k_{st})$ and unlike the benchmark case, she also gets back S_t^D – a part of her savings with the MFI till date (along with interest). She, then, saves a part σ_t^D of $f(k_{st}) + S_t^D$ with the SI and consumes the rest instantaneously.
She withdraws her savings from the SI at $T_B^D(t)$, if any.

In case of successful termination of the contract at T_{sM}, the borrower gets back her entire savings along with interest, from the MFI. She saves a part $\sigma^R_{T_{sM}}$ of that return with the SI and consumes the rest instantaneously. She withdraws her savings from the SI, at T_B^R, if any.
Finally, she graduates, if she wishes to and has at least $\bar{S}$ amount with her.

We now define a term "money in hand" which we will be using repeatedly in the subsequent analysis. *Money in hand* at any t denotes the entire amount to which the borrower has access at t.

- At the time of successful termination of the contract T_{sM} money in hand includes her entire savings with the MFI as well as that with the SI, formally

$$\int_0^{T_{sM}} e^{r(T_{sM}-t)}\alpha_{st}[f(k_{st}) - k_{st}]dt + \int_0^{T_{sM}} e^{r(T_{sM}-t)}\sigma_t^R(1-\alpha_{st})[f(k_{st}) - k_{st}]dt.$$

- In case of repayment at t, where $t \leq T_{sM}$, money in hand includes the amount she has after repaying and saving with the MFI at that instance as well as her savings with the

[17] Observe that we have not considered the case where the borrower withdraws savings from the SI multiple times. Also we have assumed $\sigma_t^R, \sigma_t^D \leq 1$, these imply that we have not allowed dissavings in this framework also. However, due to our assumptions that the utility function is linear and that the future is discounted in the same way as the interest rate on savings, these are without loss of generality. Furthermore, at this point we are not imposing that $T_B^R \geq T_{sM}$ or $T_B^D(t) \geq t$; if $T_B^R < T_{sM}$ then $\{\sigma_t^R\}_{t=T_B^R}^{T_{sM}} = 0$ and if $T_B^D(t) < t$ then $\sigma_t^D = 0$ and $\{\sigma_{t'}^R\}_{t'=T_B^D(t)}^{t} = 0$. Also observe, T_B^R and $T_B^D(t)$ are well defined only when she has positive amount of savings with the SI. Formally, when there exists an interval $[\underline{t}, \bar{t}] \subseteq [0, T_{sM}]$ such that $[\underline{t}, \bar{t}]$ has a positive measure and the borrower's savings with the SI is positive for all $t \in [\underline{t}, \bar{t}]$.

SI till date, formally

$$(1-\alpha_{st})[f(k_{st})-k_{st}]+\int_0^t e^{r(t-t')}\sigma^R_{t'}(1-\alpha_{st'})[f(k_{st'})-k_{st'}]dt'.$$

- In case of default at t, where $t \leq T_{sM}$, money in hand includes gross return $f(k_{st})$, the savings she gets back from the MFI i.e. S^D_t and her savings with the SI till date, formally

$$f(k_{st})+\gamma\int_0^t e^{r(t-t')}\alpha_{st'}[f(k_{st'})-k_{st'}]dt'+\int_0^t e^{r(t-t')}\sigma^R_{t'}(1-\alpha_{st'})[f(k_{st'})-k_{st'}]dt'.$$

- Finally at any t after the termination date, money in hand includes her savings with the SI.

3.3.2 Analysis: The Borrower's Problem

We begin by solving the borrower's problem. Given any MFI-contract $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle$, her objective is to choose her strategy: {Repay, Default} and $\langle\{\{\sigma^R_t\}_{t=0}^{T_{sM}},\sigma^D_t\},\{T^R_B,T^D_B(t)\}\rangle$ to maximise her present discounted value of lifetime utility. Given Assumption 3.2, that is graduation is welfare improving and MFI-contract her objective is to choose her strategy such that the time required for graduation is minimised. Now it is clear that, she would withdraw her savings from the SI as soon as money in her hand becomes $\bar{S}$. More specifically, if money in her hand at the termination date of the MFI-contract, irrespective of whether that was terminated successfully or because of default, is no less than $\bar{S}$, she withdraws her savings from the SI immediately, otherwise she waits till her savings becomes $\bar{S}$ and withdraws immediately.

Turning to the choice of optimal $\{\sigma^R_t\}_{t=0}^{T_{sM}}$ and σ^D_t, note that it is weakly dominant to always save as much as possible, as that may decrease the time required for graduation. More specifically, due to our assumptions of linear utility function and that she discounts future in the same way as the interest rate on savings, she is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later. But she gets strictly better off if that savings decrease the time required for graduation. Hence, the borrower is weakly better-off when she saves the maximum amount possible with the SI, she is strictly better off if that decreases the time required for graduation.

Finally, it may happen that her choice of $\langle\{\{\sigma^R_t\}_{t=0}^{T_{sM}},\sigma^D_t\},\{T^R_B,T^D_B(t)\}\rangle$ does not affect the time of graduation. Then she is indifferent between saving and not saving with the SI. Even if she saves, the time of withdrawal of her savings from the SI does not affect her present discounted value of lifetime utility. Given these indifferences, without loss of generality, we assume that she withdraws her savings from the SI, if any, at the termination date of the MFI-contract and till then saves the maximum amount possible with the SI. Hence, the next proposition.

Proposition 3.4. *Let Assumption 3.2 hold. Given any MFI-contract $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle$, the borrower chooses $\langle\{\{\sigma^R_t\}_{t=0}^{T_{sM}},\sigma^D_t\},\{T^R_B,T^D_B(t)\}\rangle$ such that she can graduate as soon as possible.*

Therefore, the borrower does not consume anything before graduating. Hence, given any MFI-contract $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle$, the borrower's present discounted value of lifetime utility (evaluated at $t = 0$) from repayment is

$$e^{-rT_B^{R^*}}\left[\int_0^{T_{sM}} e^{r(T_B^{R^*}-t)}[f(k_{st})-k_{st}]dt - \bar{S} + V\right]. \tag{3.3.1}$$

Similarly, her present discounted value of lifetime utility (evaluated at $t = 0$) from default at t, where $0 < t \leq T_{sM}$ is

$$e^{-rT_B^{D^*}(t)}\left[e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}(1-\alpha_{st'})[f(k_{st'})-k_{st'}]dt' + \int_0^t e^{r(t-t')}\gamma\alpha_{st'}[f(k_{st'})-k_{st'}]dt' + f(k_{st})\Big] - \bar{S} + V\right],$$

where the first term i.e. $\displaystyle\int_0^t e^{r(t-t')}(1-\alpha_{st'})[f(k_{st'})-k_{st'}]dt'$ represents the amount she had already saved with the SI till t and the next two terms i.e. $\displaystyle\int_0^t e^{r(t-t')}\gamma\alpha_{st'}[f(k_{st'})-k_{st'}]dt' + f(k_{st})$ represent the amount which she saves at t. Hence, her present discounted value of lifetime utility (evaluated at $t = 0$) if she defaults at $t \in (0, T_{sM}]$ is

$$e^{-rT_B^{D^*}(t)}\left[e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}(1-\alpha_{st'}(1-\gamma))[f(k_{st'})-k_{st'}]dt' + f(k_{st})\Big] - \bar{S} + V\right]. \tag{3.3.2}$$

3.3.3 Analysis: The Optimal Contract

Next, we characterise the optimal contract. Given the optimal strategy of the borrower, the problem of the MFI is to choose $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$ such that the borrower's present discounted value of lifetime utility is maximised and the borrower always repays. As observed above given Assumption 3.2, the objective of the MFI boils down to minimizing the time required to graduate, provided that the borrower repays.

The optimal contract in this framework is very similar to that in the benchmark case, we discuss that as we go along. But an interesting point to note here is that the MFI may choose to terminate the contract, successfully, before the borrower's total savings become $\bar{S}$. Since, the borrower has access to a savings technology on her own, she would be able to save and graduate even in that case. However, that would increase the time required to graduate vis-à-vis the case where the MFI lends till the time of graduation. Hence, the MFI would terminate a contract before her total savings become $\bar{S}$ only if lending till the end is not incentive compatible.

Note that it is not obvious whether a contract where the MFI lends till the borrower's total savings become $\bar{S}$ is DIC or not, because of the following reason. Lending till the end results early graduation. But that is true not only when the borrower repays but also when she defaults – her savings with the MFI increases over time, so the amount she gets back in case of default also increases over time. So on the one hand, lending till the end improves the incentive to repay. On the other hand, towards the end of the contract incentives to default also increases. Despite this trade-off, we argue by contradiction that it is possible to construct a DIC contract where the MFI lends till the borrower's total savings become $\bar{S}$ so that she graduates immediately after the successful termination of the contract. This new contract provides her higher utility, hence, the original contract cannot have been optimum. The formal proof can be found in Appendix A.

Now given assumption 3.2, since the objective of the MFI is to minimise the time required for graduation, it terminates the contract as soon the borrower's total savings become $\bar{S}$. The proof is very similar to that of Lemma 3.1, so we skip that. The following lemma characterises the "successful" termination date.

Lemma 3.3. *Suppose Assumptions 3.1 and 3.2 hold. At the optimum, the borrower graduates as soon as the MFI terminates the contract.*

Now we introduce the following observation which argues that, in any DIC contract, the money in the borrower's hand, in case of default at any t, where $0 < t < T_{sM}$, must be less than $\bar{S}$. The reason being – given Proposition 3.4, we know that otherwise, in case of default at such a t the borrower would graduate immediately, with no less than $\bar{S}$ amount, whereas, in case of repayment she graduates at $T_{sM} > t$ with exactly $\bar{S}$. So, DIC at t will definitely be violated. Hence, the observation.

Observation 3.1. *Let, $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$ be a DIC contract. The money in the borrower's hand in case of default at any t, where $0 < t < T_{sM}$, must be less than $\bar{S}$. The money in her hand in case of default at T_{sM} can be no higher than $\bar{S}$.*

So given Proposition 3.4, DIC at any t, where $0 < t \leq T_{sM}$, boils down to

$$e^{-r(T_{sM}-t)}V \geq e^{-r(T_B^{D^*}(t)-t)}V.$$

This further implies that in a DIC contract, in case of default the borrower cannot graduate at an earlier date than that in case she repays always.

Now like before we introduce the following technical definition.

Definition 3.2. *Given a scheme $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$, let $k_{sIt}\big(\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle\big)$ denote the maximum loan amount at t, such that DIC at t holds.*

In the next lemma we identify the optimum loan amount k_{st}, the part of net return α_{st} to be saved with the MFI at any instance t, where $0 \leq t \leq T^*_{sM}$ and the part of savings γ the borrower gets back from the MFI in case of default. We find that the MFI optimally chooses a contract such that the borrower's instantaneous savings is the maximum. It involves maximizing the instantaneous net return $f(k_{st}) - k_{st}$, setting α_{st} as high as possible and γ as low possible.

Lemma 3.4. *Let Assumptions 3.1 and 3.2 hold. The optimal scheme $\langle\{\alpha^*_{st}\}_{t=0}^{T^*_{sM}}, \gamma^*, \{k^*_{st}\}_{t=0}^{T^*_{sM}}, T^*_{sM}\rangle$ satisfies the following:*

(a) It lends the efficient amount whenever doing so is DIC, otherwise it lends the maximum amount which is DIC. Formally, $k^*_{st} = \min\{k_{sIt}, k^e\}$ for all t.

(b) The borrower saves her entire net return with the MFI: $\alpha^*_{st} = 1$ for all t.

(c) In case of default, the MFI confiscates the maximum amount of savings: $\gamma^* = \underline{\gamma}$.

The intuition behind this result is in a similar vein – given Assumption 3.2 the objective of the MFI is to minimise the time required to graduate which in turn requires maximising the instantaneous savings. Thus at the optimum it lends to maximise the net return, given DIC which implies lending the efficient amount k^e whenever that is incentive compatible otherwise the maximum amount which is that.

Two points to note are as follows – First, *given* any loan scheme the choice of α_{st} does not affect the borrower's savings. This is because, the borrower saves the rest, whatever she has after repaying and saving with the MFI, with the SI. Moreover, the interest rates provided by the MFI and SI are equal. But low α_{st}, that is higher savings with the SI affects DIC adversely – the maximum loan amount which is DIC decreases with decrease in α_{st}. Hence, the optimal loan amount and correspondingly the instantaneous savings (weakly) decrease with decrease in α_{st}. Therefore, the MFI chooses α_{st} as high as possible. Given the limited liability condition it chooses $\alpha^*_{st} = 1$.

The second point of interest follows from the same line of argument. The MFI chooses γ as low as possible because increase in γ means she gets higher amount of savings from the MFI in case of default. This improves her deviation payoff and that in turn adversely affects DIC. Hence, the instantaneous savings (weakly) decreases with increase in γ. Therefore the MFI chooses γ as low as possible: $\gamma^* = \underline{\gamma}$.
We summarise the optimal contract in the following proposition.

Proposition 3.5. *Let Assumptions 3.1 and 3.2 hold. The optimal scheme $\langle\{\alpha^*_{st}\}_{t=0}^{T^*_{sM}}, \gamma^*, \{k^*_{st}\}_{t=0}^{T^*_{sM}}, T^*_{sM}\rangle$ satisfies the following:*

(a) The MFI terminates the contract as soon as the borrower's savings becomes $\bar{S}$ and she graduates immediately, that is at T^*_{sM},

(b) It lends the efficient amount k^e, unless constrained by the incentive condition,

(c) In case of repayment, the borrower saves her entire net return with the MFI,

(d) In case of default, the MFI confiscates the borrower's savings as much as it can.

Observe, the optimum loan contract is unique.

3.3.4 The Time Path of the Optimal Loan Scheme

We next turn to the dynamics of the optimal loan size. Like in the benchmark case, here, we characterise the time path of the optimal loan scheme. Unlike the benchmark case, the optimal loan scheme is not always (weakly) progressive, specifically it need not be (weakly) progressive when $\underline{\gamma}$ is very high. We provide a sufficient condition which ensures that DIC gets relaxed over time and hence the optimal loan amount (weakly) increases over time.

Assumption 3.4. $r \geq \underline{\gamma}$.

Intuitively, the borrower saves her entire net return with the MFI. Now with marginal increase in time her savings increase by the rate of interest i.e. r. The time remaining to graduate decreases and the borrower's present discounted value of lifetime utility increases marginally by the future discount rate which is again r. Due to this, the maximum incentive compatible loan amount increases by $rf(k_{sIt})$. But in case of default the borrower also gets back a part of her savings with the MFI which increases with time. To be precise, with marginal increase in time the amount she gets back from the MFI marginally increases by $\underline{\gamma}[f(k_{st}) - k_{st}]$. This dampens her incentive to repay. Assumption 3.4 ensures that the DICs get relaxed over time. Therefore, given this assumption the optimal loan scheme is weakly progressive.

Depending on the value of $\bar{S}$, there can be three cases – (a) The loan scheme is strictly increasing, (b) progressive with a cap and (c) constant. Here $\bar{S}$ plays such a crucial role because now the borrower can graduate on her own in case of default. Thus when $\bar{S}$ is "low", the efficient loan amount never becomes incentive compatible and the optimal loan scheme is strictly increasing. Converesly, when $\bar{S}$ is "large", the efficient loan amount is always incentive compatible and hence the optimal loan scheme remains constant at the efficient level throughout. Finally, when $\bar{S}$ is "moderate" the optimal loan scheme is progressive with a cap – initially increases till it reaches the efficient amount and then remains constant at that.

We say that $\bar{S}$ is "low" when $(1-\underline{\gamma})\bar{S} < f(k^e)$. Similarly we say that it is "moderate" when $\dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S} \leq f(k^e) \leq (1-\underline{\gamma})\bar{S}$, and "large" when $f(k^e) \leq \dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S}$.

Now we characterise the time path of the optimal loan scheme.

Proposition 3.6. *(The Dynamics of the Optimal Loan Scheme): Let,* $\langle \{\alpha^*_{st}\}_{t=0}^{T^*_{sM}}, \gamma^*, \{k^*_{st}\}_{t=0}^{T^*_{sM}}, T^*_{sM}\rangle$ *be the optimal contract and Assumptions 3.1, 3.2 and 3.4 hold.*

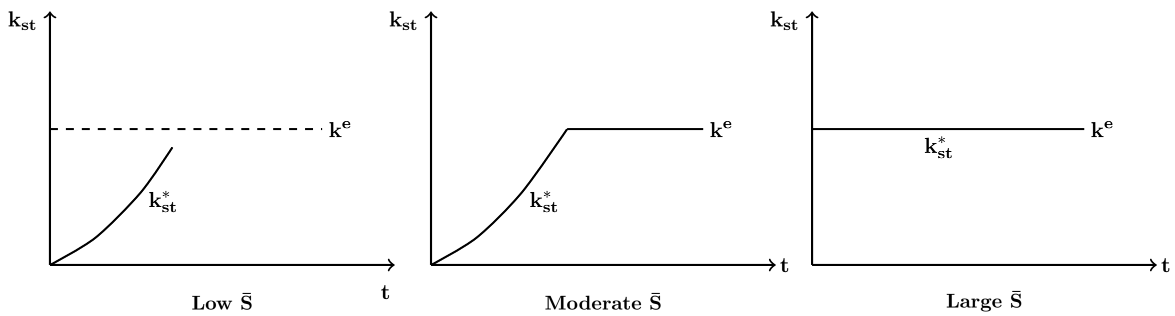
A. Weakly Progressive: The optimal loan scheme is always weakly progressive.

B. Depending on the value of $\bar{S}$ the optimal loan would be strictly progressive, progressive with a cap or constant.

(i) Strictly Progressive: The optimal loan scheme is strictly progressive if and only if $\bar{S}$ is low,

(ii) Progressive with a cap: The optimal loan scheme is progressive with a cap if and only if $\bar{S}$ is moderate,

(iii) Constant: The optimal loan scheme is constant if and only if $\bar{S}$ is large.



3.4 Comparison Between the Benchmark and the General Case: Welfare Implication

In this section, we compare the optimal outcomes and the borrower's welfare in the general case with those in the benchmark case. The borrower's outside option is higher in the general case – she can save on her own and gets back a part of her savings with the MFI in case of default. Apparently it may seem that it would improve her welfare, but actually it makes her worse off.

The reason is that the MFI is benevolent and the only problem here is that the borrower is strategic and does not repay whenever she has an incentive to do so. Under this general framework, her deviation payoff is larger which decreases her incentive to repay. The optimal loan amount and hence the instantaneous savings must then be (weakly) lower in the general case. This increases the time required for graduation. Hence, the borrower is weakly worse off in the general case. She is strictly worse off whenever $\bar{S}$, the fixed initial investment required to start the technology $\langle V, \bar{S} \rangle$ is not large, ensuring that providing a loan of k^e from $t = 0$ is not incentive compatible.

Interestingly observe, along the equilibrium path the borrower actually does not save with the SI or does not default (and hence does not get back any savings from the MFI before the successful termination date). Hence these extensions only improve her deviation payoff and that makes her (weakly) worse off.

Proposition 3.7. *Let Assumptions 3.1, 3.2, 3.3 and 3.4 hold. The borrower's present discounted value of lifetime utility is weakly lower in the general case than that in the benchmark case. It is strictly lower if and only if the investment required to start the technology $\langle V, \bar{S} \rangle$ is not large:*

$$f(k^e) > \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} \bar{S}.$$

This is not to claim that developmet of savings intitutions should not be encouraged. In fact, papers like Burgess and Pande (2005), Ashraf et al. (2006b), Dupas and Robinson (2013a,b) find positive impacts of savings on the poor people. What Proposition 3.7 shows however is that there can be some unintended consequences of doing so in this scenario.

3.5 Conclusion

Many scholars Armendàriz and Morduch (2005), Roodman (2009) among others argue that MFIs should provide not only credit but also other financial services like savings, insurance etc. In fact Rhyne (November 2, 2010) directly links *Andhra-crisis* to lack of deposit collection. Many MFIs are broadening their initial focus on microcredit to include the provision of savings (and other) products (Karlan et al., 2014).

In this paper we develop a theoretical model where the MFI provides not just credit but also access to other services in particular a savings facility. This savings service coupled with the credit service help a poor borrower to accumulate a lumpsum amount which enables her to graduate. Thus we provide *one* explanation where savings coupled with credit indeed improve borrower's utility beyond the level achievable when only credit is provided. We find that the optimal loan scheme is (weakly) progressive and when the increase in utility from graduation is "modestly positive" the optimal loan scheme is progressive with a cap – loan size initially increases and then remains constant at the efficient level of investment which conforms with reality.

Appendix A

Proof of Proposition 3.1. We show that given Assumptions 3.1, 3.2 and 3.3, at the optimum the borrower graduates. We show this by contradiction.

