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Essays on Behavioral Industrial Organization and Welfare

Priyanka Kothari

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Thesis Supervisor: Professor Prabal Roy Chowdhury

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Dedicated to Dada

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Chapter 1

Introduction

1.1 Overview

Many traits of economic agents are well studied and understood by psychologists that are yet to be captured by economic theory. This thesis, in the field of behavioral economics, is yet another attempt at unifying the different approaches of the two well-established disciplines - we ask what does economics predict of the behavior of economic agents ... once the psychology behind their decisions is accounted for. In a nutshell, this thesis is about the implications of choice-making among available alternatives.

We begin with the idea that an economic agent only derives satisfaction from what he consumes without worrying at all (let alone too much) about what he does not. This idea has been contested by celebrated psychologists like Barry Schwartz who argue that if what is bought falls below expectations, then the nagging thought about a potentially better alternative that was not purchased, takes away from the utility from consuming/using what was bought. This idea is being researched by economists, who also found that consumer benchmark is context-dependent. The use of *default options* (Park et al, 2000, and Johnson et al, 2002), *historical values* (Klein and Oglethorpe, 1987), *expectations* (Song, 2012), and *anchoring* (Chapman and Johnson, 1999 and Strack and Mussweiler, 1997) by consumer are sometimes to simplify their decision making process. The effect of these comparisons is not limited to the consumer choices, for they also influence firms' strategic decisions (Zhou, 2011; Heidhues and Kőszegi, 2005; Heidhues and Kőszegi, 2008 and Spiegler, 2012).

In fact, with durable goods the negative effect due to comparison is magnified - every time a commodity is used, an agent is reminded that he could have made a better choice. While we take up the implications of this discussion more elaborately in the final chapter of this thesis (Chapter 4) in a welfare-framework, for now it is important to note that consumers frequently compare what they buy with what they do not - particularly when there are reasons to believe that the latter is better. This is the central theme of Chapter 2 of the thesis.

1.2 Monopoly and the other: Reference dependence in vertically differentiated markets

In this chapter, we acknowledge that those who do not buy/consume the best-available alternative in a given market, are also the ones who face a utility-loss from the very fact that they are not consuming the best - that they have settled for something of a lesser value takes away from their utility from consumption. We capture this idea theoretically in a two-product setting, where the products differ in quality. We model a consumer who faces a utility loss from consumption, unless he consumes the best product available. As a first step, we ask if firms can benefit from accounting for this trait of its consumers while making its own pricing decisions. We find that the answer to this question is in the affirmative - in our duopoly setting involving one high-quality, and one low-quality manufacturing firm, we demonstrate that firms can indeed earn higher profits by accounting for this utility-loss faced by some consumers in their pricing decisions (in comparison with the profits they would earn if they did not account for the same). In the second step, we try to quantify the magnitude of the benefits that the firms can potentially enjoy if they account for this element of utility-loss by those who consume the low quality product. We find that remarkably enough, the high-quality firms can, under certain sufficient conditions, earn monopoly-level profits (despite clearly remaining in a duopoly). As a final step (where we generalise our model to accommodate more and more firms), we demonstrate that this ability to earn monopoly-level profits by high-end firms is unique to only duopolistic market-forms. With more (and more) firms, the effect of reduced market shares makes it impossible to earn monopoly-level profits.

1.3 Fairness is flexible: A study of competing focal points

At the end of this exercise, we are inclined to examine the possibility that there could be more than one reference point (Kahneman, 1992; Birkeland and Tungodden, 2014; and Neale and Bazerman, 1991). In a given context, the way two individuals process information relative to their own reference point(s) can make one happy and another sad or both. This can be very well understood using example given by Kahneman (1992), where an individual who expects a raise of $5000 but is given $3000 instead. To him it is a loss with reference to expectation, but it is a gain with reference to a raise of $0.

We study the presence of multiple reference points in a slightly different context involving bilateral bargaining. There are many ways in which focal points affect bargaining outcomes (Schmitt, 2004; Babcock and Loewenstein, 1997 and Binmore et al 1993). Many researchers have found irrelevant cues to play important roles in bargaining. Even in the absence of any cue, individuals use allocations that ease their decision making and are easily justifiable as focal/reference points. We study the role of the presence of multiple focal points in a dialogue-based dynamic unstructured bargaining game.

We examine how pairs of individuals negotiate on how to split the pie when there are at least two focal points. This can be best illustrated with an example with two agents named *X* and

Y, who must negotiate on how much to split a given pie-size z among themselves. Suppose that individually, X can earn \$50 (= d_x) and Y can earn \$100 (= d_y). We ask how X and Y should split $z = 300$ among themselves? Moulin (1988) discussed three broad classes of fair division, at least two of which, can serve as ready (focal) points of reference. The first is called the uniform gains (UG), and the second is the proportional rule (PRO) (the third one is the equal surplus (ES) akin to the Nash (1950) bargaining solution, which does not act as a ready reference point, as we will discuss later). If x and y are respectively the amounts (that sum up to z) negotiated by X and Y, then the first rule (UG) suggests that $x = y = 150$ (i.e. they share the pie regardless of their individual earning capacities). The PRO rule on the other hand, will require that $x = 100$ and $y = 200$ (i.e. the agents divide the pie in the ratio of their individual earning capacities so that $x/y = d_x/d_y$). The key point here is that each of the two can be argued to be a *fair* split. We turn to this below.

The first idea (UG) borrows from a wider notion of fairness that is desirable in many contexts, particularly in relation to justice that should ideally overlook individual differences between agents seeking judicial help. The idea that justice should be equally available for all regardless of individual differences including those in earning capacity, income, gender and so on, is aptly summed up by the Universal Declaration of Human Rights as "all are equal before the law and are entitled without any discrimination to equal protection of the law". In our example, differences in earning capacity among our agents should not matter when they can come together and achieve something greater ($z = 300 > d_x + d_y$). Anything implicitly associated with the idea of 'a greater good' should look beyond individual differences (Banerjee et al, 2012).

The second idea of proportional rule (PRO) suggests that $x = 100$, and $y = 200$. The idea that the agents could share the larger pie according to their individual capacities echoes ancient ideas such as Aristotle's notion of fairness which should be according to some measure of agents' need, ability, effort and status. In our example, the individual earning capacity can be thought of as an immediate measure of ability. Nobody contests the thought that the more able agents ultimately get access to a greater share of a society's cumulative resources. Thus this rule ties fairness to the role of deservedness.

In addition to the above two, a third notion of fairness called Equal Surplus (ES) requires that the gain from cooperation must be equal for each agent. In this example, $x = 125$ and $y = 175$, since at these values, $x - d_x = y - d_y$ is satisfied. It is clear that the ES rule strictly lies in between the other two extremes of the UG and PRO fairness rules - it is effectively the Nash-bargaining (1950) solution which picks the pair (x, y) that maximizes the joint product $(x - d_x)(y - d_y)$. What makes it less of a focal point though, is that it is harder to calculate in comparison with the other two rules (more specifically the bargaining agents must simultaneously solve $x + y = z$ and $x - d_x = y - d_y$, to arrive at this solution).

This is a first attempt at obtaining useful insights about the predictive capacity of each focal point relative to the other(s), since the existing literature (Camerer et al, 2018; Karagözoğlu and Urhan, 2017 and Karagözoğlu, 2019) offers little insights about which focal point or fairness rule to pick in a given bargain. In other words, it is *a priori* difficult to arrive at a conclusive judgment about which focal point looks more attractive than the others in a given context - we ask which focal point would turn out to be dominant over the others in terms of predictive capacity.

Pairs of individuals bargain over a given sum of money with the above disagreement payoffs in Indian Rupees (INR). We show that the pie-size itself is a source of useful information about which focal point to expect. In four treatments where pairs of individuals respectively bargain over sums of INR 180, INR 300, INR 600 and INR 900, we observe a modal split of 2:1 when the pie-size is INR 180 or INR 300, demonstrating that our bargaining agents prefer the PRO over the UG or the ES. When the pie size is INR 600 or INR 900 however, we observe a modal split of 1:1, demonstrating that the agents prefer the UG over the ES or the PRO rules. To identify a bargaining outcome as a focal point, we use two related criteria - first is the ease of calculation, and the second (also as a direct consequence of the first), that the process of negotiation was very quick. It is clear that the splits of 2:1 and 1:1 were focal points because they are easy to calculate and time taken to negotiate on these values for any given pie-size is far lesser.

We offer a useful heuristic in terms of the Nash bargaining (ES) solution. We know that the ES strictly lies between the two extremes of PRO and UG. However, for any given pie-size, if the ES is closer to the UG solution, we expect the modal offer coinciding on the UG solution, and if the ES is closer to the PRO solution, we expect the modal offer coinciding on the PRO solution. A behavioral explanation to this heuristic is that individual differences in disagreement payoffs look very insignificant (and possibly petty) in relation to larger pie-sizes. For example, if the pie size was only worth INR 153, then the 2:1 split would look naturally fairer than the equal-split, since this pie-size is not very far from the sum of the individual disagreement payoffs ($d_x + d_y = 150$). However, if the pie-size equaled (say) INR 10 million, then these individual differences in the disagreement payoffs seem very petty. In such cases, the equal split seems like a fairer outcome.

Focal points are important especially when quick decision-making is of the essence. Indeed, focal points help us skip long chains of reasoning in order to quickly arrive at a decision. In short, focal points save time and eases the cognitive load. It is therefore interesting to imagine a world where focal points are absent. Indeed, when choices are too many, the absence of focal points can be a paralyzing experience. This is the central theme of Chapter 4, which we turn to now.

1.4 Happiness in the finite: Oligopoly maximizes welfare

In day-to-day market transactions, consumers compare among available choices and decide what to consume. Taking such decisions can be quite stressful even for a perfectly healthy individual, let alone people with limited cognitive ability (Agarwal, et al, 2009 and Agarwal and Mazumder, 2013). This is one of the main reasons for the upsurge of insurance advisors, financial advisors and career counselors etc. Evaluating many possibilities can impose huge cognitive costs. Consumers have come up with ways to get around the decision making process, such as cognitive costs are very much real. The various ways used are sampling (Smallwood and Conlisk, 1979), searching (Ellison and Wolitzky, 2012; Jain and Ray, 2016), reference points (Kahneman, 1992; Zhou 2011). In fact, it has been found that offering limited choices can help both firms and consumers.[1]

The idea that 'more is better' has retained its axiomatic stature in economics. Psychologists however, have contested this idea that too many choices translate to greater welfare, by pointing out that the cognitive costs of evaluating available choices against each other often take away from agent-satisfaction. We formalize this idea and apply the same in the context of markets with welfare implications that contradict the idea that competition maximizes surplus. We demonstrate that if cognition-costs are accounted for, then welfare is maximized with a *strictly* finite number of firms, thereby mimicking an oligopolistic structure - even when cognition-costs are not very high. An immediate implication of this result for regulatory authorities is that maximizing welfare is not always the same as maximizing competition.

The central idea of this chapter can be summed up as follows. Perfect competition is characterized by free entry and exit and a large number of buyers and sellers. A large number

[1]In Psychology (Schwartz, 2004; Reutskaja and Hogarth, 2009), biology (Eagleman, 2015), marketing (Iyengar and Lepper, 2000) researchers have shown giving optimal number of choices to choose from can improve decision making and overall happiness (Jeffries, 2015).

of sellers, however, directly translates to a large number of competing products to chose from. Barry Schwartz (2004) refers to this availability of too much choice as information overload, and uses this to explain why having to many choices takes away from happiness and satisfaction - the official dogma of all western societies. We borrow this idea from psychology to account for cognition-costs in consumer choice and show that there is a strictly finite number of products (and therefore firms) beyond which the costs of evaluating, comparing and processing too many products outweigh the very benefits of having such variety (of products) in any market (and this does not even include search costs (Jain and Ray, 2016; Ellison and Wolitsky, 2012)). Thus, using welfare analysis we demonstrate that the happiest society is one where options and choice is limited.

In a nutshell, well-celebrated examples of theoretical, empirical and experimental investigations carried out over the years have clearly evolved in terms of predictive capacity. Various innovative methodologies have been introduced to capture the behavior of the imperfect agent and the implications of such behavior on other economic agents.

1.5 Summary

This thesis offers a study of three diverse, yet very much inter-linked ideas on focal points, welfare and bargaining. Each chapter is inspired by real life experience. The second chapter talks about the consumer who finds it convenient to compare every other product to the best available option and that the firms can benefit from taking into account such behavior. The third chapter analyses the presence of multiple focal points for a fair negotiation. In an experiment involving bargaining, we show how different fairness norms can act as focal points under different stake-sizes. In other words, the stake size involved can alter the preference between available focal points with embedded fairness considerations. The fourth chapter deals with another aspect of consumer behavior, where limited cognition transforms the availability of too many choices into a state of information overload, thereby, leading us to conclude that having limited number of choices can indeed be welfare maximizing.

Chapter 2

Monopoly and the other: Reference-dependence in vertically differentiated markets

2.1 Introduction

We show that in an oligopolistic market, firms can strategically benefit from internalizing the knowledge that consumers often compare any firm's product quality to that of the others'. Monopolies cannot benefit through this channel, since by definition, there is no other firm against which quality comparisons could be made by consumers. We focus on vertically-differentiated markets, where each consumer agrees on a well-defined ranking of available products, and the product of the best quality becomes a natural point of reference. We demonstrate that a firm can enjoy high (monopoly-level) profits from remaining in an oligopoly with reference-dependent consumers. We look at an incumbent monopoly that faces the potential entry of a different-quality (product manufacturing) entrant, and show that the incumbent firm can comfortably accommodate the entrant (without worrying about a loss of profits associated with a change of the market structure from a monopoly to a duopoly), if the quality differential between both the firms is sufficiently large and the consumers are bounded-rational (by which we specifically mean, reference-dependent, so that both the firms are able to account for the same in their decision making).

To present an intuition of the results, we suppose (for now) that a monopoly firm sells a high-quality product. Since this is the only firm in the market, consumers choose, either to buy the product or not buy it, and there is no question of comparison with any other firm's product.

With the entry of a low-quality product selling firm that charges a lower price, many consumers who did not buy the high-quality product now find themselves able to afford the low-quality product. This creates a (vertical) market divide between the two qualities available. The high-quality consumers do not want to consume the low-quality product, and therefore stick to the high-quality product, thereby making the demand for the same more inelastic. This allows the high-quality firm to ask for higher prices without any corresponding loss of demand, leading to higher profits (which, as we will demonstrate, can remarkably match monopoly-profits). The low-quality firm benefits because consumers are reference-dependent - a justifiably higher price for the high quality product translates to higher prices for the low quality product.

We present our primary results in the setting of a vertically differentiated duopoly with each firm i $(i = 1,2)$, producing a product of quality s_i for reference-dependent consumers. Consumers consider the high-quality product as the product of reference. While they derive some intrinsic utility from the consumption of the (low-quality) product, they also experience some loss in utility due to a comparison with the better product available (Schwartz, 2004). The consumer of the high-quality product does not experience this disutility from comparison. Our assumption of considering the high-quality product as the reference point can be explained in the words of Garcia et al. (2013), according to whom, "people commonly seek to achieve a *superior* position vis-à-vis others in a variety of contexts, from daily social situations to organizational settings and market transactions." This explains why the *'superior'* naturally becomes the reference point, since the role of individual and situational factors often increase social comparison concerns, and thus competitiveness. This sums up the idea of the 'unidirectional drive upward'. In our framework, we show that reference-dependence softens price competition enabling both the firms to charge higher prices than in the setting where the consumers do not experience reference-dependence. Centrally, we demonstrate that effect of reference-dependence can be powerful enough to nullify the profit differential between a duopoly and monopoly.

In economics, general insight suggests that an increase in competition lowers both prices and profits. However, in real life, there could be instances where a monopoly is replaced by a duopoly without any corresponding loss in the profits of the formerly incumbent monopoly. Many empirical studies have confirmed that incumbents often welcome entrants. In the anti-ulcer drug market, for instance, it has been found that prices rise as a result of new entry (Perloff et al, 2005). Similarly, in the food industry, an introduction of private labels increases prices of national labels (Ward et al. 2002; and Thomadsen, 2007). Economists have often attempted to theorize such behavior. Gelman and Salop (1983) demonstrated that an incumbent might be less aggressive if the entrant limits its capacity. Alternatively, the incumbent may give the entrant the license to its own technology to crowd out the incentive of the latter to develop a new technology (Gallini, 1984). In this paper, we provide a theoretical explanation for why prices and profits rise with an increase in competition. We derive sufficient conditions to show that firms can earn duopoly profits that match that of a monopoly; and we demonstrate this in the context of reference-dependent consumers.

This work adds to the existing research on the effect of reference points. As defined by Spiegler (2011) a reference point is "an action or a consequence (or some aspect of either) which acts as a 'frame' of a choice problem and affects choices in a way that is ruled out by conventionally rational decision making." The idea that people see things in context is manifested in the effects of *default options* (amounting to the singling out of a feasible alternative as in Park et al, 2000, and Johnson et al, 2002 among others), *historical values* (particularly in the context of consumer experience in dynamic choice under uncertainty as in Klein and Oglethorpe, 1987), *expectations* (in response to the realization of an exogenous uncertain effect as in Song, 2012), and *anchoring* (which exerts a 'pull' on the decision makers' judgment as in Chapman and Johnson, 1999; Strack and Mussweiler, 1997; and Mussweiler and Strack, 1999) in agent behavior. Such effects of references on consumer preference have been extensively observed in laboratory experiments. More extreme cases of reference-dependence leading to preference instability have been reported by Samuelson and Zeckhauser (1988), and Huber et al, (1982) where the choice between two bundles is reversed by the introduction of reference bundles, and in Shampanier et al, 2007, where such a reversal is due to a reference price (zero). Reference-dependence is also known to cause differences between willingness to pay and willingness to accept (see Thaler, 1980; Kahneman et al, 1990; and Carmon and Ariely, 2000).

In the area of consumer behavior, many experiments have demonstrated that the earlier theoretical constructs of choice and preference are not complete and need to be enriched further. Various axiomatic (see Tversky and Kahneman, 1991; Ok, Ortoleva, and Riella, 2012; and Masatlioglu and Ok, 2005, for an analysis on the determination of choices over non-risky alternatives) and non axiomatic models (see Kőszegi and Rabin, 2006) have been proposed to incorporate interesting aspects of reference-dependence that qualify as anomalous to the earlier constructs. Reference points for example, are context dependent. While Spiegler (2012) takes consumers to treat sample prices as their references, others (Heidhues and Kőszegi, 2008; and Karle and Peitz, 2012) model consumers to take rational expectations-based reference points.

We have so far, discussed the literature on reference-dependence in relation to consumer behavior. The impact of references has also been studied in many areas such as the choice over *risky alternatives* (Wedell, 1991; and Herne, 1997), choice of *job candidates* (Highhouse, 1996; and Slaughter, Sinar and Highhouse, 1999); *auctions* (Ariely and Simonson, 2003) and so on.

Our research focuses on the implication of reference-dependent consumer behavior on firm strategy. Previous research comprises the works of Heidhues and Kőszegi (2014), and Speigler (2012), who (among others) investigated the impact of expected prices as reference points on monopoly pricing; and those like Zhou (2011), that analyze the effect of reference-dependence on firms' pricing and advertising strategies in horizontally product differentiated duopoly models.

In section 2 that follows, we will present a motivation behind our model, before we formalize the same in Section 3. In section 4 we briefly discuss the welfare implications of reference-dependence before extending our discussion to the *n*-firm case in section 5. Section 6 concludes.

2.2 A motivation

As a first example, in a country like India where a career in engineering is much sought after, the advent of tuition centers becomes unquestionable. What is interesting is that the ranking of all these tuition centers are unanimous i.e. for any two centers, if one is preferred to another by a given individual then it is true for every other individual. In other words, the ranking (which is a measure of the quality of the center) between the tuition centers is common knowledge. We show that in such markets, where consumers experience loss due to reference-dependence in the product dimension, competition is softened so that both the lower-quality centre and the high-quality centre are able to raise their prices, without a corresponding loss of demand, which allows them to enjoy higher profits.

As a second example, in the Indian cosmetic industry (Rattanani, 1994), immediately after the new economic reforms that encouraged trade liberalization, the managing director of Tips

& Toes was quoted as saying "let them come and develop the market. We are not afraid. We know the Indian market" when he hinted on potential polarization of Indian consumers into two segments, the first of which would switch to buying the 'high branded' and 'more expensive' foreign products, and the second, that would continue to buy from the incumbent(s). The managing director summed up his thoughts when he said that "the rest will still come to us". Clearly, we are again in a vertically differentiated market, since a high-quality global brand like L'Oréal (for instance) is a 'preferred (generally by all) brand' compared with Lakmé (a more locally known brand then). Rattanani (1994) sums up the cosmetic industry story when she says "the price difference is likely to divide the market. A 14ml bottle of Fidji perfume for instance is priced at Rupees (INR) 2100 while a local brand sells for INR 250. The foreign brands will rule the upmarket brands while the Indians will cater to the lower end of the market. Finally, it is the customer who will decide the winner."

The above stories relate well with the words of Binkley (2007), who exclaims that "some people cut and run when confronted with prices that seem crazy. But many of us experience a sudden emotional-mathematical transformation. We set a new ceiling for a 'reasonable' price. Disinclined to go all the way to buy the *trophy*, we instead settle for a *consolation prize*. This concept is one of the reasons for the proliferation of $300 designer sunglasses these days. The fact that Ralph Lauren is charging $14,000 or so for an alligator 'Ricky' handbag makes it easier for a consumer to justify in her mind paying $300 for a rather simple sweater. Many Chanel sunglass owners are actually would-be owners of Chanel suits. Something similar has happened to many owners of Tiffany key-chains, Prada leg-warmers, Coach wallets, and Frette tea towels."

In this paper we present a model that unifies and explains the above stories. The findings employ the models of consumer behavior that account for reference-dependence. We argue that when a firm producing a high-quality product enters a market, consumers tend to compare the products of the existing (low-quality) firm with it. They consequently feel a sense of 'quality-compromise' on buying a lower quality product.[1] However, if the price charged by the high-quality product significantly exceeds that of the low-quality product, against the given (perceived) quality differential, then there remains an incentive for the low-quality firm to raise its price sufficiently high without a compensating decline in its market share (due to polarization), thereby raising its overall profits. In other words, once the consumers 'update' their willingness to pay for the higher quality product, given the information on the price(s) quoted by the high-quality firm, the lower quality firm(s) stand to gain from it, because the higher-new price seems to justify the value of the product.

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2.3 The formulation

2.3.1 The benchmark case: monopoly

Consider the case when the low-quality *(s_1)* producing firm is the monopolist in the market. A consumer's utility is characterized as follows

$$U_1 = \begin{cases} V + \theta s_1 - p_1 & if\ he\ buys\ the\ good \\ 0 & otherwise \end{cases}$$

where, V is the intrinsic utility obtained from consuming the product itself, the parameter θ captures how much the consumer values quality, and the quality parameters_1 is defined as in the introductory section, so that $V + \theta s_1$ can be thought of as the consumer's maximum willingness to pay for the product with quality s_1. For this quality, a consumer with high θ is willing to pay more than a consumer with low θ. Without loss of generality, for now, we

[1]This is different from a setting where consumers are known to have cognitive limitations which lead them to pay higher for given quality of the products sold under different brands (Sürücü, 2016). In our setting, consumers have no cognitive limitation(s) in perceiving the product the way they are, but they compare products against a reference product (of higher quality).

normalize $V = 0$ and defer the simulations for general cases with positive values of V to the penultimate section. We model a continuum of consumers, each associated with a unique θ, which is drawn from a uniform distribution with the supports $\underline{\theta}$ and $\bar{\theta}$. The consumer who is indifferent between buying and not buying from this low-quality monopolist is of the type $\hat{\theta}$, such that $\hat{\theta}s_1 - p_1 = 0$ or $\hat{\theta}s_1 = p_1$. Since all the consumers with $\theta \in [\hat{\theta}, \bar{\theta}]$ will buy from this monopolist, the proportion of consumers who will buy from this firm is given by $\frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}}$, and the profits by $\pi_1^m = \frac{p_1(\bar{\theta}-\hat{\theta})}{\bar{\theta}-\underline{\theta}} = \frac{\hat{\theta}s_1(\bar{\theta}-\hat{\theta})}{\bar{\theta}-\underline{\theta}}$, maximizing which, with respect to $\hat{\theta}$, gives us $\hat{\theta} = \bar{\theta}/2$, so that $p_1 = \bar{\theta}s_1/2$. Thus, the monopoly profits equal $\bar{\theta}^2 s_1/4(\bar{\theta} - \underline{\theta})$. Similarly, if the monopolist was, a high-quality (with $s_2 > s_1$) firm, then its profit will be $\pi_2^m = \bar{\theta}^2 s_2/4(\bar{\theta} - \underline{\theta})$ and price $p_2 = \bar{\theta}s_2/2$. We will continue to assume that if a consumer is indifferent between buying and not buying a product (i.e. each decision is associated with the exact same level of utility), then he will choose to buy the product.

2.3.2 A model of perfect information: duopoly

Two firms (1 and 2) sell a single product of distinct quality s_i $(i = 1, 2)$. Quality (known to every agent) is given exogenously such that $s_1 < s_2$. For every consumer, the higher quality *(s_2)* product is preferred over lower quality product *(s_1)*. Thus, the products are vertically differentiated i.e. all consumers unanimously prefer s_2 to s_1. However, consumers are heterogeneous in the way they value quality, represented by θ. As before, we assume that θ is uniformly distributed in the interval $[\underline{\theta}, \bar{\theta}]$. Additionally, to explicitly capture the level of consumer heterogeneity, we assume that $\bar{\theta} = k\underline{\theta}$, for some k, which can be thought of the measure of consumer-heterogeneity. We will later discuss the extension of our model to the *n*-firm scenario. A consumer with higher θ, values quality improvements more strongly. Each consumer has unit demand for the product in question, which is manufactured at a unit cost c, which we normalize to zero. Firms set prices simultaneously. The analysis that we present here is based on the models initially developed by Gabszewicz and Thisse, 1979; and Shaked and Sutton, 1982 (additionally see Bandyopadhyay and Acahryya, 2004).

When a consumer with the taste parameter θ consumes quality s_i for price p_i, she gets a material utility (with $V = 0$) of $\theta s_i - p_i$. In general, consumers also exhibit reference-dependent preferences – they have a tendency to compare, evaluate products and feel loss or gain depending on whether they buy lower or higher than their reference levels of quality. We assume that the consumer takes the high-quality firm's product (more specifically, the utility derived from it) as the reference point. Two questions arise here:

1. Why should the high-quality product serve as a reference point?

2. Why should consumers compare *utilities* derived from the consumption of goods and not something else (for example *prices*)?

The first question above has been addressed in the introductory note to this paper that sums up the idea of the 'unidirectional drive upward'. Further, in Zhou (2011) the prominent firms' product becomes the reference point.[2] It has been observed that in luxury brand product lines, the most exclusive brand serves as an anchor (Binkley, 2007). In our model, the ranking of products (say, provided by experts), is a source of prominence which is considered more reliable than any advertisement, which only focuses on the positive aspects of the product. The second question above can be answered in the context of Kőszegi and Rabin (2006), who argue that an approach that explicitly incorporates consumption utility into the analysis is clearly more complete in terms of both *behavior* and *welfare* than formulations with single

[2]We implicitly take 'prominence' in this sense to be exogenous. In general, prominence may come from firms' advertising efforts etc (for example, see Zhou, 2011).

