# Unionization, Optimal Fiscal Policy and Endogenous Economic Growth

Chandril Bhattacharyya

A Dissertation submitted to the Indian Statistical Institute in partial fulfilment of the Requirement for the Award of the Degree of Doctor of Philosophy

INDIAN STATISTICAL INSTITUTE
KOLKATA

July 2016

**To this magnificent trio.**

## Acknowledgements

This thesis is the result of my research work done at Economics Research Unit (ERU), Indian Statistical Institute, Kolkata for the last six years. I utilize this space to express my deepest gratitude to all of those who have been the source of my support and inspiration for writing this thesis.

The person who first comes in my mind from this list is my supervisor Professor Manash Ranjan Gupta. I am greatly indebted to him for his advice, suggestions and inspiration throughout these six years. Without his intellectual guidance and effort, this research work truly would not have been completed. I also like to express my gratitude to Professor Anjan Kumar Chakrabarti of University of Calcutta and Professor Soumyen Sikdar of IIM Kolkata for various kinds of suggestions and critical remarks on my research work. Thanks are also due to the esteemed faculty members of ERU like Professor Satya Ranjan Chakrabarty, Professor Abhirup Sarkar, Professor Tarun Kabiraj, Professor Manipuspak Mitra, Professor Nityananda Sarkar, Dr. Brati Sankar Chakraborty for their invaluable suggestions. I am also thankful to Professor Neri Salvadori, editor of the international journal 'Metroeconomica' and two anonymous referees of this journal for their comments and constructive criticism of the Chapter 2 of this thesis.

Throughout all these six years, I have felt blessed to have My ERU colleagues and friends like Sattwik da, Rajit da, Debojyoti, Parikshit, Tanmoy, Priya Brata da, Sandip da, Kushal da, Srikanta da, Debasmita di, Mannu di, Conan da, Trishita di, Mahamitra, Arindam, Gopa Kumar, Aditya, Avinandan, Mridu da, Pinaki, Sreoshi, Chayanika, Dripto, Soumyarup, Ranajoy and Anamitra. They have not only tolerated me but have also made this journey memorable. I am thankful to them for all their supports and inspirations. I also thank all staff members of ERU office specially, Swarup Da, Satyajit da, Chunu da, Alok da, Chandana di and Bhupal da for various kinds of help during the period of this work.

I am also thankful to my school teachers and university teachers for providing me the building blocks to build up. Dr. Arnab Chakraborty of this institute and Dr. Sukanta Bhattacharya of University of Calcutta deserves a special mention. I am also obliged to all my friends, especially Soumya for standing by me. I am grateful for the inspiration, love, care, support and enormous sacrifice of my family and my relatives.

Finally my gratitude goes to the almighty God for his or her blessings. Perhaps this is the sole reason for arrival of all these wonderful people in my life.

Chandril Bhattacharyya

Kolkata,

July, 2016.

# Contents

# Chapter 1: Introduction and Literature Survey

## 1.1 Modern Growth Theory

Countries with high per capita real GDP are usually associated with higher standard of living of its citizens. In fact, the greater the aggregate Pie grows into, it becomes easier for the government to subsidize people of the bottom section of income distribution. As a result, the theory of economic growth receives a great importance in the Economics literature. Economic growth rate is defined as the rate of increase in per capita real GDP over time; and a very small difference in growth rate may result into a huge difference in per capita real GDP in the long run due to its cumulative effect.

The literature of neoclassical growth theory starts with Solow (1956) and Swan (1956). The key aspect of the Solow–Swan model is the use of neoclassical production function which satisfies constant returns to scale, diminishing returns to each input and substitutability between inputs. These models assume an exogenously given constant savings - output ratio and build extremely simple general equilibrium growth models. The key result of these models is that in the absence of technological progress, output per capita cannot grow forever and it converges to its steady state equilibrium value. The basic exogenous savings Solow – Swan model has been extended by various authors in various directions. Despite of the path breaking contribution, these models belong to the set of exogenous growth models because the steady state equilibrium growth rate of the economy in these models is exogenously given. Later Cass (1965) and Koopmans (1965) incorporate Ramsey's analysis of consumers' life time utility maximization behaviour in the neoclassical growth model dropping the assumption of constant savings rate. However, these extensions also fail to determine economic growth rate endogenously in the long run and thus belong to the set of exogenous growth models.

A new wave in growth theory comes with works of Romer (1986), Lucas (1988) and Rebelo (1991) etc. In these models, the rate of economic growth is endogenous due to endogenous human capital accumulation. Other endogenous growth models are built by Arrow (1962), Sheshinski (1967) and Uzawa (1965) etc. Growth models of Arrow (1962) and Sheshinski (1967) are based on the concept of 'learning by doing' where the technological progress is a by-product of investment. Uzawa (1965) and Lucas (1988) focus on the effect of human capital accumulation on growth. Barro (1990) develops an endogenous growth model focusing on the role of tax financed productive public expenditure on capital accumulation.

Romer (1987, 1990), Aghion and Howitt (1992) and Grossman and Helpman (1991) incorporate R&D theories and imperfect competition into the framework of endogenous economic growth. After these major contributions, role of various other aspects such as law and order, protection of intellectual property rights, international trade, financial markets, competition in market, imitation of new technology etc. on economic growth are also analysed in various models. Labour market imperfections caused by the presence of labour union and its bargaining power is one such factor affecting economic growth.

## 1.2 Labour Union and Economic Growth

### 1.2.1 Empirical Literature

There exists a substantial empirical literature investigating the effect of existence of unionized labour market on economic growth. However, no unambiguous conclusion can be drawn from these studies because different studies show different results. These findings are mentioned in next two paragraphs. Numerous empirical studies have examined the extent and direction of the effects of unionization at firm level or at industry level[1]. Most of those studies find negative union effect on economic performance of these firms and industries. The present literature survey considers only those studies which investigate effect of labour unions only on the economic growth rate. Studies investigating effects of unions at a micro level using firm level or industry level data are exempted from this survey. This survey also does not incorporate studies focusing on the effect of corporatism on economic growth. Corporatism describes wage setting behaviour of the economy as a whole; and it includes not only degree of unionization but also level of centralization, domination of large export oriented firms and state's involvement[2].

The empirical studies made by Kim (2005), Pantuosco et al. (2001), Vedder and Gallaway (2002), Galli and Padovano (1999), Padovano and Galli (2003), Akinci et al. (2014), Carmeci and

[1] See for example Doucouliagos and Laroche (2003), Tzannatos (2008), Tzannatos and Aidt (2006), Clark (1980), Hirsch and Link (1984), Addison and Hirsch (1989), Nickell et al. (1992), Denny (1997), Chezum and Garen (1998), Hirsch (1991, 1992), Menezes-Filho et al. (1998), Allen (1988), Betts et al. (2001), Bronars and Deere (1993), Addison and Wagner (1994), Denny and Nickell (1992) etc.

[2] See for example Padovano and Galli (2003), Heitger (1987), Dowrick (1993) etc.

Mauro (2003), Adjemian et al. (2010), Carmeci and Mauro (2002), Çetintaş et al. (2008) show a negative impact of labour unions on the economic growth. Kim (2005) uses Korean data ranging from 1970 to 2002 and shows that unionization in the labour market negatively affects both employment and economic growth in Republic of Korea. Pantuosco et al. (2001) analysing a panel data set of 48 U.S. states for the period 1978-1994 shows that unions adversely affect unemployment rate as well as economic growth rate. Vedder and Gallaway (2002) uses U.S. data for the period 1964 – 1999 and finds a statistically significant negative relationship between unionization and economic growth. Galli and Padovano (1999) also finds a negative correlation between union influence and economic growth analysing data for 18 OECD countries in the 1960-1993 time interval. Padovano and Galli (2003) uses data for 18 OECD countries for the time period 1960 - 1998 and finds out a statistically significant negative effect of unionization on economic growth. Akinci et al. (2014) using data of 33 OECD countries for the period 1970 to 2011 shows that an increase in unionist movements has a negative impact on the process of economic growth. Carmeci and Mauro (2003) uses a data set of 18 OECD countries for the period 1960 -1990 and shows a negative impact of labour market imperfections on economic growth. The labour market imperfection is measured by unemployment replacement ratio and union density. Adjemian et al. (2010) using data from 183 European regions for the time period between 1980 - 2003 shows that unionization has a negative effect on growth rate. Carmeci and Mauro (2002) uses a balanced panel of 19 Italian regions covering the period 1965 -1995 and shows a negative impact of labour market imperfection on economic growth. Çetintaş et al. (2008) using data of Turkey for the period of 1984 – 2004 shows that unionization affects both economic growth and employment negatively.

However, many empirical studies do not show such unambiguous negative effects of labour unions on economic growth. These studies find either insignificant effects or positive effects. Jaoul-Grammare and Terraz (2013) uses data of 11 European countries for the period 1960 – 2009 and find that, only in case of France, union density positively affects labour productivity and thereby economic growth. For other countries, effect of union density on economic growth is not statistically significant. Cole (2014) uses data from a panel of 48 U.S. states for the years 1975 – 2004 and shows the evidence of an inverted U-shape relationship between economic growth rate and ‘Special interest groups’ (the percentage of each state’s public and private non-agricultural wage and salary employees who are union members) lobbying and rent-seeking activities. Traxler and Brandl (2009) uses data covering 18 countries for the time period from 1980 to 2000 and concludes that labour unions' bargaining power does not have any significant impact on the growth of GDP. Pantuosco et al. (2002) uses data for 48 contiguous U.S. states from 1983 to 1996; and show that public unions have a small positive effect on economic growth while private unions have little discernible effect. Pantuosco and Seyfried (2008) uses data of 48 states of U.S. for the period 1992 – 2005 and concludes that public unions have no discernible effect on economic growth while private unions have a negative effect on economic growth. Georgiou (2010) uses data of 17 Western countries for the period 1999 – 2007 and shows that labour unions have a positive impact on economic growth. Ng and McCallum (1989) examines data of 17 OECD countries from 1960 to 1979 and concludes that the effect of labour unions on economic growth depends upon the ideology of the government in power. Under 'non-socialist' ('socialist') governments, increased union density reduces (increases) economic growth rate. OECD (2004) uses data for 1970 – 2000 and finds a negative but insignificant relationship between growth in per capita real GDP and labour union density as well as between growth in per capita real GDP and collective bargaining coverage in OECD countries. Asteriou and Monastiriotis (2004) investigates the long-run relationship between labour unionism and economic growth using a panel data set covering 18 OECD economies for the time period 1960-1992; and this study finds a statistically significant positive relationship between them. Nickel and Layard (1999) uses data of 20 OECD countries for two time periods 1983-1988 and 1989-1994; and then shows that unionization raises unemployment but does not lower growth rate.

### 1.2.2 Theoretical Literature

There exists a substantial set of theoretical models analysing the effect of labour unions on economic growth. In most models, except Carmeci and Mauro (2002, 2003), this effect is analysed on the long run equilibrium growth path; whereas, Carmeci and Mauro (2002, 2003) analyses this effect on the transitional growth path of the economy. Some models define unionization as an exogenous increase in the relative bargaining power of the labour union and then analyse the effect of the increase in that power on the economic growth rate. Other models compare the growth rates of the economy with competitive labour market to that with unionized labour market. We divide this subsection into two parts. In the first part, we survey R&D Based Growth Models. Other growth models are surveyed in other part.

#### 1.2.2.1 R&D Based Growth Models

In an R&D based growth model, R&D section develops new technologies; and technological progress is the source of economic growth. Growth models surveyed in this subsection are R&D based growth models and these models analyse how unionism affects R&D and economic growth rate.

Palokangas (1996) extends the famous Romer (1990) model of endogenous growth introducing collective bargaining between employers' federation and labour union. There are three sectors and two types of labours in this model. The competitive final good sector employs both skilled and unskilled workers whereas the growth generating competitive R&D sector employs only skilled workers. Outputs of the R&D sector are bought by monopolistic firms who use foregone final output as input to produce producer durables and sell them to the final good sector as input. The rate of technological progress varies proportionately with the number of skilled workers employed in the R&D sector. The economy wide labour union represents both types of workers and bargains with employers' federation representing final good producing firms. The labour union's utility function is the discounted value of total labour income. Here, bargaining is done over wages of both types of workers. This paper shows that, in the negotiation process, the labour union does not accept any contract with unemployment of skilled workers, because, the excess skilled workers could always be absorbed in the R&D sector due to its linear production function. This raises total labour income and union's utility. If two types of workers are complements in the final good sector, then an increase in the relative bargaining power of the labour union raises the wage rate of unskilled workers; and this, in turn, raises its unemployment level. This results a fall in the demand for skilled workers in the final good sector because two types of labour are complementary. So we find a fall in their wage rate too. So the cost of producing a new design falls and its production rises due to absorption of surplus skilled workers in the R&D sector. So the rate of technological progress and hence the economic growth rate are raised. This paper also shows that unionization is welfare enhancing (reducing) if the full employment growth rate is less (more) than the socially optimal growth rate.

De Groot (1996) model is a two sector endogenous growth model with homogeneous labour. The low wage competitive traditional sector produces a homogeneous good using labour; and the high wage primary high-tech sector produces varieties of high tech product. A pool of unemployment workers exists because unemployed workers have a higher probability to get high wage jobs. Operating under monopolistic competition, 'Rent maximizing' labour unions bargain with high-tech firms over the wage rate only; and a rise in the union power raises the relative wage of high tech workers to that of traditional sector workers. As a result, more workers join the pool of unemployed rather than the traditional sector as the probability of entering the high tech sector from the secondary sector is lower than that from the unemployment pool. On the other hand, a rise in the high tech sector wage rate reduces the employment in this sector. As a result, production in that sector falls and R&D expenditures can be spread over less output. This reduces the incentive for high-tech firms to engage in R&D activities. This, in turn, lowers the equilibrium rate of growth.

Stadler (1998) uses homogeneous labour and then follows the style of Palokangas (1996). It considers a three sector economy consisting of a competitive final good sector which uses labour as well as variety of differentiated intermediate goods as inputs, a non-competitive intermediate good sector and a competitive R&D sector which use labour as the only input. The quality of these intermediate goods can be upgraded by innovative activities in the R&D sector. The rent maximizing labour union operates in the intermediate goods sector and bargains with firms over the wage rate only. A rise in the labour union's bargaining power shifts the labour resource to the competitive sectors and thereby causes expansion of R&D activity. As a result, pace of quality improvement as well as economic growth rate is increased.

De Groot (2001) has an almost similar structure as in De Groot (1996). Here also the 'Right to Manage' model of bargaining is considered. The labour union's objective function consists of the total level of employment in monopolistic firms as well as of the wage mark up over the secondary sector's competitive wage rate. In this set up, an increase in the bargaining power of the labour union raises the bargained wage rate and thereby reduces employment in the primary sector. It is assumed that the firm's R&D input is proportional to production input. So a decrease in employment also reduces R&D and thereby the rate of economic growth. However, the effect of a change in the labour union's bargaining power on the unemployment level consists of three effects. First, an increase in the relative wage in the primary sector raises wait unemployment as the probability of getting a high paid job in the primary sector for an unemployed worker is higher than that of a secondary sector worker. Secondly, it lowers employment level in the high paid primary sector and thereby lowers the probability of getting a job there. This has an adverse effect on the level of wait unemployment. Thirdly, as economic growth rate is decreased with unionization, people become less eager about their future jobs; and thus the attractiveness of becoming unemployed is also decreased. An extensive numerical simulation of the model shows that unemployment is likely to rise with unionization.

Wapler (2001) considers an economy where two final goods are produced and consumed. One of them is a traditional good which is produced in a competitive sector; and the other is a composite high tech manufacturing good produced using a multitude of intermediate goods whose qualities improve over time. The R&D sector contributes to quality improvement. Market structure of intermediate goods are monopolistically competitive. Both low skilled workers and high skilled workers are used in every sector with varying intensities. Labour unions are present only in the intermediate goods sector and represent low skilled workers only. This paper considers only 'Right to Manage' model of bargaining where unions try to maximise rent of low skilled workers. In this model, a rise in the union bargaining power leads to an increase in wage rate of low skilled workers in all sectors. This leads to a fall in the level of employment of low skilled workers in the R&D sector. This lowers productivity of high skilled workers in that sector because two types of workers are complement to each other. As a result, economic growth rate is decreased.

In Lingens (2002), the consumption good is produced using different varieties of intermediate goods which in turn are produced by monopolistic firms using labour only. These monopolistic firms buy technology produced by the competitive research sector which uses labour and human capital as inputs. Labour unions bargain over wage with both intermediate good firms and R&D firms to maximise the life time utility of its members. An increase in the union power in the R&D sector reduces employment in the R&D sector raising the wage rate and thus produces a negative growth effect. However, an increase in the union power in the intermediate good sector, on the one hand, generates a positive growth effect raising the relative labour demand in the R&D sector and, on the other hand, generates a negative growth effect raising wage rate in the R&D sector. The negative growth effect always dominates the positive growth effect.

Quang and Vousden (2002) uses a simple two period overlapping generations model where technological progress is the engine of growth. In this model, a competitive final good sector as well as a competitive R&D sector uses two intermediate goods as inputs. Of these two intermediate goods, one is produced under competitive conditions using non-unionized labour and backward technology; and the other one is produced by a monopolist, the patent holder, using unionized labour and the state of the art technology. The bargaining over wage takes place between the firm and the rent maximizing labour union. Wage bargaining raises the labour cost as well as unemployment and reduces profits of the monopolist. This lowers the incentive to invest in R&D and thus lowers the rate of technological progress and growth rate. This result is also valid in the case where the competitive intermediate good sector also uses unionized labour and where the labour union is of open-shop type.

Lingens (2003a) uses a simple three sector Schumpeterian growth model based on the works of Aghion and Howitt (1992). The competitive R&D sector produces new technology using skilled workers only. A researcher, who becomes successful to invent a new intermediate product, becomes the new monopolist in the intermediate goods market and replaces the incumbent one. The monopolistic intermediate goods producing sector uses both skilled as well as unskilled workers as inputs and supplies the most modern intermediate goods to the final goods sector producing the consumption goods. The rent maximizing labour union represents only the unskilled workers and bargains with the new monopolist in the intermediate goods market. This paper shows that, in the case of 'Right to Manage' model, the effect of unionization on the economic growth rate is ambiguous and depends on two opposite effects. First, there is a negative effect of unionization causing a reduction in the profit of the intermediate good producing monopolist. This lowers the incentive to perform R&D; and so level of employment in that sector declines. Secondly, the wage of skilled labour falls in the intermediate good sector. This is so because unionization lowers employment of low skilled workers; and this, in turn, lowers the productivity of high skilled workers due to complementary effect of inputs. This leads to migration of former intermediate skilled workers to the research sector and thereby raises the growth rate of the economy. The aggregate effect depends on the relative strength of these two effects which depends on the elasticity of substitution between two types of labour in the production of intermediate goods. When elasticity of substitution is less than (greater than) (equal to) unity, then unionization raises (lowers) (does not affect) the rate of technological progress as well as the growth rate of the economy.

Lingens (2003b) introduces labour union's wage bargaining into a growth model with endogenous skill formation. A competitive consumption good sector uses varieties of intermediate goods as inputs produced by monopolistically competitive firms using unskilled labour as the only input. These firms also do R&D activities to improve production efficiency with skilled workers as only inputs. There is a positive spillover effect in the sense that an increase in the efficiency to produce intermediate good also raises productivity in the R&D department. This spillover effect generates increasing returns to scale but it coexists with static diminishing marginal productivity of skilled workers; and this coexistence gives birth to a U-shaped demand curve for skilled workers. Workers being heterogeneous in their abilities decide whether to invest in skill formation. Since the supply curve of skilled workers is positively sloped, the economy is characterized by two locally stable equilibrium. The rent maximizing labour union represents unskilled workers only and bargains only over wage. Unionization raises unskilled wage and thereby reduces unskilled employment as well as the marginal productivity of skilled workers. Thus the demand curve for skilled labour shifts downward. However, unionization ambiguously affects expected return from remaining unskilled by raising the unskilled wage rate. If the unemployment benefit rate is high, then the incentive to invest into skill formation is reduced. As a result, effect of unionization on skilled employment and on economic growth depends on the nature of initial equilibrium. If it is a low (high) skilled equilibrium, unionization lowers (raises) the level of skilled employment as well as the economic growth rate. Effect of unionization on unskilled employment is also ambiguous and different for different equilibriums.

Palokangas (2004a) develops a R&D growth model with two sectors and homogeneous labour. The high-tech sector consists of monopolistic firms producing varieties of differentiated consumer goods as well as firm specific technology and they use labour and a fixed input. The competitive traditional sector uses labour as the only input to produce traditional consumer good. Firm specific labour unions bargain with high-tech firms over the wage only and tries to maximise the discounted present value of labour income. In such a setup, a rise in the union power raises wage rate and makes firms to improve technology to raise productivity and to lower this cost. As a result, growth rate is increased. It is also shown that a welfare maximizing level of union power exists; and it is welfare enhancing to strengthen (weaken) unions below (above) this level.

Palokangas (2004b) constructs a Schumpeterian growth model where there is a common market with a given number of member economies. Competitive firms in the common market produce the consumption good using land and intermediate goods of all member economies. Intermediate good firms in the common market are subject to oligopolistic competition; and one monopolist at a time produces the economy specific intermediate good using labour only until a new innovation is made by a research firm of that economy and the incumbent firm is replaced by that new one. Households can either act as workers or become researchers at some cost; and researchers are employed only in the R&D sector. Only workers are unionized and the country specific labour union tries to maximise the expected discounted value of the flow of the workers' wages. Labour union and the employers' federation bargain over wage rate only. In such a setup, a rise in the union power raises wage but reduces employment and thus expected wage. As a result, more households choose to become researchers rather than workers. This expands R&D activities and thus raises the economic growth rate.

Palokangas (2005) constructs a multi-economy Schumpeterian growth model where economies are interdependent only through technology transfer. Country specific competitive firms produce the final good from the country specific current intermediate goods, each of which is produced by one monopolist using labour. Households can either act as workers or become researchers at some cost. Several firms do R&D by using researchers; and as soon as any of them completes a new innovation, it takes over the whole production of the intermediate good and drives all old producers out of the market. The labour union bargains with the employers' federation over the wage rate in order to maximize the expected value of the stream of its members' real wages. In such a setup, a rise in union power raises workers' wage but lowers employment. As a result, expected wage falls and more households choose to become researchers rather than workers. This promotes R&D and thus raises economic growth rate. However, when technological change in an economy depends more on technology spillovers from abroad and less on domestic R&D, an equal increase in workers' wage yields a smaller increase in the growth rate.

In Zagler (2005) model, households consume all types of differentiated products and each such variety is produced by a monopolist using unskilled labour as the only input. New innovations are created in the competitive R&D sector using researchers (skilled workers) as the only input. Only unskilled workers are represented by labour union which tries to maximise welfare of its members and bargains only over the wage rate. In equilibrium, the unemployment rate and the growth rate are determined by the intersection between the labour resource constraint, which shows a negative effect of unemployment on economic growth, and the incentive constraint, which shows a positive effect of economic growth on unemployment. Competitive labour market leads to full employment and thereby to highest growth rate. However, firm-level bargaining results into the highest unemployment rate and thereby leads to the lowest growth rate. In the case of centralized bargaining, both growth rate and unemployment rate remain in between these two extremes.

Lingens (2007) develops a general three sector two factor Romer (1990) type endogenous growth model. The consumption good is produced using varieties of intermediate goods; and the monopolistically competitive intermediate good sector as well as the competitive R&D sector use both skilled and unskilled workers as inputs. A centralized labour union representing only unskilled workers bargains over wage in order to maximise aggregate utility of all unskilled workers. Here unionization generates two different effects. First, it reduces the profit of intermediate good firms and thereby the price of blueprints. This reduces R&D activities and thereby lowers the growth rate. Secondly, it alters the factor intensity in the economy and thus generates a potentially growth enhancing Rybczynski effect whose magnitude depends on the institutional bargaining framework. The overall effect partially depends on the elasticities of substitution between unskilled and skilled labour in both sectors of the economy.

Adjemian et al. (2010) develops a Schumpeterian model of endogenous growth with homogeneous labour. The final good is produced by competitive firms using the latest vintage of intermediate goods which are produced by monopolistic firms using labour as the only input. Upgradation of existing vintages of intermediate goods are done by the competitive R&D sector with labour as the only input; and the success replaces the existing firm. The rent maximizing labour union bargains with the monopolistic firm over wage. An increase in the union's bargaining power raises the wage rate and thereby leads to an increase in the unemployment level. However, monopolist's profit as well as the expected value of an innovation is reduced; and so R&D output and the rate of economic growth are diminished.

In Lai and Wang (2010) model, the final good producing competitive firms use 'state-of-the-art' intermediate goods and labour as inputs; and monopolistically competitive firms produce intermediate goods using capital as input. The competitive R&D sector uses final good as input to produce new blueprints. The managerial labour union with Stone-Geary utility function in terms of wage and employment, bargains with employers' federation of final goods producing firms over both employment and wage. Stability properties of the long run equilibrium influence relationship between unionization and economic growth. An increase in the labour union's bargaining power raises (reduces) the equilibrium level of employment as well as the balanced growth rate, if and only if the balanced growth equilibrium is locally determinate (indeterminate).

Peretto (2011) uses a creative accumulation growth model where innovation by a new entrant firm does not replace the existing firm. The competitive final good sector uses differentiated intermediate goods as inputs; and the monopolistically competitive intermediate firms use only labour as input. Existing intermediate firms use final good as input to run their in-house R&D subsector in order to raise productivity. However, the new intermediate goods firms are created by entrepreneurs who develop new product and their manufacturing process. New firms also need final good for entering the market. The rent maximizing labour union bargains with the intermediate firms over both wage and employment ('Efficient Bargaining' model). In such a setup, a rise in labour union's bargaining power, raises the labour cost and thereby reduces employment. This shrinks the scale of the economy and generate lesser competition in the product market. This lowers growth rate because, in "creative accumulation" growth models, product market competition and growth rate exhibit a positive relationship.

Grieben and Sener (2012) develops a North-South product cycle model of endogenous growth in which labour markets of both the countries are centrally unionized. Northern entrepreneurs participate in industry-specific R&D races to innovate products of superior quality; and successful innovators produce their top quality products using Northern labour and become global monopolists. Northern technologies can be imitated by Southern firms with lower production costs; and imitation also requires labour as the input. Successful imitation causes shifts in production from North to South. In the global markets, firms face Bertrand price competition and offer the lowest quality-adjusted price given their state of technology and regional labour costs. Northern (Southern) labour union bargains with the successful innovator (imitator) over the wage rate maximizing its 'Stone – Geary' utility function with wage and employment as arguments. In such a setup, an increase in the Northern labour union's bargaining power reduces Northern innovation, worldwide economic growth, and also Southern imitation but raises both Northern and Southern unemployment. However, an increase in the Southern labour union's bargaining power lowers Southern imitation but raises Northern innovation and worldwide economic growth. It also reduces (raises) Northern (Southern) unemployment rate.

Ji et al. (2016) introduces unionized labour market in an endogenous growth model with an "endogenous market structure" as developed by Peretto (1996). Competitive firms uses labour and differentiated non-durable intermediate goods to produce final goods that can be used for consumption and as inputs in the production of intermediate goods and as an investment in R&D. The wage oriented labour union tries to maximise its Stone – Geary form of utility function with wage and employment as arguments. The union bargains with employers' federation over both wage and employment. In the intermediate good sector, there are two dimensions of technological change: (i) vertical or quality improvement and (ii) horizontal or variety expansion. Due to the complementary relationship between labour input and differentiated intermediate goods in the production of final good, a decrease in the level of employment due to unionization results into lower demand for intermediate goods. This lowers the number of intermediate good producing firms in the same proportion keeping market size per firm unchanged. As a result, firm's intensity of R&D activities does not change; and so economic growth rate remains unchanged.

Chu et al. (2016) develops a two country R&D based growth model where final goods are produced combining intermediate goods from two countries. The competitive R&D sector innovates new blueprints of varieties of differentiated inputs using final good; and each differentiated input is also produced using final good by a monopolist who owns the patent. Intermediate goods in each country are produced using domestic labour and differentiated monopolistic inputs; and, in this monopolistically competitive sector, employers' federation and the labour union bargain with each other over both employment and labour. Following Chang et al. (2007), this paper also considers a managerial labour union. An increase in the bargaining power of a wage (employment) oriented labour union leads to a decrease (increase) in employment, growth rate and welfare. An increase in the degree of wage orientation of the union results into a decrease in employment, growth rate and welfare.

#### 1.2.2.2 Non - R&D Based Growth Models

This subsection briefly surveys the substantial set of Non – R&D growth models with unionized labour markets. In these models, economic growth take place through channels other than R&D.

Agell and Lommerud (1993) develops a model with two competitive final good production sectors - one traditional and another modern. Capital and labour are common inputs to both the sectors, but human capital is specific to the modern sector. Growth of the economy is originated from human capital accumulation which takes place over time through the process of learning by doing in the modern sector. Capital is perfectly mobile between these two sectors but labour is imperfectly mobile due to workers' locational preferences towards the traditional sector. In such a setup, the paper argues that, if the increase in wage caused by unionization is accompanied by the reduction in the degree of imperfection in labour mobility, then unionized economy may grow at a higher rate than a competitive economy. This is so because the increase in wage lowers the labour intensity of traditional sector; and so a part of this withdrawn labour force is absorbed in the modern sector. So employment in modern sector is increased; and this raises the rate of human capital accumulation and the rate of economic growth.

Sorensen (1997) uses a one sector two period overlapping generations model with two inputs – skilled labour and entrepreneurial skill. In the first period, individuals are workers and are trained by firms; and, in the second period, they are either entrepreneurs or retired. An old individual becomes an entrepreneur when she gets “a good idea” and the number of entrepreneurs varies positively with the level of skill of the old generation. An ‘Efficient Bargaining’ model as well as a ‘Right to Manage’ model is used to solve the negotiation problem between the labour union and the firm. The labour union is firm specific; and it wants to maximise workers’ discounted present value of expected income. Here, in both the models, unionization raises wage rate and reduces the level of employment at the steady state equilibrium. However, in the case of ‘Efficient Bargaining’ model (‘Right to Manage’ model), workers’ skill in the steady state level falls (rises) with unionization in the labour market. As growth rate varies positively with the level of skill, so unionization reduces (raises) economic growth rate in the steady state equilibrium in the case of ‘Efficient Bargaining’ (‘Right to Manage’) model. As a result, in the case of ‘Efficient Bargaining’ (‘Right to Manage’) model, welfare implication of unionism is negative (ambiguous).

Palokangas (1997) analyses the effects of the level of organization of labour unions on the economy using a product variety model. Production of the final composite good requires all varieties of intermediate goods as inputs and each intermediate good is produced using labour, capital and all intermediate goods as inputs. Each industry specific monopoly labour union attempts to maximise the welfare of its members with respect to wage rate subject to the behaviour of firms. In this model, a lower wage implies a higher level of employment, a higher rate of profit and a higher rate of investment leading to a higher rate of growth. When the elasticity of substitution between labour and intermediate inputs is not high, a better macroeconomic consequences of unionization is found if unions are organized at a central or at a local level than if they are organized at the medium level. This is so because the degree of centralization of unionization has two opposite impacts on wages. The first effect comes through the internalization of the macroeconomic effects of a single union's wage policy and the other effect comes through the elasticity of the demand for labour that a single union faces. Since a large union can easily internalize the benefit of a lower price level due to lower wages for their members, so they moderate their wage claims. On the other hand, a small union faces a small part of the production sector and then faces highly elastic demand for labour.

Faini (1999) uses a simple two region small open dual economy model to analyse the impact of union activity on regional growth. In the rural region, the agricultural good is produced with unskilled labour but, in the urban region, the manufacturing good is produced with both skilled and unskilled labour. Workers determine their skill level maximizing their utility and the level of skill accumulation in the economy varies positively with the wage rate of skilled workers. So the economy's growth rate being dependent on the speed of skill accumulation varies positively with the wage rate of skilled workers. The monopoly labour union operates only in the manufacturing sector and covers unskilled workers only. Union activity boosts the unskilled wage and thereby reduces both unskilled employment and productivity of skilled labour which is complementary to unskilled labour. As a result, skilled wage rate falls and this leads to a lower level of skill accumulation and thus lowers the growth rate. If the two regions are asymmetric in technologies, then technically advanced sector will specialize in skill intensive good. In such a situation, union activity will depress skilled wage more in the backward region as that region is unskilled labour intensive. So the growth rate will decline at a higher rate in the backward region than that in the advanced region. This result remains qualitatively unchanged if the assumption of regional unions is replaced by centralized union.

In Brauninger (2000b), monopolistically competitive firms produce imperfect substitute goods using capital and labour. Labour unions are firm specific and they maximise the expected income of its members and bargain with the firm over wage. The firm itself determines its employment level from the labour demand function. In such a setup, unionisation reduces employment level as well as returns to capital. This reduces the rate of capital accumulation and thereby the growth rate. The basic model is also extended by incorporating the process of creative destruction in which growth creates new technology and destructs olds. Therefore, an increase in the growth rate raises the job market flow and thereby increases workers' reservation wage rate. As a result, union's bargaining results into higher wage and lower employment. In this extended model too, an increase in union bargaining power lowers employment level as well as the growth rate. However, in this case, the fall in the growth rate lowers unemployment and thereby indirectly raises growth rate. The direct effect is dominant and hence unionisation reduces both employment level and economic growth rate.

In Boone (2000), unionisation in the labour market affects firm's choice between two types of technological progress. One type of technological progress raises the product quality whereas the other type reduces firm's fixed labour cost. Due to the fixed supply of human capital, there exists a trade-off in the choice of these two. Monopolistically competitive firms face both variable and fixed cost in terms of labour; and the fixed labour component is interpreted as management. In such a setup, an exogenous rise in labour union bargaining power raises wage rate as well as cost for paying fixed labour. As a result, firm's incentive to invest in downsizing is increased and that in quality improvement is reduced. These, in turn, lower long run growth rate, raise the unemployment rate and lower social welfare.

Corneo and Marquardt (2000) develops a two period overlapping generations model of endogenous growth with a special focus on the interaction between public pensions and unemployment insurance programs in the presence of a unionised labour market. In this model, the competitive final good sector produces a single good using capital and labour. A monopoly labour union determines the wage rate maximising a 'Stone-Geary' utility function defined over wage and employment; and the firm chooses labour from its labour demand function. Capital accumulation is the source of economic growth. In such a set-up, the rate of unemployment increases with the union's preferences for high wages. However, this has no impact on the growth rate of the economy. This is the result of two equal but opposite effects of wage increase on aggregate savings. On the one hand, a wage increase induces the employees to save more; and, on the other hand, the associated increase in unemployment rate lowers aggregate savings.

Daveri et al. (2000) shows the harmful effect of labour unions when taxes on labour income are high. They use a two period overlapping generations model where identical competitive firms use capital and labour as inputs. Wages are set by monopolistic unions who maximise the expected income of their members. If wages are set by strong and decentralized unions, then an increase in tax rate on labour income or an increase in unemployment subsidy rate raises wages. This lowers employment as well as makes firms to substitute capital for labour. As a result, marginal product of capital falls and this lowers the rate of investment and the growth rate. However, if wages are set by a large centralized labour union, then it takes into account the adverse consequences and therefore moderates the wage claim.

Irmen and Wigger (2002/2003) develops an overlapping generations model where the final good is produced by competitive firms with labour and capital as inputs and a monopoly labour union derives utility from the level of employment as well as from the wage mark up over competitive wage rate. In such a setup, it is shown that the unionised economy grows at a higher rate than the competitive one (i) if the labour union puts limited weight on the wage hike and (ii) the sum of the elasticity of substitution between capital and labour and the output elasticity of labour is less than one. This is so because, in such an OLG setup, only workers save and this savings is used for capital formation. Unionisation raises the wage rate but reduces the level of employment. So, if the elasticity of substitution between capital and labour is very small, then employment cannot fall substantially due to the complementary relationship between two inputs. On the other hand, if the output elasticity of labour is not very high, then aggregate output shall be reduced marginally. So a rise in the labour share overweighs the effect of the fall in output; and thus results into a rise in wage bill as well as into a rise in the growth rate. Irmen and Wigger (2002/2003) also analyse the effect of unionisation on the welfare level of different generations. Since it reduces employment and thereby reduces marginal productivity of capital and thus rental income. So the old generation who are dependent on rental income necessarily gets worse off. On the other hand, young generation who are dependent on wage income may be better off if a rise in wage income overweighs the future reduction in capital income.

Ramos Parreno and Sanchez-Losada (2002) uses a two sector overlapping generations model of endogenous growth with altruistic agents, accumulation of human capital, and with decentralised rent maximising monopoly labour unions. Both the consumption good sector and the education sector are competitive; and there are two alternative production functions in the education sector. In the first case, human capital is the only input in the linear production function in the education sector. In this case, the rate of economic growth with labour unions in the consumption good sectors is higher than that with competitive labour markets. Unionisation raises wage in the consumption good sector and this reduces employment in that sector. The excess workers are absorbed in the education sector; and so, production of human capital as well as the growth rate are increased. In the second case, physical capital enters as an input in the production function of education sector. In this setup, following alternative cases are considered – competitive labour markets in both sectors, union in one of the two sectors, and unions in both the sectors. Here, increase in wage rate due to unionisation not only lowers employment in the unionised sector and raises it in the other sector; but also raises the demand for physical capital in the unionised sector and lowers the same in the other sector. As a result, the net effect on the growth rate is ambiguous; and a partially unionised economy may grow at a higher rate than a competitive economy in some special cases. However, the growth rate of a completely unionised economy is always lower than that of a competitive economy.

Carmeci and Mauro (2002, 2003) incorporate a monopoly labour union in a neoclassical growth framework in order to assess the relationship between economic growth and labour market imperfections. In both of these two simple models, the labour union maximises the expected utility of its representative member who receives a wage mark-up over the reservation wage. This mark-up lowers the growth rate along the transitional path. Any increase in the mark-up over the reservation wage lowers the steady state equilibrium values of capital and the employment rate. The growth rate of the economy along the transitional path varies positively with its distance from the steady state equilibrium. Since an increase in the mark-up lowers the steady state values of the determinants of output, so economic growth rate also falls along the transitional path. They also do an empirical analysis to support the existence of this inverse theoretical relationship between growth and labour market imperfections.

In Chang et al. (2007), competitive firms produce a single good using labour and physical capital; and an economy-wide labour union is involved in centralized bargaining with the economy-wide employers' federation over both wage and employment. The labour union derives utility from wage rate and from level of employment. However, its degree of orientation towards wage may be different from that towards employment. In the Balanced Growth equilibrium of the model, unionisation raises (lowers) (does not affect) employment rate, economic growth rate as well as level of welfare if and only if the labour union is employment oriented (wage oriented) (neutral). This is so because, if the labour union is employment oriented, then an increase in its relative bargaining power results not only into a higher wage rate but also into a higher employment level. This, in turn, raises the marginal productivity of capital as well as the economic growth rate because, in an AK model, marginal productivity of capital is proportional to the level of employment. On the contrary, when a wage oriented labour union becomes more powerful, it tries to extract higher wages even at the cost of lowering employment level. As a result, growth rate is reduced.

Gori and Fanti (2009) uses a two period one sector overlapping generations model where, in the first period, individuals either earn wage income or receives unemployment benefit financed by taxing consumption. The competitive final good sector uses capital and labour as inputs; and a monopoly labour union maximises a Stone-Geary form of utility function with employment level and wage rate as arguments. It is shown that a unionised-wage economy with unemployment may grow faster than a competitive-wage economy with full employment if the replacement rate and the labour union's wage intensity are high enough to raise savings. This positive effect outweighs the negative effect of unemployment on savings; and thus generates capital accumulation and growth. This model solves for the growth rate maximising wage intensity of labour union. It also shows that the welfare level of a unionised-wage economy with unemployment may exceed that of a competitive-wage economy with full employment.

Roberts (2010) presents an OLG model where capital accumulation is the source of economic growth and households differ in entrepreneurial abilities. Firms produce final good using capital, labour and firm specific entrepreneurial ability. Each young household knows her own entrepreneurial ability but has incomplete information about other's ability. At this stage, she chooses to become either an entrepreneur or a worker in a firm comparing the expected incomes. The author considers two cases - one with full mobility of workers and another with no mobility after this initial allocation. The labour union is firm specific and its objective is to maximise the wage bill. It bargains with the firm over the wage rate only. A rise in the relative bargaining power of the labour union lowers profit but raises wage rate. As a result, number of entrepreneurs falls with unionisation; and, through this channel, unionisation affects growth. This is so because, in such a setup, number of entrepreneurs has two opposite effects on the aggregate output and thereby on the savings, capital accumulation and economic growth rate. The positive effect exists due to the presence of decreasing returns to scale in the production function; and the negative effect exists due to the heterogeneity of entrepreneurial ability because large number of firms also implies the existence of lower ability firms too. As a result, there exists an inverted U-shaped relationship between the number of firms and aggregate output; and the position of an economy on this inverted-U shaped curve depends on the degree of union bargaining power because it is the key variable governing the entry of firms. It is also shown that the growth rate maximising relative bargaining power of the union is lower under ex post labour mobility than under no ex post labour mobility; and this is due to two reasons. First same union power leads to smaller number of firms in the case of ex post labour mobility because lower ability firms get less workers and have lower profit. This acts as an entry deterrent. The growth rate maximising number of firms is higher under ex post labour mobility because, in this case, lower ability firms get less workers. Entrepreneurial ability positively affects workers' productivity; and this lowers the negative effect of the number of firms on the growth rate.

Savabi et al. (2011) does not model the behaviour of the labour union formally but assumes that it raises the wage rate above the market clearing level. The final good is produced with capital and labour by competitive firms. The policy of continuous increase in real wage adopted by the labour union causes firms to substitute labour by capital; and this results into a decrease in the employment rate as well as the output level from its potential level. As a result, savings, capital accumulation and growth rate are decreased.

