Three essays on the political economics of conflict

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Dedicated to my mother Madhumita Bakshi, my brother Deepit Bakshi and my friends Sayantan Datta and Arusharka Bhattacharya who have been with me through thick and thin

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Chapter 1

Introduction and chapter-wise summary

1.1 Introduction

A standard practice in the sociological and comparative-politics literature is to interpret ethnicity broadly as identity cleavages deriving from *non-class* categories such as language, religion, race and caste. In recent decades, conflicts between 'ethnic' communities, i.e., groups divided along non-class identity dimensions such as race, language, and, in particular, religion, have attained increasing salience in many countries. In response to this, a large analytical and empirical literature has developed both in political science and in political economics that seeks to explicate various aspects of this phenomenon. This thesis aims to throw light on two major aspects of ethnic conflicts. (1) The inter temporal and inter spatial nature of conflict, along with spillovers and linkages (2) The inter-dynamics of vertical unity in a community (characterized by a non-excludable community specific public good) and class solidarity in ethnic conflicts.

In the domains of Economics and Political Science we often come across situations where two or more agents are engaged in multiple conflicts, which Kovenock and Roberson (2010) refer to as "battlefields". The linkages amongst these battlefields have been an area of interest for game theorists for quite a while now. Borel (1921) formulated the foundational model, known as the Colonel Blotto Game, which dealt with multiple contests with linkages. Colonel Blotto Game involves two players who have fixed resources at their disposal. Each of these players are supposed to allocate resources across a finite number of battlefields, without knowing their rival's allocation of resources across battlefields. In each battlefield the player who allocates the higher amount of resources, wins. The total payoff to a player is the sum of the payoffs from the individual battlefields. Borel's Colonel Blotto game highlights how the budget constraint of each player acts as a linkage between the battlefields. Some recent work in this domain concentrates on illustrating structural linkages which are predominantly of the following types. (i) Cost function linkages and budget constraint linkages (ii) Objective side linkages (majoritarian objective, best shot objective etc). Researchers have taken combinations of these linkages and tried to analyse the multiple battlefield scenario under those conditions. Golman and Page (2009), Kyasov (2007), Kovenock and Roberson (2008), Roberson and Kyasov (2008) have extensively dealt with structural linkages with different contest success functions (mainly the Tullock CSF or the Lottery CSF). But the battlefield "linkage", which has not received its due attention is the linkage induced by outcome interdependence. If winning in one battlefield significantly enhances the chances of succeeding in another battlefield or otherwise, how would the competing players (which could be nations, firms or political factions) behave? We call this effect the "probability spill-over effect", since probability of winning in a battlefield is influenced by what's happening in another. Natalija Novta (2013) elucidates how conflicting groups battle for control over strategic territories because it might help them have an upper hand in future conflicts over other territories. She substantiated her findings with empirical evidence from Bosnia. We can observe this phenomenon in several domains including industrial organisation and political conflicts.

The inter-temporal spillover finds application in dynamic conflict in ethnocracies. In recent years, the concept of *ethnocracy* has received growing attention in political theory and conflict studies (see, for example, Howard 2012; Yiftachel 2006; Yiftachel and Ghanem 2004; and Kedar 2003). An ethnocracy is a political system in which political and social organizations are founded on ethnic belonging. An ethnocracy can be characterized by the following features: 1) political parties that are based foremost on ethnic interests; 2) ethnic quotas to determine the allocation of key posts; and 3) state institutions, especially in education and the security sector, that are segmented by ethnic group. Ethnocracies are generally parliamentary systems with proportional or semi proportional representation according to ethnic classifications. 'Contrasting political platforms ... are of secondary importance to ethnic-group membership' (Howard 2012, 155-56). Thus, politics in an ethnocratic state is organized primarily along ethnic lines, with ethnic groups organized to shift resources in their favour. Key state institutions are *ethnicized*: they are run by personnel who actively seek to use them to the benefit of their respective ethnic clientele. Control over state institutions therefore *complements* mass political mobilization in inter-ethnic resource conflicts. Standard examples include Bosnia, Lebanon, Northern Ireland and Belgium (since the 1970s). However, many other countries also exhibit one or more of the abovementioned features or *tendencies* of the ethnocratic state. States with strong ethnocratic tendencies often appear also to exhibit watershed points in history, where an *initial ethnic settlement*, in the sense of an ethnic division of state institutions intended to hold over time, is arrived at. Such initial settlements are often brokered and imposed by militarily dominant external powers.[1] These settlements are open to subsequent adjustment on the basis of mass political pressure from ethnic mobilizations, but with a temporal lag, and then too, imperfectly. Thus, the outcome of mass ethnic political conflict over resource sharing is influenced by the current relative ethnic influence within institutions; but this outcome, in turn, affects such influence in future periods, albeit imperfectly. Relative ethnic influence over institutions thus evolves over time as a joint consequence of the initial settlement and ethnic conflict: the evolution of ethnic conflict and inter-ethnic distribution is in turn determined jointly by relative ethnic influence over institutions and relative structural strength of the contending communities. The inter-spatial linkage finds application in cross territorial spillovers, in inter – community conflicts, taking place in multiple locations or countries. A large analytical and empirical literature has developed both in political science and in political economics that seeks to explicate various aspects of ethnic conflicts. This literature however largely focuses on the *internal* drivers of ethnic conflict within a society. What has received much less attention is how ‘foreign’, or extra-territorial, influences affect and condition such conflict. The same, or closely related, rival ethnic groups are often spread over multiple countries. Shias and Sunnis spread over all countries in the Middle East constitute an example, as do Hindus and Muslims spread over Bangladesh and India. A conceptually similar situation obtains when the same religious group constitutes the overwhelming majority in two different territories, but is cleaved between antagonistic