"value functions" that evaluate gains and losses relative to a reference point, and ignore or suppress the role of consumption *utility* in the evaluation of outcomes.[3] This discussion above motivates us to model the consumers' utilities as follows.

The utility of consumer with θ as her taste parameter, from consuming quality s_2 at price p_2 is given by

$$U_2 = \theta s_2 - p_2 \tag{2.1}$$

The utility of consumer with θ as her taste parameter, from consuming quality s_1 at price p_1 is given by

$$U_1 = \theta s_1 - p_1 - \lambda\theta(s_2 - s_1) \tag{2.2}$$

Here, the first two terms on the right hand side represent the material utility from product 1 and the last term captures the loss in utility due to reference-dependence (since product 2 acts as a reference point for product 1). The parameter $\lambda\,(> 0)$ captures the strength of the reference-dependence effect. If $\lambda = 0$ then there is no reference-dependence effect, and we get back the standard vertical product-differentiation model. The above (reference-dependent) utility function has yet another interesting interpretation. We can argue that given reference level quality (s_2), the consumer with a higher taste for quality (i.e. higher θ) will experience a larger loss than a consumer with a lower θ. This feature is captured by the interaction of θ with $(s_2 - s_1)$. Thus, the impact of reference-dependence for our consumer is magnified by θ.

[3] Kőszegi and Rabin (2006) consider a hypothetical experiment in which a consumer is endowed with a hundred paper clips and a hundred $10 bills as part of her reference point, and must choose between two gambles: a 50-50 chance of gaining a paper clip or losing a paper clip, and the comparable gamble involving $10 bills. It seems likely that she would risk losing the paper clip rather than the money, and do so because her sensation of gains and losses is generally likely to be smaller for a good whose consumption utility is smaller.

2.3.3 Demand

Whether a consumer with θ as her taste parameter will buy from firm 1 or firm 2 is just a matter of utility comparison. If we call our indifferent consumer as the type $\hat{\theta}$, then all the consumers with $\theta \leq \hat{\theta}$ will buy the low-quality product and all those with $\theta \geq \hat{\theta}$ will buy the high-quality product. The indifferent consumer satisfies $U_1 = U_2$, and his type is therefore, given by

$$\hat{\theta} = \frac{(p_2 - p_1)}{(s_2 - s_1)(1+\lambda)} \tag{2.3}$$

Thus, the demand that Firm 1 (the lower quality firm) faces is

$$D_1(p_1, p_2) = \frac{\hat{\theta} - \underline{\theta}}{\bar{\theta} - \underline{\theta}} = \left[\frac{(p_2 - p_1)}{(s_2 - s_1)(1+\lambda)} - \underline{\theta}\right]\left(\frac{1}{\bar{\theta} - \underline{\theta}}\right) \tag{2.4}$$

and the demand that Firm 2 (the higher quality firm) faces is

$$D_2(p_1, p_2) = \frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}} = \left[\bar{\theta} - \frac{(p_2 - p_1)}{(s_2 - s_1)(1+\lambda)}\right]\left(\frac{1}{\bar{\theta} - \underline{\theta}}\right) \tag{2.5}$$

It is clear that a rise in consumers' loss due to reference-dependence (captured by the parameter λ), raises the demand for product 2 and reduces that of product 1. The profit functions of the firms are

$$\pi_1(p_1, p_2) = p_1\left(\frac{(p_2 - p_1)}{(s_2 - s_1)(1+\lambda)} - \underline{\theta}\right)\left(\frac{1}{\bar{\theta} - \underline{\theta}}\right) \tag{2.6}$$

$$\pi_2(p_1, p_2) = p_2\left(\bar{\theta} - \frac{(p_2 - p_1)}{(s_2 - s_1)(1+\lambda)}\right)\left(\frac{1}{\bar{\theta} - \underline{\theta}}\right) \tag{2.7}$$

The first order condition (FOC) for each firm (as shown below) involves the equality of its marginal profit (conditional on the other firm's price), to its own marginal cost (equal to zero in our model).

$$\frac{\partial \pi_1}{\partial p_1} = \left[\frac{p_2 - 2p_1}{(s_2 - s_1)(1+\lambda)} - \underline{\theta}\right]\left(\frac{1}{\bar{\theta} - \underline{\theta}}\right) = 0$$

$$\frac{\partial \pi_2}{\partial p_2} = \left[\bar{\theta} - \frac{2p_2 - p_1}{(s_2 - s_1)(1+\lambda)}\right]\left(\frac{1}{\bar{\theta} - \underline{\theta}}\right) = 0$$

The reaction functions are

$$p_1(p_2) = \frac{p_2}{2} - \frac{\underline{\theta}(s_2 - s_1)(1+\lambda)}{2} \tag{2.8}$$

$$p_2(p_1) = \frac{p_1}{2} + \frac{\bar{\theta}(s_2 - s_1)(1+\lambda)}{2} \tag{2.9}$$

It is clear that each firm would want to raise its own price when the other does, since the loss in profits due to a corresponding reduction in demand is more than compensated by the rise in price. In other words, prices are strategic compliments. In the equilibrium, the prices depend on the quality levels and the reference-dependence parameter λ.

$$p_1^* = \frac{(\bar{\theta} - 2\underline{\theta})(s_2 - s_1)(1+\lambda)}{3} \tag{2.10}$$

$$p_2^* = \frac{(2\bar{\theta} - \underline{\theta})(s_2 - s_1)(1+\lambda)}{3} \tag{2.11}$$

These equations summarize the main intuition of our paper. With sufficient consumer heterogeneity (captured by the first terms involving $\bar{\theta}$ and $\underline{\theta}$ in the equilibrium prices), the 'demand territory' of each firm becomes prominent due to market-segregation. The high-quality firm can afford to increase prices without losing demand. The low-quality firm is thus faced with a higher 'ceiling' because of which it can raise its own price (since prices are strategic compliments) without worrying about a corresponding loss of demand. Thus, in a nutshell, there is a tendency for equilibrium prices to go up when consumers become more heterogeneous. The above effect becomes even stronger, when the quality differential his higher to begin with – and still stronger when the measure of reference-dependence λ is higher. The last point here, only establishes that the high-quality preferring consumers would experience a significant disutility from switching to the lower-quality product, and are therefore 'locked in' the market catered to by only the high-quality firm, allowing it to raise its prices substantially relative to the price offered by the low-quality firm. Thus, a rise in λ benefits the higher quality firm more than the lower quality firm. This is easily verified by the fact that $p_2^* - p_1^*$ is strictly increasing in λ.

Additionally, we require the following conditions to ensure full market coverage and sufficient consumer heterogeneity (but limited product-heterogeneity) to characterize the above equilibrium.

Lemma 1: $U_1(\theta)$is non-decreasing in θ.

Proof: Deferred to the appendix. ■

Lemma 2: There exists a cap on the product-heterogeneity levels_2/s_1, so that$U_1(\underline{\theta})$ is non-negative.

Proof: Deferred to the appendix. ■

Lemma 3: $U_2(\theta)$is non-decreasing in θ.

Proof: Trivial. ■

Theorem 1: Every consumer has a non-negative utility level from buying a product (high or low quality) in the equilibrium (there is full market coverage).

Proof: From *Lemma 1* above, it is clear that U_1attains a minimum at $\underline{\theta}$ with a minimum value of $U_1(\underline{\theta})$. Since $U_1(\underline{\theta}) \geq 0$ (from *Lemma 2*), $U_1(\theta) \geq U_1(\underline{\theta}) \geq 0$, for all $\theta \in [\underline{\theta}, \hat{\theta}]$. Thus, each consumer (in the relevant range) ends up buying from firm 1 and attains a strictly non-negative utility. Finally, we recognize that $U_2(\hat{\theta}) = U_1(\hat{\theta})$ by construction of $\hat{\theta}$, so that $U_2(\hat{\theta}) = U_1(\hat{\theta}) \geq U_1(\underline{\theta}) \geq 0$ (from *Lemma 1* and *Lemma 2*), and use the result of *Lemma 3*, that $U_2(\theta) \geq U_2(\hat{\theta})$, for all $\theta \in [\hat{\theta}, \bar{\theta}]$, to conclude that each consumer (in the relevant range) ends up buying from firm 2 and attains a strictly non-negative utility. This establishes the condition of full market coverage. ■

Before we proceed further, we present an intuitive explanation behind *Lemma 2*. Since there is a direct advantage of market-power associated with higher product-heterogeneity (which is a common feature of Hotelling-type price-competitive models), the latter must have a strict upper bound, beyond which, there may be an incentive to raise prices so much that the consumers with lower θ find the product(s) unaffordable and are therefore necessarily left out of the market. There is a way to unify all the above full-market coverage conditions in terms of λ, as follows.

Corollary 1: With $k = \bar{\theta}/\underline{\theta}$, and $m = s_2/s_1$, the following condition is sufficient to guarantee full-market coverage

$$\lambda \leq \frac{\left(\frac{3}{m-1}\right) - (k-2)}{k+1} = \lambda_1$$

Proof: From $U_1(\underline{\theta}) \geq 0$, we get $\lambda \leq \lambda_1$ above. $U_2(\hat{\theta}) \geq 0$, gives us $\lambda \leq \left(\frac{k+1}{2k-1}\right)\left(\frac{m}{m-1}\right) - 1 = \lambda_2$, and $U_2(\bar{\theta}) \geq 0$, gives us $\lambda \leq \left(\frac{3k}{2k-1}\right)\left(\frac{m}{m-1}\right) - 1 = \lambda_3$. Now, recognizing that $\lambda_3 - \lambda_2 =$

$\frac{m}{m-1} > 0$, so that $\lambda_3 > \lambda_2$, and that $\lambda_2 - \lambda_1 = \left(\frac{(k-2)^2}{(2k-1)(k+1)}\right)\left(\frac{m}{m-1}\right) > 0$(since $k > 2$), so that $\lambda_2 > \lambda_1$,we get $\lambda_3 > \lambda_2 > \lambda_1$. This completes the proof. ■

While the derivations of the above conditions are trivial, it is important to note that the prices and profits are increasing in λ. It is not surprising to observe that (under full market coverage) the price of the high-quality firm is increasing in λ. A high λ corresponds to more (utility) loss and therefore induces less tolerance for the consumption of product 1 in the presence of the higher quality product 2. This (relatively) high demand makes it advantageous for firm 2 to charge a higher price. Further, since the reaction function of firm 1 responds positively to the price charged by firm 2, a high λ is (indirectly) associated with a high p_1. A rise in p_1 is perhaps more surprising since the demand for firm 1 falls with a high λ. Intuitively, it can be seen that firm 2 caters to the consumers with high levels of θ, thereby creating some polarization, which in turn softens the competition between both the firms. Thus, even profits increase in λ. A further impact on market polarization is seen in the fact that both p_1 and p_2 are increasing in the quality differential $(s_2 - s_1)$. An increase in this quality differential increases the market power of the firms by relaxing competition, which enables both the firms to raise their prices.

$$p_2^* - p_1^* = \frac{(\bar{\theta} + \underline{\theta})(s_2 - s_1)(1 + \lambda)}{3} \tag{2.12}$$

As discussed before, the price differential itself is increasing in λ. Therefore, it must be the case that the increase in p_2 is more than the increase in p_1. The equilibrium profits are

$$\pi_1^* = \frac{(\bar{\theta} - 2\underline{\theta})^2 (s_2 - s_1)(1 + \lambda)}{9(\bar{\theta} - \underline{\theta})} \tag{2.13}$$

$$\pi_2^* = \frac{(2\bar{\theta} - \underline{\theta})^2 (s_2 - s_1)(1 + \lambda)}{9(\bar{\theta} - \underline{\theta})} \tag{2.14}$$

We immediately see that both firms' profits are increasing in the reference-dependence parameter λ, and the quality differential. In general, the quality differential and the parameter of reference-dependence are sources of market polarization which is similar to maximal differentiation in the literature on price-competing duopolies.

Example 1: With $k = 3$, so that $\bar{\theta} = 3\underline{\theta}$, and $m = s_2/s_1 = 3/2$ with $\lambda = 1$, we get $p_1^* = \underline{\theta}s_1/3$; $p_2^* = 5\underline{\theta}s_1/3 = 5p_1^*$; $\pi_1^* = \underline{\theta}s_1/18$; $\pi_2^* = 25\underline{\theta}s_1/18 = 25\pi_1^*$. Further, $U_1(\underline{\theta}) = \underline{\theta}s_1/6$; $U_2(\bar{\theta}) = 17\underline{\theta}s_1/6$; and $U_2(\hat{\theta}) = \underline{\theta}s_1/3$. The demand structure is given by: $D_1^* = 1/6$, $D_2^* = 5/6$.

2.3.4 Monopoly versus duopoly (full market coverage)

Comparing the profits of both the firms under the monopoly and duopoly cases, we examine if a firm in the incumbent role would hesitate to accommodate the entrant. The equality of duopoly and monopoly profits would be an interesting result. According to general wisdom, an increase in competition is immediately associated with corresponding decreases in prices and profit. The existing literature, as discussed in the introductory note, sheds light on some of the cases where entrants might be welcomed by existing monopolists. We will later demonstrate that duopoly profits can match monopoly profits since monopolists cannot capture the benefits arising from the quality differential and the disutility experienced by consumers (in the case of duopoly). It is also interesting to note that the smaller the difference in quality, the greater will be the λ required to support the above results. It must also be noted that the monopolist loses out on a portion of the market that is not strategically catered to.

However, before we formally show the workings of the above intuitions, in what follows, we demonstrate that duopoly profits cannot equal monopoly profits under full-market coverage.

Intuitively, in order to capitalize on reference-dependence fully, a firm must necessarily raise its price to a level at which some consumers find the product unaffordable, and are therefore left out of the market.

Lemma 4: Under full-market coverage, the high-quality firm will want to remain a monopoly.

Proof: Deferred to the appendix. ■

Lemma 5: Under full-market coverage, the low-quality firm will want to remain a monopoly.

Proof: Deferred to the appendix. ■

Theorem 2: Under full market coverage, duopoly profits never exceed monopoly profits.

Proof: Follows directly from *Lemma 4* and *Lemma 5* above. If the duopoly-level profit exceeds the monopoly-level profit for any firm, then it would want to invite the entrant and therefore cease to remain a monopoly. ■

Now, we come to the case of partial market coverage, where our consumers are no longer required to buy (any variant of) the product.

2.3.5 Demand (partial market coverage)

Over here, we allow for the possibility that there are some consumers who cannot afford either of the variants of the product. If a consumer does not buy any variant then his utility is zero. Rest of the model structure remains as it was described in the previous subsections. From hereon, we define $\Delta s = s_2 - s_1$. The consumer type who is indifferent between buying nothing and buying from firm 1 is given by $\hat{\theta}_1 = p_1/(s_1 - \lambda\Delta s)$, whereas $\hat{\theta}_2 = (p_2 - p_1)/\Delta s(1+\lambda)$ denotes the consumer type who is indifferent between buying from firm 1 and firm 2. Thus, the demand that firm 1 faces is

$$D_1^{pm}(p_1,p_2) = \frac{\hat{\theta}_2 - \hat{\theta}_1}{\bar{\theta} - \underline{\theta}} = \left(\frac{(p_2 - p_1)}{\Delta s(1+\lambda)} - \frac{p_1}{(s_1 - \lambda\Delta s)}\right)\frac{1}{(\bar{\theta} - \underline{\theta})} \tag{2.15}$$

and the demand that firm 2 faces is

$$D_2^{pm}(p_1,p_2) = \frac{\bar{\theta} - \hat{\theta}_2}{\bar{\theta} - \underline{\theta}} = \left(\bar{\theta} - \frac{(p_2 - p_1)}{\Delta s(1+\lambda)}\right)\frac{1}{(\bar{\theta} - \underline{\theta})} \tag{2.16}$$

The profit functions of the firms are

$$\pi_1^{pm}(p_1,p_2) = p_1 D_1^{pm}(p_1,p_2) = \frac{p_1}{(\bar{\theta} - \underline{\theta})}\left(\frac{(p_2 - p_1)}{\Delta s(1+\lambda)} - \frac{p_1}{(s_1 - \lambda\Delta s)}\right) \tag{2.17}$$

$$\pi_2^{pm}(p_1,p_2) = p_2 D_2^{pm}(p_1,p_2) = \frac{p_2}{\bar{\theta} - \underline{\theta}}\left(\bar{\theta} - \frac{(p_2 - p_1)}{\Delta s(1+\lambda)}\right) \tag{2.18}$$

The reaction functions are:

$$p_1(p_2) = \frac{p_2(s_1 - \lambda\Delta s)}{2s_2} \tag{2.19}$$

$$p_2(p_1) = \frac{p_1}{2} + \frac{\bar{\theta}\Delta s(1+\lambda)}{2} \tag{2.20}$$

With $k = \bar{\theta}/\underline{\theta}$, and $m = s_2/s_1$, the equilibrium prices and profits are:

$$p_1^{pm} = \frac{\bar{\theta}\Delta s(1+\lambda)(s_1 - \lambda\Delta s)}{4s_2 - s_1 + \lambda\Delta s} = \frac{k\underline{\theta}(m-1)(1+\lambda)\big(1 - \lambda(m-1)\big)s_1}{3m + (m-1)(1+\lambda)}$$

$$p_2^{pm} = \frac{2s_2\bar{\theta}\Delta s(1+\lambda)}{4s_2 - s_1 + \lambda\Delta s} = \frac{2k\underline{\theta}m(m-1)(1+\lambda)s_1}{3m + (m-1)(1+\lambda)}$$

$$\pi_1^{pm}\left(p_1^{pm}, p_2^{pm}\right) = \frac{s_2\bar{\theta}^2\Delta s(1+\lambda)(s_1 - \lambda\Delta s)}{(\bar{\theta} - \underline{\theta})(4s_2 - s_1 + \lambda\Delta s)^2} = \frac{k^2\underline{\theta}m(m-1)(1+\lambda)\big(1 - \lambda(m-1)\big)s_1}{(k-1)\big(3m + (m-1)(1+\lambda)\big)^2}$$

$$\pi_2^{pm}(p_1^{pm}, p_2^{pm}) = \frac{(2s_2\bar{\theta})^2 \Delta s(1+\lambda)}{(\bar{\theta} - \underline{\theta})(4s_2 - s_1 + \lambda\Delta s)^2} = \frac{(2km)^2 \underline{\theta}(m-1)(1+\lambda)s_1}{(k-1)\big(3m + (m-1)(1+\lambda)\big)^2}$$

Even in the cases of partial market (*pm*) coverage, we see that the price of the high-quality firm responds positively to λ (details in the appendix). The intuition governing this is exactly the same as that for full market coverage. The price of the low-quality firm, however, strictly decreases in λ for sufficiently large levels of product-heterogeneity (specifically, $m > 1/(4 - 2\sqrt{3}) \simeq 1.87$, which is a minimum quality-differential threshold in our story). This is because, high reference-dependence increases the demand for the (convincingly) high-quality product and reduces the relative demand for the low-quality product (when the option of buying nothing at all is already available to the consumers), to which the price of the firm must immediately respond. We formally summarize these intuitions in *Proposition 1* below.

Proposition 1: Under partial-market coverage, the price of the low-quality product (under sufficient product-heterogeneity) is strictly decreasing in λ and the price of the high quality firm is strictly increasing in λ.

Proof: Deferred to the appendix. ■

In addition to the above, we look at the following proposition to understand how profits respond to λ.

Proposition 2: Under partial-market coverage, the profit of the low-quality firm (again with sufficient product heterogeneity), is strictly decreasing in λ, and the profit of the high-quality firm is strictly increasing in λ.

Proof: Deferred to the appendix. ■

It is clear that the low-quality firm can no longer afford to raise prices because now the consumers have an option of not buying a good at all, and a higher price can only be justified by a sufficiently higher quality. Thus, the effect of price on profit is dominated by the corresponding effect of demand-loss. The intuition behind why the profit of the high-quality firm responds positively to λ, is similar to the full market-coverage scenario. It is interesting to note that even though the reaction function of firm 1 is positively sloped, the loss in demand due to the (now more sensitive, given higher λ) consumers' perception of the value of the low-quality product does not permit increased prices unlike in the case of full market coverage.

Now we come to a comparison of profits under monopoly and duopoly for both firms, when the consumers have the option to buy none of the available product-varieties. We show that a firm can exploit this situation to retain monopoly-level profits if the consumers are reference-dependent.

2.3.6 Monopoly versus duopoly (partial market coverage)

We now compare the monopoly profits of each firm type (high-quality or low-quality) against the profits that each could earn as a duopoly. The monopoly profit for each firm has been worked out in the benchmark case. We immediately see that generally, the monopoly profits for each firm can exceed the duopoly profits. It is interesting to see the profit differential (between the scenarios of monopoly and duopoly) for each firm. The (monopoly versus duopoly) profit differential for the high-quality firm is given as

$$\pi_2^{pm} - \pi_2^m = \frac{(2km)^2(m-1)(1+\lambda)\underline{\theta}s_1}{(k-1)\big(3m+(m-1)(1+\lambda)\big)^2} - \frac{k^2\underline{\theta}ms_1}{4(k-1)}$$

$$= \frac{k^2\underline{\theta}ms_1}{(k-1)}\left(\frac{4m(m-1)(1+\lambda)}{\big(3m+(m-1)(1+\lambda)\big)^2} - \frac{1}{4}\right)$$

It is clear from *Proposition 2* that, since π_2^{pm} (and therefore the profit-differential above) is strictly increasing in λ, the highest duopoly-level profit is attained at the highest value of λ permissible under the (binding) constraint $U_1'(\theta) \geq 0$. We now observe that the term $\frac{4m(m-1)(1+\lambda)}{\left(3m+(m-1)(1+\lambda)\right)^2}$ above, attains a maximum value of 1/4 at λ = 1/($m-1$), under the (binding) constraint $U_1'(\theta) \geq 0$. It is therefore clear that $\pi_2^{pm} - \pi_2^m$ above attains a maximum value of *0*.

Thus, we have established that a high-quality incumbent firm can recover its monopoly profits after a low-quality firm enters the market. The following example illustrates this point.

Example 2: With $k = 5$, so that $\bar{\theta} = 5\underline{\theta}$, and m = $s_2/s_1 = 3$ with $\lambda =$ 0.5, we get $p_1^{pm} = 0$; $p_2^{pm} = 15\underline{\theta}s_1/2$; $\pi_1^{pm} = 0$; $\pi_2^{pm} = \pi_2^m = 75\underline{\theta}s_1/16$; $p_1^m = 5\underline{\theta}s_1/2$; $p_2^m = 15\underline{\theta}s_1/2$. Further, $U_1(\hat{\theta}_1) = U_1(\hat{\theta}_2) = U_2(\hat{\theta}_2) = 0$; $U_2(\bar{\theta}) = 15\underline{\theta}s_1/2$; and $\pi_1^m = 25\underline{\theta}s_1/16$. The demand structures are: $D_1^m = D_2^m = 5/8$; $D_1^{pm} = 5/16$, $D_2^{pm} = 5/8$ (this leaves 1/16$^{\text{th}}$ of the consumer base that does not consume/buy anything).

The above example is particularly interesting because in a natural duopoly setting, one firm (of the high-quality product) earns monopoly profits and the other firm (of the low-quality product) simultaneously earns only competitive level normal profits, by equating price to its own marginal cost (equal to zero).

Now we compare the monopoly-level and duopoly-level profits of the low-quality firm as below.

$$\pi_1^{pm} - \pi_1^m = \frac{k^2\underline{\theta}m(m-1)(1+\lambda)\left(1-\lambda(m-1)\right)s_1}{(k-1)\left(3m+(m-1)(1+\lambda)\right)^2} - \frac{k^2\underline{\theta}s_1}{4(k-1)}$$

$$= \frac{k^2\underline{\theta}s_1}{(k-1)}\left(\frac{m(m-1)(1+\lambda)\left(1-\lambda(m-1)\right)}{\left(3m+(m-1)(1+\lambda)\right)^2} - \frac{1}{4}\right)$$

In the proposition that follows, we establish that in the case of partial market-coverage (just as in the case with full market-coverage), the duopoly-level profits never exceed monopoly-level profits for the low quality firm.

Proposition 3: For all permissible values of *m* (> *1*), the maximum value of $\pi_1^{pm} - \pi_1^m$, is negative.

Proof: Recognizing that the maximum value of $\pi_1^{pm} - \pi_1^m$, above (under the binding constraint $U_1'(\theta) \geq 0$), is $-\left(3k^2\underline{\theta}s_1\right)/\left(16(k-1)\right) < 0$ (the details are deferred to the appendix), completes the proof. ■

We now come to an analysis of welfare in our duopoly setting before extending our model to the *n*-firm case.

2.4 Welfare analysis

For now, we look at the scenario of full market coverage. The welfare analysis under partial market coverage (although quite involving)is qualitatively similar. In line with the existing literature, we define social welfare as the sum of profits of both firms and consumers' surplus. Therefore the social welfare, denoted by *W* is given by

$$W = \pi_1 + \pi_2 + \frac{1}{(\bar{\theta} - \underline{\theta})}\int_{\underline{\theta}}^{\hat{\theta}} \left(\theta s_1 - \frac{\left(\bar{\theta} - 2\underline{\theta}\right)(1+\lambda)\Delta s}{3} - \lambda\theta\Delta s\right) d\theta$$

$$+ \frac{1}{(\bar{\theta} - \underline{\theta})}\int_{\hat{\theta}}^{\bar{\theta}} \left(\theta s_2 - \frac{\left(2\bar{\theta} - \underline{\theta}\right)(1+\lambda)\Delta s}{3}\right) d\theta$$

We will show that both consumer and aggregate social welfare are decreasing in λ. Given the individual quality levels, and hence the quality differential (or product-heterogeneity), as λ goes up, so do the prices of both the products. Clearly, each consumer becomes worse off with this. This loss in utility (net of prices) due to λ exceeds the gain in profits due to higher prices (we will shortly demonstrate that the demand structure remains unchanged), because of the additional component involving λ in U_1 (as far as prices go, the loss to consumers due to higher prices, in response to higher λ, is compensated by what the producers gain - what consumers pay to producers is what the producers earn). Moreover the equilibrium is characterized by unchanged demand structure, since the cut-off $\hat{\theta} = (\bar{\theta} + \underline{\theta})/3$ is independent of λ. This means that consumers do not switch their consumption (from low to high quality, or conversely) regardless of the strength of reference-dependence. Thus, in the equilibrium, the overall demand-structure remains unchanged leading to an aggregate decline in consumer welfare with a higher λ. We sum this idea up more concretely in the following theorem.

Theorem 3: Consumer welfare and social welfare are decreasing in λ.

Proof: Using $\hat{\theta} = (\bar{\theta} + \underline{\theta})/3$, and differentiating W with respect to λ, we get

$$\frac{\partial W}{\partial \lambda} = \frac{(s_2 - s_1)}{18(\bar{\theta} - \underline{\theta})}\left(-\bar{\theta}^2 + 8\underline{\theta}^2 - 2\bar{\theta}\underline{\theta}\right) = \frac{(s_2 - s_1)}{18(\bar{\theta} - \underline{\theta})}\left\{\underbrace{-\left(\bar{\theta}^2 - (2\underline{\theta})^2\right)}_{negative}\underbrace{-\left(2\underline{\theta}(\bar{\theta} - 2\underline{\theta})\right)}_{negative}\right\}$$

$$< 0$$

The above inequality immediately follows from the fact that $\bar{\theta} > 2\underline{\theta} > 0$. This completes the proof. ■

It is clear that more severe loss due to reference-dependence reduces price competition, thereby enabling firms to charge higher prices, leaving the consumers worse off. The total welfare is reduced because of the reference-dependence component in the consumers' aggregate utility magnified by λ.