Tsoukis and Tournemaine (2011) develops a simple AK type growth model where capital accumulation is the source of economic growth. Capital and labour are used to produce the final good. Workers consume their whole wage income and only capitalists save. The labour union's objective function is same as the discounted present value of lifetime utility of workers. Four different equilibria are compared. These are (i) Competitive equilibrium, (ii) Stackelberg equilibrium, where workers unilaterally decide their share of output subject to the capitalist's reaction function, (iii) Non-cooperative equilibrium where unions and firms bargain over the labours' share and (iv) Cooperative equilibrium where union and firms bargains over both labours' share and the growth rate. Labours' share in these equilibriums are decreasingly ordered as follows – Stackelberg, Cooperative and Non-cooperative, competitive. Since growth rate depends on savings and therefore on capitalists' share, so ordering for growth rate is reversed except for the Cooperative equilibrium case. This yields the maximum growth rate as it involves unconstraint maximization.

Fanti and Gori (2011) develops an overlapping generations model where a monopoly labour union sets wage rate for the whole economy maximising utility derived from wage rate and employment rate. The competitive final good sector uses capital and labour as inputs; and the government finances unemployment benefit expenditure by levying a proportional consumption tax on the young. In such a setup, a unionised economy grows at a faster (an equal) rate than a competitive economy in the presence (absence) of unemployment benefit scheme. A rise in the union's relative wage intensity produces a twofold effect on savings and therefore on capital accumulation and growth - a positive wage effect and a negative unemployment effect. These two effects cancel out each others in the absence of unemployment benefit. However, in the presence of unemployment benefit, the positive wage effect on savings dominates the negative unemployment effect and, as a result, unionised economy grows at a faster rate than the competitive one.

Mauro and Pigliaru (2013) extends the Futagami et al. (1993) model of endogenous growth incorporating social capital and imperfect labour market. The competitive final good sector uses labour, private capital and public capital; and a monopolistic and myopic labour union maximises the expected utility of its members subject to firm's labour demand function and sets the wage rate accordingly. As the labour union sets a higher wage mark-up, level of employment is decreased; and as a result, growth rate falls due to lower utilisation rate of the productive input.

Liu (2014) constructs an endogenous growth model, in which, competitive firms produce goods using labour and physical capital as inputs and using a Cobb-Douglas production technology. A part of workers' compensation is financed by the revenue share of the firm. An economy-wide labour union, who maximises the real expected income of its members, is engaged in centralized bargaining with an economy-wide employers' federation. Two scenarios are considered: – (i) bargaining takes place over employees' revenue share, employment and capital; (ii) Employees' share and employment is determined in the bargaining process but capital is determined unilaterally by firms. In the first scenario, bargaining is Pareto efficient. So the firms' demand for labor and capital are similar to those obtained in a competitive market and is not affected by workers' revenue share. Thus, unionisation has no impact on the equilibrium level of employment even though it raises workers' revenue share. As a result, economic growth rate is not affected by unionisation in an AK growth model. However, in the second scenario, unionisation raises employment but produces ambiguous effect on workers' revenue share due to following reasons. First, an increase in employment level lowers per head revenue share of workers and therefore union demands a higher share. Secondly, a higher share discourages firms to accumulate capital and this, in turn, lowers firm's output and thereby lowers workers' share. Since this is an AK growth model, the growth rate depends on both employment level as well as on the net (after share) marginal product of capital. As a result, growth effect of unionisation is also ambiguous in this case.

Ono (2015) considers a two-period Overlapping Generations model where competitive firms produce the final good using capital and labour and bargain with a 'Managerial' labour union over the wage rate. The labour union's utility function is Cobb-Douglas in terms of wage mark-up and employment. The increase in the bargaining power of the labour union, on the one hand, lowers employment; and on the other hand, raises wage rate. However, the former negative effect always outweighs the latter positive effect; and this, in turn, lowers the rate of capital accumulation and the rate of growth.

Chang and Hung (2016) considers a one sector growth model where the final good is produced by competitive firms using labour and capital. The labour union has a Stone-Geary utility function similar to that in Chang et al. (2007) and it bargains with the economywide employer's federation over the wage and the number of workers. However, unlike Chang et al. (2007), they consider a wage oriented labour union. Workers unilaterally decide the number of hours to work by maximizing their utility; and the employer has the right to set capital levels unilaterally. In the Balanced Growth equilibrium of this model, unionization rises the wage rate and thereby lowers the number of workers. However, this rise in the wage rate induces employed workers to raise their working hours. As a result, the effective employment, i.e., the number of workers times their working hours, rises; and this leads to an increase in the growth rate.

## 1.3 Optimal Taxation and Productive Public Spending

In this section, we briefly discuss few endogenous growth models which analyse properties of growth rate maximising or welfare maximising fiscal instruments used for financing productive public expenditure. The pioneering work of Barro (1990) considers an AK growth model, where private capital as well as non – durable productive public expenditure are used as inputs to produce the final good. This expenditure is financed by an income tax. In the steady state growth equilibrium, the growth rate maximising tax rate and the welfare maximising tax rates are same; and it is equal to the competitive output share of the productive public input. Futagami et al. (1993) extends the Barro (1990) model by replacing the perishable public input assumption by a durable public capital. Unlike the Barro (1990) model, this model shows transitional dynamic properties. Like Barro (1990), here also the growth rate maximising tax rate as well as the welfare maximising tax rate in the steady state equilibrium is equal to the competitive output share of the productive public capital. However, this result does not hold in the transitional phase of economic growth.

Later, both Barro (1990) model and Futagami et al. (1993) model have been extended and reanalysed in various directions by various authors; and of this vast literature, we mention here only a few works. Glomm and Ravikumar (1994) analyses optimum taxation in an endogenous growth model with infrastructure which is nonexclusive but may exhibit varying degrees of nonrivalry. Turnovsky (1996) incorporates convex adjustment costs of private capital investment in an endogenous growth model with productive public expenditure and then analyse properties of optimal taxation. Cazzavillan (1996) extends Barro (1990) model in the direction where public good creates positive externalities on production as well as on utility of the consumer. Loayza (1996) and Penalosa and Turnovsky (2005) develop two-sector models with a formal sector and an informal sector and then derive the properties of optimal taxation when taxes from the formal sector income finances productive public services but the informal sector remains untaxed. Turnovsky (1997) extends Futagami et al. (1993) by introducing congestion effect on productive public capital and analyse properties of fiscal policy while maximising growth rate as well as welfare. Greiner and Hanusch (1998) extends Futagami et al. (1993) model with various fiscal instruments. Here, the government uses its tax revenue to finance investment in public capital, subsidy for private investment and transfer payments. Tanaka (2002) extends Barro (1990) model with decision making over a finite horizon. Dasgupta (1999) constructs a two sector model of endogenous growth with durable public capital. Both private capital and public capital are used in production of both the final good and public investment good. Government imposes a proportional tax on the household's aggregate capital income and charges a price per unit of the infrastructural service to producers of the final good. Turnovsky (1999a) studies the role of productive public expenditure in a stochastic version of the endogenous growth model with public input. Turnovsky (1999b) analyses optimum tax rate in an extended Barro (1990) model with international openness and elastic labour supply. Eicher and Turnovsky (2000) focuses on the distinction between relative and aggregate congestion effects of public capital due to private capital accumulation and explores their implications on optimum tax rate. Baier and Glomm (2001) analyses this tax issue in an endogenous growth model where government finances infrastructure investment as well as utility enhancing government services and transfer payments. Tsoukis and Miller (2003) and Ghosh and Roy (2004) analyse optimum taxation in an endogenous growth model with flow public expenditure as well as with durable public capital. Kalaitzidakis and Kalyvitis (2004) incorporates maintenance expenditure to reduce the depreciation of existing public capital and analyses properties of optimum fiscal policy. Park and Philippopoulos (2003) extends Barro (1990) model incorporating moral hazard of redistributive transfers and analyses its implication for growth and fiscal policy. This model also considers heterogeneous capital endowments across individuals who obtain utility from the consumption of both final good and public services. Hosoya (2003) analyses growth rate maximising tax rate in a two-sector endogenous growth model where public expenditure on health input helps accumulation of health capital through a flow channel while a physical capital deepening externality helps accumulate it through a stock channel. Marrero and Novales (2005) examines the optimality issue of alternative tax policies in an endogenous growth model with productive public expenditure as well as with public consumption expenditure. In Greiner (2005), production process creates environmental pollution, which negatively affects the utility of the households. In such a setup, growth maximising as well as welfare maximising income tax rate and pollution tax rate are analysed. Ott and Turnovsky (2006) analyses optimal tax and user cost structure in an extension of Barro (1990) model with excludable and non-excludable productive public inputs where both public inputs are subject to congestion effect. Cassou and Lansing (2006) investigates optimal taxation in an endogenous growth model with human capital and with two types of public expenditures. Gomez (2008) investigates optimal tax rate in an endogenous growth model with absolute as well as relative congestion of productive public capital and with Lucas (1988) type of human capital accumulation. Agenor (2008) studies growth and welfare maximising income taxation and the allocation of public spending in an endogenous growth framework where infrastructure affects not only the production of goods but also the supply of health services. Dioikitopoulos and Kalyvitis (2008) introduces a congestion effect of public capital and the problem of depreciation of public capital with the role of maintenance expenditure in a Futagami et al. (1993) type of model; and investigates optimal and growth-maximizing fiscal policies. In Agenor (2009) model, properties of growth-maximizing tax rate and share of infrastructure investment are analysed when the maintenance expenditure plays a dual role of increasing the durability as well as the efficiency of public capital. Agenor (2011) analyses growth rate maximising tax rate in an endogenous growth model with human capital where public capital in infrastructure affects human capital accumulation. Gupta and Barman (2009, 2010, 2013, 2015) and Barman and Gupta (2010) investigate growth rate maximising income tax rate when government finances both productive public expenditure as well as abatement expenditure for cleaning environment. Ni and Wang (1994), Corsetti and Roubini

(1996), Glomm and Ravikumar (1997), Bandyopadhyay and Basu (2001), Blankenau and Simpson (2004), Chakraborty and Gupta (2009), Tournemaine and Tsoukis (2015) etc. search for optimum taxation in endogenous growth models focusing on human capital formation.

However, all these models dealing with the issue of optimum taxation in an endogenous growth model assume competitive labour market. Only Raurich and Sorolla (2003, 2004), Kitaura (2010) and Chang and Chang (2015) deal with this problem in the presence of unionised labour market. Raurich and Sorolla (2003) attempts to analyse effects of fiscal policies on economic growth when income taxes finance productive public capital accumulation as well as unemployment benefit. However, their model fails to derive any analytical solution and finally relies on numerical techniques to obtain solutions. In Raurich and Sorolla (2004), the government finances the unemployment benefit with a part of the tax revenue earned from wage income; and its other part as well as the total revenue earned from private capital income taxation are used to provide public input. Since this is an OLG model, so capital income taxation does not reduce savings and capital accumulation. However, it increases public capital accumulation. As a result, growth rate is increased due to capital income taxation. However, labour income taxation reduces savings but raises public capital accumulation. As a result, growth effect of wage income taxation is ambiguous.

Kitaura (2010) uses a simple overlapping generations endogenous growth model where productive public expenditure is the source of economic growth. Government finances both unemployment benefit and productive public expenditure using taxes on wage income. This model shows that the growth rate maximising tax rate is higher than (equal to) the elasticity of output with respect to public input in the presence (absence) of unemployment benefit scheme. This tax rate also varies positively with the proportion of revenue used to finance unemployment benefit scheme. Using numerical values, this paper also shows that the welfare maximising tax rate is lower than the growth rate maximising one.

Chang and Chang (2015) extends Barro (1990) model with labour unions and monopolistic competition in the goods market. Here also the government finances both investment in public capital and expenditure on unemployment benefit. This model analytically determines the growth rate maximizing ratio of government spending to GDP when the government charges two different tax rates on capital income and on labour income. They also examine whether this growth rate maximizing ratio also maximizes social welfare or not.

## 1.4 Existing Research Gaps

There may exist various types of research gaps to be fulfilled in the future works. However, only a few research gaps are to be addressed in the present thesis. In these discussed models of subsection 1.2.2, only Mauro and Pigliaru (2013) incorporates productive public capital as an input in the production function. However, their main objective is to study the interaction between social capital endowment and decentralization; and they show that decentralization can be a source of regional divergence in countries characterized by a highly heterogeneous distribution of social capital. Since they use Italian data for calibration of the model, they are forced to incorporate labour unions in their model; and as a result, their modelling of labour union is very simple to model the reality. They consider monopoly union model and does not consider bargaining between union and firm. The objective of the union is to maximise the expected utility of its members; and it does not care for the size of the membership. Here, the government does not finance unemployment benefit expenditure in that model while, in unionised economies, governments spend a significant share of budget to finance unemployment benefit. As a result, this model cannot analyse the role of interaction between a tax financed public expenditure policy and an unemployment benefit policy on the growth effect of unionisation.

The existing literature also does not consider labour union's concern about workers' health and safety and environmental protection while analysing the growth effect of unionisation. However, there exists enough evidence pointing out that many labour unions fight hard for protection of workers' health and improvement of working environment. For example, Gahan (2002) shows that workplace safety always remains in the set of priorities of the union. Khan et al. (2012a) presents evidences to show that labour unions struggle for environmental protection. Khan (2010) and Khan et al. (2012b) also justify trade unions' role to protect environment. Valenduc (2001) points out that labour unions in Belgium have environmental awareness projects. Kawakami et al. (2004) points out that trade unions in Asia organize training workshop to improve workers' safety and health. Stevis (2011) shows that, over the last two decades, labour unions have developed their environmental agendas consistent with their concerns about safety and health. There are many other evidences to establish that labour unions negotiate for workers' health and safety and for environment protection. Gray et al. (1998) studies many private-sector collective bargaining agreements in which health and safety provisions frequently appear. Magane et al.

(1997) provides evidences of firm's switching to eco-friendly production techniques due to struggle of labour unions for health and safety. Since, in many cases, workers first experience negative effects of industrial hazards which are going to pollute the environment, union's effort can lead to improvement in the broader natural environment. Magane et al. (1997), Davies (1993) and Dembo et al. (1988) also think that workplace environment should be seen as part of the broader natural environment. The disasters of Thor Chemicals in South Africa, Union Carbide plant in Bhopal, India, Sandoz warehouse in Basel, Switzerland, Nuclear power plant in Chernobyl, Soviet Union etc. also support the link between the global natural environmental disasters and industrial environment problem in the workplace. Due to the productive role of environment on labour productivity, unionisation can affect the growth rate of the economy through its positive role on environmental protection.

Mentioned works of subsection 1.2.2 also do not focus on the role of 'Efficiency Wage Hypothesis' to study the effect of unionisation on growth. 'Efficiency Wage Hypothesis' points out that there are costs as well as benefits to firms for paying higher wages to workers. There are many explanations for this hypothesis. It may hold in developing countries because a rise in income may result a rise in food consumption and thereby a rise in the productivity of workers. In developed countries, firms may pay higher wages to their workers in order to make the jobs valuable for them which makes 'getting fired from the job' as a punishment to them. This prevents the workers from shirking even when effort cannot be monitored perfectly by the employer. Offering a higher wage raises the average quality of the applicant pool and thus raises the average ability of the workers the firm hires. Higher wage can also build loyalty among workers and hence make them put more effort.[3] The empirical literature also confirms the strong existence of 'Efficiency Wage Hypothesis'.[4] Since unionisation raises the wage rate, and since according to 'Efficiency Wage Hypothesis', effort (efficiency) level per worker varies positively with the wage rate, unionisation may produce an overall positive effect on the production level. So, it is very important to analyse the effect of unionisation in the labour market in the presence of this hypothesis. Mentioned works of subsection 1.2.2 also ignore the government's role to raise workers' efficiency through investment in human capital accumulation. In many countries, the

[3] See, for example, Solow (1979), Yellen (1984), Stiglitz (1976), Shapiro and Stiglitz (1984), Akerlof (1982, 1984), Akerlof and Yellen (1986) etc. for a discussion on the 'Efficiency Wage Hypothesis'.
[4] See for example Peach and Stanley (2009).

government spends a huge amount for education to raise the efficiency of workers. So we should study the effect of unionisation on economic growth with a special focus on the government's role on human capital accumulation.

On the other hand, as mentioned in the section 1.3, a few models consider the issue of optimal taxation in a unionised economy. In Raurich and Sorolla (2003), analytical properties of optimal taxation are not derived; and this model finally relies on numerical techniques to obtain solutions. Kitaura (2010) derives analytical properties of optimum fiscal policies in the presence of labour unions, productive public capital accumulation and unemployment subsidy policies. However, this model does not establish any link between the unemployment rate and the optimal tax policy. A very recent paper by Chang and Chang (2015) extends Barro (1990) model introducing monopolistic competition into the product market and unions into the labour market; and analytically determines the growth rate maximizing ratio of government spending to GDP when the government charges two different tax rates on capital income and on labour income. They also examine whether this growth rate maximizing ratio also maximizes social welfare or not. However, the model cannot determine the optimum tax rate on labour income and that on capital income separately; and thus remains incomplete determining only the optimum sum of those two components. Additionally, there are a few major limitations of each of these three works. In each of these three models, the labour union maximizes only the average income of workers but does not care for the size of membership except Raurich and Sorolla (2004). Raurich and Sorolla (2003, 2004) and Kitaura (2010) do not introduce bargaining problem between the labour union and the employers' association; and so they cannot analyse the growth effect of unionization. Each of them develops Overlapping Generation model, and hence, cannot analyse Ramsey optimal solutions. Although Chang and Chang (2015) considers 'Efficient Bargaining' between the employers' union and the employees' union, it does not analyse the effect of unionization on economic growth.

The theoretical literature on endogenous growth models with productive public capital also has two major problems. First of all, these models assume that the production functions of both goods are identical. So it is important to derive properties of optimal income tax rate where private goods and public goods are produced with different production technologies. Few works like Dasgupta (1999, 2001), Dasgupta and Shimomura (2006), Pintea and Turnovsky (2006), Turnsovsky and Pintea (2006) consider different production technologies to produce private goods and public goods. However, Pintea and Turnovsky (2006) and Turnsovsky and Pintea (2006) do not derive optimal tax rate analytically. On the other hand, Dasgupta (2001) and Dasgupta and Shimomura (2006) do not consider income taxation. Dasgupta (1999) shows that the optimal income tax rate is zero and the government should earn entire revenue by charging the private sector firms for usage of services of public capital on per unit basis. This may be impossible to implement when public services are non-excludable in nature; and firms will try to take a free ride. So one should stick to the idea of Barro (1990) of freely distributing services of public capital and of charging income taxes to finance its cost. The second problem with Barro (1990) type of modelling is more severe because it is assumed that the government buys public inputs from private producers at a given price; and this price is equal to the price of the final good in the case of identical production functions. However, why the government should act as a price-taker is not clear. The government is the only buyer; and so it should act as a monopsonist and should use the relative price as a tool to maximize its objective.

A few models surveyed in section 1.3 such as Gupta and Barman (2009, 2010, 2013, 2015), Barman and Gupta (2010), Greiner (2005) etc. analytically derive properties of growth rate maximising income tax rate in the presence of environmental pollution and abatement expenditure. All these models assume competitive labour market with full employment equilibrium. However, as mentioned above, there exists enough empirical evidence to show that labour unions force firms to spend for improvement in workers' health and safety condition in the workplace which, in turn, can lead to improvement in the broader natural environment. So it is important to analyse whether an optimal tax policy used to finance public abatement expenditure in a unionized economy differs from that in a competitive economy especially when the labour union can force firms to spend for environmental development.

In a subset of the literature, consisting of works of Blankenau and Simpson (2004), Ni and Wang (1994), Corsetti and Roubini (1996), Glomm and Ravikumar (1997), Chakraborty and Gupta (2009), Bandyopadhyay and Basu (2001), Tournemaine and Tsoukis (2015) etc., properties of optimal tax rate are analysed in models with competitive labour market. This tax finances human capital accumulation. However, growth maximising tax rates in unionised economy can differ significantly from those obtained in the case of competitive labour markets. This is so because, according to 'Efficiency Wage Hypothesis', higher wage rate raises workers' efficiency and labour unions bargain for higher wage. So, the interaction of the 'Efficiency Wage Hypothesis' and spending on human capital accumulation can play an important role to determine the nature of fiscal policy.

## 1.5 A Summary of the Present Thesis

In addition to the present introductory chapter, the thesis consists of five other chapters in which we develop different theoretical models attempting to fill up the research gaps pointed out in section 1.4.

Chapter 2 attempts to combine two different strands of literature. On the one hand, it investigates the growth effect and the welfare effect of unionization with a special focus on the role of interaction between a tax financed public expenditure policy and an unemployment benefit policy. However, on the other hand, it attempts to analyse the optimality of an income tax policy designed to finance productive public expenditure in the presence of an unemployment benefit policy. The model developed here is an otherwise identical Barro (1990) model where the assumption of competitive labour market is replaced by the unionized labour market with bargaining between a labour union and an employers' association. This leads to an unemployment equilibrium causing a leakage of tax revenue from productive public expenditures to unemployment allowances. In this modified Barro (1990) framework, we use two alternative versions of bargaining models —'Efficient Bargaining' model of McDonald and Solow (1981) and 'Right to Manage' model of Nickell and Andrews (1983). Productive public expenditure is defined as it is in Barro (1990) model.

We derive many interesting results from this model. First, the optimum income tax rate in this model appears to be higher than (equal to) that obtained in Barro (1990) model in the presence (absence) of unemployment allowances. This optimum tax rate varies positively with the rate of unemployment benefit and with the level of unemployment. Second, the endogenous growth rate varies inversely with the rate of unemployment benefit. However, the level of welfare may not vary inversely with this rate; and there may exist a positive welfare maximizing rate of unemployment benefit. These two results are valid in each of these two bargaining models. Third, how unionization in the labour market affects various macroeconomic variables depends on the type of the bargaining model considered. In the case of a 'Right to Manage' model, unionization must have a negative effect on the level of employment as well as on the rate of economic growth irrespective of the orientation of the labour union. However, this may not be true for the effect on the level of welfare. In the case of an ‘Efficient Bargaining’ model, unionization affects employment level and growth rate ambiguously; and the nature of this effect on employment (growth rate and welfare) depends solely (partially) on the nature of orientation of the labour union. Fourth, effects of unionization on the optimum income tax rate are also different in these two models. In ‘Right to Manage’ model, the optimum tax rate varies positively with the degree of unionization. However, in the ‘Efficient Bargaining’ model, this may not be true when the labour union is employment oriented.

Chapter 3 attempts to analyse properties of optimal income tax rate used to finance investment in public capital in a two sector economy with different production functions for producing final good and public investment good. In this model, the private sector produces public investment good and sells it to the government who has a monopsony power to set the buying price. Otherwise, our model is similar to Futagami et al. (1993) model. In the basic model, we follow Barro (1990) and Futagami et al. (1993) to assume a competitive labour market with full employment equilibrium. However, in the extended model, we consider a unionized labour market with unemployment equilibrium.

We derive many interesting results from the basic model. First of all, the growth rate maximising income tax rate is equal to the elasticity of output with respect to public capital in the production of final private goods only but is independent of the production technology to produce public investment good. Secondly, welfare maximising solutions are different from growth rate maximising solutions even in the steady state growth equilibrium. A few interesting results are also obtained after introducing unionisation and unemployment. First, economic growth rate is always higher in the case of competitive labour markets because marginal productivity of capital varies positively with the level of employment and unionisation creates unemployment. Finally, the steady state growth rate maximising allocation of private capital and the steady state growth rate maximising income tax rate are independent of unionisation in the labour market.

In chapter 4, we, on the one hand, make an attempt to analyse the effect of unionisation on the long run economic growth rate in the presence of environmental pollution, and, on the other hand, to analyse properties of an optimum income tax policy designed to finance public abatement expenditure when labour unions bargain for workers’ health and safety and for environment development. We consider two alternative bargaining models to analyse the negotiation problem

– the ‘Efficient Bargaining’ model of McDonald and Solow (1981) and the ‘Right to Manage’ model of Nickell and Andrews (1983).

We derive interesting results from this model. First, growth rate maximising rate of income tax used to finance public abatement expenditure varies inversely with the relative bargaining power of the labour union. Secondly, how unionisation affects employment depends on the nature of bargaining. In the ‘Efficient Bargaining’ model, unionisation raises employment level only if the labour union is highly employment oriented. Otherwise, it always lowers the level of employment. Thirdly, the effect on economic growth depends partly on the employment effect and partly on the effect of employer’s spending to protect environment; and this is valid for each of the two bargaining models. Since the environmental protection effect is always positive, it may outweigh the employment effect even if it is negative; and thus unionisation may have a positive effect on economic growth even when unions are wage oriented.

Chapter 5 attempts to develop a model to analyse the effect of unionisation in the labour market on economic growth in the presence of ‘Efficiency Wage Hypothesis’. The model developed here is an AK model with a unionised labour market and with an unemployment benefit scheme. In this model also, we use two alternative versions of bargaining models – the ‘Efficient Bargaining’ model of McDonald and Solow (1981) and the ‘Right to Manage’ model of Nickell and Andrews (1983).

We derive interesting results from this model. In the ‘Efficient Bargaining’ model, unionisation in the labour market reduces the number of workers unless the labour union is highly employment oriented but always raises the effort (efficiency) level per worker. As a result, effective employment must (may) increase for employment oriented and neutral (wage oriented) labour union. The effect of unionisation on the growth rate as well as on the level of welfare are same as that on the effective employment. However, in the ‘Right to Manage’ model, unionisation raises worker’s effort level but lowers the number of workers irrespective of the orientation of the labour union; and raises effective employment, balanced growth rate and welfare level if and only if the wage elasticity of effort is greater than the unemployment rate. This sufficient condition is always valid when the rate of income tax used to finance unemployment benefit expenditure is very low.

Chapter 6 extends chapter 5 by incorporating the government’s role in human capital accumulation and this chapter develops a simple endogenous growth model with a special focus on the 'Efficiency Wage Hypothesis' and on the government's role in human capital accumulation. In this model, we analyse the effect of unionisation on the economic growth rate as well as on the optimum tax rate to finance public education when the educational expenditure is financed by taxation only on labour income. We use two different bargaining models to solve the negotiation problem between the employers' association and the labour union - 'Efficient Bargaining' model of McDonald and Solow (1981) and 'Right to Manage' model of Nickell and Andrews (1983).

Our main findings are as follows. First, in each of these two bargaining models, for a given tax rate on labour income, unionisation lowers the number of employed workers but raises their effort level. However, when the government imposes the growth rate maximising tax rate on labour income, then the number of employed workers becomes independent of labour union's bargaining power but varies inversely with the elasticity of efficiency with respect to human capital. Secondly, this growth rate maximising tax rate varies positively with the elasticity of worker's efficiency with respect to human capital as well as with the budget share of investment in human capital accumulation; and, on the other hand, varies inversely with the degree of unionisation in the labour market. Thirdly, the growth rate maximising tax rate is different from the corresponding welfare maximising tax rate; and the welfare effect of unionisation is also different from the growth effect of unionisation in each of these two bargaining models. Lastly, growth effect of unionisation consists of a positive effort effect and an ambiguous human capital accumulation effect. In the case of 'Efficient Bargaining' ('Right to Manage') model, a higher value of the elasticity of worker's efficiency with respect to the wage premium than the value of that elasticity with respect to human capital is a sufficient but not a necessary (both necessary and sufficient) condition to ensure a positive growth effect of unionisation.

Concluding remarks are made in chapter 7.

## Chapter 2: Unemployment Allowances and Productive Public Expenditure[5]

### 2.1 Introduction

In this chapter, we develop an endogenous growth model with special focus on the role of unionized labour market and on the interaction between the tax financed productive public expenditure and unemployment benefit policy of the government. Here, productive public expenditure means government expenditure to develop physical infrastructure like roads, communications etc. This infrastructural development generates positive external effects on private production; but does not produce any external effect on consumption.

There exists strong unionised labour market as well as huge amount of spending to run the unemployment benefit scheme in many countries, especially in the OECD countries. For example, union densities were 74.1%, 78%, 53.3% and 70.4% in Finland, Sweden, Norway and Denmark respectively in 2003; and it was 55.4% in Belgium in 2002. On the other hand, bargaining coverages were 82% and 95% in Netherlands and Finland respectively in 2001 and were 92% and 81% in Sweden and Spain respectively in 1997; and were 77% and 99% in Norway and Austria respectively in 1998. In France, unions' bargaining coverage was 95% in 2003.[6] The evolution of union density for a few European countries[7] from 1993 to 2012 is shown graphically in figure 2.1.

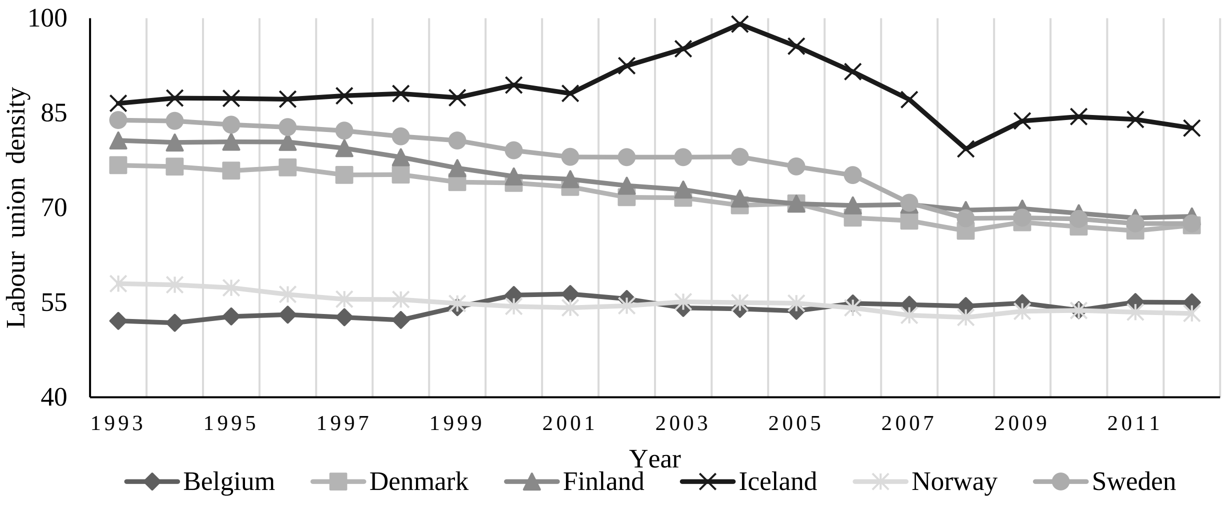


Figure – 2.1: Evolution of labour union density.

---

[5] A related version of this chapter is published in Metroeconomica.
[6] See table 3 and table 4 of Visser (2006).
[7] Data are available from (http://stats.oecd.org/Index.aspx?DataSetCode=UN_DEN).

The percentage rate of public spending on unemployment benefit to GDP in Austria, Belgium, Denmark, Finland, Ireland, Spain, Netherlands and France were 1.1, 3.7, 2.3, 2.0, 2.6, 3.5, 1.4 and 1.5 respectively in 2009.[8] We also show graphically the evolution of unemployment benefit rate during 1998 to 2011 for those countries in figure 2.2[9].

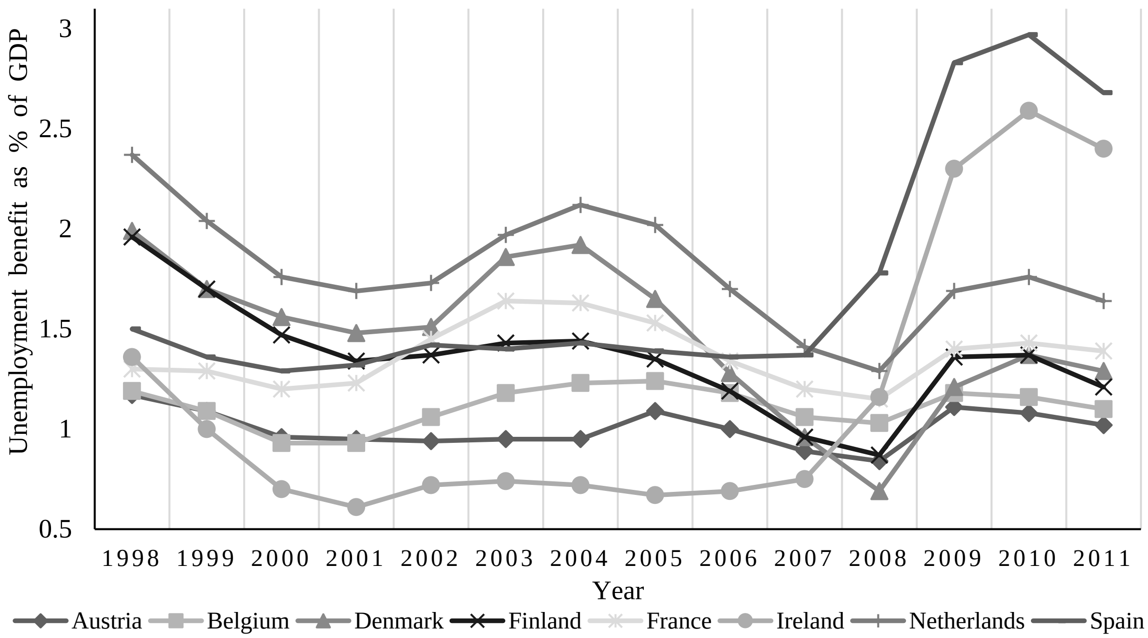


Figure – 2.2: Evolution of unemployment benefit rate.

These empirical findings help to develop the motivation of this model. The present chapter attempts to combine two different strands of literature. On the one hand, it investigates the growth effect and the welfare effect of unionisation in the labour market using an aggregate one sector dynamic framework with a special focus on the role of interaction between a tax financed public expenditure policy and an unemployment benefit policy. However, on the other hand, it attempts to analyse the optimality of an income tax policy designed to finance productive public expenditure in the presence of an unemployment benefit policy. We consider unemployment benefit policy because, on the one hand, it is widely discussed; and, on the other hand, it is simple to introduce

[8] Data are available from (http://data.oecd.org/socialexp/public-unemployment-spending.htm).
[9] Data are available from (http://stats.oecd.org/index.aspx?DataSetCode=LMPEXP). In figure 2.2, only 'Full unemployment benefits' is considered and no other component of 'Out-of-work income maintenance and support' is considered.

in a one sector model with unemployment. In reality, the concept of welfare state takes the form of a package of various social development programmes.

The model developed here is an otherwise identical Barro (1990) model where the assumption of competitive labour market is replaced by the unionised labour market with bargaining between a labour union and an employers' association. This leads to an unemployment equilibrium causing a leakage of tax revenue from productive public expenditures to unemployment allowances. In this modified Barro (1990) framework, we use two alternative versions of bargaining models – 'Efficient Bargaining' model of McDonald and Solow (1981) and 'Right to Manage' model of Nickell and Andrews (1983). Productive public expenditure is defined as it is in Barro (1990) model.

We derive many interesting results from this model. First, the optimum income tax rate in this model appears to be higher than (equal to) that obtained in Barro (1990) model in the presence (absence) of unemployment allowances. This optimum tax rate varies positively with the rate of unemployment benefit and with the level of unemployment. Secondly, the endogenous growth rate varies inversely with the rate of unemployment benefit. However, the level of welfare may not vary inversely with this rate; and there may exist a positive welfare maximising rate of unemployment benefit. These two results are valid in each of these two bargaining models. Thirdly, how unionisation in the labour market affects various macroeconomic variables depends on the type of the bargaining model considered. In the case of a 'Right to Manage' model, unionisation must have a negative effect on the level of employment as well as on the rate of economic growth irrespective of the orientation of the labour union. However, this may not be true for the effect on the level of welfare. In the case of an 'Efficient Bargaining' model, unionisation affects employment level and growth rate ambiguously; and the nature of this effect on employment (growth rate and welfare) depends solely (partially) on the nature of orientation of the labour union. Fourthly, effects of unionisation on the optimum income tax rate are also different in these two models. In 'Right to Manage' model, the optimum tax rate varies positively with the degree of unionisation. However, in the 'Efficient Bargaining' model, this may not be true when the labour union is employment oriented.

The chapter is organised as follows. In section 2.2, we describe the basic model with 'Efficient Bargaining' and then derive various theoretical results. These results are compared to corresponding results obtained from 'Right to Manage' model in section 2.3.

## 2.2 The 'Efficient Bargaining' Model

### 2.2.1 Firms

The representative competitive firm produces the final good, *Y*, using private capital, *K*, labour, *L*, and public input, *G*. Only one commodity is produced with homogeneous capital and homogeneous labour. The production function of the final good is given by

$$Y = F(K, L, G) = AK^{\alpha}L^{\beta}G^{1-\alpha} \text{ where A > 0; } \alpha, \beta \in (0,1) \text{ and } \alpha + \beta < 1 \quad . \quad (2.2.1)$$

Here *G* enters as an argument into the production function because it represents infrastructure. Following Barro (1990), we assume it to be a flow variable though, in reality, it is a durable input. Here $A$ is time independent. $\alpha$ and $\beta$ represent the capital elasticity of output and the labour elasticity of output respectively**.** The Cobb-Douglas production function satisfies increasing returns to scale in terms of all inputs but decreasing returns in terms of private inputs. So a positive bargaining power of employers' association leads to a positive profit (rent) generated from the bargaining between the labour union and the employers' association. Following Chang et al. (2007), we assume that a fixed factor exists and is needed to set up a plant. So the number of firms is fixed in the short-run equilibrium; and, for the sake of simplicity, it is normalised to unity.

The representative firm's objective is to maximise its profit, $\pi$, defined as

$$\pi = (1-\tau)Y - wL - rK \quad . \quad (2.2.2)$$

Here *w*, *r* and $\tau$ stand for the wage rate of labour, the rental rate on private capital and the income tax rate respectively[10].

### 2.2.2 Capital Market

Private capital market is perfectly competitive. So the rental rate on capital is determined by demand supply equality in this market. Profit - maximising behaviour of the competitive firm leads to the following demand function for capital.

$$r = (1-\tau)A\alpha K^{\alpha-1}L^{\beta}G^{1-\alpha} = \frac{(1-\tau)\alpha Y}{K} \quad . \quad (2.2.3)$$

[10] Here we are assuming that all firms and all inputs of production are owned by households. So profit income is also taxable. As there is a single final good, so its price is normalised to unity.

### 2.2.3 Labour Union's Objective Function

Following Pemberton (1988) and Chang et al. (2007), we consider a 'managerial' labour union that takes care of members' interests as well as leadership's interest. Members want to earn higher income and the leadership wants to increase the number of members. So the utility function of the labour union is given by[11]

$$u_T = (w - w_c)^m L^n \quad \text{with} \quad m, n > 0 \ . \tag{2.2.4}$$

Here $u_T$ and $w_c$ denote the utility of the labour union and the competitive wage rate respectively and $(w - w_c)$ is defined as the worker's income gain due to unionisation. The union derives utility from the additional income of members and from the size of the membership. All workers are members of the union as we assume closed shop labour union. $m$ and $n$ are two non negative preference parameters representing elasticities of utility with respect to income gain and the level of employment respectively. If $m$ > ( < ) ( = ) $n$, then the labour union is said to be "wage oriented" ("employment oriented") ("neutral").[12]

In a competitive labour market, wage is equated to the marginal product of labour when firms maximise profit; and the labour force, normalised to unity, is fully employed[13]. So the competitive wage rate is given by the following equation.

$$w_c = (1 - \tau)\beta A K^\alpha G^{1-\alpha} \quad . \tag{2.2.5}$$

### 2.2.4 Employment and Wage Determination

In the basic model, we introduce the 'Efficient Bargaining' case. Both the wage rate and the level of employment are determined by bargaining between the nationwide labour union and the nationwide employers' association.[14] The result of the bargaining process can be obtained maximising the 'generalised Nash product' function which is given by

$$\psi = (\ u_T)^\theta (\pi)^{(1-\theta)} \quad \text{with} \quad 0 < \theta < 1 \quad . \tag{2.2.6}$$

[11] Some models like Chang et al. (2007), Adjemian et al. (2010) etc. take the difference between bargained wage rate and the rate of unemployment benefit as the argument in the labour union's utility function. Contrary to this, the difference between bargained wage rate and competitive wage rate is used as an argument in the works of Irmen and Wigger (2002/2003), Lingens (2003a) and Lai and Wang (2010).
[12] See Chang et al. (2007) to know more about these parameters.
[13] We assume that the population does not grow overtime.
[14] Details can be seen from Booth (1995). The 'Right to Manage' model case is discussed in the next section.

Bargaining disagreement results to zero employment, which, in turn, implies zero profit and zero utility. The parameter, $\theta$, represents the relative bargaining power of the labour union. Using equations (2.2.2) and (2.2.3), we have

$$\pi = (1-\tau)(1-\alpha)Y - wL \ . \tag{2.2.7}$$

Finally, incorporating equations (2.2.1), (2.2.4), (2.2.5) and (2.2.7) into equation (2.2.6), we obtain

$$\psi = \{(w - [1-\tau]\beta A K^{\alpha} G^{1-\alpha})^{m} L^{n}\}^{\theta}\{(1-\tau)(1-\alpha)AK^{\alpha}L^{\beta}G^{1-\alpha} - wL\}^{(1-\theta)} \quad . \tag{2.2.8}$$

Here $\psi$ is to be maximised with respect to $w$ and $L$. Assuming an interior solution, we obtain[15]

$$\hat{L} = \left\{\frac{(1-\alpha)\{\theta n + \beta(1-\theta) - \theta m(1-\beta)\}}{\beta\{\theta n + (1-\theta)\}}\right\}^{\frac{1}{1-\beta}} \quad ; \tag{2.2.9}$$

and

$$w = \frac{\{\theta n + (1-\theta)\beta\}(1-\tau)\beta A K^{\alpha} G^{1-\alpha}}{\{\theta n + (1-\theta)\beta - \theta m(1-\beta)\}} = X w_c \quad . \tag{2.2.10}$$

Here we assume that $\theta n + (1-\theta)\beta > \theta m(1-\beta)$; and hence

$$X = \frac{\{\theta n + (1-\theta)\beta\}}{\{\theta n + (1-\theta)\beta - \theta m(1-\beta)\}} > 1 \quad . \tag{2.2.11}$$

$X$ represents the ratio of the bargained wage rate to the competitive wage rate; and $X = 1$ when the union has no bargaining power. We assume the following parametric restriction.