Suppose not and there exists an optimum contract $\langle \{k_t^*\}_{t=0}^{T_M^*}, \{\alpha_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$[18] such that the borrower does not graduate. So, her present discounted value of lifetime utility is

$$\int_0^{T_M^*} e^{-rt}(1-\alpha_t^*)[f(k_t^*)-k_t^*]dt + e^{-rT_M^*}\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*)-k_t^*]dt$$

And, DIC at any t, where $0 \leq t \leq T_M^*$, is

$$\int_t^{T_M^*} e^{-r(t'-t)}(1-\alpha_{t'}^*)[f(k_{t'}^*)-k_{t'}^*]dt' + e^{-r(T_M^*-t)}\int_0^{T_M^*} e^{r(T_M^*-t')}\alpha_{t'}^*[f(k_{t'}^*)-k_{t'}^*]dt' \geq f(k_t^*).$$

Now there can be two cases (a) $\exists \tilde{t} \leq T_M^*$ such that $\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(k_t^*)-k_t^*]dt \geq \bar{S}$ and (b) there does not exist any such $\tilde{t}$. We argue that in each of the cases, it is possible to construct another DIC contract such that the borrower graduates and her lifetime utility is higher, so the original contract cannot have been optimum.

Case (a) $\boldsymbol{\exists \tilde{t} \leq T_M^*}$ **such that** $\boldsymbol{\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(k_t^*)-k_t^*]dt \geq \bar{S}}$

Define a new contract $\langle \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M\rangle$ as follows

$$\begin{cases} \hat{k}_t = k_t^* \text{ and } \hat{\alpha}_t = 1 & \forall t \in [0, \hat{T}_M] \\ \text{and } \hat{T}_M \text{ is such that} & \int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(k_t^*)-k_t^*]dt = \bar{S}. \end{cases}$$

Observe, by Intermediate Value Theorem such a $\hat{T}_M$ exists and $\hat{T}_M \leq T_M^*$. Under this new contract, the borrower graduates at $\hat{T}_M$ as that gives her higher utility:

$$\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt - \bar{S} + V > \int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt.$$

The left hand side is her lifetime utility if she graduates and the right hand side is that from consuming the amount immediately. We get this inequality because $V - \bar{S} > 0$. Now we show that her present discounted value of lifetime utility under this new contract is higher than

[18] Here we want to point out that none of the results depend on our restriction $\alpha_t \geq 0$, i.e. even if we allow for dissaving all the results go through. This is of particular interest because this restriction implies that in case the borrower does not graduate i.e. $T_M = \infty$ there will be no savings. But if we allow for dissaving, the MFI may take deposits even when $T_M = \infty$ and return at certain instances which may relax DIC and improve borrower welfare. However, this assumption is without loss of generality because even if we allow for dissaving, the MFI would enable the borrower to graduate as soon as possible and set α_t as high as possible. The proof is available on request.

that from the original contract. Her present discounted value of lifetime utility under this new contract is

$$\begin{aligned}
& e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt - \bar{S} + V\right] \\
> & e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt\right] + e^{-r\hat{T}_M}\frac{1}{r}[f(k^e)-k^e] \\
= & e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(k^*_t)-k^*_t]dt\right] + e^{-r\hat{T}_M}\int_0^{\infty} e^{-rt}[f(k^e)-k^e]dt \\
\geq & \int_0^{\hat{T}_M} e^{-rt}[f(k^*_t)-k^*_t]dt + \int_{\hat{T}_M}^{T^*_M} e^{-rt}[f(k^*_t)-k^*_t]dt \\
= & \int_0^{T^*_M} e^{-rt}[f(k^*_t)-k^*_t]dt \\
= & \int_0^{T^*_M} e^{-rt}(1-\alpha^*_t)[f(k^*_t)-k^*_t]dt + e^{-rT^*_M}\int_0^{T^*_M} e^{r(T^*_M-t)}\alpha^*_t[f(k^*_t)-k^*_t]dt.
\end{aligned}$$

where the first inequality is coming from Assumption 3.2 and the second inequality is coming from the definition of k^e.[19] The last expression is the borrower's present discounted value of lifetime utility under the original contract. Now it is immediate that DICs at all t where $0 \leq t \leq \hat{T}_M$ are satisfied. So, the original contract cannot have been optimum.

Case (b) $\nexists \tilde{\mathbf{t}} \leq \mathbf{T^*_M}$ such that $\int_0^{\tilde{\mathbf{t}}} \mathbf{e^{r(\tilde{t}-t)}[f(k^*_t)-k^*_t]dt \geq \bar{S}}$

An optimum contract must be non-trivial in that there must exist finite $t \leq T^*_M$ such that $k^*_t > 0$. To construct the new contract, we follow the algorithm below. Consider any finite T and define $\hat{t} = \{\text{minimum } t \in [0,T] | f(k^*_t) - k^*_t \geq f(k^*_{t'}) - k^*_{t'} \ \forall t' \in [0,T]\}$.[20] In other words, $f(k^*_{\hat{t}}) - k^*_{\hat{t}} > f(k^*_t) - k^*_t \ \forall t \in [0,\hat{t})$ and $f(k^*_{\hat{t}}) - k^*_{\hat{t}} \geq f(k^*_t) - k^*_t \ \forall t \in [\hat{t},T]$. Now compute

$$\int_0^{\hat{t}} e^{r(T-t)}[f(k^*_t)-k^*_t]dt + \int_{\hat{t}}^{T} e^{r(T-t)}[f(k^*_{\hat{t}})-k^*_{\hat{t}}]dt.$$

(A) If that amount is no less than $\bar{S}$, stop the algorithm and define the new contract as follows:

$$\begin{cases}
\hat{k}_t = k^*_t \quad \forall t \in [0,\hat{t}) \quad \text{and } \hat{k}_t = k^*_{\hat{t}} \ \ \forall t \in [\hat{t},\hat{T}_M] \\
\hat{\alpha}_t = 1 \quad \forall t \in [0,\hat{T}_M] \\
\text{and } \hat{T}_M \text{ is such that } \quad \int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt = \bar{S}.
\end{cases}$$

(B) If that amount is less than $\bar{S}$, increase T and follow the same algorithm until we get (A). Observe, since we consider only non-trivial contracts, that such a T exists follows from the the Intermediate Value Theorem since,

$$\operatorname*{Limit}_{T\to 0}\int_0^{T} e^{r(T-t)}[f(\hat{k}_t)-\hat{k}_t]dt = 0 \quad \text{and} \quad \operatorname*{Limit}_{T\to \infty}\int_0^{T} e^{r(T-t)}[f(\hat{k}_t)-\hat{k}_t]dt = \infty.$$

[19] Recall, k^e solves $\operatorname*{argmax}_k \ [f(k)-k]$.

[20] k_t is continuous in t and $[0,T]$ is bounded, so from Weierstrass Theorem such a t exists.

Also observe, since $\nexists \tilde{t} \leq T_M^*$ such that $\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(k_t^*) - k_t^*]dt \geq \bar{S}$, $\hat{t} < \hat{T}_M$.

Mimicing the argument in case (a) it can be shown that under this new contract the borrower graduates at $\hat{T}_M$. Now we show that her present discounted value of lifetime utility under this new contract is higher than that from the original contract. Her present discounted value of lifetime utility under this new contract is

$$
\begin{aligned}
& e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t) - \hat{k}_t]dt\right] + e^{-r\hat{T}_M}(V - \bar{S}) \\
> & e^{-r\hat{T}_M}\left[\int_0^{\hat{t}} e^{r(\hat{T}_M-t)}[f(\hat{k}_t) - \hat{k}_t]dt + \int_{\hat{t}}^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t) - \hat{k}_t]dt\right] + e^{-r\hat{T}_M}\int_0^{\infty} e^{-rt}[f(k^e) - k^e]dt \\
\geq & \int_0^{\hat{t}} e^{-rt}[f(k_t^*) - k_t^*]dt + \int_{\hat{t}}^{\hat{T}_M} e^{-rt}[f(k_t^*) - k_t^*]dt + \int_{\hat{T}_M}^{T_M^*} e^{-rt}[f(k_t^*) - k_t^*]dt \\
= & \int_0^{T_M^*} e^{-rt}[f(k_t^*) - k_t^*]dt \\
= & \int_0^{T_M^*} e^{-rt}(1-\alpha_t^*)[f(k_t^*) - k_t^*]dt + e^{-rT_M^*}\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*) - k_t^*]dt.
\end{aligned}
$$

where again the first inequality is coming from Assumption 3.2, the second inequality is coming from the construction and the definition of k^e.[21] The last expression is the borrower's present discounted value of lifetime utility under the original contract. Now it is immediate that DICs at all t where $0 \leq t \leq \hat{T}_M$ are satisfied. So, the original contract cannot have been optimum. ■

Proof of Lemma 3.1. Let $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle$ be an optimum contract, and suppose to the contrary the accumulated savings at T_M exceeds $\bar{S}$. We construct a new contract $\langle \{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M \rangle$ such that both the constraints are satisfied and the borrower's present discounted value of lifetime utility is higher under the new scheme, so that the original contract cannot have been the optimum. The new scheme is as follows:

$$
\begin{cases}
\hat{\alpha}_t = \alpha_t \quad \forall t \in [0, \hat{T}_M], \\
\hat{k}_t = k_t \quad \forall t \in [0, \hat{T}_M], \text{ and} \\
\hat{T}_M = T_M - \Delta, \quad \text{such that } \Delta > 0 \text{ and } \displaystyle\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}\hat{\alpha}_t[f(\hat{k}_t) - \hat{k}_t]dt \geq \bar{S}.
\end{cases}
$$

In step 1, we show that the borrower's present discounted value of lifetime utility is higher under this new contract and then in step 2, we show that the new contract is DIC.

Step 1. The borrower's present discounted value of lifetime utility, if she always repays, is

$$
\int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t) - k_t]dt + e^{-rT_M}\left[\int_0^{T_M} e^{r(T_M-t)}\alpha_t[f(k_t) - k_t]dt - \bar{S} + V\right]
$$

Due to our assumptions that future is discounted similarly as the interest on savings and that

[21] By construction, $f(k_t^*) - k_t^* = f(\hat{k}_t) - \hat{k}_t \; \forall t \in [0, \hat{t}]$ and $f(k_t^*) - k_t^* \leq f(\hat{k}_t) - \hat{k}_t \; \forall t \in (\hat{t}, \hat{T}_M]$. And from the definition of k^e, $\int_{\hat{T}_M}^{\infty} e^{-rt}[f(k^e) - k^e]dt \geq \int_{\hat{T}_M}^{T_M^*} e^{-rt}[f(k_t^*) - k_t^*]dt$.

the borrower's utility function is linear, observe it can be written as

$$\int_0^{T_M} e^{-rt}[f(k_t) - k_t]dt + e^{-rT_M}\Big[V - \bar{S}\Big]. \tag{3.5.1}$$

This is essentially saying that the borrower is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later.
Partially differentiating (3.5.1) with respect to T_M from the left we get:

$$e^{-rT_M}[f(k_{T_M}) - k_{T_M}] - re^{-rT_M}(V - \bar{S}) = -re^{-rT_M}\Big[V - \bar{S} - \frac{[f(k_{T_M}) - k_{T_M}]}{r}\Big] < 0,$$

where the inequality follows since given Assumption 3.2 and the definition of k^e

$$V - \bar{S} > \frac{f(k^e) - k^e}{r} \geq \frac{f(k_{T_M}) - k_{T_M}}{r}.$$

Step 2. Finally, we argue that the DICs for the new scheme $\langle\{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M\rangle$ hold for all $t \leq \hat{T}_M$. Consider some $t \leq \hat{T}_M$, the L.H.S. of the DIC at t equals

$$\begin{aligned}
&\int_t^{\hat{T}_M} e^{-r(t^{'}-t)}(1-\hat{\alpha}_{t^{'}})[f(\hat{k}_{t^{'}}) - \hat{k}_{t^{'}}]dt^{'} + e^{-r(\hat{T}_M - t)}\Bigg[\int_0^{\hat{T}_M} e^{r(\hat{T}_M - t^{'})}\hat{\alpha}_{t^{'}}[f(\hat{k}_{t^{'}}) - \hat{k}_{t^{'}}]dt^{'} - \bar{S} + V\Bigg] \\
= &\int_t^{\hat{T}_M} e^{-r(t^{'}-t)}(1-\alpha_{t^{'}})[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} + e^{-r(\hat{T}_M - t)}\Bigg[\int_0^{\hat{T}_M} e^{r(\hat{T}_M - t^{'})}\alpha_{t^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} - \bar{S} + V\Bigg] \\
> &\int_t^{T_M} e^{-r(t^{'}-t)}(1-\alpha_{t^{'}})[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} + e^{-r(T_M - t)}\Bigg[\int_0^{T_M} e^{r(T_M - t^{'})}\alpha_{t^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} - \bar{S} + V\Bigg] \\
\geq &\, f(k_t) = f(\hat{k}_t)
\end{aligned}$$

where the first equality follows from construction, the second inequality follows from the construction, in particular $\hat{T}_M < T_M$, and the argument in Step 1, and the final inequality follows from the DICs for the original scheme. Hence, the original scheme cannot have been the optimum. ■

Proof of Lemma 3.2. Let $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$ be an optimum contract. In Step 1, we show that $k_t^* = \min\{k_{It}, k^e\}$ for all t, and then in Step 2, we show that $\alpha_t^* = 1$ for all t, where $0 \leq t \leq T_M^*$.
Step 1. $\mathbf{k_t^* = \min\{k_{It}, k^e\}}$ for all t. Observe that $k_t^* \leq k_{It}$, $\forall t \leq T_M^*$ (otherwise, given the definition of k_{It}, DIC at t will be violated). Next, consider the set $\mathcal{M} = \{t \leq T_M^* : \text{Either } k_t^* < \min\{k_{It}, k^e\}, \text{ or } k_t^* \in (k^e, k_{It}]\}$. In order to prove this lemma, it is sufficient to show that the measure of the set $\mathcal{M}$ is zero.[22] Suppose not. Then $\exists \mathcal{M}'$ and $T_M^{'} < T_M^*$ such that (i) $\mathcal{M}' \subsetneq \mathcal{M}$, (ii) $t \leq T_M^{'}$ for all $t \in \mathcal{M}'$, and (iii) the measure of $\mathcal{M}' > 0$.

We then construct another scheme $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^{'}\}_{t=0}^{T_M^*}, T_M^*\rangle$ such that:

$$k_t^{'} = \begin{cases} k_t^*, & \text{when } t \notin \mathcal{M}' \text{ and } t \leq T_M^*, \\ \dfrac{k_t^* + \min\{k_{It}, k^e\}}{2}, & \text{when } t \in \mathcal{M}' \text{ and } k_t^* < \min\{k_{It}, k^e\}, \\ \dfrac{k^e + k_t^*}{2}, & \text{when } t \in \mathcal{M}' \text{ and } k_t^* \in (k^e, k_{It}]. \end{cases}$$

[22]Given our assumption of continuous k_t, this ensures that $k_t^* = \min\{k_{It}, k^e\}$ for *all* t.

Hence by construction $\forall t \in \mathcal{M}'$, $[f(k_t^{'}) - k_t^{'}] > [f(k_t^*) - k_t^*]$.[23] Further, $\forall t \leq T_M^*$ and $t \notin \mathcal{M}'$, we have $k_t^{'} = k_t^*$, and thus $[f(k_t^{'}) - k_t^{'}] = [f(k_t^*) - k_t^*]$. Therefore,

$$\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^{'}) - k_t^{'}]dt > \int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*) - k_t^*]dt = \bar{S},$$

where the inequality is strict since $\mathcal{M}'$ has a positive measure. Now we check the DICs of the new contract.

i) DIC at any $t \notin \mathcal{M}'$ is satisfied because we started with a DIC contract and the continuation payoff at t under this new contract is no less than that under the original contract whereas the deviation payoff has not changed.

ii) Consider the change at t where $k_t^{'} = \dfrac{k_t^* + \min\{k_{It}, k^e\}}{2}$. The continuation payoff as well as the deviation payoff at t have increased, but that does not violate DIC because $k_t^{'} \leq k_{It}$.

iii) Similarly, consider the change at t where $k_t^{'} = \dfrac{k^e + k_t^*}{2}$. The continuation payoff at t has increased but the deviation payoff at t has decreased. So, DIC at such a t is satisfied.

Next since time is continuous, $\exists \Delta^{'} > 0$ such that

$$\int_0^{T_M^*-\Delta^{'}} e^{r(T_M^*-\Delta^{'}-t)}\alpha_t^*[f(k_t^{'}) - k_t^{'}]dt \geq \bar{S} \text{ and } \Delta^{'} < T_M^* - T_M^{'}.$$

Finally, mimicing the argument of Step 2 of the proof of Lemma 3.1, it can be shown that, for $\Delta^{'}$ small enough, DICs are not violated in this new scheme. Thus, for $\Delta^{'}$ small enough, we have constructed another scheme $\langle\{\alpha_t^*\}_{t=0}^{T_M^*-\Delta^{'}}, \{k_t^{'}\}_{t=0}^{T_M^*-\Delta^{'}}, T_M^* - \Delta^{'}\rangle$ that satisfies the DICs and the GC, and ends earlier than T_M^*. Hence, given Lemma 3.1, the scheme $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$ cannot have been optimal, which is a contradiction.

Step 2. $\alpha_t^* = 1$ for all t. The proof is immediate from the argument above. ■

Proof of Proposition 3.3. To prove this proposition we introduce the following two lemmas.

Lemma 3.5. *Let Assumptions 3.1, 3.2 and 3.3 hold.*

(i) The optimal loan scheme is always weakly progressive.

(ii) The optimal loan scheme can never be strictly progressive.

Proof. (i)Note that k_{It} is strictly increasing over time. This follows since $f(k_{It}) = e^{-r(T_M^*-t)}V$ $\forall t \leq T_M^*$, so differentiating it with respect to t we get $re^{-r(T_M^*-t)}V > 0$. So given Lemma 3.2, the optimal loan scheme is also strictly increasing over time unless it becomes constant at the efficient amount k^e. Hence, the optimal loan scheme is weakly progressive.

(ii) To prove the claim, we need to show that any progressive loan scheme must be capped at k^e that is k^e becomes DIC at some $t < T_M^*$. For that it is sufficient to argue that $k_{IT_M^*} > k^e$. This follows since $f(k_{IT_M^*}) = V$[24] and from assumption 3.3 we have $f(k^e) < \bar{S}$. □

Hence, the optimum loan scheme can be either progressive with a cap or constant. Next, we characterise the corresponding parametric conditions. We find that the optimal loan scheme

[23] Recall k^e maximises $f(k) - k$. Now first consider any $t \in \mathcal{M}'$ and $k_t^* < \min\{k_{It}, k^e\}$, $f(k_t) - k_t$ increases as we increase k_t. Next consider any $t \in \mathcal{M}'$ and $k_t^* \in (k^e, k_{It}]$, $f(k_t) - k_t$ increases as we decrease k_t.

[24] Recall, k_{It} denotes the maximum amount of loan which is DIC at t, where $0 \leq t \leq T_M$. Now, DIC at T_M^* is $V \geq f(k_{T_M^*})$, hence $f(k_{IT_M^*}) = V$

is "progressive with a cap" if and only if the increase in utility from graduation is *modestly positive*. In that event, the efficient level k^e cannot be sustained from the very beginning. Thus the loan amount keeps on increasing till it reaches k^e and remains constant thereafter. Finally, when this increase in utility from graduation is *transformative* k^e becomes DIC from the very beginning, hence the optimal loan amount remains constant at k^e. Hence the following lemma.

Lemma 3.6. *Let Assumptions 3.1, 3.2 and 3.3 hold.*

(i) The optimal loan scheme is progressive with a cap if and only if the increase in utility from graduation is not too large:

$$\frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V < f(k^e),$$

(ii) Otherwise, the optimal loan scheme is constant.

Proof. From the preceding lemma we know that the optimal loan scheme is either progressive with a cap or constant:

(i) If $k_{I0} < k^e$ then from Lemma 3.2, and the argument in Lemma 3.5, the optimal loan scheme must be "progressive with a cap".

(ii) And similarly if $k_{I0} \geq k^e$ then the optimal loan scheme must be constant at k^e.

So we characterise the parametric conditions under which the optimal loan scheme is progressive with a cap, that is $k_{I0} < k^e$. We show that

$$k_{I0} \geq k^e \text{ if and only if } \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V \geq f(k^e).$$

- Suppose $k_{I0} \geq k^e$: This implies k^e is DIC at $t = 0$. So from Lemma 3.5 we have $k_t^* = k^e$ $\forall t \in [0, T_M^*]$. Then the graduation constraint GC can be written as $\int_0^{T_M^*} e^{r(T_M^* - t)}[f(k^e) - k^e]dt = \bar{S}$, which in turn implies that $e^{-rT_M^*} V = \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V$.