We now come to the effect of product heterogeneity m on welfare. To make the intuition clearer, it will help to assume (for the moment) that $\lambda = 0$. In this case, welfare strictly increases in m only due to the consumers with higher θ, who consume the high-quality product (the effect of the changes in prices due to m get cancelled out – higher prices reduce consumers' surplus, but add to profits in the same magnitude). We now provide a formal argument for why welfare increases in the degree of product-heterogeneity m (for moderate to large levels of product-heterogeneity – specifically $m > 1.146$).

Theorem 4: Social welfare increases in m for sufficient ($m > 1.146$) levels of product-heterogeneity.

Proof: For the sake of contradiction, we assume that $\partial W/\partial m < 0$, which implies the statement below

$$\lambda > \frac{(4k+1)(2k-1)}{(k+4)(k-2)} = g(k) \geq 6.854$$

where the latter part of the inequality comes from the fact that the minimum value of $g(k)$ (attained at $k = 6.855$) is 6.854. From $U_1'(\theta) \geq 0$, we get $1/(m-1) \geq \lambda$. On combining these two inequalities, we get $1/(m-1) > g(k) \geq 6.854$, which gives us $m < 1.146$. This contradicts the assumption of sufficient product-heterogeneity. ■

We show the plot of welfare against λ and m in Figure 1 (following suitable normalizations) and immediately see that W decreases in λ and increases in m.[4] Also, W is more responsive to changes in m than in λ.

[4]Note that an increase in m could also be brought about by a decline in s_1 for a fixed s_2.In this case, W must decrease in m. Given the triviality of this issue, we stick to the case of a fixed s_1 throughout this paper, with flexible values of m (and hence s_2), and present all our terms (prices, profits etc.) in terms of s_1.

Since the expression of welfare in the partial-market coverage is fairly involving (shown below), we will resort to analyzing a plot of welfare against the values of m and λ (shown in Figure 2). We immediately see that W continues to increase in m and decrease in λ.

$$W^{pm} = \frac{\overline{\theta}^2 m s_1}{2(\overline{\theta} - \underline{\theta})}\left[\frac{3m - 2 + 4m(3m-1) + \lambda(m-1)\big(4 + m - 2\lambda(m-1)\big)}{\big(4m - 1 + \lambda(m-1)\big)^2}\right] \qquad (2.21)$$

Figure 1: Plot of welfare under full market coverage

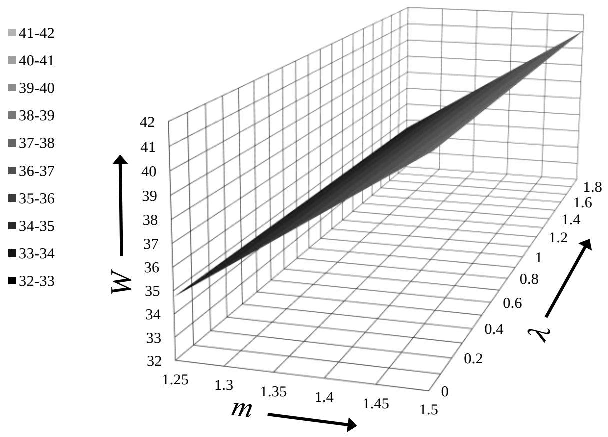
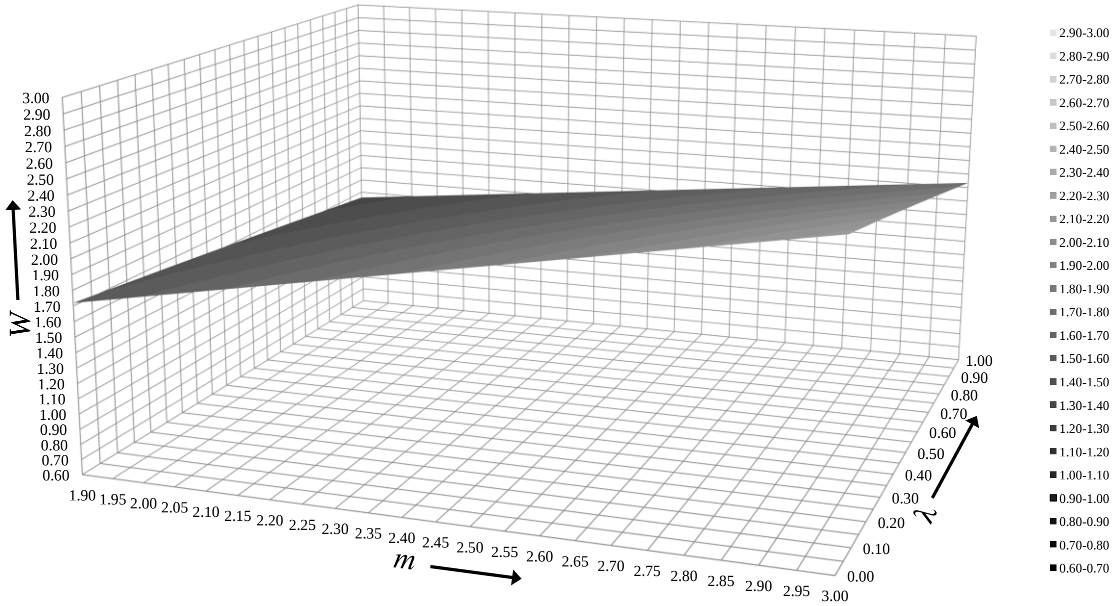


Figure 2: Plot of welfare under partial market coverage



The explanation behind why W is more responsive to λ under partial market coverage comes from the liberty that firms have. The firms have no obligation to cover market fully so they maximize their profits by raising prices (as a response to increase in λ) to a level where some consumers (with lower valuation for quality) find the product unaffordable and therefore experience reduced welfare.

2.5 An extension to the *n*-firm situation

We now check the robustness of our finding to the *n*-firm case and see if there is a critical number of oligopoly firms, beyond which, each firm's individual profit in the oligopoly will necessarily be exceeded by the counterfactual profits earned if it were a monopoly. Formally, we will demonstrate that for any firm $i \in \{1, 2, \ldots, n\}$, producing quality s_i, the profit π_i^m that would arise if it were a monopoly, will exceed the profit π_i from remaining in an oligopoly (with $n > 2$). We begin with the following utility specifications with the assumption (without loss of generality) that $s_1 < s_2 < \cdots < s_n$

$$U_i = \theta s_i - p_i - \lambda\theta(s_n - s_i), \ \ \forall i \in \{1, 2, \dots, n-1\}$$

$$U_n = \theta s_n - p_n$$

Since the algebra for a large number of firms is fairly involved, we will resort to numerical simulations for $n > 2$, under reasonably general conditions with $V > 0$. However, we will present these simulations following a general algebraic representation of our *n*-firm model. For a given number of firms *n*, each firm *i*, offering quality s_i, at price p_i, has a reaction function of the type $p_i = R_i(\boldsymbol{p}_{-i})$, where $\boldsymbol{p}_{-i}$ is the vector of prices offered by all firms other than *i*. We have already argued that the highest quality level $s_n(> s_{n-1} > \cdots > s_1)$, serves as a natural reference point for all the consumers. Additionally, in what follows, we also assume (for tractability) that the quality differential between any two closest quality levels is a constant i.e.$s_i - s_{i-1} = \Delta s, \forall i$. In this setting, the reaction functions assume the following specifications:

$$p_1 = \frac{p_2 - \underline{\theta}(1+\lambda)\Delta s}{2}$$

$$p_i = \frac{p_{i+1} + p_{i-1}}{4}, \ \ \forall i \in \{2, \dots, n-1\}$$

$$p_n = \frac{p_{n-1} + \bar{\theta}(1+\lambda)\Delta s}{2}$$

We demonstrate using simulations for $2 < n < 6$ that the solution to the above equations above exist and are stable. Stability is easily verified by introducing a disturbance $\boldsymbol{\varepsilon}$ to the equilibrium price vector $\boldsymbol{p}^* = \{p_1^*, \dots, p_n^*\}$, in period 0, denoted as $\boldsymbol{p}_0^*$ and subsequently observing the fixed point of the iterative function $g: \mathbb{R}^n \mapsto \mathbb{R}^n$ with $\boldsymbol{p}_1 = g(\boldsymbol{p}_0 + \boldsymbol{\varepsilon})$ and

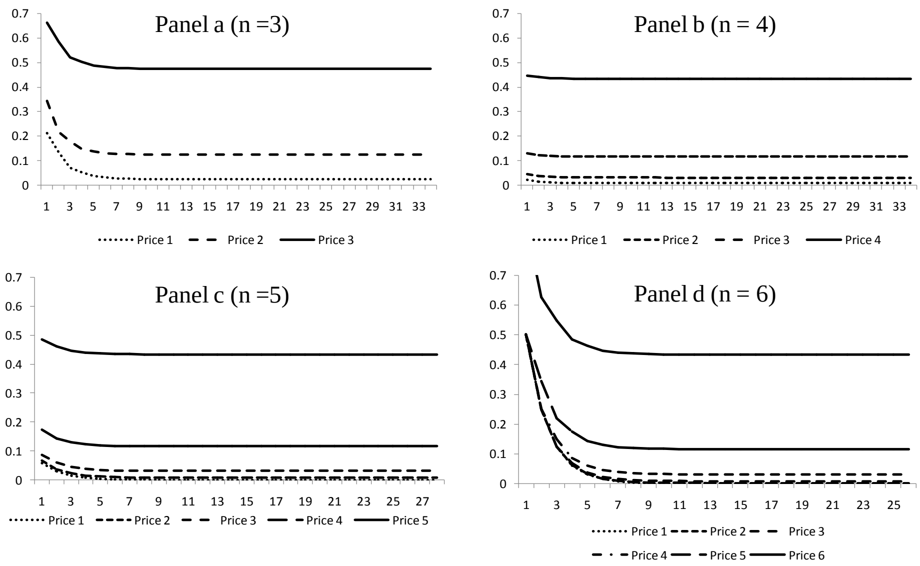
$\boldsymbol{p}_{t+1} = g(\boldsymbol{p}_t)$ for every subsequent period/iteration. The function g is associated with the reaction function in that it captures how each firm individually responds to price changes in the market. We immediately see a progressive convergence to the Nash equilibrium $\boldsymbol{p}^*$.

2.5.1 Stability in the n-firm case

Figure 3 shows that not only do equilibrium prices exist, but they remain largely insensitive to small perturbations. Panels a, b, c, and d, in Figure 3, show how prices respond over time (shown in the horizontal axes) for perturbations in period zero for oligopolies with three, four, five and six firms respectively. It is easy to verify that the price level to which each firm's price converges indeed defines the (unique) Nash equilibrium in prices. Thus, we see that not only do the equilibrium prices exist, but they are also stable. Interestingly, in all the cases, prices of the highest quality offering firm remain significantly higher (more than ten times) than the prices offered by the lowest quality firms. With such lower price levels, coupled with shrinking demand (and therefore market share), it is clear that at least for the lowest quality-firms, profits under oligopoly are exceeded by each firm's respective monopoly profit-levels (which score both on market shares and price levels). It is therefore, sufficient to show that if, for three or more firms, the profit of the highest-quality manufacturer is exceeded by its corresponding monopoly-level profits, then the only oligopolistic market form, where some firms can hope to match monopoly profits is uniquely a duopoly.[5] We turn to this discussion now.

[5]More formally, once it is recognized that the difference between oligopoly and monopoly profits is strictly decreasing in the firm quality level, then demonstrating that the oligopoly profit level for the highest quality firm is strictly less than its corresponding monopoly level profit, will be the first step at establishing this key insight. The second (and the more crucial) step will be to show that both prices and market share of the highest quality firm will be strictly lower than those in the corresponding monopoly situation.

Figure 3: Stable prices for oligopolies with different number of firms



2.5.2 Comparison with monopoly

We immediately see that for $n > 2$, i.e. with at least three firms in the market, it is impossible to sustain the advantage that a duopoly can enjoy in terms of monopoly-level profits. This result follows trivially from a direct comparison of individual profits for each firm in an oligopoly with that if it were a monopolist without any change in its own product quality. This comparison helps us understand that the only oligopolistic market that can match (in terms of profits) a monopoly due to reference-dependence is uniquely a duopoly. The intuition governing this result has already been presented earlier in this paper. Higher duopoly profits (equaling monopoly profits) are directly contingent upon the strength of reference-dependence and existing consumer heterogeneity (in terms of preferences). This aggregate degree of heterogeneity is narrowed down for each firm, as more firms enter the market. Further, the level of competition is higher (since each firm now competes with at least one other firm that provides an immediately near quality level - see Appendix) which keeps a check on prices from being quoted too high (this is illustrated for $n = 3$ and $n = 5$ in the Appendix). This interplay of shrinking demand and a check on prices (in comparison to a situation involving lesser number of firms) make sure that monopoly profits remain higher than oligopoly level profits. These results are also partially driven from our choice of construction where the quality differential between any two closest quality levels is a constant i.e.$s_i - s_{i-1} = \Delta s, \forall i$. We formalize these ideas in the last results below.

Lemma 6: Under an oligopoly, there is a natural upper bound on the price charged by the highest-quality firm.

Proof: Using the fact that $p_n \geq p_{n-1}$; along with the reaction function p_n = $[p_{n-1} + \bar{\theta}(1+\lambda)\Delta s]/2$, gives us

$$\bar{\theta}(1+\lambda)\Delta s \geq p_{n-1}$$

Finally, we invert the reaction function to write p_{n-1} in terms of p_n, to get write the above inequality as $\bar{\theta}(1+\lambda)\Delta s \geq 2p_n - \bar{\theta}(1+\lambda)\Delta s$. This can be simplified to $\bar{\theta}(1+\lambda)\Delta s \geq p_n$. This completes the proof. ■

Lemma 7: In an *n*-firm vertically differentiated oligopoly, the profit of the highest-quality manufacturing firm is given by π_n = $\frac{1}{(\bar{\theta}-\underline{\theta})}\left[\frac{p_n^2}{(1+\lambda)\Delta s}\right]$.

Proof: We begin by identifying that the demand faced by the highest-quality firm is given by $D_n = \frac{1}{(\bar{\theta}-\underline{\theta})}\left[\bar{\theta} - \frac{p_n - p_{n-1}}{(1+\lambda)\Delta s}\right]$. Inverting the reaction function p_n= $[p_{n-1} + \bar{\theta}(1+\lambda)\Delta s]/2$, gives us $p_{n-1} = 2p_n - \bar{\theta}(1+\lambda)\Delta s$, which we use in the demand specification D_n to get $D_n = \frac{1}{(\bar{\theta}-\underline{\theta})}\left[\frac{p_n}{(1+\lambda)\Delta s}\right]$, and $\pi_n = p_n D_n = \frac{1}{(\bar{\theta}-\underline{\theta})}\left[\frac{p_n^2}{(1+\lambda)\Delta s}\right]$. This completes the proof. ■

Theorem 5: The profit of the highest-quality firm in the *n*-firm vertically differentiated oligopoly never exceeds its monopoly level profit.

Proof: We assume for the sake of contradiction that $\pi_n^* - \pi_n^m = \frac{1}{(\bar{\theta} - \underline{\theta})}\left[\frac{p_n^2}{(1+\lambda)\Delta s}\right] - \frac{\bar{\theta} p_n^m}{2(\bar{\theta} - \underline{\theta})} \leq 0$ this requires that

$$2\bar{\theta}\Delta s(1+\lambda)p_n^m - p_n^2 \leq 0 \tag{2.22}$$

Now using the fact that $p_n^m \geq p_n$ (see Appendix), and $2\bar{\theta}\Delta s(1+\lambda) > \bar{\theta}\Delta s(1+\lambda) \geq p_n$ (from *Lemma 6*). Multiplying these two inequalities gives us $2\bar{\theta}\Delta s(1+\lambda)p_n^m > p_n^2$. This contradicts the in equation (2.22) above, thereby completing the proof. ■

2.6 Conclusion

This paper contributes to the growing literature that tries to understand the implication of a particular consumer behavior on the functioning of oligopolies. We have analyzed the implications of consumer reference-dependence on firms' strategy and consumer welfare, in a vertically differentiated market. We have shown that both firms' prices and profits increase with greater reference-dependence and consumer welfare decreases with the same. We also provide an additional reason why the profit of the higher quality firm in a vertically differentiated duopoly can exactly match that in monopoly, provided that the degree of differentiation is sufficiently high.

It will be interesting to extend this model to the `*n*' product case, where there is no restriction on $s_i - s_{i-1}$, so that Δs can vary. Alternatively, the reference-point(s) for consumption could depend on a consideration set of alternatives instead of the just the best alternative itself. It would also be desirable to introduce uncertainty in the quality dimension i.e. where consumers only know the product ranking and not the exact qualities. In general, it is hard to infer the exact quality of a product prior to its purchase, although there are several sources (such as expert ranking) that help consumers to rank products on the basis of quality. So, *ex* *ante* it is possible to have some ranking of the products. Further, reference-dependence may influence investment by firms on R&D, which in turn may affect the quality of products available. Thus, one can even think of endogenizing the choice of quality *s* by each firm (Bandyopadhyay and Acharyya, 2004).

A caveat to the results concerning the monopoly cases, is that due to the mathematical nature of our problem, where the consumers are represented as a continuum on the unit line, we get the result that those who choose to purchase a unit good of the low-quality product are identically the same as those who choose the high-quality. While this minor simplification does not substantially take away from our central conclusions, we emphasize that such consumer behavior should not always be taken for granted.

Appendices to Chapter 2

Appendix 2A

Proof of theorems and lemmas

Lemma 1: $U_1(\theta)$is non-decreasing in *θ.*

Proof: We begin by assuming that U_1 is strictly decreasing in θ, which means that: $\frac{\partial U_1(\theta)}{\partial \theta} = s_1 - \lambda(s_2 - s_1) < 0$. This condition can be re-written as $\frac{s_2}{s_1} > 1 + \frac{1}{\lambda}$. Now it is sufficient to show that $U_1(\hat{\theta}) \geq 0$ to guarantee full-market coverage in duopoly, since every consumer to the left of $\hat{\theta}$ will have a strictly greater utility (than this marginal consumer) from the consumption of product 1. Now we know that in equilibrium, $\hat{\theta} = (\bar{\theta} + \underline{\theta})/3$, so that $U_1(\hat{\theta}) \geq 0$, can be re-written as under:

$$U_1(\hat{\theta}) = \frac{(\bar{\theta} + \underline{\theta})}{3} s_1 - p_1^* - \lambda \frac{(\bar{\theta} + \underline{\theta})}{3} (s_2 - s_1) \geq 0$$

which can be further simplified to

$$1 + \frac{k+1}{(1+\lambda)(k-2) + \lambda(k+1)} \geq \frac{s_2}{s_1}$$

Finally, on combining the above inequality with $\frac{\partial \mathrm{U}_1(\theta)}{\partial \theta} < 0$, we get

$$1 + \frac{k+1}{(1+\lambda)(k-2) + \lambda(k+1)} \geq \frac{s_2}{s_1} > 1 + \frac{1}{\lambda}$$

The extreme left and right hand side of the above equation lead us to $\lambda(k+1) > (1+\lambda)(k-2) + \lambda(k+1)$, which is a contradiction. Thus, $U_1(\theta)$ must be non-decreasing in θ. This completes the proof. ■

Lemma 2: There exists a cap on the product-heterogeneity level s_2/s_1, so that $U_1(\underline{\theta})$ is non-negative.

Proof: At equilibrium prices, the non-negativity of $U_1(\underline{\theta})$ will require the following:

$$U_1(\underline{\theta}) = \underline{\theta}s_1 - \frac{(\bar{\theta} - 2\underline{\theta})(s_2 - s_1)(1+\lambda)}{3} - \lambda\underline{\theta}(s_2 - s_1) \geq 0$$

which can be further simplified to

$$\frac{s_2 - s_1}{s_1} \leq \frac{3\underline{\theta}}{\bar{\theta}(1+\lambda) + \underline{\theta}\lambda - 2\underline{\theta}}$$

using $\bar{\theta} = k\underline{\theta}$, we get

$$\lambda \leq \frac{3s_2}{(k+1)(s_2 - s_1)} - 1 \equiv \lambda^*$$

Thus, for all $\lambda \leq \lambda^*$, $U_1(\underline{\theta})$ is non-negative. Now, since we want $\lambda^* > 0$, the right hand side above should be positive. Thus

$$1 \leq \frac{s_2}{s_1} \leq \frac{k+1}{k-2}$$

If the above condition is violated, i.e. if product heterogeneity exceeds the above sufficiency level, then firm 1 will find it profitable to raise prices as a response to firm 2 (which will capitalize on the higher consumer heterogeneity) to a level where some consumers will be left out of the market. ■

Lemma 4: Under full-market coverage, the high-quality firm will want to remain a monopoly.

Proof: For the high-quality incumbent to accommodate a low-quality entrant, the profit it will earn as a duopoly must at least equal that which it would earn as a monopoly. Then the following inequalities must hold true

$$\left(\frac{3k}{2(2k-1)}\right)^2\left(\frac{s_2}{s_2-s_1}\right)-1\leq\lambda\leq\frac{3s_2}{(k+1)(s_2-s_1)}-1$$

where, the latter half of the inequality above comes directly from the full-market coverage condition in *Theorem 1* (more specifically *Lemma 2*). The former half of the inequality comes directly from the comparison of the profits of the high-quality product manufacturing firm under monopoly and duopoly. The above inequality can be further simplified to

$$\left(\frac{3k}{2(2k-1)}\right)^2\leq\frac{(\lambda+1)(s_2-s_1)}{s_2}\leq\frac{3}{(k+1)}$$

Since the function $g(k)=\frac{3}{(k+1)}-\left(\frac{3k}{2(2k-1)}\right)^2$ attains a maximum value of *0* at $k = 2$, for any value of $k \neq 2, g(k) < 0$, meaning that duopoly-level profit will always be lower than monopoly-level profit for the high-quality firm. Note that at $k = 2$, $\hat{\theta}=\underline{\theta}$. Thus, with full market coverage, the high-quality firm will strategically capture the entire market, and therefore continue to remain a monopoly. ■

Lemma 5: Under full-market coverage, the low-quality firm will want to remain a monopoly.

Proof: For the low-quality incumbent to accommodate a high-quality entrant, its own duopoly-profit level must be at least as good as its monopoly-profit level. Under full market coverage, this requirement can be expressed in the following inequalities

$$\left(\frac{3k}{2(k-2)}\right)^2\left(\frac{s_1}{s_2-s_1}\right)-1\leq\lambda\leq\frac{3s_2}{(k+1)(s_2-s_1)}-1$$

Combining the extreme right-hand (market-coverage condition) and left-hand (duopoly versus monopoly profit) sides of the above inequalities, leads us to the simplified expression below.

$$\frac{s_2}{s_1} \geq \frac{3k^2(k+1)}{4(k-2)^2}$$

On combining the above inequality with $\lambda^* > 0$ from *Lemma 2*, we get

$$\frac{k+1}{k-2} \geq \frac{s_2}{s_1} \geq \frac{3k^2(k+1)}{4(k-2)^2}$$

Finally, on simplifying the extreme left and right hand sides of the above inequality we get

$$(3k-2)^2 + 20 \leq 0$$

The impossibility of the above statement establishes the proof. ■

Proposition 1: Under partial-market coverage, the price of low-quality product (under sufficient product-heterogeneity) is strictly decreasing in λ and the price of high quality firm is strictly increasing in λ.

Proof: We immediately see that

$$\frac{\partial ln\left(p_1^{pm}\right)}{\partial \lambda} = \frac{1}{(1+\lambda)} - \frac{(m-1)}{\left(1-\lambda(m-1)\right)} - \frac{(m-1)}{3m+(m-1)(1+\lambda)}$$

$$\Leftrightarrow \frac{\partial ln\left(p_1^{pm}\right)}{\partial \lambda} = -\frac{(m-2)+2\lambda(m-1)}{(1+\lambda)\left(1-\lambda(m-1)\right)} - \frac{(m-1)}{3m+(m-1)(1+\lambda)}$$

It is easy to see that the above term is strictly negative for any $m \geq 2$. For the above derivative $\partial ln\left(p_1^{pm}\right)/\partial \lambda$, to be strictly positive, the following condition must be met

$$\frac{1 - (4 - 2\sqrt{3})m}{m-1} > \lambda$$

Finally, recognizing that $\lambda > 0$, a negative left-hand-side above (which is equivalent to saying that $m > 1/(4 - 2\sqrt{3}) \simeq 1.87$) will immediately violate the above condition, thereby making the required derivative $\partial ln(p_1^{pm})/\partial\lambda$, negative for all m greater than this threshold.

Further,

$$\frac{\partial ln(p_2^{pm})}{\partial\lambda} = \frac{1}{(1+\lambda)} - \frac{(m-1)}{3m + (m-1)(1+\lambda)} > 0$$

The above statement directly follows from $3m(m-1) > 0$. ■

Proposition 2: Under partial-market coverage, the profit of low-quality firm (again with sufficient product heterogeneity), is strictly decreasing in λ and the profit of high quality firm is strictly increasing in λ.

Proof: We see that

$$\frac{\partial ln(\pi_1^{pm})}{\partial\lambda} = \frac{1}{(1+\lambda)} - \frac{(m-1)}{\big(1 - \lambda(m-1)\big)} - \frac{2(m-1)}{3m + (m-1)(1+\lambda)}$$

simplifies to,

$$\frac{m\big(3 - (m-1)(4+7\lambda)\big)}{(1+\lambda)\big(1-\lambda(m-1)\big)\big(3m + (m-1)(1+\lambda)\big)}$$

Now, $\partial ln(\pi_1^{pm})/\partial\lambda < 0$, requires just the numerator above to be negative (since the denominator is positive) which follows from the following inequality(using the fact that $m > 1/(4 - 2\sqrt{3}) \simeq 1.87 > 7/4$, as shown in *Proposition 1*, above).

$$\lambda > 0 > \frac{7-4m}{7(m-1)}$$

Further,

$$\frac{\partial ln\left(\pi_2^{pm}\right)}{\partial \lambda} = \frac{1}{(1+\lambda)} - \frac{2(m-1)}{3m+(m-1)(1+\lambda)} < 0$$

requires that $3m/(m-1) < (1+\lambda)$. But this contradicts the condition of $U_1'(\theta) \geq 0$.[6] Thus, profit of the high quality firm must be necessarily increasing in λ. ∎

Proposition 3: For all permissible values of *m* (> *1*), the maximum value of $\pi_1^{pm} - \pi_1^{m}$, is negative.

Proof: Since $s_1 \leq s_2$, our permissible region is$m \geq 1$. We consider two cases: First, when $1 \leq m \leq 7/4$ then the difference between firm 1's profit under partial market duopoly and monopoly given by following function

$$\pi_1^{pm} - \pi_1^{m} = \frac{k^2\underline{\theta}s_1}{k-1}\left(\frac{m(m-1)(1+\lambda)\big(1-\lambda(m-1)\big)}{\big(3m+(m-1)(1+\lambda)\big)^2} - \frac{1}{4}\right)$$

This difference is maximized at $\lambda = \frac{m(7-4m)}{(m-1)(2+5m)}$, plugging which, back to the above expression, gives us $\pi_1^{pm} - \pi_1^{m} = \frac{k^2\underline{\theta}s_1}{k-1}\left(\frac{m(m^2+4m-2)(4m^2-2m+2)}{(16m^2+10m-2)^2} - \frac{1}{4}\right)$. This expression always remains negative in the interval $1 \leq m \leq 7/4$, as can be verified from a simple plot of the term within the brackets.