Condition 2.A: $$-\left(\frac{1-\theta}{\theta} + m\right) < \frac{n-m}{\beta} < \left(\frac{\alpha}{1-\alpha-\beta}\right)\left[\frac{1-\theta}{\theta} + m\right] \ .$$

This ensures that $0 < \hat{L} < 1$ and $w > 0$. Second order conditions of maximisation of $\psi$ are also satisfied[16].

Equation (2.2.9) shows that equilibrium level of employment is time independent; and this must be so because the size of the labour endowment is normalised to unity. Equation (2.2.10) shows that the bargained wage rate is greater than the competitive wage rate. This also must be so because otherwise workers do not have any income gain from unionisation.

### 2.2.5 The Government

---

[15] Derivation of equations (2.2.9) and (2.2.10) from the first order conditions are shown in the appendix 2.A.

[16] For details, see appendix 2.A.

Proportional income tax is the only source of revenue; and the government spends the entire tax revenue to finance the unemployment benefit scheme as well as the productive public expenditure. So the balanced budget equation is given by

$$\tau Y = G + s(1 - L) \quad . \tag{2.2.12}$$

We introduce proportional income tax due to two reasons. First, the working of the model avoids technical complication in this case. Secondly, Barro (1990) model and majority of its extensions assume proportional income tax.[17] Here $s$ is the amount of benefit given to an unemployed worker. Unemployment benefit policy is the only instrument to reduce the degree of income inequality in this model. For the sake of technical simplicity, we rule out other welfare programmes.

### 2.2.6 The Household

The representative household derives instantaneous utility from consumption of the final good only and not from leisure. Also this utility function is independent of public services because physical productive infrastructures do not affect consumers' utility directly.[18] The household chooses the time path of consumption to maximise her discounted present value of instantaneous utility subject to her intertemporal budget constraint. Mathematically the household's problem is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} \, dt \tag{2.2.13}$$

subject to, $\dot{K} = wL + rK + \pi + s(1 - L) - c$ ; (2.2.14)

and $K(0) = K_0$ .

Here $c$ is the control variable and $K$ is the state variable. Here $\sigma$ is the elasticity of marginal utility with respect to consumption; and $\rho$ is the constant rate of discount. Capital does not depreciate over time. It is assumed that the rate of unemployment is same for all households; and the representative household saves and invests the rest of his income left after consumption.

Solving this dynamic optimisation problem we obtain the growth rate of consumption[19], denoted by $g$, as given below:

---

[17] Some works such as Doménech and García (2008), Chang et al. (2007) etc. consider many other alternative taxes.

[18] Barro (1990) and many of its extensions also assume that productive public expenditure does not enter into the utility function of the representative household.

[19] Derivation of equation (2.2.15) is given in the appendix 2.A.

$$g = \frac{\dot{c}}{c} = \frac{r - \rho}{\sigma} \quad . \tag{2.2.15}$$

Equation (2.2.15) shows that the rate of growth of consumption is equal to the excess of the rate of return on capital over the rate of discount of consumption normalised with respect to the elasticity of marginal utility of consumption.

### 2.2.7 Optimum Tax Rate

For the sake of simplicity, we assume that unemployment benefit per worker is proportional to the wage rate. So

$$s = bw \tag{2.2.16}$$

where $b$ is a positive fraction. Using equations (2.2.1), (2.2.10), (2.2.12) and (2.2.16), we obtain

$$\frac{G}{K} = \left\{A\tau L^{\beta} - Ab(1-L)\beta X(1-\tau)\right\}^{\frac{1}{\alpha}} . \tag{2.2.17}$$

Using equations (2.2.3), (2.2.15) and (2.2.17) and then putting $L = \hat{L}$, we obtain

$$g = \frac{A^{\frac{1}{\alpha}}(1-\tau)\alpha\hat{L}^{\,\beta}\left\{\tau\hat{L}^{\,\beta} - b\left(1-\hat{L}\,\right)\beta X(1-\tau)\right\}^{\frac{1-\alpha}{\alpha}} - \rho}{\sigma} \quad . \tag{2.2.18}$$

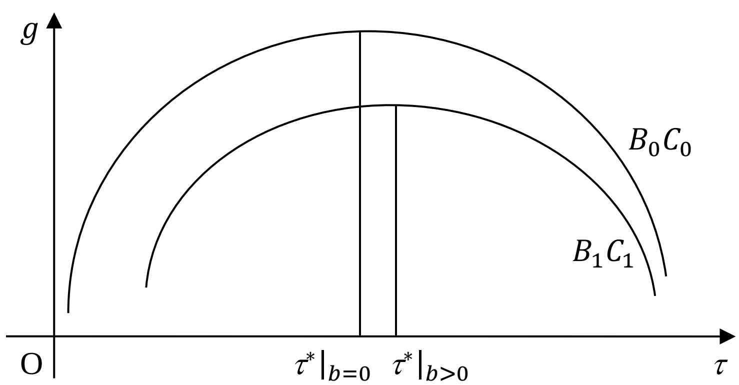


Figure – 2.3: Growth maximising tax rate.

This equation (2.2.18) represents the ‘Barro Curve (*BC*)’ which shows the relationship between the rate of growth, $g$, and the tax rate, $\tau$. It takes an inverted U – shape as shown in figure 2.3; and is drawn for a given value of $b$. For $b > 0$, we have a lower Barro Curve than that with $b = 0$, i.e., in the absence of unemployment benefit. We now turn to derive the growth rate maximising income tax rate; and so we maximise the right - hand side of equation (2.2.18) with respect to $\tau$ and then obtain

$$\tau^* = \frac{(1-\alpha)\hat{L}^{\,\beta} + b(1-\hat{L}\,)\beta X}{\hat{L}^{\,\beta} + b(1-\hat{L}\,)\beta X} = 1 - \frac{\alpha\hat{L}^{\,\beta}}{\hat{L}^{\,\beta} + b\left(1-\hat{L}\,\right)\beta X} \quad . \qquad (2.2.19)$$

Equation (2.2.19) shows that the growth rate maximising income tax rate, $\tau^*$, varies positively with the rate of unemployment benefit, $b$. Here $\tau^* = 1-\alpha$ when $b = 0$. It may be noted that, in the original Barro (1990) model with competitive labour market and full employment equilibrium, we obtain the highest possible Barro Curve because $\hat{L} \leq 1$ in the case of unemployment equilibrium. Optimum value of $\tau^*$ is independent of $\hat{L}$ when $b$ = 0.

This is an important result because it differs from the corresponding result of Barro (1990) in the presence of an unemployment benefit scheme. The Barro (1990) result states that growth rate maximising tax rate is identical to the elasticity of output with respect to productive public services. Barro (1990) does not consider unionised labour market and unemployment equilibrium. Our analysis shows that Barro (1990) result is valid even if there is a unionised labour market with unemployment equilibrium when the government does not introduce any unemployment benefit scheme[20]. This result is obtained from the initial Barro Curve $B_0C_0$ which exists in the absence of unemployment benefit. However, if the government finances unemployment benefit with a part of its tax revenue, then the growth rate maximising tax rate will be higher than the elasticity of output with respect to productive public services. This is obvious because this tax revenue not only finances productive public expenditure but also finances unemployment benefit which does not contribute to economic growth. When $b$ > 0, then employment and unemployment benefit rate enter into the expression of post tax marginal productivity of private capital non-multiplicatively. As a result, $\tau^*$, obtained by maximising the post tax marginal productivity of private capital becomes a function of $\hat{L}$ and $b$. The 'Barro Curve' shifts downward to $B_1C_1$ in this case; and the maximum point of this new 'Barro Curve' corresponds to a higher tax rate as shown in figure 2.3.

From equation (2.2.19), we have

[20] Barro (1990) model is an AK model. The labour input enters into the production function in a multiplicative way. So when $b$ = 0, then post tax marginal productivity of private capital varies multiplicatively with a function of employment. Since maximisation of growth rate implies maximisation of post tax marginal productivity of private capital, so introduction of labour union and bargaining solution lowers employment but does not affect the optimum income tax rate in the absence of unemployment benefit.

$$\frac{\partial \tau^*}{\partial \hat{L}} = -\frac{\alpha b \beta X \hat{L}^{\beta-1}\left(\hat{L} + \beta[1-\hat{L}]\right)}{\left\{\hat{L}^{\beta} + b\left(1-\hat{L}\right)\beta X\right\}^2} < 0 \quad . \tag{2.2.20}$$

This implies that $\tau^*$ varies inversely with the level of employment. This is so due to two reasons: (i) Higher level of employment leads to lower expenditure to provide unemployment benefit. (ii) Employment and output and hence employment and tax revenue (given the tax rate) are positively related to each others. The 'Barro Curve' is drawn for a given level of employment; and it shifts upward when the level of employment is increased. So $\tau^*$ varies inversely with the level of employment. In Kitaura (2010) too, the growth rate maximising tax rate is higher than the elasticity of output with respect to productive public services. However, Kitaura (2010) does not show how $\tau^*$ varies with the level of unemployment and with the bargaining power of the labour union.

We now turn to analyse its effect on the level of welfare, $\hat{\omega}$. Here

$$\hat{\omega} = \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\, dt \quad ; \tag{2.2.21}$$

and it can be shown[21] that

$$\hat{\omega} = \frac{{K_0}^{1-\sigma}\left\{\frac{\rho\left[\hat{L}^{\beta} + b\beta X\left(1-\hat{L}\right)\right] + g\left[\sigma\hat{L}^{\beta} + \sigma b\beta X\left(1-\hat{L}\right) - \alpha\hat{L}^{\beta}\right]}{\alpha\hat{L}^{\beta}}\right\}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \tag{2.2.22}$$

Equation (2.2.22) shows that $\hat{\omega}$ varies positively with $g$ if $\sigma\hat{L}^{\beta} + \sigma b\beta X\left(1-\hat{L}\right) > \alpha\hat{L}^{\beta}$ and $\rho > g(1-\sigma)$. Since $\hat{L}$ is independent of the rate of tax, the growth rate maximising tax rate is identical to the social welfare maximising tax rate. We now can establish the following proposition.

**Proposition 2.2.1:** The growth rate maximising income tax rate and the welfare maximising income tax rates are identical; and this optimum tax rate exceeds (equals to) the elasticity of output with respect to productive public service when the rate of unemployment benefit is positive (zero). This optimum tax rate varies positively with the rate of unemployment benefit and with the level of unemployment.

[21] Derivation is found in appendix 2.A.

This result is partly sensitive to the assumption of proportional income tax. The result holds true with any form of tax which shows tax revenue to be proportional to income but is not valid where the tax rate does not affect the rate of return on capital. This result remains valid with taxes on capital income when the production function satisfies Cobb – Douglas property. However, this is not true when we consider taxes on consumption and / or on labour income.

### 2.2.8 Growth Effect and Welfare Effect of Unemployment Benefit

We now turn to analyse the effect of providing unemployment benefit on the growth rate of the economy. From equation (2.2.18), we obtain

$$\frac{\partial g}{\partial b} = -\frac{A^{\frac{1}{\alpha}}(1-\tau)(1-\alpha)\hat{L}^{\beta}(1-\hat{L})\beta X(1-\tau)}{\sigma\{\tau\hat{L}^{\beta} - b(1-\hat{L})\beta X(1-\tau)\}^{\frac{2\alpha-1}{\alpha}}} < 0 \quad . \tag{2.2.23}$$

Equation (2.2.23) shows that, given the tax rate, the growth effect of providing unemployment benefit is always negative. This is so because the denominator of equation (2.2.23) is positive as shown by equation (2.2.17). A rise in $b$ raises expenditure to finance unemployment benefit; and given the tax rate, it causes productive public expenditure to fall[22]. So the growth rate declines with a rise in the rate of unemployment benefit; and hence the growth rate maximising unemployment benefit rate is either zero or equal to a lower limit, $\bar{b}$, imposed by political considerations.

This theoretical result receives partial empirical support. Empirical works of Awaworyi and Yew (2014), Nordstrom (1992), and Weede (1986, 1991) find out an inverse relationship between the public social security transfer and the growth rate of the economy.[23]

From equation (2.2.22), we obtain

$$\frac{\partial\hat{\omega}}{\partial b} = \hat{\omega}\left\{\begin{array}{c}\frac{\partial g}{\partial b}\left[\frac{(1-\sigma)}{[\rho-g(1-\sigma)]} + \frac{(1-\sigma)[\sigma\hat{L}^{\beta}+\sigma b\beta X(1-\hat{L})-\alpha\hat{L}^{\beta}]}{\rho[\hat{L}^{\beta}+b\beta X(1-\hat{L})]+g[\sigma\hat{L}^{\beta}+\sigma b\beta X(1-\hat{L})-\alpha\hat{L}^{\beta}]}\right] \\ +\frac{(1-\sigma)\{\rho\beta X(1-\hat{L})+g\sigma\beta X(1-\hat{L})\}}{\rho[\hat{L}^{\beta}+b\beta X(1-\hat{L})]+g[\sigma\hat{L}^{\beta}+\sigma b\beta X(1-\hat{L})-\alpha\hat{L}^{\beta}]}\end{array}\right\} .$$

---

[22] See equation (2.2.17).

[23] These empirical works consider all components of social security transfers rather than only the unemployment benefit.

(2.2.24)

Equation (2.2.24) shows that the welfare effect of providing unemployment benefit consists of two different effects - a negative growth effect and a positive effect obtained from the increase in initial disposable income. So optimum $b$ is not necessarily equal to $\bar{b}$; and there may be an interior solution of $b$ satisfying $1 > b > \bar{b}$ while maximising welfare. We now establish the following proposition.

**Proposition 2.2.2:** Providing unemployment benefit must have a negative effect on economic growth but its welfare effect is not necessarily negative.

The intuition behind this proposition can be explained as follows. Income tax revenue is the common source of providing public infrastructure and unemployment benefit. So additional unemployment benefit lowers fund to finance public infrastructure; and this lowers marginal productivity of capital because a strong complementary relationship exists between capital and productive public expenditure. This, in turn, produces a negative effect on the growth rate. However, unemployment benefit is a source of consumption; and the level of utility, in this model, is a positive and concave function of the level of consumption only. So providing unemployment benefit raises the level of consumption; and hence the welfare effect of this policy is not necessarily negative.

### 2.2.9 Effects of Unionisation[24]

The economy is always in the steady state equilibrium without any transitional dynamics. In equilibrium, $\hat{L}$, $\tau^*$ and $\bar{b}$ all are time-independent; and hence $g$ and $G/K$ are also so. So $G$, $K$ and $Y$ grow at the rate, $g$. $w$ and $\pi$ also grow at the same rate but $r$ remains time-independent. $\tau^* Y$ and $bw(1-\hat{L})$ also grow at the same rate.

We now turn to analyse how unionisation defined as an exogenous increase in the relative bargaining power of the labour union affects economy's employment level, growth rate and welfare level in the steady-state equilibrium. Chang et al. (2007), Palokangas (1996) etc. also make similar analysis in their models without considering the role of productive public expenditure.

[24] In this section, we assume that optimum $b = \bar{b}$.

From equation (2.2.9), we have

$$\frac{\partial \hat{L}}{\partial \theta} = \frac{(n-m)\hat{L}}{(\theta n + 1 - \theta)\{\theta n + (1-\theta)\beta - \theta m(1-\beta)\}} \gtreqless 0 \quad for \quad n \gtreqless m \quad . \qquad (2.2.25)$$

Equation (2.2.25) shows that an increase in the relative bargaining power of the labour union will raise (lower) (not affect) the employment level of the economy if the labour union is employment oriented (wage oriented) (neutral). Chang et al. (2007) also obtains the same result.

From equations (2.2.18) and (2.2.19) and putting $b = \bar{b}$, we obtain

$$g|_{b=\bar{b}} = \frac{A^{\frac{1}{\alpha}}\alpha^2 \hat{L}^{\frac{\alpha\beta+\beta}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{[\hat{L}^{\beta} + \bar{b}(1-\hat{L})\beta X]\sigma} - \frac{\rho}{\sigma} \quad . \qquad (2.2.26)$$

From equation (2.2.11), we obtain

$$\frac{\partial X}{\partial \theta} = \frac{\beta m(1-\beta)}{\{\theta n + (1-\theta)\beta - \theta m(1-\beta)\}^2} > 0 \quad . \qquad (2.2.27)$$

From equation (2.2.26), we have

$$\left.\frac{\partial g}{\partial \theta}\right|_{b=\bar{b}} = \left(\frac{A^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\sigma}\right)\left(\frac{E_1\frac{\partial \hat{L}}{\partial \theta} - E_2\frac{\partial X}{\partial \theta}}{[\hat{L}^{\beta} + \bar{b}(1-\hat{L})\beta X]^2}\right) \quad ; \qquad (2.2.28)$$

where

$$E_1 = \hat{L}^{\frac{\alpha\beta+\beta}{\alpha}}\bar{b}\beta X + \frac{\beta}{\alpha}\hat{L}^{\frac{2\alpha\beta+\beta-\alpha}{\alpha}} + \bar{b}(1-\hat{L})\beta X\left(\frac{\alpha\beta+\beta}{\alpha}\right)\hat{L}^{\frac{\alpha\beta+\beta-\alpha}{\alpha}} > 0 \qquad (2.2.29)$$

and

$$E_2 = \hat{L}^{\frac{\alpha\beta+\beta}{\alpha}}\bar{b}(1-\hat{L})\beta > 0 \quad . \qquad (2.2.30)$$

Equation (2.2.28) shows that the growth effect of unionisation is ambiguous in sign. It partly depends on the nature of orientation of the labour union. The first term of the last bracket in the R.H.S. of equation (2.2.28) depends solely on the sign of $\frac{\partial \hat{L}}{\partial \theta}$ whereas the second term inside that bracket is always negative. So the employment orientation property of the labour union is necessary but not sufficient to ensure a positive growth effect of unionisation; and the growth effect is always negative if the labour union is not employment oriented. However, in Chang et al. (2007), an employment oriented labour union must ensure a positive growth effect of unionisation; and the growth effect is negative if and only if the union is wage oriented.

The intuition behind this result can be explained as follows. The growth effect of unionisation in this model consists of two parts. First one comes from the employment effect whose sign depends on the nature of orientation of the labour union; and this is same as that found in Chang et al. (2007). The second one is a negative tax effect; and it is special to the present model. Unionisation in the labour market raises the negotiated wage rate. So the unemployment benefit per worker, $\bar{b}w$, goes up. So government's expenditure to provide unemployment benefit is increased; and, to finance that expenditure, income tax rate has to rise. This reduces the after tax marginal productivity of private capital; and this, in turn, leads to the negative growth effect. This second effect does not exist in Chang et al. (2007) because they do not consider the role of productive public expenditure.

We now turn to analyse its effect on the level of welfare, $\widehat{\omega}$. From equation (2.2.22), we obtain

$$\left.\frac{\partial\widehat{\omega}}{\partial\theta}\right|_{b=\bar{b}} = \widehat{\omega}\left(\frac{E_3\frac{\partial\hat{L}}{\partial\theta} + E_4\frac{\partial X}{\partial\theta}}{\left[\hat{L}^{\beta} + \bar{b}(1-\hat{L})\beta X\right]^2}\right) \quad ; \qquad (2.2.31)$$

where

$$E_3 = \frac{(1-\sigma)\{\rho[\beta\hat{L}^{\beta-1} - \bar{b}\beta X] + g[\sigma\beta\hat{L}^{\beta-1} - \sigma\bar{b}\beta X - \alpha\beta\hat{L}^{\beta-1}]\}}{\rho[\hat{L}^{\beta} + \bar{b}\beta X(1-\hat{L})] + g[\sigma\hat{L}^{\beta} + \sigma\bar{b}\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]} - \frac{(1-\sigma)\beta}{\hat{L}}$$

$$+\left(\frac{(1-\sigma)}{[\rho - g(1-\sigma)]} + \frac{(1-\sigma)[\sigma\hat{L}^{\beta} + \sigma\bar{b}\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}{\rho[\hat{L}^{\beta} + \bar{b}\beta X(1-\hat{L})] + g[\sigma\hat{L}^{\beta} + \sigma\bar{b}\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}\right)$$

$$. \quad \left(\frac{A^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}E_1}{\sigma\left[\hat{L}^{\beta} + \bar{b}(1-\hat{L})\beta X\right]^2}\right) \quad . \qquad (2.2.32)$$

and

$$E_4 = \frac{(1-\sigma)\{\rho\bar{b}\beta(1-\hat{L}) + g\sigma\bar{b}\beta(1-\hat{L})\}}{\rho[\hat{L}^{\beta} + \bar{b}\beta X(1-\hat{L})] + g[\sigma\hat{L}^{\beta} + \sigma\bar{b}\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}$$

$$-\left(\frac{(1-\sigma)}{[\rho - g(1-\sigma)]} + \frac{(1-\sigma)[\sigma\hat{L}^{\beta} + \sigma\bar{b}\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}{\rho[\hat{L}^{\beta} + \bar{b}\beta X(1-\hat{L})] + g[\sigma\hat{L}^{\beta} + \sigma\bar{b}\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}\right)$$

$$. \quad \left(\frac{A^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}E_2}{\sigma\left[\hat{L}^{\beta} + \bar{b}(1-\hat{L})\beta X\right]^2}\right) \quad . \qquad (2.2.33)$$

We cannot sign $E_3$ and $E_4$ when $\sigma \neq 1$. In Chang et al. (2007), welfare effect of unionisation depends solely on the employment effect. However, our analysis shows that this is not necessarily true in the presence of optimal taxation to finance productive public expenditure as well as unemployment benefit policy. The following proposition summarises the major result.

**Proposition 2.2.3:** Unionisation raises (lowers) (does not affect) the level of employment when the labour union is employment oriented (wage oriented) (neutral). However, the growth effect of unionisation depends not only on the nature of orientation of the labour union but also on the negative taxation effect. An employment (wage or neutrally) oriented labour union is necessary but not sufficient (sufficient but not necessary) to have a positive (negative) growth effect.

Now we analyse the effect of unionisation on the optimal tax rate. From equation (2.2.19), we have

$$\left.\frac{\partial \tau^*}{\partial \theta}\right|_{b=\bar{b}} = \frac{\alpha \hat{L}\bar{b}(1-\alpha)\begin{Bmatrix} m(1-\beta)\beta\left(1-\hat{L}\right)[1-\theta+\theta n] \\ -(n-m)[\theta n+\beta(1-\theta)]\left[\hat{L}+\beta\left(1-\hat{L}\right)\right]\end{Bmatrix}}{\begin{matrix}\{\theta n+\beta(1-\theta)-\theta m(1-\beta)\} \\ \left\{\hat{L}[1-\theta+\theta n]+\bar{b}(1-\alpha)\left(1-\hat{L}\right)[\theta n+\beta(1-\theta)]\right\}^2\end{matrix}} \quad . \tag{2.2.34}$$

The two terms of the denominator and the first term of the numerator in the R.H.S. of equation (2.2.34) are positive in sign but the sign of the second term of the numerator depends on the nature of the orientation of the labour union. This equation (2.2.34) shows that an increase in $\theta$ leads to an increase (ambiguous change) in the optimal tax rate when the labour union is wage oriented or neutral (employment oriented).

The optimum tax rate and the level of employment are inversely related. As the labour union becomes more powerful, then the negotiated wage rate and hence the unemployment benefit per worker are increased. This requires an increase in the optimum tax rate to finance the additional unemployment benefit. However, it may raise or lower the employment level depending on labour unions' orientation; and thus it may affect the optimum tax rate ambiguously. The level of employment is decreased when the labour union is wage oriented. Models available in the existing literature do not incorporate the role of productive public input and of unionised labour market simultaneously; and hence the question of the effect of unionisation on optimum taxation does not arise there.[25] The innovative result is stated in the following proposition.

**Proposition 2.2.4:** Unionisation in the labour market raises optimal tax rate if the labour union is wage oriented or neutral. Otherwise, unionisation affects optimal tax rate ambiguously.

## 2.3 The 'Right to Manage' Model

In the 'Right to Manage' model of bargaining, firm's association and labour union bargain over wage only; and employment is determined from the labour demand function derived from the profit maximisation exercise of the firm. The inverted labour demand function is given by

$$w = (1-\tau)\beta A K^{\alpha} G^{1-\alpha} L^{\beta-1} = w_c L^{\beta-1} \quad . \tag{2.3.1}$$

Using equations (2.3.1) and (2.2.7), we have

$$\pi = (1-\tau)(1-\alpha-\beta) A K^{\alpha} L^{\beta} G^{1-\alpha} \quad . \tag{2.3.2}$$

So the 'generalised Nash product' function is modified as

$$\psi = \{(w-w_c)^m L^n\}^{\theta}\{(1-\tau)(1-\alpha-\beta)AK^{\alpha}L^{\beta}G^{1-\alpha}\}^{(1-\theta)} \quad . \tag{2.3.3}$$

Here $\psi$ is to be maximised with respect to *w* only, subject to equation (2.3.1). Since equation (2.3.1) implies an inverse relationship between *w* and *L*, one can maximise $\psi$ with respect to *L* instead of *w* subject to equation (2.3.1). From the first order condition of maximisation, we derive the level of employment and negotiated wage rate as given by[26]

$$L^* = \left\{\frac{\{\theta n + \beta(1-\theta) - \theta m(1-\beta)\}}{\{\theta n + \beta(1-\theta)\}}\right\}^{\frac{1}{1-\beta}} \quad ; \tag{2.3.4}$$

and

$$w = w_c (L^*)^{\beta-1} \quad . \tag{2.3.5}$$

Condition 2.A ensures that the negotiated wage rate is positive and the level of employment is a positive fraction.

Negotiated wage rates are same in both these two bargaining models; and this can be checked easily using equations (2.2.5), (2.2.10), (2.3.4) and (2.3.5).

---

[25] Since Raurich and Sorolla (2003) and Kitaura (2010) consider monopoly labour union and Chang and Chang (2014) do not find optimum income tax rates on wage income and on capital income, so analysing the effect of unionisation on the optimum tax rate is not possible in these three works.

[26] Second order condition of maximisation of $\psi$ is also satisfied.

Government's budget balance equation is same as equation (2.2.12). Representative household's optimisation problem is also represented by equations (2.2.13) and (2.2.14). Solving this dynamic optimisation problem, we obtain the similar expression of growth rate as given by equation (2.2.15). Using equations (2.2.1), (2.2.5), (2.2.12), (2.2.16) and (2.3.5), we obtain

$$\frac{G}{K} = \left\{A\tau L^{\beta} - Ab(1-L)\beta L^{\beta-1}(1-\tau)\right\}^{\frac{1}{\alpha}} \quad . \tag{2.3.6}$$

Using equations (2.3.6), (2.2.3) and (2.2.15) we obtain

$$g = \frac{\dot{c}}{c} = \frac{A^{\frac{1}{\alpha}}(1-\tau)\alpha {L^*}^{\beta}\left\{\tau {L^*}^{\beta} - b(1-L^*)\beta {L^*}^{\beta-1}(1-\tau)\right\}^{\frac{1-\alpha}{\alpha}} - \rho}{\sigma} \quad . \tag{2.3.7}$$

Government chooses the tax rate to maximise the growth rate of consumption. Assuming an interior solution, we derive the following optimal tax rate

$$\bar{\tau} = \frac{(1-\alpha)L^* + b(1-L^*)\beta}{L^* + b(1-L^*)\beta} = 1 - \frac{\alpha L^*}{L^* + b(1-L^*)\beta} \quad . \tag{2.3.8}$$

From equations (2.2.1), (2.2.2), (2.2.3), (2.2.5), (2.2.14), (2.2.15), (2.2.16), (2.2.21) and (2.3.5), we obtain

$$\widehat{\omega} = \frac{{K_0}^{1-\sigma}\left\{\frac{\rho + g\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - g\right\}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \tag{2.3.9}$$

Like equation (2.2.22), equation (2.3.9) also shows that there exists a positive monotonic relationship between the welfare level and the growth rate since if $\frac{\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] > 1$; and this is always true for $\sigma > \alpha$. Since $L^*$ is independent of tax rate, so the growth rate maximising tax rate is identical to the social welfare maximising tax rate.

Equation (2.3.7) shows that $g$ varies inversely with $b$. However, equation (2.3.9) shows that there may exist a welfare maximising interior solution of $b$. So propositions 1 and 2 are valid here too.

From equation (2.3.4), we have

$$\frac{\partial L^*}{\partial \theta} = -\frac{\beta m L^*}{[\theta n + \beta(1-\theta)]\{\theta n + (1-\theta)\beta - \theta m(1-\beta)\}} < 0 \quad . \tag{2.3.10}$$

This equation (2.3.10) implies that an increase in $\theta$ unambiguously lowers $L^*$ for any set of values of parameters $m$ and $n$. This is contrary to the corresponding result obtained in the earlier model where the nature of the effect depends on the mathematical sign of $(n-m)$. This is so because, in the 'Right to Manage' model, two parties bargain only over wage rate and the employer solely determines the level of employment according to its downward sloping labour demand function. Now, a rise in the relative bargaining power of labour union leads to a rise in the bargained wage rate and this leads to a fall in the level of employment.

Now using equations (2.3.7) and (2.3.8), we obtain

$$g|_{b=\bar{b}} = \frac{\dot{c}}{c} = \frac{A^{\frac{1}{\alpha}}\alpha^2 L^{*\frac{(\beta+\alpha)}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\left[L^* + \bar{b}(1-L^*)\beta\right]\sigma} - \frac{\rho}{\sigma} \quad . \tag{2.3.11}$$

From equation (2.3.11), we have

$$\left.\frac{\partial g}{\partial \theta}\right|_{b=\bar{b}} = \frac{A^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\sigma}\left(\frac{\left[\frac{\beta}{\alpha}L^{*\frac{\beta}{\alpha}}\{L^* + \bar{b}(1-L^*)\beta\} + \bar{b}\beta L^{*\frac{\beta}{\alpha}}\right]}{\left[L^* + \bar{b}(1-L^*)\beta\right]^2}\right)\frac{\partial L^*}{\partial \theta} \quad . \tag{2.3.12}$$

Since all the terms of the right hand side of equation (2.3.12) are positive and $\frac{\partial L^*}{\partial \theta} < 0$, so equation (2.3.12) implies that unionisation unambiguously lowers the growth rate of the economy for any set of values of *m* and *n* whereas the sign of the effect in the 'Efficient Bargaining' model depends partly on the sign of $(n-m)$. The intuition for different growth effects of unionisation in these two bargaining models is described as follows. As shown in the subsection 2.2.9, the growth effect of unionisation consists of employment effect and negative tax effect. The ambiguous employment effect in the 'Efficient Bargaining' model leads to the ambiguity in the growth effect of unionisation in this model. However, in the 'Right to Manage' model, both effects are strictly negative; and as a result, the combined growth effect is also strictly negative.

To analyse its welfare effect, once again we assume that $\frac{\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] > 1$. So from equation (2.3.9), we obtain

$$\left.\frac{\partial \widehat{\omega}}{\partial \theta}\right|_{b=\bar{b}} = \widehat{\omega} \cdot \left(\frac{\partial g}{\partial \theta}\left[\frac{(1-\sigma)}{[\rho - g(1-\sigma)]} + \frac{(1-\sigma)\left\{\frac{\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - 1\right\}}{\left\{\frac{\rho + g\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - g\right\}}\right] - \frac{(1-\sigma)\left(\frac{\rho + g\sigma}{\alpha}\right)\frac{b\beta}{L^{*2}}}{\left\{\frac{\rho + g\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - g\right\}}\frac{\partial L^*}{\partial \theta}\right) \quad . \tag{2.3.13}$$

Equation (2.3.13) shows that welfare effect of unionization is independent of labour union's orientation towards wage or employment which is not true in the 'Efficient Bargaining' model. However, in this model also, welfare effect of unionisation is ambiguous.

Equations (2.3.8) and (2.3.10) show the inverse relationship between $\bar{\tau}$ and $L^*$ and the inverse relationship between $L^*$ and $\theta$ respectively. So there is a positive relationship between $\bar{\tau}$ and $\theta$. This result is also different from the corresponding one obtained in the 'Efficient Bargaining' model where the result depends on labour union's orientation. The reason for this difference in optimum tax effect of unionisation in two bargaining models is described as follows. As stated in the subsection 2.2.9, unionisation affects optimum tax rate through two different channels. Unionisation always raises unemployment benefit per worker, and as a result, this channel always generates a positive effect. However, contrary to 'Efficient Bargaining' model, unionisation always lowers employment in the 'Right to Manage' model; and as a result, the other channel also generates a positive effect. So, ambiguity in the optimum tax effect of unionisation does not arise here.

We now state the major result in the form of the following proposition.

**Proposition 2.3.1:** In the 'Right to Manage' model, unionisation always lowers the level of employment as well as the rate of endogenous growth but raises the optimal tax rate. However, the welfare effect of unionisation, though independent of union's orientation, is ambiguous in sign.

## Appendix

### Appendix 2.A

Derivation of equations (2.2.9) and (2.2.10):

From equations (2.2.8) and (2.2.5), we have

$$log\psi = \theta m \, log(w - w_c) + \theta n \, logL$$
$$+ (1-\theta) \, log\{(1-\tau)(1-\alpha)AK^{\alpha}L^{\beta}G^{1-\alpha} - wL\} \quad . \qquad (2.A.1)$$

The first order optimality conditions of maximization of $log\psi$ with respect to $w$ and $L$ are given by

$$\frac{\theta m}{w - w_c} + \frac{(1-\theta)(-L)}{\pi} = 0 \quad ; \qquad (2.A.2)$$

and

$$\frac{\theta n}{L} + \frac{(1-\theta)\{\frac{(1-\tau)(1-\alpha)\beta Y}{L} - w\}}{\pi} = 0 \quad . \qquad (2.A.3)$$

Using equations (2.A.2) and (2.A.3), we obtain

$$(m-n)w = m(1-\tau)(1-\alpha)\beta\frac{Y}{L} - nw_c \quad . \qquad (2.A.4)$$

Using equations (2.2.1), (2.2.5), (2.2.7), (2.A.2) and (2.A.4) we obtain equation (2.2.9) in the body of the chapter. Using equations (2.2.1), (2.2.7), (2.2.9) and (2.A.3) we obtain equation (2.2.10) in the body of the chapter.

Second order conditions:

From equations (2.A.2) and (2.A.3), we obtain respectively

$$\frac{\partial^2 log\psi}{\partial w^2} = -\frac{\theta m}{(w-w_c)^2} - \frac{(1-\theta)L^2}{\pi^2} < 0 \quad ; \qquad (2.A.5)$$

and

$$\frac{\partial^2 log\psi}{\partial L^2} = -\frac{\theta n}{L^2} - \frac{(1-\theta)}{\pi}\left[(1-\tau)(1-\alpha)\beta(1-\beta)\frac{Y}{L^2}\right]$$
$$-\frac{(1-\theta)}{\pi^2}\left[(1-\tau)(1-\alpha)\beta\frac{Y}{L} - w\right]^2 < 0 \quad . \qquad (2.A.6)$$

Again from equation (2.A.2), we have

$$\frac{\partial^2 log\psi}{\partial L\partial w} = -\frac{(1-\theta)}{\pi^2}\{(1-\tau)(1-\alpha)AK^{\alpha}L^{\beta}G^{1-\alpha}(1-\beta)\} \quad . \qquad (2.A.7)$$

From equations (2.2.7), (2.2.1), (2.2.5), (2.2.9) and (2.2.10), we have

$$\pi^2 = {w_c}^2 L^2 \left\{\frac{(1-\beta)(1-\theta)}{\{\theta n + \beta(1-\theta) - \theta m(1-\beta)\}}\right\}^2 \quad . \qquad (2.A.8)$$

Using equations (2.2.5), (2.2.9), (2.A.7) and (2.A.8), we have

$$\frac{\partial^2 log\psi}{\partial L\partial w} = -\frac{[1-\theta+\theta n]\{\theta n + \beta(1-\theta) - \theta m(1-\beta)\}}{(1-\beta)Lw_c(1-\theta)} \quad . \qquad (2.A.9)$$

From equations (2.2.5) and (2.2.10), we have

$$w - w_c = w_c\left\{\frac{\theta m(1-\beta)}{\theta n + \beta(1-\theta) - \theta m(1-\beta)}\right\} \quad . \qquad (2.A.10)$$

Using equations (2.A.5), (2.A.8) and (2.A.10), we have

$$\frac{\partial^2 log\psi}{\partial w^2} = -\frac{\{\theta n + \beta(1-\theta) - \theta m(1-\beta)\}^2}{{w_c}^2(1-\beta)^2\theta m(1-\theta)}[1-\theta+\theta m] \quad . \qquad (2.A.11)$$

From equations (2.A.6), (2.2.1), (2.2.5), (2.A.8) and (2.2.10), we have

$$\frac{\partial^2 log\psi}{\partial L^2} = -\frac{1}{L^2}\left[\frac{(1-\theta+\theta n)[\theta n + \beta(1-\theta)]}{(1-\theta)}\right] \quad . \qquad (2.A.12)$$

Now using equations (2.A.9), (2.A.11) and (2.A.12), we have

$$\Rightarrow \left\{\frac{\partial^2 log\psi}{\partial w^2}\right\}.\left\{\frac{\partial^2 log\psi}{\partial L^2}\right\} - \left\{\frac{\partial^2 log\psi}{\partial L\partial w}\right\}^2 = \frac{\{\theta n + \beta(1-\theta) - \theta m(1-\beta)\}^2(1-\theta+\theta n)}{L^2{w_c}^2(1-\beta)^2(1-\theta)^2}$$

$$.\left\{\frac{[1-\theta+\theta m][\theta n + \beta(1-\theta)]}{\theta m} - [1-\theta+\theta n]\right\} \quad . \qquad (2.A.13)$$

Here, by assumption, $\theta n + \beta(1-\theta) - \theta m(1-\beta) > 0$ .

$$\Rightarrow \frac{(1-\theta+\theta m)[\theta n + \beta(1-\theta)]}{\theta m} > [\theta n + (1-\theta)] \quad . \qquad (2.A.14)$$

Equation (2.A.13) and inequality (2.A.14) imply that

$$\left\{\frac{\partial^2 log\psi}{\partial w^2}\right\}.\left\{\frac{\partial^2 log\psi}{\partial L^2}\right\} - \left\{\frac{\partial^2 log\psi}{\partial w\partial L}\right\}^2 > 0 \quad .$$

Derivation of equation (2.2.15):

Using equations (2.2.13) and (2.2.14), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda[wL + rK + \pi + s(1-L) - c] \quad . \qquad (2.A.15)$$

Here $\lambda$ is the co-state variable. Maximising equation (2.A.15) with respect to $c$, we obtain the following first order condition.

$$c^{-\sigma} - \lambda = 0 \quad ; \tag{2.A.16}$$

Again from equation (2.A.15), we have

$$\frac{\dot{\lambda}}{\lambda} = \rho - r \quad ; \tag{2.A.17}$$

and from equation (2.A.16), we have

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{c}}{c} \quad . \tag{2.A.18}$$

Using equations (2.A.17) and (2.A.18), we have equation (2.2.15) in the body of the chapter.

<u>Derivation of equation (2.2.22):</u>

From equation (2.2.21), we obtain

$$\widehat{\omega} = \frac{{c_0}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \tag{2.A.19}$$

Here, $c(0) = c_0$.

From equations (2.2.2), (2.2.16), (2.2.14), (2.2.1) and (2.2.10), we obtain

$$c_0 = K_0\left\{(1-\tau)A\hat{L}^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\alpha} + (1-\tau)b\beta X(1-\hat{L})A\left(\frac{G_0}{K_0}\right)^{1-\alpha} - g\right\} \quad . \tag{2.A.20}$$

Using equations (2.2.3) and (2.2.15), we obtain

$$\frac{\rho + \sigma g}{\alpha\hat{L}^{\beta}} = \left(\frac{G_0}{K_0}\right)^{1-\alpha}(1-\tau)A \quad . \tag{2.A.21}$$

Using equations (2.A.20) and (2.A.21), we obtain

$$c_0 = K_0\left\{\frac{\rho\left[\hat{L}^{\beta} + b\beta X(1-\hat{L})\right] + g\left[\sigma\hat{L}^{\beta} + \sigma b\beta X\left(1-\hat{L}\right) - \alpha\hat{L}^{\beta}\right]}{\alpha\hat{L}^{\beta}}\right\} \quad . \tag{2.A.22}$$

Using equations (2.A.19) and (2.A.22), we obtain

$$\widehat{\omega} = \frac{{K_0}^{1-\sigma}\left\{\frac{\rho\left[\hat{L}^{\beta} + b\beta X\left(1-\hat{L}\right)\right] + g\left[\sigma\hat{L}^{\beta} + \sigma b\beta X\left(1-\hat{L}\right) - \alpha\hat{L}^{\beta}\right]}{\alpha\hat{L}^{\beta}}\right\}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \tag{2.A.23}$$

Equation (2.A.23) is identical to the equation (2.2.22) in the body of the chapter.

Derivations of equations in section 2.3 are similar to that in section 2.2.

# Chapter 3: Difference in Production Technology

## 3.1 Introduction

In the earlier chapter, we have assumed that final good and public investment good are produced with same production - technology. In this chapter, we attempt to analyse the properties of optimal income tax rate used to finance investment in public capital in a two sector economy with different production functions for producing final good and for producing public investment good. In this model, the private sector produces public investment good and sells it to the government who has a monopsony power to set the buying price. Thus this price is also used to control allocation of resources between these two sectors. Otherwise, our model has a framework similar to what Futagami et al. (1993) model has. In the basic model, we follow Barro (1990) and Futagami et al. (1993) to assume a competitive labour market with full employment equilibrium. However, in the extended model, we consider a unionized labour market with unemployment equilibrium.

We derive many interesting results from the basic model. First of all, the growth rate maximising income tax rate is equal to the elasticity of output with respect to public capital in the production of final goods only but is independent of the production technology to produce public investment good. Secondly, welfare maximising solutions are different from growth rate maximising solutions even in the steady state growth equilibrium. These results are different from the corresponding results obtained from Barro (1990), Futagami et al. (1993) etc.