  Now observe, $f(k_{I0}) = e^{-rT_M^*} V$. Hence, $k_{I0} \geq k^e$ implies $\frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V \geq f(k^e)$.

- Similarly $f(k^e) \leq \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V$ implies $k_{I0} \geq k^e$. Hence, the lemma. □

Hence, the proposition. ■

Proofs of the General Framework.

Proof of Proposition 3.4. For this we need the following two lemmas. The first one characterises $\langle T_B^R, T_B^D(t) \rangle$ the time of withdrawal of savings from the SI in case of repayment and default at t, where $0 \leq t \leq T_{sM}$. The second lemma characterises $\langle \{\sigma_t^R\}_{t=0}^{T_{sM}}, \sigma_t^D \rangle$ where σ_t^R denotes the part she wants to save with the SI at any arbitrary t, after repaying and saving with the MFI (or getting back her savings from the MFI which happens at T_{sM}), and σ_t^D denotes the part of $f(k_{st}) + S_t^D$ she wants to save with the SI after defaulting at t; where $0 \leq t \leq T_{sM}$ and $0 \leq \sigma_t^R, \sigma_t^D \leq 1$.

Lemma 3.7. *Let Assumption 3.2 hold. Given any MFI-contract* $\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$

1. *Suppose the borrower always repays, the optimum* $T_B^{R^*}$ *satisfies the following:*

   i) *When money in her hand at the termination date of the contract* T_{sM} *is at least* $\bar{S}$*, she withdraws her savings from the SI at that termination date, that is* $T_B^{R^*} = T_{sM}$

   ii) *When money in her hand at* T_{sM} *is less than* $\bar{S}$*, she chooses* $T_B^{R^*}$ *in such a way that at* $T_B^{R^*}$ *her savings with the SI becomes exactly equal to* $\bar{S}$*.*

2. *Suppose the borrower defaults at some* t*, where* $0 \le t \le T_{sM}$*, the optimum* $T_B^{D^*}(t)$ *satisfies the following:*

   i) *When money in her hand at the termination date of the contract* t *is at least* $\bar{S}$*, she withdraws her savings from the SI at that termination date, that is* $T_B^{D^*}(t) = t$

   ii) *When money in her hand at* t *is less than* $\bar{S}$*, she chooses* $T_B^{D^*}(t)$ *in such a way that at* $T_B^{D^*}(t)$ *her savings with the SI becomes exactly equal to* $\bar{S}$*.*

Proof. This proof follows from the facts that graduation is welfare improving so the borrower wants to graduate as soon as possible and that she is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) in future. Fomally, we show this in three steps.

Step 1. We show that the borrower prefers to save and graduate over consuming that amount. Denote the money in the borrower's hand at the termination date T, irrespective of whether the contract was terminated successfully or due to default, by M and the time of graduation by τ, where $\tau \ge T$. If she does not graduate her present discounted value of lifetime utility at T is M. If she graduates at τ her present discounted value of lifetime utility at T is

$$e^{-r(\tau-T)}\left[e^{r(\tau-T)}M + (V - \bar{S})\right].$$

Given Assumption 3.2, $V - \bar{S} > 0$ hence her utility is higher when she graduates.

Step 2. We show that her welfare increases as the time of graduation decreases. For that, we show that the borrower's present discounted value of lifetime utility is decreasing in τ. Differentiating present discounted value of the borrower's lifetime utility at T from graduation with respect to τ we get $-re^{-r(\tau-T)}(V - \bar{S}) < 0$. Therefore, she optimally chooses the time of withdrawal of her savings from the SI as soon as that becomes $\bar{S}$.

Step 3. We show that when her savings with the SI is not required to graduate, time of withdrawal of her savings from the SI does not affect her utility. Given our assumptions that the borrower's utility function is linear and that the future is discounted in the same way as the interest rate this is immediate. Recall without loss of generality, we assume that in such cases the borrower chooses the termination date of the contract as the time of withdrawal of her savings from the SI. From these three steps the lemma is immediate. □

Lemma 3.8. *Given an MFI-contract* $\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$*, the borrower saves as much as she can with the SI, in case of repayment as well as in case of default:*

1. $\sigma_t^{D^*} = 1$ *for all* t*, where* $0 \le t \le T_{sM}$*,*

2. $\sigma_t^{R^*} = 1$ *for all* t*, where* $0 \leq t \leq T_{sM}$*.*

Proof. 1. For that we fix the borrower's strategy in case of repayment at $\langle \sigma_t^R, T_B^R \rangle$ and from Lemma 3.7 we know that $T_B^{D^*}(t)$ satisfies the following:

i) $T_B^{D^*}(t) = t$ when the amount with which she defaults is at least $\bar{S}$

ii) Otherwise $T_B^{D^*}(t)$ is such that

$$e^{r(T_B^D(t)-t)}\left[\sigma_t^D\left[f(k_{st}) + \gamma\int_0^t e^{r(t-t')}\alpha_{st'}[f(k_{st'}) - k_{st'}]dt'\right] + \int_0^t e^{r(t-t')}\sigma_{t'}^R(1-\alpha_{st'})[f(k_{st'}) - k_{st'}]dt'\right] = \bar{S}.$$

In (i) she graduates immediately, using the amount she gets from the lender, this is same as saying that she saves the entire amount with the SI and withdraws immediately. To keep the notation similar we say that in this case $\sigma_t^{D^*} = 1$.

Now for (ii) – as argued above, a borrower is indifferent between consuming an amount now, and saving and consuming that amount later (along with interest). However, increase in σ_t^D decreases $T_B^{D^*}(t)$ which implies that the borrower graduates at an earlier date. Hence, her present discounted value of lifetime utility increases with increase in σ_t^D. Given limited liability $\sigma_t^{D^*} = 1$.

2. Now we show that $\sigma_t^{R^*} = 1$ for all $t \in [0, t_{sM}]$. Observe that following repayment, at any $t < T_{sM}$ there can be two cases: The borrower defaults at some $\tau \in (t, T_{sM}]$ or she repays at all $t \leq T_{sM}$. So we consider both the cases.

   (i) Suppose the borrower defaults at some $\tau \in (t, T_{sM}]$. Recall, the borrower's optimum strategy in case of default at τ is given by $\sigma_\tau^{D^*} = 1$ and $T_B^{D^*}(\tau)$ as characterised in Lemma 3.7. Now as observed above, the borrower's present discounted value of lifetime utility is decreasing in $T_B^{D^*}(\tau)$. Next given a contract, the borrower's savings with the SI (weakly) increases as σ_t^R increases and that (weakly) decreases $T_B^{D^*}(\tau)$. So given limited liability constraint, the measure of the set Ω^D is zero; where $\Omega^D = \{t \leq \tau : \sigma_t^{R^*} < 1\}$.

   (ii) The borrower repays at all $t \leq T_{sM}$. From Lemma 3.4 and the argument made above it is obvious that the measure of the set Ω^R is zero; where $\Omega^R = \{t \leq T_{sM} : \sigma_t^{R^*} < 1\}$. Hence, the lemma. □

Hence, the proposition. ■

Proof of Lemma 3.3. To prove this lemma we need to show that at the optimum, (a) the MFI provides loans at all instances till the borrower graduates i.e. she graduates at the successful termination date of the contract T_{sM}^* and (b) the graduation constraint binds. We prove (a) and skip (b) as that is very similar to the proof of Lemma 3.1.

(a) Suppose not, the optimal contract be $\langle \{\alpha_{st}^*\}_{t=0}^{T_{sM}^*}, \gamma^*, \{k_{st}^*\}_{t=0}^{T_{sM}^*}, T_{sM}^* \rangle$ and the borrower graduate at some $T = T_B^{R^*} > T_{sM}^*$. We construct another DIC contract $\langle \{\hat{\alpha}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{\gamma}, \{\hat{k}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{T}_{sM} \rangle$

such that the borrower's present discounted value of lifetime utility is higher under this new contract. So the original contract cannot have been optimum. The new contract $\langle\{\hat{\alpha}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{\gamma}, \{\hat{k}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{T}_{sM}\rangle$ is as follows

$$\begin{cases} \hat{T}_{sM} = T^*_{sM} + \Delta \quad \text{where } \Delta \text{ is such that } \displaystyle\int_0^{\hat{T}_{sM}} e^{r(\hat{T}_{sM}-t)}[f(\hat{k}_{st}) - \hat{k}_{st}]dt < \bar{S} \\ \hat{k}_{st} = k^*_{st} \quad \forall t \in [0, T^*_{sM}] \quad \text{and} \quad \hat{k}_{st} = k^*_{sT^*_{sM}} \, \forall t \in (T^*_{sM}, \hat{T}_{sM}] \\ \hat{\alpha}_{st} = \alpha^*_{st} \quad \forall t \in [0, T^*_{sM}] \quad \text{and} \quad \hat{\alpha}_{st} = \alpha^*_{sT^*_{sM}} \, \forall t \in (T^*_{sM}, \hat{T}_{sM}] \\ \hat{\gamma} = \gamma^*. \end{cases}$$

We first show that this new contract provides higher utility. Recall, given the borrower's optimum strategy she graduates at $\hat{T}_B^{R^*}$ where it is given by $\int_0^{\hat{T}_{sM}} e^{r(\hat{T}_B^{R^*}-t)}[f(\hat{k}_{st}) - \hat{k}_{st}]dt = \bar{S}$

So, she is better off under this new scheme as she graduates at an earlier date. But she becomes better off even when she defaults at some $t \in (T^*_{sM}, \hat{T}_{sM}]$, so it may not be obvious that DICs at all $t \in (T^*_{sM}, \hat{T}_{sM}]$ are satisfied. However, we show that DICs at all $t \in [0, \hat{T}_{sM}]$ are satisfied which implies that the original scheme cannot have been optimum.

DICs of the new contract at any $t \in [0, T^*_{sM}]$ are satisfied because we started with a DIC contract and under this new scheme at any $t \in [0, T^*_{sM}]$, the present discounted value of lifetime utility from repayment has increased whereas that from default has not changed. Now we argue that DIC at any $\tilde{t} \in (T^*_{sM}, \hat{T}_{sM}]$ is also satisfied. For that it is sufficient to show that the borrower's total savings till that $\tilde{t}$ is higher than the money in her hand in case of default.[25] So we want to show that

$$\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(\hat{k}_{st}) - \hat{k}_{st}]dt + f(\hat{k}_{s\tilde{t}}) - \hat{k}_{s\tilde{t}} \geq \int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[1 - \hat{\alpha}_{st}(1-\hat{\gamma})][f(\hat{k}_{st}) - \hat{k}_{st}]dt + f(\hat{k}_{s\tilde{t}})^{[26]}$$
$$\Rightarrow \int_0^{T^*_{sM}} e^{r(\tilde{t}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt + \int_{T^*_{sM}}^{\tilde{t}} e^{r(\tilde{t}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt \geq k^*_{sT^*_{sM}}. \tag{3.5.2}$$

where the second expression follows from the construction. We establish this from the dynamic incentive compatibility constraint, of the original contract, at T^*_{sM} which implies money in hand at T^*_{sM} in case of repayment must be no less than that in case of default. Otherwise given the borrower's strategy she would graduate at an earlier date[27] in case of default which will violate

[25] It is sufficient because in case of default the borrower has that money in hand only, to graduate – she saves that amount with the SI and graduates as soon as that becomes $\bar{S}$. Now in case of repayment she also gets loan till $\hat{T}_{sM}$. Hence the amount which helps her to graduate not only includes savings till that instance, but also the net return at each instance in future.

[26] Recall money in hand in case of default at $\tilde{t}$ is

$$f(\hat{k}_{s\tilde{t}}) + \int_0^{\tilde{t}} e^{r(\tilde{t}-t)}\hat{\gamma}\hat{\alpha}_{st}[f(\hat{k}_{st}) - \hat{k}_{st}]dt + \int_0^{\tilde{t}} e^{r(\tilde{t}-t)}(1-\hat{\alpha}_{st})[f(\hat{k}_{st}) - \hat{k}_{st}]dt.$$

[27] That is $T_B^{R^*} < T_B^{D^*}(T^*_{sM})$.

DIC at T^*_{sM}. So, we have

$$\int_0^{T^*_{sM}} e^{r(T^*_{sM}-t)}[f(k^*_{st}) - k^*_{st}]dt + f(k^*_{sT^*_{sM}}) - k^*_{sT^*_{sM}}$$
$$\geq \int_0^{T^*_{sM}} e^{r(T^*_{sM}-t)}[1-\alpha^*_{st}(1-\gamma^*)][f(k^*_{st}) - k^*_{st}]dt + f(k^*_{sT^*_{sM}})$$
$$\Rightarrow \int_0^{T^*_{sM}} e^{r(T^*_{sM}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt \geq k^*_{sT^*_{sM}}.$$

Now expression (3.5.2) is immediate as $\tilde{t} > T^*_{sM}$ and $\int_{T^*_{sM}}^{\tilde{t}} e^{r(\tilde{t}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt > 0$.

(b) The problem of the MFI thus becomes

$$\underset{\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle}{\text{Maximise}} \quad e^{-rT_{sM}}\left[\int_0^{T_{sM}} e^{r(T_{sM}-t)}[f(k_{st}) - k_{st}]dt - \bar{S} + V\right]$$

$$\text{Subject to: } \mathrm{GC}_R : \int_0^{T_{sM}} e^{r(T_{sM}-t)}[f(k_{st}) - k_{st}]dt \geq \bar{S},$$

$$\text{DIC:} \quad \forall t \leq T_{sM}; \quad e^{-r(T_{sM}-t)}\left[\int_0^{T_{sM}} e^{r(T_{sM}-t')}[f(k_{st'}) - k_{st'}]dt' - \bar{S} + V\right]$$
$$\geq e^{-r(T_B^{D^*}(t)-t)}\left[e^{r(T_B^{D^*}(t)-t)}\left[\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'}) - k_{st'}]dt' + f(k_{st})\right] - \bar{S} + V\right].$$

We want to show that GC_R binds. The proof is similar to that of Lemma 3.1, so we skip it here. ■

Proof of Observation 3.1. Suppose not. Then $\exists t \in [0, T_{sM})$, such that money in the borrower's hand in case she defaults at t is no less than $\bar{S}$, that is, $\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'}) - k_{st'}]dt' + f(k_{st}) \geq \bar{S}$. Given Lemma 3.4, in case of default she immediately graduates and her utility is

$$\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'}) - k_{st'}]dt' + f(k_{st}) - \bar{S} + V$$

which is higher than $e^{-r(T_{sM}-t)}V$ – present discounted value of lifetime utility from repayment. Hence, DIC at t cannot be satisfied.

Following the same argument, we get that the money in her hand in case of default at T_{sM} can be no higher than $\bar{S}$. ■

Proof of Proposition 3.6. A. From Lemma 3.4 we know $k^*_{st} = \min\{k_{sIt}, k^e\}$, so all we need to show is that, given assumption 3.4, k_{sIt} is (weakly) increasing in t, where $0 \leq t < T^*_{sM}$. Now, given observation 3.1 and the borrower's strategy identified in Proposition 3.4, we can write the following:

$$e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt'+f(k^*_{st})\Big]=\bar{S}$$

$$\Rightarrow\ e^{-r(T_B^{D^*}-t)}=\frac{1}{\bar{S}}\Big[\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt'+f(k^*_{st})\Big]$$

So, DIC at any $t\in[0,T^*_{sM})$ can be written as

$$e^{-r(T^*_{sM}-t)}V\geq\frac{1}{\bar{S}}\Big[\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt'+f(k^*_{st})\Big]V$$

From this we can write for any $t\in[0,T^*_{sM})$

$$f(k_{sIt})=\bar{S}e^{-r(T^*_{sM}-t)}-\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt' \tag{3.5.3}$$

So, to prove that the optimum loan scheme is weakly progressive it is sufficient to show that the R.H.S of (3.5.3) is increasing in t. Differentiating (3.5.3) with respect to t we get

$$r\bar{S}e^{-r(T^*_{sM}-t)}-r\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt'-\underline{\gamma}[f(k^*_{st})-k^*_{st}]$$
$$=rf(k_{sIt})-\underline{\gamma}[f(k^*_{st})-k^*_{st}]>0.$$

where the inequality is coming from Assumption 3.4 and $k^*_{st}=\min\{k_{sIt},k^e\}$.
B. This implies that the optimal loan scheme is either constant or progressive which may or may not be capped.

(i) Given Lemma 3.4 and part A of this proposition, a loan scheme is "strictly progressive" if and only the efficient loan amount is not incentive compatible even at T^*_{sM} i.e. $k_{sIT^*_{sM}}<k^e$. Now, observe $f(k_{sIT^*_{sM}})=(1-\underline{\gamma})\bar{S}$. Hence the loan scheme is "strictly progressive" if and only

$$(1-\underline{\gamma})\bar{S}<f(k^e).$$

This implies that the loan scheme is either "progressive with a cap" or "constant" over time if and only $(1-\underline{\gamma})\bar{S}\geq f(k^e)$. Mimicing the steps used in the Proof of 3.3, it can be shown that

(ii) The optimal loan scheme is "constant" if and only if $f(k^e)\leq\dfrac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S}$.

(iii) It is "progressive with a cap" if and only if $\dfrac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S}\leq f(k^e)\leq(1-\underline{\gamma})\bar{S}$. ■

Proof of Proposition 3.7. To prove this proposition we first introduce the following observation.

Observation 3.2. *The necessary and sufficient condition for the constant scheme, in the general case is implied by that in the benchmark case.*

Proof. The necessary and sufficient condition for k^e to be DIC from the very first instance, in the benchmark case, is $f(k^e) < \dfrac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}V$ whereas that in the general case is given by $f(k^e) \leq \dfrac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S}$. Since $V > \bar{S}$, this observation is immediate. □

Recall, we say that the investment required to start the technology $\langle V, \bar{S}\rangle$ is not large when

$$f(k^e) > \frac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S}.$$

Now we prove the proposition in three steps. In the first step, we show that at any t, where $0 \leq t \leq T^*_M$, the optimal loan amount is weakly lower in the general case in comparison to that in the benchmark case. It is strictly lower if and only if $\bar{S}$ is not large. The next two steps are obvious. In the second step we show that the time required to graduate is weakly higher in the general case, it is strictly higher if and only if $\bar{S}$ is not large. Finally in the third step, we show that the borrower's present discounted value of lifetime utility is weakly lower in the general case than that in the benchmark case. It is strictly lower if and only if $\bar{S}$ is not large.

Step 1. Since the deviation payoff is higher in the general case where the borrower gets back a part of her savings with the MFI till date, and can graduate by saving that amount with the SI, the amount which is DIC in the general case is also DIC in the benchmark case.

Now, the optimum loan amount is the minimum of the efficient amount and the maximum amount which is DIC, so when k^e is not DIC in the benchmark case it is not DIC in the general case as well, so in that case, the optimum loan amount is higher in the benchmark case than that in the general case.

Also, when the efficient amount is DIC in the benchmark case but not in the general case, the optimum loan amount is higher in the former case.

The optimum loan amounts are equal only in those instances where k^e is DIC in the general case. So, the optimal loan schemes under these two cases are identical when k^e is DIC at $t = 0$ in the general case. Recall, that happens if and only if $f(k^e) \leq \dfrac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S}$. Therefore, the optimal loan amount at any t, where $0 \leq t \leq T^*_M$, is weakly lower in the general case in comparison to that in the benchmark case.

Step 2. This proof follows from the preceding step.

Step 3. Since the time required to graduate is weakly lower in the benchmark case than that in the general case and the borrower's present discounted value of lifetime utility increases with a decrease in the time required to graduate, the result is immediate. ■

Appendix B

Here we provide some evidence that support various modelling assumptions made in the paper.

A. **Outreach.** "In FY 2015, 1033 institutions reported an outreach of *116.6 million borrowers* who have access to credit products, corresponding to a *gross loan portfolio of USD 92.4 billion...* and *98.4 million depositors* and *account for USD 58.9 billion of deposits*". In Table 3.1 we provide some more details. *Source:* MIX (2017).

B. **Near Perfect Repayment Rate in Microfinance.** Table 3.2 shows that repayment rates are very high. *Portfolio at Risk (PAR)* is one of the indicators of repayment rate. Low PAR indicates high repayment rate. *Source:* MIX (2017).