Second, we now consider the case where $m \geq 7/4$. The function takes the maximum value when $\lambda = 0$ (since, we know from the proof of *Proposition 2* above, that duopoly profit level for the low-quality firm is strictly decreasing in λ in this range of values for *m*). Thus, given any $m \geq 7/4$, the maximum value (at $\lambda = 0$) of $\pi_1^{pm} - \pi_1^{m} = \frac{k^2\underline{\theta}s_1}{k-1}\left(\frac{m(m-1)}{(4m-1)^2} - \frac{1}{4}\right)$, is strictly

[6] Note that $U_1'(\theta) \geq 0$ requires $\lambda \leq 1/(m-1)$. Recognizing that $1/(m-1) < (2m+1)/(m-1)$, and combining these inequalities lead us to$3m/(m-1) > (1+\lambda)$, which is equivalent to$\partial ln(\pi_2)/\partial\lambda > 0$.

increasing in m. So we let m to be arbitrarily large enough to attain the maximum value as $m \to \infty$. This maximum value is negative and equals $-\left(3k^2\underline{\theta}s_1\right)/\left(16(k-1)\right)$. Thus, the low quality firm, cannot attain its own monopoly-level profits under partial market coverage.

■

Appendix 2B

The key results from the oligopoly set up

Case I: When n = 3

We consider three firms producing quality levelss_1, s_2 and s_3 $(s_1 < s_2 < s_3)$, respectively. The utility of consumer with θ as her taste parameter, from consuming quality s_i at price p_i for $i \in \{1,2\}$ is given by

$$U_i = \theta s_i - p_i - \lambda\theta(s_3 - s_i)$$

We define $\Delta s = s_2 - s_1 = s_3 - s_2$. The utility of consumer with θ as her taste parameter, from consuming quality s_3 at price p_3 is given by

$$U_3 = \theta s_3 - p_3$$

The consumer type who is indifferent between buying from firm 1 and buying from firm 2 is given by $\hat{\theta}_{12} = (p_2 - p_1)/\Delta s(1+\lambda)$, whereas $\hat{\theta}_{23} = (p_3 - p_2)/\Delta s(1+\lambda)$ denotes the consumer type who is indifferent between buying from firm 2 and firm 3. Thus, the demand that the firms face are

$$D_1(p_1, p_2, p_3) = \frac{\hat{\theta}_1 - \underline{\theta}}{\bar{\theta} - \underline{\theta}} = \left(\frac{(p_2 - p_1)}{\Delta s(1+\lambda)} - \underline{\theta}\right)\frac{1}{(\bar{\theta} - \underline{\theta})}$$

$$D_2(p_1, p_2, p_3) = \frac{\hat{\theta}_2 - \hat{\theta}_1}{\bar{\theta} - \underline{\theta}} = \left(\frac{(p_3 - p_2)}{\Delta s(1+\lambda)} - \frac{(p_2 - p_1)}{\Delta s(1+\lambda)}\right)\frac{1}{(\bar{\theta} - \underline{\theta})}$$

$$D_3(p_1, p_2, p_3) = \frac{\bar{\theta} - \hat{\theta}_2}{\bar{\theta} - \underline{\theta}} = \left(\bar{\theta} - \frac{(p_3 - p_2)}{\Delta s(1+\lambda)}\right)\frac{1}{(\bar{\theta} - \underline{\theta})}$$

The profit functions of the firms are:

$$\pi_1(p_1, p_2, p_3) = \ p_1 D_1(p_1, p_2, p_3) = \frac{p_1}{(\bar{\theta} - \ \underline{\theta})}\left(\frac{(p_2 - p_1)}{\Delta s(1+\lambda)} - \underline{\theta}\right)$$

$$\pi_2(p_1, p_2, p_3) = p_2 D_2(p_1, p_2, p_3) = \frac{p_2}{\bar{\theta} - \ \underline{\theta}}\left(\frac{(p_3 - p_2)}{\Delta s(1+\lambda)} - \frac{(p_2 - p_1)}{\Delta s(1+\lambda)}\right)$$

$$\pi_3(p_1, p_2, p_3) = p_3 D_3(p_1, p_2, p_3) = \frac{p_3}{\bar{\theta} - \ \underline{\theta}}\left(\bar{\theta} - \frac{(p_3 - p_2)}{\Delta s(1+\lambda)}\right)$$

The reaction functions are:

$$p_1(p_2, p_3) = \frac{p_2 - \underline{\theta}(\Delta s)(1+\lambda)}{2};\ p_2(p_1, p_3) = \frac{(p_3 + p_1)\Delta s}{4\Delta s};\ p_3(p_1, p_2) = \frac{p_2 + \bar{\theta}\Delta s(1+\lambda)}{2}$$

With $k = \bar{\theta}/\underline{\theta}$, the equilibrium prices and profits are:

$$p_1^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-7)}{12};\ p_2^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-1)}{6};\ p_3^* = \frac{\Delta s(1+\lambda)\underline{\theta}(7k-1)}{12}$$

$$\pi_1^* = \frac{\Delta s(1+\lambda)\underline{\theta}}{(k-1)}\left(\frac{k-7}{12}\right)^2;\ \pi_2^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-1)}{18};\ \pi_3^* = \frac{\Delta s(1+\lambda)\underline{\theta}}{(k-1)}\left(\frac{7k-1}{12}\right)^2$$

We now compare the profits of the highest quality firm under monopoly and oligopoly.

Lemma 2B.1: The profit of the highest quality firm under monopoly exceeds its profits under oligopoly (i.e.$\pi_3^* < \pi_3^m$).

Proof: Let us assume for the sake of contradiction that $\pi_3^* \geq \pi_3^m$, which implies that $\left(\frac{7k-1}{k}\right)^2 \frac{(1+\lambda)\Delta s}{36} \geq s_3 = s_1 + 2\Delta s$, which further simplifies to $\left[\left(\frac{7k-1}{k}\right)^2 \frac{(1+\lambda)}{36} - 2\right]\Delta s \geq s_1$. Now, $U_1(\underline{\theta}) \geq 0$, implies that $s_1 \geq \frac{\Delta s}{12}[(1+\lambda)(k+17) - 24]$. We combine these inequalities to get $[7-(1/k)]^2 \geq 3(k+17)$, which is a contradiction for all values of $k > 7$ (required in the equilibrium, since $p_1^* > 0$). Thus we establish that $\pi_3^* < \pi_3^m$, thereby completing the proof. ■

The equilibrium market share of this highest quality firm in the oligopoly is *(7k – 1)/[12(k – 1)]*, which has a limiting value of *7/12* as $k \to \infty$. This means that for all permissible values of *k*, the market share it enjoys exceeds the monopoly market share. Therefore, it must be the case that the price that this oligopolist is able to charge does not exceed that which would emerge if this firm were a monopoly. We formally show this below.

Lemma 2B.2: The price of the highest quality firm under monopoly exceeds its profits under oligopoly (i.e. $p_3^* < p_3^m$).

Proof: Let us assume for the sake of contradiction, that $p_3^* \geq p_3^m$, which implies that $\frac{1}{(1+\lambda)\left(7-\frac{1}{k}\right)} \leq \frac{s_3-s_1}{12s_3}$. This inequality can be simplified (using $s_3 - s_1 = \Delta s$) to $s_1 \leq \frac{\Delta s}{6}\left[(1+\lambda)\left(7-\frac{1}{k}\right)-12\right]$. Now, $U_1(\underline{\theta}) \geq 0$, implies that $s_1 \geq \frac{\Delta s}{12}[(1+\lambda)(k+17)-24]$. We combine these inequalities to get $2[7-(1/k)] \geq (k+17)$, which is a contradiction for all values of $k > 7$ (required in the equilibrium, since $p_1^* > 0$). Thus we establish that $p_3^* < p_3^m$, thereby completing the proof. ■

The primary intuition behind why price under oligopoly remains strictly less than the monopoly level is that now the highest-quality firm (in the oligopoly) must worry about losing a part of its market to a firm that offers a closer substitute (s_2 instead of a more distant s_1). The above two lemmas (*2B.1* and *2B.2*) together illustrate that the effect of price is more dominant (than market share) in the determination of profits. A similar reasoning makes it clear that the price of the lowest quality (s_1) manufacturing firm diminishes with an increase in the number of firms (*n*). This is formally, shown in the next part of this Appendix (where a brief analysis is presented for *n* = *5*), and has immediate implications on the difference between full market coverage and partial market coverage for $n \geq 3$, since the prices that naturally emerge in the equilibrium are so low (also verified in Figure 3) that practically every consumer in the market can afford the product (more specifically for *n* = *3*, $U_1(\underline{\theta}) \geq 0$ requires that $\Delta s \leq \frac{12s_1}{[k-7+\lambda(k+17)]}$: a condition that is easily satisfied). Thus, it is sufficient to study the full market coverage cases for *n* > *2*. Indeed, even with *n* = *2*, under partial market coverage, it is seen that only 1/16th of the market (as shown in example 2) remains uncaptured by the firms.[7]

Since it is clear that both the prices and market shares of the lowest quality firms (s_1 and s_2) are less than their respective monopoly levels (shown below), it is only sufficient to compare the profits of the highest-quality firm under monopoly and oligopoly.

Case II: When n = 5

We retain the structure for *Case I* above here and use *n* = *5* to get the following equilibrium.

$$p_1^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-97)}{168};\ p_2^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-13)}{84};\ p_3^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-1)}{24};$$

$$p_4^* = \frac{\Delta s(1+\lambda)\underline{\theta}(13k-1)}{84};\ p_5^* = \frac{\Delta s(1+\lambda)\underline{\theta}(97k-1)}{168}$$

$$\pi_1^* = \frac{\Delta s(1+\lambda)\underline{\theta}}{(k-1)}\left(\frac{k-97}{168}\right)^2;\ \pi_2^* = \frac{\Delta s(1+\lambda)\underline{\theta}}{2(k-1)}\left(\frac{k-13}{42}\right)^2;\pi_3^* = \frac{\Delta s(1+\lambda)\underline{\theta}(k-1)}{288};$$

$$\pi_4^* = \frac{\Delta s(1+\lambda)\underline{\theta}}{2(k-1)}\left(\frac{13k-1}{42}\right)^2;\pi_5^* = \frac{\Delta s(1+\lambda)\underline{\theta}}{(k-1)}\left(\frac{97k-1}{168}\right)^2$$

[7] The general term for the uncaptured segment equals *[(m – 1)(1 +λ)(k – 1) – 3m]/[(m – 1)(1 + λ)(k – 1) + 3m(k – 1)]*.

We now compare the prices and market shares of each firm under different (oligopolistic) market forms

Lemma 2B.3: The market shares of lower quality firms shrink as an oligopoly expands.

Proof: We recognize that $D_1^m - D_1^*|_{n=2} = \frac{k+4}{6(k-1)} \geq 0$; $D_1^*|_{n=2} - D_1^*|_{n=3} = \frac{3k-1}{12(k-1)} \geq 0$; $D_1^*|_{n=3} - D_1^*|_{n=5} = \frac{13k-1}{168(k-1)} \geq 0$; $D_2^m - D_2^*|_{n=3} = \frac{k+2}{6(k-1)} \geq 0$; $D_2^*|_{n=3} - D_2^*|_{n=5} = \frac{13k-1}{42(k-1)} \geq 0$; $D_3^m - D_3^*|_{n=5} = \frac{5k+1}{12(k-1)} \geq 0$

Lemma 2B.4: The prices of lower quality firms shrink as an oligopoly expands.

Proof: We recognize that $P_1^m - P_1^*|_{n=2} > 0$; $P_1^*|_{n=2} - P_1^*|_{n=3} = \frac{\Delta s(1+\lambda)\underline{\theta}(3k-1)}{3} > 0$; $P_1^*|_{n=3} - P_1^*|_{n=5} = \frac{\Delta s(1+\lambda)\underline{\theta}(13k-1)}{168} > 0$; $P_2^m - P_2^*|_{n=3} > 0$; $P_2^*|_{n=3} - P_2^*|_{n=5} = \frac{\Delta s(1+\lambda)\underline{\theta}(13k-1)}{84} > 0$; $P_3^m - P_3^*|_{n=5} > 0$

Let say $P_3^m - P_3^*|_{n=5} \leq 0$ that means $s_3 \leq \frac{\Delta s(1+\lambda)(k-1)}{12k}$. Now, in the equilibrium, $U_3(\hat{\theta}_2)|_{n=5} \geq 0$ implies that $s_3 \geq \Delta s\left[\frac{7(1+\lambda)(k-1)}{5k+19} + 2\lambda\right]$. These inequalities can be combined to get $\frac{(1+\lambda)(k-1)(79k-19)}{12k(5k+19)} + 2\lambda \leq 0$. This is not possible for any permissible value of *k*. Thus $P_3^m - (P_3^*|n=5) > 0$. In similar manner, it can be shown that $P_1^m - P_1^*|_{n=2} > 0$ and $P_3^m - P_3^*|_{n=5} > 0$. ■

Both Lemmas *2B.3* and *2B.4* together guarantee that the profits of the firms of less-than-highest quality firms under oligopoly do not exceed their respective monopoly profits.

Chapter 3

Fairness is flexible: A study of competing focal points

3.1 Introduction

We address one of the most fundamental questions about the (non-)uniqueness of fairness ideals. The idea that negotiated outcomes between two or more agents should be *fair* is universally agreed upon - and for good reason. Ken Binmore (1994, 1998) argues that fairness is nature's solution to the allocation problem of limited resources. The problem of resource allocation is two-fold: the first level addresses ways to achieve (higher) efficiency (i.e. the movement from lower to higher payoff frontiers – a class of folk theorems can guarantee the sustainability of the higher frontiers); and the second level addresses the problem of picking an allocation among a set of competing allocations that are efficient. The focus of this paper is on the second level that has witnessed contributions in collective decision making in the form of bargaining problems (e.g. Nash) and those that involve the aggregation of individual preferences (e.g. Arrow), among other approaches to collective decision making that aim to achieve outcomes deemed apt by each consequential agent (e.g. Coase).

Fairness is often a crucial starting point to the development of such approaches. For example, implicit notions of fairness are embedded in Nash's (1950) axiom of symmetry, Arrow's axiom of non-dictatorship, and can generally be thought to be embedded in the preferences of an impartial arbitrator who must decide the outcome of negotiation between two agents (Thomson, 1994).

Fairness, as a key requirement, however, faces a central problem – there is no unique way to define fairness – the nature of fairness ideals is often contextual. We show that the size of the bargaining pie in relation to that of the disagreement payoff, provides such context. More specifically, we demonstrate that the strength of the disagreement payoff (in the determination of bargaining outcomes) decreases as the pie-size increases.

Real-life negotiation often witnesses several competing ideals of fairness. The fact that these ideals need not be unique, motivates our current investigation. Consider a situation where two agents who can individually earn $100 and $50 can come together and earn $300. One can argue that a fair split will require that the joint pie is divided equally (each agent gets $150). One can also argue that a fair split should be according to (in proportion to) the agents' individual earning capacities ($200 and $100). Yet another way to describe a fair split is where only the surplus beyond the individual earnings is shared equally ($175 and $125). To the best of our knowledge, this is a first attempt at a conclusive understanding of which fairness ideal to expect in a given context. Thus, our purpose is to provide insights on which fairness ideal should prevail as a given outcome of unstructured bargaining.

We formally examine the above three classes of fairness ideals (known as *Uniform Gains (UG), Proportional Gains (PRO)* and *Equal Surplus (ES)* respectively, in the spirit of Moulin, 2003). Our aim is to provide useful insights on exactly which fairness ideal to expect in a given negotiation-environment. We demonstrate that each of the above notions of fairness can become more relevant than the others according to the specifics of a bargain. Through our experiment, we demonstrate that in unstructured bargaining, the disagreement payoff becomes less important as the stake size increases - different fairness ideals become more apt for different stake sizes, and in the process, become focal points.

In a nutshell, the notions of fairness are not devoid of context. As an extreme example, Birkeland and Tungodden (2014), show that bargaining outcomes could be sensitive to the fairness motivation of the negotiating agents – disagreement on what a fair outcome is, may result in a disagreement outcome (i.e. a loss of efficiency). The task of unifying fairness ideals is challenging because it is well recognized that fairness perceptions frequently respond to changes in strategic environments (Schmitt, 2004). This malleability of ideals makes it possible for agents to (unknowingly) have self-serving biases in the idea of fairness (Babcock and Loewenstein, 1997; Feng et al 2013). For example, ultimatum games and dictator games are frequently used to examine the effect of the pie-size and the results known in the literature are diverse. Given the prevalence of discussions about fairness in the literature on bargaining (and its contextual nature), the question that naturally arises is if something can be specifically said about the fairness ideal to expect in a given context.

We present the findings of an experiment, in which each treatment uniquely corresponds to exactly one of the three families of fairness solutions (discussed through our example) that is deemed most suitable. To be specific, when the pie size is smaller relative to the disagreement payoff then the PRO is the most popular choice, as pie size increases the UG becomes the most popular choice. In other words, we explicitly show that in one treatment, one family of fairness ideals is the most popular choice, in a second treatment, a second family of fairness ideals is the most popular choice (and so on). Therefore, the most popular outcomes in each treatment is consistent with exactly one fairness solution.

As a first step, we will establish that the pie-size (the only source of change between our treatments) is itself a good indicator of which fairness ideal will be empirically dominant. In this sense, our study is related to the research of List and Cherry (2008); and Andersen et al (2011), where it is shown that the monies transferred by the more consequential subjects respond less-than-proportionately to the stakes in hand. These results can be explained by

Rabin's (1993) attempt to incorporate fairness in game theory, in his stylized requirement that the willingness (of the responder) to punish (by rejecting the offer) should diminish with higher stakes. In general, it is agreed that fairness considerations can play an important role in the determination of bargaining outcomes when stake sizes vary (Karagözoğlu and Urhan, 2017). We go a step further to single out the fairness ideal that is the most dominant (in comparison with other fairness ideals) in explaining observed outcome in each context.

It is clear that fairness considerations are important to agents on any matter involving an allocation of resources. Fehr and Schmidt (2003) explicitly argue that many people are strongly motivated by concerns for fairness and reciprocity - not just material self interest. Therefore, even if an agent does not feel strongly about fairness considerations, he may want to make higher offers to mitigate the chances of rejection by another agent who is known to value fairness strongly (Carpenter, 2003).

When notions of fairness are well-defined, any deviation(s) from the same often trigger feelings of shame and guilt for the deviating agents. These notions of fairness can be so powerful that if an agent is known to treat other interacting agents in a fair manner, then it is an immediate guarantee of loyalty and reciprocated fairness for this agent from the receivers of fair treatment (Chiu et al, 2009).

In conclusion, fairness is a key to pro-sociality (Henrich et al, 2010; Charness and Rabin, 2002), and is a driver of societal and institutional progress (Janssen, 2000), and can even shape regulatory stance (Banerjee, 2015). Therefore, we can consequently benefit from a unifying fairness ideal, in which many contextual ideas of fairness are nested. The additional merit of our study is that we look at relatively less explored unstructured or free-form bargaining (Karagözoğlu, 2019; Anbaric and Feltovich 2013) - which is what we observe in the real world.

3.2 The theory

In what follows, we now formalize the motivating example in the introductory note in the spirit of Moulin (2003), which in turn, will help us motivate the rationale behind our treatment groups.

3.2.1 The formulation

Two individuals X and Y (both from the same homogenous population) have the following two options.

Option 1: Individually earn $d(x)$ and $d(y)$, respectively.

Option 2: Cooperate and generate a pie of size $z > d(x) + d(y)$, and share the same. Their respective shares are x and y (both non-negative), so that $x + y = z$.

In the event Option 2 is chosen, Moulin (2003) offers the following three classes of solutions (the last one is an intermediate between the first two extremes).

1. *Uniform Gains (UG):* X and Y share z equally – i.e. $x = y = z/2$.

2. *Proportional Gains (PRO):* X and Y share z in proportion to their individual earnings

– i.e. $x/y = d(x)/d(y)$.

3. *Equal Surplus (ES):* X and Y share z, such that the gains from cooperation are matched

– i.e. $x – d(x) = y – d(y)$.

It is clear that when $d(x) = d(y)$, all the three solution concepts yield the same result. The blur between the fairness ideals occur when $d(x)$ and $d(y)$ are different.

3.2.2 A discussion

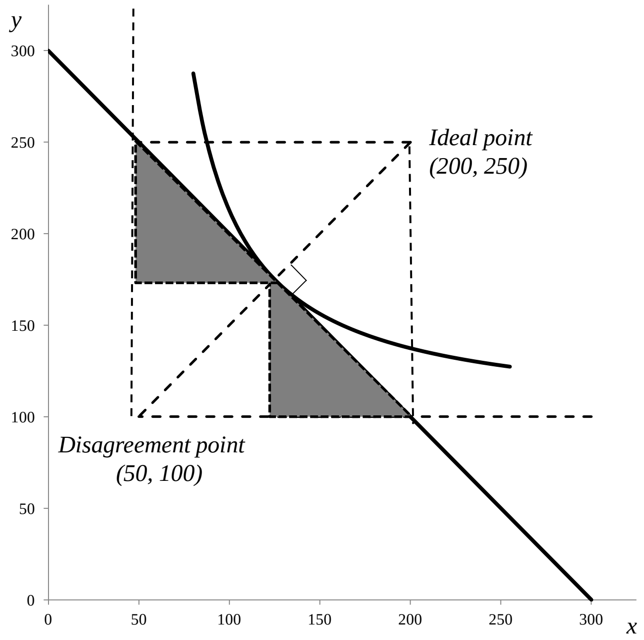
In what follows, without loss of generality, we assume that $d(y) > d(x)$. Now each of the three solution concepts can be thought of as a fair way to distribute z. For example, fixing $d(x) = 50$, $d(y) = 100$, and $z = 300$, will give us $(x = 150, y = 150)$ under the *Uniform Gains* (*UG*) protocol; $(x = 100, y = 200)$ under the *Proportional Gains (PRO)* protocol; and $(x = 125, y = 175)$ under the *Equal Surplus (ES)* protocol. It is clear that the *ES* allocation rules will always remain somewhere midway between the *UG* and the *PRO* protocols for any value of z, $d(x)$, and $d(y)$.

Before proceeding further, it helps to clarify that each of the above three formalizations of fairness (*UG*, *PRO* and *ES*) includes a class of allocation rules/bargaining solutions that have implicit notions of fairness. For example, in Figure 1, the *ES* fairness ideal introduced above can be interpreted as the outcome of Nash (1950) bargaining (which maximizes $[x - d(x)][y - d(y)]$, subject to $x + y = z$); the Kalai-Smorodinsky (1975) solution (KS hereafter, which requires (x, y) to be along the line that joins the disagreement point to the *ideal point*); the (discrete) Raiffa (1953) solution (which bisects the line segment of Pareto optimal points above each agents' disagreement payoffs); the Equal Area solution (EA hereafter which equates the surpluses (shown by the shaded area) given up by each agent in order to reach an agreement); the Yu (1973) solution (which chooses (x, y) closest to the *ideal point* of KS).[1] All these bargaining solutions are individually discussed in Thomson (1994), and have been presented in Figure 1 for the values for disagreement payoffs and the pie size assumed in the previous paragraph. Thus, since the *ES,* as a fairness rule, is a class of bargaining solutions covering the Nash, KS, Yu, and Raiffa solutions (among others), we do not need separate

[1] In Figure 1, the set of feasible alternatives is shown by the region bounded by the axes (which represent the monetary payoffs for *X* and *Y*) and the negative-45° line (which is the set of Pareto efficient points). The *ideal point* for each agent can be thought of as that point the agent will be able to achieve if he/she were a dictator. It is more formally defined as the most favorable Pareto optimal point subject to the requirement that each other agent receives at least their disagreement payoff levels.

discussions around these solutions of cooperative bargaining game theory.[2] What follows now is a brief discussion about the *UG* and *PRO* fairness rules.

Figure 1: ES is a class of bargaining solutions



The *Uniform Gains* protocol is the egalitarian solution discussed in Thomson, 1994, is the first extreme, where the differences between *X* and *Y*, if any, are inconsequential in terms of the final solution. For instance, the idea that all the rich and poor are equal in the eyes of the law is thought of a *fair* way to disseminate justice. The *Uniform Gains* solution may however, be questionable for it completely ignores (as it should) systematic differences between the agents in question. In our example, since *Y* can individually earn twice that of *X*, (i.e., clearly $d(y) = 100 = 2d(x)$) then, *Y* may feel entitled to a higher share in *z*, since *Y* is (say) more efficient than *X*. This is true of the world we live in – Cardenas and Carpenter

[2]Similarly, note that the *UG* rule is a class of solutions covering the Egalitarian-type solutions discussed in Thomson (1994). Interestingly, for $z = 180$, it also covers the dictatorial solution of Thomson, 1994, where agent *X* is the dictator.

(2008), for example, point out that the perception of how deserving recipients of ultimatum games are, is a strong predictor of altruism. We turn to this idea of fairness now.

Proportional Gains (*PRO*) requires that the agents share the pie in proportion to their perceived individual capacities (in this case, 2:1), and has appealed to theorists who have modeled the same. These interpretations are consistent with Aristotle's idea of fairness which should be proportional to some measure of agents' need, ability, effort and status (additionally see Harsanyi 1962, 1966 for one of the first theoretical approaches to bargaining). The effects of ability, status, and effort on bargaining outcomes have been frequently demonstrated experimentally (Hoffman et al 1994; Ball et al 2001).

In short, all the classes of fairness rules require a sense of equality. Under the *UG* protocol, it is the equality of the *shares* (of the total pie-size); under the *PRO* protocol, it is the equality of *proportions* (between the outcomes of agreement and disagreement); and under the *ES* protocol, it is the equality of *gains* (from cooperation – i.e. the transition from disagreement to agreement). We are now in a position to describe our experiment designed to disentangle one fairness solution concept from another.

3.3 The experiment

3.3.1 An overview

A total of 452 undergraduate and post-graduate students, aged between 18 to 28 years, from five institutes across India were the subjects of our experiment. Each individual received a show-up fee of INR 200, in addition to which, they retained the amount they could bargain for themselves in the experiment. They were randomly assigned to one of four treatments, after which they took a test. After the test, each treatment group was divided into two subgroups of top half and bottom half performers – that is, the tests were graded and ranked according to the subjects' performances, and then split into a top-half group and a bottom-half group in each treatment. Finally, in each treatment, each subject among the top half performers was randomly paired with a subject among the bottom half performers for the purpose of bargaining. As we will see below, the only distinguishing characteristic between our treatments is the pie-size. What follows is a description of the treatments.