A few interesting results are also obtained after introducing unionisation and unemployment. First, economic growth rate is always higher in the case of competitive labour markets because marginal productivity of capital varies positively with the level of employment and unionisation creates unemployment. Finally, the steady state equilibrium growth rate maximising allocation of private capital as well as the steady state equilibrium growth rate maximising income tax rate are independent of unionisation in the labour market.

Rest of the chapter is organized as follows. Section 3.2 describes the basic model with competitive labour market. Subsection 3.2.1 describes the structure of the model; and subsection 3.2.2 deals with the properties of steady state growth equilibrium and growth rate maximizing policies in the basic model. Subsection 3.2.3 compares between growth rate maximizing fiscal policies and optimal (welfare maximizing) fiscal policies in the steady state equilibrium. Section

3.3 extends the basic model with unionized labour market and compares its results to those in the basic model.

## 3.2 The Basic Model

### 3.2.1 Structure of the Model

The representative household-producer produces both final good and public investment good using private capital and public capital. Public investment good is defined as the additional stock of non-rival public capital. Production functions of two sectors with different technologies are given by

$$Y = A(\phi K)^{\alpha} G^{1-\alpha} \quad \text{where} \quad \alpha \in (0,1) \quad \text{and} \quad A > 0 \quad ; \tag{3.2.1}$$

and

$$\dot{G} = B[(1-\phi)K]^{\beta} G^{1-\beta} \quad \text{where} \quad \beta \in (0,1) \quad \text{and} \quad B > 0 \quad . \tag{3.2.2}$$

Here, $Y$, $K$, $G$ and $\phi$ denote level of output of final good, stock of private capital, stock of public capital and the share of private capital allocated to final goods sector respectively. $\dot{G}$ represents the level of output of public investment good. The government sets the relative price of $\dot{G}$; and the household–producer determines the allocation of resources between two sectors. Public capital does not depreciate over time.

The government buys all $\dot{G}$ at the relative price, $\mu$; and freely provides the whole stock of $G$ to the household-producers. An income tax at the rate, $\tau$, is charged to finance the production of public investment good; and the balanced budget equation is given by

$$\tau Y + \tau\mu\dot{G} = \mu\dot{G} \ . \tag{3.2.3}$$

The representative household is infinitely lived; and she derives instantaneous utility from consumption of final goods only; and maximizes her discounted present value of instantaneous utility subject to her intertemporal budget constraint. The optimization problem is given by the following.

$$Max \int_0^{\infty} \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{3.2.4}$$

subject to, $\dot{K} = (1-\tau)Y + (1-\tau)\mu\dot{G} - c \quad ;$ (3.2.5)

$K(0) = K_0 \quad ;$

and $\phi \in [0\,,1]$ .

Here $c$ is the level of consumption of the final good and $K_0$ is historically given initial private capital stock. $\sigma$ represents the elasticity of marginal utility with respect to consumption and $\rho$ denotes the constant rate of discount. Savings is always invested; and there is no depreciation of private capital.

Here $c$ and $\phi$ are two control variables and $K$ is the only state variable. Solving this dynamic optimisation problem, we obtain[27]

$$(1-\tau)A\alpha\phi^{\alpha-1}K^{\alpha}G^{1-\alpha} = \mu(1-\tau)B\beta(1-\phi)^{\beta-1}K^{\beta}G^{1-\beta} \quad ; \qquad (3.2.6)$$

and

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A\alpha\phi^{\alpha}K^{\alpha-1}G^{1-\alpha} + \mu(1-\tau)B\beta(1-\phi)^{\beta}K^{\beta-1}G^{1-\beta} - \rho}{\sigma} \quad . \qquad (3.2.7)$$

Equation (3.2.6) shows the efficient allocation of private capital between the two sectors. It implies that the after tax value of the marginal product of private capital is same in both these two sectors. Equation (3.2.7) describes the demand rate of growth of consumption which is defined as the excess of after tax marginal return of private capital over the rate of discount normalized with respect to the elasticity of marginal utility.

### 3.2.2 The Steady State Equilibrium

The equations of motion of the system are given by equations (3.2.2), (3.2.5) and (3.2.7). In the steady-state growth equilibrium,

$$g = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{c}}{c} \quad , \qquad (3.2.8)$$

where $g$ is the balanced growth rate of the economy.

Using equations (3.2.1), (3.2.2) and (3.2.3), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\mu B(1-\phi)^{\beta}}{A\phi^{\alpha}}\left(\frac{K}{G}\right)^{\beta-\alpha} \quad . \qquad (3.2.9)$$

Using equations (3.2.2) and (3.2.8), we have

$$\left(\frac{G}{K}\right) = \frac{(1-\phi)B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}} \quad . \qquad (3.2.2a)$$

[27] Derivation of equations (3.2.6) and (3.2.7) are shown in the appendix 3.A.

Using equations (3.2.2a) and (3.2.6), we have

$$\frac{(1-\phi)^{1-\alpha}}{\phi^{1-\alpha}} = \frac{\mu\beta B^{\frac{\alpha}{\beta}}}{A\alpha} g^{\frac{\beta-\alpha}{\beta}} \quad . \tag{3.2.10}$$

This equation (3.2.10) shows that the allocation share of private capital to the final goods sector, $\phi$, varies inversely with the buying price of public investment good, $\mu$, for a given balanced growth rate, $g$. However, $g$ is not given in this model. It is endogenously determined.

Now using equations (3.2.2), (3.2.6), (3.2.7), (3.2.8) and (3.2.9), we have[28]

$$\rho + \sigma g = \frac{\beta B^{\frac{1}{\beta}} \mu g^{\frac{\beta-1}{\beta}}}{1+\alpha\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \quad . \tag{3.2.11}$$

Equation (3.2.11) solves for the balanced growth rate, $g$, and this solution is unique. This equation also shows the nature of the relationship between the buying price of the public investment good, $\mu$, and the balanced growth rate, $g$.

Hence equations (3.2.10) and (3.2.11) show that a change in $\mu$ has two different effects on $\phi$ in the steady – state equilibrium. First, it has a direct negative effect obtained for a given value of $g$. Secondly, it has an ambiguous indirect effect which works through change in $g$ via equation (3.2.11).

Now using equations (3.2.2a), (3.2.9) and (3.2.10), we find that

$$\left(\frac{\tau}{1-\tau}\right) = \mu^{\frac{1}{1-\alpha}} g^{\frac{\beta-\alpha}{\beta(1-\alpha)}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{B^{\frac{\alpha}{\beta}}}{A}\right)^{\frac{1}{1-\alpha}} \quad . \tag{3.2.9a}$$

Equations (3.2.9a) and (3.2.11) simultaneously show how a change in the buying price of public investment good, $\mu$, affects the tax rate, $\tau$, in the steady – state equilibrium. This change in $\mu$ has a direct positive effect obtained for a given growth rate, $g$, and an ambiguous indirect effect working through change in $g$ via equation (3.2.11). The final effect will depend on the relative strength of these direct and indirect effects and also on the nature of capital – intensity ranking between the two sectors, i.e., on the mathematical sign of ($\alpha$ - $\beta$).

[28] Derivation of equation (3.2.11) is shown in appendix 3.A.

However, $\mu$ is not a parameter in this model. $\mu$ is an instrument to solve the optimisation problem of the government. Ideally, the government's objective should be to maximise the welfare level of the representative household, $\omega$, given by

$$\omega = \underset{\{c\}}{Max} \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\, dt \qquad . \qquad (3.2.12)$$

Unfortunately, we cannot solve for the welfare maximising buying price of the public investment good due to technical complications. Rather, we solve for its steady-state equilibrium growth rate maximising solution in this subsection; and examine, in the next subsection, whether it deviates from its welfare maximising solution. Now we maximize $g$ given by equation (3.2.11) with respect to $\mu$; and, using the first order condition, we obtain the following.[29]

$$\mu = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} g^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} \qquad . \qquad (3.2.13)$$

Using equations (3.2.11) and (3.2.13), we have

$$(\rho + \sigma g) g^{\frac{1-\alpha}{\beta}} = A(1-\alpha)^{1-\alpha} \beta^{1-\alpha} \alpha^{2\alpha} B^{\frac{1-\alpha}{\beta}} \qquad . \qquad (3.2.14)$$

Equation (3.2.14) solves for the maximum value of $g$, which is the endogenous rate of growth of the economy in the steady-state equilibrium.

Denoting this maximum value of $g$ by $g^*$ and putting it in equation (3.2.13), we obtain[30]

$$\mu^* = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} (g^*)^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} \qquad . \qquad (3.2.15)$$

This equation (3.2.15) shows the steady state equilibrium growth rate maximising buying price of the public investment good.

Using equations (3.2.2), (3.2.6), (3.2.9) and (3.2.15), we obtain

$$\phi^* = \frac{\alpha^2}{\alpha^2 + \beta(1-\alpha)} \qquad ; \qquad (3.2.16)$$

and

$$\tau^* = 1 - \alpha \qquad . \qquad (3.2.17)$$

[29] Derivation of equation (3.2.13) is shown in the appendix 3.A.

[30] The second order condition of maximisation of growth rate with respect to $\mu$ is satisfied. From equation (3.2.11), it can be shown very easily that $\frac{\partial^2 g}{\partial \mu^2} < 0$ when equation (3.2.13) holds.

Here $\phi^*$ represents the growth rate maximising allocation of private capital to the final goods producing sector in the steady state growth equilibrium. Equation (3.2.16) shows that $\phi^*$ varies inversely with $\beta$ and positively with $\alpha$. This is so because, as $\beta$ ($\alpha$) rises, productivity of private capital rises in the public investment good (final good) sector relative to the other sector; and, as a result, allocative share of private capital to public investment good (final good) sector goes up. In the case of identical production technology, $\phi^* = \alpha$.

Equation (3.2.17) does not involve $\beta$. So this leads to the following proposition.

**Proposition 3.2.1:** The steady state equilibrium growth rate maximising income tax rate is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology in the public investment good producing sector.

Public investment good sector uses public capital as input only to produce additional public capital. There exists only one final good sector to receive the service of public capital free of cost. If there is exchange, it is optimal for the final good sector to buy public investment good at the competitive price. So in the absence of exchange, it is optimal to charge a tax rate which is equal to the competitive output share of public capital in the final good sector.

In Barro (1990) and Futagami et al. (1993), input elasticities of output are same in both the sectors. So this problem does not arise.

### 3.2.3 Welfare Maximization

In this subsection, we examine whether the growth rate maximising buying price of public investment good is identical with the welfare maximising buying price of public investment good. We use equations (3.2.1), (3.2.2), (3.2.5), (3.2.6), (3.2.7), (3.2.9) and (3.2.12) to obtain the welfare level of the representative household, denoted by $\omega$. This is identical to her discounted present value of instantaneous utilities over the infinite horizon. It is derived as[31]

[31] See appendix 3.A for derivation of equation (3.2.18).

$$\omega = \frac{\left[\frac{\rho}{\alpha} + g\left(\frac{\sigma}{\alpha} - 1\right) + \frac{(\alpha-\beta) g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right]^{1-\sigma}}{{K_0}^{\sigma-1}(1-\sigma)[\rho - g(1-\sigma)]}$$

$$+ \, constant \quad . \tag{3.2.18}$$

If $\sigma > \alpha$ and if $\rho - g(1-\sigma) > 0$, then equation (3.2.18) shows that $\omega$ varies positively with $g$ when $\alpha = \beta$. So the growth rate maximising solution is identical to the welfare maximising solution in the steady state equilibrium when $\alpha = \beta$, i.e., when production technologies are identical in these two sectors. However, when $\alpha \neq \beta$, i.e., when production technologies are not identical, then the welfare maximising solution is not identical to the growth rate maximising solution even in the steady state equilibrium. From equation (3.2.18), we differentiate $\omega$ with respect to $\mu$ and then evaluate it at $\mu = \mu^*$. Hence we obtain[32]

$$\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*} = \left\{\frac{\left[\frac{\rho}{\alpha} + g^*\left(\frac{\sigma}{\alpha} - 1\right) + \frac{A(\alpha-\beta) {g^*}^{\frac{\alpha-1}{\beta}} B^{\frac{1-\alpha}{\beta}} \beta^{1-\alpha}}{[\alpha^2 + \beta(1-\alpha)](1-\alpha)^{\alpha-2}\alpha^{1-2\alpha}}\right]^{-\sigma}}{{K_0}^{\sigma-1}[\rho - g^*(1-\sigma)]}\right\}$$

$$. \quad \left\{\frac{(\alpha-\beta){g^*}^{\frac{\beta-1}{\beta}} B^{\frac{1}{\beta}} \alpha^2 \beta}{[\alpha^2 + \beta(1-\alpha)]^2}\right\} \quad . \tag{3.2.19}$$

We assume $\sigma > \alpha$ and $\rho > g^*(1-\sigma)$. This ensures that the right hand side of equation (3.2.19) is positive (zero) (negative) when $\alpha$ > (=) (<) $\beta$[33]. This implies that the welfare maximising value of $\mu$ is higher (lower) than the growth rate maximising value of $\mu$ even in the steady state equilibrium when the final private good sector is more (less) private capital intensive than the public investment good sector. We refer welfare maximising $\mu$ as $\bar{\mu}$ .

Now, we compare growth rate maximising solutions $\tau^*$ and $\phi^*$ to welfare maximising solutions $\bar{\tau}$ and $\bar{\phi}$. When $\alpha > \beta$ , then $\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*}$ is positive and as a result, $\bar{\mu} > \mu^*$. So the growth

[32] See appendix 3.A for derivation of equation (3.2.19).

[33] When $\beta > \alpha$, then also the first term in the R.H.S. of equation (3.2.19) is positive as $c_0$ cannot be negative.

rate corresponding to $\bar{\mu}$, i.e., $\bar{g}$, is less than $g^*$ as $g^*$ is the maximum value of balanced growth rate. As a result, $\bar{\mu}\bar{g}^{\frac{\beta-\alpha}{\beta}} > \mu^* g^{*\frac{\beta-\alpha}{\beta}}$.

Using equations (3.2.2), (3.2.6) and (3.2.9), we obtain[34]

$$\phi = \frac{1}{1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}} \quad ; \tag{3.2.20}$$

and

$$\tau = \frac{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}} + \beta} < 1 \quad . \tag{3.2.21}$$

Since equations (3.2.20) and (3.2.21) show that $\phi$ and $\tau$ vary inversely and positively with $\mu g^{\frac{\beta-\alpha}{\beta}}$ respectively, so welfare maximising $\phi$, i.e., $\bar{\phi}$, is less than $\phi^*$ but welfare maximising $\tau$, i.e., $\bar{\tau}$, is higher than $\tau^*$. Similarly, when $\beta > \alpha$, then $\mu^* > \bar{\mu}$ and $g^* > \bar{g}$. So, $\bar{\mu}\bar{g}^{\frac{\beta-\alpha}{\beta}} < \mu^* g^{*\frac{\beta-\alpha}{\beta}}$; and as a result, $\bar{\phi}$ is greater than $\phi^*$ but $\bar{\tau}$ is less than $\tau^*$.

Barro (1990) and Futagami et al. (1993) show that growth rate maximising income tax rate is identical to the welfare maximising income tax rate in the steady state equilibrium. However, we find that the welfare maximising solution is different from the growth rate maximising solution even in the steady state equilibrium when we consider different production functions for different goods. However, two solutions are always identical with identical production technology. So our result generalises the result of Barro (1990) and Futagami et al. (1993). This result is stated in the following proposition.

**Proposition 3.2.2:** When the final good sector is more (less) private capital intensive than the public investment good sector, welfare maximising buying price of public investment good, income tax rate and the allocation share of private capital to the public investment good sector

[34] See appendix 3.A for derivation of equations (3.2.20) and (3.2.21).

exceeds (falls short of) their corresponding growth rate maximising values even in the steady state equilibrium.

### 3.3 Extension with Labour Union

In this section, we extend the basic model with unionised labour market and then analyse the effect of unionisation on the growth rate. Unionisation is defined as the transformation of a competitive labour market into a unionised labour market.[35] To introduce labour union, we incorporate sector specific labour inputs[36] in the production of each of the two goods. We also assume that production process is carried out by competitive firms rather than by household producers. So the modified production functions are given by

$$Y = A(\phi K)^{\alpha} G^{1-\alpha} N^{1-\alpha} \quad \text{where} \quad \alpha \in (0,1) \quad \text{and} \quad A > 0 \quad ; \qquad (3.3.1)$$

and

$$\dot{G} = B[(1-\phi)K]^{\beta} G^{1-\beta} S^{1-\beta} \quad \text{where} \quad \beta \in (0,1) \quad \text{and} \quad B > 0 \quad . \qquad (3.3.2)$$

Here $N$ and $S$ are sector specific labour inputs employed in the final good sector and in the public investment good sector respectively. We assume that endowments of these two types of labour inputs are fixed and are given by $\bar{N}$ and $\bar{S}$ respectively.

The input demand functions are obtained from profit maximisation exercise of firms; and they are given by

$$r = A\alpha(\phi K)^{\alpha-1} G^{1-\alpha} N^{1-\alpha} \quad ; \qquad (3.3.3)$$

$$r = \mu B\beta[(1-\phi)K]^{\beta-1} G^{1-\beta} S^{1-\beta} \quad ; \qquad (3.3.4)$$

$$w_N = (1-\alpha)A(\phi K)^{\alpha} G^{1-\alpha} N^{-\alpha} \quad ; \qquad (3.3.5)$$

and

$$w_S = \mu B(1-\beta)[(1-\phi)K]^{\beta} G^{1-\beta} S^{-\beta} \quad . \qquad (3.3.6)$$

Here $r$ stands for the rental rate on mobile private capital; and $w_N$ and $w_S$ represent wage rate in the final good sector and in the public investment good sector respectively. Now, we assume

[35] Unlike other chapters, here we do not consider bargaining between labour union and firms' association or the firm itself. As a result, here we cannot define unionisation as an increase in the relative bargaining power of the labour union.

[36] Our basic model as well as the model of Dasgupta (1999) assumes sector specific labour and normalises each type of labour endowment to unity. Perfectly flexible wage rate in each of the two competitive labour markets ensures full employment equilibrium. So the labour input does not explicitly enter into the production functions given by equations (3.2.1) and (3.2.2) of the basic model.

monopoly labour union[37]; and introduce it in both the labour markets. First, we consider the behaviour of the labour union in the labour market specific to the final goods sector. The closed shop labour union[38] tries to maximise its utility subject to the representative firm's labour demand function given by equation (3.3.5). The labour union takes care of members' interests as well as of leadership's interest. Members want to earn higher income and the leadership wants to increase the number of members. So the utility function of the labour union is given by[39]

$$u_{TN} = (w_N - w_{N_c})^{m_N} N^{n_N} \quad \text{with} \quad m_N, n_N > 0 \quad . \tag{3.3.7}$$

Here, $u_{TN}$ denotes the utility level of the labour union and $w_{N_c}$ is the competitive wage rate in the final good sector. So $(w_N - w_{N_c})$ is defined as the worker's income gain due to the existence of labour union. $m_N$ and $n_N$ represent elasticities of labour union's utility with respect to wage gain and with respect to the level of employment respectively. If $m_N$ > ( < ) ( = ) $n_N$, then the labour union is said to be "wage oriented" ("employment oriented") ("neutral"). The competitive wage rate, $w_{N_c}$, that would have prevailed in the absence of labour union, is given by

$$w_{N_c} = (1-\alpha)A(\phi K)^{\alpha} G^{1-\alpha} \overline{N}^{-\alpha} \quad . \tag{3.3.8}$$

Here the Right hand side of equation (3.3.8) represents the marginal productivity of labour in the final good sector in the presence of full employment. From the labour union's behaviour, i.e., from maximisation of equation (3.3.7), subject to equation (3.3.5), we obtain the equilibrium level of employment and the unionised wage rate in the final good sector, as given by

$$N^* = \overline{N}\left(\frac{n_N - m_N\alpha}{n_N}\right)^{\frac{1}{\alpha}} \quad ; \tag{3.3.9}$$

and

[37] Unlike other chapters, here we do not consider bargaining between the labour union and the representative of firms. We do so because, in other chapters, decreasing returns to private inputs in the production function is assumed such that positive profit remains after paying private inputs according to their marginal contributions. This positive profit is treated as the rent in the bargaining process. However, in those chapters, we consider one sector models such that the existence of a fixed factor such as land, which is essential for production process, is sufficient to fix the total number of firms even in the presence of positive profit. However, in this two sector model, this assumption can fix the total number of firms in the economy but cannot fix the number of firms in each sector. So any change in the economy will also alter the number of firms in each sector; and this will make the model analytically complicated. So to avoid the above mentioned problem and to keep our model simple, we assume constant returns to private inputs such that no positive profit remains after paying private inputs according to their marginal contributions. As a result, we do not consider bargaining models but consider more simple monopoly labour union model.

[38] In case of closed shop labour union, the number of union members is equal to the number of workers.

[39] Some models like Chang et al. (2007), Adjemian et al. (2010) etc. take the difference between bargained wage rate and the rate of unemployment benefit as the argument in the labour union's utility function. Contrary to this, the difference between bargained wage rate and competitive wage rate is used as an argument in the works of Irmen and Wigger (2002/2003), Lingens (2003a) and Lai and Wang (2010).

$$w_N{}^{*} = (1-\alpha)A(\phi K)^{\alpha}G^{1-\alpha}N^{*-\alpha} \quad . \tag{3.3.10}$$

We assume that the labour union is not very wage oriented, i.e., $n_N > m_N\alpha$; and hence equation (3.3.9) ensures positive employment level. Equation (3.3.9) also shows that this employment level varies positively (inversely) with $n_N$ ($m_N$) and is always less than the full employment level, $\bar{N}$. Equation (3.3.10) along with equation (3.3.8) shows that the unionised wage rate is higher than the competitive wage rate because labour demand function depicts an inverse relationship between wage rate and employment level.

Next we consider the behaviour of the labour union in the labour market specific to the public investment good producing sector. The utility function of the labour union is given by

$$u_{TS} = (w_S - w_{S_c})^{m_S}S^{n_S} \quad \text{with} \quad m_S, n_S > 0 \quad . \tag{3.3.11}$$

Here $u_{TS}$, $w_{S_c}$, $m_S$ and $n_S$ represent the utility level of the labour union, the competitive wage in this sector, elasticity of labour union's utility with respect to wage gain, and that with respect to the level of employment respectively. The competitive wage, $w_{S_c}$, is given by

$$w_{S_c} = \mu B(1-\beta)[(1-\phi)K]^{\beta}G^{1-\beta}\bar{S}^{-\beta} \quad . \tag{3.3.12}$$

So, from maximisation of equation (3.3.11) subject to equation (3.3.6), we obtain the equilibrium level of employment and the unionised wage rate in the public investment good producing sector; and they are given by

$$S^{*} = \bar{S}\left(\frac{n_S - m_S\beta}{n_S}\right)^{\frac{1}{\beta}} \quad ; \tag{3.3.13}$$

and

$$w_S{}^{*} = \mu B(1-\beta)[(1-\phi)K]^{\beta}G^{1-\beta}S^{*-\beta} \quad . \tag{3.3.14}$$

Here also, we assume, $n_S > m_S\beta$. So equations (3.3.13) and (3.3.14) also show that level of employment (wage rate) in the unionised labour market is lower (higher) than that in the competitive labour market.

Private capital market is competitive; and private capital is perfectly mobile between these two sectors. So the intersectoral allocation of private capital is determined from equalisation of Value of marginal productivity of Capital (VMPK) in these two sectors. It is given by

$$\frac{[1-\phi]^{1-\beta}}{(\phi)^{1-\alpha}} = \frac{\mu B\beta}{A\alpha}\left(\frac{G}{K}\right)^{\alpha-\beta}\frac{S^{1-\beta}}{N^{1-\alpha}} \quad . \tag{3.3.15}$$

In the case of competitive labour market in both the sectors, $S = \bar{S}$ and $N = \bar{N}$ and we come back to equation (3.2.6) of the basic model when $\bar{S} = \bar{N} = 1$.

The balanced budget equation of the government is same as given by equation (3.2.3). So, from equations (3.2.3), (3.3.1) and (3.3.2), we obtain

$$\tau A(\phi K)^{\alpha} G^{1-\alpha} N^{1-\alpha} = (1-\tau)\mu B[(1-\phi)K]^{\beta} G^{1-\beta} S^{1-\beta} \quad . \tag{3.3.16}$$

The representative household's utility function is same as given in subsection 3.2.1. Only her budget constraint is changed because she is no more a producer[40]. She plays the role of a factor owner. She earns rental income as well as wage income from both types of labour. So the optimization problem of the representative household is given by the following.

$$Max \int_0^{\infty} \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{3.3.17}$$

subject to, $\dot{K} = (1-\tau)rK + (1-\tau)w_N N + (1-\tau)w_S S - c$ ; (3.3.18)

and $K(0) = K_0$ .

Here $c$ is the only control variable and $K$ is the only state variable. Solving this dynamic optimisation problem, we obtain

$$\frac{\dot{c}}{c} = \frac{(1-\tau)r-\rho}{\sigma} \quad . \tag{3.3.19}$$

In the steady-state growth equilibrium,

$$g = \frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} \quad . \tag{3.3.20}$$

Using equations (3.3.2) and (3.3.20), we have

$$\left(\frac{G}{K}\right) = \left[\frac{B(1-\phi)^{\beta} S^{1-\beta}}{g}\right]^{\frac{1}{\beta}} \quad . \tag{3.3.21}$$

Using equations (3.3.3) and (3.3.19), we obtain

$$\rho + \sigma g = (1-\tau)A\alpha\phi^{\alpha-1}\left(\frac{G}{K}\right)^{1-\alpha} N^{1-\alpha} \quad . \tag{3.3.22}$$

Now using equations (3.3.15), (3.3.16) (3.3.21) and (3.3.22), we obtain

$$(\rho+\sigma g) g^{\frac{1-\alpha}{\beta}} = \frac{\beta A\alpha\phi^{\alpha}}{[\alpha(1-\phi)+\beta\phi]} B^{\frac{1-\alpha}{\beta}} (1-\phi)^{1-\alpha} S^{\frac{(1-\alpha)(1-\beta)}{\beta}} N^{1-\alpha} \quad . \tag{3.3.23}$$

Now, using equations (3.3.15), (3.3.21) and (3.3.23), we obtain

[40] She also does not determine the allocation of private capital as she is no more a producer. It is determined by the firms as shown by equation (3.3.15).

$$(\rho+\sigma g)g^{\frac{1-\beta}{\beta}}=\frac{\beta^{2}B^{\frac{1}{\beta}}S^{\frac{1-\beta}{\beta}}\mu A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}N}{\left[A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}N\beta+\alpha\mu^{\frac{1}{1-\alpha}}\beta^{\frac{1}{1-\alpha}}B^{\frac{\alpha}{(1-\alpha)\beta}}S^{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}}g^{\frac{\beta-\alpha}{\beta(1-\alpha)}}\right]} \quad . \quad (3.3.24)$$

So the growth rate maximising buying price of public investment good is obtained from maximising the R.H.S. of equation (3.3.24) with respect to $\mu$; and the solution is given by

$$\mu=\frac{A(1-\alpha)^{1-\alpha}N^{1-\alpha}}{\alpha^{1-2\alpha}B^{\frac{\alpha}{\beta}}g^{\frac{\beta-\alpha}{\beta}}\beta^{\alpha}S^{\frac{\alpha(1-\beta)}{\beta}}} \quad . \qquad (3.3.25)$$

Equation (3.3.25) shows that this growth rate maximising $\mu$ becomes identical to that given by equation (3.2.15) if we assume $N = S = 1$. This expression also shows that the growth rate maximising $\mu$, given by equation (3.3.25), varies positively with $N$ and inversely with $S$ for a given balanced growth rate, $g$. However, $g$ is not given in this model. It is endogenously determined.

Now incorporating the growth rate maximising value of $\mu$ from equation (3.3.25) in the equation (3.3.24), we obtain

$$(\rho+\sigma g)g^{\frac{1-\alpha}{\beta}}=\beta^{1-\alpha}B^{\frac{1-\alpha}{\beta}}N^{1-\alpha}\alpha^{2\alpha}(1-\alpha)^{(1-\alpha)}AS^{\frac{(1-\alpha)(1-\beta)}{\beta}} \quad . \qquad (3.3.26)$$

Equation (3.3.26) solves for the maximum value of $g$, which is the endogenous rate of growth of the economy in the steady-state equilibrium. Here the L.H.S. of equation (3.3.26) is a positive function of $g$. Also $0 < \alpha$ , $\beta < 1$. Hence equation (3.3.26) shows that $g$ varies positively with $N$ and / or $S$. Hence the economic growth rate, $g$, varies positively with the level of employment of both types of labour. So economic growth rate is higher with competitive labour markets than that with unionised labour markets in any sector because full employment is attained in the competitive labour market.

Now, using equations (3.3.15), (3.3.16), (3.3.21) and (3.3.25), we obtain the steady state growth rate maximising allocation of private capital to the final goods producing sector and the steady state growth rate maximising income tax rate respectively. They are identical to those obtained in the basic model and are given by equations (3.2.16) and (3.2.17) respectively. So unionisation in the labour market does not affect the steady state growth rate maximising allocation of private capital between two production sectors as well as the steady state growth rate maximising income tax rate. We summarise these results in the following proposition.

**Proposition 3.3.1:** The endogenous rate of growth is higher with competitive labour markets than that with unionised labour markets in any sector. However, the steady state growth rate maximising allocation of private capital as well as the steady state growth rate maximising income tax rate are independent of unionisation in the labour market.

In this extended model, each of the two sector specific labour inputs enters into the corresponding production function multiplicatively. So marginal productivity of capital varies positively with the level of employment in each of these two sectors. Unionisation in the labour market lowers level of employment which, in turn, lowers marginal productivity of capital in both the sectors as well as the balanced growth rate. Since labour inputs enter production functions in a multiplicative way, they also enter the growth equation (given by equation (3.3.23)) in a multiplicative way; and, as a result, unionisation does not affect the growth rate maximising value of intersectoral allocation of private capital. So the growth rate maximising value of buying price of public investment good adjusts with unionisation in such a way that the growth rate maximising allocation of private capital between two sectors remains unchanged. Unionisation also does not alter technology of production in two sectors; and, as a result, the growth rate maximising rate of income tax used to finance investment in public good remains unaffected by unionisation in the labour market and also remains equal to the elasticity of output of final good with respect to public capital.

## Appendix

### Appendix 3.A

Derivation of equations (3.2.6) and (3.2.7):

Using equations (3.2.4) and (3.2.5), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda\big[(1-\tau)Y + (1-\tau)\mu\dot{G} - c\big] \quad . \qquad (3.A.1)$$

Here $\lambda$ is the co-state variable. Incorporating equations (3.2.1) and (3.2.2) in equation (3.A.1); and then maximising it with respect to $c$ and $\phi$, we obtain following first order conditions.

$$c^{-\sigma} - \lambda = 0 \quad ; \qquad (3.A.2)$$

and

$$\lambda(1-\tau)A(K)^{\alpha}G^{1-\alpha}\alpha\phi^{\alpha-1} = \lambda\mu(1-\tau)B[K]^{\beta}G^{1-\beta}\beta(1-\phi)^{\beta-1} \quad . \qquad (3.A.3)$$

From equation (3.A.3), we obtain equation (3.2.6) in the body of the chapter.

Again from equation (3.A.1), we have

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau)AK^{\alpha-1}G^{1-\alpha}\alpha\phi^{\alpha} - \mu(1-\tau)BK^{\beta-1}G^{1-\beta}\beta(1-\phi)^{\beta} \quad ; \qquad (3.A.4)$$

and from equation (3.A.2), we have

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{c}}{c} \quad . \qquad (3.A.5)$$

Using equations (3.A.4) and (3.A.5), we have equation (3.2.7) in the body of the chapter.

Derivation of equation (3.2.11):

From equation (3.2.7), we have

$$\rho + \sigma g = (1-\tau)A\alpha\phi^{\alpha}\left(\frac{G}{K}\right)^{1-\alpha} + \mu(1-\tau)B\beta(1-\phi)^{\beta}\left(\frac{G}{K}\right)^{1-\beta} \quad . \qquad (3.A.6)$$

From equation (3.2.2), we have

$$\left(\frac{G}{K}\right) = \frac{B^{\frac{1}{\beta}}(1-\phi)}{g^{\frac{1}{\beta}}} \quad . \qquad (3.A.7)$$

From equations (3.2.2), (3.2.6), (3.2.9), (3.A.6) and (3.A.7), we obtain equation (3.2.11) in the body of the chapter.

Derivation of equation (3.2.13):

Taking log on both sides of equation (3.2.11) and then differentiating it with respect to $\mu$ and assuming $\frac{dg}{d\mu} = 0$, we obtain

$$\frac{1}{\mu} = \frac{\frac{\alpha}{1-\alpha}\mu^{\frac{\alpha}{1-\alpha}}\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{1}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}{1+\alpha\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \quad . \quad (3.A.8)$$

From equation (3.A.8), we obtain equation (3.2.13) in the body of the chapter.

Derivation of equation (3.2.18):

From equation (3.2.12), we obtain

$$\omega = \frac{{c_0}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \quad (3.A.9)$$

Here, $c(0) = c_0$.

From equation (3.2.5), we obtain

$$c_0 = K_0\left\{(1-\tau)A(\phi)^{\alpha}\left(\frac{G_0}{K_0}\right)^{1-\alpha} + (1-\tau)\mu B(1-\phi)^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\beta} - g\right\} \quad . \quad (3.A.10)$$

Using equations (3.2.7) and (3.A.10), we obtain

$$c_0 = K_0\left\{\frac{\rho+\sigma g}{\alpha} + (1-\tau)\mu B(1-\phi)^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\beta}\left(\frac{\alpha-\beta}{\alpha}\right) - g\right\} \quad . \quad (3.A.11)$$

Using equations (3.2.2) and (3.A.11), we obtain

$$c_0 = K_0\left\{\frac{\rho+\sigma g}{\alpha} + (1-\tau)\mu B(1-\phi)\left(\frac{B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}}\right)^{1-\beta}\left(\frac{\alpha-\beta}{\alpha}\right) - g\right\} \quad . \quad (3.A.12)$$

Using equations (3.2.2), (3.2.6), (3.2.9) and (3.A.12), we obtain

$$c_0 = K_0 \left\{ \frac{\rho}{\alpha} + g\left(\frac{\sigma}{\alpha} - 1\right) + \frac{(\alpha-\beta) g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \right\} . \quad (3.A.13)$$

Using equations (3.A.9) and (3.A.13), we obtain equation (3.2.18) in the body of the chapter.

<u>Derivation of equation (3.2.19):</u>

Differentiating equation (3.2.18) with respect to $\mu$ and evaluating it at $\mu = \mu^*$, we obtain

$$\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*}$$

$$= \left\{ \frac{\left[ \frac{\rho}{\alpha} + g^*\left(\frac{\sigma}{\alpha} - 1\right) + \frac{(\alpha-\beta) {g^*}^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} {\mu^*}^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \right]^{-\sigma}}{{K_0}^{\sigma-1}[\rho - g^*(1-\sigma)]} \right\}$$

$$\cdot \left\{ \frac{(\alpha-\beta) {g^*}^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} {\mu^*}^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \right\}$$

$$\cdot \left\{ \left(\frac{2-\alpha}{1-\alpha}\right)\left(\frac{1}{\mu^*}\right) - \frac{\frac{{\mu^*}^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} - \frac{\frac{{\mu^*}^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \right\}$$

$$. \qquad (3.A.14)$$

Now, from equations (3.2.2), (3.2.6) and (3.2.9), we find that the last bracket term is equal to $\left(\frac{1}{\mu^*}\right)\left\{\frac{2-\alpha}{1-\alpha} - \frac{1}{1-\alpha}[(1-\phi^*)+\tau^*]\right\}$. Again, from equations (3.2.2) and (3.2.6), it appears that $\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\left(\frac{1}{\phi^*}\right)$; and from equations (3.2.2), (3.2.6) and (3.2.9), we find that $\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\frac{\alpha(1-\phi^*)}{\phi^*\tau^*}$. Incorporating all these equalities and putting values of $\mu^*$, $\tau^*$ and $\phi^*$ from equations (3.2.15), (3.2.16) and (3.2.17), we obtain equation (3.2.19).

Derivations of equations (3.2.20) and (3.2.21):

From equations (3.2.2) and (3.2.9), we obtain

$$\frac{G}{K} = B^{\frac{1}{\beta}}(1-\phi)g^{-\frac{1}{\beta}} \quad . \qquad (3.A.15)$$

Using equations (3.2.6) and (3.A.15), we obtain

$$\frac{(1-\phi)^{1-\alpha}}{\phi^{1-\alpha}} = \mu g^{\frac{\beta-\alpha}{\beta}}\frac{B^{\frac{\alpha}{\beta}}}{A}\frac{\beta}{\alpha} \quad . \qquad (3.A.16)$$

From equation (3.A.16), we obtain equation (3.2.20) in the body of the chapter.

Now, from equation (3.2.9) and (3.A.15), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\mu(1-\phi)^{\alpha}}{A\phi^{\alpha}}g^{\frac{\beta-\alpha}{\beta}}B^{\frac{\alpha}{\beta}} \quad . \qquad (3.A.17)$$

Using equations (3.A.16) and (3.A.17), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\alpha}{\beta}\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}} \quad . \qquad (3.A.18)$$

From equation (3.A.18), we obtain equation (3.2.21) in the body of the chapter.

# Chapter 4: Problem of Environmental degradation

## 4.1 Introduction

In the earlier chapters, we have not introduced the problem of environmental pollution and labour unions role to fight with this problem. The present chapter is an attempt to introduce this problem. This chapter, on the one hand, analyses the effect of unionisation in the labour market on the long run economic growth rate in the presence of environmental pollution; and, on the other hand, analyses properties of an optimum income tax policy designed to finance public abatement expenditure when labour unions bargain for workers' health and safety and for environment development. We consider two alternative bargaining models to analyse the negotiation problem – the 'Efficient Bargaining' model of McDonald and Solow (1981) and the 'Right to Manage' model of Nickell and Andrews (1983).

We derive interesting results from this model. First, growth rate maximising rate of income tax used to finance public abatement expenditure varies inversely with the relative bargaining power of the labour union. Secondly, how unionisation affects employment depends on the nature of bargaining. In the 'Efficient Bargaining' model, unionisation raises employment level only if the labour union is highly employment oriented. Otherwise, it always lowers the level of employment. Thirdly, the effect on economic growth depends partly on the employment effect and partly on the effect of employer's spending to protect environment; and this is valid for each of the two bargaining models. Since the environmental protection effect is always positive, it may outweigh the employment effect even if it is negative; and thus unionisation may have a positive effect on economic growth even when unions are wage oriented.

The chapter is organized as follows. In section 4.2, we describe the basic model with an 'Efficient Bargaining' theory. This section also analyses the properties of the growth rate maximising tax policy and effects of unionisation on growth rate. These results are compared to the corresponding results obtained from the 'Right to Manage' model analysed in section 4.3.

## 4.2 The Model

### 4.2.1 Firms

The representative competitive firm produces the final good, $Y$, using private capital, $K$, labour, $L$, average economy wide stock of capital, $\bar{K}$, and environmental quality, $E$.[41] The production function of the final good is given by[42]

$$Y = F(K, L, \bar{K}, E) = AK^{\alpha} L^{\beta} \bar{K}^{1-\alpha} E^{\delta} \qquad (4.2.1)$$

satisfying $\alpha, \beta, \delta \in (0,1)$ , $\alpha + \beta < 1$ and $A > 0$ .

Existence of decreasing returns to private inputs leads to a positive profit if employers' association owns a positive degree of bargaining power. Following Chang et al. (2007), we assume that a fixed quantity of land is essential for a firm; and thus the number of firms is fixed even in the presence of positive profit.[43]

The firm maximises profit, $\pi$, defined as

$$\pi = Y - \gamma Y - wL - rK \quad . \qquad (4.2.2)$$

Here $\gamma$ is firm's rate of expenditure incurred to protect environment. $w$ and $r$ represent the wage rate and rental rate on private capital respectively.

### 4.2.2 Capital Market

Capital market is perfectly competitive; and so the supply-demand equality determines the equilibrium value of the perfectly flexible rental rate on capital. Demand function for capital is derived from firms' profit maximizing behaviour; and the inverted demand function is given by

$$r = (1-\gamma) A\alpha K^{\alpha-1} L^{\beta} \bar{K}^{1-\alpha} E^{\delta} = \frac{(1-\gamma)\alpha Y}{K} \quad . \qquad (4.2.3)$$

### 4.2.3 Environment

[41] An improvement in environmental quality leads to an improvement of health capital of workers and an increase in efficiency of public capital. As Gupta and Barman (2009) writes "There are various ways by which degradation of environmental quality reduces the effective benefit of public investment expenditure. For example, deforestation reduces rainfall; and this, in turn, reduces the efficiency of the public irrigation programme by reducing the canals' water flow and lowering the recharging rate of groundwater. Poor quality of natural resources (coal) and the lack of current in the river water negatively affect the generation of electricity. Global warming leads to natural disasters like floods, earthquakes, cyclones, etc.; and these, in turn, cause severe damage to infrastructural capitals like roads, electric lines, power plants, buildings, industrial plants, etc. Water pollution and air pollution cause various diseases; and hence the public health expenditure programme fails to provide the desired benefit to the workers which, in turn, lowers their efficiency.". Gupta and Barman (2009, 2010, 2013), Barman and Gupta (2010), Economides and Philippopoulos (2008), Greiner (2005) etc. include environmental quality as an input in the production function.

[42] Chang et al. (2007) also assumes similar production function where average economy wide stock of capital enters as an input in the production function. However, they do not consider the productive role of environmental quality in the production process.

[43] Number of firms is normalised to unity.