C. **Progressive Lending with a Cap.** Almost all the MFIs practise progressive lending. Many of those MFIs set caps as well – loan size cannot increase beyond that. Here we provide some examples from India, Bangladesh and Vietnam – top three countries by number of active borrowers. Table 3.3 shows that all the top five MFIs (by number of active borrowers) of India practise "progressive lending with a cap". Table 3.4 shows that all the top five MFIs (by number of active borrowers) of Bangladeh practise "progressive lending with a cap". Vietnam is the third largest country by active borrowers and Vietnam Bank of Social Policies is the largest MFI. In their website it is not mentioned whether they practise progressive lending or not, but each of the products offered by them has a cap. Table 3.5 documents that.

D. **Savings.** We then discuss deposit collection in various parts of the World.

South Asia. Due to regulation, deposit collection in India is low. In fact, Kline and Sadhu (2015) point out "No microfinance institution registered as an NBFC, currently accepts deposits because regulation requires that institutions must obtain an investment grade rating, which no microfinance institution has obtained." In table 3.6 we document the savings products offered by SEWA Bank, the largest Indian MFI (by number of depositors).[28] In table 3.7 savings products offered by the top five MFIs in Bangladesh are documented.

LAC is covered in table 3.8. Colombia, Peru and Bolivia are top three countries by number of depositors in Latin America and Carribean (LAC).

EAP, Africa and ECA are covered in table 3.9. Philippines, Indonesia and Vietnam are top three countries by the number of depositors in East Asia and the Pacific (EAP). Nigeria and Mongolia are top countries by the number of depositors in Africa and Eastern Europe and Central Asia (ECA) respectively.

[28] We do not consider Bandhan here, as it has become a bank now and in the website it is not mentioned which savings products are for poor people.

A. Global Outreach

Table 3.1: **Global Outreach: Borrower-Depositor** *Source* MIX (2017)

Regions	Number of Active Borrowers '000	Percentage of Total Borrowers	Number of Depositors '000	Percentage of Number of Depositors	Deposits USD m	Percentage of Total Deposits
Africa	5,778.2	5%	17,928.0	18%	9,212.1	16%
EAP	16,257.5	14%	16,117.9	16%	7,687.2	13%
ECA	3,082.6	3%	5,091.0	5%	7,664.3	13%
LAC	22,495.3	19%	23,708.6	24%	27,293.1	46%
MENA	2,148.4	2%	465.1	0%	251.0	0%
South Asia	66,929.3	57%	35,109.2	36%	6,885.8	12%
Grand Total	116,691.3	100%	98,419.8	100%	58,993.6	100%

B. Near Perfect Repayment Rate in Microfinance – Evidence

Table 3.2: **Near Perfect Repayment Rate in Microfinance – Evidence**

Regions	Percentage of Total Borrowers	Percentage of Gross Loan Portfolio (GLP)†	**Portfolio at Risk**> 30 **Days (PAR)**‡
Africa	5%	9%	**10.60%**
East Asia and the Pacific (EAP)	1%	16%	**3.40%**
Eastern Europe and Central Asia (ECA)	3%	11%	**10.00%**
Latin America and the Carribean (LAC)	19%	42%	**5.40%**
Middle East and North Africa (MENA)	2%	1%	**3.60%**
South Asia	57%	20%	**2.60%**

† "Gross Loan Portfolio (GLP)": All outstanding principals due for all outstanding client loans. This includes current, delinquent, and renegotiated loans, but not loans that have been written off.
‡ "Portfolio at Risk (PAR)": is one of the indicators of repayment rate. PAR [*xx*] days is defined as the value of all loans outstanding that have one or more installments of principal past due more than [*xx*] days.
Source: MIX (2017): Global Outreach and Financial Performance Benchmark Report 2015.

C. Progressive Lending with a Cap – Evidence

Table 3.3: **India – The Largest Country by Number of Active Borrowers**

MFI	No. of Active Borrowers '000	Gross Loan Portfolio (GLP) m	Description			
			Product Name	Progressive Lending?	Maximum Loan Amount INR	Reference/url (accessed on 8th May, 2018)*
Bandhan	–	2,596.22	Suchana	Yes	25,000	https://www.bandhanbank.com/Microloans.aspx
			Srishti	Yes	1,00,000	
Jana Small Finance (Formerly known as Janalakshmi)	5,888.75	1,974.73	Small Batch Loans	Yes	50,000	http://www.janalakshmi.com/products-services/loans-for-individuals *(Accessed in January, 2018 before it became a Bank.)
			Jana Kisan Loan	Yes	1,00,000	
Bharat Financial Inclusion Limited (Formerly known as SKS Microfinance Limited)	5,323.06	1,413.30	Income Generation Loans (IGL) – Aarambh	Yes	29,565	http://www.bfil.co.in/our-products/
			Mid-Term Loans (MTL) – Vriddhi	Yes	15,010	
			Long Term Loans (LTL)	Yes	49,785	
Share	3,740.00	251.68	General Loans	Yes	60,000	http://www.sharemicrofin.com/products.html
			Micro Enterprise Loans	Yes	2,50,000	
Shree Kshethra Dharmasthala Rural Devt. Project (SKDRDP)	3,013.18	986.55	Pragathi Nidhi Programme	Yes	50,000 (collateralized thereafter)	Rao (2005) and https://skdrdpindia.org/programmes/microfinance/

India: No. of active borrowers 43,153,000 and gross loan portfolio 14,901m. Top 5 MFIs from India by the no. of active borrowers, except Bandhan as number of active borrowers is not available in MIX Market data, however it is well known that this is the largest MFI in India (Gross Loan Portfolio is the maximum). Source MIX (2017).

Table 3.4: **Bangladesh – The Second Largest Country by Number of Active Borrowers**

MFI	No. of Active Borrowers ’000	Gross Loan Portfolio (GLP) m	Description			
			Product Name	Progressive Lending?	Maximum Loan Amount	Reference/url (accessed on 8th May, 2018)
Grameen	7,290.00	1,498.47	Basic Loan	Yes	No (but an individual gets a loan as long as she is below poverty line)	http://www.grameen.com/wp-content/uploads/bsk-pdf-manager/GB-2015_33.pdf https://grameenfoundation.org/sites/default/files/books/GrameenGuidelines.pdf
ASA	6,794.85	1,919.02	Primary Loan	Yes Constant at the max. when the economic potential is large. Otherwise increasing.	BDT 99,000	http://www.asa.org.bd/loan-products/
			Special Loan		BDT 10,00,000	
BRAC	5,356.52	1,768.61	Microloans (DABI)	Yes	USD 2,500	http://www.brac.net/images/factsheet/MF_Briefing_Doc_English.pdf
			Small enterprise loans (PROGOTI)	Yes	USD 13,000	
			Agriculture Loan	Yes	USD 1,500	
BURO Microfinance Program	996.22	406.58	Micro-Enterprise Loan	Yes	BDT 300,000	https://www.burobd.org/microfinance-loan-product.php?id=11
			Agriculture Loan	Yes	BDT 50,000	
Thengamara Mohila Sabuj Sangha (TMSS)	739.80	231.92	Loan for Enterprise Advancement and Development (LEAD)	Yes	BDT 10,00,000	http://tmss-bd.org/loan-for-enterprise-advancement-and-development-lead
			Rural Micro Credit (Jagaron)	Not Mentioned		http://tmss-bd.org/annual-report-2016
			Ultra Poor Program (Buniad)			
			Micro Enterprise SME Program (Agroshar)			

Bangladesh: No. of active borrowers 25,671,000 and gross loan portfolio 7,206m. Top 5 MFIs from Bangladesh by the no. of active borrowers. Source MIX (2017).

Table 3.5: **Vietnam – The Third Largest Country by Number of Active Borrowers**

Vietnam Bank of Social Policies (VBSP)			
Product Name	Maximum Loan Amount	Progressive Lending?	Reference/url (accessed on 9th May, 2018)
Poor Households Lending	VND 30 million/household	Not Mentioned	http://eng.vbsp.org.vn/poor-households-lending.html
Job Creation	Enterprises: VND 500,000,000/project. Households: VND 20,000,000/household		http://eng.vbsp.org.vn/job-creation.html
Overseas Workers	VND 30,000,000/labor		http://eng.vbsp.org.vn/overseas-workers-lending.html
Business& Production Households in Disadvantaged Areas	Generally VND 30 million. In some specific cases, loan amount can be over VND 30,000,000 to under VND 100,000,000		http://eng.vbsp.org.vn/business-production-households-in-disadvantaged-areas.html
Small and Medium Enterprises	VND 500,000,000/enterprise		http://eng.vbsp.org.vn/small-and-medium-enterprises.html
Extremely Disadvantaged Ethnic Minority Households	VND 5,000,000		http://eng.vbsp.org.vn/extremely-disadvantaged-ethnic-minority-households.html

Vietnam: No. of active borrowers 7,394,000 and gross loan portfolio 7,937m. VBSP is the largest MFI by the no. of active borrowers: No. of active borrowers 6,784740 and gross loan portfolio 6,911.69m. VBSP is the largest single microcredit lender in the world (Haughton and Khandker (2016)).
Source: MIX (2017).

D. Savings

Demand for Savings Service among Poor People and Lack of that

- "The commitment savings account gives you the chance to make a really long-term high-value swap, suitable for family ambitions like education, marriages and jobs for the youngsters, land and housing, and more distant anxieties like how to survive after you are too old and weak to work." (Rutherford (2009))).
- Poor people save even at a negative interest rate (for example with Jyothi in India (Rutherford (2009))), and with the Susu men in Africa (Besley (1995)).

Deposit Collecting MFIs

- "...(M)any MFIs have become true microbanks, doing both credit and voluntary savings. Their savings accounts take various forms. Some are completely liquid, allowing deposits and withdrawals of any amount at any amount, or nearly. Others are time deposits, like certificates of deposit, which are locked up for agreed periods and pay higher interest in return. In between there are semi-liquid accounts.... which limit the number, amount, or both of transactions per month through rules of penalties." (Roodman (2009) p 261.)

  "This counterintuitive combination of saving and borrowing accelerates loan repayment so that toward the end of a loan cycle, the MFI is actually in debt to its clients" (Roodman (2009) p 124.)

- Village Banking Institutions "typically require each village bank member to save. These forced savings are often a significant percentage of the amount the member has borrowed from the VBI. For example, forced savings range from 10% to 32% of the amount borrowed in the four leading Latin American VBIs analyzed in this study. Forced savings serve at least two major purposes. First, they act as cash collateral... The second purpose ... is to introduce (the bank members) to the discipline and habit of saving and to the possibilities that having a sizable savings balance could open up for them. For example, a sizable pool of savings could be used for emergencies, to pay school fees and other large household expenditures, to buy tools or machinery, or to start another business" (Westley (2004)).
- "Thus, in effect, the funds serve as a form of partial collateral." (Morduch (1999)).
- "(C)ollateralizing mandatory savings could offer a win-win solution for both lender and borrower by providing the MFP" (Microfinance Providers) "with security while at the same time building the asset base of the client." (Aslam and Azmat (2012)).

Table 3.6: **South Asia: India – Savings Services provided by the Top MFI (by no. of depositors)**

MFI No. of Depositors Deposits (USD m)	Product	Terms	Reference/url (accessed on 20th May, 2018)
Shri Mahila Sewa Sahakari Bank Ltd. 2,00,660 14.08	Regular Savings Product		https://www.sewabank.com/saving.html
	Fixed Deposit		https://www.sewabank.com/fixed-deposit.html
	Chinta Nivaran Yojana (Worry Riddance Scheme)	Deposits are made every month up to Five Years. In any emergency, after one year of joining in the scheme they can get an overdraft loan.	https://www.sewabank.com/recurring.html
	Kishori Gold Yojana	To encourage member to save money for special occasion. This was aimed at meeting expenses towards buying gold and gold ornaments during the wedding of their progeny.	
	Mangal Prasang Yojana	Help members during wedding of their sons and daughters.	
	Ghar Fund Yojana (Housing Fund Scheme)	To enable the member to have a house of their own. Maturity after 5/10 years	
	National Pension Scheme		https://www.sewabank.com/pension.html

India: No. of active depositors 374,000 and total deposit USD 329.65m. (We have not considered Bandhan here, as it has become a Bank now and in the website it is not mentioned which savings products are for the poor people.)
Source: MIX (2017).

Table 3.7: **South Asia: Bangladesh – Savings Services provided by the Top Five MFIs**

MFI	Product	Terms	Reference/url (accessed on 8th May, 2018)
Grameen Deposits (USD m) 2,604.93	Personal Savings	**Weekly compulsory savings.** Withdrawal at any time is allowed.	Alam and Getubig (n.d.), Rutherford (2010)
	Grameen Pension Scheme (GPS)	**For five to ten years.** Higher interest rate. Not restricted to retirement needs: Many younger families see the program as a means to save for medium-term expenses, such as school fees or weddings in the future for recently born children.	
ASA No. of Depositors 7,843960 Deposits (USD m) 826.34	Regular Savings: Clients belonging to Loan Programs need to deposit a regular fixed amount.	**Min. savings: Tk. 10 per week and Tk. 50 per month for primary loan; Tk. 50 per week and Tk. 100 per month for special loan.Members may withdraw from their savings any time maintaining a balance of at least 10% of their loan outstanding.**	http://www.asa.org.bd/savings-products/
	Voluntary Savings: Excess of Mandatory/Regular savings is treated as voluntary savings.	May deposit any amount above their *mandatory weekly savings*. Members may withdraw from their savings anytime **maintaining a balance of at least 10% of their loan outstanding.**	
	Long Term Savings: Any client can participate in this product.	Members deposit from Tk. 50 to Tk. 1000. Members can withdraw from their savings anytime at an interest rate calculated on monthly basis. **For withdrawal before maturity she is given lower rate of return.**	
	Capital Buildup Savings Fund	Weekly premium is BDT 10 or monthly premium BDT 50.The duration of CBSF is 400 weeks. For withdrawal before its maturity the borrower is given interest benefit on deposited amount at a special rate. On death of a borrower his/her family is given twice the deposited amount as security.	
BRAC No. of Depositors 5,957950 Deposits (USD m) 635.14	General Savings		http://www.brac.net/program/microfinance/
	*Safe*save	Longer-term “commitment savings” account. Deposit regularly for a defined term of up to ten years and receive higher rates of interest.	
BURO Bangladesh No. of Depositors 1,449090 Deposits (USD m) 128.14	General Savings	The general savings account is like a current account, where customers can save or withdraw on demand.	https://www.burobd.org/microfinance-savings-product.php?id=12
	Contractual Savings	A way of building up useful lump sums: This savings can be invested or used for social obligations such as marriages,funeral or childrens education. Higher interest than general savings. **In the contractual savings account clients agree to regularly deposit a set amount for a set period of time after which they can withdraw the entire amount plus the interest.**	
TMSS No. of Depositors 879600 Deposits (USD m) 73.10	General Savings, Special Savings, Monthly Savings		http://tmss-bd.org/annual-report-2016

Bangladesh: No. of active depositors 24,353,000 and total deposit USD 4,884m. *Source:* MIX (2017).

Table 3.8: **Evidence for Savings in Latin America and the Caribbean (LAC)**

Country No. of Depositors Deposits (USD m)	MFI No. of Depositors Deposits (USD m)	Product	Terms	Reference/url (accessed on 9th May, 2018)
Colombia 7,274,000 4,598	Banco Caja Social 4,655,300 3,352.44	Data not found	–	–
Peru 5,835,000 9,476	MiBanco 631,770 4,655.30	MiBanco (has) moved strongly into savings		Roodman (2009)
Bolivia 3,992,000 6,741	BancoSol 847,660 1,114.13	Liquid Savings Product: Cuenta de Ahorro		https://translate.googleusercontent.com/translate_c?depth=1&hl=en&prev=search&rurl=translate.google.com&sl=es&sp=nmt4&u=https://www.bancosol.com.bo/productos-y-servicios/ahorros/cuentas-de-ahorro&xid=17259,15700019,15700124,15700149,15700168,15700173,15700186,15700189,15700201&usg=ALkJrhg1Y7nbNtRsqFOn1UWWp17OjVAokg
		Semi-liquid Savings Product: Cuenta de Mayor	**A minimum balance has to be maintained. The maximum number of withdrawal is 4 per month.**	https://translate.googleusercontent.com/translate_c?depth=1&hl=en&prev=search&rurl=translate.google.com&sl=es&sp=nmt4&u=https://www.bancosol.com.bo/productos-y-servicios/ahorros/cuenta-de-ahorro-mayor&xid=17259,15700019,15700124,15700149,15700168,15700173,15700186,15700189,15700201&usg=ALkJrhhJorHTWqzPeD-8_tUdMto06rnJHw
		Fixed Term Deposit: Deposito A Plazo Fizo		https://translate.googleusercontent.com/translate_c?depth=1&hl=en&prev=search&rurl=translate.google.com&sl=es&sp=nmt4&u=https://www.bancosol.com.bo/productos-y-servicios/ahorros/depositos-a-plazo-fijo&xid=17259,15700019,15700124,15700149,15700168,15700173,15700186,15700189,15700201&usg=ALkJrhhDx-HNwDYl2933OvWISb38AUHZXw
		Sol Seguro: "(N)icely combines the virtues of insurance with an incentive to save." Some other products include Savings for children: Solecito (0-12 years) and SolGeneracion(13-17 years)		https://translate.googleusercontent.com/translate_c?depth=1&hl=en&prev=search&rurl=translate.google.com&sl=es&sp=nmt4&u=https://www.bancosol.com.bo/productos-y-servicios/ahorros/solecito-2&xid=17259,15700019,15700124,15700149,15700168,15700173,15700186,15700189,15700201&usg=ALkJrhjzlXN1DAVeE2UrGYxgv-RXbg8Chw https://translate.googleusercontent.com/translate_c?depth=1&hl=en&prev=search&rurl=translate.google.com&sl=es&sp=nmt4&u=https://www.bancosol.com.bo/productos-y-servicios/ahorros/sol-generacion-1-7-2&xid=17259,15700019,15700124,15700149,15700168,15700173,15700186,15700189,15700201&usg=ALkJrhh-RLBB7E3bN-AQO2zFi9eIpKEtiQ

Table 3.9: **Evidence for Savings contd.**

Region	Country No. of Depositors Deposits (USD m)	MFI No. of Depositors Deposits (USD m)	Product	Terms	Reference/url (accessed on 9th May, 2018)
East Asia and The Pacific (EAP)	Philippines 7,244,000 734	ASA Philippines 1,532,700 135.40	Capital Build-Up (CBU)	CBU is an alternative micro savings service for clients designed to promote the idea of poor families saving for the future in order to meet family emergencies and other needs. Withdrawable at any time.	http://asaphil.org/about/who-we-are/primary-services.aspx
			Locked-in Capital Build Up (LCBU)	**LCBU is fixed and mandatory, and serves as a monitoring tool of a client's performance and a basis for determining a clients loan renewal and any increase in loan amount. Non-withdrawable although it is 100% refundable.**	
	Indonesia 782000 29	Bank of Rakyat Indonesia (BRI) Unit Desa – –	"The global goliath of microsavings, BRI, offers all three" (liquid savings, locked-up savings and in between).		Roodman (2009)
	Vietnam 556000 3,404	Vietnam Bank of Social Policies (VBSP) — 2,463.85	Savings Deposit through savings and credit group (SCG) (An individual gets a loan from VBSP only if she is a member of SCG and saves)		http://eng.vbsp.org.vn/terms-savings-deposits.html
			Demand savings deposit		http://eng.vbsp.org.vn/demand-savings-deposits.html
			Term savings deposit		
Africa	Nigeria 4,240,000 184	Life Above Poverty Organization (LAPO) Microfinance Bank 2,631,980 90.97	Offers different savings product including Regular Savings, Savings Plan Account, Term Deposit Savings, Voluntary Savings, Individual Savings, Festival Savings, My Pikin Savings		Roodman (2009) and http://www.lapo-nigeria.org/
Eastern Europe and Central Asia (ECA)	Mongolia 2,987,000 2,404	Khan Bank 2,397,570 2,001.23	Data not found	–	–

For the Referee

Proof of Remark 3.3. We prove the progressivity of the optimal loan scheme in a restrictive framework where (a) dissaving is not allowed and (b) the interest rate z charged by the MFI on loan repayment is time-invariant and exogenously given.[29]

Let Assumptions 3.1, 3.2 and 3.3 hold and the optimal scheme be $\langle \{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^* \rangle$. We want to show that the optimal loan scheme (weakly) increases over time. Note that in this framework T_M^* can be ∞ which implies that the borrower never graduates. The profit of the MFI from this borrower at any $t \leq T_M^*$ is $(z-1)k_t$ and that is increasing in k_t.

Let us denote the borrower's present discounted value of lifetime utility from repayment at any t by V_t, where $0 \leq t \leq T_M$. To show that the optimal loan size is (weakly) progressive it is sufficient to show that the measure of either $[\underline{t}, \hat{t}]$ or $(\hat{t}, \bar{t}]$ such that $k_{t'}^* > k_{t''}^*$ $\forall t' \in [\underline{t}, \hat{t}]$, $t'' \in (\hat{t}, \bar{t}]$ and $\bar{t} \leq T_M^*$ is zero. We prove this by contradiction.