Treatment 180 (T180): In this treatment of a total of 116 individuals (58 pairs), each subject from the top half (Y) is paired with a subject in the bottom half (X) and each pair formed of individuals X and Y are asked to split INR 180 (= z) among themselves.[3] They are given a time period of ten minutes to reach an agreement, failing which, the outcome is treated as a disagreement, in which case the high-ranker in each pair is given INR 100 (= $d(y)$), and the low ranker is given INR 50 (= $d(x)$).[4] Both negotiating agents in a pair knew their own and each other's disagreement payoffs. Note that, in this treatment, the size of the pie is only marginally higher than the sum of the disagreement payoffs ($d(x) + d(y)$ = INR 150). We observe that X and Y share the pie-size of INR 180 mostly according to the *PRO* rule in this treatment. There were no disagreements.

Treatment 300 (T300): In this treatment of a total of 110 individuals (55 pairs), X and Y are asked to split a sum of INR 300 (= z) between themselves. Everything else remains the same including the disagreement payoffs. In this treatment too, we saw no disagreements and a majority bargained according to the *PRO* rule.

Treatment 600 (T600): In this treatment of 108 individuals (54 pairs), X and Y are asked to split a sum of INR 600 (= z) between themselves. Everything else remains the same including the disagreement payoffs. Note that in this treatment, the gains from cooperation are fairly

[3] As of September 1, 2019, US$1 = INR 72.
[4] The suitability of ten minutes was determined from prior pilot studies.

high ($z - d(x) - d(y)$ = INR 450). We observe that a majority of the agents *X* and *Y* go for the *UG* solution. There were no disagreements.

Treatment 900 (T900): In this treatment of 118 individuals (59 pairs), *X* and *Y* are asked to split a sum of INR 900 (= z) between themselves. Everything else remains the same. Note that in this treatment, the gains from cooperation are the largest ($z - d(x) - d(y)$ = INR 750). We observe that a majority of the agents *X* and *Y* settle by the *UG* solution. There were no disagreements.

3.3.2 Further details

In each treatment, bargaining happened between *X* and *Y* over Skype with rank revealing IDs such as Rank.001, Rank.002 and so on. This helped in preserving anonymity, which is desirable because the knowledge of who each subject was paired with could mitigate the effect of the test.[5] Further, it retained a key feature of real-life bargaining and negotiation processes – *dialogue*. In the real world, economic agents give away apt reactions through either their vocal tone or facial expressions when they are pleased or displeased with the direction of negotiations. Our subjects were found to frequently use apt emoticons according to whether they felt that the bargains suggested by their partners were fair or unfair. The process of communication used text-chat instead of voice-chat to mitigate the possibility of any identification, since the subjects came from a homogeneous population (i.e. the same

[5] While more natural forms of negotiation involve face to face interaction (Nydeger and Owen, 1975; Chakravarty et al, 2013), allowing for a bargaining protocol where anonymity is compromised could be counterproductive for our setting.

institution) and were likely to be friends. The subjects were instructed to bargain only in English.[6]

Finally, the test (given in the appendix) was a compilation of 20 extremely difficult questions for which, a time limit of 10 minutes was given. Each question was followed by four possible answers of which, only one was correct. There was no negative marking and the instructions explicitly required the subjects to maximize their total score of right answers. The extreme difficulty level coupled with the limited time to solve would have ensured that the subjects were forced to resort to random marking of the answers.[7] Thus, effectively, each question had a one-fourth probability of being answered correctly. Clearly, on an average, therefore, we should expect one-fourth of the given questions to be correctly answered. We see that the students got an average score of 4.96 out of 20, which is not significantly different from what is expected. Therefore, we are confident that the ranking on the basis of the test is as good as random. Thus, having the actual rank of bargaining agents as a potential determinant of bargaining outcomes is least likely to be correlated with unobserved ability - moreover we explicitly examine the same by checking for possible correlation between our test ranks and the actual academic performances of our subjects in their respective courses provided by the institutes where we undertook our experiments (practically negligible R-squared of 0.008). Note that we can only compare our findings with models of cooperative bargaining since the negotiation process of our experiment is unstructured (see Karagözoğlu and Urhan 2017).

[6]The alternative language was Hindi. The issue is that speaking in the first-person language reveals the gender of the individual. Male agents for example will say "I *think* this is a fair split" differently from female agents.
[7]For example, the second question in the test (called Monila's urn) is based on the Fermat's last theorem, which took over three centuries for mathematicians to solve.

3.3.3 Testable hypotheses

We now set up regression functions that models the (functions of) expected share of the high-ranked agents conditional on the pie size as follows:

$$E(y|z) = \alpha_0 + \alpha_1 z \tag{3.1}$$

This leads us to the following hypotheses of interest for our chosen values of $d(x) = 50$ and $d(y) = 100$

1. *Hypothesis UG* (H_{UG}): $\alpha_0 = 0$ and $\alpha_1 = 1/2$

2. *Hypothesis PRO* (H_{PRO}): $\alpha_0 = 0$ and $\alpha_1 = 2/3$

3. *Hypothesis ES* (H_{ES}): $\alpha_0 = 25$ and $\alpha_1 = 1/2$

It should be noted that for the treatment T180, the *UG* rule requires that agent *Y* be given at least his/her disagreement payoff level, so that $y = 100$ and $x = 80$ (Moulin, 1988). This is discussed more closely and in greater detail when we discuss our non-parametric sampling methodology and the empirical analyses that follow. The power analyses (deferred to the appendix) presented in relation to this, accounts for the potential existence of mass-points when the underlying random variable is non-Gaussian. For now, we note that our game-structure itself gives strong hints about how to conduct power analyses for sample sizes.

3.4 Descriptive statistics

3.4.1 An overview

Table 1 presents a brief summary of the four treatments. Most of the participants were female (just over 55%). The high-ranked subjects were able to negotiate, on an average about 58% of the respective pie-sizes (Table 1 reports the figures by treatment). Apart from T180, no other treatment saw individual shares exceeding two-thirds of the given pie-size. The average time to negotiate remained between three and four minutes in each treatment.

Table 1. Summary of the treatments

	T180	T300	T600	T900
Observations	116	110	108	118
Number of pairs	58	55	54	59
Number of males	52	52	47	50
Mean Share (High-ranked)	0.640	0.600	0.547	0.524
(Standard Error)	(0.005)	(0.009)	(0.008)	(0.005)
Maximum Share (High-ranked)	0.694	0.667	0.667	0.667
Average time taken (Seconds)	214.586	219.382	199.370	194.424
(Standard Error)	(17.145)	(18.149)	(16.747)	(16.533)

Figure 2: Distribution of shares of high-ranked individuals

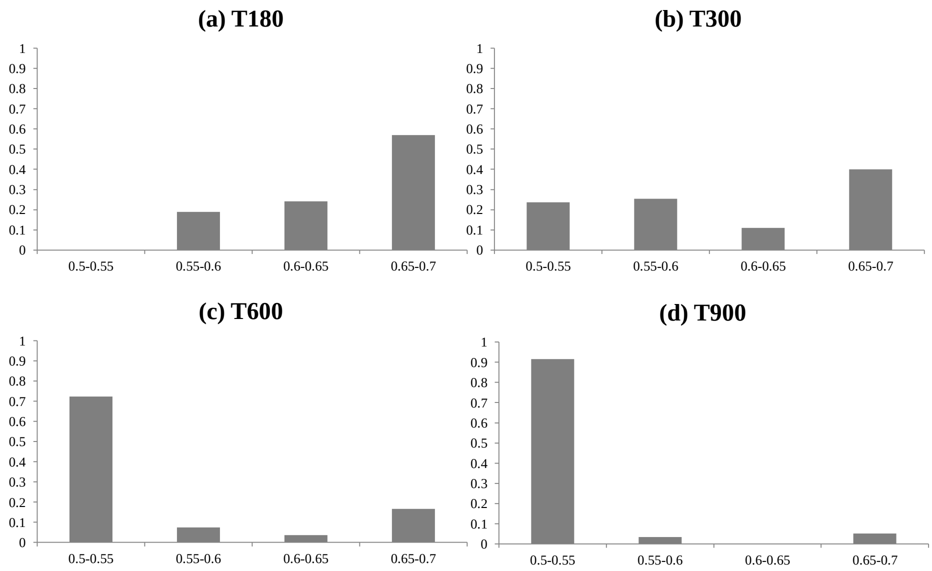
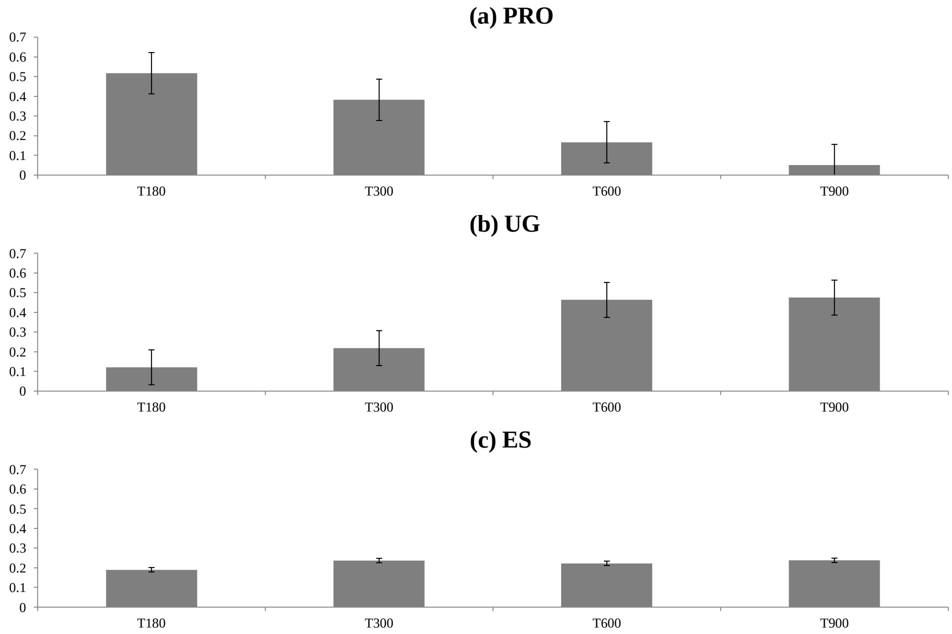


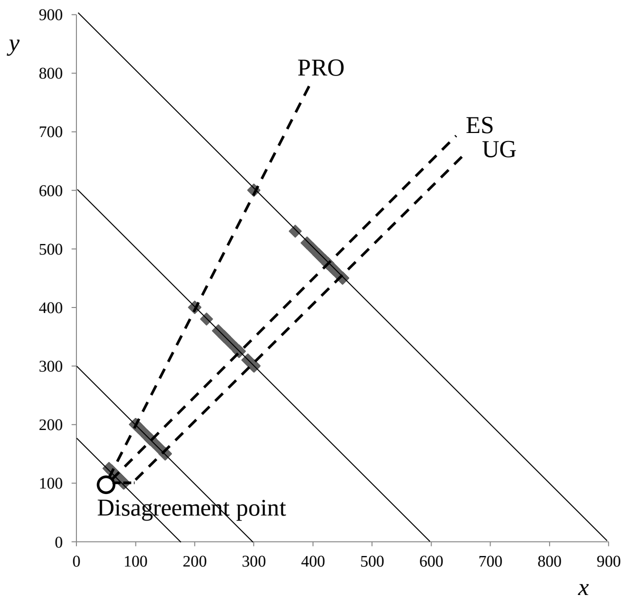
Figure 2 displays the distribution of shares of the high-ranked individuals in each treatment (the distribution of the shares of the low-ranked individuals is a mirror-image of this). It is immediately seen that the density of shares gravitate (away from about two-thirds) towards half as the pie-size increases from T180 (panel a) to T300 (panel b) to T600 (panel c) to T900 (panel d). From Figure 3, it becomes immediately clear why this is so. We see in Figure 3, that the results are primarily driven by focal-points (which become statistical mass-points, and therefore define the modal class of observations - all this in turn, influences the mean). If we, for now, accept the exact values assumed by our hypotheses (*UG*, *PRO* and *ES*) as focal points, then it will help to look into the proportion of individuals agreeing on each focal-point in each treatment. This is the purpose of Figure 3 below.

Figure 3: Fraction of focal point agreements



We begin by pointing out that focal points are *exact* values that are immediate and easy to spot and calculate. For example, in T180, an agreement like *(x, y) = (62, 118)*, would not qualify as a focal point even though it is very close to the *PRO* solution of *(60, 120)*, which becomes easy to spot and calculate since it easily replicates the ratio of the disagreement payoffs. Panel (a) in Figure 3 reports the proportion of negotiating pairs who exactly went for the *PRO* rule in each treatment. For example, among the total number of negotiations in T180, about 52% settled on the *PRO* rule. The corresponding figures for T300, T600 and T900 are 38%, 17% and 5% respectively, which makes it clear that lesser and lesser proportion of negotiating pairs deemed the *PRO* rule to be apt as the pie-size increased. The exact opposite can be argued of the *UG* rule in which case, the proportion of negotiating agents who deem the *UG* rule to be apt increases with the pie size (12% in T180, to 22% in T300, to 46% in T600, to 47% in T900). The proportion of negotiating pairs that saw the *ES* rule as apt remained relatively stable (in comparison to the above rules) around 20% to 25% throughout the four treatments. Before we examine these focal points any closer, it will help to put data and theory together - we look at the scatter-plot of negotiated outcomes against the requirements of the *UG*, *PRO* and *ES* fairness rules in Figure 4 below. The four negative-45° lines are the set of Pareto optimal points on our payoff-frontiers corresponding to our four treatments. The disagreement point *(d(x), d(y)) = (50, 100)* is marked with ○.

Figure 4: Scatter-plot of negotiated outcomes against fairness ideals



The set of *PRO* agreements are along the line $y = 2x$ as shown above. Similarly, the set of *ES* agreements are along the line $y = x + 50$, and the set of *UG* agreements are along the line $y = x$ above $x = 100$ (up to which *y* remains *100* - since the *UG* rule requires that the agents share the pie equally subject to each agent receiving at least the disagreement level payoff). It is immediately seen that the scatter gravitates towards equality as the pie-size increases in the north-east direction. For the sake of robustness, in the subsection that follows, we impose stringent requirements on what qualifies as a focal point.

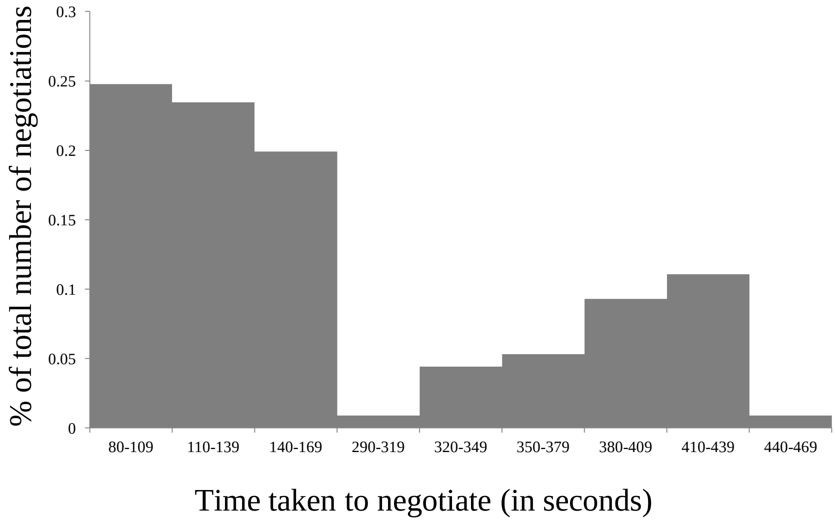
3.4.2 A closer look at focal points

The natural choice for focal points in each treatment is immediately driven by the ease with which they are spotted instinctively - and since focal points are natural enough to be instinctively spotted, they are also likely to lead to relatively quicker agreements between bargaining agents. The process of bargaining itself (in the real world) can be costly and stressful (Banerjee, 2015), and therefore there is value in spotting focal points that facilitate quicker agreements between agents. As a first step, we look at the distribution of the time taken to reach a negotiation between the pairs of agents (Figure 5). Since our subjects engaged in chat-based negotiation over Skype, we were able to record the time the negotiation started and compare the same to the exact time when they struck a deal. Figure 5 shows the distribution of the time taken (in seconds) by our negotiating agents to strike a deal.

The distribution of the time taken to negotiate (Figure 5) clearly shows that a significant proportion (32% or about one-third) of our negotiating pairs did bargain hard and argue till the very end, averaging over six and a half minutes (more precisely, 390.19 seconds) to reach an agreement (this makes up the right-cluster in our distribution). A majority (the remaining 68%, or about two-thirds, making up the left-cluster in our distribution) of our negotiating pairs however, chose to strike a deal quickly averaging at about two minutes (more precisely, 121.14 seconds). Thus, Figure 5 shows a clear distinction between pairs that arrived at quick decisions and the pairs that did not.[8] Since we are looking at pairs that engage in a natural process of social contemplation (through a verbal exchange which we call *dialogue*), and telling them apart from pairs that display a natural social preference embedded in their instincts, our work is also related to that of Rubinstein (2016), which classify agents as contemplative or instinctive depending on their individual response times in their choice(s) of action.

[8]Indeed, any value between 180 to 270 seconds can comfortably act as a natural split between the quick and the not-so-quick negotiating pairs.

Figure 5: Distribution of time taken to reach agreement



It should be noted that not all negotiations that ended quickly ended as focal-point agreements (for example, there were negotiating pairs where, after one of the agents proposed a focal-point split, the bargaining agents quickly negotiated their way to a final settlement that remained around, but different from the initial focal point identified. Similarly, not all focal-point negotiations ended quickly (for example, many bargaining pairs negotiated their way to finally reach splits that looked like focal-points). Table 2, displays this clearly. Overall, about 59% of all agreements were focal point agreements. It is also clear from Table 2 that people generally prefer quicker negotiations.

Table 2: Distribution of focal-point agreements by time taken

	Focal-point agreement	Non-focal-point agreement	**Total**
Quicker negotiation	106	48	**154**
Longer negotiation	28	44	**72**
Total	**134**	**92**	**226**

We now accept the following two criteria in order to identify focal points (in the interest of more stringent requirement for a negotiation to qualify as focal point):

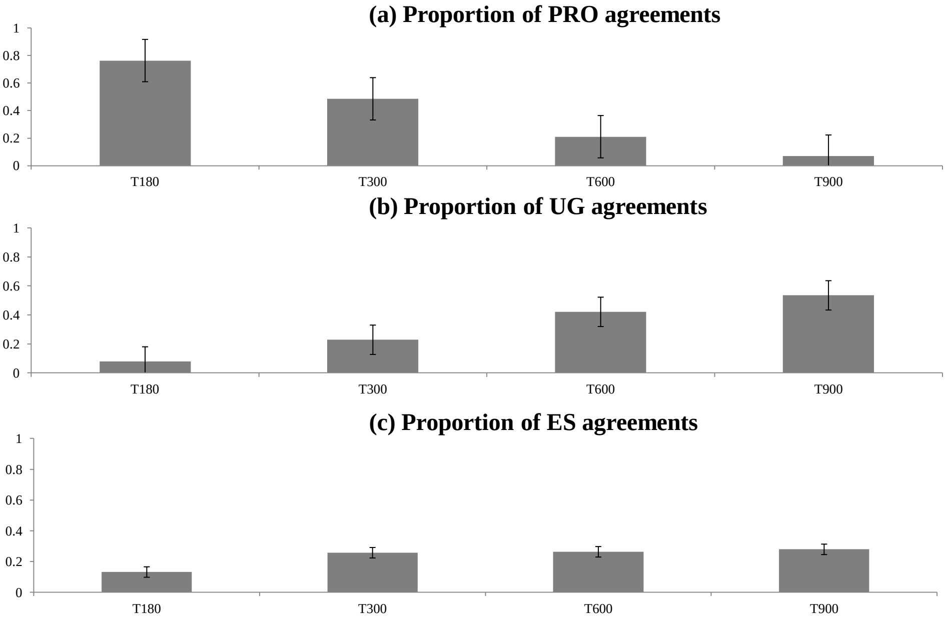
1. Easy to locate, identify, and calculate

2. Reduce the time to negotiate.

In Figure 6, we look at focal-point agreements as a proportion of the quickest (that make up the left cluster) conversations in each treatment. For example, from Panel (a), we learn that of all the negotiations that concluded quickly in T180, 76% were *PRO* outcomes. Similarly 49%, 21%, and 7% of the quickest negotiations in T300, T600 and T900 respectively, were *PRO* outcomes. It is clear that as the pie-size increased, lesser and lesser number of bargaining pairs felt that the *PRO* rule was apt. Similarly, the fact that among the quickest negotiations in T180, T300, T600 and T900, respectively 8%, 23%, 42% and 53% were according to the *UG* rule. Clearly, more and more negotiating pairs saw this to be more apt as the pie size increased. As the pie size increased from T180 to T300 to T600 and finally to T900, 13%, 26%, 26% and 28% of the quickest decisions were as per the *ES* fairness rule.[9]

[9]This is not surprising since in T180, an allocation like $(x, y) = (65, 115)$ does not look like a focal point. The *PRO* fairness rule which is close to this would have looked more natural.

Figure 6: Fraction of focal points amongst quickest decision



Thus, it is relatively easy to see that some fairness rules are more apt than others for a given pie size potentially because they naturally stem from instincts, revealing joint/social preferences.

In the section that follows, we will put forward a behavioral explanation for why some focal points are naturally more attractive to negotiating instincts. When we present the central conclusions, we will propose a general fairness solution that unifies all the fairness rules discussed so far (and by extension, the bargaining solutions from cooperative bargaining game theory) and then argue that *general* social preferences are different from *aggregate* social preferences.

3.5 Empirical strategy and results

3.5.1. Determination of sample-sizes

We non-parametrically determine our sample-sizes for each treatment without making any assumption on the distribution of the underlying outcome variables (negotiated outcomes).

Since our final hypotheses of interest are based on the shares of the high-ranked individuals in a bargaining pair, our sampling methodology is centered around the same (this prepares us for our final analyses). In general, it is desirable that sampling methodology be consistent with the technique of estimation (as is often the case with the purer sciences including experimental physics and biology).

Our choice(s) of null and alternate hypotheses for power analyses in sample size determination come directly from theory. The knowledge that *ES* always lies between the *PRO* and the *UG* rules aids our two-fold strategy for determining sample sizes for each treatment: first we take the H_{UG} to be the null and H_{ES} to be the alternate hypothesis; and second we treat the H_{ES} to be the null and H_{PRO} to be the alternate hypothesis.[10] For each treatment, these test of hypotheses are conducted for a given (lower bound of the) power of our tests, based on which we get two sample sizes (one from testing H_{UG} against H_{ES} and the other from testing H_{ES} against H_{PRO}), and we choose the larger of the two sample sizes as the desired sample size for that particular treatment. This prepares us for all contingencies in relation to statistical inference.

Our method of determining sample sizes is consistent (in fact, intertwined) with our technique of estimation which is primarily based on the shares of the high-ranked individuals in each negotiating pair. We account for the possibility of mass-points in our method of determining sample sizes and make no assumption on the distribution of the underlying random variable(s) associated with observed negotiated outcomes. The exact details of arriving at the sample sizes are detailed, and are therefore, deferred to the appendix.

[10] Note that this is necessary since, it is impossible to convincingly define the power of our test for three competing hypotheses together. Calculating sample sizes from competing pairs of hypotheses prepares us for the worst case and guarantees a minimum power as shown in the appendix.

3.5.2 Hypotheses tests and regression results

In what follows, we use the information presented in Table 1, and use our empirical confidence intervals to see if they include the hypothesized values of the *UG*, *ES* and the *PRO* fairness rules under each treatment. Table 3 below presents the same.

Table 3. Observed data against fairness rules

Treatment	**N**	**Mean**	**95% CI**	**PRO**	**ES**	**UG**
(1)	(2)	(3)	(4)	(5)	(6)	(7)
T180	58	0.640 (0.005)	0.629 – 0.650	2/3 ☒	23/36 ☑	5/9 ☒
T300	55	0.600 (0.008)	0.582 – 0.617	2/3 ☒	7/12 ☑	1/2 ☒
T600	54	0.547 (0.008)	0.530 – 0.563	2/3 ☒	13/24 ☑	1/2 ☒
T900	59	0.524 (0.005)	0.514 – 0.534	2/3 ☒	19/36 ☑	1/2 ☒

Column (1) above enlists the treatments, and in column (2), we report the number of pairs in each treatment. Column (3) reports the mean share of the high-ranked individual *Y* observed in each of the treatments followed by the standard errors in the parentheses, which in turn are used for the confidence intervals in column (4). Columns (5), (6) and (7) show the expected share for the high-ranked subject *Y* in each treatment under the *PRO, ES*, and the *UG* allocation rules, followed by a ☒ or a ☑, depending on whether observations in the relevant treatment groups are inconsistent or consistent with the allocation rules in question (i.e. the 95% confidence intervals around the observed mean in the given treatment contains the predicted value of the said allocation rule). For example, the entry of 2/3 under the *PRO* rule in column (5) is clearly outside the 95% confidence intervals (shown in column 4) around the observed mean share of 60% in T300. Thus, we reject the hypothesis that the *PRO* rule explains the observed shares in T300, and represent the same with a ☒ sign (underneath the expected value under the *PRO* rule). Since each of the columns (5) and (7), has at least one ☒, it means that all of the observed experimental data is not *strictly* being explained by any of these rules.

In a nutshell, we see in Table 3 that even though the *PRO* fairness rule was the most popular (modal) choice in the T180 and the T300 treatments, it lies outside the 95% confidence interval around the observed means of those two treatments. Similarly, even though the *UG* fairness rule was the most popular choice in the T600 and the T900 treatments, it remains significantly distinct from the average choice of those treatment groups. The *ES* fairness rule, however, remains only insignificantly far from the observed mean in each treatment despite not being the most popular choice in any. In what follows, we formally examine this using the following regression equation in accordance with our testable hypotheses.

$$NegotiAmt_i = \alpha_0 + \alpha_1 PieSize_i + \alpha_2 T180_i + \boldsymbol{W_i \beta} + \varepsilon_i \qquad (3.2)$$

where $NegotiAmt_i$ (associated with the coefficient α_1) is the amount that the high-ranked agent *Y* negotiates (with the low-ranked agent *X*) for himself/herself in the *i*th pair; $PieSize_i$ is the size of the pie that our agents bargain over; $\boldsymbol{W_i}$ (associated with the coefficient vector $\boldsymbol{\beta}$) is a vector of other covariates that could potentially influence our bargaining outcomes; α_0 is the constant of regression and ε_i is the error specific to the negotiating pair. Lastly, we include a treatment dummy for T180 ($T180_i$ equals 1 if the *i*th bargaining pair belongs to T180, and 0 otherwise) as per the requirements of our hypotheses in Section 3. Since, if the *UG* solution were to explain the data, then agent *Y* must (on an average) get half the share in T300, T600, and T900 and an additional INR10 in T180, for he/she must at least earn his/her disagreement payoff subject to negotiation. The treatment dummy $T180_i$ solves this problem because, together with $\alpha_2 = 10$, the hypothesized values, $\alpha_0 = 0, \alpha_1 = 1/2$ continue to represent the *UG* fairness rule. Panels 1, 2 and 3 in Table 4 summarize the results from this regression.

In panel 1, we show the regression results without any additional controls. The F-tests for H_{UG} and H_{PRO} (shown in the lower panel) suggest that our observed regression coefficients are significantly different from what is required by the *UG* and the *ES* fairness rules. At this point we do not have sufficient evidence against the *ES* fairness rule, so we do not reject that. It seems that if Thomson's (1994) *impartial arbitrator* cared for *majority decision rule*, then he/she would specifically recommend the *PRO* rule or the *UG* rule depending on the pie-size - i.e. he/she would look for *general* preference (he/she will observe the mode and pick the modal preference). However, if he/she believed in some form of a social *aggregation* of the preferences of all agents (say, as in Arrow), then he/she would propose the *ES* fairness rule.