Following Greiner (2005), we consider environmental quality, $E$, as a flow variable satisfying public input properties. Following Gupta and Barman (2010), Barman and Gupta (2010), Economides and Philippopoulos (2008) and Greiner (2005), we assume that production of the final good is the only source of emission[44]; and public abatement expenditure incurred by the government can improve environmental quality. However, we also consider firms' role to protect environment. Firms are forced to spend for environmental development due to bargaining power of the labour union. For example, firms may use costly eco-friendly techniques of production or may allocate resources to non-productive activities for the sake of workers' health and safety. This aspect has not been considered in the existing theoretical literature on economic growth with labour unions. The environmental quality function is given by

$$E = E((\tau_E + \gamma)Y, vY) \text{ with } E_1 > 0 \text{ and } E_2 < 0 \quad . \qquad (4.2.4)$$

Here $\tau_E$ is the rate of income tax used to finance public abatement expenditure and hence $(\tau_E + \gamma)$ is the combined rate of abatement expenditure. $v$ is the emission - output coefficient. We specify a simple functional form given by[45]

$$E = (\tau_E Y + \gamma Y)^{\mu}(vY)^{-\mu} = (\tau_E + \gamma)^{\mu} v^{-\mu} \quad . \qquad (4.2.4.a)$$

Here $((\tau_E + \gamma)/v)$ represents the effective abatement activity per unit of pollution; and $\mu > 0$ is the elasticity of environmental quality with respect to this argument. This environmental quality function is a technological constraint and valid at the macro level. However, for an individual competitive firm, environmental quality, $E$, is an externality in the production function. An individual competitive firm cannot firm cannot affect $E$ through the choice of $\gamma$.

### 4.2.4 Labour Union's Utility Function

The labour union derives utility from three arguments: (i) worker's income gain measured by the bargained wage rate over the competitive wage rate,[46] (ii) level of employment and (iii)

[44] There may be many other sources, for example - consumption of pollution intensive goods, extracting natural resources etc.

[45] There is a technical problem associated with this form. If $(\tau_E + \gamma) = 0$, then $E = 0$; and then equation (4.2.1) implies that $Y = 0$. In order to avoid this we assume that $\tau_E > 0$.

[46] Some works assume that the difference between the bargained wage rate and the unemployment benefit is an argument in the labour union's utility function. Contrary to this, Irmen and Wigger (2003), Lingens (2003a) and Lai and Wang (2010) assume that the difference between the bargained wage rate and the competitive wage rate is an argument in the labour union's utility function. Since the provision of unemployment benefit is not considered in this model, so we incorporate the difference between the bargained wage rate and the competitive wage rate as an argument in the labour union's utility function.

firm's spending rate to protect environment. The third argument is generally not considered in the existing literature. The utility function is given by

$$u_T = (w - w_c)^{\varepsilon_1} L^{\varepsilon_2} \gamma^{\varepsilon_3} \quad \text{with} \quad \varepsilon_j > 0 \quad \text{for} \quad j = 1,2,3 \quad . \tag{4.2.5}$$

Here $u_T$ and $w_c$ stand for the utility of the labour union and the competitive wage rate respectively. $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are three non-negative parameters representing degrees of orientation of the labour union towards those arguments. If $\varepsilon_j = Max\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, then the labour union is called *jth* argument oriented.

In a competitive labour market, perfectly flexible wage rate is equated to the marginal productivity of labour and labour is fully employed. The competitive firm does not spend for environmental development because $E$ is external and the firm is too small to internalize the externality. Hence $\gamma = 0$ for a competitive firm. So, with labour endowment being normalized to unity, we have

$$w_c = \beta A K^{\alpha} \overline{K}^{1-\alpha} E^{\delta} \quad . \tag{4.2.6}$$

### 4.2.5 The 'Efficient Bargaining' Model

In this section, we consider a bilateral monopoly labour market where labour union and employers' association are two parties. We choose the 'Efficient Bargaining' model where wage rate, level of employment and the rate of firm's spending to protect environment are determined jointly by the labour union and the employer's association; and they maximize the 'generalised Nash product' function given by

$$\psi = (\, u_T - \overline{u_T})^{\theta} (\pi - \bar{\pi})^{(1-\theta)} \quad . \tag{4.2.7}$$

Here $\overline{u_T}$ and $\bar{\pi}$ symbolize the reservation utility level of the labour union and the reservation profit level of the firm respectively. Bargaining disagreement stops production and hence results into zero employment, which, in turn, implies that $\overline{u_T}$ = 0 and $\bar{\pi}$ = 0. $\theta \in (0,1)$ represents the relative bargaining power of the labour union. Unionisation in the labour market implies an exogenous increase in the value of $\theta$. However, the capital market is competitive.

Using equations (4.2.2) and (4.2.3), we obtain

$$\pi = (1 - \gamma)(1 - \alpha)Y - wL \quad . \tag{4.2.8}$$

Finally, using equations (4.2.5), (4.2.7) and (4.2.8), we obtain

$$\psi = \{(w - w_c)^{\varepsilon_1} L^{\varepsilon_2} \gamma^{\varepsilon_3}\}^{\theta} \{(1 - \gamma)(1 - \alpha)Y - wL\}^{(1-\theta)} \quad . \tag{4.2.9}$$

Here $\psi$ is to be maximised with respect to $w$, $L$ and $\gamma$. Using equations (4.2.1) and (4.2.6), and the three first order conditions of optimisation, we solve for optimal $w$, $L$ and $\gamma$. These are given by[47]

$$L^* = \left\{\frac{(1-\alpha)\{\beta(1-\theta+\theta\varepsilon_1)-\theta(\varepsilon_1-\varepsilon_2)\}}{\beta\{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)\}}\right\}^{\frac{1}{1-\beta}} \quad ; \qquad (4.2.10)$$

$$w^* = \frac{\{\beta(1-\theta)+\theta\varepsilon_2\}w_c}{\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}} \quad ; \qquad (4.2.11)$$

and

$$\gamma^* = \frac{\theta\varepsilon_3(1-\beta)}{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)} \quad . \qquad (4.2.12)$$

To ensure positive values of $L^*$ and $w^*$ and to ensure $L^* < 1$, we need a parametric restriction. This is given by

$$Condition\ 4.A: \quad -\beta(1-\theta+\theta\varepsilon_1) < \theta(\varepsilon_2-\varepsilon_1) < \frac{\alpha\beta(1-\theta+\theta\varepsilon_1)+\theta\varepsilon_3\beta(1-\beta)}{(1-\alpha-\beta)} \ .$$

From equation (4.2.10), we obtain

$$\frac{\partial L^*}{\partial\theta} = \frac{[\varepsilon_2-\varepsilon_1-\beta\varepsilon_3]L^*}{\{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)\}\{\beta(1-\theta+\theta\varepsilon_1)-\theta(\varepsilon_1-\varepsilon_2)\}} \quad . \qquad (4.2.10.a)$$

Here the denominator in the R.H.S. of equation (4.2.10.a) is always positive. So $\frac{\partial L^*}{\partial\theta} > 0$ if and only if $\varepsilon_2 > \varepsilon_1 + \beta\varepsilon_3$. This means that unionisation in the labour market raises the employment level if the union is highly employment oriented.

Chang et al. (2007) does not consider trade union's concern about environment development; and hence $\varepsilon_3 = 0$ there. So sign of $\left(\frac{\partial L^*}{\partial\theta}\right)$ depends solely on the sign of $(\varepsilon_2 - \varepsilon_1)$. However, in our analysis, $\varepsilon_3 > 0$; and so the nature of the employment effect depends on the magnitude of $\varepsilon_3$.

The intuition behind this can be explained as follows. The labour union wants to raise $L$ due to a rise in $\theta$ if it obtains a marginal utility higher than its marginal opportunity cost. In the case of Chang et al. (2007), i.e., in the absence of trade union's concern for environment development, opportunity cost of raising $L$ is same as the loss in utility from not raising $w$. However, in the present model, this opportunity cost also includes the loss in utility from not raising $\gamma$. Hence $\varepsilon_3$ enters into the picture.

[47] See Appendix 4.A for derivation.

Equation (4.2.11) shows that the negotiated wage rate, $w^*$, exceeds the competitive equilibrium wage rate, $w_c$. From this equation, we obtain

$$\frac{\partial w^*}{\partial \theta} = \frac{\{\beta\varepsilon_1(1-\beta)\}w_c}{\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}^2} > 0 \quad . \tag{4.2.11.a}$$

So $w^*$ varies positively with $\theta$. This is obvious because the labour union always derives higher utility from a higher wage; and so it bargains for a higher wage with a greater bargaining power.

From equation (4.2.12), we obtain

$$\frac{\partial \gamma^*}{\partial \theta} = \frac{\varepsilon_3(1-\beta)}{\{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)\}^2} > 0 \quad . \tag{4.2.12.a}$$

Unionisation forces the firm to spend a higher fraction of output on environment development because the labour union always derives higher utility from a higher value of $\gamma$.

Second order conditions of maximization of $\psi$ are also satisfied[48]; and we now state the following proposition.

**Proposition 4.2.1:** Unionisation in the labour market always raises the wage rate as well as the firm's spending rate to protect environment but raises employment level only if the labour union is highly employment oriented.

### 4.2.6 Households

The representative household derives instantaneous utility only from consumption of the final good.[49] She maximises her discounted present value of instantaneous utility over the infinite time horizon subject to the intertemporal budget constraint. The household's problem is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{4.2.13}$$

subject to, $\dot{K} = (1-\tau_E)Y - c$ ; (4.2.14)

and $K(0) = K_0$ ($K_0$ is historically given) .

[48] See Appendix 4.A for derivation.

[49] We assume that households supply constant amount of labour; and so labour - leisure choice of representative household is ruled out. We also do not consider environmental quality as an argument in household's utility function for the sake of simplicity even though we incorporate trade union's concern about environmental effects. This exclusion is a restrictive one.

Here $c$ denotes the level of consumption of the representative household. $c$ is the control variable and $K$ is the state variable. $\sigma$ and $\rho$ are the elasticity of marginal utility with respect to consumption and the constant rate of discount of consumption respectively. Savings is always invested; and capital does not depreciate.

Solving this dynamic optimisation problem, we obtain the growth rate of consumption given by

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau_E)A\alpha\left(\frac{\bar{K}}{K}\right)^{1-\alpha} L^{\beta}E^{\delta} - \rho}{\sigma} \quad . \qquad (4.2.15)$$

### 4.2.7 Equilibrium

At the symmetric equilibrium, $\bar{K} = K$; and hence, from equations (4.2.3), (4.2.6) and (4.2.15), we obtain

$$r = (1-\gamma)A\alpha L^{\beta}E^{\delta} \quad ; \qquad (4.2.3.a)$$

$$w_c = \beta AKE^{\delta} \qquad (4.2.6.a)$$

and

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau_E)A\alpha L^{\beta}E^{\delta} - \rho}{\sigma} \quad . \qquad (4.2.15.a)$$

It looks like an *AK* model and there is no transitional dynamics. At equilibrium, employment of labour, tax rate, rental rate of capital, environmental quality and firm's abatement expenditure rate - all are time-independent. So the growth rate of consumption given by equation (4.2.15.a) is also time-independent. Capital stock, *K*, final output, *Y*, negotiated wage rate, $w^*$, firm's profit, $\pi$, also grow at that rate in the steady-state equilibrium.

### 4.2.8 Optimal Tax Rate

We first derive the growth rate maximising income tax rate. Using equations (4.2.4.a) and (4.2.15.a), we obtain the growth rate maximising income tax rate, ${\tau_E}^*$, given by[50]

$${\tau_E}^* = \left(\frac{\mu\delta - \gamma^*}{1+\mu\delta}\right) \quad . \qquad (4.2.16)$$

We assume the following parametric restriction to ensure that $0 < {\tau_E}^* < 1$.

[50] Second order condition of maximisation is also satisfied.

$$\mu\delta > \frac{\theta\varepsilon_3(1-\beta)}{\theta\varepsilon_3(1-\beta)+(1-\theta+\theta\varepsilon_2)} \quad .$$

Using equations (4.2.12) and (4.2.16), we obtain

$$\tau_E{}^* = \left\{\frac{\mu\delta[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]-\theta\varepsilon_3(1-\beta)}{(1+\mu\delta)[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]}\right\} \quad . \qquad (4.2.16.a)$$

From equation (4.2.16.a), we obtain

$$\frac{\partial\tau_E{}^*}{\partial\theta} = -\frac{\varepsilon_3(1-\beta)}{\{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)\}^2(1+\mu\delta)} < 0 \quad . \qquad (4.2.17)$$

Equation (4.2.17) implies that an increase in $\theta$ lowers $\tau_E{}^*$. This is so because this raises $\gamma^*$ which, in turn, upgrades environmental quality and thus lowers the marginal benefit of public abatement expenditure.

**Proposition 4.2.2:** Optimum rate of income tax used to finance public abatement expenditure varies inversely with the bargaining power of the labour union.

Here we do not incorporate environmental quality in the household's utility function. So there exists a positive monotonic relationship between the growth rate and the welfare level. So the growth rate maximising tax rate is identical to the welfare maximising tax rate.

### 4.2.9 Growth Effect of Unionisation

We now analyse the effect of an increase in $\theta$ on the endogenous growth rate. Using equations (4.2.4.a), (4.2.16) and (4.2.15.a) and putting $\gamma = \gamma^*$ and $L = L^*$, we obtain

$$g^* = \frac{A\alpha L^{*\beta}\left(\frac{\mu\delta}{v}\right)^{\mu\delta}\frac{(1+\gamma^*)^{\mu\delta+1}}{(1+\mu\delta)^{\mu\delta+1}} - \rho}{\sigma} \quad . \qquad (4.2.18)$$

From equation (4.2.18), we have

$$\frac{\partial g^*}{\partial\theta} = \frac{A\alpha L^{*\beta}\left(\frac{\mu\delta}{v}\right)^{\mu\delta}\frac{(1+\gamma^*)^{\mu\delta+1}}{(1+\mu\delta)^{\mu\delta+1}}}{\sigma}\left[\beta\frac{\frac{\partial L^*}{\partial\theta}}{L^*} + (1+\mu\delta)\frac{\frac{\partial\gamma^*}{\partial\theta}}{(1+\gamma^*)}\right] \quad . \qquad (4.2.19)$$

Equation (4.2.19) shows that the effect of an increase in $\theta$ on the growth rate, $g^*$, is ambiguous. The first term inside the bracket on the right hand side of equation (4.2.19) represents the employment effect on growth due to unionisation. Its sign is determined by the sign of $\frac{\partial L^*}{\partial\theta}$; and so it depends on the nature of labour union's orientation towards arguments in its utility function. However, the second term inside this bracket is definitely positive because equation (4.2.12.a) shows that $\frac{\partial \gamma^*}{\partial \theta} > 0$. This term represents the environment development effect on growth due to unionisation. So the effect of unionisation on the growth rate may be qualitatively different from its employment effect.

In Chang et al. (2007), the environment development effect on growth does not exist. Hence the growth effect of unionisation is qualitatively similar to the employment effect in that model; and hence the nature of growth effect is determined by the nature of orientation of the labour union. So, in that model, the growth effect is positive (negative) when the union is employment (wage) oriented.

However, in the present model where environment development effect exists, nature of the orientation of the labour union alone cannot determine the nature of the growth effect. If the environment development effect dominates the employment effect, then unionisation always raises the growth rate regardless of the nature of orientation of the labour union. Growth effect may be positive even if the employment effect is negative, i.e., if the union is wage oriented.

We can establish the following proposition.

**Proposition 4.2.3:** Unionisation in the labour market must (may) raise the endogenous growth rate if the labour union is highly employment (wage or neutrally) oriented.

Since there exists a positive monotonic relationship between growth rate and welfare, welfare effects of unionisation are qualitatively identical to its growth effects.

## 4.3 The 'Right to Manage' Model

In the 'Right to Manage' model, two parties bargain over $w$ and $\gamma$. The individual firm determines $L$ from its labour demand function derived from its profit maximising behaviour; and it is given by

$$w = (1-\gamma)\beta A K^{\alpha} \overline{K}^{1-\alpha} E^{\delta} L^{\beta-1} = (1-\gamma) w_c L^{\beta-1} \quad . \qquad (4.3.1)$$

Using equations (4.2.8) and (4.3.1), we have

$$\pi = (1-\gamma)(1-\alpha-\beta)Y \quad ; \qquad (4.3.2)$$

and hence the 'generalised Nash product' function is obtained as follows.

$$\psi = \{(w - w_c)^{\varepsilon_1} L^{\varepsilon_2} \gamma^{\varepsilon_3}\}^{\theta} \{(1-\gamma)(1-\alpha-\beta)Y\}^{(1-\theta)} \quad . \tag{4.3.3}$$

In this model, $\psi$ is to be maximised with respect to $w$ and $\gamma$, subject to equation (4.3.1). Optimum values of $w$ and $\gamma$ are same as those obtained in the 'Efficient Bargaining' model. However, employment level is different; and is given by[51]

$$L^{**} = \left\{ \frac{(1-\theta+\theta\varepsilon_2)\{\beta(1-\theta+\theta\varepsilon_1) - \theta(\varepsilon_1 - \varepsilon_2)\}}{\{\beta(1-\theta)+\theta\varepsilon_2\}[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]} \right\}^{\frac{1}{1-\beta}} \quad . \tag{4.3.4}$$

Comparing equation (4.3.4) to equation (4.2.10) we find that $L^{**} \neq L^{*}$.

*Condition 4.A* guarantees that $L^{**} > 0$; equation (4.3.4) clearly shows that $L^{**} < 1$ because $\beta(1-\theta+\theta\varepsilon_1) > \theta(\varepsilon_1 - \varepsilon_2)$. Second order conditions of maximisation are also satisfied.

The government's objective as well as the representative household's objective are same in both the models. So equations and solutions derived are also same in these two models. Optimal tax rate is also identical.

From equation (4.3.4), we have

$$\frac{\partial L^{**}}{\partial \theta} = -\left[ \frac{\varepsilon_3(1-\beta)}{(1-\theta+\theta\varepsilon_2)[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]} + \frac{\beta\varepsilon_1(1-\beta)}{\{\beta(1-\theta+\theta\varepsilon_1)-\theta(\varepsilon_1-\varepsilon_2)\}\{\beta(1-\theta)+\theta\varepsilon_2\}} \right] < 0 \quad . \tag{4.3.5}$$

So an increase in $\theta$ always lowers $L^{**}$; and this implies that the employment effect of unionisation is always negative. This result is contradictory to the corresponding result obtained in the 'Efficient Bargaining' model where the nature of the employment effect depends on the mathematical sign of $(\varepsilon_2 - \varepsilon_1 - \beta\varepsilon_3)$.

Effects of unionisation on the wage rate, on firm's environment development expenditure, and on optimum tax rates in this model are qualitatively similar to corresponding effects obtained in the previous model. Equation (4.2.18) is otherwise valid here except that $L^{*}$ is replaced by $L^{**}$. So the effect of unionisation on the rate of growth is given by

$$\frac{\partial g^{**}}{\partial \theta} = \frac{A\alpha L^{**\beta} \left(\frac{\mu\delta}{v}\right)^{\mu\delta} \frac{(1+\gamma^{*})^{\mu\delta+1}}{(1+\mu\delta)^{\mu\delta+1}}}{\sigma} \left[ \beta \frac{\frac{\partial L^{**}}{\partial \theta}}{L^{**}} + (1+\mu\delta) \frac{\frac{\partial \gamma^{*}}{\partial \theta}}{(1+\gamma^{*})} \right] \quad . \tag{4.3.6}$$

[51] See Appendix 4.B for detailed derivations of the section 4.3.

Here the first term and the second term inside the bracket of the right hand side of equation (4.3.6) are negative and positive respectively. So unionisation on the one hand lowers the growth rate through negative employment effect and on the other hand raises it through positive environment development effect; and the net effect depends on the relative strength of these two. Important results are summarized in the following proposition.

**Proposition 4.3.1:** In the 'Right to Manage' model of bargaining, unionisation in the labour market always lowers employment level; and so unionisation raises the growth rate if and only if the positive environment development effect dominates the negative employment effect.

## Appendix

### Appendix 4.A: Derivation of section 4.2

Derivation of first order conditions:

From equations (4.2.9) and (4.2.1), we have

$$log\psi = \theta\varepsilon_1\, log(w - w_c) + \theta\varepsilon_2\, logL + \theta\varepsilon_3\, log\gamma$$
$$+(1-\theta)\, log\left\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\overline{K}^{1-\alpha}E^{\delta} - wL\right\} \quad . \qquad (4.A.1)$$

The first order conditions of maximization of log $\psi$ are given by the followings.

$$\frac{\theta\varepsilon_1}{w-w_c} + \frac{(1-\theta)(-L)}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\overline{K}^{1-\alpha}E^{\delta} - wL\}} = 0 \quad ; \qquad (4.A.2)$$

$$\frac{\theta\varepsilon_2}{L} + \frac{(1-\theta)\left\{(1-\gamma)(1-\alpha)\beta AK^{\alpha}L^{\beta-1}\overline{K}^{1-\alpha}E^{\delta} - w\right\}}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\overline{K}^{1-\alpha}E^{\delta} - wL\}} = 0 \quad ; \qquad (4.A.3)$$

and

$$\frac{\theta\varepsilon_3}{\gamma} + \frac{(1-\theta)\left\{-(1-\alpha)AK^{\alpha}L^{\beta}\overline{K}^{1-\alpha}E^{\delta}\right\}}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\overline{K}^{1-\alpha}E^{\delta} - wL\}} = 0 \quad . \qquad (4.A.4)$$

Now using equations (4.A.2), (4.A.3) and (4.2.6), we have

$$(\varepsilon_1 - \varepsilon_2)w = \varepsilon_1(1-\gamma)(1-\alpha)L^{\beta-1}w_c - \varepsilon_2 w_c \quad . \qquad (4.A.5)$$

From equations (4.A.2) and (4.2.6), we obtain

$$\theta\varepsilon_1\left\{(1-\gamma)(1-\alpha)L^{\beta-1}\frac{w_c}{\beta} - w\right\} = (1-\theta)(w - w_c) \quad . \qquad (4.A.6)$$

Using Equations (4.A.5) and (4.A.6), we obtain

$$L^{\beta-1} = \frac{\beta\theta\varepsilon_2 + \beta(1-\theta)}{(1-\gamma)(1-\alpha)\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}} \quad . \qquad (4.A.7)$$

From equation (4.A.4), we obtain

$$\frac{\theta\varepsilon_3}{\gamma} = \frac{(1-\theta)\left\{(1-\alpha)AK^{\alpha}L^{\beta-1}\overline{K}^{1-\alpha}E^{\delta}\right\}}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta-1}\overline{K}^{1-\alpha}E^{\delta} - w\}} \quad . \qquad (4.A.8)$$

Using equations (4.A.5), (4.A.7), (4.A.8) and (4.2.6), we obtain

$$\gamma = \frac{\{1-\beta\}\theta\varepsilon_3}{\{\theta\varepsilon_2 + (1-\theta)\} + \theta\varepsilon_3\{1-\beta\}} \quad . \qquad (4.A.9)$$

Equation (4.A.9) is identical to equation (4.2.12) in the body of the chapter.

Using equations (4.A.9) and (4.A.7), we have

$$L^{\beta-1} = \frac{\beta[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3\{1-\beta\}]}{(1-\alpha)\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}} \quad . \tag{4.A.10}$$

From equation (4.A.10), we obtain equation (4.2.10) in the body of the chapter.

Using equations (4.A.7) and (4.A.5), we have

$$w = \frac{\beta(1-\theta) + \theta\varepsilon_2}{\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}} w_c \quad . \tag{4.A.11}$$

Equation (4.A.11) is identical to equation (4.2.11) in the body of the chapter.

Derivation of Condition 4.A:

We derive Condition 4.A as follows. To ensure positive values of $L^*$ and $w^*$, we need the following parametric restriction.

$Condition\ 4.A_1$: $\quad \beta(1-\theta+\theta\varepsilon_1) > \theta(\varepsilon_1 - \varepsilon_2)$[52] .

Again, labour employment has to be less than its endowment i.e., $L^* < 1$; and so the following parametric restriction is needed.

$Condition\ 4.A_2$: $\quad (1-\alpha-\beta)\theta(\varepsilon_2 - \varepsilon_1) < \alpha\beta(1-\theta+\theta\varepsilon_1) + \theta\varepsilon_3\beta(1-\beta)$ .

Combining these two conditions $4.A_1$ and $4.A_2$, we obtain condition 4.A given in the body of the chapter.

Second order conditions:

Using equations (4.A.2) and (4.2.6), we have

$$\frac{\partial^2 Log(\psi)}{\partial w^2} = -\frac{\theta\varepsilon_1}{(w-w_c)^2} - \frac{(1-\theta)}{\left\{(1-\gamma)(1-\alpha)L^{\beta-1}\frac{w_c}{\beta} - w\right\}^2} < 0 \quad . \tag{4.A.12}$$

From equations (4.A.3), (4.2.6), (4.A.7) and (4.A.11), we have

$$\frac{\partial^2 Log(\psi)}{\partial L^2} = -\frac{\theta\varepsilon_2}{L^2} - (1-\theta)\left[\frac{\left\{\begin{array}{c}\frac{(1-\gamma)(1-\alpha)(1-\beta)w_c L^{\beta-1}(1-\beta)(1-\theta)w_c}{\{\beta(1-\theta)+\beta\theta\varepsilon_1-\theta(\varepsilon_1-\varepsilon_2)\}} \\ +\left\{(1-\gamma)(1-\alpha)L^{\beta-1}w_c - w\right\}^2\end{array}\right\}}{\left\{(1-\gamma)(1-\alpha)L^{\beta}\frac{w_c}{\beta} - wL\right\}^2}\right] < 0 \quad . \tag{4.A.13}$$

[52] Condition $4.A_1$ also ensures that negotiated wage rate is higher than the competitive wage rate.

From equations (4.A.4) and (4.2.6), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} = -\frac{\theta\varepsilon_3}{\gamma^2} - \frac{(1-\theta)\left\{(1-\alpha)L^{\beta-1}\frac{w_c}{\beta}\right\}^2}{\left\{(1-\gamma)(1-\alpha)L^{\beta-1}\frac{w_c}{\beta} - w\right\}^2} < 0 \quad . \qquad (4.A.14)$$

Now, using equations (4.A.12), (4.A.11) and (4.A.7), we have

$$\frac{\partial^2 Log(\psi)}{\partial w^2} = -\frac{\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}^2(1-\theta+\theta\varepsilon_1)}{{w_c}^2(1-\beta)^2\theta\varepsilon_1(1-\theta)} \quad . \qquad (4.A.15)$$

Using equations (4.A.13), (4.A.11) and (4.A.7), we have

$$\frac{\partial^2 Log(\psi)}{\partial L^2} = -\frac{(1-\theta+\theta\varepsilon_2)[\beta(1-\theta)+\theta\varepsilon_2]}{L^2(1-\theta)} \quad . \qquad (4.A.16)$$

Again using equations (4.A.14), (4.A.9), (4.A.11) and (4.A.7), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} = -\frac{[\{\theta\varepsilon_2 + (1-\theta)\} + \theta\varepsilon_3\{1-\beta\}]^2(1-\theta+\theta\varepsilon_3)}{(1-\beta)^2(1-\theta)\theta\varepsilon_3} \quad . \qquad (4.A.17)$$

Now from equations (4.A.2), (4.A.7), (4.A.11) and (4.2.6), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\,\partial \mathrm{w}} = -\frac{[1-\theta+\theta\varepsilon_2]\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1-\varepsilon_2)\}}{L(1-\theta)w_c(1-\beta)} \quad . \qquad (4.A.18)$$

Using equations (4.A.15), (4.A.16) and (4.A.18), we have

$$\frac{\partial^2 Log(\psi)}{\partial L^2} . \frac{\partial^2 Log(\psi)}{\partial w^2} - \left[\frac{\partial^2 Log(\psi)}{\partial L\,\partial \mathrm{w}}\right]^2$$

$$= \frac{\{\beta(1-\theta) + \theta\varepsilon_2 - \theta\varepsilon_1(1-\beta)\}^3[\theta\varepsilon_2 + (1-\theta)]}{L^2{w_c}^2(1-\beta)^2(1-\theta)\theta\varepsilon_1} > 0 \quad . \qquad (4.A.19)$$

Again from equations (4.2.6), (4.A.3), (4.A.7), (4.A.10) and (4.A.11), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\,\partial \gamma} = -\frac{[\beta(1-\theta)+\theta\varepsilon_2][\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3\{1-\beta\}]}{L(1-\theta)(1-\beta)} < 0 \quad ; \qquad (4.A.20)$$

and from equations (4.2.6), (4.A.2), (4.A.7) and (4.A.11), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma \partial w} = \frac{\{\beta(1-\theta) + \theta\varepsilon_2 - \theta\varepsilon_1(1-\beta)\}[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3\{1-\beta\}]}{-(1-\theta)w_c\{1-\beta\}^2}$$

$$< 0 \quad . \qquad (4.A.21)$$

From equations (4.A.18), (4.A.15), (4.A.21) and (4.A.20), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\,\partial \gamma} . \frac{\partial^2 Log(\psi)}{\partial w^2} - \frac{\partial^2 Log(\psi)}{\partial L\,\partial \mathrm{w}} . \frac{\partial^2 Log(\psi)}{\partial w \partial \gamma}$$

$$= \left\{ \frac{\{\beta(1-\theta) + \theta\varepsilon_2 - \theta\varepsilon_1(1-\beta)\}^3 [\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3\{1-\beta\}]}{\theta\varepsilon_1(1-\theta)L(1-\beta)^3 {w_c}^2} \right\} > 0 \quad . \quad (4.A.22)$$

From equations (4.A.18), (4.A.20), (4.A.16) and (4.A.21), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\, \partial w} . \frac{\partial^2 Log(\psi)}{\partial L \partial \gamma} - \frac{\partial^2 Log(\psi)}{\partial L^2} . \frac{\partial^2 Log(\psi)}{\partial w \partial \gamma} = 0 \quad . \qquad (4.A.23)$$

So from equations (4.A.17), (4.A.19), (4.A.20), (4.A.22), (4.A.21) and (4.A.23), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} \left\{ \left(\frac{\partial^2 Log(\psi)}{\partial w^2}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial L^2}\right) - \left(\frac{\partial^2 Log(\psi)}{\partial w\, \partial L}\right)^2 \right\}$$

$$- \frac{\partial^2 Log(\psi)}{\partial \gamma\, \partial L} \left\{ \left(\frac{\partial^2 Log(\psi)}{\partial w^2}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma\, \partial L}\right) - \left(\frac{\partial^2 Log(\psi)}{\partial w \partial L}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma \partial w}\right) \right\}$$

$$+ \frac{\partial^2 Log(\psi)}{\partial \gamma \partial w} \left\{ \left(\frac{\partial^2 Log(\psi)}{\partial w \partial L}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma\, \partial L}\right) - \left(\frac{\partial^2 Log(\psi)}{\partial L^2}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma \partial w}\right) \right\}$$

$$= - \frac{[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3\{1-\beta\}]^3 \{\beta(1-\theta) + \theta\varepsilon_2 - \theta\varepsilon_1(1-\beta)\}^3}{\theta^2 \varepsilon_3 \varepsilon_1 {w_c}^2 L^2 (1-\beta)^4 (1-\theta)} < 0. \quad (4.A.24)$$

**Appendix 4.B: Derivation of equations of section 4.3**

Bargaining: First order conditions:

From equations (4.2.1), (4.3.1) and (4.3.3), we obtain

$$\psi = (w - w_c)^{\theta\varepsilon_1} \left[\frac{(1-\gamma)w_c}{w}\right]^{\frac{\theta\varepsilon_2 + \beta(1-\theta)}{1-\beta}} \gamma^{\theta\varepsilon_3} (1-\gamma)^{(1-\theta)} (1-\alpha-\beta)^{(1-\theta)}$$

$$\left[A K^{\alpha} \bar{K}^{1-\alpha} E^{\delta}\right]^{(1-\theta)} \quad . \qquad (4.B.1)$$

The first order conditions of maximisation $\psi$ with respect to *w* and $\gamma$ are given by

$$\frac{\partial Log(\psi)}{\partial w} = \frac{\theta\varepsilon_1}{(w - w_c)} - \frac{\theta\varepsilon_2 + \beta(1-\theta)}{(1-\beta)w} = 0 \quad ; \qquad (4.B.2)$$

and

$$\frac{\partial Log(\psi)}{\partial \gamma} = - \frac{\theta\varepsilon_2 + \beta(1-\theta)}{(1-\gamma)(1-\beta)} - \frac{(1-\theta)}{(1-\gamma)} + \frac{\theta\varepsilon_3}{\gamma} = 0 \quad . \qquad (4.B.3)$$

From equation (4.B.2), we obtain

$$w = \frac{[\theta\varepsilon_2 + \beta(1-\theta)]}{[\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]} w_c \quad . \qquad (4.B.4)$$

Equation (4.B.4) is identical to equation (4.2.11) in the body of the chapter.

From equation (4.B.3), we obtain

$$\gamma = \frac{\theta\varepsilon_3(1-\beta)}{[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3(1-\beta)]} \quad . \tag{4.B.5}$$

Equation (4.B.5) is same as equation (4.2.12) in the body of the chapter.

Using equations (4.B.4), (4.B.5) and (4.3.1), we obtain

$$L = \left[\frac{[\theta\varepsilon_2 + (1-\theta)][\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]}{[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3(1-\beta)][\theta\varepsilon_2 + \beta(1-\theta)]}\right]^{\frac{1}{1-\beta}} \quad . \tag{4.B.6}$$

Equation (4.B.6) is same as equation (4.3.4) in the body of the chapter.

<u>Second order conditions:</u>

From equations (4.B.2) and (4.B.4), we obtain

$$\frac{\partial^2 Log(\psi)}{\partial w^2} = -\frac{[\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]^3}{w_c{}^2(1-\beta)^2\theta\varepsilon_1[\theta\varepsilon_2 + \beta(1-\theta)]} < 0 \quad . \tag{4.B.7}$$

From equation (4.B.3), we obtain

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} = -\frac{\theta\varepsilon_2 + (1-\theta)}{(1-\gamma)^2(1-\beta)} - \frac{\theta\varepsilon_3}{\gamma^2} < 0 \quad . \tag{4.B.8}$$

and

$$\frac{\partial^2 Log(\psi)}{\partial \gamma w} = 0 \quad . \tag{4.B.9}$$

Using equations (4.B.7), (4.B.8) and (4.B.9), we have

$$\frac{\partial^2 Log(\psi)}{\partial w^2} \cdot \frac{\partial^2 Log(\psi)}{\partial \gamma^2} - \left[\frac{\partial^2 Log(\psi)}{\partial \gamma w}\right]^2$$

$$= \frac{[\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]^3}{w_c{}^2(1-\beta)^2\theta\varepsilon_1[\theta\varepsilon_2 + \beta(1-\theta)]}\left\{\frac{\theta\varepsilon_2 + (1-\theta)}{(1-\gamma)^2(1-\beta)} + \frac{\theta\varepsilon_3}{\gamma^2}\right\} > 0 \quad . \tag{4.B.10}$$

<u>Employment effect:</u>

From equation (4.3.4), we obtain

$$(1-\beta)\frac{\frac{\partial L^{**}}{\partial \theta}}{L^{**}} = \frac{\varepsilon_2 - 1}{[\theta\varepsilon_2 + (1-\theta)]} + \frac{\varepsilon_2 - \beta - \varepsilon_1(1-\beta)}{[\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]}$$

$$- \frac{\varepsilon_2 - \beta}{[\theta\varepsilon_2 + \beta(1-\theta)]} - \frac{\varepsilon_2 - 1 + \varepsilon_3(1-\beta)}{[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3(1-\beta)]}$$

$$\Rightarrow \quad \frac{\frac{\partial L^{**}}{\partial \theta}}{L^{**}} = \frac{-\beta\varepsilon_1}{[\theta\varepsilon_2 + \beta(1-\theta)][\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]}$$

$$- \frac{\varepsilon_3}{[\theta\varepsilon_2 + (1-\theta)][\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3(1-\beta)]} \quad . \qquad (4.\mathrm{B}.11)$$

From equation (4.B.11), we obtain equation (4.3.5) in the body of the chapter.

Derivations of other equations in this section are similar to those in the previous model.

# Chapter 5: Role of Efficiency Wage

## 5.1 Introduction

In the earlier chapters of the thesis we have not considered the role of ‘Efficiency Wage Hypothesis’ on the bargaining problem of the union. The present chapter attempts to develop a model to analyse the effect of unionisation in the labour market on economic growth in the presence of ‘Efficiency Wage Hypothesis’. The model developed here is an AK model with a unionised labour market and with an unemployment benefit scheme. However, in this model, we do not incorporate the role of productive public expenditure and environmental problem for the sake of simplicity. In this model, we use two alternative versions of bargaining models – the ‘Efficient Bargaining’ model of McDonald and Solow (1981) and the ‘Right to Manage’ model of Nickell and Andrews (1983).

We combine the ‘Efficiency Wage Hypothesis’ and union firm wage bargaining theories in a unified model because they may be either mutually reinforcing[53] or conflicting[54]. The reinforcing effect takes place because ‘Efficiency Wage Hypothesis’ makes it easier for the union to raise wage in a bargaining environment. The adverse effect of rent sharing is reduced because higher labour productivity is associated with higher wage. In contrast, the conflicting effect indicates that the greater is the labour union’s bargaining strength, the less are incentives for firms to drive up wages due to efficiency – wage consideration. Our analysis in this chapter provides support to the reinforcing effect hypothesis.

We derive interesting results from this model. In the ‘Efficient Bargaining’ model, unionisation in the labour market reduces the negotiated number of workers unless the labour union is highly employment oriented; but always raises workers’ effort level. As a result, effective employment, growth rate as well as the level of welfare must (may) increase for employment oriented and neutral (wage oriented) labour union. However, in the ‘Right to Manage’ model, unionisation raises worker’s effort level but lowers the number of workers irrespective of the orientation of the labour union; and raises effective employment, balanced growth rate and welfare level if and only if the wage elasticity of effort is greater than the unemployment rate. This

[53] See, for example, Summers (1988), Garino and Martin (2000), Meeusen et al. (2011).
[54] See, for example, Lindbeck and Snower (1991).

sufficient condition is always valid when the rate of income tax used to finance unemployment benefit expenditure is very low.

The chapter is organised as follows. In section 5.2, we describe the basic model and analyse the effect of unionisation with 'Efficient Bargaining'. In section 5.3, we do the same with a 'Right to Manage' model.

## 5.2 The 'Efficient Bargaining' Model

### 5.2.1 Firms

The representative competitive firm produces the final good, *Y*, using private capital, *K*, and effective labour, e*L*; and its production function[55] is given by

$$Y = AK^{\alpha}(eL)^{\beta}\bar{K}^{1-\alpha} \tag{5.2.1}$$

satisfying $\alpha, \beta, \alpha+\beta \in (0,1)$ .

Here $A > 0$ is a time independent technology parameter. $\bar{K}$ represents the average amount of capital stock of all firms available in the economy; and $0 < \alpha < 1$ ensures that external effect of capital is positive. Here *L* denotes the number of workers and *e* represents the efficiency (effort) level of the representative worker.[56] Existence of decreasing returns to private inputs in the production technology leads to a positive super normal profit in the competitive equilibrium; and this acts as the rent in the bargaining process to be negotiated between the employers' association and the labour union. Following Chang et al. (2007), we assume that a fixed quantity of land is necessary for a firm to operate; and thus the number of firms is fixed even in the presence of positive profit.[57]

We introduce the 'Efficiency Wage Hypothesis' which states that the efficiency level of a worker, *e*, varies positively with the premium of wage over his alternative reservation income. Since wage income as well as reservation income are taxed at equal rates, the relative wage income in terms of reservation income is independent of the tax rate. For simplicity, we assume that the

[55] This production function is identical to that in Chang et al. (2007) except for the fact that Chang et al. (2007) does not consider effort of workers, *e*.

[56] We assume that all workers have identical effort levels.

[57] Number of firms is normalized to unity. The equilibrium in the product market is always a short run competitive equilibrium with positive profit. Lai and Wang (2010) and Chang et al. (2007) also assume that union – firm bargaining takes place in competitive production sector. However, Adjemian et al. (2010), Bräuninger (2000b) and Lingens (2003b) assume a monopolistically competitive production sector; and Lingens (2003a) assumes a monopoly product market. Ramos-Parreño and Sánchez-Losada (2002) and Irmen and Wigger (2002/2003) consider monopoly labour union model.

reservation income is equal to the unemployment benefit per unemployed worker, $b$. So the worker's effort function is given by [58]

$$e = h\left(\frac{[1-\tau]w}{[1-\tau]b}\right)^{\delta} = h\left(\frac{w}{b}\right)^{\delta} \quad . \tag{5.2.2}$$

Here $\tau$ is the rate of income tax; and $h$ is a positive parameter representing worker's effort level when $\delta$ = 0. Here $\delta$ represents the elasticity of effort with respect to the relative wage rate; and it is assumed to satisfy 0 < $\delta$ < 1. Chang et al. (2007) does not consider 'Efficiency Wage Hypothesis'. In Chang et al. (2007), $e \equiv 1$, i.e., $\delta$ = 0 and $h$ = 1.

However, we consider a static efficiency function in which the relationship between efficiency, $e$, and relative wage, ($w/b$), is instantaneous. In a dynamic intertemporal model, it is more appropriate to consider a dynamic efficiency function at which efficiency level depends not only on the current wage but also on the past wage[59].

The firm maximises profit, $\pi$, defined as

$$\pi = Y - wL - rK \quad . \tag{5.2.3}$$

Here $w$ and $r$ represent the wage rate and the rental rate on capital respectively.

Capital market is perfectly competitive. The equilibrium value of the rental rate on capital is determined by the supply-demand equality in the capital market. The demand function for capital is derived from firms' profit maximisation exercise; and it is given by

$$r = \alpha A K^{\alpha-1}(eL)^{\beta}\bar{K}^{1-\alpha} = \frac{\alpha Y}{K} \quad . \tag{5.2.4}$$

### 5.2.2 Government

The government finances the unemployment benefit scheme by imposing an exogenously given rate of income tax, $\tau$; and balances its budget at each point of time. The budget balancing equation is given by

$$\tau Y + \tau b(1-L) = b(1-L) \quad . \tag{5.2.5}$$

Here $(1-L)$ is the unemployment level.

### 5.2.3 Labour Union and Bargaining

[58] Danthine and Kurmann (2006) has also used almost similar functional form.