Suppose not. Measure of both $[\underline{t}, \hat{t}]$ and $(\hat{t}, \bar{t}]$ are positive. We construct another DIC contract such that the profit of the MFI is higher under this new contract, so the new contract cannot have been optimal. Before that consider DIC at any $t' \in [\underline{t}, \hat{t}]$: $V_{t'} \geq f(k_{t'}^*)$ and similarly DIC at any $t'' \in (\hat{t}, \bar{t}]$: $V_{t''} \geq f(k_{t''}^*)$. Now as observed above the profit of the MFI is increasing in k_t, so $k_{t'}^* > k_{t''}^*$ and DICs imply $V_{t'}^* > V_{t''}^*$ $\forall t'$ and t'', where $t' \in [\underline{t}, \hat{t}]$ and $t'' \in (\hat{t}, \bar{t}]$ and $\bar{t} \leq T_M^*$.

The new contract $\langle \{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M \rangle$ is as follows:

$$
\begin{cases}
\hat{k}_t = k_t^* \text{ and } \hat{\alpha}_t = \alpha_t^* & \forall t \leq \hat{t} \\
\hat{k}_t = k_{\hat{t}}^* \text{ and } \hat{\alpha}_t = \alpha_{\hat{t}}^* & \forall t \in (\hat{t}, \bar{t}] \\
\hat{k}_t = k_{t-\hat{t}}^* \text{ and } \hat{\alpha}_t = \alpha_{t-\hat{t}}^* & \forall t \in (\bar{t}, \hat{T}_M] \\
\hat{T}_M = T_M^* + [\bar{t} - \hat{t}] & \text{if } T_M^* = \text{ finite} \\
\hat{T}_M = \infty & \text{otherwise.}
\end{cases}
$$

First observe the profit of the MFI is higher here. Now from the observation above that $V_{t'}^* > V_{t''}^*$ $\forall t'$ and t'', where $t' \in [\underline{t}, \hat{t}]$, $t'' \in (\hat{t}, \bar{t}]$ and $\bar{t} \leq T_M^*$, and the construction it is easy to check that the new contract is DIC $\forall t \leq \hat{T}_M$: For that first observe DICs at all $t \leq \hat{t}$ are satisfied because the original contract is DIC and the utility from default at any such t is the same whereas that from repayment has increased. Second, DICs at any $t \in (\hat{t}, \bar{t}]$ is satisfied because the original contract is DIC at $\hat{t}$. Finally, DICs at any $t \in (\bar{t}, \hat{T}_M]$ is satisfied because the original contract is DIC at $t \in (\hat{t}, T_M^*]$. ■

[29] These two restrictions are *with* loss of generality. First, not allowing dissaving is with loss of generailty because in case the borrower does not graduate, this restriction implies that at the optimum there will be no savings at all. That may impose a serious restriction as we are ruling out the possibility of cycles in the optimum loan scheme. Second, time-independent, exogenous interest rate is with loss of generailty because this is actually a choice variable of the MFI, even if we assume that there is an exogenously given upper bound on the interest rate, the profit-maximizing MFI may optimally choose that to be lower than the upper bound at some instances.

Part II

Competition Among MFIs

Chapter 4

How Does Competition among MFIs affect Moneylenders and Borrowers?

4.1 Introduction

This chapter seeks to understand the consquences of competition among MFIs on the borrowers and the informal moneylenders. We develop a theoretical model which studies the effects of increase in the number of the MFIs on the size of the aggregate loan provided by the MFIs, that provided by the moneylender and the interest rate charged by the moneylender. We also study the welfare implication of the increase in competition among the MFIs.

The moneylender in our model, like in Jain and Mansuri (2003), provides a bridge-loan to repay the MFI-loan(s). However in their model bridge-loans are required because of frequent repayment schedule of the MFIs. In our framework, the bridge-loan is required because the borrower invests in a risky project, outcome of which is observed by the moneylender but not by the MFI(s), so in case of project failure the moneylender provides bridge-loan to the poor borrower which enables her to repay MFI loan(s). In case the borrower does not repay the loan of a particular MFI, that MFI terminates the contract and renegotiation is not allowed.

We find that the aggregate loan amount provided by the MFIs as well as that provided by the moneylender decrease and the interest rate charged by the moneylender increases with the increase in the number of MFIs. The welfare of the borrower decreases and the profit of the moneylender increases as a consequence of the increase in the competition among MFIs. These predictions of our theoretical model are supported by empirical findings, in particular an empirical study conducted by Chowdhury et al. (2017) find similar results.

Intuitively, we get the results above because the MFIs do not share information among themselves. As a result, with increase in the number of MFIs the threat of termination of contract by a single MFI becomes less effective. This decreases the maximum amount of loan which is incentive compatible and hence, the aggregate loan amount provided by the MFIs decreases. The moneylender, in our framework, only provides bridge-loan, so with increase in MFI competition size of the loan provided by the moneylender also decreases.

Turning to the intuition behind the welfare implication – we assume that the moneylender is a profit maximizer and hence he extracts the entire borrower surplus, whenever that is possible. This implies that the difference in moneylender interest rate, as a result of increase in the number of MFIs, is equal to the difference in the proportion of return from total investment to total investment. Now we assume that the production function in concave which implies that with a decrease in loan amount viz. investment, the proportion of return to investment increases. So, the moneylender's interest rate increases with increase in MFI competition. The increase in profit due to the increase in interest rate is dominated by the decrease in profit due to the decrease in loan size. Hence, the moneylender's profit increases with increase in MFI competition. The welfare of the borrower decreases with MFI competition because her present discounted value of lifetime utility is an increasing function of the loan size which decreases with increase in competition among MFIs.

Finally we find that when the agents (we assume that the moneylender and the borrower have common discount factor) are very patient, there is over borrowing - this happens because the objective of the MFI is to increase borrower's present discounted value of lifetime utility which is an increasing function of investment till it reaches the efficient amount (without any risk) and the risk of the project is borne by the moneylender.

The rest of the chapter is organized as follows: Next we describe the basic framework of the model. Our analysis starts with the case where there is only one MFI and then we allow for more than one MFIs. In section 4 we compare the optimum outcomes of these two cases. Finally we conclude.

4.2 The Framework

4.2.1 Payoffs and Technology

Consider an economy with one agent, one moneylender,[1] and n MFIs. We analyze the lending dynamics between these agents in an infinite horizon, discrete time framework. For ease of exposition, we assume that all the agents are risk neutral, and have the same discount factor δ, where $0 < \delta < 1$. The agent has access to a non-deterministic production technology. We also assume that access to this technology is tied with the MFI contract, that is the agent can invest in this technology only if she is in a relationship with at least one MFI. An investment of k in this technology yields $f(k)$ with probability p, $(0 < p < 1)$ and zero with the complementary probability. The production function $f(\cdot)$ satisfies the usual assumptions.

Assumption 4.1. $f(0) = 0,\ f^{'}(k) > 0,\ f^{''}(k) < 0,\ \lim_{k\to\infty} f^{'}(k) = 0,$ *and* $\lim_{k\to 0} f^{'}(k) = \infty.$

Given this assumption we define the efficient level of capital as follows

Definition 4.1. *The efficient level of capital $k^E(p)$ solves $pf^{'}(k) = 1$, given assumption 4.1 it is unique.*

4.2.2 MFIs and the Moneylender

In our framework the main differences between the MFIs and the moneylender are:

1. *Benevolence:* MFIs are more benevolent than the moneylender, we take an extreme case where the MFI is benevolent and the moneylender is profit maximizer.

[1] Like Mookherjee and Motta (2016) we assume that if there are more than one moneylender within a village, then they collude.

2. *Information:* Due to thick interactions, the moneylender has all the information - borrower's project return, MFI contract, borrower's repayment decision regarding the MFI loan and can also enforce repayment perfectly. On the other hand, the MFIs cannot observe borrower's project return and other MFI or moneylender's contract.

3. *Cost of Lending:* Lending cost of the MFI is lower than that of the moneylender, we normalise MFI's lending cost to zero and let that of the moneylender be c, where $c > 0$. We assume this because MFIs get subsidised/ cheap funding from outside donors and that decreases their lending cost.

4. *Break even Condition:* The MFIs must break even *ex post* in every period, while the moneylender only breaks even *ex ante*. We assume this because the donors who are not aware of the local situations may impose *ex post* break even condition.

5. *Bridge-loan:* Due to this informational advantage, the moneylender may choose to provide "bridge-loans" to the agent when the project fails. The agent may choose to use this bridge-loan to repay the MFI loans.

6. *Commitment:* We assume that the MFIs can commit whereas the borrower defaults on MFI loan(s) whenever she has incentive to do so.

   Due to his informational advantage the moneylender can dictate the borrower about the number of the MFIs with which she can be in relationships, if at all. The moneylender chooses this number strategically, in that he let the borrower repay to only those many number of MFIs for which his present discounted value of lifetime utility is maximized. We discuss this in details in later section, after stating the framework and assumptions of our model.

7. *Information Sharing:* The MFIs do not share any information among themselves.

8. Moneylender has the power to extract any agreed upon surplus from the borrower.

4.2.3 Contracts

Suppose there are n MFIs. A contract of any MFI i, where $i \in \{1, ..., n\}$, starting from that period t is given by $\xi^i(t) \equiv \langle k^i(t), r^i(t) \rangle$. It has two components: $k^i(t) \equiv \{k^i_\tau\}^\infty_{\tau=t}$ and $r^i(t) \equiv \{r^i_\tau\}^\infty_{\tau=t}$ where k^i_τ and r^i_τ denote the loan amount and the rate of interest respectively, at τ^{th} period of time. ξ^i is contingent on the repayment history, the belief about the number of MFIs from which the borrower takes loans and belief about other MFIs' and the moneylender's contracts. Now due to our assumption of *ex post* break even condition, MFI i terminates the ongoing contract whenever the borrower defaults on it's loan.[2] So, only relevant history of repayment is that of the last period.

For expositional simplicity, we only consider symmetric, stationary contracts.[3] The immediate implication of considering only stationary contracts is that MFI i's loan amount and interest

[2]Observe due to *ex post* break even condition, partial repayment is not allowed in that the MFI terminates the contract even if the borrower repays a part of the specified amount. Also renegotiation is not allowed.

[3]Observe we assume that the MFIs can commit and MFI contracts are chosen from the class of stationary and symmetric contracts. Checking the robustness of the analysis to these assumptions are part of my future research agenda.

rate are time independent:

$$k_t^i = k^i \text{ and } r_t^i = r^i \quad \forall i \in \{1, .., n\}.$$

And similarly implication of considering only symmetric equilbria is that the loan amount and the interest rate of each MFI are equal: $k^i = \dfrac{k}{n}$ where k is the total MFI loan at any period t and n is the number of MFI and $r^i = r$.

As stated above in our analysis, the role of the moneylender is to provide "bridge-loans" when the project fails. Since the access to technology $f(\cdot)$ is assumed to be tied with the MFI contracts, *this* relationship between the moneylender and the agent is also tied with the MFI contracts. In other words, if all the MFIs terminate their contracts, the borrower would not take any bridge-loan from the moneylender. Now this bridge-loan is over and above any other ongoing loans (or transactions) between the moneylender and the borrower. So the moneylender being strategic and profit-maximizer provides bridge-loan contracts if and only if his present discounted vaule of lifetime utility is (weakly) higher from that (in comparison to that from not providing any bridge-loan at all).

Formally, the moneylender offers a menu of contracts $\langle \{R(x), \dfrac{x}{n}(1+r)k\}_{x \in X} \rangle$ where $x \in X \subseteq \mathcal{X} \equiv \{0, 1, ..., n\}$ is the number of MFIs whose loans the moneylender wants the borrower to repay with the bridge-loan and $R(x)$ is the rate of interest in that case. Observe since the moneylender only provides bridge-loan and partial repayment of any particular MFI loan is not allowed, he can choose the amount of bridge-loan only by choosing the number of MFIs.

Let us try to understand with an example: Suppose at t^{th} period there are six MFIs, so $\mathcal{X} \equiv \{0, 1, ..., 6\}$. Now the moneylender may post any subset of $\langle \{R(0), 0\}$; $\{R(1), \dfrac{1}{6}(1+r)k\}$; $\{R(2), \dfrac{2}{6}(1+r)k\}$; $\{R(3), \dfrac{3}{6}(1+r)k\}$; $\{R(4), \dfrac{4}{6}(1+r)k\}$; $\{R(5), \dfrac{5}{6}(1+r)k\}$; $\{R(6), (1+r)k\} \rangle$ where $\{R(3), \dfrac{3}{6}(1+r)k\}$ corresponds to the case where the moneylender provides $\dfrac{3}{6}(1+r)k$ amount of bridge loan using which the borrower can repay any three of the six MFI loans and in case of success in the next period she has to repay $(1 + R(3))\dfrac{3}{6}(1+r)k$ to the moneylender.

Finally, we assume that the borrower is protected by limited liability. We start our analysis with one MFI, this not only helps us to understand but also provides us with a benchmark case. Then we analyse the general case, where there are more than one MFIs. The main objective of this paper is to compare the equilibrium outcomes of this benchmark case with those of the general case.

4.3 One MFI

4.3.1 Timeline

Having there been no default till date, the following things happen at any t:

Stage 1: The MFI provides k amount of loan at an interest rate r

Stage 2: The borrower invests k in $f(\cdot)$ technology and obtains either $f(k)$ with probability p or zero otherwise.

Stage 3: The moneylender observes what happened in stages 1 and 2.

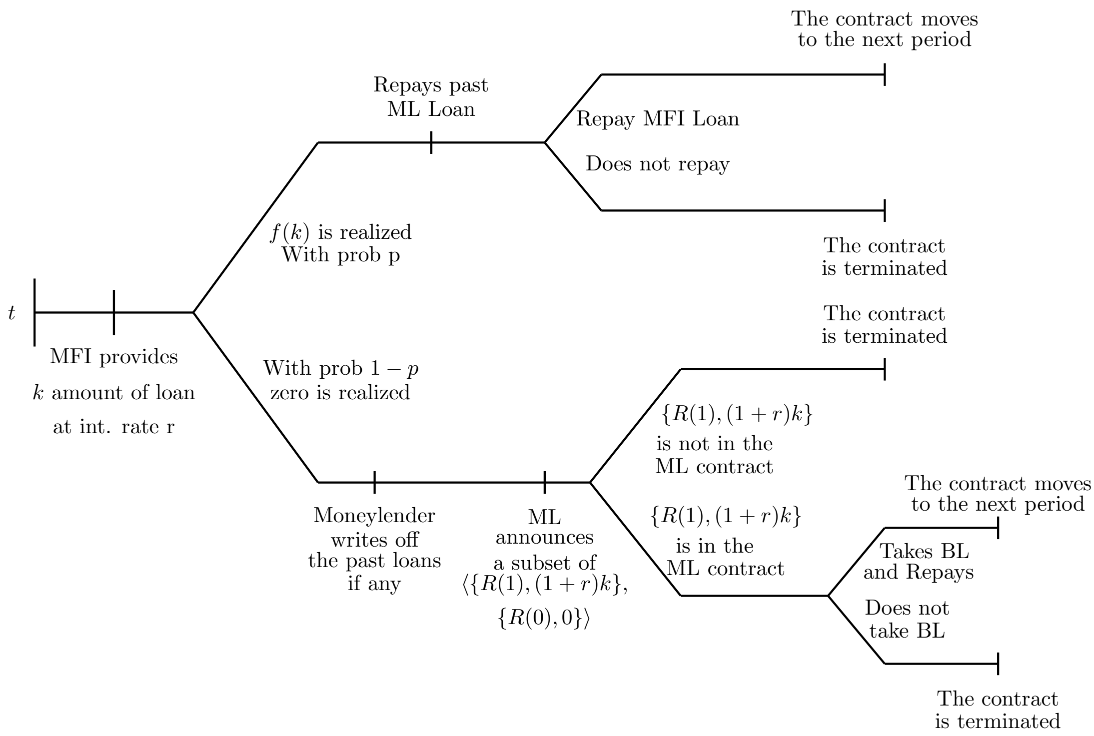
(i) In case the project fails, the moneylender announces a subset of $\langle\{R(0), 0\};$ $\{R(1),(1+r)k\}\rangle$ where $\{R(0), 0\}$ denotes the case where the moneylender does not provide any bridge-loan and $\{R(1),(1+r)k\}$ denotes the case where the moneylender provides $(1+r)k$ amount of loan (which is equal to the amount to be repaid to the MFI) and $R(1)$ is the rate of interest on that. So in case of success in the next period, the borrower has to repay $(1+R(1))(1+r)k$ to the moneylender. Since we are confining ourselves only to the stationary contracts, the moneylender chooses $R(1)$ in such a way that in case of success, the borrower can repay MFI loan after repaying moneylender's loan, if she is willing to do so.

- If the moneylender's contract includes $\{R(1),(1+r)k\}$, the borrower decides whether to take that bridge-loan at all or not. In case she decides to take the bridge-loan, she makes a binding commitment to repay the moneylender in case of success in the next period[4] and the games moves to the next period. In case she defaults, the MFI terminates the contract. Observe, irrespective of repayment decision, agent's instantaneous utility is zero, in case of project failure.

(ii) In case of project success, the future events depend on the previous period's project outcome.

- The project was a failure at $t-1^{th}$ period: Observe the agent must have taken the bridge loan otherwise the game wouldn't have reached t^{th} period. So first she repays the moneylender $(1+R(1))(1+r)k$ and then decides whether to repay MFI loan or not.
  * If she repays the MFI loan, she repays $(1+r)k$ to the MFI. The game moves to the next period providing her with $f(k)-(1+R(1))(1+r)k-(1+r)k$ instantaneous utility.
  * If she does not repay the MFI loan, the MFI terminates the contract and the agent's instantaneous utility becomes $f(k)-(1+R(1))(1+r)k$.
- The project was a success at $t-1^{th}$ period, the agent decides whether to repay the MFI loan or not.
  * In case she repays, the game moves to the next period providing her with $f(k)-(1+r)k$ instantaneous utility
  * In case of default the game is terminated and her instantaneous utility in that case becomes $f(k)$.

[4]As mentioned above, due to thick interaction the agent cannot (strategically) default on moneylender loan, also it is not possible for her to take bridge-loan from the moneylender and not repay the MFI.



4.3.2 Moneylender's Problem

Let us first consider the problem where the moneylender chooses $R(1)$ in order to maximize his present discounted value of lifetime utility. Now given limited liability constraint of the borrower, the moneylender has to choose $R(1)$ in such a way that, in case of success in the next period it is feasible for the borrower to repay MFI loan.[5] So, the problem of the moneylender is to

$$\begin{aligned}
\underset{R(1)}{\text{Maximize}} \quad & \delta p(1+R(1))(1+r)k - (1+c)(1+r)k \\
& + \delta^2 p(1-p)(1+R(1))(1+r)k - \delta(1-p)(1+c)(1+r)k \\
& + \delta^3 p(1-p)(1+R(1))(1+r)k - \delta^2(1-p)(1+c)(1+r)k + ...
\end{aligned}$$

Subject to:

$$\text{RC:} \quad f(k) - (1+R(1))(1+r)k \geq (1+r)k.$$

First, let us explain the Repayment Constraint (RC) which is an immediate outcome of our assumption of limited liability. The left hand side of this constraint is the amount the borrower has after repaying the moneylender: $f(k) - (1+R(1))(1+r)$ and the right hand side is the amount to be repaid to the MFI ($(1+r)k$) if the borrower wants to continue the relationship with the MFI. So, this constraint ensures that the amount the borrower has after repaying the moneylender is high enough to repay the MFI loan.

Next we explain the moneylender's objective function while choosing $R(1)$ at any period (say at t^{th} period). The first two terms correspond to the expected present discounted value from providing loan at this period: with probability p the project becomes successful at $t+1^{th}$ period

[5] Observe this is due to the fact that we restrict ourselves to the stationary contracts.

and the moneylender gets $(1+R(1))(1+r)k-(1+r)k$ whereas his cost of providing the bridge-loan is $(1+c)(1+r)k$. So his expected present discounted value from providing loan at t^{th} period is $\delta p(1+R(1))(1+r)k-(1+c)(1+r)k$. Similarly the third and fourth terms represent expected present discounted value from providing bridge-loan at $t+1^{th}$ period: The bridge-loan at $t+1^{th}$ period is provided when the project fails, probability of which is $1-p$. So the moneylender's expected present discounted value from that event is $\delta(1-p)[\delta p(1+R(1))(1+r)k-(1+c)(1+r)k]$. Likewise we get the following terms.