In panel 2, we introduce additional controls for the individual rank of the high-ranked individual, how many ranks ahead is he/she of his/her negotiating opponent (Rank Difference), the gender of the high-ranked individual, that of his/her negotiating opponent (opponent's gender), the high-ranked agent's academic aptitude and the time taken to negotiate (in seconds). In panel 3, in addition to these covariates, we include institution dummy variables. We see that the higher the rank of the high-ranked individual, the greater is his expected negotiated amount (note that Rank 1 is better than Rank 10, so higher values of the Rank variable correspond to lower ranks, thereby explaining the observed negative coefficients). Chakravarty and Somanathan (2008) demonstrate that high ranks are linked to high pay (additionally see Banerjee and Dey, 2014; Sharma, 2015). We also see that the greater the rank difference between the two agents in a pair, the greater, on an average, is the gap between their respective shares (the share of *Y* moves upward, so that of *X* must necessarily move downward given the pie-size).

Table 4. Pie-Size as a determinant of the share of high-ranked agents

Dependent variable: y	(1)	(2)	(3)
Pie-Size (z)	0.487***	0.486***	0.484***
	(0.009)	(0.007)	(0.007)
T180	-7.189*	-8.512***	-8.850***
	(3.716)	(2.917)	(2.917)
Rank		-2.269***	-1.925***
		(0.660)	(0.668)
Rank Difference		2.455***	2.677***
		(0.524)	(0.573)
Gender (Male = 1)		1.385	-1.504
		(3.478)	(3.829)
Opponent's Gender (Male = 1)		-1.103	-2.596
		(3.304)	(3.357)
Academic Aptitude		0.528	0.618
		(0.549)	(0.540)
Time Taken		0.010	0.011
		(0.011)	(0.012)
Constant	34.724***	19.055*	19.668*
	(4.926)	(10.473)	(0.071)
N(umber of pairs)	226	226	226
Institution dummies	No	No	Yes
F-Test for H_{UG}	$F(3, 223) = 139.61$	$F(3, 217) = 15.55$	$F(3, 213) = 15.36$
(p-value)	(0.0000)	(0.0000)	(0.0000)
F-Test for H_{PRO}	$F(2, 223) = 521.23$	$F(2, 217) = 380.76$	$F(2, 213) = 408.73$
(p-value)	(0.0000)	(0.0000)	(0.0000)
F-Test for H_{ES}	$F(2, 223) = 2.10$	$F(2, 217) = 3.43$	$F(2, 213) = 3.79$
(p-value)	(0.1251)	(0.0342)	(0.0241)
Decision rule	Reject H_{UG} and H_{PRO}	Reject all	Reject all

Notes: ***, **, and * mark out significance at 1%, 5% and 10%; Robust standard errors in parentheses.

To get uniformity in academic scores (which were in the form of aggregate percentages or as GPAs with different denominators for each institution as per its own norms), we split this data (for each institution individually) into ten groups of roughly equal size. Thus, for each institution we coded those in the top ten percent as 10, the next ten percent as 9 and so on till the bottom ten percent (coded as 1).[11] The fact that this variable called Academic Aptitude is not correlated with our test-ranks also verifies that our assignment of ranks (based on the test) is indeed random.

The strength of anonymity is reflected in the fact that none of the observed agent-characteristics, such as academic aptitude and gender turned out to be significant determinants of the observed shares. In general, that bargaining outcomes could significantly depend on the gender of the powerful agent (e.g. the proposer of an ultimatum game, or the high-ranked agent in our experiment), has been widely documented (Castillo et al, 2013, Banerjee, 2015; Chakravarty et al, 2011). Many market outcomes can be shaped based on attitudes towards gender (Gangadharan et al, 2016; Gangadharan et al 2019 and Gangadharan et al, 2015)

Finally, with the inclusion of other covariates (panels 2 and 3) in our regression specification, we find evidence even against the *ES* solution at the 5% level (see the F-tests in the lower panel). Note that since our reported p-values are upper bounds on the actual p-values (see Appendix), it is clear that our standard errors are also upper bounds on the 'true' (and unobserved) standard errors (in general the smaller the standard errors, the smaller are the p-

[11]This data on academic scores were recorded right at the end of our experiment under the conditions of strict anonymity. The arrangement was that our subjects were identified throughout by their experiment reference identities (also on the first page of the test on the appendix). These IDs were then handed over to our local institute contacts who provided us the academic scores against the IDs on our dataset without revealing any names. This data was recorded right at the end of our experiment and we do not retain any copies of our subjects' workings on our test (as per the requirement that all information that could identify our subjects be destroyed). We are not even aware if the percentages/GPAs we received were cumulative or just the scores for the last semester/trimester our subjects had been a part of - it just remained consistent for all the agents who came from the same institution.

values). Before we propose a fairness rule that unifies all the rules discussed here, it will help to look at a behavioral explanation behind the observations we make.

3.5.3 Why some focal points are naturally more apt than the others - A behavioral explanation

Before we formally present our explanation, it will help to consider two thought experiments. In the first, *ceteris paribus,* suppose we had a treatment with a pie-size equaling INR153 - so that this treatment was called T153. The *UG,* the *ES* and the *PRO* rules will respectively suggest $(x, y) = (53, 100)$, $(x, y) = (51.5, 101.5)$, and $(x, y) = (51, 102)$. In all these cases, the actual ratio y/x is remarkably close to $2/1$. Thus, the ratio between our disagreement payoffs serves as a natural point of reference for bargaining making it a strong focal point. In general, whenever the joint surplus is not significantly different from the sum of our agents' individual earning capacities, we expect the ratio of our disagreement points to be a strong predictor of our negotiated outcomes. This is what we observe in T180, where the size of the surplus is only INR30 more than the sum of our disagreement payoffs.

In the second thought experiment, we look at another extreme example. Suppose agents *X* and *Y* could earn \$1 and \$2 by themselves. Suppose that they could earn a joint sum of \$1 billion *only if* they worked together. How likely will they be to split the joint pie in the ratio 2:1 (the ratio of their disagreement payoffs)? We emphasize that the size of the joint pie is so enormously far from the (sum of) individual payoffs that *nothing* about the individual payoffs (let alone the ratio between them) any longer matters. In this case, therefore the joint pie is more likely to be split equally. This is indeed what we observe in T600 and T900.

The above two thought experiments motivate us to propose the following fairness rule in which the agents weigh the size of they are jointly earning against their own individual payoffs. If we define 'surplus from cooperation' s to be the difference between a given pie-size and the sum of individual earning capacities, i.e. $s = z - [d(x) + d(y)]$, then our allocation rule is given as follows:

$$\frac{y}{x} = \left[\frac{s}{a} + d(y)\right] / \left[\frac{s}{a} + d(x)\right] \tag{3.3}$$

where a is nonnegative constant. The intuition based on the two thought experiments is simple - if the gains from cooperation (s) are low in relation to the disagreement payoffs, then the subjects will still tend to use the *PRO* rule and share in proportion to their disagreement payoffs. However, if the gains from cooperation are so high that the effect of individual capacities blur away, then there will be a convergence toward equality. More formally, the *PRO* rule is attained such that

$$y/x = \lim_{s \to 0}[(\frac{s}{a}) + d(y)]/[(\frac{s}{a}) + d(x)] = d(y)/d(x)$$

and the *UG* rule is attained such that

$$y/x = \lim_{s \to \infty}[(\frac{s}{a}) + d(y)]/[(\frac{s}{a}) + d(x)] = 1.$$

Our fairness rule above unifies all the fairness solutions known in the literature. In fact, different values of a specifically characterize different fairness solutions. For example $a = 0$ gives us *exactly* the *UG* fairness rule; $a = 2$ gives us *exactly* the *ES* fairness rule; $a = \infty$ gives us *exactly* the *PRO* fairness rule; $a = 1$ gives us a modified version of the Kalai-Smorodinsky solution where the final outcome is observed along the line joining the disagreement point to the *ideal point* of each agent if the other became absent immediately after the joint surplus was created (in Figure 1, this *ideal point* will be *(300, 300)* instead of *(200, 250)* shown).

Table 5. Observed data against fairness rules and our allocation rule

Treatment (1)	N (2)	Mean (3)	95% CI (4)	PRO (5)	ES (6)	UG (7)	Our allocation rule
T180	58	0.640	0.629 – 0.650	0.667	0.638	0.556	0.641
T300	55	0.600	0.582 – 0.617	0.667	0.583	0.500	0.590
T600	54	0.547	0.530 – 0.563	0.667	0.542	0.500	0.546
T900	59	0.524	0.514 – 0.534	0.667	0.528	0.500	0.530

Our experimental data shows that the observed share of the high-raked individual gets closer to 50% as the pie size z (and therefore s) increases. This convergence to equality is different from what the *ES* rule suggests. For final robustness, we estimate the value of a from our data and examine if the 95% confidence interval around our point estimate $\hat{a}$ excludes the values that correspond to the *UG*, *ES* and the *PRO* fairness rules (as per what we should expect from the F-tests corresponding to the regressions in panels 2 and 3 in Table 4). The point estimate $\hat{a} = 2.28$, with a 95% confidence interval of 2.15 to 2.42 excludes the values assumed by the *ES* (a = 2), the *UG* (a = 0) and the *PRO* ($a = \infty$) rules. In Table 5, we display the estimated shares according to our allocation rule in comparison to the actual observations and the predicted values under the *ES, UG* and the *PRO* rules. Thus, we have proposed an allocation rule that not only unifies the individual fairness rules discussed, but also refines the predictive capacity of fairness rules.

3.6 Conclusion

We have addressed one of the most frequently arising issues in the understanding of bargaining outcomes in relation to the uniqueness of fairness ideals. In the experiment, it was observed that the amount of stakes involved may in itself provide useful hints on what is perceived as fair by the agents, as the disagreement payoffs (and hence their sum) are dwarfed in relation to the pie-sizes, our agents move away from choosing *PRO* toward *UG* to determine the final outcome(s). We also proposed a solution concept that unifies all the three fairness rules and individually displays better predictive capacity in comparison to all the fairness rules discussed. Our results are primarily driven by focal points which we identify using the following two criteria

- Its precise and easy number (a value like 112.5 is unlikely to be a focal point)
- It reduces the time taken to bargain

The most famous example of the *Equal Surplus* allocation rule is the Nash bargaining (Nash, 1950) solution, which maximizes $(x - d(x))(y - d(y))$ with respect to x and y subject to the constraint: $x + y = z$, and leads to a first order condition: $x - d(x) = y - d(y)$, which is, in fact, the very requirement of the *ES* protocol. These solutions can also be perceived to be fair since there is a sense of equality in gains.

Appendices to Chapter 3

Appendix 3A

Test

Instructions: You have 15 minutes to complete this test. There are 20 questions: Each question (marked 1, 2, 3, etc.) is immediately followed by four options (marked a, b, c, and d). Only one of the options correctly answers the associated question. Your task is to mark a tick on what you believe to be the correct answer and maximize your score. Each correct entry carries one point. There is no negative marking. You may begin. All the best.

Name:

Gender (M/F):

Course:

Please leave the following spaces blank.

Time:

Score:

Experimental Reference ID:

1. A truel is similar to a duel, except that there are three participants rather than two. One morning Mr. Black, Mr. Grey, and Mr. White decide to resolve a conflict by truelling with pistols until only one of them survives. Mr. Black is the worst shot, hitting his target on average only one time in three. Mr. Grey is a better shot hitting his target two times out of three. Mr. White is the best shot hitting his target every time. To make the truel fairer, Mr. Black is allowed to shoot first, followed by Mr. Grey (if he is still alive), followed by Mr. White (if he is still alive) and round again (and again) until only one of them survives. Where should Mr. Black aim his first shot?

(a) He should aim at Mr. White

(b) He should aim at Mr. Grey

(c) He should shoot himself

(d) He should shoot in the air

2. Two urns contain the same total number of balls - each ball is either black or white, and these urns have different compositions of black and white balls. There is at least one ball of each color in each urn. From each urn, n (≥ 3) balls are drawn with replacement. We are interested in the number of drawings and the composition of black and white balls in the two urns, such that the probability that all the balls drawn from the first urn are white, is equal to, the probability that either all balls drawn from the second urn are white or all are black. Which of the following statements is true?

(a) This will never be possible

(b) Number of white balls in the first urn must be greater than the number of both white and black balls in the second urn

(c) Number of black balls in the second urn ≥ number of white balls in the first urn ≥ number of white balls in the second urn

(d) Number of white balls in the second urn ≥ number of white balls in the first urn ≥ number of black balls in the second urn

Answer the next two questions (3 and 4) based on the information in the following question:

3. If $N_K = \{1, \dots, K\}$, then how many sets $X = \{x_i \in N_{K^*} | i \in N_{K_*}\}$ solve the following problem when $K^* > K_* > 2$?

$$\text{maximize:} \quad [max(X) - min(X)] - [max(X\backslash\{max(X)\}) - min(X\backslash\{min(X)\})]$$

(a) There exists only one unique set solving the above problem

(b) $K^* - K_*$ sets

(c) There are exactly two sets that solve the above problem

(d) $K^* - K_* + 1$ sets

4. The maximum value in the above problem is

(a) $K^* - K_* - 1$

(b) $K^* - K_*$

(c) $K^* - K_* + 1$

(d) $K^* - K_* + 2$

Answer the next three questions (5 to 7) based on the information in the following question:

5. Let the function $f: (1, \infty) \mapsto (0, \infty)$ satisfy the property $f(xy) = f(x) + f(y)$; $\forall x, y \in (1, \infty)$, we look at the set of equations below

$$f(y) = f(2) + f(x)$$

$$yf(x) = xf(y)$$

the pair (x, y) that solves the above set of equations is

(a) not unique, there are infinitely many such pairs

(b) the information is insufficient to even determine if (x, y) is unique or not

(c) $x = 2, y = 4$

(d) unique, but there is insufficient information to arrive at the actual values of x and y

6. Now, alter the domain of the function f to $[1, \infty)$, and its range to $[0, \infty)$. Define a function

$$g: (-\infty, \infty) \mapsto (0, \infty)$$

which satisfies the property $g(x + y) = g(x)g(y)$. The value of $f(g(0)) + g(f(1))$ always equals

(a) 0

(b) 0.5

(c) 1

(d) Cannot be determined

7. Alter again the domain of f above to $\mathbb{R} - \{0\}$ and its range to $(-\infty, \infty)$. Consider the following statements.

Statement 1: $f((1/x)) = -f(-x)$

Statement 2: $f(-1) = 0$

Mark the correct option.

(a) Only Statement 1 is true

(b) Only Statement 2 is true

(c) Both the statements are true

(d) Neither of them is true

Answer the following two questions (8 and 9) based on the following information. Jack is captured by a tribe. Whether or not he gets to live is decided by the tribe members based on the outcome of the following exercise. There are 50 black and 50 white balls, which Jack must distribute between two identical and opaque boxes (that the tribe provides to him) in any way he wishes, but with the requirement that each ball must be put into one of the two boxes. The tribe then secretly allocates the balls among the two boxes as instructed by Jack and closes them before putting them in front of him. Jack gets to randomly pick a box before they blindfold him and make him draw a ball from it. If the ball is white, he survives, otherwise they execute him.

Answer the following two questions.

8. Jack's maximum probability of survival is

(a) $1/2$

(b) $74/99$

(c) $3/4$

(d) $71/100$

9. If Jack were offered five boxes instead of just two above, then his maximum probability of survival will

(a) definitely increase

(b) definitely decrease

(c) remain the same

(d) well ... cannot say

10. Which of the following events is more likely than the others?

(a) Getting at least 1 six when 6 dice are rolled

(b) Getting at least 2 sixes when 12 dice are rolled

(c) Getting at least 3 sixes when 18 dice are rolled

(d) All the three events above are equally likely

11. A professor chooses two consecutive numbers from the following set $\{1, 2, 3, \ldots, 10\}$. A is told the first number and B, the other. The following conversation takes place:

A: I do not know your number.

B: Neither do I know your number.

A: Now I know.

In how many ways can the professor choose the numbers so that this exact conversation between A and B is possible?

(a) 1

(b) 5

(c) 2

(d) 4

12. The number of linear functions $f: \mathbb{R} \mapsto \mathbb{R}$ that satisfy the property $f(x + f(x)) = x$ is

(a) 1

(b) 2

(c) 3

(d) 4

13. You want to find someone whose birthday matches yours. What is the least (expected) number of strangers whose birthdays you need to ask to have a (greater than) 50% chance of finding a match? (Assume a year of 365 days.)

(a) *23*

(b) *183*

(c) *253*

(d) *364*

14. Alice was first to arrive at a theatre with 98 seats. She forgot her seat number and picks a random seat for herself. After this, every single person who get to the theatre sits on his seat if its available else chooses any available seat at random. Charles is last to enter the theatre and 97 seats were occupied. With what probability does he get to sit in his own seat?

(a) *1*

(b) *1/2*

(c) *1/3*

(d) *1/4*

15. Shuffle an ordinary deck of 52 playing cards containing four aces. Then turn up cards from the top until the first ace appears. On the average, how many cards are required to produce the first ace?

(a) *10.6*

(b) *9.6*

(c) *13.0*

(d) *13.4*

16. If a stick is broken in two at random, what is the average (expected) ratio of the length of the smaller piece to the larger?

(a) *0.333*

(b) *0.386*

(c) *0.301*

(d) *0.441*

17. A player tosses a coin from a distance of about five feet onto the surface of a table ruled in one-inch squares. If the coin (3/4 inches in diameter) falls entirely inside a square, the player wins a holiday package; otherwise he loses. If the penny lands on the table, what is his probability of winning?

(a) *1/2*

(b) *1/4*

(c) *1/8*

(d) *1/16*

18. To encourage Bob's promising tennis career, his father offers him a prize if he wins (at least) two tennis sets in a row in a three-set series to be played with his father and a club champion alternately: father-champion-father or champion-father-champion according to Bob's choice. The champion is a better player than Bob's father. Which series should Bob choose (assume that the outcome of each game in a given series in independent of another)?

(a) father-champion-father

(b) champion-father-champion

(c) He will be indifferent between the two

(d) There is no definite answer

19. For any function f with $f' > 0$, and $f'' < 0$, the maximum value of $f(x)f(1-x)$ is attained at

(a) the maximum of $f[x(1-x)]$

(b) $x = 1/2$

(c) both (a) and (b) above

(d) Cannot be determined

20. A three-man jury has two members, each of whom independently has a probability p of making the correct decision and a third juror who flips a coin for each decision (majority rules). A one man jury has the probability p of making the correct decision. Which jury has the better probability of making the correct decision?

(a) Both of them are equally good

(b) The three-man jury is better than the one-man jury

(c) The one-man jury is better than the three-man jury

(d) There is no conclusive answer

Appendix 3B

Working of sample sizes for our treatment groups

Let the *i*th pair of shares be (x_i, y_i), where $x_i + y_i = 1$. Thus, we are expressing *x* and *y* as a fraction of the pie size in this entire section. For any pair *i*, the share *y* of the high-ranked individual *Y* is sufficient to uniquely characterize the shares of both the agents.[12] Therefore we define the negotiated outcome of any bargaining pair as $z_i = y_i$, and then let $\bar{Z} = \frac{Z_1 + ... + Z_n}{n}$ (where *n* is the number of observed pairs). Thus, $\bar{Z}$ measures the average share of the high-ranked individuals.

In what follows, we will demonstrate the process of calculation explicitly for *T300* and just state the desirable sample-size values for our other treatments. Our aim is to answer how large our sample sizes should be to tell our fairness solution concepts apart. In each stage we will suppose that the population mean our random variable $\bar{Z}$ is μ.

We first consider the test of the null hypothesis that $\mu = \mu_0 = 1/2$ (i.e. the *UG* fairness solution is indeed what the population of pairs choose on average). The question is: what would be the minimum sample that is required for such a test to have reasonable power against an alternative hypothesis that the population mean is $\mu = \mu_1 = 7/12 > \mu_0 = 6/12$ (i.e. the *ES* fairness solution is indeed what the population of pairs choose on average)?[13] We do not make any assumption(s) on the distribution of Z_i (and therefore $\bar{Z}$) under the null or the alternate hypothesis.

Let α be the (maximum permissible) size of the type-I error. Let *c* be a non-negative constant such that $P(\bar{Z} - \mu_0 > c \mid \mu = \mu_0) \leq \alpha$. In other words, the null is rejected whenever $\bar{Z} > \mu_0 + c$.

---

[12] Clearly, for example, from $y = 0.6$, we immediately know that $x = 0.4$ since, $x + y = 1$.

[13] Note that for a pie-size equalling *300*, the *ES* fairness rule suggests that *X* gets 125 and *Y* gets *175*. In proportions this translates to $x = 5/12$ and $y = 7/12$, which is the chosen value for μ_1 here.

To determine c as a function of α and n, we note the following inequalities (the first one of which is $P(\bar{Z} \leq \mu_0 + c) \geq P(\mu_0 - c \leq \bar{Z} \leq \mu_0 + c) \geq P(\mu_0 - c < \bar{Z} < \mu_0 + c)$).

$P(\bar{Z} \leq \mu_0 + c) \geq P(\mu_0 - c < \bar{Z} < \mu_0 + c)$; {∵ LHS spans more values}

$P(\mu_0 - c < \bar{Z} < \mu_0 + c) = P(|\bar{Z} - \mu_0| < c) \geq 1 - \frac{\sigma_Z^2}{nc^2}$; {∵ Chebyshev's inequality}

We combine the two inequalities above as follows

$$P(\bar{Z} \leq \mu_0 + c) \geq 1 - \frac{\sigma_Z^2}{nc^2}$$

$$\Rightarrow \quad P(\bar{Z} - \mu_0 > c | \mu = \mu_0) \leq \frac{\sigma_Z^2}{nc^2}$$

$$\Rightarrow \quad \text{P(Type I error)} \leq \frac{\sigma_Z^2}{nc^2} = \alpha$$

$$\Rightarrow \quad c = \frac{\sigma_Z}{\sqrt{\alpha n}} \tag{3B.1}$$

Thus, the probability of a Type-I error does not exceed α when $c = \frac{\sigma_Z}{\sqrt{\alpha n}}$. Now we turn to Type II error (which should not exceed β).

$$P(\text{Type II error}) = P(\bar{Z} < \mu_0 + c | \mu = \mu_1)$$

Now $\mu_0 = 0$, and we substitute for c from (3B.1), we get

$$P(\text{Type II error}) = P(\bar{Z} < \mu_0 + \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1)$$

Note that for any k, we know from Chebyshev's inequality that

$$P(\mu_1 - k < \bar{Z} < \mu_1 + k | \mu = \mu_1) \geq 1 - \frac{\sigma_Z^2}{nk^2}$$

We now take $k = \mu_1 - \mu_0 - c = \mu_1 - \mu_0 - \frac{\sigma_Z}{\sqrt{\alpha n}}$ in the above inequality (with $\mu_0 = 0$) to get

$$P(\frac{\sigma_Z}{\sqrt{\alpha n}} < \bar{Z} < 2\mu_1 - \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq 1 - \frac{\sigma_Z^2}{nk^2} \tag{3B.2}$$

But

$$P(\bar{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq P(\frac{\sigma_Z}{\sqrt{\alpha n}} < \bar{Z} < 2\mu_1 - \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1); \text{ {∵ LHS spans more values}} \tag{3B.3}$$

The LHS above spans more values since:

$P(\overline{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq P(\overline{Z} > \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq P(\frac{\sigma_Z}{\sqrt{\alpha n}} < \overline{Z} < 2\mu_1 - \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1)$

On combining the inequalities (3B.2) and (3B.3), we get

$$P(\overline{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq 1 - \frac{\sigma_Z^2}{nk^2}$$

$$\Rightarrow \quad 1 - P(\overline{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \leq 1 - (1 - \frac{\sigma_Z^2}{nk^2}) = \frac{\sigma_Z^2}{nk^2}$$

$$\Rightarrow \quad P(\overline{Z} < \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \leq \frac{\sigma_Z^2}{nk^2}$$

$$\Rightarrow \quad P(\text{Type II error}) \leq \frac{\sigma_Z^2}{nk^2} = \beta \tag{3B.4}$$

Thus, the probability of a Type II error does not exceed β when $\frac{\sigma_Z^2}{nk^2} = \beta$. Substituting for $k = \mu_1 - \mu_0 - c = \mu_1 - \mu_0 - \frac{\sigma_Z}{\sqrt{\alpha n}}$, we get

$$\beta = \frac{\sigma_Z^2}{n(\mu_1 - \mu_0 - \frac{\sigma_Z}{\sqrt{\alpha n}})^2} \tag{3B.5}$$

$$\Rightarrow \quad n = \frac{\sigma_Z^2}{(\mu_1 - \mu_0)^2}\left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}\right)^2 \tag{3B.6}$$

In this expression, we fix the probabilities of α and β, (the upper bounds on our Type I and II errors) to be 0.05 and 0.20 respectively. With $\mu_0 = 6/12$ and $\mu_1 = 7/12$, the only limitation is that we do not know the value of σ_Z.[14] To estimate σ_Z, we use a pilot study in which, $\hat{\sigma}_Z = \hat{\sigma}_y = 0.06$.[15] Using this value gives us $n^* = 23.33 \approx 24$ pairs (48 subjects). Note that c equals 0.05 for this value of n. In other words, if the mean is indeed $\mu_0 = 1/2$ (as per the *UG* rule which requires that $x = y = 50\%$), then with 24 negotiating pairs, the probability that we will observe an average $\bar{z} > \mu_0 + c = 0.55$ (i.e. the average agent *X* gets at most 45%, and the

[14]According to Thompson (2012), an "aspect of sample size formulas such as these is that they depend on the population variance, which generally is unknown. In practice, one may be able to estimate the population variance using a sample variance from past data from the same or a similar population."

[15]Note that Var[z] = Var[y]. Thus, $\sigma_Z = \sigma_y$.

average agent Y gets at least 55%) will be *at most* 5%.[16] This sums up how we work out the sample size when we take our null hypothesis to be H_{UG}, and our alternate hypothesis to be H_{ES}.

Similarly, in T300, if we took our null hypothesis to be H_{ES}, and our alternate hypothesis to be H_{PRO}, then our values for μ_0 and μ_1 will respectively be *7/12* (as calculated above), and *2/3* (= *8/12*).[17] For our choice of values of α (= 0.05), β (= 0.20) and, $\hat{\sigma}_Z$ (= 0.06), we again get n^* = $23.33 \approx 24$ pairs (48 subjects). In general, these two sample sizes need not be equal for any given treatment, in which case, we choose the larger of the two values of n^* as our chosen sample size for that treatment. Table 3B.1. below, summarizes the above calculations for all our treatments. For each treatment (column), for the ease of comparison, we have represented the hypothesized mean values with a common denominator. For example, under H_{PRO}, the hypothesized value of the average share of the high-ranked agent Y, is 2/3 - this is written as 24/36 for T180, 8/12 for T300, 16/24 for T600, and 24/36 for T900 (our μ_1 entries for H_{ES} vs H_{PRO}). We do the same for H_{UG} (representing 1/2 as 6/12, 12/24 and 24/36 for T300, T600 and T900), with the exception of T180, where the high-ranked agent Y should earn at least his/her disagreement payoff of 100.