[59] See, for example, Banerji and Gupta (1997), Ljungqvist (1993), Ray (1993), Ray and Streufert (1993) etc.

The labour union in this model derives utility from the hike in the wage rate over the unemployment benefit rate[60] as well as from the size of the membership. Since wage income and unemployment benefit income are taxed at equal rates, the trade union does not distinguish between post-tax income and pre-tax income. All employed workers are assumed to be members of the union. The utility function of the labour union is given by

$$u_T = ([1-\tau]w - [1-\tau]b)^m L^n = [1-\tau]^m (w-b)^m L^n \quad . \qquad (5.2.6)$$

Here $u_T$ stands for the level of utility of the labour union. $m$ and $n$ represent elasticities of labour union's utility with respect to wage premium and with respect to number of members respectively. The labour union is said to be 'wage oriented' ('employment oriented') ('neutral') if $m$ > (<) (=) $n$. Chang et al. (2007) contains a brief discussion about these parameters. Others works including Lingens (2003a, 2003b) and Adjemian et al. (2010) assume $m = n = 1$, i.e., the labour union to be neutral.

We now consider the 'Efficient Bargaining' model where the wage rate and the number of employed workers are determined jointly by the labour union and the employer's association. They maximise the generalised Nash product function given by

$$\psi = (u_T - \overline{u_T})^{\theta} (\pi - \bar{\pi})^{(1-\theta)} \quad \text{satisfying} \quad 0 < \theta < 1 \quad . \qquad (5.2.7)$$

Here $\overline{u_T}$ and $\bar{\pi}$ stand for the reservation utility level of the labour union and the reservation profit level of the firm respectively. Bargaining disagreement discontinues production process and this implies $\overline{u_T} = \bar{\pi} = 0$. The relative bargaining power of the labour union is represented by $\theta$. Unionisation is defined as an exogenous increase in the relative bargaining power of the labour union, i.e. in the value of $\theta$.

Finally, using equations (5.2.3), (5.2.6) and (5.2.7), we obtain

$$\psi = \{(1-\tau)^m (w-b)^m L^n\}^{\theta} \{Y - wL - rK\}^{(1-\theta)} \quad . \qquad (5.2.8)$$

Here $\psi$ is to be maximised with respect to $w$ and $L$. Using equations (5.2.1), (5.2.2), (5.2.4) and (5.2.5), and two first order conditions of optimisation, we solve for optimal $w$ and $L$.[61] These are given by

[60] Irmen and Wigger (2003), Lingens (2003a) and Lai and Wang (2010) assume that the difference between the bargained wage rate and the competitive wage rate is an argument in the labour union's utility function. Contrary to this, Adjemian et al. (2010) and Chang et al. (2007) consider the difference between the bargained wage rate and the unemployment benefit; and we follow them.

[61] Derivation of optimal $w$ and $L$ is provided in Appendix 5.A.

$$L^* = \frac{(1-\tau)\Theta_2\Theta_4}{(1-\tau)\Theta_2\Theta_4 + \tau\Theta_1\Theta_3} \quad ; \tag{5.2.9}$$

and

$$w^* = \frac{AK^\alpha h^\beta \overline{K}^{1-\alpha}(1-\tau)^{\beta-1}}{([1-\tau]\Theta_2\Theta_4 + \tau\Theta_1\Theta_3)^{\beta-1}}\left(\frac{{\Theta_2}^\beta {\Theta_3}^{\beta\delta}}{\Theta_1 {\Theta_4}^{1-\beta(1-\delta)}}\right) \quad . \tag{5.2.10}$$

Here,

$$\Theta_1 = (1-\theta+\theta n) > 0 \quad , \tag{5.2.11}$$

$$\Theta_2 = [\theta n(1-\alpha) + \beta(1-\theta)] \quad > 0 \quad , \tag{5.2.12}$$

$$\Theta_3 = [\theta n(1-\alpha-\beta) + \beta(1-\delta)(1-\theta+\theta n)] \quad > 0 \tag{5.2.13}$$

and

$$\Theta_4 = [\theta(n-m)(1-\alpha-\beta) + \beta(1-\delta)(1-\theta+\theta n)] \quad . \tag{5.2.14}$$

We assume $\Theta_4$ to be positive to ensure $0 < L^* < 1$. This assumption implies that the elasticity of union's utility with respect to relative wage cannot be far greater than the corresponding elasticity with respect to the size of membership. If the union is neutral or employment oriented, i.e., $m \leq n$, then $\Theta_4$ is always positive. From equation (5.2.9), we obtain

$$\frac{\partial L^*}{\partial \tau} = -\frac{\Theta_2\Theta_4\Theta_1\Theta_3}{[(1-\tau)\Theta_2\Theta_4 + \tau\Theta_1\Theta_3]^2} < 0 \quad . \tag{5.2.9.a}$$

Equation (5.2.9.a) shows that the number of employed workers varies inversely with the rate of income tax used to finance unemployment benefit. This is so because a rise in the tax rate raises unemployment benefit per worker; and this lowers union's utility from the wage hike. As a result, the wage rate is increased and the employment level is reduced.

Now, from equations (5.2.2), (5.2.5), (5.2.9) and (5.2.10), we obtain the effort level per worker as given by[62]

$$e^* = h\left(\frac{\Theta_3}{\Theta_4}\right)^\delta \quad . \tag{5.2.15}$$

From equations (5.2.9) and (5.2.15), we obtain effective level of employment i.e., the level of employment in efficiency unit. It is given by

$$e^*L^* = h\frac{(1-\tau)\Theta_2{\Theta_4}^{1-\delta}{\Theta_3}^\delta}{(1-\tau)\Theta_2\Theta_4 + \tau\Theta_1\Theta_3} \quad . \tag{5.2.16}$$

[62] Derivation is provided in Appendix 5.A.

### 5.2.4 Households

The representative household derives instantaneous utility only from consumption of the final good. She maximises her discounted present value of instantaneous utility over the infinite time horizon subject to the intertemporal budget constraint. The household's problem is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\, dt \tag{5.2.17}$$

subject to, $\dot{K} = (1-\tau)Y - c + (1-\tau)b(1-L)$ ; (5.2.18)

and $K(0) = K_0$ ($K_0$ is historically given) .

Here $c$ denotes the consumption level of the representative household; and $\sigma$ and $\rho$ are the two parameters representing the elasticity of marginal utility of consumption and the rate of discount respectively. It is assumed that the rate of unemployment is same for all households. The representative household saves and invests the rest of his income left after consumption and there is no depreciation of private capital.

Solving this dynamic optimisation problem, we obtain the growth rate of consumption as given by

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau)\alpha A K^{\alpha-1}(eL)^{\beta}\bar{K}^{1-\alpha} - \rho}{\sigma} \quad . \tag{5.2.19}$$

### 5.2.5 Equilibrium

We assume a symmetric equilibrium where $\bar{K} = K$, i.e., all firms have equal amount of capital; and hence, from equation (5.2.19), we obtain the growth rate of consumption given by

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau)\alpha A(eL)^{\beta} - \rho}{\sigma} \quad . \tag{5.2.20}$$

The economy is always in the steady state equilibrium; and so $g$ is always time-independent. It does not have transitional dynamic properties because this is an AK model. In equilibrium, all variables like number of workers, $L$, income tax rate, $\tau$, rental rate on capital, $r$, effort level of worker, $e$, and effective employment, $eL$, are time-independent. Capital stock, $K$, final output, $Y$, negotiated wage rate, $w^*$, firm's profit, $\pi$, and unemployment benefit, $b$, grow at equal rates in the steady-state equilibrium.

### 5.2.6 Effect of Unionisation

From equations (5.2.9), (5.2.11), (5.2.12), (5.2.13) and (5.2.14), we obtain

$$\frac{\partial L^*}{\partial \theta} = \frac{\tau(1-\tau)(1-\alpha-\beta)\left[(n-m)\{\beta(1-\delta)\Theta_1\Theta_2+n\theta(1-\alpha-\beta)\Theta_3\}-\beta^2\delta n(1-\delta){\Theta_1}^2\right]}{[(1-\tau)\Theta_2\Theta_4+\tau\Theta_1\Theta_3]^2} \quad . \qquad (5.2.21)$$

Equation (5.2.21) shows that the effect of unionisation on the employment of workers consists of two components. First component is union's orientation effect on employment. It is ambiguous in sign and depends on the nature of orientation of the labour union. Second component is the substitution effect on employment. An increase in worker's efficiency lowers the employer's demand for workers. So the second component is always negative. The net effect depends on the relative strength of these two effects. We find that employment orientation property of the labour union is necessary but not sufficient to establish a positive relationship between unionisation and the number of workers (members). When the labour union is wage oriented or even neutral, unionisation must reduce the number of workers. In Chang et al. (2007), the effect of unionisation on employment consists only of orientation effect, i.e., it depends only on the nature of orientation of the labour union.

When the labour union is neutral, i.e., $m = n$, then

$$\frac{\partial L^*}{\partial \theta} = -\frac{\tau(1-\tau)(1-\alpha-\beta)\beta^2\delta n(1-\delta)}{[(1-\tau)\Theta_2\beta(1-\delta)+\tau\Theta_3]^2} < 0 \quad \text{for} \quad 0<\delta<1 \quad . \qquad (5.2.21.a)$$

When the labour union is neutral, employment effect is nil in Chang et al. (2007). This is so because $\delta = 0$ in that model. However, in our model, $0 < \delta < 1$. So the effect on employment of workers is negative even if the labour union is neutral.

Now, from equations (5.2.13), (5.2.14) and (5.2.15), we obtain

$$\frac{\partial e^*}{\partial \theta} = \delta h \left(\frac{\Theta_3}{\Theta_4}\right)^{\delta-1} \frac{m(1-\alpha-\beta)\beta(1-\delta)}{{\Theta_4}^2} > 0 \quad . \qquad (5.2.22)$$

Equation (5.2.22) shows that the efficiency level of the representative worker varies positively with the degree of unionisation in the labour market. Negotiated wage rate is increased with the increase in the relative bargaining power of the labour union; and this induces the worker to put greater effort. This positive relationship between unionisation and effort level is valid only in the presence of 'Efficiency Wage Hypothesis'.

Again, from equations (5.2.11), (5.2.12), (5.2.13), (5.2.14) and (5.2.16), we obtain

$$\frac{\partial e^* L^*}{\partial \theta} = \frac{(n-m)(1-\tau)(1-\alpha-\beta)\tau h {\Theta_3}^{\delta+2}}{[(1-\tau)\Theta_2\Theta_4+\tau\Theta_1\Theta_3]^2 {\Theta_4}^{\delta}}$$

$$+\frac{h(1-\tau)\beta\delta(1-\delta)(1-\alpha-\beta)m\{(1-\tau){\Theta_2}^2\Theta_4+\tau\Theta_1\Theta_3\theta n(1-\alpha-\beta)\}}{{\Theta_3}^{1-\delta}{\Theta_4}^{\delta}[(1-\tau)\Theta_2\Theta_4+\tau\Theta_1\Theta_3]^2} \quad . \quad (5.2.23)$$

Equation (5.2.23) shows that unionisation affects effective employment through two channels – changing the number of workers (members) and changing the effort level of the representative worker. The effect on the number of workers depends partially on the orientation of the labour union. However, the other effect is originated from the rise in the effort level of the worker; and hence this effect is always positive. So employment orientation property or neutrality property of the labour union is sufficient but not necessary to establish a positive relationship between effective employment and unionisation in the presence of 'Efficiency Wage Hypothesis'. This implies that, in the presence of 'Efficiency Wage Hypothesis', unionisation may raise effective employment through a rise in the efficiency level even if the number of workers is reduced. However, in the absence of this hypothesis, i.e., when $\delta = 0$, unionisation does not raise workers' effort level; and its effect on employment (number of workers) depends solely on the orientation of the labour union.

Now, equation (5.2.20) shows that the balanced growth rate, $g$, varies positively with the level of effective employment. So the effect of unionisation on the growth rate is qualitatively similar to that on effective employment. From equation (5.2.20), we obtain

$$\frac{\partial g}{\partial \theta}=\left(\frac{(1-\tau)\alpha A\beta(e^*L^*)^{\beta-1}}{\sigma}\right)\frac{\partial e^*L^*}{\partial \theta} \quad . \quad (5.2.24)$$

Sign of $\frac{\partial g}{\partial \theta}$ depends on the sign of $\frac{\partial e^*L^*}{\partial \theta}$. In Chang et al. (2007), the nature of the growth effect of unionisation depends totally on the nature of orientation of the labour union because $e \equiv 1$ there. However, our model incorporates 'Efficiency Wage Hypothesis'; and so the effect on effective employment is crucial rather than the effect on the employment of workers.

The welfare level of the representative household, $\omega$, is obtained from equations (5.2.1), (5.2.17), (5.2.18) and (5.2.20); and it is given by

$$\omega=\frac{{K_0}^{1-\sigma}\left[\frac{\rho+\sigma g-(1-\tau)\alpha g}{\alpha(1-\tau)}\right]^{1-\sigma}}{1-\sigma}\left[\frac{1}{\rho-g(1-\sigma)}\right]+constant \quad . \quad (5.2.25)$$

Equation (5.2.25) shows that the level of welfare varies positively with the growth rate as we assume $1 > \sigma > (1-\tau)\,\alpha$ and $\rho > g(1\text{-}\sigma)$. These assumptions are made for technical simplicity in the literature on 'Endogenous Growth' theory. Hence the welfare effect of unionisation is qualitatively similar to its growth effect. However, this welfare effect is restricted to those households who include employed workers.

Chang et al. (2007) shows that the employment effect, growth effect and welfare effect of unionisation are nil when labour union is neutral. However, in our model, each of them is positive in the presence of 'Efficiency Wage Hypothesis' even in this case.

So we can establish the following proposition.

**Proposition 5.2.1:** In the presence of 'Efficiency Wage Hypothesis', unionisation in the labour market reduces the negotiated number of workers unless the labour union is highly employment oriented; but always raises workers' effort level. As a result, effective employment must (may) increase for employment oriented and neutral (wage oriented) labour union. The effect of unionisation on the growth rate as well as on the level of welfare are same as that on the effective employment.

## 5.3 The 'Right to Manage' Model

In this section, we use the 'Right to Manage' model of bargaining where the two parties bargain only over the wage rate. The firm unilaterally decides the level of employment from its labour demand function obtained from its profit maximising behaviour. The inverted labour demand function of the representative firm is given by

$$w = \left[\beta A K^{\alpha} \bar{K}^{1-\alpha} L^{\beta-1} h^{\beta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} \quad . \qquad (5.3.1)$$

So the firms' association and the labour union jointly maximise the 'generalised Nash product' function given by equation (5.2.8), with respect to *w* only, subject to the firm's labour demand function given by equation (5.3.1). Using the first order condition and equations (5.2.1), (5.2.2), (5.2.4), (5.2.5) and (5.3.1), optimum values of $L$ and $w$ are obtained as[63]

$$L^{**} = \frac{\beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\tau\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\} + \beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}} < 1 \quad ; \qquad (5.3.2)$$

and

$$w^{**} = A K^{\alpha} \bar{K}^{1-\alpha} h^{\beta} \beta^{1+\beta\delta} {L^{**}}^{\beta-1-\beta\delta} \tau^{-\beta\delta} (1-L^{**})^{\beta\delta} (1-\tau)^{\beta\delta} \quad . \qquad (5.3.3)$$

We assume

[63] We assume that second order condition of maximisation is satisfied.

$$\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}>\theta m(1-\beta)(1-\alpha-\beta)$$

to ensure that $L^{**}>0$.

From equations (5.2.1), (5.2.2), (5.2.5) and (5.3.3), we obtain the efficiency level of the representative worker as given by

$$e^{**}=h\left[\frac{\beta}{\tau}\frac{(1-L^{**})(1-\tau)}{L^{**}}\right]^{\delta} \quad . \tag{5.3.4}$$

The government's budget balance equation as well as the representative household's behaviour in this model is identical to that in the 'Efficient Bargaining' model. So equations and solutions derived here are same as those obtained in section 5.2 except that $L^{*}$ is replaced by $L^{**}$ and $e^{*}$ is replaced by $e^{**}$.

Now, from equation (5.3.2), we have

$$\frac{\partial L^{**}}{\partial\theta}=\frac{-(1-\tau)\tau m\beta^{2}(1-\delta)(1-\alpha-\beta)(1-\beta)^{2}}{\left\{\begin{matrix}\tau\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}\\+\beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}\end{matrix}\right\}^{2}}<0 \quad . \tag{5.3.5}$$

So, in this model, unionisation in the labour market lowers the number of workers irrespective of the orientation of the labour union.

Again, from equation (5.3.4), we obtain

$$\frac{\partial e^{**}}{\partial\theta}=-\delta h\left[\frac{\beta(1-\tau)}{\tau}\right]^{\delta}\left[\frac{(1-L^{**})}{L^{**}}\right]^{\delta-1}\frac{1}{L^{**2}}\frac{\partial L^{**}}{\partial\theta}\geq 0 \quad \text{for} \quad \delta\geq 0 \quad . \tag{5.3.6}$$

and

$$\frac{\partial(e^{**}L^{**})}{\partial\theta}=\frac{e^{**}(1-\delta-L^{**})}{(1-L^{**})}\frac{\partial L^{**}}{\partial\theta} \quad . \tag{5.3.7}$$

Equation (5.3.6) shows that, in the presence (absence) of 'Efficiency Wage Hypothesis', efficiency level of a worker goes up (does not change) with unionisation in the labour market. This result is similar to that obtained in the 'Efficient Bargaining' model. However, contrary to the 'Efficient Bargaining' model, equation (5.3.7) shows that the effect of unionisation on effective employment depends not on the orientation of the labour union but on the mathematical sign of $(1-\delta-L^{**})$. If $(1-\delta-L^{**})$ is negative (positive), then the level of effective employment varies positively (inversely) with unionisation in the labour market because the number of workers varies inversely with that unionisation. This implies that unionisation raises (lowers) effective employment if the rate of unemployment, $(1-L^{**})$, is less (greater) than the wage elasticity of effort, $\delta$.

The intuition behind this result is as follows. Unionisation affects effective employment by changing both the efficiency level of the representative worker and the number of workers. Since the first effect is positive and the second effect is negative, the aggregate effect depends on the relative strength of these two effects. When unionisation raises the wage rate and thereby the efficiency level of the worker, the wage elasticity of efficiency parameter, $\delta$, captures the strength of this effect. However, when unionisation raises the number of unemployed workers, the strength of this effect is captured by $(1 - L^{**})$. Hence $(1 - L^{**}) < \delta$ implies that the first effect dominates the second effect.

This condition $(1 - L^{**}) < \delta$ has an important implication for policy prescription. Using equation (5.3.2), we find that $(1 - L^{**}) < \delta$

$$\Rightarrow \quad \tau < \bar{\tau} = \frac{\beta\delta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\beta\delta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\} + (1-\delta)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}} \quad . \tag{5.3.8}$$

Here $\bar{\tau} > 0$ if

$$\theta(1-\alpha-\beta)[m(1-\beta) - n(1-\beta\delta)] < \beta(1-\delta)(1-\theta)(1-\beta) \quad .$$

If $m \leq n$, then this inequality is always valid.

So, if the tax rate is very low, then unionisation raises effective employment. The level of employment varies inversely with the tax rate, $\tau$. So a low value of $\tau$ leads to a low rate of unemployment such that a rise in the efficiency level of the representative worker compensates the fall in employment of workers due to unionisation. So if the rate of income tax used to finance unemployment benefit is very low, then unionisation may have a positive effect on effective employment in the presence of 'Efficiency Wage Hypothesis'. However, in the absence of 'Efficiency Wage Hypothesis', i.e., when $\delta$ = 0, unionisation always lowers employment level (number of workers).

The rate of growth and the level of welfare in this model are identical to those given by equations (5.2.20) and (5.2.25) in the 'Efficient Bargaining' model except that $L^{*}$ and $e^{*}$ are replaced by $L^{**}$ and $e^{**}$. So the effect of unionisation on the growth rate and on the welfare level are qualitatively similar to its effect on effective employment. So we can conclude that unionisation raises the growth rate and the welfare level if $\delta > (1 - L^{**})$. This result is different from that obtained in the case of 'Efficient Bargaining' model where the effect of unionisation on growth and welfare depends partly on the nature of orientation of the labour union.

Important results derived in this section are summarised in the following proposition.

**Proposition 5.3.1:** In the 'Right to Manage' model of bargaining, unionisation in the labour market raises the efficiency level of the representative worker but lowers the number of workers irrespective of the orientation of the labour union. However, it raises (lowers) effective employment, balanced growth rate and welfare level if the wage elasticity of effort is greater (less) than the unemployment rate.

## Appendix

### Appendix 5.A

<u>Derivation of optimal *w* and *L*:</u>

From equations (5.2.1) and (5.2.8), we obtain two first order conditions given by

$$\frac{\theta m}{(w-b)}+\frac{(1-\theta)\left[\beta\delta\frac{Y}{w}-L\right]}{Y-wL-rK}=0 \quad . \qquad (5.A.1)$$

$$\frac{\theta n}{L}+\frac{(1-\theta)\left[\beta\frac{Y}{L}-w\right]}{Y-wL-rK}=0 \quad . \qquad (5.A.2)$$

From equations (5.A.2) and (5.2.4), we obtain

$$\frac{Y}{wL}=\frac{(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]} \quad . \qquad (5.A.3)$$

From equations (5.A.1), (5.2.4) and (5.2.5), we obtain

$$\frac{\theta m}{1-\left(\frac{\tau Y}{w[1-L]}\right)}=\frac{(1-\theta)\left[1-\beta\delta\frac{Y}{wL}\right]}{\left[(1-\alpha)\frac{Y}{wL}-1\right]} \quad . \qquad (5.A.4)$$

Incorporating equation (5.A.3) in equation (5.A.4), we obtain

$$\frac{\theta m}{1-\left(\frac{\tau L}{[1-L]}\frac{(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]}\right)}=\frac{(1-\theta)\left[1-\frac{\beta\delta(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]}\right]}{\left[\frac{(1-\alpha)(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]}-1\right]} \quad .$$

(5.A.4a)

Solving equation (5.A.4a), we obtain the optimal value of *L* as given in equation (5.2.9) in the body of the chapter.

Now, using equations (5.2.1) and (5.2.5), we obtain

$$Y=\left[AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}L^{\beta}w^{\beta\delta}\tau^{-\beta\delta}(1-L)^{\beta\delta}\right]^{\frac{1}{1+\beta\delta}} \quad . \qquad (5.A.5)$$

Using equations (5.A.3) and (5.A.5), we obtain

$$\frac{\left[AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}L^{\beta}w^{\beta\delta}\tau^{-\beta\delta}(1-L)^{\beta\delta}\right]^{\frac{1}{1+\beta\delta}}}{wL}=\frac{(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]} \quad .$$

$$\Rightarrow \quad w = \left[AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}L^{\beta-(1+\beta\delta)}\tau^{-\beta\delta}(1-L)^{\beta\delta}\right]\left\{\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}^{1+\beta\delta} . \quad (5.A.6)$$

Using equations (5.A.6) and (5.2.9), we obtain the optimal value of *w* as given in equation (5.2.10) in the body of the chapter.

<u>Second order conditions:</u>

From equations (5.A.1) and (5.A.2), we obtain

$$\frac{\frac{\partial^2\psi}{\partial w^2}\psi-\left(\frac{\partial\psi}{\partial w}\right)^2}{\psi^2}$$

$$= -\frac{\theta m}{(w-b)^2}+\frac{(1-\theta)\left[\beta\delta(\beta\delta-1)\frac{Y}{w^2}(Y-wL-rK)-\left\{\beta\delta\frac{Y}{w}-L\right\}^2\right]}{(Y-wL-rK)^2} \quad ; \quad (5.A.7)$$

$$\frac{\frac{\partial^2\psi}{\partial L^2}\psi-\left(\frac{\partial\psi}{\partial L}\right)^2}{\psi^2}$$

$$= -\frac{\theta n}{L^2}+\frac{(1-\theta)\left[\beta(\beta-1)\frac{Y}{L^2}(Y-wL-rK)-\left\{\beta\frac{Y}{L}-w\right\}^2\right]}{(Y-wL-rK)^2} \quad ; \quad (5.A.8)$$

and

$$\frac{\frac{\partial^2\psi}{\partial L\partial w}\psi-\frac{\partial\psi}{\partial L}\frac{\partial\psi}{\partial w}}{\psi^2}=\frac{\left[\left(\beta^2\delta\frac{Y}{wL}-1\right)(Y-wL-rK)-\left\{\beta\delta\frac{Y}{w}-L\right\}\left\{\beta\frac{Y}{L}-w\right\}\right]}{(1-\theta)^{-1}(Y-wL-rK)^2} .$$

$$(5.A.9)$$

Using equations (5.2.1), (5.2.4), (5.2.5), (5.2.9), (5.2.11), (5.2.12), (5.2.13), (5.2.14), (5.A.3), (5.A.7), (5.A.8), (5.A.9) and $\frac{\partial\psi}{\partial L}=\frac{\partial\psi}{\partial w}=0$, we obtain respectively

$$\frac{\frac{\partial^2\psi}{\partial w^2}}{\psi}=-\frac{(1-\theta+\theta m)\Theta_3{}^2+(1-\alpha-\beta)(1-\theta)\beta\delta\Theta_1(1-\beta\delta)\theta m}{w^2(1-\alpha-\beta)^2\theta m(1-\theta)}<0 \ ; \quad (5.A.10)$$

$$\frac{\frac{\partial^2\psi}{\partial L^2}}{\psi}=-\frac{\theta n(1-\alpha-\beta)\Theta_1+\Theta_1\beta(1-\theta)(1-\beta)}{(1-\alpha-\beta)(1-\theta)L^2}<0 \quad ; \quad (5.A.11)$$

and

$$\frac{\frac{\partial^2\psi}{\partial L\partial w}}{\psi}=\frac{[\theta n+\beta(1-\theta)-\Theta_1\Theta_2]}{(1-\alpha-\beta)(1-\theta)wL} . \quad (5.A.12)$$

Now using equations (5.A.10), (5.A.11) and (5.A.12), we have

$$\frac{\frac{\partial^2\psi}{\partial w^2}\frac{\partial^2\psi}{\partial L^2}-\left(\frac{\partial^2\psi}{\partial L\partial w}\right)^2}{\psi^2}$$

$$=\frac{\begin{matrix}\left\{(1-\theta+\theta m){\Theta_3}^2+(1-\alpha-\beta)(1-\theta)\beta\delta\Theta_1(1-\beta\delta)\theta m\right\}\\ \{\theta n(1-\alpha-\beta)\Theta_1+\Theta_1\beta(1-\theta)(1-\beta)\}\\ -\{\theta n+\beta(1-\theta)-\Theta_1\Theta_2\}^2(1-\alpha-\beta)\theta m\end{matrix}}{w^2(1-\alpha-\beta)^3\theta m(1-\theta)^2L^2} \quad . \qquad (5.A.13)$$

We assume that the R.H.S. of equation (5.A.13) is positive in order to satisfy the second order conditions.

Derivation of equation (5.2.15):

From equations (5.2.2) and (5.2.5), we obtain

$$e=h\left(\frac{w(1-L)}{\tau Y}\right)^{\delta} \quad . \qquad (5.A.14)$$

Using equations (5.A.3) and (5.2.9), we obtain equation (5.2.15) in the body of the chapter.

Derivations of equations in the section 5.3 are similar to that in the section 5.2.

# Chapter 6: Efficiency Wage with Human Capital Accumulation

## 6.1 Introduction

The present chapter extends the previous chapter 5 by incorporating the government's role in human capital accumulation and thereby in raising the workers' efficiency. In most of the countries, the government spends a vast amount for education to raise the efficiency of workers. The government not only spends for primary education, secondary education and higher education but also spends for training of unskilled workers[64]. The figure 6.1 presented below[65] shows percentages of total government expenditure allocated for education in a few developed countries for the year 2011.

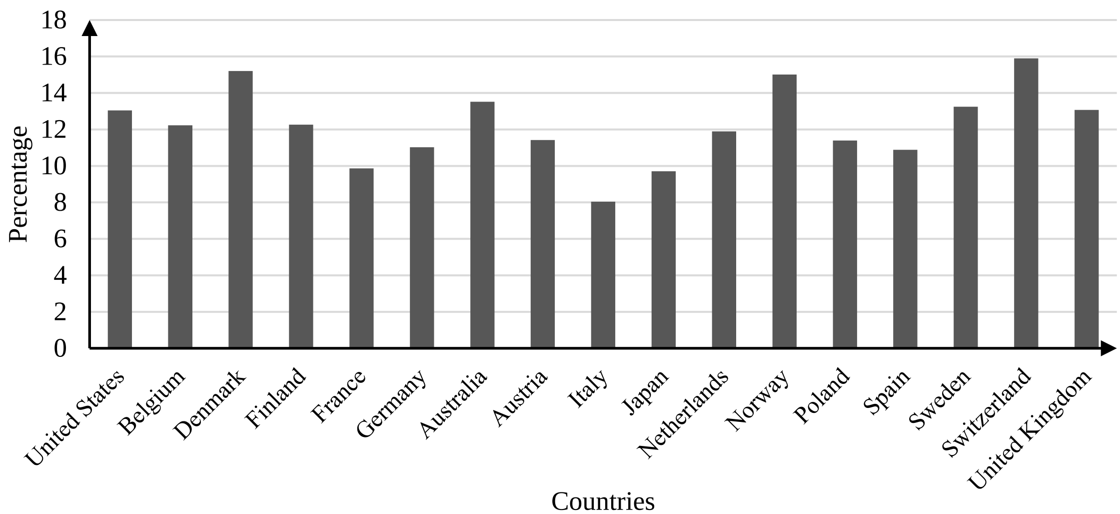


Figure - 6.1: Expenditure on education as a percentage of total government expenditure.

From the figure 6.1, government's priority towards skill formation can be easily understood. The budgeted share of education varies from country to country in between 8% and 16%. The government is a very powerful economic institution and can play an important role to raise the level of efficiency of workers. So in this chapter, we develop a simple endogenous growth model

[64] Government also spends for health development of the people to raise their efficiency. However, in this model we overlook the health aspect of workers. As a result, skill level becomes equivalent to the stock of human capital in this model.

[65] The graph is drawn on the basis of data provided by World Bank web-site.

with a special focus on the 'Efficiency Wage Hypothesis' and on the government's role in human capital accumulation to study the effect of unionisation on economic growth. Here we also analyse the properties of optimum income tax rate[66] to finance investment in human capital accumulation when the labour market is unionised. In this model, public education is financed by labour income taxation only. We use two different bargaining models to solve the negotiation problem between the employers' association and the labour union - 'Efficient Bargaining' model of McDonald and Solow (1981) and 'Right to Manage' model of Nickell and Andrews (1983).

Our main findings are as follows. First, in each of these two bargaining models, for a given tax rate on labour income, unionisation lowers the number of employed workers but raises their effort level. However, when the government imposes the growth rate maximising tax rate on labour income, then the number of employed workers becomes independent of labour union's bargaining power but varies inversely with the elasticity of efficiency with respect to human capital. Secondly, this growth rate maximising tax rate varies positively with the elasticity of worker's efficiency with respect to human capital as well as with the budget share of investment in human capital accumulation; and, on the other hand, varies inversely with the degree of unionisation in the labour market. Thirdly, the growth rate maximising tax rate is different from the corresponding welfare maximising tax rate; and the welfare effect of unionisation is also different from the growth effect of unionisation in each of these two bargaining models. Lastly, growth effect of unionisation consists of a positive effort effect and an ambiguous human capital accumulation effect. In the case of 'Efficient Bargaining' model, a value of the elasticity of worker's efficiency with respect to the wage premium higher than the value of that elasticity with respect to human capital is a sufficient but not a necessary condition to ensure a positive growth effect of unionisation. However, this condition is both necessary and sufficient in the case of 'Right to Manage' model.

Rest of the chapter is organized as follows. In section 6.2, we describe the basic model with 'Efficient Bargaining'; and analyse the existence, uniqueness and stability of the balanced growth equilibrium. We also analyse properties of growth rate maximising tax rate and the growth effect of unionisation in this section. In section 6.3, same issues are dealt with a 'Right to Manage' model.

---

[66] The optimal income tax rate to finance productive public expenditure in a unionised economy should be different from the optimal income tax rate to finance investment in human capital accumulation in a unionised economy. This is so because, the positive externality of productive public capital enjoyed by the private producers is independent of the number of employed workers. Contrary to this, the amount of benefit enjoyed by producers due to rise in the efficiency level of workers depends on the number of employed workers which is very much affected by unionisation in the labour market.

## 6.2 The Model

### 6.2.1 Production of Final Good

The representative competitive firm[67] produces the final good, $Y$, with the following production function[68].

$$Y = AK^{\alpha}(eL)^{\beta}\overline{K}^{\xi} \quad ; \qquad \alpha, \beta, \xi, \alpha+\beta \in (0,1) \qquad . \qquad (6.2.1)$$

Here $A > 0$ is a time independent technology parameter and $K$ denotes the amount of capital used by the representative firm. $eL$ represents firm's effective employment in efficiency unit where $L$ stands for the number of workers employed and $e$ stands for the efficiency per worker.[69] $\overline{K}$ symbolizes average quantity of capital stock existing in the economy; and $0 < \xi < 1$ ensures that the external effect of capital is positive. The Cobb – Douglas production function satisfies private diminishing returns. However, social returns to scale may not be diminishing. Decreasing returns to private inputs in the production function results into a positive profit in equilibrium; and this profit is the rent in the bargaining to be negotiated between the employers' association and the labour union. Following Chang et al. (2007), we assume that fixed amount of land is necessary to setup a firm; and as a result, the number of firms remains unchanged even in the presence of positive profit.[70]

We assume that net efficiency per worker, $e$, depends on his accumulated stock of efficiency, $e_1$, as well as on his effort level, $e_2$. Efficiency stock of a worker, $e_1$, varies positively with his level of human capital. This is consistent with the assumptions made in Lucas (1988), Uzawa (1965), Caballé and Santos (1993), Bucci (2008), Docquier et al. (2008) etc. His effort level, $e_2$, varies positively with his net wage relative to his net reservation income. This keeps consistency with the assumption made by the 'Efficiency Wage Hypothesis'. For simplicity, we assume that a worker's net reservation income is the after tax unemployment benefit given to an unemployed worker. So the worker's net efficiency, $e$, is given by [71]

---

[67] Following Chang et al. (2007), here also free entry assumption of perfect competition is restricted by the existence of a fixed factor land. Necessity of this assumption will be discussed in a little while.

[68] Chang et al. (2007) does not consider efficiency of workers, $e$. Otherwise, this production function is identical to that in Chang et al. (2007).

[69] It is assumed that all workers have identical efficiency level.

[70] Number of firms is normalized to unity.

[71] Danthine and Kurmann (2006) has also used similar functional form of effort function.

$$e = e_1 e_2 \quad . \tag{6.2.2}$$

Here

$$e_1 = h^{\eta} \quad \text{with } 0 < \eta < 1 \quad ; \tag{6.2.2.a}$$

and

$$e_2 = \left(\frac{[1-\tau]w}{[1-\tau]b}\right)^{\delta} = \left(\frac{w}{b}\right)^{\delta} \quad \text{with } 0 < \delta < 1 \quad . \tag{6.2.2.b}$$

Here $h$ and $w$ denote the level of human capital and the wage rate respectively; and $b$ stands for the rate of unemployment benefit. $\eta$ and $\delta$ represent elasticities of net efficiency with respect to the stock of human capital and with respect to the relative wage rate respectively; and they are assumed to be positive fractions. Chang et al. (2007) does not distinguish between labour time and labour efficiency. So, in Chang et al. (2007), $e \equiv e_1 \equiv e_2 \equiv 1$, i.e., $\eta = \delta = 0$.

The firm maximises profit, $\pi$, given by

$$\pi = Y - wL - rK \tag{6.2.3}$$

where $r$ represents rental rate on capital.

Capital market is perfectly competitive; and so the equilibrium value of rental rate on capital is determined by the supply-demand equality in this market. The inverted demand function for capital is obtained from firm's profit maximization exercise; and it is given by

$$r = \alpha A K^{\alpha-1}(eL)^{\beta}\overline{K}^{\xi} = \frac{\alpha Y}{K} \quad . \tag{6.2.4}$$

### 6.2.2 Government

The government finances investment in human capital accumulation (educational expenditure) as well as benefit given to unemployed workers. To finance these expenditures, a proportional tax on wage income as well as on unemployment benefit is imposed at the rate $\tau$; and the budget remains balanced at each point of time. The total tax revenue is allocated to these two types of expenditures in an exogenously given proportion.[72] For the sake of simplicity, it is also assumed that the rate of human capital accumulation is proportional to the educational expenditure of the government. So we have

$$\lambda[\tau w L + \tau b(1-L)] = \dot{h} \quad ; \tag{6.2.5}$$

[72] Since we do not consider any productive role of unemployment benefit in this model, so endogenous determination of this proportion by maximising the economic growth rate is beyond the scope of this model.

and

$$(1-\lambda)[\tau wL+\tau b(1-L)]=b(1-L) \quad . \qquad (6.2.6)$$

Here $(1-L)$ is the unemployment level and $\lambda$ is the fraction of revenue allocated to finance investment in human capital.

We consider taxation only on labour income but not on capital income. This is due to three reasons. First, both the channels of expenditure provide benefits to workers and not to capitalists. Secondly, taxation on capital income makes the analysis complicated. Thirdly, capital income taxation reduces the net marginal productivity of capital and thereby reduces the rate of growth. A set of works on public economics consisting of Bräuninger (2000a, 2005), Crossley and Low (2011), Landais et al. (2010), Davidson and Woodbury (1997) etc. also considers taxation only on wage income to finance unemployment benefit scheme.

### 6.2.3 Labour Union and 'Efficient Bargaining'

In this model, the labour union derives utility from the net wage premium defined as the difference between the after tax bargained wage rate and the after tax unemployment benefit rate[73] as well as from the number of members of the union. All employed workers are assumed to be members of the union.[74] The utility function of the labour union is defined as follows.

$$u_T=[(1-\tau)w-(1-\tau)b]^m L^n=(1-\tau)^m(w-b)^m L^n \quad \text{with} \quad m,n>0 \quad . \qquad (6.2.7)$$

Here $u_T$ symbolizes the level of utility of the labour union. Two parameters, $m$ and $n$ represent elasticities of labour union's utility with respect to wage premium and with respect to number of members respectively. If $m$ > (<) (=) $n$, then the labour union is said to be wage oriented (employment oriented) (neutral). Chang et al. (2007) contains a brief discussion of these parameters.

We now consider the 'Efficient Bargaining' model where both the wage rate and the number of employed workers are determined mutually by the labour union and the employer's association. To obtain these results of bargaining, we maximize the 'generalised Nash product' function given by

[73] In Irmen and Wigger (2002/2003), Lingens (2003a) and Lai and Wang (2010), the difference between the bargained wage rate and the competitive wage rate is an argument in the labour union's utility function. Contrary to this, in Adjemian et al. (2010) and Chang et al. (2007), the difference between the after tax bargained wage rate and the net unemployment benefit is an argument in the labour union's utility function. So, this work belongs to the second group.

[74] This is due to our assumption of a closed shop labour union.

$$\psi = ( u_T - \overline{u_T})^{\theta}(\pi - \bar{\pi})^{(1-\theta)} \quad \text{satisfying} \quad 0 < \theta < 1 \quad . \qquad (6.2.8)$$

Here $\overline{u_T}$ and $\bar{\pi}$ represent the reservation utility level of the labour union and the reservation profit level of the firm respectively. $\overline{u_T}$ and $\bar{\pi}$ are assumed to be zero as, bargaining disagreement stops production and hence employment. $\theta$ represents the relative bargaining power of the labour union. Unionisation is defined as an exogenous increase in the value of $\theta$.

Now, using equations (6.2.3), (6.2.7) and (6.2.8), we obtain

$$\psi = \{(1-\tau)^m (w-b)^m L^n\}^{\theta}\{Y - wL - rK\}^{(1-\theta)} \quad . \qquad (6.2.9)$$

Here $\psi$ is to be maximised with respect to $w$ and $L$. The first order conditions of maximization are given by

$$\frac{\theta m}{w-b} + \frac{(1-\theta)}{[Y - wL - rK]}\left\{\frac{\beta\delta Y}{w} - L\right\} = 0 \quad ; \qquad (6.2.10)$$

and

$$\frac{\theta n}{L} + \frac{(1-\theta)}{[Y - wL - rK]}\left\{\frac{\beta Y}{L} - w\right\} = 0 \quad . \qquad (6.2.11)$$

Using equations (6.2.4) and (6.2.11) we obtain

$$\frac{wL}{Y} = \frac{[\theta n(1-\alpha) + \beta(1-\theta)]}{(1-\theta+\theta n)} \quad . \qquad (6.2.11.a)$$

Equation (6.2.11.a) shows that the labour share of income is time independent and it varies positively with the relative bargaining power of the union.[75] If the labour union has no bargaining power, i.e., if $\theta$ = 0, then this labour share of income is equal to its competitive share, i.e. $\beta$. However, if the labour union is a monopolist, i.e., if $\theta$ = 1, then it takes away all the income left after paying return on capital; and hence the labour share is equal to (1-$\alpha$).

Using equations (6.2.1), (6.2.2), (6.2.2.a), (6.2.2.b), (6.2.4), (6.2.6), (6.2.10) and (6.2.11), we obtain[76]

$$L^* = \frac{[1-(1-\lambda)\tau]}{[1-(1-\lambda)\tau] + \Theta_5(1-\lambda)\tau} \quad ; \qquad (6.2.12)$$

and

$$w^* = b\Theta_5 \quad , \qquad (6.2.13)$$

where,

---

75 $\frac{\partial\left(\frac{wL}{Y}\right)}{\partial\theta} = \frac{n(1-\alpha-\beta)}{(1-\theta+\theta n)^2} > 0$ .