So the moneylender's job is to choose $R(1)$ in order to maximize this objective function given RC. Observe the moneylender's problem can alternatively written as:

$$\begin{aligned}
&\underset{R(1)}{\text{Maximize}} \quad \Big[1+\frac{\delta(1-p)}{1-\delta}\Big]\Big[\delta p(1+R(1))(1+r)k-(1+c)(1+r)k\Big] \\
&\text{Subject to:} \\
&\text{RC:} \quad f(k)-(1+R(1))(1+r)k \geq (1+r)k. \qquad (4.3.1)
\end{aligned}$$

Lemma 4.1. *Let assumption 4.1 hold*

1. $1+R(1)^* = \dfrac{f(k)-(1+r)k}{(1+r)k}$
2. *The moneylender offers* $\{R(1)^*,(1+r)k\}$ *if and only if present discounted value of his lifetime utility from offering this is (weakly) higher than that from offering* $\{R(0),0\}$.

Proof.

1. Moneylender's objective function (4.3.1) is increasing and the constraint gets tighter with increase in $R(1)$. So, the moneylender would optimally choose $R(1)$ such that RC binds. Hence the result.

2. Observe, the moneylender can always offer $\{R(0),0\}$ and in our simple stationary equilibrium $R(0)$ is zero. So, his present discounted value of lifetime utility from offering $\{R(0),0\}$ is zero. So he offers $\{R(1)^*,(1+r)k\}$ only if his present discounted value of lifetime utility from offering this is at least zero. So, he offers $\{R(1)^*,(1+r)k\}$ only if

$$\Big[1+\frac{\delta(1-p)}{1-\delta}\Big]\Big[\delta p[f(k)-(1+r)k]-(1+c)(1+r)k\Big] \geq 0$$

∎

4.3.3 MFI's problem

We next turn to the problem of the MFI. As stated above, the MFI is benevolent in that it wants to maximize the present discounted value of lifetime utility of the agent subject to *ex post* break even condition. The MFI is aware of the fact that the project gets successful only with probability p, but cannot observe the actual outcome.[6] So the MFI has to design a contract such that both the borrower and the moneylender have incentives to continue the relationship:

[6] Alternatively, we can say that the MFI can observe the project outcome by incurring huge cost whereas the moneylender, due to thick interaction, observes the project outcome costlessly.

In case of failure of the project the moneylender should have incentive to offer $\{R(1)^*, (1+r)k\}$ such that the borrower can repay the MFI loan if she has incentive to do so. On the other hand, the borrower should also have the incentive to take the bridge-loan to repay the MFI loan. Also in case of success the borrower should have incentive to repay the MFI loan either after repaying the moneylender (in case the project was a failure in the previous period) or otherwise. As it can be seen that the incentive of the moneylender who has all the information and provides the bridge-loan in case of failure, plays an important role. Though the MFI does not provide any contract to the moneylender directly but it knows the moneylender's strategic reponses to its contracts, so the MFI has to incorporate those while designing the contract. We will come back to this again after we write down the incentive constraint of the borrower as well as that of the moneylender.

Let us start with the incentive problem of the borrower: In case of success at any period t, the borrower must (weakly) prefer to repay the MFI loan and continue the relationship over defaulting. There can be two cases: the project was a success at $t-1^{th}$ period and it was a failure at $t-1^{th}$ period. Suppose the project was a success at $t-1^{th}$ period, then the borrower's present discounted value of lifetime utility from repayment is

$$f(k) - (1+r)k + \delta p[f(k) - (1+r)k] + \delta^2(1-p)p[f(k) - (1+R(1))(1+r)k - (1+r)k] \\ +\delta^2 p^2[f(k) - (1+r)k] + ...$$

where $[f(k) - (1+r)k]$ is the instantaneous utility from repayment at t^{th} period; in the next period the project becomes successful with probability p, borrower's instantaneous utility in that case is $[f(k) - (1+r)k]$, so the expected present discounted value of that is $\delta p[f(k) - (1+r)k]$ on the other hand at $t+1^{th}$ period the project fails with probability $1-p$ so the borrower's instantaneous utility at $t+1^{th}$ in this case is zero, given all the other incentives she takes a bridge-loan and repays the MFI loan and with probability p her project becomes a success at $t+2^{th}$ period, in which case her instantaneous utility becomes $f(k)-(1+R(1))(1+r)k-(1+r)k$, hence, the expected present discounted value of that is $\delta^2(1-p)p[f(k) - (1+R(1))(1+r)k - (1+r)k]$ and so on... Now the borrower's utility from default is $f(k)$. So the incentive compatibility constraint for repayment when the project becomes successful at t^{th} period and also was a success at $t-1^{th}$ period is

$$f(k) - (1+r)k + \delta p[f(k) - (1+r)k] \\ +\delta^2(1-p)p[f(k) - (1+R(1))(1+r)k - (1+r)k] + ... \geq f(k).$$

Now consider the case where the project was a failure at $t-1^{th}$ period (success at t^{th}) period. The borrower's present discounted value of lifetime utility from repayment is

$$f(k) - (1+R(1))(1+r)k - (1+r)k + \delta p[f(k) - (1+r)k] \\ +\delta^2(1-p)p[f(k) - (1+R(1))(1+r)k - (1+r)k] + ...$$

where $f(k) - (1+R(1))(1+r)k - (1+r)k$ is instantaneous utility at t^{th} period from repayment: the return of the project is $f(k)$ out of which she repays $(1+R(1))(1+r)k$ to the moneylender,

$(1+r)k$ to the MFI and then she consumes the rest. The continuation payoff from $t+1^{th}$ period is same as the previous case. Now the borrower's utility from default is $f(k)-(1+R(1))(1+r)k$. So the incentive compatibility constraint for repayment when the project becomes successful at t^{th} period and was a failure at $t-1^{th}$ period is

$$\begin{aligned}&f(k)-(1+R(1))(1+r)k-(1+r)k+\delta p[f(k)-(1+r)k]+...\\ \geq& f(k)-(1+R(1))(1+r)k\end{aligned}$$

Now incorporating moneylender's strategy given by $1+R(1)^{*}=\dfrac{f(k)-(1+r)k}{(1+r)k}$, observe we can write the borrower's incentive compatibility constraint in case of project success, irrespective of whether the project was successful or not at $t-1^{th}$ period as

$$\big[\delta p+\frac{\delta^2p^2}{1-\delta}\big][f(k)-(1+r)k]\geq(1+r)k. \qquad (\ IC_M^B\ )$$

Another incentive constraint of the borrower is that the borrower would (weakly) prefer to take a bridge-loan over not taking that in case of project failure.

$$\begin{aligned}&\delta p[f(k)-(1+R(1))(1+r)k-(1+r)k]+\delta^2p^2[f(k)-(1+r)k]\\ &\delta^2(1-p)p[f(k)-(1+R(1))(1+r)k-(1+r)k]+\delta^3p^2[f(k)-(1+r)k]+...\geq 0.\end{aligned}$$

The left hand side of the above constraint denotes the expected present discounted value of lifetime utility of the borrower in case she takes a bridge-loan from the moneylender (and repays the MFI loan in case her project fails) whereas the right hand of the same represents that from deviation. Let us explain the left hand side first. At period t, the borrower's project return is zero, she takes bridge-loan $(1+r)k$ and repays the MFI loan. So her instantaneous utility at t is zero. Then at $t+1^{th}$ period, her project becomes successful with probability p, she repays the moneylender and MFI loans and consumes the rest. So, the expected present discounted value from this contingency is $\delta p[f(k)-(1+R(1))(1+r)k-(1+r)k]$ which is the first term of the LHS of the constraint. Again her utility at $t+1^{th}$ period becomes zero in case of failure, probability of which is $1-p$. Then the second term of the LHS of the constraint corresponds to the instantaneous utility of the borrower at $t+2^{th}$ period when her project becomes successful at $t+1^{th}$ as well as $t+2^{th}$ periods, probability of that event being p^2. Hence the expected present discounted value due to this is $\delta^2p^2[f(k)-(1+r)k]$. The third term corresponds to the instantaneous utility of the borrower at $t+2^{th}$ period when her project fails at $t+1^{th}$ period and succeeds at $t+2^{th}$ period, probability of that event is $(1-p)p$. So her expected present discounted value of lifetime utility due to this event is $\delta^2(1-p)p[f(k)-(1+R(1))(1+r)k-(1+r)k]$. Similarly we get the following terms. Finally the right hand side of the constraint denotes her continuation payoff from not taking the bridge-loan from the moneylender (and hence not repaying the MFI loan). Hence this constraints ensures that she takes the bridge-loan from the moneylender to continue the relationship with the MFI forever. Like before, incorporating moneylender's strategy, we can write borrower's incentive compatibility constraint in case of project failure as

$$\frac{\delta^2 p^2}{1-\delta}[f(k) - (1+r)k] \geq 0.$$

Observation 4.1. *Incentive compatible constraint for taking a bridge-loan is implied by the incentive compatible constraint for repayment in case of success of the project*

Proof.

$$f(k) - (1+r)k \geq \frac{(1+r)k}{\delta p + \dfrac{\delta^2 p^2}{1-\delta}} > 0.$$

■

This is quite intuitive in that the borrower's present discounted value of not taking bridge-loan in case of project failure is zero, so she takes the bridge-loan whenever continuation payoff is non-negative. Now in case of success the borrower has to repay $(1+r)k$, she repays that amount only if continuation payoff is higher than $(1+r)k$ which is positive.

Given this observation we ignore the borrower's incentive constraint for taking a bridge-loan. Finally as discussed above the moneylender offers $\{R(1), (1+r)k\}$ only if he has incentive to do so, that is only if his present discounted value of lifetime utility from offering $\{R(1), (1+r)k\}$ is as high as that from offering $\{R(0), 0\}$. As discussed above, the incentive compatibility constraint[7] of the moneylender becomes

$$\left[1 + \frac{\delta(1-p)}{1-\delta}\right]\left[\delta p[f(k) - (1+r)k] - (1+c)(1+r)k\right] \geq 0 \qquad (\; IC_M^{ML} \;)$$

Given all these constraints discussed above, the MFI's objective is to maximize present discounted value of lifetime utility of the borrower:

$$p^2[f(k) - (1+r)k] + p(1-p)[f(k) - (1+R(1))(1+r)k - (1+r)k] + \delta p^2[f(k) - (1+r)k] + ...$$

where $p^2[f(k) - (1+r)k]$ is the instantaneous utility of the borrower when the project was a success at $t-1$ as well as at t, $p(1-p)[f(k) - (1+R(1))(1+r)k$ is the instantaneous utility of the borrower when the project was a failure at $t-1$ and success at t and so on. Incorporating the moneylender's strategy given by $1 + R(1)^* = \dfrac{f(k) - (1+r)k}{(1+r)k}$, the objective function becomes

$$\frac{p^2}{1-\delta}[f(k) - (1+r)k].$$

Hence the problem of the MFI, ignoring incentive compatibility constraint of the borrower in case the project was a failure at $t-1$ and moneylender's break even constraint, is to

$$\underset{\langle k,r \rangle}{\text{Maximize}} \quad \frac{p^2}{1-\delta}[f(k) - (1+r)k]$$

Subject to:

$$\text{IC}_M^B : \; \left[\delta p + \frac{\delta^2 p^2}{1-\delta}\right][f(k) - (1+r)k] \geq (1+r)k$$

$$\text{IC}_M^{ML} : \; \delta p[f(k) - (1+r)k] - (1+c)(1+r)k \geq 0.$$

[7]Observe in one MFI case this is just the moneylender's breakeven condition.

Lemma 4.2. *IC_M^B is implied by IC_M^{ML}.*

Proof. This is immediate from the observation that

$$[\delta p + \frac{\delta^2 p^2}{1-\delta}][f(k) - (1+r)k] > \delta p[f(k) - (1+r)k] \geq (1+c)(1+r)k > (1+r)k.$$

■

So, the problem of the MFI becomes

$$\begin{aligned} &\underset{\langle k,r\rangle}{\text{Maximize}} \quad \frac{p^2}{1-\delta}[f(k) - (1+r)k] \\ &\text{Subject to: } \text{IC}_M^{ML}: \ \delta p[f(k) - (1+r)k] - (1+c)(1+r)k \geq 0. \end{aligned}$$

Observe that IC_M^{ML} can be written as

$$\frac{\delta p}{1+c} \geq \frac{(1+r)k}{f(k) - (1+r)k}$$

Now we introduce a definition, basically this definition specifies the maximum amount of loan (k_{IM}) for which the moneylender offers $\{R(1)^*, (1+r)k\}$, if loan amount is increased beyond k_{IM}, then the moneylender incurs loss from offering the bridge-loan and hence does not offer any. Formally,

Definition 4.2. *Given δ, $k_{IM}(\delta)$ is the amount of loan such that a moneylender with discount factor δ is indifferent between offering $\{R(1)^*, (1+r)k\}$ and $\{R(0), 0\}$. Hence,*

$$\frac{\delta p}{1+c} = \frac{(1+r)k_{IM}}{f(k_{IM}) - (1+r)k_{IM}}.$$

Lemma 4.3. *Let assumption 4.1 hold*

1. $r^M = 0$
2. $k^M = min\{k^E(1), k_{IM}\}$[8].

Proof.

1. It is obvious from the observation that a decrease in r, not only increases the objective function but also relaxes moneylender's constraint.
2. Putting $r^M = 0$, IC_M^{ML} can be written as

$$\frac{\delta p}{1+c} \geq \frac{k}{f(k) - k}.$$

Observe, the right hand side is increasing in k and the left hand side is constant, so IC_M^{ML} is violated for any $k > k_{IM}$, given the parameters of the model.

[8] Recall definition 4.1 $k^E(p)$ solves $pf^{'}(k) = 1$, so $f^{'}(k^E(1)) = 1$

Now the objective function of the MFI is increasing in k till $k^E(1)$, so whenever IC_M^{ML} is satisfied at $k^E(1)$, that is $k_{IM} \geq k^E(1)$ the MFI lends $k^E(1)$. When IC_M^{ML} is not satisfied at $k^E(1)$, the MFI lends the highest amount which is incentive compatible, so the MFI lends k_{IM} in that case. Hence, the result

$$k^M = \min\{k^E(1), k_{IM}\}.$$

■

So, there exists parametric conditions under which the borrower gets more than the efficient amount of loan, as defined in the definition 4.1. We will discuss the intuition behind this, but before that let us introduce the following corollary and definition.

Corollary 4.1. *The optimum loan amount (weakly) increases with future discount factor.*

The immediate corollary of the above lemma is, as δ increases, given other parameters of the model, k_{IM} increases. Next we introduce a definition:

Definition 4.3. *δ^M is defined as the future discount factor such that $\forall \delta \geq \delta^M$ $k^M = k^E(1)$. Formally,*

$$\delta^M \equiv \delta = \frac{1+c}{p} \frac{k^E(1)}{f(k^E(1)) - k^E(1)}.$$

So a borrower gets $k^E(1)$, which is higher than the efficient amount when $p < 1$, when $\delta \geq \delta^M$ and $\delta^M \in (0, 1)$.

Alternatively, there is over-borrowing whenever the common discount factor and probability of success are high. The intuition behind this result is that the risk of the project is borne by the moneylender and the MFI which designs the contract cares only about the welfare of the borrower.

The above discussions can be summarised in the following proposition.

Proposition 4.1. *Suppose assumption 4.1 hold. There exists a unique equilibrium where*

1. *MFI offers $\langle \{k^M, r^M\} \rangle$ where r^M is zero and $k^M = min\{k^E(1), k_{IM}\}$, and k_{IM} is given by* $\dfrac{\delta p}{1+c} = \dfrac{k_{IM}}{f(k_{IM}) - k_{IM}}$

2. *Moneylender offers $\langle \{R(1)^*, k^M\}, \{R(0), 0\} \rangle$ where $1 + R(1)^* = \dfrac{f(k^M) - k^M}{k^M}$ and $R(1)^* =$ 0*

3. *Borrower takes the bridge-loan, repays the MFI always and moneylender in case of success.*

4. *When $p < 1$, $\delta \geq \delta^M$ and $\delta^M \in (0, 1)$ there is over-borrowing: the borrower gets more than the efficient amount $k^E(p)$.*

4.4 MFI Competition

Now we address the case where there are n MFIs. The MFIs do not share informations among themselves, however each MFI knows that the borrower gets loans from n MFIs. So the borrower does not loses her access to MFI loans even if she defaults on one of the n MFI loans (where $n > 1$). Even moneylender needs higher incentive to provide bridge-loan so that the borrower can repay all the n MFIs instead of x MFIs where $x < n$.

4.4.1 Timeline

Suppose there have been no default on any MFI loan, till date. So, the borrower gets loans from all the n MFIs. Since we are confining ourselves to only symmetric, stationary case, each MFI provides $\frac{k}{n}$ amount of loan where k is the total amount of loan.

Stage 1: MFI i provides $\frac{k}{n}$ amount of loan at an interest rate r, where $i \in \{1, ..., n\}$.

Stage 2: The borrower invests k in $f(\cdot)$ technology and obtains either $f(k)$ with probability p or zero otherwise.

Stage 3: The moneylender observes what happened in stages 1 and 2.

(i) In case the project fails, the moneylender announces $\langle\{R(x), \frac{(1+r)xk}{n}\}_{x\in X}\rangle$ where $X \subseteq \mathcal{X} = \{0, 1, ..., n\}$.

- Next, the borrower observes the menu of the moneylender's contract and decides whether to take a bridge loan or not and if there are more than one options which one to take. She accordingly makes a binding commitment to repay the moneylender in case of success in the next period.[9] The games moves to the next period with those MFIs whose loans the borrower repays using the bridge loan. The MFIs whose loans the borrower does not repay terminate their contracts. Observe, irrespective of repayment decision, agent's instantaneous utility is zero, in case of project failure.

So taking the example we discussed, suppose the moneylender announces $\langle\{R(0), 0\}$; $\{R(2), \frac{(1+r)2k}{6}\}$; $\{R(5), \frac{(1+r)5k}{6}\}\rangle$, and the borrower chooses $\{R(5), \frac{(1+r)5k}{6}\}\rangle$.[10] Then she repays any five out of the six MFIs using the bridge loan and the game moves to the next period with those five MFIs.

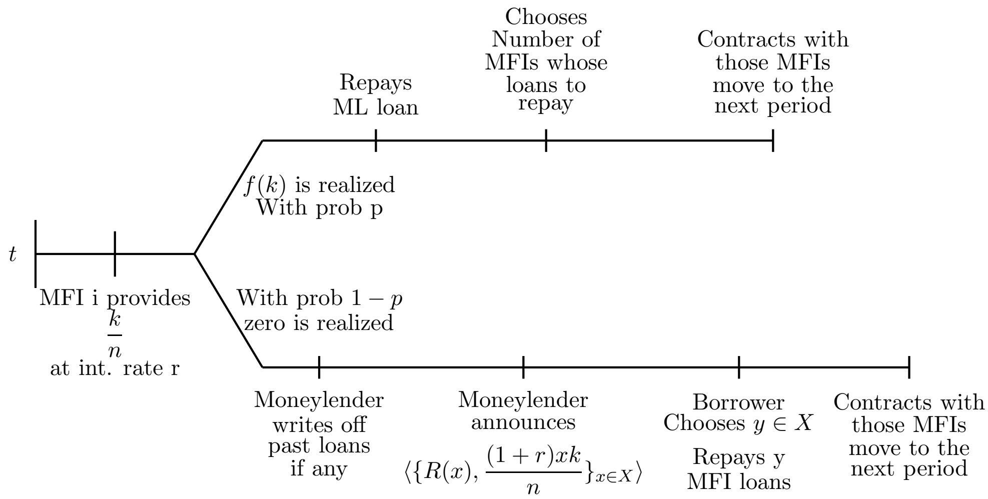
(ii) In case of project success, the future event depends on the previous period's project outcome.

- The project was a failure at $t-1^{th}$ period: Observe the agent must have taken the bridge loan $\{R(n), (1+r)k\}$ at $t-1^{th}$ period, otherwise the game wouldn't have reached t^{th} period with all the n MFIs. So, the borrower repays $(1+R(n))(1+r)k$ to the moneylender.
  * Then decides the number of MFIs whose loans she wants to repay. She then chooses those many MFIs randomly and repays $(1+r)\frac{k}{n}$ to each one of them. The game moves to the next period with those MFIs providing the borrower with $f(k) - (1+R(n))(1+r)k - (1+r)\frac{x}{n}k$ instantaneous utility, where x is the number of MFIs whose loans the borrower repaid.

[9] As mentioned above, due to thick interaction the agent cannot (strategically) default on moneylender loan.