The intuition behind why the sample-sizes vary for different pairs of competing hypotheses across different treatments is noteworthy. Our desired sample-size expression is inversely related to the number of standard deviations that can be fitted between the means assumed under the two hypotheses (also known as Cohen's d - see Cohen, 1977). The gap between the population mean values assumed under H_{UG} and H_{ES} ($\frac{23}{36} - \frac{20}{36} = \frac{3}{36}$) in T180 is wider than the

[16]In fact, the probability of our type-I error will be significantly less than 5% (note that Chebyshev's bounds are never very tight). The exact same argument holds for our type-II error as well (our power is way higher than 80%)

[17]Note that for a pie-size equalling *300*, the *PRO* fairness rule suggests that X gets 100 and Y gets *200*. In proportions this translates to $x = 1/3$ and $y = 2/3$, which is the chosen value for μ_1 here.

gap between the population means assumed under H_{UG} and H_{ES} ($\frac{24}{36} - \frac{23}{36} = \frac{1}{36}$) in the same treatment. Thus, clearly more data is required in the latter to tell the competing hypotheses apart. The logic is that the standard error of the sample-mean is inversely related to the (square-root of) the sample-size. This argument extends to the varying calculated sample-sizes for all the other treatments.

Table 3B.1. Sample size determination for $\alpha = 0.05$, and $\beta = 0.20$

	T180	**T300**	**T600**	**T900**
μ_0 (H_{UG} vs H_{ES})	20/36	6/12	12/24	18/36
μ_1 (H_{UG} vs H_{ES})	23/36	7/12	13/24	19/36
μ_0 (H_{ES} vs H_{PRO})	23/36	7/12	13/24	19/36
μ_1 (H_{ES} vs H_{PRO})	24/36	8/12	16/24	24/36
Pilot $\hat{\sigma}_Z$	0.03	0.06	0.04	0.03
n^* (H_{UG} vs H_{ES})	6	24	42	53
n^* (H_{ES} vs H_{PRO})	53	24	5	3
Chosen n^* (greater of the above two)	53	24	42	53

Appendix 3C

Experimental instructions

GENERAL INSTRUCTIONS

Hello and welcome to this experiment. You will receive a sum total of INR 200 as a show-up fee. This is the minimum amount you will get (provided you stick to the rules of this experiment). In today's session you have to bargain over a certain amount of money with someone you will be paired with. Any amount you earn here will be additional earnings. For purposes of confidentiality you will be identified only by your identity (ID) numbers which will be provided to you.

You will be given a form that requests your consent for participating in the experiment. You will have to sign it and return it to us. Please read it carefully. (Consent forms are distributed). The amount that is due to you will be filled in after the experiment in the cash receipt, when we can determine your total earnings (show-up fee plus amount earned in today's exercise). You are free to quit the experiment at any point of time. Please raise your hands if you have any questions, otherwise we are ready to move on to the next part of the experiment.

You will now be divided into different groups. Please come one by one, with your consent form, to the computer screen and press 'enter'; and give your names. (We run the command one by one, on R for each student to hit enter and record their names in the reference sheet T180, T300, T600 or T900 depending on the output.)

Stay in this room (if the output is 1, signifying T180).

Go to Room 2 (if the output is 2, signifying T300; the research assistants guide them to the room).

Go to Room 3 (if the output is 3, signifying T600; the research assistants guide them to the room).

Go to Room 4 (if the output is 4, signifying T900; the research assistants guide them to the room).

INSTRUCTIONS TO THE INR180 TREATMENT GROUP (T180)

Instructions in the waiting room **before** the test

1. Please read the instructions carefully and fill in your details.
2. Your aim is to score as high as possible, as this can put you on a better bargaining position relative to the individual you are paired with.
3. There are twenty questions, each of which is worth one point. You will be marked according to the number of points you score. If two (or more) individuals score the same number of points, then they will be ranked according to whoever hands over their test booklet first (second and so on).
4. Do you have any questions? Please raise your hands.
5. You may begin now.

(Test begins.)

(Test is over and answer scripts are collected.)

Instructions in the waiting room **after** the test

1. Your tests will now be evaluated.

2. We will now require you to fill in some of your details in the sheet provided. Please read and fill carefully

(Subject-detail sheet are distributed.)

(The sheet are collected.)

Instructions in the waiting room **after test evaluation**

1. Your tests have now been evaluated.

2. Based on your test performances, you have all been ranked.

3. Each candidate in the top half will be randomly paired with a candidate in the bottom half.

4. You will move to the experimental lab.

5. Once just outside the experimental lab, you will be called in one by one by your names and seated on your allotted workstations.

6. On your workstations, you will get to know your Rank which will also be your Skype Username.[18]

7. You will have to **chat** in **English** on **Skype** with the candidate you have been paired with to decide on how to split INR 180 between yourselves.

8. You will have only **ten minutes** to complete this conversation.

9. Should you disagree or not reach an agreement in ten minutes, then the high rank individual in the pair will be given INR 100 (plus, the show-up fee) and the low rank will receive INR 50 (plus, the show-up fee).

10. Do you have any questions? Please raise your hands.

11. More instructions will be given to you once you are in the lab.

[18] For example, if your Skype Username is Rank.002 then you have the second highest score. If your partner's Skype Username is Rank.014 then, it is clear that in your pair you are the high rank agent and your partner is the low rank agent.

Instructions **in the lab**

(Candidates find that the 'Cash Receipts' are already kept on their workstations.

They also see that they have already been logged in to Skype and the chat-windows of the subjects they have been paired with, are also open.)

(The following instructions are given.)

1. **Do not disclose your identities**. Any implicit or explicit attempt to do so will lead to the cancellation of both the show-up fee and the negotiated amount.

2. **Do not misreport your negotiated amounts** in the cash receipts. Any attempt to do so will lead to the **immediate cancellation** of both the show-up fee and the negotiated amount.

3. Please remember that your responses are confidential and the raw data collected from this experiment will not be given to anyone outside this project.

4. Do you have any questions? Please raise your hands.

5. You may begin now.

(Chatting commences and the candidates finalize their negotiations over Skype.)

(Chatting ends.)

Please fill in your details in the cash receipt patiently now.

(At this stage, the candidates already know their own (negotiated) shares/earnings.)

(Once the total amount filled, we make them sign the receipts, and pay them accordingly.)

INSTRUCTIONS TO THE INR300 TREATMENT GROUP (T300)

Same as instructions to T180, except the point 7 in lab instructions:

7. You will have to **chat** in **English** on **Skype** with the candidate you have been paired with to decide on how to split INR 300 between yourselves.

INSTRUCTIONS TO THE INR600 TREATMENT GROUP (T600)

Same as instructions of T180, except the point 7 in lab instructions:

7. You will have to **chat** in **English** on **Skype** with the candidate you have been paired with to decide on how to split INR 600 between yourselves.

INSTRUCTIONS TO THE INR900 TREATMENT GROUP (T900)

Same as instructions to T180, except the point 7 in lab instructions:

7. You will have to **chat** in **English** on **Skype** with the candidate you have been paired with to decide on how to split INR 900 between yourselves.

Chapter 4

Happiness in the finite: Oligopoly maximizes welfare

4.1 Introduction

We re-examine one of the central and important results of welfare economics, that competition maximizes welfare. We argue that the validity of this known result implicitly rests on the assumption that consumers have infinite cognitive capacity. We show that in a real world of consumers with limited cognition, a welfare-maximizing market has a *strictly* oligopolistic structure.

We model decision-makers' cognitive limitations directly from the biology of the human brain (Sapolsky, 2017; Eagleman 2015) and study the welfare implications of cognitively limited agents in markets. This immediately relates to the limited mental bandwidth problem of Mullainathan and Shafir, 2013. To the best of our knowledge, this is a first attempt at the endeavor of uncovering the welfare implications of Mullainathan and Shafir's new '*science in the making*'.

The idea that the availability of greater choices can only improve welfare, is embedded in economic theory.[1] The empirical truth that the Western world is flooded with choices, is however, seen as the *official dogma* of the industrialized world by psychologists (Schwartz, 2004; Iyengar and Lepper, 2000). Specifically, what economists call *variety* is formally termed as *choice overload* by psychologists.

[1] The intuition is that any additional item in the choice basket will be consumed only if it enhances utility, otherwise that additional choice will be redundant. Thus, additional choices can only be welfare improving ... even if the number of choices is infinite.

If we accept that utility is derived not only from final consumption but also from the process of determining what to consume (called respectively as 'outcome satisfaction' and 'process satisfaction' - see Reutskaja and Hogarth, 2009), then the cognitively-taxing process of evaluating too many choices against each other may very well take away from satisfaction. On the aggregate, therefore, welfare will reduce beyond the number of products where the consumers' benefit from available product options and the crunch/pressure/tax on cognitive capacity balance each other out. Thus, we show that a finite number of *products*, and therefore a finite number of *producers*, is consistent with welfare maximization. [2] Additionally, firms benefit from reduced competition - thus, the additions to both consumers' and producers' surplus contribute to welfare maximization.

From economic theory we know that it is sufficient for a consumer to be able to make pair-wise comparisons in order to arrive at the complete preference ordering between a certain number n of commodities/bundles. The number of such pair-wise comparisons increases at a much higher rate than n itself (specifically it is $n(n - 1)/2$). The cognitive costs of comparison therefore, increase according to the number of possible comparisons rather than just on the basis of the number of options. Even binary ($n = 2$) choices that involve only one (= $n(n - 1)/2$) possible comparison (between the two available alternatives) could be mentally taxing in some cases. For example, a diabetic who decides whether or not to consume the cake in front of him, experiences an internal conflict. The amygdala (or the 'reptilian brain' governing individual instincts) gives in to the temptation of immediate (tasty) benefits from consumption, but the pre-fontal cortex (PFC hereafter, also the most recently evolved part of the brain that is broadly responsible for reasoning), interferes with this instinct by making the individual aware of the threat of future health-related costs. This 'tug of war' between the two brains consumes a sufficient amount of mental energy, leaving the agent mentally exhausted

[2] The exact finite number will, among other things, depend on the nature and the strength of the cognitive costs of comparison.

at the end of this decision-making process (Sapolsky, 2017). Even if the amygdala wins, the still-active PFC takes away from the utility from immediate consumption.

For a sufficiently large number of available choices, Schwartz (2004) provides two (additional) psychological channels through which, decision-making becomes costly (in utility terms). First, there is an automatic upward revision of expectations from the very availability of too many alternatives. More specifically, it is easy to imagine that a better choice was possible whenever one is made. The psychological comparison (between the choice made and the imagined alternative) that follows, reduces the utility from the choice made. Second, if a bad choice is made, then the blame is on the 'world' when the alternatives to choose from are very few (in the extreme case with $n = 1$, the decision-maker can reason to himself that he could do nothing about what was available), but there is an element of *self-blame* when there are too many alternatives to choose from. Further, if the consumable in question is durable, then the memory of this *self-blame* is triggered (in the hippocampus - the region responsible for memory storage) every time the good is used, which further reduces utility. Therefore, in addition to the biological cognitive costs of making a choice between too many alternatives, there are psychological costs that take away from utility - a world with too many choices is ultimately unhappy.

The comparison of too many alternatives directly translates to the processing of too much information. The human brain has evolved to avoid the time-costs of too much processing and focuses on only a subset of information that is deemed most important at any given point of time (Sapolsky, 2017). This is the reason why one may forget his wallet when in a hurry to make it to an important meeting. The mind creates a *tunnel* of focus on the key objective at hand (in this case, to reach the meeting on time) and all the mental bandwidth is used up to think about booking a cab, organizing the meeting briefs, and so on. Therefore, even basic (and often important) things that remain out of the tunnel (in this case, remembering to carry wallet) are immediately ignored by the brain — the sense of urgency created in the limited time leads an agent to subliminally economize on his brain capacity (Mullainathan and Shafir, 2013). Since cognition is a scarce resource, in order to facilitate quicker decision making when desirable, any agent's actions are frequently dependent on the information that his senses can immediately access (Sapolsky, 2017 and Eagleman, 2015). This information frequently translates to focal or reference points, thereby explaining why the *decoy effect* works in marketing (Ariely, 2008), or why economic agents are frequently *present-biased* (see Camerer, 2003; Bardsley et al 2009; Frey and Stutzer, 2007), or why they rely on expert opinion (Smith, 2007; Beattie et al, 1994), or even why they can be *nudged* to achieve greater levels of welfare (Thaler and Sunstein, 2008) with an external influence on their choice architecture. In a nutshell, limited cognition against too many alternatives has well-established empirical consequences, and is therefore worthy of a theoretical examination from a welfare perspective.

Regulatory authorities frequently try to increase welfare by promoting competition to counter the market power possessed by monopolies. This gives any consumer, a sense of freedom of choice accompanied with autonomy, thereby making the experience of shopping in the presence of several available options very pleasant. So long as the individual evaluation of each option is costless, more choices can only increase welfare (if not leave the same unaltered). Therefore, any regulation that promotes diversity in choices is expected to be welfare enhancing. We argue that this traditional view of welfare only holds when consumers have infinite cognitive capacity to arrive at the best decisions, after meticulously evaluating all the available alternatives.

Section 2 presents our multi-agent model and establishes the presently known welfare results. In section 3, we incorporate the cognitive costs of decision making in our framework, and establish new welfare results, and we conclude with section 4.

4.2 The model

To begin with, we assume that there are k identical utility-maximizing (and perfectly rational) consumers, and n identical profit maximizing firms who engage in Cournot competition. Our choice of Cournot competition is simply based on the flexibility to look at the extreme cases of monopoly ($n = 1$) and competitive ($n = \infty$) scenarios and show the established welfare implications of each. This also requires us to assume perfect product homogeneity so that our results are immediately comparable (and consistent) with the known welfare implications of perfect competition.

4.2.1 The consumers' problem

We are interested in the market for a given commodity, the quantity (consumed) of which is represented by x. Each identical consumer must decide how to allocate his/her given income M, between x units of this commodity and y units of all other goods. The price for the latter is normalized to unity. We assume that utility U has a quasi-linear specification and is specifically additive in the functional components of x and y. The representative consumer faces the following (simple) utility maximization problem:

$$\begin{aligned} &Maximize: \quad U = ax - bx^2 + y \\ &Subject\ to: \qquad px + y = M \end{aligned} \tag{4.1}$$

with the latter being the budget constraint: $px + y = M$, where p is the price of the commodity of interest, and therefore, px is the total expenditure on the commodity. We look at a quadratic specification in x (with $a > 0$ and $b > 0$) particularly for two reasons. First, quadratic expressions can uniquely approximate any other well behaved utility specification remarkably well.[3] Second, and more importantly, we naturally arrive at a *linear* demand curve to keep things tractable when we progressively introduce complications in sections 3 (consumers with limited cognition).[4]

The solution to (1) involves the substitution of y from the budget constraint into the objective function and equating the derivative (of U w.r.t. x) to zero.[5] This gives us the following (individual) demand curve for our representative consumer.

$$x^* = \frac{a - p}{2b}$$

Finally, the aggregate market demand for k consumers $(X = kx^*)$ is attained from the horizontal summation of k identical individual demand curves. Thus, the market demand X is given by:

$$X = kx^* = k\left(\frac{a - p}{2b}\right) = A - Bp$$

with $A = (ak/2b)$, and $B = (k/2b)$.

We now invert the above and write the inverse demand curve as

$$p = \frac{A - X}{B} = \alpha - \beta X \tag{4.2}$$

with $\alpha = (A/B)$, and $\beta = (1/B)$. This is the demand curve faced by our n identical profit-maximizing firms. We now turn to the supply-side of our story.

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[3] For example, the function $f(x) = ln(ax + b)$ can be approximated remarkably well by the quadratic expression $g(x) = (-a^2/8b^2)x^2 + (3a/4b)x + [ln(2) + ln(b) - (5/8)]$ in the vicinity of $x = b/a$. Additionally see Small (2007) and Ball (2003) for beautiful discussions on quadratic functions and approximations.

[4] The discipline of industrial organization has, since long, acknowledged the merits of quadratic utility specifications in the tractability of subsequent market and welfare analyses (Häckner, 2000; Singh and Vives, 1984; and Dixit, 1979).

[5] Note that $U = ax - bx^2 + (M - px)$, after substitution. Maximization is guaranteed from the global concavity of U.

4.2.2 The producers' problem

Just like X represents the total market demand, we define the total market supply as $Q = \sum_{i=1}^{n} q_i$, where q_i is the quantity produced and supplied by *Firm i* ($i \epsilon \{1, ... , n\}$). Using the fact that market supply Q will exactly equal market demand X in the equilibrium, so that the market clearing condition ($X = Q$) can be incorporated in the inverse demand function (2) as

$$p = \alpha - \beta Q = \alpha - \beta \sum_{i=1}^{n} q_i \tag{4.3}$$

The profit function of a typical firm *i* is given by $\pi_i = (p - c)q_i$. Assuming zero costs ($c = 0$) in the interest of simplicity, the problem of *Firm i* is to maximize its own profit shown below.[6]

$$Maximise\colon\ \pi_i = pq_i = (\alpha - \beta Q)q_i \tag{4.4}$$

This leads us to the first order condition $\alpha - \beta Q - \beta q_i = 0$. Finally, we use symmetry (our firms are identical) and use $q_i = (Q/n)$ to get

$$Q^* = \frac{n}{(n+1)}\frac{\alpha}{\beta}\ ;\ q_i^* = \frac{1}{(n+1)}\frac{\alpha}{\beta}\ ; and\ p^* = \frac{\alpha}{(n+1)} \tag{4.5}$$

Which is the standard *n-firm* Cournot outcome. We now come to a discussion of total utility received by *k* consumers and *n* firms in the following subsection.

4.2.3 Welfare implications

Using (4.5), we can immediately work out individual profits ($\pi^* = p^*q^*$) and the total producers' surplus ($n\pi^* = np^*q^*$) as follows:

[6] Even with zero costs, we will be dealing with exceedingly complex expressions in Sections 3 and 4. The results presented here easily extend to general (upward sloping or constant) cost structures.

$$\pi^* = \frac{1}{(n+1)^2}\frac{\alpha^2}{\beta}\ ;\ n\pi^* = \frac{n}{(n+1)^2}\frac{\alpha^2}{\beta}$$

Finally, replacing α and β above by *(A/B)* and *(1/B)* respectively, and replacing *A* and *B* in turn, by *(ak/2b)*, and *(k/2b)* respectively (i.e. working back the transformations introduced in subsection 4.2.1), we re-write the (market and individual) quantities, the price, and the total producers' surplus above as:

$$Q^* = \frac{akn}{2b(n+1)}\ ;\ q_i^* = \frac{ak}{2b(n+1)}\ ;\ p^* = \frac{a}{(n+1)}\ ; n\pi^* = \frac{ka^2n}{2b(n+1)^2} \tag{4.6}$$

We now use the market clearing condition to replace Q^* by *X*, (so that $Q^*/k = X/k$), to work out individual consumption as follows:

$$x^* = \frac{an}{2b(n+1)} \tag{4.7}$$

To explain this equilibrium with an example, suppose eighty units of output are produced and traded in a market comprising twenty consumers and five producers so that $k = 20$ and $n = 5$, with $X = Q = 80$, then, each consumer must consume four units and each producer must produce (and sell) sixteen units, so that $x^* = 4$, and $q^* = 16$. The equilibrium level of utility for each consumer is given by

$$U(x^*) = \frac{a^2n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + y$$

Now, we replace $y = (M - px) = \left(M - \frac{a^2n}{2b(n+1)^2}\right)$ in the above expression to get

$$U(x^*) = U^* = \frac{a^2n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + \left(M - \frac{a^2n}{2b(n+1)^2}\right) \tag{4.8}$$

Finally, we sum up the individual utilities for k consumers, and add to that, the total profits of the firms to define the total surplus $S(n) = kU^* + n\pi^*$ (which is our measure of total welfare) as follows:

$$S(n) = \frac{ka^2n}{2b(n+1)} - bk\left(\frac{an}{2b(n+1)}\right)^2 + k\left(M - \frac{a^2n}{2b(n+1)^2}\right) + \frac{ka^2n}{2b(n+1)^2}$$

which in turn, can be further simplified (after accounting for the fact that $kp^*x^* = n\pi^*$, i.e. what consumers pay to producers is the latter's earning, and thus gets cancelled out from the last two terms above) to:

$$S(n) = \frac{ka^2n(n+2)}{4b(n+1)^2} + kM \tag{4.9}$$

It is immediately clear that welfare increases with more firms, since $S(n+1) - S(n)$ is always positive as shown below:

$$S(n+1) - S(n) = \frac{ka^2(2n+3)}{4b(n+1)^2(n+2)^2} > 0\ for\ \{n > 0\} \tag{4.10}$$

Thus, it always benefits to accommodate more and more firms, thereby encouraging the competitive removal of barriers to entry. Welfare in this world, has a maximum possible value of $(ka^2/4b) + kM$ (with an infinitely large number n of firms), and our model structure is consistent with the established welfare results. We now introduce cognitive costs in our model while retaining the current feature of product-homogeneity.

4.3 Markets with consumers of limited cognition

We now introduce bounds on perfect reasoning (i.e. perfect cognition is not costless) in a manner, systematically different from those discussed in Spiegler (2011).[7] More specifically

[7] More specifically, we distinguish Spiegler's (2011) bounded rational agents from our agents of limited cognition. It is the former that implies the latter, and the latter does not systematically rule out rationality.

we model a (cognitive/biological and psychological) 'cost' of deciding what choice(s) to make, the basis for which has already been presented in the introductory section.

It may well be argued that if products are indeed homogeneous, then there is no reason for consumers to 'choose between' what different firms make available in the market. In other words, the consumers can simply buy the product without incurring any psychological/biological costs (for example, without having to worry about comparing what they have bought against what they have not, since the products are homogeneous). Therefore, with homogeneous products, the need for comparison may not arise (thereby associating welfare maximization with the perfectly competitive result as in the previous section). It is here, however, where we stress on a realism that we intend to (at least partially) capture in our model. In the real world, consumers learn about the homogeneity between two or more brands (even if there is any) only after they have already incurred the cognitive cost of comparison. In other words, people need to first compare two things even to realise that they are indeed identical. Thus, the cognitive costs of comparison (and therefore deciding what to buy) are not necessarily lost even when agents deal with (comparing two or more) homogeneous products. If anything, it is much easier to rank two items when one is distinctly (and significantly) better than the other. Things become difficult (i.e. more cognitive resources are required) when one has to choose between two very similar items (e.g. choosing between two very similar jobs). The second reason why we want to incorporate cognitive costs here is that we want our welfare results to be directly comparable with those in the previous section (with homogeneous products) and therefore keep our results consistent with the existing requirements of perfectly competitive markets (including product homogeneity).

We begin with a general cognition-cost function $G(n)$ of evaluating and comparing n brands. For example, if the consumer has a task of (pair-wise) comparing n 'brands' (to completely specify the ordering required by economic theory), the *information size* (in the computing nomenclature) is *n*. In this case, for any given constant μ (> *0*), the function $G(n) = \mu C(n, 2)$, that involves the combinatorial number $C(n, 2) = n(n - 1)/2$, could be a possible choice for a function that captures the cognitive (Mullainathan and Shafir, 2013), biological (Sapolsky, 2017; and Eagleman, 2015) and psychological (Schwartz, 2004) costs of evaluating and comparing *n* objects (the constant μ can be thought of as the *degree or measure of cognitive paralysis* of an agent). This composite *cost* of processing *n* 'information items' is also sufficiently in line with economic theory that in general requires costs to be increasing and convex. To begin with, we will assume this functional form for *G(n)*, and then endogenously determine the critical functional form of cognition-costs that will support our results. We now look at the modified consumers' and producers' problems in the subsection that follows.

4.3.1 Agent behaviour in the market

We borrow from psychology that the cognitive costs involving too many choices, *take away* from the utility from consumption in the following modified specification of the representative consumer's problem.[8]

$$\begin{aligned} &Maximize: \quad U = ax - bx^2 + y - G(n) \\ &Subject\ to: \qquad px + y = M \end{aligned} \tag{4.11}$$

The solution to the above problem gives us the exact demand structure and market outcomes of the previous section specified from equations (4.2) to (4.7). With this additional negative utility component though, the welfare conditions change. We turn to this now.

4.3.2 Analysis of welfare

The equilibrium utility level for our representative consumer is now given as follows

[8] Minsky (1986) gives compelling argument that support the structure of the cognitive cost that we propose.

$$U(x^*) = U^* = \frac{a^2 n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + y - G(n)$$

Just as in the previous section, the total utility of *k* consumers and the total producers' surplus amounts to $S(n) = kU^* + n\pi^*$, and results in the following specification (after some steps, assuming that $G(n) = \mu n(n - 1)/2$).

$$S(n) = \frac{ka^2 n(n+2)}{4b(n+1)^2} + kM - \frac{k\mu n(n-1)}{2} \tag{4.12}$$

The steps to arrive at the above expression are deferred to the Appendix. It is clear that

$$S(1) = \frac{3ka^2}{16b} + kM > 0; and \lim_{n\to\infty} S(n) = -\infty < 0$$

It is therefore, easily seen that the combined costs of cognition for *k* consumers must eventually outweigh the welfare benefits of competition. We conclude that social welfare must necessarily be at a positive maximum at a finite value of *n*, which in turn, will depend on the value of μ for our current choice of cognition-cost specification. We now progressively examine the restrictions on μ for a given number of firms *n*, such that the entry of the next firm (i.e. the $(n + 1)^{\text{th}}$ firm) is necessarily welfare improving. More specifically, we see (after a few complex algebraic steps deferred to the Appendix) that

$$S(n+1) - S(n) = k\left(\frac{a^2(2n+3)}{4b(n+1)^2(n+2)^2} - \mu n\right) \tag{4.13}$$

Thus, $S(n + 1) - S(n) > 0$ means that the cognitive costs should be lower than a critical value. More precisely, μ should not exceed $a^2(2n + 3)/4bn(n + 1)^2(n + 2)^2$. This means that the latter should be a strict lower bound for μ for maximum welfare to be attained with exactly *n* firms. Thus, we have the following result.

Proposition 1: If exactly n firms maximize social welfare, then it must be the case that μ is at least $a^2(2n + 3)/4bn(n + 1)^2(n + 2)^2$.

Proof: Trivial. It follows directly from (4.13). ■

In Figure 1, we plot the socially most desirable number of firms against these lower bounds on μ after normalizing $a^2/b = 100$ and fixing $k = M = 10$.