76 See appendix 6.A for derivation of optimal *w* and *L*.

$$\Theta_5 = \frac{[\theta n(1-\alpha-\beta) + \beta(1-\delta)(1-\theta+\theta n)]}{[\theta(n-m)(1-\alpha-\beta) + \beta(1-\delta)(1-\theta+\theta n)]} \quad . \tag{6.2.14}$$

$\Theta_5$ represents the equilibrium value of the negotiated wage rate relative to the unemployment benefit rate. We assume the denominator of $\Theta_5$ to be positive in order to ensure $0 < L^* < 1$. When the labour union is neutral or employment oriented, i.e., when $m \leq n$, the denominator of $\Theta_5$ is always positive. However, when the union is wage oriented, i.e., when $m > n$, $\Theta_5 > 0$ implies that the labour union can not be highly biased for wage premium. This assumption also implies that $\Theta_5 > 1$,[77] which further implies that $w^* > b$. Now from equations (6.2.2.b) and (6.2.13), we obtain the effort level per worker as given by

$$e_2{}^* = (\Theta_5)^\delta \quad . \tag{6.2.15}$$

Equation (6.2.12) shows that $L^*$ varies inversely with $\Theta_5$. As $\Theta_5$ is increased, the union claims for a higher wage; and so the number of employed workers is reduced. Equation (6.2.12) also shows that $L^* \to 1$ as $(1-\lambda) \to 0$. This implies that unemployment does not exist when there is no unemployment benefit. The number of employed workers, $L^*$, varies inversely with $(1-\lambda)$ as well as with $\tau$. It can be easily shown that

$$\frac{\partial L^*}{\partial \tau} = -\frac{(1-\lambda)\Theta_5}{\{[1-(1-\lambda)\tau] + \Theta_5(1-\lambda)\tau\}^2} < 0 \quad ; \tag{6.2.16}$$

and

$$\frac{\partial L^*}{\partial \lambda} = \frac{\tau\Theta_5}{\{[1-(1-\lambda)\tau] + \Theta_5(1-\lambda)\tau\}^2} > 0 \quad . \tag{6.2.17}$$

As the tax rate is increased and the proportion for funding unemployment benefit remains unchanged, unemployment benefit per worker, $b$, is also increased. This unemployment benefit is the reservation income of the worker. So the labour union wants a higher wage rate and the employer lowers the number of employed workers in this case. By the similar logic, the number of employed workers is reduced when the proportion for funding unemployment benefit is increased but the tax rate remains unchanged.

Now from equation (6.2.14), we obtain

$$\frac{\partial \Theta_5}{\partial \theta} = \frac{m\beta(1-\alpha-\beta)(1-\delta)}{[\theta(n-m)(1-\alpha-\beta) + \beta(1-\delta)(1-\theta+\theta n)]^2} > 0 \quad ; \tag{6.2.18}$$

---

[77] If the denominator of $\Theta_5$ is positive, then $\Theta_5$ is greater than unity as the numerator of $\Theta_5$ is obviously greater than the denominator of $\Theta_5$.

and from equation (6.2.15), we have

$$\frac{\partial {e_2}^*}{\partial \theta} = \delta(\Theta_5)^{\delta-1}\frac{\partial \Theta_5}{\partial \theta} > 0 \qquad . \qquad (6.2.19)$$

As the labour union becomes more powerful, it claims a higher wage relative to the alternative income of the worker. As a result of this, equation (6.2.19) implies that the effort level per worker varies positively with the degree of unionisation.

Now, from equations (6.2.12) and (6.2.18), we obtain

$$\frac{\partial L^*}{\partial \theta} = -\frac{[1-(1-\lambda)\tau](1-\lambda)\tau}{\{[1-(1-\lambda)\tau]+\Theta_5(1-\lambda)\tau\}^2}\frac{\partial \Theta_5}{\partial \theta} < 0 \qquad . \qquad (6.2.20)$$

Equation (6.2.20) shows that, given the tax rate, the negotiated number of employed workers varies inversely with the degree of unionisation. This is so because unionisation raises the negotiated wage rate as well as the ratio of that wage to the unemployment benefit; and, as a result, effort level per worker is increased[78]. This rise in the wage rate reduces the demand for labour and the rise in worker's effort level substitutes the number of employed workers. As a result, number of employed workers declines due to unionisation. We summarize this result in the following proposition.

**Proposition 6.2.1:** For a given tax rate, unionisation lowers the number of employed workers but raises the wage rate as well as the effort level of the worker irrespective of the orientation of the labour union.

Lingens (2003a, 2003b) and Adjemian et al. (2010) consider a neutral labour union and show that unionisation reduces the number of employed workers due to rise in the wage rate. On the contrary, Chang et al. (2007) shows that unionisation does not necessarily lower the number of employed workers; and the employment effect of unionisation is positive for an employment oriented labour union. However, we consider 'Efficiency wage hypothesis' and show that unionisation leads to a decline in the number of employed workers irrespective of the orientation of the labour union. Our result is due to the substitution effect resulting from an increase in the effort of the worker.

[78] See equation (6.2.19).

### 6.2.4 The Representative Household

The representative household derives instantaneous utility only from consumption of the final good[79]. She maximises her discounted present value of instantaneous utility over the infinite time horizon subject to her intertemporal budget constraint. So her dynamic optimisation problem is defined as follows.

$$Max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{6.2.21}$$

subject to, $\dot{K} = (1-\tau)wL + rK + \pi + (1-\tau)b(1-L) - c$ ; (6.2.22)

and $K(0) = K_0$ ($K_0$ is historically given) .

Here $c$ is the consumption level of the representative household; and $\sigma$ and $\rho$ are two parameters representing elasticity of marginal utility of consumption and the rate of discount respectively. We assume entire savings to be invested and rule out depreciation of capital. We also assume that unemployment rate is equal among households. Here $c$ is the control variable and $K$ is the state variable.

Solving this dynamic optimisation problem, we derive the rate of growth of consumption as given by[80]

$$g = \frac{\dot{c}}{c} = \frac{\alpha A K^{\alpha-1}(eL)^{\beta}\bar{K}^{\xi} - \rho}{\sigma} \quad . \tag{6.2.23}$$

### 6.2.5 Existence and Stability of Steady State Equilibrium

The symmetric steady state growth equilibrium satisfies following properties:

(i) $\frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{\dot{h}}{h} = \frac{\dot{Y}}{Y} = \frac{\dot{w}^*}{w^*} = \frac{\dot{\pi}}{\pi} = \frac{\dot{b}}{b} = g$ ;

(ii) $K = \bar{K}$; and

(iii) $r$, $L^*$, $\tau$, $\lambda$, ${e_2}^*$ and $g$ are time independent. To ensure that $h$, $K$ and $Y$ grow at the same rate, i.e., to satisfy property (i), we further assume that $\xi = 1 - \alpha - \beta\eta$. This implies that the production function satisfies the property of social constant returns to scale.

[79] For technical simplicity, we assume that the representative household does not obtain utility from her human capital stock. In reality, people enjoy good health as well as respect from others due to his/her skill level.

[80] See appendix 6.A for derivation of equation (6.2.23).

Using equations (6.2.1), (6.2.2), (6.2.2.a), (6.2.5), (6.2.6), (6.2.11.a), (6.2.15), (6.2.22), (6.2.23), and putting $\xi = 1 - \alpha - \beta\eta$, $L = L^*$ and $K = \overline{K}$, we obtain

$$g = \frac{\dot{c}}{c} = \frac{\alpha A L^{*\beta}\left(\frac{h}{K}\right)^{\beta\eta}[\Theta_5]^{\beta\delta} - \rho}{\sigma} \quad ; \tag{6.2.24}$$

$$g = \frac{\dot{h}}{h} = \frac{\lambda\tau[\theta n(1-\alpha) + \beta(1-\theta)]A\left(\frac{K}{h}\right)^{1-\beta\eta} L^{*\beta}[\Theta_5]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)} \quad ; \tag{6.2.25}$$

and

$$g = \frac{\dot{K}}{K} = AL^{*\,\beta}\left(\frac{h}{K}\right)^{\beta\eta}[\Theta_5]^{\beta\delta}\left[1 - \frac{\lambda\tau[\theta n(1-\alpha) + \beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right] - \frac{c}{K} \quad . \tag{6.2.26}$$

We define two new variables *M* and *N* such that *M* = (*c/K*) and *N* = (*h/K*). So using equations (6.2.24), (6.2.25) and (6.2.26), we obtain

$$\frac{\dot{M}}{M} = \frac{\alpha A L^{*\beta}(N)^{\beta\eta}[\Theta_5]^{\beta\delta} - \rho}{\sigma} - AL^{*\,\beta}(N)^{\beta\eta}[\Theta_5]^{\beta\delta}\left[1 - \frac{\lambda\tau[\theta n(1-\alpha) + \beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right] + M \quad ; \tag{6.2.27}$$

and

$$\frac{\dot{N}}{N} = \frac{\lambda\tau[\theta n(1-\alpha) + \beta(1-\theta)]A(N)^{\beta\eta-1}L^{*\beta}[\Theta_5]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)} - AL^{*\,\beta}(N)^{\beta\eta}[\Theta_5]^{\beta\delta}\left[1 - \frac{\lambda\tau[\theta n(1-\alpha) + \beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right] + M \quad . \tag{6.2.28}$$

In the steady state growth equilibrium, $\frac{\dot{M}}{M} = \frac{\dot{N}}{N} = 0$; and this implies that

$$\frac{\alpha A L^{*\beta}(N)^{\beta\eta}[\Theta_5]^{\beta\delta} - \rho}{\sigma} = \frac{\lambda\tau[\theta n(1-\alpha) + \beta(1-\theta)]A(N)^{\beta\eta-1}L^{*\beta}[\Theta_5]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)} \quad . \tag{6.2.29}$$

Equation (6.2.29) is solely a function of *N*. We now turn to show the existence and uniqueness of the steady state equilibrium; i.e., a unique solution of equation (6.2.29). For this purpose, we use a diagram. In figure 6.2, L.H.S. and R.H.S. of equation (6.2.29) are measured on the vertical axis and *N* on the horizontal axis.

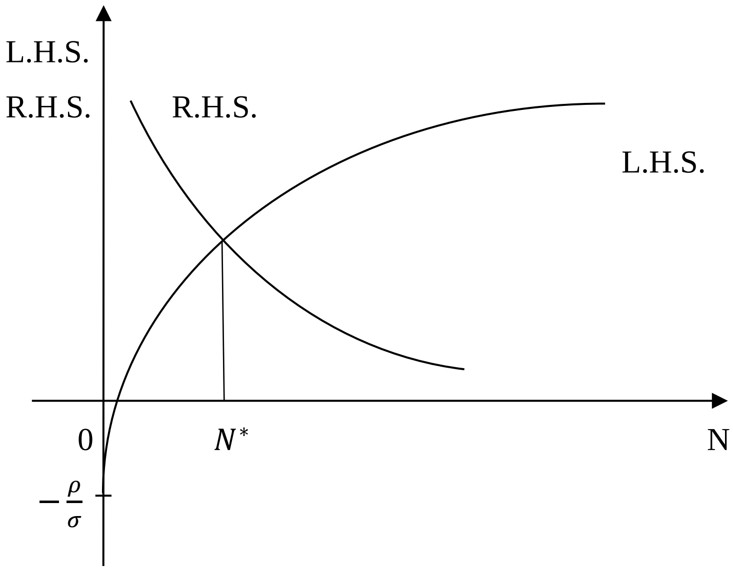


Figure - 6.2: Existence of a unique steady state equilibrium.

The L.H.S. curve is positively sloped and is concave to the horizontal axis with a point of intersection on that axis. However the R.H.S. curve is negatively sloped, convex to the origin and asymptotic to both axes. The unique point of intersection of these two curves at $N^*$ shows the existence of a unique steady state growth equilibrium.

To analyse stability of the system, we use equations (6.2.27) and (6.2.28). The mathematical sign of the Jacobian determinant, given by

$$|J| = \begin{vmatrix} \frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial M} & \frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial N} \\ \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial M} & \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial N} \end{vmatrix} \quad ,$$

is to be evaluated. It can be easily shown that[81]

$$|J| = -\left[(1-\beta\eta)\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]A(N)^{\beta\eta-2}{L^*}^{\beta}[\Theta_5]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)} + \frac{\beta\eta\alpha A{L^*}^{\beta}(N)^{\beta\eta-1}[\Theta_5]^{\beta\delta}}{\sigma}\right] < 0 \quad ; \qquad (6.2.30)$$

[81] See appendix 6.A for derivation of equation (6.2.30).

and the negative sign of $|J|$ implies that the two latent roots of $J$ matrix are of opposite sign. This implies that the unique steady state growth equilibrium is saddle point stable.

### 6.2.6 Growth Rate Maximising Tax Rate

We assume that the government wants to maximise the rate of growth in the steady state equilibrium[82]; and now turn to analyse properties of growth rate maximising tax rate. Substituting (*h/K*) from equation (6.2.25) into equation (6.2.24), we obtain

$$(\rho+\sigma g)[g]^{\frac{\beta\eta}{1-\beta\eta}}=\alpha\left[AL^{*\beta}{\Theta_5}^{\beta\delta}\right]^{\frac{1}{1-\beta\eta}}\left\{\lambda\tau\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right\}^{\frac{\beta\eta}{1-\beta\eta}}. \quad (6.2.31)$$

The L.H.S. of equation (6.2.31) is a monotonically increasing function of $g$. So the tax rate, which maximises the R.H.S. of equation (6.2.31), also maximises the growth rate. So from equations (6.2.12) and (6.2.31), we obtain the growth rate maximising tax rate as given by

$$\tau^*=\frac{\eta}{[(1-\eta)\Theta_5+\eta](1-\lambda)} \quad . \quad (6.2.32)$$

Equation (6.2.32) shows that $\tau^{*}$[83] varies positively with $\eta$ and $\lambda$. If human capital is not productive, i.e., if $\eta=0$, then no tax should be imposed in order to maximise the growth rate. Equation (6.2.31) clearly shows that the rate of growth varies inversely with the tax rate when $\eta=0$. This is so because $L^*$ varies inversely with $\tau$. Again, from equation (6.2.32), we obtain

$$\frac{\partial\tau^*}{\partial\theta}=-\frac{\eta(1-\eta)}{[(1-\eta)\Theta_5+\eta]^2(1-\lambda)}\frac{\partial\Theta_5}{\partial\theta}<0 \quad . \quad (6.2.33)$$

Equation (6.2.33) shows that growth rate maximising tax rate varies inversely with the degree of unionisation in the labour market. The intuition behind this result is as follows. The change in tax rate has two opposite effects on the growth rate. The first effect works by reducing employment level and the second effect works by increasing human capital accumulation. These two effects balances each other at $\tau=\tau^*$. Now, a rise in $\theta$ lowers employment level; and to raise it back to its previous level, $\tau^*$ should decline due to the inverse relationship between $L^*$ and $\tau$. So the growth

[82] Usually it is assumed that the objective of the government is to maximise social welfare. However, for technical simplicity, here we consider growth rate maximization. Agénor and Neanidis (2014) also focuses on growth rate maximisation rather than on welfare maximisation on the ground that, in practice, imperfect knowledge about household preferences makes it easier to measure their income level rather than their welfare level.

[83] We assume that the second order condition is satisfied.

rate maximising tax rate is reduced due to unionisation in this case. These properties of growth rate maximising tax rate is summarised in the following proposition.

**Proposition 6.2.2:** The growth rate maximising tax rate on labour income, on the one hand, varies positively with the elasticity of efficiency with respect to human capital as well as with the budget share of investment in human capital accumulation; and, on the other hand, varies inversely with the degree of unionisation in the labour market.

Incorporating the value of $\tau^*$ from equation (6.2.32) in equation (6.2.12), we obtain

$$L^* = 1 - \eta \quad . \tag{6.2.34}$$

Equation (6.2.34) shows that the rate of employment of workers is independent of the degree of unionisation when government imposes the growth rate maximising tax rate; and it varies inversely with the elasticity of efficiency with respect to human capital stock. This is so because unionisation has two different effects on employment. One is the direct effect; and the other is the indirect effect operating through the change in the tax rate. Equations (6.2.20) and (6.2.33) show that both $L^*$ and $\tau^*$ vary in the same direction with unionisation; and equation (6.2.16) shows that $L^*$ varies inversely with $\tau^*$. As a result, these two effects of unionisation on $L^*$ cancel out each other; and thus employment level becomes independent of the degree of unionisation. $L^*$ varies inversely with $\eta$ because a higher value of $\eta$ indicates a higher level of efficiency and the efficiency gain always substitutes the number of employed workers. This is stated in the following proposition.

**Proposition 6.2.3:** When the government imposes the growth rate maximising tax rate on labour income, rate of employment becomes independent of the degree of unionisation in the labour market but varies inversely with the elasticity of efficiency with respect to human capital.

The welfare level of the representative household, $\omega$, is defined as her discounted present value of instantaneous utility over the infinite time horizon. It is obtained from equations (6.2.1), (6.2.3), (6.2.6), (6.2.11.a), (6.2.21), (6.2.22) and (6.2.23) and is given by[84]

[84] See appendix 6.A for derivation of equation (6.2.35).

$$\omega = \frac{K_0{}^{1-\sigma}\left[\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau}{[1-(1-\lambda)\tau]}\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}-g\right]^{1-\sigma}}{(1-\sigma)[\rho-g(1-\sigma)]} + \ constant \quad . \qquad (6.2.35)$$

We assume $1 > \sigma$ and $\rho > g(1\text{-}\sigma)$. Since initial consumption $c_0$ is positive, so $\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau}{[1-(1-\lambda)\tau]}\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}$ has to be greater than $g$. Here, we do not attempt to derive the welfare maximising income tax rate on labour income for technical complexity. Rather, here we check whether the growth rate maximising income tax rate on labour income, given by equation (6.2.32), is identical to the welfare maximising labour income tax rate or not. For this purpose, we differentiate $\omega$ with respect to $\tau$ at $\tau = \tau^*$ and obtain

$$\left.\frac{\partial\omega}{\partial\tau}\right|_{\tau=\tau^*} = -\frac{K_0{}^{1-\sigma}\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{\frac{[\theta n(1-\alpha)+\beta(1-\theta)]\lambda}{(1-\theta+\theta n)[1-(1-\lambda)\tau^*]^2}\right\}[\rho-g^*(1-\sigma)]^{-1}}{\left[\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{1-\frac{\lambda\tau^*}{[1-(1-\lambda)\tau^*]}\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}-g^*\right]^{\sigma}} < 0 \quad . \qquad (6.2.36)$$

Here $g^* = g|_{\tau=\tau^*}$. Equation (6.2.36) implies that the welfare maximising tax rate on labour income is lower than the growth rate maximising tax rate. This is so because, given the allocation of tax revenue between investment in human capital accumulation and unemployment subsidy, initial consumption level of the representative household falls with increase in the labour income tax rate. Since the economic growth rate in the steady state equilibrium does not depend on the level of initial consumption[85], so the growth rate maximising tax rate, $\tau^*$, does not take into account this negative effect of taxation on initial consumption. On the other hand, welfare level depends on the level of initial consumption; and so the welfare maximising labour income tax rate takes into account this negative effect. This result is stated in the following proposition.

**Proposition 6.2.4:** Welfare maximising tax rate on labour income is lower than the corresponding growth rate maximising tax rate in the presence of public investment in human capital accumulation.

### 6.2.7 Effect of Unionisation

---

[85] See equation (6.2.31).

We now turn to analyse the effect of an increase in $\theta$ on the endogenous growth rate when the government charges the growth rate maximising labour income tax rate[86]. Using equations (6.2.31) and (6.2.32), we obtain

$$(\rho+\sigma g^*)[g^*]^{\frac{\beta\eta}{1-\beta\eta}}$$
$$=A^{\frac{1}{1-\beta\eta}}\alpha(1-\eta)^{\frac{\beta}{1-\beta\eta}}[\Theta_5]^{\frac{\beta\delta}{1-\beta\eta}}\left\{\frac{\eta\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\eta)\Theta_5(1-\lambda)(1-\theta+\theta n)}\right\}^{\frac{\beta\eta}{1-\beta\eta}}. \quad (6.2.37)$$

From equation (6.2.37), we have

$$\left[\frac{\sigma g^*}{(\rho+\sigma g^*)}+\frac{\beta\eta}{1-\beta\eta}\right]\frac{\frac{\partial g^*}{\partial\theta}}{g^*}=\left(\frac{\beta\eta}{1-\beta\eta}\right)\left\{\frac{n(1-\alpha-\beta)}{(1-\theta+\theta n)[\theta n(1-\alpha)+\beta(1-\theta)]}\right\}$$
$$-\frac{\beta^2 m\eta(1-\alpha-\beta)(1-\delta)}{(1-\beta\eta)[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}$$
$$+\frac{\beta^2 m\delta(1-\alpha-\beta)(1-\delta)}{(1-\beta\eta)[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}. \quad (6.2.38)$$

Equation (6.2.38) shows that the growth effect of unionisation is ambiguous. It consists of two effects – (i) the effort effect and (ii) the human capital accumulation effect. The first effect is operated through the change in the effort level of the worker. It is positive and is captured by the third term in the R.H.S. of equation (6.2.38). The second effect is operated through the change in the rate of human capital accumulation. It is ambiguous in sign and is captured by the first term as well as by the second term in the R.H.S. of equation (6.2.38). On the one hand, unionisation raises labour share of income and thereby the tax base[87]. This positive effect is captured by the first term. However, on the other hand, unionisation lowers the growth rate maximising tax rate; and this negative effect is captured by the second term. So the net effect on tax revenue generation is ambiguous. Since a fixed fraction of tax revenue is spent to finance human capital accumulation, the effect on human capital accumulation is also ambiguous. If human capital is not productive, i.e., if $\eta=0$, then only the positive effort effect remains and unionisation always raises the rate of economic growth. Similarly, if the effort level is independent of the wage rate, i.e., if $\delta=0$, then the third term is vanished and the effect of unionisation depends only on the human capital accumulation effect. However, if we ignore the entire dynamic 'Efficiency Wage Hypothesis', i.e.,

[86] Since we cannot derive the welfare maximising labour income tax rate, so we are unable to derive the growth effect of unionisation when the government charges the welfare maximising labour income tax rate.
[87] See footnote 75.

if we assume that $\delta = \eta = 0$, then unionisation does not affect the growth rate of the economy. This result is valid regardless of the nature of orientation of the labour union. This happens because unionisation does not affect the level of employment when government chooses the growth rate maximising tax rate.

Combining the second and the third term in the R.H.S. of equation (6.2.38), we find that the positive work effort effect dominates the negative component of human capital accumulation effect if the elasticity of worker's efficiency with respect to the wage premium rate, $\delta$, is higher than the elasticity of worker's efficiency with respect to the stock of human capital, $\eta$. So unionisation in this case is definitely growth generating as the other component of human capital accumulation effect is always positive. However, the converse is not necessarily true. So, $\delta > \eta$ is a sufficient condition but not a necessary condition to ensure positive growth effect of unionisation. These results are summarised in the following proposition.

**Proposition 6.2.5:** Growth effect of unionisation consists of a positive work effort effect and an ambiguous human capital accumulation effect. If the elasticity of worker's efficiency with respect to the stock of human capital is not higher than the elasticity of worker's efficiency with respect to the wage premium, then unionisation always raises the economic growth rate.

Now, we analyse the effect of unionisation in the labour market on the welfare level of the representative household, $\omega$, when the government imposes the economic growth rate maximising tax rate on labour income. For this purpose, we use equations (6.2.32) and (6.2.35); and obtain

$$\left.\frac{\partial \omega}{\partial \theta}\right|_{\tau=\tau^*}$$

$$= \frac{\partial g^*}{\partial \theta} \frac{\left[\left\{\frac{\rho + \sigma g^*}{\alpha}\right\}\left\{1 - \frac{\lambda\eta[\theta n(1-\alpha) + \beta(1-\theta)]}{\Theta_5(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\} - g^*\right]^{1-\sigma}}{{K_0}^{\sigma-1}[\rho - g^*(1-\sigma)]} \left\{\frac{1}{[\rho - g^*(1-\sigma)]}\right.$$

$$\left. + \frac{\left[\frac{\sigma}{\alpha}\left\{1 - \frac{\lambda\eta[\theta n(1-\alpha) + \beta(1-\theta)]}{\Theta_5(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\} - 1\right]}{\left[\left\{\frac{\rho + \sigma g^*}{\alpha}\right\}\left\{1 - \frac{\lambda\eta[\theta n(1-\alpha) + \beta(1-\theta)]}{\Theta_5(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\} - g^*\right]}\right\}$$

$$- \frac{\left\{\frac{(n-m)(1-\alpha-\beta)\theta n[\theta n(1-\alpha-\beta)+2\beta(1-\delta)(1-\theta+\theta n)]+\beta^2(1-\delta)(1-\theta+\theta n)^2[n(1-\delta)-m]}{(1-\theta+\theta n)^2[\theta n(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}\right\}}{\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}^{-1}\left[\frac{\lambda\eta(1-\alpha-\beta)}{(1-\eta)(1-\lambda)}\right]^{-1}{K_0}^{\sigma-1}\left[\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{1-\frac{\lambda\eta[\theta n(1-\alpha)+\beta(1-\theta)]}{\Theta_5(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\}-g^*\right]^{\sigma}[\rho-g^*(1-\sigma)]} \quad . \qquad (6.2.39)$$

Equation (6.2.39) shows that welfare effect of unionisation consists of two effects. One of them is the growth effect of unionisation and it is captured by the first term in the R.H.S. of equation (6.2.39). The second effect comes from the change in initial consumption level of the household due to change in the educational expenditure; and it is captured by the second term in the R.H.S. of equation (6.2.39). This effect is ambiguous because the term $\left[(n-m)(1-\alpha-\beta)\theta n[\theta n(1-\alpha-\beta)+2\beta(1-\delta)(1-\theta+\theta n)]+\beta^2(1-\delta)(1-\theta+\theta n)^2[n(1-\delta)-m]\right]$ is ambiguous in sign. This is so because, on the one hand, unionisation lowers the tax rate and thereby lowers investment in human capital accumulation. This can be easily understood from the term $\lambda\tau/[1-(1-\lambda)\tau]$ in the R.H.S. of equation (6.2.35). On the other hand, unionisation raises the income share of labour and thereby the tax base. This can be easily understood from the term $[\theta n(1-\alpha)+\beta(1-\theta)]/(1-\theta+\theta n)$ in the R.H.S. of equation (6.2.35). So if $m \geq n$, then the effect on tax rate dominates the other effect and the initial consumption effect becomes positive. So the welfare effect of unionisation is stronger than its growth effect in this case. The major result is stated in the following proposition.

**Proposition 6.2.6:** The welfare effect of unionisation is different from its growth effect when the government invests in human capital accumulation; and is stronger than the growth effect if $m \geq n$.

In Chang et al. (2007), growth effect as well as welfare effect of unionisation solely consists of the employment effect of unionisation, which depends only on the orientation of the labour union. However, there is no employment effect in our model; and hence the growth effect as well as the welfare effect of unionisation does not depend on the orientation of labour union.

## 6.3 The 'Right to Manage' Model

In this case, the employers' union and the employees' union bargain only over the wage rate; and the firm determines the number of employed workers from its labour demand function obtained from its profit maximisation exercise. So, from equations (6.2.1), (6.2.2), (6.2.2.a), (6.2.2.b) and (6.2.3), we obtain the inverted labour demand function of the representative firm as given by

$$w = \left[\beta A K^{\alpha} \bar{K}^{\xi} L^{\beta-1} h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} \quad . \qquad (6.3.1)$$

So the firms' association and the labour union jointly maximise the 'generalised Nash product' function given by equation (6.2.9) with respect to $w$ only subject to equation (6.3.1). Using the first order condition of maximisation and equations (6.2.1), (6.2.2), (6.2.2.a), (6.2.2.b), (6.2.4), (6.2.6) and (6.3.1), optimum values of $L$ and $w$ are obtained as[88]

$$L^{**} = \frac{[1-\tau(1-\lambda)]\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-[1-\tau(1-\lambda)]\theta m(1-\beta)(1-\alpha-\beta)} < 1 \quad ; \qquad (6.3.2)$$

and

$$w^{**} = \beta A K^{\alpha} \bar{K}^{\xi} h^{\beta\eta} {L^{**}}^{\beta-1-\beta\delta} \tau^{-\beta\delta} (1-L^{**})^{\beta\delta} (1-\lambda)^{-\beta\delta} [1-\tau(1-\lambda)]^{\beta\delta} \quad . \qquad (6.3.3)$$

We assume the following parametric restriction to be valid in order to ensure that $L^{**} > 0$.

$$\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\} > \theta m(1-\beta)(1-\alpha-\beta) \quad .$$

This restriction implies that the labour union can not be highly wage oriented. In this model also, $L^{*}$ varies inversely with $\theta$ when $\tau$ and $\lambda$ are given. This is shown by

$$\frac{\partial L^{**}}{\partial \theta} = -\frac{[1-(1-\lambda)\tau](1-\lambda)\tau\beta m(1-\beta)^2(1-\alpha-\beta)(1-\delta)}{[\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-[1-\tau(1-\lambda)]\theta m(1-\beta)(1-\alpha-\beta)]^2} < 0 \quad . \qquad (6.3.4)$$

Now, from equations (6.2.2.b), (6.2.6) and (6.3.2), representative worker's effort level is obtained and is given by

$$e_2^{**} = \left[\frac{[1-(1-\lambda)\tau](1-L^{**})}{(1-\lambda)\tau L^{**}}\right]^{\delta}$$

$$= \left[\frac{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}\right]^{\delta} \quad . \qquad (6.3.5)$$

From equation (6.3.5), we have

$$\frac{\partial e_2^{**}}{\partial \theta} = \frac{\delta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}^{\delta-1}\beta m(1-\beta)^2(1-\alpha-\beta)(1-\delta)}{[\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)]^{\delta+1}} > 0 \quad . \qquad (6.3.6)$$

Equation (6.3.6) implies that effort level of the worker varies positively with the degree of unionisation. Since, in this model, the government's budget balancing equations as well as the representative household's behaviour are identical to those given in the 'Efficient Bargaining' model, so the existence and stability properties of the steady state equilibrium derived in that model will remain unchanged here.

[88] Derivations of equations (6.3.2), (6.3.3) and (6.3.5) are provided in appendix 6.B. We assume that second order condition of maximisation is satisfied.

Now, using equations (6.2.2), (6.2.2.a), (6.2.2.b), (6.2.5), (6.2.6), (6.2.23), (6.3.3) and (6.3.5), we obtain the balanced growth equation given by

$$(\rho+\sigma g)[g]^{\frac{\beta\eta}{1-\beta\eta}} = A^{\frac{1}{1-\beta\eta}}\alpha L^{**\frac{\beta(1-\delta)}{1-\beta\eta}}\left[\frac{(1-L^{**})}{(1-\lambda)}\right]^{\frac{\beta\delta}{1-\beta\eta}}[\beta\lambda]^{\frac{\beta\eta}{1-\beta\eta}}\left\{\frac{[1-(1-\lambda)\tau]}{\tau}\right\}^{\frac{\beta\delta-\beta\eta}{1-\beta\eta}} \quad . \qquad (6.3.7)$$

Using equations (6.3.2) and (6.3.7), we obtain the growth rate maximising tax rate given by

$$\tau^{**} = \frac{\eta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\eta\theta m(1-\beta)(1-\alpha-\beta)\}(1-\lambda)} \quad . \qquad (6.3.8)$$

From equation (6.3.8), we obtain

$$\frac{\partial\tau^{**}}{\partial\theta} = -\frac{\eta(1-\eta)\beta m(1-\beta)^2(1-\alpha-\beta)(1-\delta)}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\eta\theta m(1-\beta)(1-\alpha-\beta)\}^2(1-\lambda)} < 0 \quad . \qquad (6.3.9)$$

So the growth rate maximising tax rate varies inversely with the degree of unionisation. Incorporating the value of $\tau^{**}$ from equation (6.3.8) in equation (6.3.2), we obtain the same value of $L^{**}$ as that is given in equation (6.2.34). Now, to check the equivalence between the growth rate maximising labour income tax rate and the welfare maximising labour income tax rate, we use equations (6.2.1), (6.2.3), (6.2.6), (6.2.21), (6.2.22), (6.2.23) and (6.3.1); and thus obtain

$$\omega = \frac{{K_0}^{1-\sigma}\left[\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau\beta}{[1-(1-\lambda)\tau]}\right\}-g\right]^{1-\sigma}}{(1-\sigma)[\rho-g(1-\sigma)]} + constant \quad . \qquad (6.3.10)$$

We assume $1 > \sigma$ and $\rho > g(1\text{-}\sigma)$. Since initial consumption, $c_0$, is positive, so $\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau\beta}{[1-(1-\lambda)\tau]}\right\}$ has to be greater than $g$. From equation (6.3.10), we obtain

$$\left.\frac{\partial\omega}{\partial\tau}\right|_{\tau=\tau^{**}} = -\frac{\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{\frac{\beta\lambda}{[1-(1-\lambda)\tau^{**}]^2}\right\}[\rho-g^{**}(1-\sigma)]^{-1}}{{K_0}^{\sigma-1}\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]^{\sigma}} < 0 \quad ; \qquad (6.3.11)$$

Equation (6.3.11) shows that here also the welfare maximising tax rate falls short of the growth rate maximising tax rate due to the negative effect of taxation on initial consumption.

Now, using equations (6.2.34), (6.3.7) and (6.3.8), we obtain

$$\left[\frac{\sigma g^{**}}{(\rho+\sigma g^{**})}+\frac{\beta\eta}{1-\beta\eta}\right]\frac{\frac{\partial g^{**}}{\partial\theta}}{g^{**}} = -\left(\frac{\beta[\delta-\eta]}{1-\beta\eta}\right)\left\{\frac{(1-\lambda)}{[1-(1-\lambda)\tau^{**}]}+\frac{1}{\tau^{**}}\right\}\frac{\partial\tau^{**}}{\partial\theta} \gtreqless 0 \ \ iff \ \ \delta \gtreqless \eta \quad . \qquad (6.3.12)$$

Equation (6.3.12) shows that the sign of the growth effect of unionisation depends solely on the sign of $(\delta-\eta)$. So, if the elasticity of worker's efficiency with respect to the wage premium, $\delta$, is higher than (equal to) (lower than) the elasticity of worker's efficiency with respect to the stock of human capital, $\eta$, then unionisation in the labour market raises (does not affect) (lowers) the rate of economic growth. In the ‘Efficient Bargaining’ model, the growth effect of unionisation partially depends on the mathematical sign of $(\delta - \eta)$. However, in the ‘Right to Manage’ model, growth effect of unionisation fully depends on the mathematical sign of $(\delta - \eta)$. So in this model, $\delta > \eta$ is a necessary as well as a sufficient condition to ensure positive growth effect of unionisation. Important results derived in this section are summarized in the following proposition.

**Proposition 6.3.1:** In the ‘Right to Manage’ model of bargaining, unionisation in the labour market raises (does not change) (lowers) the rate of economic growth if the elasticity of worker’s efficiency with respect to the wage premium is higher than (equal to) (lower than) the elasticity of worker’s efficiency with respect to the stock of human capital.

To analyse the welfare effect of unionisation, we use equation (6.3.10) and obtain

$$\left.\frac{\partial \omega}{\partial \theta}\right|_{\tau=\tau^{**}} = \frac{{K_0}^{1-\sigma}[\rho - g^{**}(1-\sigma)]^{-1}\frac{\partial g^{**}}{\partial \theta}}{\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\beta\tau^{**}}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]^{\sigma-1}}\left\{\frac{\left[\frac{\sigma}{\alpha}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-1\right]}{\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]} + \frac{1}{[\rho - g^{**}(1-\sigma)]}\right\} - \frac{[\rho - g^{**}(1-\sigma)]^{-1}\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\frac{\lambda\beta}{[1-(1-\lambda)\tau^{**}]^2}\frac{\partial \tau^{**}}{\partial \theta}}{{K_0}^{\sigma-1}\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]^{\sigma}} \quad . \qquad (6.3.13)$$

Equation (6.3.13) implies that here also the welfare effect of unionisation consists of growth effect as well as initial consumption effect. However, the initial consumption effect is always positive here because, unlike the previous model, income share of labour in this model is independent of the level of unionisation. So the welfare effect of unionisation is always stronger than its growth effect.

**Appendix**

**Appendix 6.A**

<u>Derivation of optimal *w* and *L*:</u>

From equations (6.2.4) and (6.2.10), we obtain

$$\theta m[(1-\alpha)Y - wL] = (1-\theta)(w-b)\left\{L - \frac{\beta\delta Y}{w}\right\} \quad . \quad (6.A.1)$$

Now from equation (6.2.6), we obtain

$$b(1-L) = \frac{(1-\lambda)\tau wL}{[1-(1-\lambda)\tau]} \quad . \quad (6.A.2)$$

Using equations (6.A.1) and (6.A.2), we obtain

$$\theta m[(1-\alpha)Y - wL] = (1-\theta)\left(w - \frac{(1-\lambda)\tau wL}{[1-(1-\lambda)\tau](1-L)}\right)\left\{L - \frac{\beta\delta Y}{w}\right\} \quad . \quad (6.A.3)$$

Using equations (6.2.11.a) and (6.A.3), we obtain

$$\frac{(1-L)[1-(1-\lambda)\tau]}{(1-\lambda)\tau L} = \Theta_5 \quad . \quad (6.A.4)$$

From equation (6.A.4), we obtain equation (6.2.12) in the body of the chapter.

Incorporating the value of $L^*$ from equation (6.2.12) in equation (6.A.2), we obtain equation (6.2.13) in the body of the chapter. We assume that second order conditions of maximisation is satisfied.

<u>Derivation of equation (6.2.23):</u>

Using equations (6.2.21) and (6.2.22), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma} - 1}{1-\sigma} + \mu[(1-\tau)wL + rK + \pi + (1-\tau)b(1-L) - c] \quad . \quad (6.A.5)$$

Here $\mu$ is the co-state variable. Maximising equation (6.A.5) with respect to *c*, we obtain the following first order condition.

$$c^{-\sigma} - \mu = 0 \quad . \quad (6.A.6)$$

Again from equation (6.A.5), we have

$$\frac{\dot{\mu}}{\mu} = \rho - r \quad ; \quad (6.A.7)$$

and from equation (6.A.6), we have

$$\frac{\dot{\mu}}{\mu} = -\sigma \frac{\dot{c}}{c} \quad . \tag{6.A.8}$$

Using equations (6.A.7) and (6.A.8), we have equation (6.2.23) in the body of the chapter.

Derivation of the Jacobian determinant:

The Jacobian determinant is given below.

$$|J| = \begin{vmatrix} \frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial M} & \frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial N} \\ \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial M} & \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial N} \end{vmatrix} \quad .$$

From equations (6.2.27) and (6.2.28), we have

$$\frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial M} = \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial M} = 1 \quad ;$$

$$\frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial N} = \frac{\beta\eta\alpha A L^{*\beta}[\Theta_5]^{\beta\delta}}{\sigma N^{1-\beta\eta}} - \frac{\beta\eta A L^{*\beta}[\Theta_5]^{\beta\delta}}{N^{1-\beta\eta}}\left[1 - \frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right] \quad ;$$

and

$$\frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial N} = -\frac{(1-\beta\eta)\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]A L^{*\beta}[\Theta_5]^{\beta\delta}}{N^{2-\beta\eta}[1-(1-\lambda)\tau](1-\theta+\theta n)}$$
$$- \frac{\beta\eta A L^{*\beta}[\Theta_5]^{\beta\delta}}{N^{1-\beta\eta}}\left[1 - \frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right] \quad .$$

Using these equations, we obtain equation (6.2.30) in the body of the chapter.

Derivation of equation (6.2.35):

From equation (6.2.21), we obtain

$$\omega = \frac{{c_0}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \tag{6.A.9}$$

Here, $c(0) = c_0$.

Now, from equations (6.2.22) and (6.2.3), we obtain

$$\dot{K} = (1-\tau)wL + Y - wL + (1-\tau)b(1-L) - c \quad . \tag{6.A.10}$$

Using equations (6.A.10) and (6.2.6), we obtain

$$\dot{K} = (1-\tau)wL + Y - wL + \frac{(1-\tau)(1-\lambda)\tau wL}{[1-(1-\lambda)\tau]} - c \quad . \qquad (6.A.11)$$

Using equations (6.A.11) and (6.2.11.a), we obtain

$$\dot{K} = Y\left\{1 - \frac{\tau\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right\} - c \quad . \qquad (6.A.12)$$

From Equation (6.A.12), we obtain

$$c_0 = K_0\left\{\frac{Y_0}{K_0}\left[1 - \frac{\tau\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right] - g\right\} \quad . \qquad (6.A.13)$$

Using equations (6.A.13), (6.2.1) and (6.2.23), we obtain

$$c_0 = K_0\left\{\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left[1 - \frac{\tau\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right] - g\right\} \quad . \qquad (6.A.14)$$

Using equations (6.A.9) and (6.A.14), we obtain equation (6.2.35) in the body of the chapter.

## Appendix 6.B

Derivation of equations (6.3.2), (6.3.3) and (6.3.5):

Incorporating the inverted labour demand function of the representative firm from equation (6.3.1) in equation (6.2.9) and obtain

$$\psi = \{(1-\tau)^m\left(\left[\beta A K^\alpha \overline{K}^\xi L^{\beta-1} h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} - b\right)^m L^n\}^\theta$$

$$. \ \{(1-\beta)\left[\beta A K^\alpha \overline{K}^\xi h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} L^{\frac{\beta(1-\delta)}{1-\beta\delta}} - rK\}^{(1-\theta)} \quad . \qquad (6.B.1)$$

Since equation (6.3.1) shows a monotonic relationship between *w* and *L*, so we maximise equation (6.B.1) with respect to *L*. Using this first order condition and equation (6.2.4), we obtain

$$\frac{\frac{\theta m(\beta-1)}{1-\beta\delta}\left[\beta A K^\alpha \overline{K}^\xi h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} L^{\frac{\beta(1+\delta)-2}{1-\beta\delta}}}{\left[\beta A K^\alpha \overline{K}^\xi L^{\beta-1} h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} - b} + \frac{\theta n}{L}$$

$$+ \frac{(1-\theta)(1-\beta)\left[\beta^{\beta\delta} A K^\alpha \overline{K}^\xi h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} \frac{\beta(1-\delta)}{1-\beta\delta} L^{\frac{\beta-1}{1-\beta\delta}}}{(1-\alpha-\beta)\left[\beta^{\beta\delta} A K^\alpha \overline{K}^\xi h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} L^{\frac{\beta(1-\delta)}{1-\beta\delta}}} = 0 \quad . \qquad (6.B.2)$$

From equation (6.2.6), we have

$$b = \frac{(1-\lambda)\tau w L}{[1-(1-\lambda)\tau](1-L)} \quad . \tag{6.B.3}$$

Now, using equations (6.3.1) and (6.B.3), we obtain

$$b = \frac{(1-\lambda)\tau\left[\beta A K^{\alpha} \bar{K}^{\xi} L^{\beta(1-\delta)} h^{\beta\eta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}}}{[1-(1-\lambda)\tau](1-L)} \quad . \tag{6.B.4}$$

Using equations (6.B.2) and (6.B.4), we obtain the equation (6.3.2) in the body of the chapter. Now, using equations (6.B.3) and (6.3.2), we obtain the equation (6.3.5) in the body of the chapter. We obtain the equation (6.3.3) of the main chapter using equations (6.B.3) and (6.3.1). Derivations of other equations in section 6.3 are similar to that in section 2.2.