[10] making a binding commitment to the moneylender that in case of success she will repay $(1+R(5))\frac{(1+r)5k}{6}$

– In case the project was a success at $t-1^{th}$ period, similarly the agent decides the number of MFIs whose loans she wants to repay. She then chooses those many MFIs randomly and repays $(1+r)\frac{k}{n}$ to each one of them. The game moves to the next period with those MFIs providing the borrower with $f(k)-(1+r)\frac{x}{n}k$ instantaneous utility, where x is the number of MFIs whose loans the borrower repays.



4.4.2 Moneylender's Problem

Moneylender, like before, wants to maximize his own present discounted value of lifetime utility by offering $\langle\{R(x), x(1+r)\frac{k}{n}\}_{x\in X}\rangle$ where $X \subseteq \mathcal{X} \equiv \{0,1,...,n\}$. Observe $\frac{k}{n}$ is given, so given MFI's contract the moneylender first optimally chooses $R(x)$ for each $x \in \{1,..,n\}$, then he compares the present discounted value of his lifetime utility from offering each of those x's. After that, he chooses X such that the present discounted value of his lifetime utility from offering $\{R(x), x(1+r)\frac{k}{n}\}$ for each $x \in X$ are equal and that from offering $\{R(y), y(1+r)\frac{k}{n}\}$ where $y \notin X$ is strictly lower.

If there are more than one such $\{R(x), x(1+r)\frac{k}{n}\}$ that is $|X| > 1$, then the borrower chooses one of them to maximize her present discounted value of lifetime utility. Each MFI knows these and offers $\frac{k}{n}$ such that $n \in X$, the borrower accepts $\{R(n), (1+r)k\}$ and the borrower's utility is maximized.

We first discuss the problem of the moneylender, the problem of the moneylender while choosing $R(x)$ is to

$$\begin{aligned}
\underset{R(x)}{\text{Maximize}} \quad & \delta p(1+R(x))x(1+r)\frac{k}{n} - (1+c)(1+r)x(1+r)\frac{k}{n} \\
& + \delta^2 p(1-p)(1+R(x))x(1+r)\frac{k}{n} - \delta(1-p)(1+c)x(1+r)\frac{k}{n} \\
& + \delta^3 p(1-p)(1+R(x))x(1+r)\frac{k}{n} - \delta^2(1-p)(1+c)x(1+r)\frac{k}{n} + ...
\end{aligned}$$

Subject to:

$$\text{RC:} \quad f(x\frac{k}{n}) - (1+R(x))x(1+r)\frac{k}{n} \geq x(1+r)\frac{k}{n}$$

First let us explain the Repayment Constraint (RC) which is an immediate outcome of our assumptions of limited liability and stationary contract. The left hand side of this constraint is the amount the borrower has after repaying the moneylender: $f(x\frac{k}{n}) - (1+R(x))x(1+r)\frac{k}{n}$ and the right hand side is the amount to be repaid to the MFI if the borrower wants to continue the relationship with x number of MFI: $x(1+r)\frac{k}{n}$. So, this constraint ensures that the amount the borrower has after repaying the moneylender is high enough to repay x MFI loan.

Now we explain the moneylender's objective function while choosing $R(x)$ at any period (say at t^{th} period). The first two terms correspond to the expected present discounted value from providing loan at this period: with probability p the project becomes successful at $t+1^{th}$ period and the borrower repays $(1+R(x))x(1+r)\frac{k}{n}$ whereas the moneylender's cost of providing the bridge-loan is $(1+c)x(1+r)\frac{k}{n}$. So the moneylender's expected present discounted value from providing loan at t^{th} period is $\delta p(1+R(x))x(1+r)\frac{k}{n} - (1+c)x(1+r)\frac{k}{n}$. Similarly the third and the fourth terms represent expected present discounted value from providing bridge-loan at $t+1^{th}$ period. The borrower takes the bridge-loan at $t+1^{th}$ period when her project fails, probability of which is $1-p$. So the moneylender's expected present discounted value from providing loan at $t+1^{th}$ period is $\delta(1-p)[\delta p(1+R(x))x(1+r)\frac{k}{n} - (1+c)x(1+r)\frac{k}{n}]$. Likewise we get the following terms.

The moneylender's job is to choose $R(x)$ in order to maximize this objective function given RC. Observe that the moneylender's problem can alternatively written as:

$$\underset{R(x)}{\text{Maximize}} \quad \left[1+\frac{\delta(1-p)}{1-\delta}\right]\left[\delta p(1+R(x))x(1+r)\frac{k}{n} - (1+c)x(1+r)\frac{k}{n}\right]$$

Subject to:

$$\text{RC:} \quad f(x\frac{k}{n}) - (1+R(x))x(1+r)\frac{k}{n} \geq x(1+r)\frac{k}{n}. \tag{4.4.1}$$

Lemma 4.4. *1. The moneylender chooses $R(x)^*$ such that*

$$1+R(x)^* = \frac{f(x\frac{k}{n}) - x(1+r)\frac{k}{n}}{x(1+r)\frac{k}{n}}$$

2. The moneylender optimally chooses X where $X \subseteq \mathcal{X} \equiv \{0,1,...,n\}$, that is at optimum he offers $\langle\{R(x)^*, x(1+r)\frac{k}{n}\}_{x\in X^*}\rangle$ such that

(i) Present discounted value of lifetime utility from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ are equal for each $x \in X^*$ and

(ii) Present discounted value of lifetime utility from offering $\{R(y)^*, y(1+r)\frac{k}{n}\}$ where $y \notin X^*$ is strictly lower than that from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ where $x \in X^*$.

Proof.

1. Moneylender's objective function is increasing and the constraint gets tighter with increase

in $R(x)$. So, the moneylender would optimally choose $R(x)$ such that RC binds. Hence the result.

2.(i) Suppose not.

a) $\exists x^{'} \in X^*$ such that the present discounted values of lifetime utility from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ are equal $\forall x \in X^* \setminus x^{'}$ and the present discounted value of lifetime utility from offering $\{R(x^{'})^*, x^{'}(1+r)\frac{k}{n}\}$ is strictly higher than that from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ for any $x \in X^* \setminus x^{'}$.

Then observe it is (weakly) utility improving for the moneylender to offer $\{R(x^{'})^*, x^{'}(1+r)\frac{k}{n}\}$ only, it is strictly utility improving when the borrower chooses $\{R(x)^*, x(1+r)\frac{k}{n}\}$ where $x \in X^* \setminus x^{'}$. This is a contradiction.

b) $\exists x^{'} \in X^*$ such that the present discounted values of lifetime utility from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ are equal $\forall x \in X^* \setminus x^{'}$ and the present discounted value of lifetime utility from offering $\{R(x^{'})^*, x^{'}(1+r)\frac{k}{n}\}$ is strictly lower than that from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ for any $x \in X^* \setminus x^{'}$.

Then observe it is (weakly) utility improving for the moneylender to offer $X^* \setminus x^{'}$, it is strictly utility improving when the borrower chooses $\{R(x^{'})^*, x^{'}(1+r)\frac{k}{n}\}$. This is a contradiction.

2.(ii) Similarly suppose not. $\exists y \notin X^*$ such that the present discounted value of lifetime utility from offering $\{R(y)^*, y(1+r)\frac{k}{n}\}$ is strictly higher than that from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ where $x \in X^*$. Then it is strictly utility improving for the moneylender to offer $\langle\{R(y)^*, y(1+r)\frac{k}{n}\}\rangle$.

We assume that when the moneylender is indifferent between two contracts $\{R(x)^*, x(1+r)\frac{k}{n}\}$ and $\{R(y)^*, y(1+r)\frac{k}{n}\}$, that is when present discounted lifetime utility from offering $\{R(y)^*, y(1+r)\frac{k}{n}\}$ is equal to that from offering $\{R(x)^*, x(1+r)\frac{k}{n}\}$ where $x \in X^*$, the moneylender offers $\{R(y)^*, y(1+r)\frac{k}{n}\}$ as well. Hence the result.

■

4.4.3 MFI's problem

Let us now consider the problem of the MFI. The objective of the MFI is to maximize the present discounted value of lifetime utility of the borrower subject to *ex post* break even constraint. As discussed above, the project gets successful only with probability p, given limited liability, in case of failure the borrower cannot repay on her own. So the moneylender has to have incentives to provide bridge-loans . Now observe since there are n MFIs, moneylender may choose to provide bridge-loans such that it is only possible to repay a subset of MFIs. Since we have confined ourselves to symmetric, stationary environment, in that case the borrower would choose the subset of MFIs randomly. So, in order to avoid default, each MFI has to design contracts such that both the borrower and the moneylender have incentives to continue the relationships with all the n MFIs. The moneylender must have incentive to provide $\{R(n), k\}$ contract for the bridge-loan when the borrower's project fails. And the borrower should also have incentive to repay all the n MFIs, in case of success on her own and in case of failure by taking bridge-loan from the moneylender.

Let us first consider the incentive of the borrower in case of success at any period t : The borrower must (weakly) prefer to repay n MFI loans over repaying x MFI loans where $x \in \{0, ..., n-1\}$. In the following lemma we show that it is sufficient to consider that the borrower does not have incentive to deviate and repay $n-1$ MFIs instead of n MFIs. Alternatively, we show that the condition which ensures that the borrower prefers to repay all n MFI loans over $n-1$ MFI loans also ensures that she prefers to repay x MFI loans over $x-1$ MFI loans, where $x \in \{0, ..., n-1\}$. Hence recursively, this condition is sufficient to incentivise the borrower to repay n MFI loans over any x MFI loans. First let us state and explain the incentive compatibility constraint of the borrower in case of success. The incentive compatibility constraint of the borrower at any period t when the project becomes successful in both t and $t-1^{th}$ period is given by:

$$\begin{aligned}
&f(k) - (1+r)k + \delta p[f(k) - (1+r)k] \\
&\quad + \delta^2 p^2[f(k) - (1+r)k] + \delta^2(1-p)p[f(k) - (1+r)(1+R(n))k - (1+r)k] + ... \\
&\geq f(k) - x(1+r)\frac{k}{n} + \delta p[f(x\frac{k}{n}) - x(1+r)\frac{k}{n}] \\
&\quad + \delta^2 p^2[f(x\frac{k}{n}) - x(1+r)\frac{k}{n}] + \delta^2(1-p)p[f(x\frac{k}{n}) - x(1+r)(1+R(x))\frac{k}{n} - x(1+r)\frac{k}{n}] + ... \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall x \in \{0, ..., n-1\}.
\end{aligned}$$

The left hand side of the above incentive compatibility constraint denotes the borrower's present discounted value of lifetime utility from repaying all the n MFIs. On the other hand, the right hand side denotes her present discounted value of lifetime utility from deviation by repaying only x number of MFIs instead of n MFI where $x < n$. Now let us explain the incentive compatibility constraint in more details.

The first two terms of the left hand side of the constraint denote the borrower's instantaneous utility after repaying all the n MFI: $f(k)$ is the project return and she repays $(1+r)\frac{k}{n}$ to each of the n MFIs. Similarly in case of success in the next period, which happens with probability with p, her instantaneous utility is $f(k) - (1+r)k$, so her expected present discounted value of utility from that is given by $\delta p[f(k) - (1+r)k]$. Again following this event, at $t+2^{th}$ period with probability p the agent becomes successful, expected present discounted value from that contingency is given by $\delta^2 p^2[f(k) - (1+r)k]$. On the other hand, in case her project fails at $t+1^t$ period which happens with probability $1-p$ she takes a bridge-loan and repays MFI loans.[11] Then in case of success at $t+2^{th}$ period, which happens with probability p, she first repays the moneylender and then the MFIs. Expected present discounted value from that contingency is

[11]This is ensured by incentive compatibility constraint in case of failure which we discuss later.

$\delta^2(1-p)p[f(k)-(1+r)(1+R(n))k-(1+r)k]$. Similarly we can get the following terms.

The first two terms of the right hand side of the incentive compatibility constraint denote the borrower's instantaneous utility from repaying x number of MFIs: $f(k)$ is the project return and she repays $(1+r)\frac{k}{n}$ to each of the x MFIs. Since the borrower is repaying only x MFIs all other $n-x$ MFIs terminate their contracts. So, the borrower gets loans from only those x MFIs from $t+1^{th}$ period. As discussed above for the case of n MFIs, present discounted value of lifetime utility from continuing the relationship with x MFIs can be interpreted.

The incentive compatibility constraint of the borrower at any period t when the project becomes successful at t^{th} but was a failure at $t-1^{th}$ period, is given by:

$$\begin{aligned}
&f(k)-(1+r)(1+R(n))k-(1+r)k+\delta p[f(k)-(1+r)k]\\
&\quad+\delta^2p^2[f(k)-(1+r)k]+\delta^2(1-p)p[f(k)-(1+r)(1+R(n))k-(1+r)k]+...\\
\geq& f(k)-(1+r)(1+R(n))k-x(1+r)\frac{k}{n}+\delta p[f(x\frac{k}{n})-x(1+r)\frac{k}{n}]\\
&\quad+\delta^2p^2[f(x\frac{k}{n})-x(1+r)\frac{k}{n}]+\delta^2(1-p)p[f(x\frac{k}{n})-x(1+r)(1+R(x))\frac{k}{n}-x(1+r)\frac{k}{n}]+...\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\forall x\in\{0,...,n-1\}.
\end{aligned}$$

Observe the difference between this incentive compatibility constraint with the previous one comes from t^{th} period's instantaneous utility. This is because the project was a failure at $t-1^{th}$ period and since the contract has reached t^{th} period with all the n MFIs, the borrower must have taken $\{R(n),k\}$ bridge-loan. So after realization of the prject return $f(k)$ she has to repay $(1+r)(1+R(n))k$ to the moneylender, then she decides the number of MFI whose loans she wants to repay. The left hand side represents the present discounted value of lifetime utility of the borrower when she repays all the n MFIs and the right hand side represents that when she decides to repay x number of MFIs. Now the terms of both the sides are immediate from the previous discussion.

Finally incorporating the moneylender's strategy

$$x(1+R(x)^*)(1+r)\frac{k}{n}=f(x\frac{k}{n})-x(1+r)\frac{k}{n}$$

the incentive compatibility constraints of the borrower at tth period, when the project is successful, can be written as: $\forall x\in\{0,...,n-1\}$

$$\big[\delta p+\frac{\delta^2p^2}{1-\delta}\big]\big[[f(k)-(1+r)k]-[f(x\frac{k}{n})-x(1+r)\frac{k}{n}]\big]\geq(1+r)\frac{n-x}{n}k.$$

Now let us turn to the incentive compatibility constraint in case the project fails at t^{th} period. This constraint ensures that the borrower takes the bridge-loan $\{R(n),k\}$ and continues the relationship with all the n MFIs rather than taking any other bridge-loan contract $\{R(x),x\frac{k}{n}\}$ and continues the relationship with only x MFIs, where $x\in\{0,1,...,n-1\}$. Observe this also captures the scenario where the borrower does not take any bridge-loan at all.

$$\begin{aligned}
&\delta p\big[f(k)-(1+r)(1+R(n))k-(1+r)k\big]+\delta^2p^2[f(k)-(1+r)k]\\
&\ +\delta^2(1-p)p[f(k)-(1+r)(1+R(n))k-(1+r)k]+...\\
\geq&\delta p\big[f(x\frac{k}{n})-(1+r)(1+R(x))k-x(1+r)\frac{k}{n}\big]+\delta^2p^2[f(x\frac{k}{n})-x(1+r)\frac{k}{n}]\\
&\ +\delta^2(1-p)p[f(x\frac{k}{n})-x(1+r)(1+R(x))\frac{k}{n}-x(1+r)\frac{k}{n}]+... \qquad \forall x\in\{0,...,n-1\}.
\end{aligned}$$

Given the discussion above, this incentive compatibility constraint is obvious. Like before we incorporate moneylender's strategy and rewrite the incentive compatibility constraint for taking bridge-loan as: $\forall x\in\{0,...,n-1\}$

$$\frac{\delta^2p^2}{1-\delta}\big[[f(k)-(1+r)k]-[f(x\frac{k}{n})-x(1+r)\frac{k}{n}]\big]\geq 0.$$

Observation 4.2. *Incentive compatibility constraint for repaying n MFIs rather than x MFIs by taking bridge-loan in case of failure is implied by that in case of success, where $x\in\{0,1,...,n-1\}$.*

Proof.

$$[f(k)-(1+r)k]-[f(x\frac{k}{n})-x(1+r)\frac{k}{n}]\geq\frac{(1+r)\frac{n-x}{n}k}{\delta p+\frac{\delta^2p^2}{1-\delta}}>0.$$

■

So, the only incentive compatibility constraint to induce the borrower to maintain relationship with all the n MFIs is given by $\forall x\in\{0,...,n-1\}$

$$\big[\delta p+\frac{\delta^2p^2}{1-\delta}\big]\big[[f(k)-(1+r)k]-[f(\frac{x}{n}k)-x(1+r)\frac{k}{n}]\big]\geq(1+r)\frac{n-x}{n}k.$$

In the next lemma we show that it is sufficient to consider deviation from n to $n-1$ MFIs only.

Lemma 4.5. *If the borrower does not have incentive to deviate from n MFIs to $n-1$ MFIs, then she does not also have any incentive to deviate from n to x MFIs where $x\in\{0,1,...,n-2\}$.*

Proof. We prove this lemma recursively. We show that the condition which ensures that the borrower does not have any incentive to deviate from n MFIs to $n-1$ MFIs also ensures that she does not have any incentive to deviate from $n-1$ to $n-2$ MFIs. Then we show that this condition also ensures that the borrower does not have any incentive to deviate from any general x to $x-1$ MFIs, given that she did not have that to deviate from $x+1$ to x MFIs. So the lemma is obvious from the way we show this.

So we want to show that

$$
\begin{aligned}
&[\delta p + \frac{\delta^2 p^2}{1-\delta}]\,[[f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k]] \\
\geq &(1+r)\frac{n-(n-1)}{n}k = (1+r)\frac{1}{n}k \\
\Rightarrow &[\delta p + \frac{\delta^2 p^2}{1-\delta}]\,[[f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k] - [f(\frac{n-2}{n}k) - (1+r)\frac{n-2}{n}k]] \\
\geq &(1+r)\frac{n-1-(n-2)}{n}k = (1+r)\frac{1}{n}k
\end{aligned}
$$

Observe in order to show this it is sufficient to show that

$$
\begin{aligned}
&[f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k] - [f(\frac{n-2}{n}k) - (1+r)\frac{n-2}{n}k] \\
\geq &[f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k]
\end{aligned}
$$

But it is obvious given our assumption that $f(\cdot)$ is a strict concave function. From this it is obvious that it can be shown for any general x where $x \in \{1, ..., n-1\}$. So we get

$$
\begin{aligned}
&f(\frac{1}{n}k) - (1+r)\frac{1}{n}k > [f(\frac{2}{n}k) - (1+r)\frac{2}{n}k] - [f(\frac{1}{n}k) - (1+r)\frac{1}{n}k] \\
> &[f(\frac{3}{n}k) - (1+r)\frac{3}{n}k] - [f(\frac{2}{n}k) - (1+r)\frac{2}{n}k] \\
&\vdots \\
> &[f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k] \geq \frac{(1+r)\frac{1}{n}k}{\delta p + \frac{\delta^2 p^2}{1-\delta}}.
\end{aligned}
$$

Now it is obvious that

$$
[f(k) - (1+r)k] - [f(\frac{x}{n}k) - (1+r)\frac{x}{n}k] > \frac{(1+r)\frac{n-x}{n}k}{\delta p + \frac{\delta^2 p^2}{1-\delta}}
$$

Hence, the lemma. ■

So, the only incentive compatible constraint for the borrower an MFI needs to consider is given by

$$
[f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k] \geq \frac{(1+r)\frac{1}{n}k}{\delta p + \frac{\delta^2 p^2}{1-\delta}} \qquad (\ IC_c^B\ )
$$

Next we turn to the moneylender's incentive problem: The moneylender must (weakly) prefer to offer bridge-loan $\{R(n), k\}$ over any other bridge-loan contract $\{R(x), \frac{x}{n}k\}$ where $x \in \{0, ..., n-$

$1\}$. As discussed above the moneylender offers $\{R(n), k\}$ only if

$$\begin{aligned}&\delta p[f(k) - (1+r)k] - (1+c)(1+r)k \\ \geq\ &\delta p[f(\frac{x}{n}k) - (1+r)\frac{x}{n}k] - (1+c)(1+r)\frac{x}{n}k \qquad \forall x \in \{0, 1, ..., n-1\}.\end{aligned}$$

Similar to the case of borrower's incentive problem, here also we show that it is sufficient to consider that the moneylender (weakly) prefers to offer $\{R(n), k\}$ over $\{R(n-1), \frac{n-1}{n}k\}$.