Figure 1: Socially desirable number of firms against the measure of cognitive paralysis μ

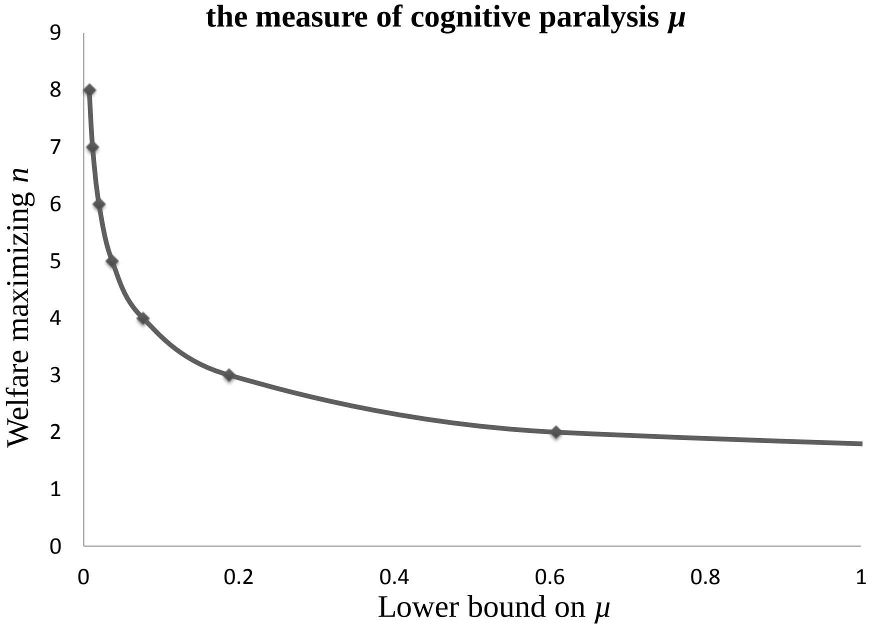


Figure 1 shows that as μ increases beyond each given critical bound, the number of firms that maximize welfare progressively diminishes. Indeed, in Figure 2, we see that welfare is maximized for a *strictly finite* value of n, for carefully chosen values of μ between the critical bounds. The panels (a), (b), (c) and (d) show how $S(n)$ is maximized at $n = 3, 4, 5$ *and* 6 respectively for different values of μ (*0.20*, *0.10*, *0.05* and *0.02* in that order).

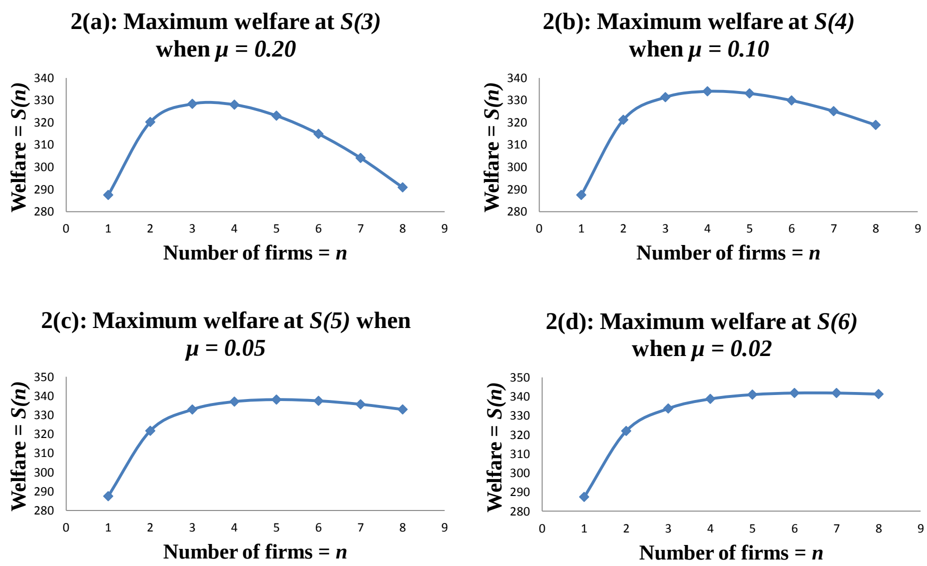
We now come to the final result of this subsection.

Proposition 2: If consumers have infinite cognitive capacity ($\mu = 0$), then welfare is the maximum with an infinite number of firms.

Proof: Trivial. It follows directly from the previous section where μ is indeed zero. ■

The importance of the above proposition becomes clear here where we have discussed cognitive costs of comparison. The fact that utility is strictly increasing in n, gives an insight into the consumers' implicit love for the *flexibility* to choose his optimal consumption amount from *any* of the n available firms. This is because U is strictly dependent on the *amount* (x) of consumption of the commodity in question, and there is no *a priori* reason for U to be strictly increasing in n as well. The latter just happens to be a consequence of equilibrium.

Figure 2: Finite number of firms maximize welfare



Finally, before concluding this subsection we present one final result.

Proposition 3: The consumption component of utility is strictly positive in equilibrium. It is also *strictly* increasing in consumption x.

Proof: We use the above equilibrium condition above and observe that

$$x^* = \frac{an}{2b(n+1)} < \frac{a}{2b} < \frac{a}{b}$$

which gives us $bx^* < a$, or $a - bx^* > 0$, which on multiplying by x^*, further gives us $ax^* - b(x^*)^2 > 0$, thereby completing the proof. The latter part of the theorem follows directly from recognizing that $x^* < a/2b$. ■

4.3.3 Analysis with a general cognition-cost function

Many efficient ways to evaluate, sort and compare between n pieces of information have been proposed formally in computer sciences. For some of the best computing algorithms, the complexity of processing/sorting and evaluating n inputs of information, increases proportional to $n \cdot ln(n)$, and not necessarily $C(n, 2) = n(n - 1)/2$. It is clear that since the welfare benefits from increases in n are bounded above (i.e. $S(n)$ in equation (4.9) is bounded above by $(ka^2/4b) + kM$), any cognition-cost function $G(n)$, that increases in n without bound will necessarily lead us to a welfare-maximizing oligopoly. We now endogenously determine a critical function $G^*(n)$ such that any $G(n) \geq G^*(n)$ uniformly for all n, will necessarily give us a welfare-maximizing oligopoly. We will demonstrate that such a critical cognition-function, belongs to a more general family of cognition-cost functions $G_l^*(n)$, that are not necessarily unbounded (although monotonic) in n.

Theorem 1: The critical cognition-cost function $G_l^*(n)$ is increasing and bounded in n such that any function $G(n) \geq G_l^*(n)$ ($\forall n \geq 2$) is consistent with a welfare-maximizing oligopoly.

Proof: We provide the proof in three steps. Firstly, we solve for (by construction) $G_l^*(n)$ which is uniquely determined in the appendix as $G_l^*(n) = \frac{\mu a^2(n+l)(n+2+l)}{4b(n+1+l)^2}$ (or any finite $l > 0$). The second step is to recognize that $G_l^*(n)$ is strictly increasing in n and bounded (above) by $\mu a^2/4b$.[9] Finally, we are now in a position to demonstrate that a finite n maximizes

[9] We recognize that $\frac{\mathrm{d}\ln G_l^*(n)}{\mathrm{dn}} = \frac{2+\mathrm{l}(\mathrm{n}+2+\mathrm{l})}{(\mathrm{n}+\mathrm{l})(\mathrm{n}+1+\mathrm{l})(\mathrm{n}+2+\mathrm{l})} > 0$ and $\lim_{\mathrm{n}\to\infty} \mathrm{G}^*(\mathrm{n}) = \frac{\mu \mathrm{a}^2}{4\mathrm{b}} > 0$.

welfare. We demonstrate this by specifying the overall welfare function (in terms of the general specification of our critical cost function) as follows

$$S(n) = \frac{ka^2}{4b} \cdot \frac{(n)(n+2)}{(n+1)^2} - \frac{\mu ka^2}{4b} \cdot \frac{(n+l)(n+2+l)}{(n+1+l)^2} + kM$$

Maximizing $S(n)$ above with respect to n gives us the first order condition below

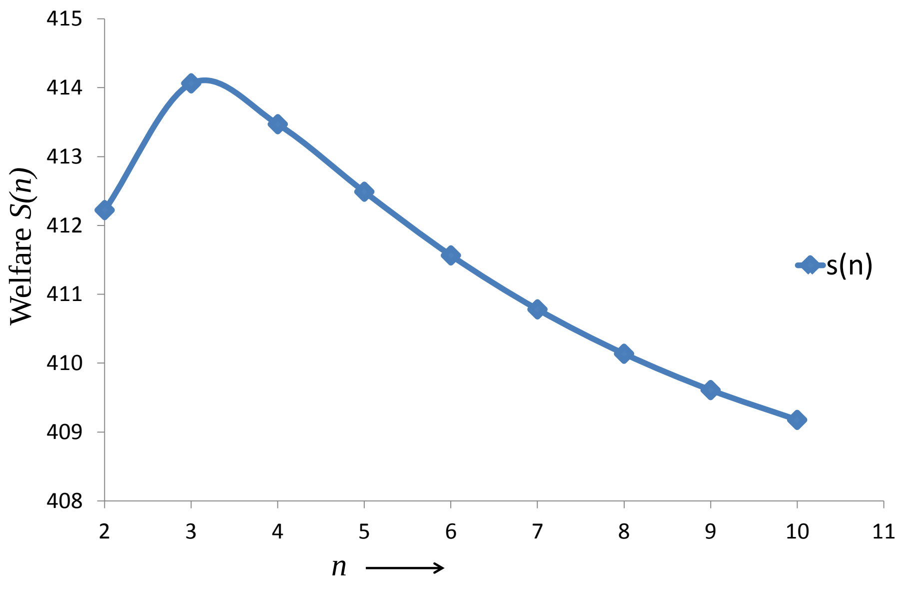
$$\tilde{n} = \frac{1 + l - \mu^{1/3}}{\mu^{1/3} - 1} \tag{4.14}$$

for any $\mu \in (1, \left(\frac{3+l}{3}\right)^3]$. Note that in this interval $\tilde{n} \geq 2$ (shown in the appendix). ■

Example 1: For our cognition-cost specification with $\mu = 3.375$ and $l = 2$, our welfare specification *S(n)* is strictly maximized at $\tilde{n} = 3$, attaining a value of $S(\tilde{n}) = S(3) = k[M - (2.34375a^2/4b)] > \lim_{n\to\infty} S(n) = k[M - (2.375a^2/4b)] > S(0) = k[M - (3a^2/4b)]$.

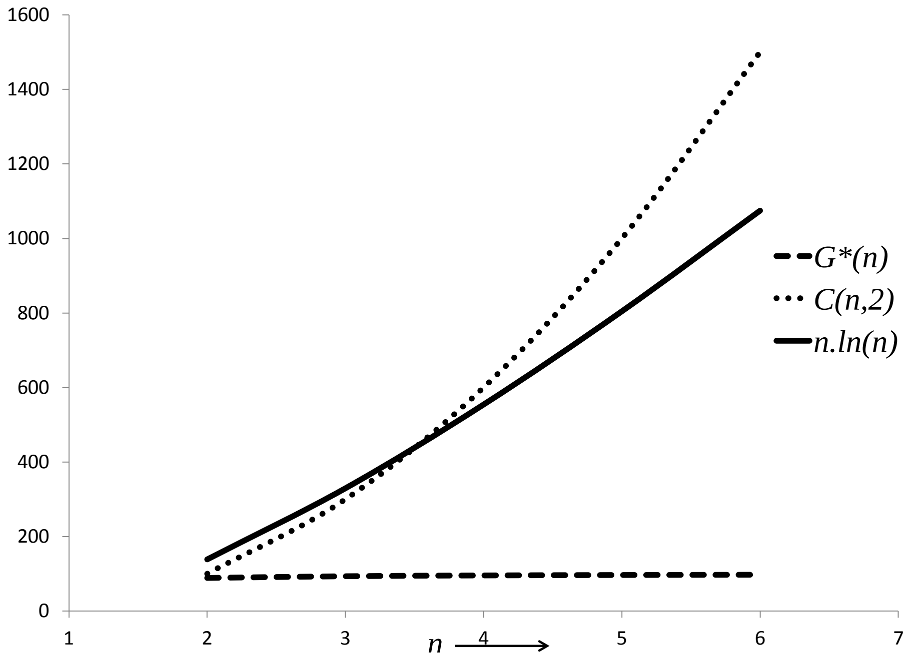
Figure 3 below, shows the welfare function for suitable values of *k (= 10)*, *M (= 100)*, and *a^2/b (= 100)* for Example 3 above.

Figure 3: Finite number of firms maximize welfare



The purpose of the above exercise (including the example) was to demonstrate that welfare can attain a maximum at a strictly oligopolistic market structure even when cognition-costs are significantly low (let alone bounded). To give an idea of how our critical cognition-cost compares with other cost-structures observed in the fields of computer sciences, we look at Figure 4, which compares two cognition-cost functions with our critical function to show that the results are indeed powerful, since even the most efficient ways of evaluating and comparing *n* alternatives are associated with costs that significantly exceed our critical cognition-costs.

Figure 4: A comparison of different cognition-costs

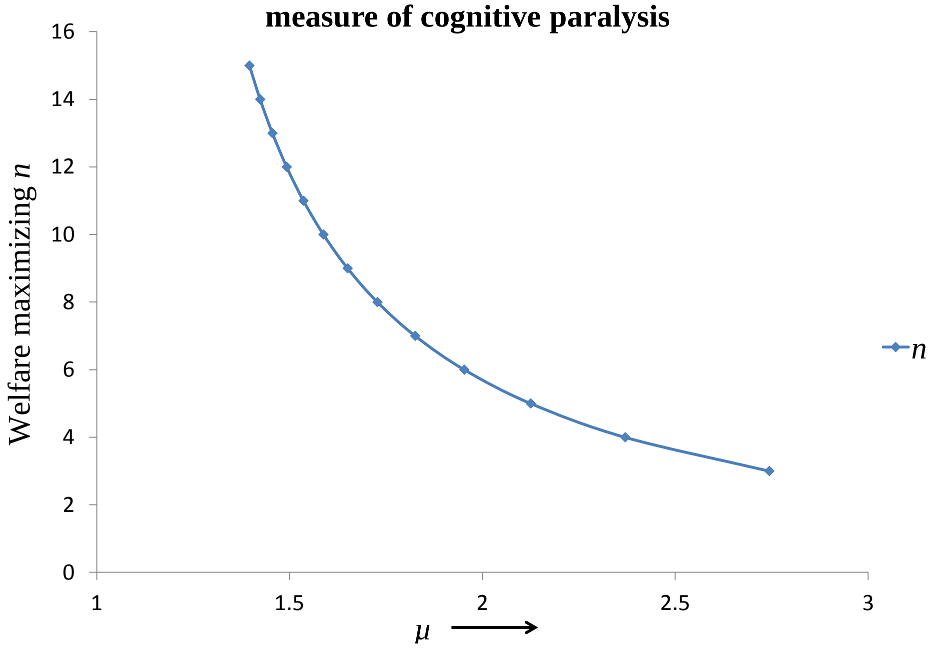


It is noteworthy that among the above structures of cognition-costs, only the critical specification G^* can accommodate an infinite number of firms when there is free entry and exit (which indeed makes our set up more comparable with the central requirements of the model of perfect competition). This is because welfare *S(n)* remains positive no matter how large *n* becomes. This is not possible with the other two cognitive-cost functions, for which there is a strictly finite *n*, beyond which, welfare can attain negative values. Note that for our critical cognition-cost function, $\lim_{n\to\infty} S(n) = k[M - ((\mu-1)a^2/4b)] > 0$. Finally, we look at the relation between the measure of cognitive paralysis and the welfare-maximizing number $\tilde{n}$ of firms. From equation (4.14), it is clear that

$$\frac{dln(\tilde{n})}{d\mu} = -\frac{l\left(\mu^{-2/3}\right)}{3\left(\mu^{1/3}-1\right)\left(1+l-\mu^{1/3}\right)} < 0$$

since $\mu \in (1, \left(\frac{3+l}{3}\right)^3]$. Note that in this interval, each term in the denominator is individually positive. Just like in Figure 1, we demonstrate the negative relation between the welfare-maximizing number of firms and the measure of cognitive-paralysis μ in Figure 5 below.

Figure 5: Socially desirable number of firms against the measure of cognitive paralysis



4.4 Conclusion

In this paper we have suggested a general theoretical approach to incorporate the cognitive cost faced by consumers (as they have limited cognitive ability) in markets. We show that in the presence of cognitive costs, welfare is maximised when the markets are oligopolistic. This also suggests that policy makers should think of both the aspects, as increasing competition need not be the best way to increase welfare, especially if consumers find options as choice overload. Although in this paper, we have imposed product homogeneity in the interest of direct comparison with established welfare results (that require the conditions of competition in the strictest sense), it will be interesting as a further extension to look at welfare implications in the heterogeneous-product scenario, where consumers display a love for variety (so that some of the results shown in this paper may weaken). The central objective of this paper is to establish that maximising welfare is not necessarily the same as maximising competition.

Appendix to Chapter 4

Appendix 4A

Proof of theorems and derivation of equations

Theorem 1: The critical cost function $G^*(n)$ is increasing and bounded in n such that any function $G(n) \geq G^*(n)$ ($\forall n \geq 2$) is consistent with a welfare-maximizing oligopoly.

Proof: We want to specify a critical cognition-cost function $G^*(n)$, such that for any general cognitive cost function $G(.) \geq G^*(.)$, we will get an welfare-maximizing oligopoly. In what follows, we will go through a step-wise construction of G^*. Firstly, we recognise that for welfare to be maximized at a finite $n = \tilde{n}$, the following conditions must be met

$$S(\tilde{n}+1) - S(\tilde{n}) \leq 0 \Rightarrow G(\tilde{n}+1) - G(\tilde{n}) \geq \frac{a^2}{4b}\left[\frac{1}{(\tilde{n}+1)^2} - \frac{1}{(\tilde{n}+2)^2}\right] \qquad (4A.1)$$

$$S(\tilde{n}) - S(\tilde{n}-1) \geq 0 \Rightarrow G(\tilde{n}) - G(\tilde{n}-1) \leq \frac{a^2}{4b}\left[\frac{1}{(\tilde{n})^2} - \frac{1}{(\tilde{n}+1)^2}\right] \qquad (4A.2)$$

Clearly, the welfare-maximizing conditions above translate to requirements on our cognition-cost function shown in (4A.1) and (4A.2). As a second step, we begin by recognizing that the function $G^*(n) = \frac{\mu a^2(n)(n+2)}{4b(n+1)^2}$ exactly satisfies the above welfare-maximizing conditions 4A.1 and 4A.2 (with equality) at $n = \tilde{n}$ and $\mu = 1$. This is verified below

$$G^*(\tilde{n}+1) - G^*(\tilde{n}) = \frac{a^2}{4b}\left(\frac{(\tilde{n}+1)(\tilde{n}+3)}{(\tilde{n}+2)^2} - \frac{(\tilde{n})(\tilde{n}+2)}{(\tilde{n}+1)^2}\right)$$

$$= \frac{a^2}{4b}\left(\frac{(\tilde{n}+2)^2 - 1}{(\tilde{n}+2)^2} - \frac{(\tilde{n}+1)^2 - 1}{(\tilde{n}+1)^2}\right) = \frac{a^2}{4b}\left(\frac{1}{(\tilde{n}+1)^2} - \frac{1}{(\tilde{n}+2)^2}\right)$$

$$G^*(\tilde{n}) - G^*(\tilde{n}-1) = \frac{a^2}{4b}\left(\frac{1}{(\tilde{n})^2} - \frac{1}{(\tilde{n}+1)^2}\right)$$

Thirdly, we recognize that G^* is monotonically increasing and bounded above by $\frac{a^2}{4b}$.[10] Finally, in order to completely specify the class of critical cognition-cost functions we consider a change of origin $G_l^*(n) = G^*(n+l) = \frac{\mu a^2(n+l)(n+2+l)}{4b(n+1+l)^2}$ (for any $l > 0$).[11] Note that the monotonocity of $G^*(n)$ implies that $G_l^* > G^*$, whenever $l > 0$. In the paper, it is demonstrated that this family G_l^* of cognition-cost functions lead us to strictly oligopolistic welfare-maximizing markets for $\mu \in (1, \left(\frac{3+l}{3}\right)^3]$ (for any $l > 0$). For any value of μ out of this interval, welfare is either maximized in a competitive market ($\tilde{n} = \infty$) or in a monopoly ($\tilde{n} = 1$). For example, if $\mu^{1/3} > \left(\frac{3+l}{3}\right)$, then $\tilde{n} = \frac{1+l-\mu^{1/3}}{\mu^{1/3}-1} < 2$. ■

Derivation of equation (4.12) :

$$kU = \frac{ka^2n}{2b(n+1)} - bk\left(\frac{an}{2b(n+1)}\right)^2 + ky - \mu k\left(\frac{n(n-1)}{2}\right)$$

$$n\pi^* = \frac{ka^2n}{2b(n+1)^2}$$

Total surplus

$$S(n) = \frac{ka^2n}{2b(n+1)^2} + \frac{ka^2n}{2b(n+1)} - bk\left(\frac{an}{2b(n+1)}\right)^2 + ky - \mu k\left(\frac{n(n-1)}{2}\right)$$

$$S(n) = \frac{ka^2n}{2b(n+1)}\left(\frac{4+n}{2(n+1)}\right) + ky - \mu k\left(\frac{n(n-1)}{2}\right)$$

Now, using $ky = k(M - px) = k(M - \frac{a^2n}{2b(n+1)^2})$

[10] We recognize that $\frac{d\ln G^*(n)}{dn} = \frac{2}{(n)(n+1)(n+2)} > 0$ and $\lim_{n\to\infty} G^*(n) = \frac{a^2}{4b} > 0$.

[11] Among the well established ways to solve for functional forms that satisfy certain critical properties, often involve a slight change of origin. Such methods are extensively discussed in Small (2007).

$$S(n) = \frac{ka^2n}{2b(n+1)}\left(\frac{4+n}{2(n+1)}\right) + k(M - \frac{a^2n}{2b(n+1)^2}) - \mu k\left(\frac{n(n-1)}{2}\right)$$

$$S(n) = \frac{ka^2n(n+2)}{4b(n+1)^2} + kM - \mu k\left(\frac{n(n-1)}{2}\right)$$

Derivation of equation (4.13) :

$$\frac{S(n+1) - S(n)}{k}$$

$$= \frac{a^2}{4b}\left(\frac{(5+n)(n+1)}{(n+2)^2} - \frac{n(n+4)}{(n+1)^2}\right) + \frac{a^2}{2b}\left(\frac{n}{(n+1)^2} - \frac{(n+1)}{(n+2)^2}\right)$$

$$- \frac{\mu(n+1)n}{2} + \frac{\mu(n-1)n}{2}$$

$$= \frac{a^2}{2b}\left(\frac{(n+3)(n+1)}{2(n+2)^2} - \frac{n(n+2)}{2(n+1)^2}\right) - \mu n$$

$$= \frac{a^2}{4b}\left(\frac{(n+1)^4 + 2(n+1)^3 - (n+2)^4 + 2(n+2)^3}{(n+2)^2(n+1)^2}\right) - \mu n$$

$$= \frac{a^2}{4b}\left(\frac{2\{(n+1)^3 + (n+2)^3\} - \{(n+2)^2 + (n+1)^2\}\{(n+2)^2 - (n+1)^2\}}{(n+2)^2(n+1)^2}\right) - \mu n$$

$$= \frac{a^2}{4b}\left(\frac{2\{(2n+3)((n+1)^2 - (n+1)(n+2) + (n+2))^2\} - \{(n+2)^2 + (n+1)^2\}\{2n+3\}}{(n+2)^2(n+1)^2}\right)$$

$$- \mu n$$

$$= \frac{a^2(2n+3)}{4b(n+2)^2(n+1)^2}\left(\{2(2n+3)^2 - 6(n+1)(n+2)\} - \{(n+2)^2 + (n+1)^2\}\right) - \mu n$$

$$S(n+1) - S(n) = k\left(\frac{a^2(2n+3)}{4b(n+2)^2(n+1)^2} - \mu n\right)$$

Chapter 5

Conclusion

The central theme of the second chapter is how oligopoly profits measure up against monopoly profits. The idea that oligopoly profits can match monopoly-level profits (at least for some firms) remained unexamined so far. We identified that reference-dependence is a channel through which only oligopoly firms can benefit in ways monopolies cannot. This is because comparison (among competing alternatives) by consumers is a key prerequisite for our model to work – and such comparison becomes even more meaningful when the products are differentiated (in our case, vertically). Our understanding of how reference-dependence actually works in the context of market outcomes, naturally leads us to a careful understanding of the limitations of the same – we see that monopoly-level profits can only be matched by a special case of oligopoly (*duopoly,* to be specific).

In summary, this chapter leaves us with the quest to explore more avenues (that do not naturally apply to monopolies) through which oligopolies can benefit in terms of higher payoffs. Such avenues, once identified, can be examined in terms of the nature of the associated payoffs – for example, how far can the payoffs go? … and can they potentially *exceed* that of monopoly-levels? The truth is that we do not have immediate answers to such questions. Such understanding is important - not just because oligopolies occur way more frequently than monopolies in reality, but the sheer value of such academic curiosity can lead us to a better understanding of markets in ways that have remained unexplored (not to mention the range of firm- strategies, welfare-effects, and policy implications that follow).

One such implication of the nature of the very *act* of *comparison* itself (not necessarily in terms of reference-dependence) has already been explored in Chapter 4, where we account for the possibility that any comparison need not be free from accompanying cognitive costs – and once we account for such costs of cognition, we realise that the existing wisdom on the relation between market-structure and welfare (specifically, the idea that *competition maximizes welfare*) needs further re-examination. In our study, we specifically show that a finite number of firms maximize welfare precisely where the marginal benefits from competition balance out the marginal costs of the process of decision-making (including cognition-costs). This welfare-maximizing market-structure mimics an oligopoly situation.

In this exercise, what is surprising is that our results go through even when these cognition-costs are strictly bounded above – and remarkably enough such a bound could be extremely small in comparison with some of the known costs of computation already known and examined in other disciplines. In short, for some form of an oligopoly to maximize welfare, the cognition-costs need not be very high. Thus, under very reasonable conditions that mimic the real world, we expect an oligopoly (and not competition) to maximize welfare.

Our results are shown in the context of perfectly homogeneous products (i.e. there is no product differentiation - horizontal or vertical), which may not characterize an oligopoly in the real world. The existing results on welfare however, have been established under strict conditions of perfect competition that require product-homogeneity in the strictest sense. A natural next step to this research will therefore involve welfare-comparisons between different market forms under conditions that do not impose (or require) strict product-homogeneity. Needless to say, we find that this is a hard problem that becomes intractable even with a small number of firms. The next step could also head toward the exploration of axioms that characterize consumer preferences under the existence of cognition-costs. All these are important areas that require further investigation.

We finally come to a few concluding remarks in relation to Chapter 3, on our laboratory experiment. This is the only empirical paper in this thesis where we look at elements of social decision-making in the presence of several competing focal-points (instead of just one focal point which also the key feature of the study in Chapter 2, where the high-quality product also becomes the point of reference). The existing literature on bargaining (in experiments) identify that certain focal points are important for they facilitate quicker negotiation between bargaining agents. We show in our experiment, that while focal points are important, what matters more is their relevance. In this chapter on the *relevance* of focal points in bargaining, we show that the 50-50 norm strongly applies to cases where the pie-size (from cooperation) is high relative to the sum of the individual disagreement-payoffs of the bargaining agents. On the other hand, bargaining agents are more likely to settle on a share that mimics the proportion of their disagreement payoffs when the sum of which closely matches the pie-size (from cooperation).

Natural extensions to this form of inquiry could look into unstructured bargaining with three or more bargaining agents. Still further, one could look into cases where disagreement points vary across treatments against a fixed pie-size. All such extensions to our form of experimentation will naturally inform theory to facilitate a more general form of bargaining solution that unifies the ideas embedded in the existing notions of fairness. All this development is further likely to guide us towards a better understanding of both individual and joint social welfare.

In a nutshell therefore, this thesis is about the welfare implications of (individual or joint) decision-making in the presence of focal points of comparison. At some level, we economists are scientists and at others, we are artists. What is more important is to remember that we are *explorers all the time*!

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