# Chapter 7: Conclusion

In earlier chapters of this thesis starting from chapter 2, we have analysed a few theoretical problems related to effects of unionisation on the economic growth and have also analysed properties of optimal income tax rate in those models. In this chapter, we summarize major results obtained in those chapters and also mention limitations of the present work.

## 7.1 Major Findings of the Present Thesis

In chapter 1 of the thesis, we have surveyed the existing empirical and theoretical works regarding the effect of unionisation on economic growth. We have also briefly surveyed the theoretical literature on endogenous growth models dealing with the issue of optimal taxation to finance productive public spending. This chapter has also pointed out a few research gaps in the existing theoretical literature.

Chapter 2, on the one hand, investigates the growth effect and welfare effect of unionisation in the labour market in the presence of productive public expenditure; and, on the other hand, analyses the properties of optimum income tax policy to finance productive public expenditure and unemployment benefit. The Barro (1990) model is extended by incorporating collective bargaining between the labour union and the employers' union resulting into an unemployment equilibrium. We use two alternative versions of bargaining models – the 'Efficient Bargaining' model of McDonald and Solow (1981) in section 2.2 and the 'Right to Manage' model of Nickell and Andrews (1983) in section 2.3.

Our major findings are as follows. First, the optimum rate of proportional income tax financing productive public expenditure as well as unemployment benefit, is found to be higher in this model than that in the models of Barro (1990) and Futagami et al. (1993); and its magnitude depends on the unemployment level and labour union's bargaining power. Secondly, the endogenous growth rate of the economy varies inversely with the rate of unemployment benefit though social welfare may not. Both these two results are valid in each of these two bargaining models. Thirdly, how unionisation affects employment, economic growth and welfare depends on the nature of the bargaining model considered. In the 'Right to Manage' model, unionisation must have a negative effect on employment and growth regardless of the orientation of the labour union. However, welfare may increase due to unionisation. On the contrary, the nature of these effects at least partially depends on the nature of orientation of the labour union in the ‘Efficient Bargaining’ model. Growth effects and welfare effects are not necessarily positive even if the labour union is employment oriented; and the growth effect is always negative if the union is neutral or wage oriented. Fourthly, unionisation raises the optimal tax rate in the ‘Right to Manage’ model but affects it ambiguously in the ‘Efficient Bargaining’ model.

Chapter 3 constructs a simple two sector endogenous growth model with public capital; and derives the properties of optimal fiscal policies in the steady state equilibrium. Both final good and public investment good are produced by the private sector using different production technologies and the government buys public good from private producers at a monopsony price. This is how the present model differs from models like Barro (1990), Futagami et al. (1993) etc. We also extend this basic model with competitive labour markets in the case of unionized labour markets.

Various interesting findings are obtained here. First, the growth rate maximising income tax rate is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology of public investment good. Secondly, welfare maximising solutions are different from growth rate maximising solutions even in the steady state equilibrium when production technologies are different in these two sectors. Lastly, economic growth rate is higher in the case of competitive labour markets than that in the case of unionised labour markets in any sector. However, the steady state growth rate maximising allocation of private capital and the steady state growth rate maximising income tax rate are independent of unionisation in the labour market.

Chapter 4 on the one hand, investigates the effect of unionisation in the labour market on the long run growth rate of an economy in the presence of environmental pollution and trade union’s concern about environment development, and, on the other hand, derives properties of the optimum income tax policy designed to finance public abatement expenditure. Here we use an AK growth model and consider two alternative versions of bargaining models – the ‘Efficient Bargaining’ model of McDonald and Solow (1981) and the ‘Right to Manage’ model of Nickell and Andrews (1983).

Major findings obtained in this chapter are as follows. First, negotiated wage rate as well as firm’s spending rate to protect environment varies positively with degree of unionisation in the labour market. Secondly, the optimum rate of income tax used to finance public abatement expenditure varies inversely with this degree of unionisation. These two results hold for both versions of bargaining models. Thirdly, effects of unionisation on employment level and on economic growth depend on the nature of the bargaining model considered. In the case of 'Efficient Bargaining' model, unionisation may raise employment level only if the labour union is highly employment oriented. Otherwise, it is always harmful for the level of employment. Effect of unionisation on the long run growth rate partly depends on its ambiguous employment effect and partly on its positive environment development effect. However, in the 'Right to Manage' model of bargaining, unionisation in the labour market always lowers employment level; and so unionisation raises the growth rate if and only if the positive environment development effect dominates the negative employment effect.

In chapter 5, we develop a model to investigate the effect of unionisation in the labour market on the long run growth rate of an economy in the presence of 'Efficiency Wage Hypothesis'. Here also we use an AK growth model and consider two alternative versions of bargaining models – the 'Efficient Bargaining' model of McDonald and Solow (1981) and the 'Right to Manage' model of Nickell and Andrews (1983).

We derive different results from these two versions of bargaining models. In the 'Efficient Bargaining' model, unionisation in the labour market reduces the negotiated number of workers unless the labour union is highly employment oriented; but always raises workers' effort level. As a result, effective employment must (may) increase for employment oriented and neutral (wage oriented) labour union. The effect of unionisation on the growth rate as well as on the level of welfare are same as that on the effective employment. However, in the 'Right to Manage' model, unionisation raises the effort level of the worker but reduces the number of workers irrespective of the orientation of the labour union. This raises effective employment, balanced growth rate and welfare level of the economy if the wage elasticity of effort (efficiency) exceeds the unemployment rate; and this sufficient condition is likely to be valid when the income tax rate is very low.

Chapter 6 extends chapter 5 by incorporating public financed human capital accumulation. In this chapter, we develop an endogenous growth model with a special focus on human capital formation and on the "Efficiency Wage Hypothesis' in order to study the effect of unionisation in the labour market on the long run economic growth rate. We also have derived properties of growth rate maximizing tax rate on labour income which is used to finance investment in human capital formation as well as unemployment benefit given to unemployed workers. We have used both the

‘Efficient Bargaining’ model of McDonald and Solow (1981) and the ‘Right to Manage’ model of Nickell and Andrews (1983) to derive the outcome of negotiation between the labour union and the employers’ association.

We have derived many interesting results. First, in each of these two type of bargaining models, for a given tax rate on labour income, unionisation lowers the number of employed workers but raises the wage rate as well as the effort level of the worker irrespective of the orientation of the labour union. The growth rate maximising tax rate on labour income varies positively with the elasticity of efficiency with respect to human capital as well as with the budget share of investment in human capital accumulation but varies inversely with the degree of unionisation in the labour market. When the government imposes the growth rate maximising tax rate on labour income, rate of employment becomes independent of the degree of unionisation in the labour market but varies inversely with the elasticity of efficiency with respect to human capital. Secondly, the growth rate maximising tax rate on labour income is different from the corresponding welfare maximising tax rate. The Welfare effect of unionisation is also different from the growth effect of unionisation in both these two models. Thirdly, in the case of the ‘Efficient Bargaining’ model, if the elasticity of worker’s effort level with respect to the wage premium is higher than the elasticity of worker’s efficiency with respect to the stock of human capital, then there exists a positive growth effect of unionisation in the labour market though this condition is not necessary. However, in case of the ‘Right to Manage’ model, this condition becomes necessary as well as sufficient to obtain a positive growth effect of unionisation.

## 7.2 Limitations

Despite of their contributions in the literature, models developed in chapters from 2 to 6 are abstract and fail to consider many aspects of reality. For example, all these models consider closed economy whereas most of the economies are reasonably open nowadays. Throughout these chapters, we do not consider population growth, technological progress, allocation of household’s income towards education, non-productive utility enhancing public services and congestion effect of capital accumulation on productivity. In all these chapters, we assume ‘closed shop’ labour union for simplicity and do not consider ‘open shop’ labour union which is more common in reality. We also do not consider union membership dynamics and intertemporal behaviour of unions. In all of these models, we assume that labour union’s utility function is of ‘Stone-Geary’ form and do not consider other forms of utility function present in the literature. Benevolent governments are assumed and therefore the political aspects of governments are ignored.

Beside these general limitations, there are also some chapter specific drawbacks. For example in the chapter 2, we assume public expenditure as a flow variable and hence do not consider the role of public capital accumulation. Chapter 4 assumes that the environment quality is a flow variable rather than a stock variable. It is also assumed that environmental quality does not affect household's utility. In chapters 5 and 6, worker's efficiency in the current period does not depend on past wages.

We plan to do further research in future to remove these limitations.

**Bibliography**


Addison, J. T., & Hirsch, B. T. (1989). Union Effects on Productivity, Profits, and Growth: Has the Long Run Arrived? *Journal of Labor Economics, 7(1)*, 72-105.

Addison, J. T., & Wagner, J. (1994). UK Unionism and Innovative Activity: Some Cautionary Remarks on the Basis of a Simple Cross-country Test. *British Journal of Industrial Relations, 32(1)*, 85-98.

Adjemian, S., Langot, F., & Quintero-Rojas, C. (2010). How do Labor Market Institutions affect the Link between Growth and Unemployment: the case of the European countries. *The European Journal of Comparative Economics, 7(2)*, 347-371.

Agell, J., & Lommerud, K. E. (1993). Egalitarianism and Growth. *The Scandinavian Journal of Economics, 95(4)*, 559-579.

Agénor, P. R. (2009). Infrastructure investment and maintenance expenditure: Optimal allocation rules in a growing economy. *Journal of Public Economic Theory, 11(2)*, 233-250.

Agénor, P. R. (2011). Schooling and public capital in a model of endogenous growth. *Economica, 78(309)*, 108-132.

Agénor, P.-R. (2008). Health and infrastructure in a model of endogenous growth. *Journal of Macroeconomics, 30(4)*, 1407–1422.

Agénor, P.-R., & Neanidis, K. C. (2014). Optimal taxation and growth with public goods and costly enforcement. *The Journal of International Trade & Economic Development, 23(4)*, 425-454.

Aghion, P., & Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica, 60(2)*, 323-351.

Akerlof, G. A. (1982). Labor Contracts as Partial Gift Exchange. *The Quarterly Journal of Economics, 97(4)*, 543-569.

Akerlof, G. A. (1984). Gift exchange and efficiency-wage theory: Four views. *The American Economic Review, 74(2)*, 79-83.

Akerlof, G. A., & Yellen, J. L. (1986). *Efficiency wage models of the labor market.* New York: Cambridge University Press.

Akıncı, M., Akıncı, G. Y., & Yılmaz, O. (2014). Sendikal Hareketlerin İki Yüzü: OECD Ülkelerinde İktisadi Büyüme Üzerindeki Etkilerinin Analizi. *Hacettepe Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi, 32(1)*, 1-27.

Allen, S. G. (1988). Productivity Levels & Productivity Change Under Unionism. *Industrial Relations, 27(1),* 94-113.

Arrow, K. J. (1962). The Economic Implications of Learning by Doing. *The Review of Economic Studies, 29(3),* 155-173.

Asteriou, D., & Monastiriotis, V. (2004). What do unions do at the large scale? Macro-economic evidence from a panel of OECD countries. *Journal of Applied Economics, 7(1),* 27-46.

Awaworyi, S., & Yew, S. L. (2014). Government Transfers and Growth: Is there Evidence of Genuine Effect? *Department of Economics Discussion Paper No. 40-14.* Monash University.

Baier, S. L., & Glomm, G. (2001). Long-run growth and welfare effects of public policies with distortionary taxation. *Journal of Economic Dynamics and Control, 25(12),* 2007–2042.

Bandyopadhyay, D., & Basu, P. (2001). Redistributive Tax and Growth in a Model with Discrete Occupational Choice. *Australian Economic Papers 40(2),* 111–132.

Banerjee, D. S., & Gaston, N. (2004). Labour market signalling and job turnover revisited. *Labour Economics, 11(5),* 599–622.

Banerji, S., & Gupta, M. R. (1997). The efficiency wage given long-run employment and concave labor constraint. *Journal of Development Economics, 53(1),* 185–195.

Barman, T. R., & Gupta, M. R. (2010). Public Expenditure, Environment, and Economic Growth. *Journal of Public Economic Theory, 12(6),* 1109–1134.

Barro, R. J. (1990). Government Spending in a Simple Model of Endogeneous Growth. *Journal of Political Economy, 98(5),* S103-S125.

Betts, J. R., Odgers, C. W., & Wilson, M. K. (2001). The effects of unions on research and development: an empirical analysis using multi-year data. *Canadian Journal of Economics, 34(3),* 785–806.

Bhanoji Rao, V. V., Banerjee, D. S., & Mukhopadhaya, P. (2003). Earnings Inequality in Singapore. *Journal of the Asia Pacific Economy, 8(2),* 210-228.

Blankenau, W. F., & Simpson, N. B. (2004). Public education expenditures and growth. *Journal of Development Economics 73(2),* 583– 605.

Boone, J. (2000). Technological progress, downsizing and unemployment. *The Economic Journal, 110(465),* 581-600.

Booth, A. L. (1995). *The Economics of the Trade Union.* Cambridge University Press, U. K.

Bräuninger, M. (2000a). Unemployment Insurance, Wage Differentials and Unemployment. *FinanzArchiv / Public Finance Analysis 57(4)*, 485-501.

Bräuninger, M. (2000b). Wage Bargaining, Unemployment, and Growth. *Journal of Institutional and Theoretical Economics, 156(4)*, 646-660.

Bräuninger, M. (2005). Social Security, Unemployment, and Growth. *International Tax and Public Finance 12(4)*, 423-434.

Bronars, S. G., & Deere, D. R. (1993). Unionization, Incomplete Contracting, and Capital Investment. *The Journal of Business, 66(1)*, 117-132.

Bucci, A. (2008). Population growth in a model of economic growth with human capital accumulation and horizontal R&D. *Journal of Macroeconomics 30(3)*, 1124–1147.

Caballé, J., & Santos, M. S. (1993). On Endogenous Growth with Physical and Human Capital. *Journal of Political Economy 101(6)*, 1042-1067.

Carmeci, G., & Mauro, L. (2002). The Convergence of the Italian Regions and Unemployment: Theory and Evidence. *Journal of Regional Science, 42(3)*, 509-532.

Carmeci, G., & Mauro, L. (2003). Imperfect Labor Market and Convergence: Theory and Evidence for Some OECD Countries. *Journal of Policy Modelling, 25(8)*, 837–856.

Cass, D. (1965). Optimum Growth in an Aggregative Model of Capital Accumulation. *Review of Economic Studies, 32(3)*, 233-240.

Cassou, S. P., & Lansing, K. J. (2006). Tax Reform with Useful Public Expenditures. *Journal of Public Economic Theory, 8(4)*, 631–676.

Cazzavillan, G. (1996). Public Spending, Endogenous Growth, and Endogenous Fluctuations. *Journal of Economic Theory, 71(2)*, 394–415.

Çetintaş, H., Şenkal, A., & Başel, H. (2008). Unionization, Unemployment and Economic Growth; Case of Turkey. *Serbian Journal of Management 3(2)*, 157-170.

Chakrabarty, D. (2012). Poverty Traps and Growth in a Model of Endogenous Time Preference. *The B.E. Journal of Macroeconomics, 12(1)*, 1-35.

Chakrabarty, D., Chanda, A., & Ghate, C. (2006). Education, Growth, and Redistribution in the Presence of Capital Flight. *Topics in Macroeconomics, 6(2)*, 1-41.

Chakraborty, B., & Gupta, M. R. (2009). Human capital, inequality, endogenous growth and educational subsidy: A theoretical analysis. *Research in Economics, 63(2)*, 77–90.

Chakraborty, S., & Das, M. (2005). Mortality, Human Capital and Persistent Inequality. *Journal of Economic Growth, 10(2)*, 159-192.

Chang, J. J., & Hung, H. W. (2016). Trade unions, unemployment, economic growth, and income inequality. *Macroeconomic Dynamics, 20(01)*, 404-428.

Chang, J.-j., Shaw, M.-f., & Lai, C.-c. (2007). A ‘‘Managerial’’ trade union and economic growth. *European Economic Review, 51(2)*, 365-384.

Chang, S.-h., & Chang, J.-j. (2015). Optimal Government Spending in an Economy with Imperfectly Competitive Goods and Labor Markets. *Southern Economic Journal, 82(2)*, 385-407.

Chezum, B., & Garen, J. E. (1998). Are union productivity effects overestimated?: evidence from coal mining. *Applied Economics, 30(7)*, 913-918.

Chu, A. C., Cozzi, G., & Furukawa, Y. (2016). Unions, innovation and cross-country wage inequality. *Journal of Economic Dynamics and Control, 64*, 104-118.

Clark, K. B. (1980). The Impact of Unionization on Productivity: A Case Study. *Industrial and Labor Relations Review, 33(4)*, 451-469.

Cole, I. M. (2014). Short- and Long-Term Growth Effects of Special Interest Groups in the U.S. States: A Dynamic Panel Error-Correction Approach. *MPRA Paper No. 54455*.

Corneo, G., & Marquardt, M. (2000). Public pensions, unemployment insurance, and growth. *Journal of Public Economics, 75(2)*, 293-311.

Corsetti, G., & Roubini, N. (1996). Optimal government spending and taxation in endogenous growth models. *NBER Working paper 5851*.

Crossley, T. F., & Low, H. (2011). Borrowing constraints, the cost of precautionary saving and unemployment insurance. *International Tax and Public Finance, 18(6)*, 658-687.

Danthine, J.-P., & Kurmann, A. (2006). Efficiency wages revisited: The internal reference perspective. *Economics Letters, 90(2)*, 278–284.

Das, M. (2003). Optimal growth with decreasing marginal impatience. *Journal of Economic Dynamics and Control, 27(10)*, 1881–1898.

Das, M. (2007). Persistent inequality: An explanation based on limited parental altruism. *Journal of Development Economics, 84(1)*, 251–270.

Dasgupta, D. (1999). Growth Versus Welfare in a Model of Nonrival Infrastructure. *Journal of Development Economics, 58(2)*, 359–385.

Dasgupta, D. (2001). Lindahl Pricing, Nonrival Infrastructure, and Endogenous Growth. *Journal of Public Economic Theory, 3(4)*, 413–430.

Dasgupta, D., & Shimomura, K. (2006). Public infrastructure, employment and sustainable growth in a small open economy with and without foreign direct investment. *The Journal of International Trade & Economic Development: An International and Comparative Review, 15(3)*, 257-291.

Daveri, F., Tabellini, G., Bentolila, S., & Huizinga, H. (2000). Unemployment, Growth and Taxation in Industrial Countries. *Economic Policy, 15(30),* 47-104.

Davidson, C., & Woodbury, S. A. (1997). Optimal unemployment insurance. *Journal of Public Economics 64(3),* 359–387.

Davies, A. (1993). The workplace environment continuum. In D. Hallowes, *Hidden faces: environment, development, justice: South Africa and the global context* (pp. 70-73). Scottsville: Earthlife Africa, Scottsville.

De Groot, H. L. (1996). The Struggle for Rents in a Schumpterian Economy. *Center Discussion Paper; Vol. 1996-51.* Tilburg University.

De Groot, H. L. (2001). Unemployment, Growth, and Trade Unions. *Growth and Change, 32(1),* 69-91.

Dembo, D., Dias, D., Kadwani, A., & Morehouse, W. (1988). *Nothing to lose but our lives: empowerment to oppose industrial hazards in a transnational world.* New Horizons Press, New York.

Denny, K. (1997). Productivity and trade unions in British manufacturing industry 1973–85. *Applied Economics, 29(10),* 1403-1409.

Denny, K., & Nickell, S. J. (1992). Unions and Investment in British Industry. *The Economic Journal, 102(413),* 874-887.

Dioikitopoulos, E. V., & Kalyvitis, S. (2008). Public capital maintenance and congestion: Long-run growth and fiscal policies. *Journal of Economic Dynamics and Control, 32(12),* 3760-3779.

Docquier, F., Faye, O., & Pestieau, P. (2008). Is migration a good substitute for education subsidies? *Journal of Development Economics, 86(2),* 263–276.

Doménech, R., & García, J. R. (2008). Unemployment, taxation and public expenditure in OECD economies. *European Journal of Political Economy, 24(1)*, 202–217.

Doucouliagos, C., & Laroche, P. (2003). Unions and productivity growth: A meta-analytic review. In T. Kato, *Advances in the Economic Analysis of Participatory & Labor-Managed Firms, Volume 7* (pp. 57 - 82). Emerald Group Publishing Limited.

Dowrick, S. (1993). Enterprise Bargaining, Union Structure and Wages. *Economic Record, 69(4)*, 393–404.

Economides, G., & Philippopoulos, A. (2008). Growth enhancing policy is the means to sustain the environment. *Review of Economic Dynamics, 11(1)*, 207–219.

Eicher, T., & Turnovsky, S. J. (2000). Scale, Congestion and Growth. *Economica, 67(267)*, 325-346.

Faini, R. (1999). Trade unions and regional development. *European Economic Review, 43(2)*, 457-474.

Fanti, L., & Gori, L. (2011). A Note on Trade Unions, Unemployment Insurance, and Endogenous Growth. *Eastern Economic Journal, 37(2)*, 270–280.

Futagami, K., Morita, Y., & Shibata, A. (1993). Dynamic Analysis of an Endogenous Growth Model with Public Capital. *The Scandinavian Journal of Economics, 95(4)*, 607-625.

Gahan, P. G. (2002). (What) do unions maximise? Evidence from survey data. *Cambridge Journal of Economics, 26(3)*, 279-297.

Galli, E., & Padovano, F. (1990). Corporatist vs. decentralized governance and economic growth. *Journal for Institutional Innovation, Development and Transition, 3*, 31-42.

Garino, G., & Martin, C. (2000). Efficiency wages and union–firm bargaining. *Economics Letters, 69(2)*, 181–185.

Georgiou M. N. (2010). Trade Unions and Economic Growth: A Discussion Based on Panel Data Analysis (1999- 2007). Retrieved from http://ssrn.com/abstract=1704324

Ghosh, S., & Roy, U. (2004). Fiscal policy, long-run growth, and welfare in a stock-flow model of public goods. *Canadian Journal of Economics/Revue canadienne d'économique, 37(3)*, 742–756.

Glomm, G., & Ravikumar, B. (1994). Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control, 18(6)*, 1173–1187.

Glomm, G., & Ravikumar, B. (1997). Productive government expenditures and long-run growth. *Journal of Economic Dynamics and Control, 21(1)*, 183–204.

Gómez, M. A. (2008). Fiscal policy, congestion, and endogenous growth. *Journal of Public Economic Theory, 10(4)*, 595-622.

Gori, L., & Fanti, L. (2009). Right-to-manage unions endogenous growth and welfare. *Economics Bulletin, 29(2)*, 903-917.

Gray, G. R., Myers, D. W., & Myers, P. S. (1998). Collective Bargaining Agreements: Safety and Health Provisions. *Monthly Labor Review, 121, 13*.

Greiner, A. (2005). Fiscal Policy in an Endogenous Growth Model with Public Capital and Pollution. *Japanese Economic Review, 56(1)*, 67–84.

Greiner, A., & Hanusch, H. (1998). Growth and Welfare Effects of Fiscal Policy in an Endogenous Growth Model with Public Investment. *International Tax and Public Finance, 5(3)*, 249-261.

Grieben, W.-H., & Şener, F. (2012). North-South Trade, Unemployment and Growth: What´s the Role of Labor Unions? *Working Paper Series 2012-06, Department of Economics, University of Konstanz.* Retrieved from https://kops.uni-konstanz.de/bitstream/handle/123456789/30412/Grieben_0-274273.pdf?sequence=3&isAllowed=y

Grossman, G. M., & Helpman, E. (1991). *Innovation and Growth in the Global Economy*. Cambridge: MIT Press.

Gupta, M. R., & Barman, T. R. (2009). Fiscal policies, environmental pollution and economic growth. *Economic Modelling, 26(5)*, 1018–1028.

Gupta, M. R., & Barman, T. R. (2010). Health, infrastructure, environment and endogenous growth. *Journal of Macroeconomics, 32(2)*, 657–673.

Gupta, M. R., & Barman, T. R. (2013). Endogenous growth with environmental pollution and depreciation of public capital : a theoretical note. *Keio economic studies, 49*, 93- 103.

Gupta, M. R., & Barman, T. R. (2015). Environmental Pollution, Informal Sector, Public Expenditure And Economic Growth. *Hitotsubashi Journal of Economics, 56(1)*, 73-91.

Heitger, B. (1987). Corporatism, technological gaps and growth in oecd countries. *Weltwirtschaftliches Archiv, 123(3)*, 463-473.

Hirsch, B. T. (1991). *Labor Unions and the Economic Performance of Firms*. Kalamazoo: W.E. Upjohn Institute for Employment Research.

Hirsch, B. T. (1992). Firm Investment Behavior and Collective Bargaining Strategy. *Industrial Relations, 31(1)*, 95-121.

Hirsch, B. T., & Link, A. N. (1984). Unions, Productivity, and Productivity Growth. *Journal of Labor Research, 5(1)*, 29-37.

Hosoya, K. (2003). Tax financed government health expenditure and growth with capital deepening externality. *Economics Bulletin, 5(14),* 1-10.

Irmen, A., & Wigger, B. U. (2002/2003). Trade Union Objectives and Economic Growth. *FinanzArchiv / Public Finance Analysis, 59(1),* 49-67.

Jaoul-Grammare, M., & Terraz, I. (2013). A Causality Analysis of Economic Growth and Union Density in European Countries. *Labour, 27(4),* 421–442.

Ji, L., Chang, J.-J., & Huang, C.-Y. (2016). Unionization, Market Structure, and Economic Growth. *Southern Economic Journal, 82(3),* 935–951.

Kalaitzidakis, P., & Kalyvitis, S. (2004). On the macroeconomic implications of maintenance in public capital. *Journal of Public Economics, 88(3-4),* 695–712.

Kawakami, T., Kogi, K., Toyama, N., & Yoshikawa, T. (2004). Participatory approaches to improving safety and health under trade union initiative--experiences of POSITIVE training program in Asia. *Industrial Health, 42(2),* 196–206.

Khan, M. T. (2010). Role of Labor Unions Beyond Collective Bargaining. *Far East Journal of Psychology and Business, 1(1),* 78-90.

Khan, M. T., Khan, N. A., Khan, T. M., & Khan, W. A. (2012a). Labor Unions' Role For Environmental Protection- By Exerting Efforts For Legislation and Alliance Making With Environmental Activists. *Far East Journal of Psychology and Business, 8(1),* 43-57.

Khan, M. T., Khan, N. A., Khan, T. M., & Khan, W. A. (2012b). Environmental Problem and Labor Unions' Efforts for Environment Friendly Generation, Management and Use of Energy – A Literature Review. *International Journal of Business and Behavioral Sciences, 2(5),* 32-40.

Kim, D. K. (2005). Unionization, unemployment, and growth in Korea: A cointegration approach. *Atlantic Economic Journal, 33(2),* 225-233.

Kitaura, K. (2010). Fiscal Policy and Economic Growth in the Imperfect Labor Market. *Metroeconomica, 61(4),* 686–700.

Koopmans, T. C. (1965). On the Concept of Optimal Economic Growth. In *The Econometric Approach to Development Planning* (pp. 225–287). Chicago: Rand McNally.

Lai, C.-H., & Wang, V. (2010). The effects of unionization in an R&D growth model with (In)determinate equilibrium. *MPRA Paper 27748.* University Library of Munich, Germany.

Landais, C., Michaillat, P., & Saez, E. (2010). Optimal Unemployment Insurance Over the Business Cycle. *NBER Working Paper No. 16526*.

Lindbeck, A., & Snower, D. J. (1991). Interactions between the efficiency wage and insider-outsider theories. *Economics Letters, 37(2)*, 193–196.

Lingens, J. (2002). Growth and employment effects of unions in a simple endogenous growth model. *Working paper, University of Kassel.*

Lingens, J. (2003a). The impact of a unionised labour market in a Schumpeterian growth model. *Labour Economics, 10(1)*, 91-104.

Lingens, J. (2003b). Unionisation, Growth and Endogenous Skill-Formation. *Economics Discussion Papers No. 45/03*. University of Kassel, Institute of Economics, Germany.

Lingens, J. (2007). Unions, wage setting, and economic growth. *Economic Modelling, 24(1)*, 167–188.

Liu, C. Y. (2014). Equilibrium Unemployment and Growth: The Role of Revenue Sharing and Bargaining Institutions. *Academia Economic Papers, 42(4)*, 485–525.

Ljungqvist, L. (1993). Economic underdevelopment: The case of a missing market for human capital. *Journal of Development Economics, 40(2)*, 219–239.

Loayza, N. V. (1996). The economics of the informal sector: a simple model and some empirical evidence from Latin America. *Carnegie-Rochester Conference Series on Public Policy, 45*, (pp. 129–162).

Lucas Jr., R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics 22(1)*, 3–42.

Magane, P., Miller, S., Goldblatt, M., & Bethlehem, L. (1997). Unions and Environment: Life, health and the pursuit of employment. In L. Bethlehem, & M. Goldblatt, *The Bottom Line: Industry and the Environment in South Africa* (pp. 176-193). UCT Press.

Marrero, G. A., & Novales, A. (2005). Growth and welfare: Distorting versus non-distorting taxes. *Journal of Macroeconomics, 27(3)*, 403–433.

Mauro, L., & Pigliaru, F. (2013). Decentralization, Social Capital and Regional Convergence. *Working Papers 57.2013*. Fondazione Eni Enrico Mattei.

McDonald, I. M., & Solow, R. M. (1981). Wage Bargaining and Employment. *The American Economic Review, 71(5)*, 896-908.

Meeusen, W., Stavrevska, V., & Rayp, G. (2011). Efficiency Wages, Unemployment Benefits and Union—Firm Wage Bargaining: The Issue of the Choice of the Outside Option. *Journal of Labor Research, 32(1),* 75-86.

Menezes-Filho, N., Ulph, D., & Reenen, J. V. (1998). The determination of R&D: Empirical evidence on the role of unions. *European Economic Review, 42(3-5),* 919–930.

Ng, I., & McCallum, J. (1989). Trade Unions, Economic Growth and Politics. *Journal of Industrial Relations, 31(3),* 372-384.

Ni, S., & Wang, X. (1994). Human capital and income taxation in an endogenous growth model. *Journal of Macroeconomics, 16(3),* 493–507.

Nickell, S. J., & Andrews, M. (1983). Unions, Real Wages and Employment in Britain 1951-79. *Oxford Economic Papers, New Series, 35,* 183-206.

Nickell, S., & Layard, R. (1999). Labor market institutions and economic performance. In O. C. Ashenfelter, & D. Card, *Handbook of Labor Economics, Volume 3, Part C* (pp. 3029-3084). Elsevier.

Nickell, S., Wadhwani, S., & Wall, M. (1992). Productivity growth in U.K. companies, 1975–1986. *European Economic Review, 36(5),* 1055-1085.

Nordstrom, H. (1992). Studies in trade policy and economic growth. *Monograph No. 20.* Stockholm: Institute for international economic studies.

OECD. (2004). Wage-setting Institutions and Outcomes. In OECD, *OECD Employment Outlook 2004* (pp. 127–181). Paris: OECD Publishing.

Ono, T. (2015). Growth, Unemployment, and Fiscal Policy: A Political Economy Analysis. *Discussion Paper 14-30-Rev.* Osaka University.

Ott, I., & Turnovsky, S. J. (2006). Excludable and Non-Excludable Public Inputs: Consequences for Economic Growth. *Economica, 73(292),* 725-748.

Padovano, F., & Galli, E. (2003). Corporatism, policies and growth. *Economics of Governance, 4(3),* 245-260.

Palokangas, T. (1996). Endogenous growth and collective bargaining. *Journal of Economic Dynamics and Control, 20(5),* 925-944.

Palokangas, T. (1997). The Centralization of Wage Bargaining, Investment, and Technological Change. *Journal of Institutional and Theoretical Economics, 153(4),* 657-673.

Palokangas, T. (2004a). Union–Firm Bargaining, Productivity Improvement and Endogenous Growth. *LABOUR, 18(2),* 191–205.

Palokangas, T. (2004b). Integration, Wage Bargaining, and Growth with Creative Destruction. *Discussion Paper No. 598:2004.* Department of Economics, University of Helsinki.

Palokangas, T. (2005). International Labor Union Policy and Growth with Creative Destruction. *Review of International Economics, 13(1)*, 90–105.

Pantuosco, L. J., & Seyfried, W. (2008). The Effect Of Public And Private Unions On State Economic Activity: Evaluating The Benefits To Organized Workers, Policymakers, And Companies. *Journal of Business & Economics Research, 6(2),* 27-40.

Pantuosco, L. J., Parker, D., Seyfried, W., & Lyman, S. (2002). Macroeconomic Differences in Public and Private Union Density: An Analysis of U.S. State Economies. *The Review of Regional Studies, 32(2),* 171-186.

Pantuosco, L., Parker, D., & Stone, G. (2001). The effect of unions on labor markets and economic growth: An analysis of state data. *Journal of Labor Research, 22(1),* 195-205.

Park, H., & Philippopoulos, A. (2003). On the dynamics of growth and fiscal policy with redistributive transfers. *Journal of Public Economics, 87(3-4),* 515–538.

Peach, E. K., & Stanley, T. D. (2009). Efficiency Wages, Productivity and Simultaneity: A Meta-Regression Analysis. *Journal of Labor Research, 30(3),* 262-268.

Pemberton, J. (1988). A `Managerial' Model of the Trade Union. *The Economic Journal, 98(392),* 755-771.

Peñalosa, C. G., & Turnovsky, S. J. (2005). Second-best optimal taxation of capital and labor in a developing economy. *Journal of Public Economics, 89(5-6),* 1045–1074.

Peretto, P. F. (1996). Sunk Costs, Market Structure, and Growth. *International Economic Review, 37(4),* 895-923.

Peretto, P. F. (2011). Market Power, Growth, and Unemployment. In O. D. Grandville (ed.), *Economic Growth and Development (Frontiers of Economics and Globalization, Volume 11)* (pp. 493 - 525). Emerald Group Publishing Limited.

Pintea, M., & Turnovsky, S. J. (2006). Congestion and Fiscal Policy in a Two-Sector Economy with Public Capital: A Quantitative Assessment. *Computational Economics, 28(2),* 177-209.

Quang, D. H., & Vousden, N. (2002). Unionisation, Unemployment and Economic Growth. *International and Development Economics Working Paper 02-10.* Australian National University.

Ramos Parreño, J. M., & Sánchez-Losada, F. (2002). The role of unions in an endogenous growth model with human capital. *Journal of Macroeconomics, 24(2)*, 171–192.

Raurich, X., & Sorolla, V. (2003). Growth, unemployment and public capital. *Spanish Economic Review, 5(1)*, 49-61.

Raurich, X., & Sorolla, V. (2004). Employment and Wage Formation in a Growth Model with Public Capital. *XI Encuentro de Economía Pública: [los retos de la descentralización fiscal ante la globalización]*. Barcelona.

Ray, D. (1993). Labor markets, adaptive mechanisms and nutritional status. In P. Bardhan, M. Datta-Chaudhuri, & T. Krishnan, *Development and Change: Essays in Honour of K.N.Raj*. Oxford: Oxford University Press.

Ray, D., & Streufert, P. A. (1993). Dynamic equilibria with unemployment due to undernourishment. *Economic Theory, 3(1)*, 61-85.

Rebelo, S. (1991). Long-Run Policy Analysis and Long-Run Growth. *The Journal of Political Economy, 99(3)*, 500-521.

Roberts, M. A. (2010). Firm formation and economic growth: the effects of labour union bargaining power and of worker mobility. *Discussion Paper No. 10/10*. University of Nottingham.

Romer, P. M. (1986). Increasing Returns and Long-Run Growth. *The Journal of Political Economy, 94(5)*, 1002-1037.

Romer, P. M. (1987). Growth Based on Increasing Returns Due to Specialization. *American Economic Review, 77(2)*, 56-62.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy, 98(5)*, S71–S102.

Savabi, F., Moniri, Z., & Hashemi, M. (2011). The Effect of Real Wage Increase on the Potential Real Output Growth Rate and Unemployment Rate: A Theoretical Approach. *Proceedings of Intellectbase International Consortium. 14,* pp. 46-54. Intellectbase International Academic Consortium.

Shapiro, C., & Stiglitz, J. E. (1984). Equilibrium Unemployment as a Worker Discipline Device. *The American Economic Review, 74(3)*, 433-444.

Sheshinski, E. (1967). Optimal Accumulation with Learning by Doing. In K. Shell ed., *Essays on the Theory of Optimal Economic Growth* (pp. 31–52). Cambridge: MIT Press.

Solow, R. M. (1956). A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics, 70(1)*, 65-94.

Solow, R. M. (1979). Another possible source of wage stickiness. *Journal of Macroeconomics, 1(1)*, 79–82.

Sorensen, J. R. (1997). Do Trade Unions Actually Worsen Economic Performance? *Working Paper No. 1997-6.* School of Economics and Management, University of Aarhus.

Stadler, M. (1998). Dual labor markets, unemployment and endogenous growth. *Tübinger Diskussionsbeiträge, No. 126.* School of Business and Economics, University of Tübingen.

Stevis, D. (2011). Unions and the Environment : Pathways to Global Labor Environmentalism . *WorkingUSA, 14(2)*, 145–159.

Stiglitz, J. E. (1976). The Efficiency Wage Hypothesis, Surplus Labour, and the Distribution of Income in L.D.C.s. *Oxford Economic Papers, 28(2)*, 185-207.

Summers, L. H. (1988). Relative wages, efficiency wages and Keynesian unemployment. *The American Economic Review, 78(2)*, 383-388.

Swan, T. W. (1956). Economic Growth and Capital Accumulation. *Economic Record, 32(2)*, 334–361.

Tanaka, J. (2002). A note on government spending on infrastructure in an endogenous growth model with finite horizon. *Journal of Economics and Business, 54(6)*, 651–654.

Tournemaine, F., & Tsoukis, C. (2015). Public Expenditures, Growth and Distribution in a Mixed Regime of Education with a Status Motive. *Journal of Public Economic Theory, 17(5)*, 673–701.

Traxler, F., & Brandl, B. (2009). *The economic effects of collective bargaining coverage: a cross-national analysis.* Geneva: International Labour Office.

Tsoukis, C., & Miller, N. j. (2003). Public services and endogenous growth. *Journal of Policy Modeling, 25(3)*, 297–307.

Tsoukis, C., & Tournemaine, F. (2011). Social conflict, growth and factor shares. *Metroeconomica, 62(2)*, 283–304.

Turnovsky, S. (1996). Fiscal policy, adjustment costs, and endogenous growth. *Oxford Economic Papers, 48(3)*, 361-381.

Turnovsky, S. (1997). Fiscal policy in a growing economy with public capital. *Macroeconomic Dynamics, 1(3)*, 615-639.

Turnovsky, S. J. (1999a). On the role of government in a stochastically growing open economy. *Journal of Economic Dynamics and Control, 23(5-6)*, 873–908.

Turnovsky, S. J. (1999b). Fiscal Policy and Growth in a Small Open Economy with Elastic Labour Supply. *The Canadian Journal of Economics, 32(5)*, 1191-1214.

Turnovsky, S. J., & Pintea, M. (2006). Public and private production in a two-sector economy. *Journal of Macroeconomics 28(2)*, 273–302.

Tzannatos, Z. (2008). The Impact of Trade Unions: What Do Economists Say? In J. Berg, & D. Kucera, *In Defence of Labour Market Institutions* (pp. 150-191). Palgrave Macmillan UK.

Tzannatos, Z., & Aidt, T. S. (2006). Unions and microeconomic performance: A look at what matters for economists (and employers). *International Labour Review, 145(4)*, 257-278.

Uzawa, H. (1965). Optimum Technical Change in An Aggregative Model of Economic Growth. *International Economic Review, 6(1)*, 18-31.

Valenduc, G. (2001). Trade unions as agents of environmental awareness : Outcomes from the RISE project. In E. Hildebrandt, B. Lorentzen, & E. Schmidt, *Towards a Sustainable Worklife* (pp. 55-66). Sigma, Berlin.

Vedder, R., & Gallaway, L. (2002). The economic effects of labor unions revisited. *Journal of labor research, 23(1)*, 105-130.

Visser, J. (2006). Union membership statistics in 24 countries. *Monthly Labor Review, 129(1)*, 38-49.

Wapler, R. (2001). Unions, Growth and Unemployment. *Tübinger Discussion Paper No. 206.* School of Business and Economics, University of Tübingen.

Weede, E. (1986). Sectoral reallocation, distributional coalitions and the welfare state as determinants of economic growth rates in industrialized democracies. *European Journal of Political Research, 14(5-6)*, 501–520.

Weede, E. (1991). The impact of state power on economic growth rates in OECD countries. *Quality and Quantity, 25(4)*, 421-438.

Yellen, J. L. (1984). Efficiency Wage Models of Unemployment. *The American Economic Review, 74(2)*, 200-205.

Zagler, M. (2005). Wage pacts and economic growth. *Journal of Economic Studies, 32(5)*, 420-434.