Lemma 4.6. *If the moneylender weakly prefers to offer bridge-loan contract $\{R(n), k\}$ over the bridge-loan contract $\{R(n-1), \frac{n-1}{n}k\}$, then he also prefers to offer $\{R(n), k\}$ over $\{R(x), \frac{x}{n}k\}$.*

Proof. Can be shown using the arguments of the previous lemma. ■

So, the only incentive compatible constraint for the moneylender an MFI needs to consider is given by

$$\delta p\big[[f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - \frac{n-1}{n}(1+r)k]\big] \geq (1+c)(1+r)\frac{1}{n}k. \qquad (IC_c^{ML})$$

So the problem of an MFI is to

$$\underset{\langle \frac{k}{n}, r\rangle}{\text{Maximize}} \quad \frac{p^2}{1-\delta}[f(k) - (1+r)k]$$

Subject to:

$$\text{IC}_c^B : \ [f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - (1+r)\frac{n-1}{n}k] \geq \frac{(1+r)\frac{1}{n}k}{\delta p + \frac{\delta^2 p^2}{1-\delta}}$$

$$\text{IC}_c^{ML} : \ \delta p\big[[f(k) - (1+r)k] - [f(\frac{n-1}{n}k) - \frac{n-1}{n}(1+r)k]\big] \geq (1+c)(1+r)\frac{1}{n}k.$$

In the following lemma we show that it is sufficient to consider IC_c^{ML} only.

Lemma 4.7. *IC_c^{ML} implies IC_c^B*

Proof. Observe IC_c^{ML} can be written as

$$\frac{\delta p}{1+c} \geq \frac{(1+r)k}{n\big[f(k) - f(\frac{n-1}{n}k)\big] - (1+r)k}$$

IC_c^B can be written as

$$\frac{(1-\delta)\delta p + \delta^2 p^2}{1-\delta} \geq \frac{(1+r)k}{n\big[f(k) - f(\frac{n-1}{n}k)\big] - (1+r)k}$$

Also observe that $\dfrac{(1+r)k}{n\big[f(k) - f(\frac{n-1}{n}k)\big] - (1+r)k}$ is an increasing function of k. So in order to prove the lemma we show that $\dfrac{(1-\delta)\delta p + \delta^2 p^2}{1-\delta} > \dfrac{\delta p}{1+c}$, hence any k for which IC_c^{ML} is satisfied, IC_c^B is also satisfied.

$$\frac{(1-\delta)\delta p+\delta^2p^2}{1-\delta}-\frac{\delta p}{1+c}=\frac{\delta^2p^2+c[(1-\delta)\delta p+\delta^2p^2]}{(1-\delta)(1+c)}>0$$

■

So, the problem of an MFI can be written as

$$\underset{\langle \frac{k}{n},r\rangle}{\text{Maximize}} \quad \frac{p^2}{1-\delta}[f(k)-(1+r)k]$$

Subject to:

$$\text{IC}_c^{ML}:\ \delta p\big[[f(k)-(1+r)k]-[f(\frac{n-1}{n}k)-\frac{n-1}{n}(1+r)k]\big]\geq(1+c)(1+r)\frac{1}{n}k$$

As observed above IC_c^{ML} can be written as

$$\frac{\delta p}{1+c}\geq\frac{(1+r)k}{n\big[f(k)-f(\frac{n-1}{n}k)\big]-(1+r)k}$$

Like before we define k_{Ic} such that the moneylender is indifferent between offering $\{R(n)^*,(1+r)k\}$ and $\{R(n-1)^*,\frac{n-1}{n}(1+r)k\}$. Formally,

Definition 4.4. *$k_{Ic}(\delta)$ is defined as,*

$$\frac{\delta p}{1+c}=\frac{(1+r)k_{Ic}}{n\big[f(k_{Ic})-f(\frac{n-1}{n}k_{Ic})\big]-(1+r)k_{Ic}}.$$

Lemma 4.8. *Let assumption 4.1 hold*

1. $r^c=0$
2. $k^c=min\{k^E(1),k_{Ic}\}$[12].

Proof. The proof is same as that of lemma 4.3, so we skip it here. ■

So, there exists parametric conditions under which the borrower gets more than the efficient amount of loan, as defined in the definition 4.1. Also like before we introduce the following definition.

Definition 4.5. *δ^c is defined as the future discount factor such that $\forall\delta\geq\delta^c$ $k^c=k^E(1)$. Formally,*

$$\delta^c\equiv\delta=\frac{1+c}{p}\frac{k^E(1)}{n\big[f(k^E(1))-f(\frac{n-1}{n}k^E(1))\big]-k^E(1)}.$$

So there is over-borrowing when $p<1$, $\delta\geq\delta^M$ and $\delta^M\in(0,1)$.
The above discussions can be summarised in the following proposition.

Proposition 4.2. *Suppose assumption 4.1 hold. There exists a unique equilibrium where*

[12]Recall definition 4.1 $k^E(p)$ solves $pf^{'}(k)=1$, so $f^{'}(k^E(1))=1$

1. *MFI offers* $\langle\{k^c, r^c\}\rangle$ *where* r^c *is zero and* $k^c = min\{k^E(1), k_{Ic}\}$, *and* k_{Ic} *is given by* $\dfrac{\delta p}{1+c} = \dfrac{k_{Ic}}{n\big[f(k_{Ic}) - f(\frac{n-1}{n}k_{Ic})\big] - k_{Ic}}$

2. *Moneylender offers* $\langle\{R(n)^*, k^c\}, \{R(n-1)^*, \frac{n-1}{n}k\}\rangle$ *where* $1 + R(n)^* = \dfrac{f(k^c) - k^c}{k^c}$ *and* $1 + R(n-1)^* = \dfrac{f(\frac{n-1}{n}k^c) - \frac{n-1}{n}k^c}{\frac{n-1}{n}k^c}$

3. *Borrower takes the bridge-loan, repays all the* n *MFI always and moneylender in case of success.*

4. *When* $p < 1$, $\delta \geq \delta^c$ *and* $\delta^c \in (0, 1)$ *there is over-borrowing: the borrower gets more than the efficient amount* $k^E(p)$.

4.5 Comparison

In this section we compare the scenario where there is one MFI with that where there are n MFIs, where $n > 1$. We compare MFI loan amount, moneylender's interest rate, bridge-loan amount and finally the welfare of the borrower and the profit of the moneylender between these two scenario.

4.5.1 MFI Loan Size

As we have observed moneylender's incentive constraint is stronger than that of the borrower irrespective of the number of MFIs present in the economy. So we try to understand how moneylender's incentive changes as the number of the MFI increases in the economy.

When there are more than one MFIs, the moneylender knows that even if he provides bridge-loan to repay only x MFIs loans, where $x < n$, then also the relationship with the MFIs will not be terminated compeletely, it will be continued with those x MFIs. So alternatively, as the number of the MFIs increases the moneylender can choose the "amount" of bridge-loan more finely, by choosing the number of MFIs whose loans to be repaid using the bridge-loan. Note that due to our assumption that $f(\cdot)$ is strictly concave and the cost of providing loan is linear, marginal profit of the moneylender is higher when k is low. Also, the moneylender incurs cost of lending one period earlier than he gets the benefit from that lending, if any. So when a moderately motivated[13] moneylender has just two options - either provide the entire amount to be repaid (say k) and continue the relationship or provide nothing which leads to the termination of the relationship, the moneylender may choose the former. However when the same moneylender has the option to provide either the entire amount and continue the relationship with all the n MFIS or provide only a part of that and continue the relationship with a subset of n MFIs, he may choose the latter. Hence k was incentive compatible with one MFI but is not incentive compatible when there are n MFIs. So, the MFIs have to decrease the loan amount to avoid default.

[13] We formalize "moderately patient" in the following paragraphs.

Hence we can see that the future discount factor plays an important role which gives us our next lemma. The minimum patience level required to provide $k^E(1)$ amount of bridge-loan is higher when there are n MFIs vis-a-vis when there is one MFI. Given this lemma, it is evident that aggregate loan amount in both the markets are equal when future discount factor is higher than δ^c.

Lemma 4.9. $\delta^c > \delta^M$.

Proof.

$$\begin{aligned}
&\delta^c - \delta^M \\
=&\frac{1+c}{p}\frac{k^E(1)}{n\big[f(k^E(1)) - f(\frac{n-1}{n}k^E(1))\big] - k^E(1)} - \frac{1+c}{p}\frac{k^E(1)}{f(k^E(1)) - k^E(1)} \\
=&\frac{(1+c)k^E(1)}{p}\frac{f(k^E(1)) - k^E(1) - nf(k^E(1)) + nf(\frac{n-1}{n}k^E(1)) + k^E(1)}{\Big[n\big[f(k^E(1)) - f(\frac{n-1}{n}k^E(1))\big] - k^E(1)\Big]\Big[f(k^E(1)) - k^E(1)\Big]} \\
=&\frac{(1+c)k^E(1)}{p}\frac{n\Big[f(\frac{n-1}{n}k^E(1)) - \frac{n-1}{n}f(k^E(1))\Big]}{\Big[n\big[f(k^E(1)) - f(\frac{n-1}{n}k^E(1))\big] - k^E(1)\Big]\Big[f(k^E(1)) - k^E(1)\Big]} \\
>&0.
\end{aligned}$$

where the last inequality is coming from our assumption that $f(\cdot)$ is a concave function.

■

Now we compare the optimum loan size when there is one MFI vis-a-vis when there are n MFIs. In fact the optimum loan size is a monotonice function of the number of the MFIs. In the next lemma we show that the optimum loan size when there is one MFI is (weakly) higher than that when there are n MFIs, they are equal when future discount factor is very high namely $\delta \geq \delta^c$, otherwise the former is strictly higher.

Lemma 4.10. *k^M is weakly higher than k^c.*

1. *When $\delta \in [\delta^c, 1]$ $k^c = k^M = k^E(1)$*
2. *When $\delta < \delta^c$ $k^M > k^c$.*

Proof.

1. Recall when $\delta \geq \delta^c$ $k^c = k^E(1)$ and when $\delta \geq \delta^M$ $k^M = k^E(1)$.

   Now, $\delta^c > \delta^M$, so when $\delta \in [\delta^c, 1]$ $k^c = k^M = k^E(1)$.

2. $\delta < \delta^c$:

a) When $\delta \in [\delta^M, \delta^c)$, $k^M = k^E(1) > k_{Ic}(\delta) = k^c$.

b) When $\delta < \delta^M$

$k^M = k_{IM}(\delta) < k^E(1)$ and $k^c = k_{Ic}(\delta) < k^E(1)$.

Let us now compare $k_{IM}(\delta)$ with $k_{Ic}(\delta)$.

Recall $\dfrac{\delta p}{1+c} = \dfrac{k_{Ic}}{n\big[f(k_{Ic}) - f(\frac{n-1}{n}k_{Ic})\big] - k_{Ic}}$.

Now observe that $\dfrac{\delta p}{1+c}$ is a constant, whereas $\dfrac{k}{n\big[f(k) - f(\frac{n-1}{n}k)\big] - k}$ is an increasing function of k and when $n > n^{'}$

$$\frac{k}{n\big[f(k) - f(\frac{n-1}{n}k)\big] - k} > \frac{k}{n'\big[f(k) - f(\frac{n^{'}-1}{n^{'}}k)\big] - k}.$$

So, $k_{Ic}(n)$ must be less than $k_{Ic}(n^{'})$ in order to maintain the equality. In particular

$$k^M = k_{IM} = k_{Ic}(1) > k_{Ic}(n) = k^c.$$

Hence, the result.

■

4.5.2 Moneylender's Loan Size and Interest Rate

Now we study the effect of the increase in the number of the MFIs on the moneylender's interest rate and on the amount of loan. The moneylender only provides bridge-loan in our framework, so as loan amount is weakly decreasing with the number of the MFIs, so is the bridge-loan.

Let us now compare $R(n)^*$ and $R(1)^*$, where $n > 1$. Since $f(\cdot)$ is a concave function and optimum loan amount is (weakly) lower when there are n MFIs in comparison to that when there is one MFI, so average net product is higher in the former scenario. Now, the moneylender's gross interest rate is the average net product which implies that $R(n)^*$ is higher than $R(1)^*$. This result is driven by the fact that the moneylender's "bargaining power" in the competitive market is higher, so loan amount is lower which in turn increases the moneylender's interest rate. Hence the following lemma.

Lemma 4.11. *1. Bridge-loan is (weakly) lower when there are more than one MFIs.*

2. Moneylender's interest rate is (weakly) higher when there are more than one MFIs.

Proof.

1. This is trivial in that the moneylender provides bridge-loan only to repay MFI loan(s). Since the aggregate loan amount is (weakly) lower when there are more than one MFIs than that when there is only one MFI, so is the bridge loan.

2. Recall $1 + R(n)^* = \dfrac{f(k^c) - k^c}{k^c}$ and $1 + R(1)^* = \dfrac{f(k^M) - k^c}{k^M}$.

   So, $R(n)^* - R(1)^* = \dfrac{f(k^c)}{k^c} - \dfrac{f(k^M)}{k^M} \geq 0$, where the last inequality is strict when $k^c < k^M$ and we get this inequality from our assumption that $f(\cdot)$ is a strict concave function.[14]

[14]Differentiating $\frac{f(k)}{k}$ with respect to k we get $\frac{f^{'}(k)k - f(k)}{k^2}$ <)0 since $f(\cdot)$ is a strict concave function.

Hence the result.

■

4.5.3 Welfare

Observe, the borrower's present discounted value of lifetime utility is increaseing in k when $k < k^E(1)$. Now when $\delta < \delta^c$, $k^c < k^M$ and k^M never exceeds $k^E(1)$. Hence borrower's present discounted value of lifetime utility is (weakly) higher when there is one MFI rather than n MFIs, strictly higher when $\delta < \delta^c$. On the other hand as it is evident from the above discussion that moneylender's present discounted value of lifetime utility is (weakly) higher when there are more than one MFI than that when there is one MFI. In the following lemma we show this formally.

Lemma 4.12. *1. The borrower's present discounted value of lifetime utility is (weakly) higher when there is one MFI in comparison to that when there are more than one MFIs in the economy.*

2. The moneylender's present discounted value of lifetime utility is (weakly) lower when there is one MFI in comparison to that when there are more than one MFIs in the economy.

Proof.

1. Observe the borrower's present discounted value of lifetime utility is given by $\frac{p^2}{1-\delta}[f(k) - k]$ which is an increasing function of when $k < k^E(1)$. So, when $\delta \in [\delta^c, 1]$ borrower's present discounted value of lifetime utility in both the cases are equal whereas when $\delta < \delta^c$ $k^c < k^M$ and k^M never exceeds $k^E(1)$. Hence the result.

2. Here also when $\delta \in [\delta^c, 1]$ moneylender's present discounted value of lifetime utility in both the cases are equal.

   Now let us consider the case $\delta < \delta^c$

$$
\begin{aligned}
&\text{ML's PDV of lifetime utility}|_{n\ \text{MFI}} - \text{ML's PDV of lifetime utility}|_{1\ \text{MFI}} \\
=&\Big[1 + \frac{\delta(1-p)}{1-\delta}\Big]\Big[\big[\delta p[f(k^c) - k^c] - (1+c)k^c\big] - \big[\delta p[f(k^M) - k^M] - (1+c)k^M\big]\Big] \\
=&\delta p[-\delta p[f(k^M) - k^M] - (1+c)k^M \\
=&\delta p[f(k_{Ic}) - nf(k_{Ic}) + nf(\frac{n-1}{n}k_{Ic})] \\
=&\delta pn\big[f(\frac{n-1}{n}k_{Ic}) - \frac{n-1}{n}f(k_{Ic})\big] \\
>&0.
\end{aligned}
$$

   where we get the second equality by putting $k^c = k_{Ic}$ and $k^M = k_{IM}$ and the last inequality from our assumption that $f(\cdot)$ is a concave function.

■

Summarizing the above discussion we get the following proposition.

Proposition 4.3. *1. As the number of the MFI increases aggregate optimum MFI loan size (weakly) decreases.*

2. As the number of the MFI increases moneylender's loan amount (weakly) decreases.

3. As the number of the MFI increases moneylender's interest rate (weakly) increases.

4. The borrower's present discounted value of lifetime utility is (weakly) higher when there is one MFI in comparison to that when there are more than one MFIs in the economy.

5. The moneylender's present discounted value of lifetime utility is (weakly) lower when there is one MFI in comparison to that when there are more than one MFIs in the economy.

4.6 Conclusion

This chapter seeks to provide a theoretical explanation of the empirical findings that as competition among MFIs increases aggregate MFI loan size, moneylender loan size decrease, borrower's welfare decreases whereas moneylender's profit increases. As stated above, we assume that the MFIs can commit and MFI contracts are chosen from the class of stationary and symmetric contracts. Checking the robustness of the analysis to these assumptions are part of my future research agenda.

This study, hence, contributes to the literature where it is argued that competition among MFIs not necessarily improve the borrower's welfare especially when the MFIs are inherently benevolent.

Chapter 5

Conclusion

This thesis studies the dynamics of credit contracts in a context where neither legal nor social enforcement of repayment is possible and the borrowers are strategic, however repayment rate is very high. So, this incidence of high repayment solely depends on the dynamic incentives.

Part I comprises two chapters and provides two explanations of progressive lending in two different frameworks. First, where the borrower has a preference for consumption smoothing and has access to a technology to transfer money from one period to another even at autarky. In the second framework, there is a possibility of graduation which requires a minimum amount of wealth and the economic opportunity of an agent with endowment below that threshold is very limited in that capital depriciates completely within a period. The benevolent lender's job is to help the borrower to accumulate that minimum wealth which is required to graduate.

Part II comprises one chapter which studies the effects of competition among MFIs on MFI loan size, informal moneylender loan size, borrower's and moneylender's welfare. We find that the aggregate MFI loan size as well as the moneylender loan size decrease with increase in the number of MFIs. However, moneylender's interest rate increases with the increase in the number of the MFIs. We also find that the borrower's welfare decreases and the moneylender's profit increases with increase in the number of MFIs.

5.1 Future Work

5.1.1 Part I

We plan to extend chapter 2 of this thesis along several dimensions. Since, longrun consumption and investment are independent of whether credit is provided or not, we want to rank the growth rate and hence, the present discounted value of lifetime utilities of the agents with different initial wealth. This is important especially when the lender is credit constrained and this will help us understand the policy implications. Next, we want to check the robustness of our analysis by changing the curvature of the utility function as we have used the concavity of utility function in our analysis.

We would also explore the case where there is nonconvexity in the transformation technology, this merges chapter 2 with chapter 3. So, alternatively we want to extend the analysis to the case where there is a possibility of graduation (as defined in chapter 3) and the agent has concave utility function.

All the analyses have been done assuming that the lender is benevolent, so by definition the lender's and the agent's objectives are alligned (given the lender's no loss condition). We think it would be interesting to see how far, qualitatively, these results continue to hold when the lender is motivated (maximizes the weighted sum of his own profit and the borrower's utility) and at extreme is a profit maximizer.

Though relaxing commitment requirement from the lender's side seems to be an interesting extension, our conjecture is that all the results go through, as far as the lender is benevolent and credit constrained (for the first chapter we need the lender to be credit constrained, otherwise he can simply write off all the past loans at the beginning of any period). This lack of commitment may alter the results when the lender is a profit maximizer and we plan to address that.

Finally, we want to explore the case where the agent has self-control problem (in the sense made precise in Gul and Pesendorfer (2001, 2004)). Our conjecture is that non-trivial, sustainable, DIC contracts, in this case, do exist,[1] even when the lender cannot provide access to any superior technology, if the time of repayment and the time of investment differ and the time of consumption lies in between those two stages (as in Duflo et al. (2011)).

5.1.2 Part II

First, as stated in the text: we assume that the MFIs can commit and MFI contracts are chosen from the class of stationary and symmetric contracts. Checking the robustness of the analysis to these assumptions are part of my future research agenda.

Second, we want to relax the commitment requirement from the MFI's side, this is a non-trivial excercise in that the moneylender provides bridge-loans in case of failure and the borrower repays him in the next period in case of success. The amount to be repaid to the moneylender depends on the return from investment which further depends on the loans provided by the MFIs in the next period.

The analyses we have done so far, assume that the moneylender can charge any interest rate he wants to. We think this is a valid assumption given his power in a village economy. But it is interesting to relax this assumption and analyze the optimum contracts and study the effects of MFI competition when the moneylender is non-strategic and the moneylender loan schedule is exogenously given.

Finally, it would be very interesting to explore the class of non-stationary contracts, however that is technically quite challenging and hence we keep that in our very longrun agenda.

[1] In fact, Gul and Pesendorfer (2004) address the Bulow and Rogoff (1989b) problem and with an example show the existence.

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