1 The US played this role in the Dayton Agreement of 1995 which ended the war in Bosnia. The Good Friday Agreement of 1998 in Northern Ireland was also actively brokered by the US, as was the post-Taliban Afghan political settlement. The last involved a power sharing deal among various ethnic groups, though without explicit ethnic quotas. Lebanon's National Pact of 1943 was imposed by France, the colonial power.The subsequent 1989 Taif Agreement, shaped by Syria, Saudi Arabia, and the US, maintained ethnocratic principles and practices, though amending details. After invading Iraq in 2003, US authorities chose the members of an Iraqi Governing Council along ethno-sectarian lines, roughly matching each group’s share of the population.

theocratic and secularizing political tendencies (as for example is the case for Hindus spread over Nepal and India). A related phenomenon is a single ethnic group spread over multiple countries, but facing different ethnic groups in different countries (such as the Pashtuns across Pakistan and Afghanistan). In these cases, the balance of power between two antagonistic groups within a territory is often likely to 'spillover' into another; i.e., to impact inter-group conflict in the latter. The spillover effect of Shia-Sunni conflict in Iraq on Syria and Lebanon, in the context of the recent rise of the Islamic State of Iraq and the Levant, is well-documented, as is the spillover effect of the war in Afghanistan on Pakistan.

The inter-dynamics of vertical unity in a community (characterized by a non excludable community specific public good) and class solidarity has received sufficient attention in the public economics literature. Dasgupta and Kanbur (2007) talk about how the distribution of nominal income could give a misleading picture of the tensions in society, both within and across communities. Ideologies of community solidarity may well trump that of class solidarity because of the implicit sharing of community resources brought about by community-specific public goods. There are umpteen number of instances, especially in the last two decades where ethno-religious identity has outshone class-based debates in elections, worldwide. A point in hand is the recently concluded national election in India, where the surge of nationalism, especially followed by the surgical strike in response to the Pulwama attacks drowned all discussions on growth and unemployment figures. The rise of the Republican Party in the USA, the Brexit referendum and the general anti – immigrant sentiment in Europe, generating from strong ethnic affiliations goes to show that national or ethno-religious identity is almost as important as class identity, if not more. And what is concomitant with that is a war for appropriation of the public sphere to assert identities. The struggle can be on symbols representing identities or social norms like food habit, characterizing ethno – religious groups or religious monuments. Hence the question which deserves attention is: how do economic class divisions and the struggle to assert one's ethno-religious identity interact with each other? That is precisely what the last chapter of the thesis attempts to address.

1.2 Organisation of the thesis

The organization of the thesis is as follows. **Chapter 2** deals with a model of dynamic conflict in ethnocracies. **Chapter 3** examines the impact of cross border spillovers in ethno – religious conflicts. **Chapter 4** is concerned with analyzing the distribution and welfare implications of ethnic conflicts. The thesis ends with a complete list of references. The next section provides a brief overview and summary of Chapters 2-4 of the thesis.

1.3 Chapter-wise summary

Chapter 2: A model of dynamic conflict in ethnocracies

This chapter develops a simple model of dynamic ethnic conflict when state institutions exhibit evolving ethnic bias. The main results involve the non-monotone nature of the relationship between conflict, distribution and social welfare on one hand, and key peace policy variables, such as the ethnic bias of an initial institutional division and the responsiveness of such bias to emerging political pressure from mass ethnic movements, on the other. These findings may be seen as highlighting the fundamental limitation of ethnocratic settlements as peace-building devices in conflict-ridden societies. The analysis suggests that, when successful in avoiding all-out civil war and state collapse, such ethnocratic settlements are likely to generate high levels of conflict over ethnic rent-seeking, within the framework of a fragile state, perpetually susceptible to external interference and intervention. This 'ethnocratic trap' (Howard 2012) can be avoided only by developing more inclusive, porous and flexible forms of identity formation and integrative institutions that cut across rigid ethnic divides.

Chapter 3: Identity conflict with cross-border spillovers

This chapter develops a parsimonious model of simultaneous between-group ('ethnic') conflict over public goods with group-specific non-monetary benefits (state 'culture' or 'religion'), decentralized intra-group distributive conflict over private consumption ('income'), and production, with conflict spillovers across politico-administrative territories such as countries or provinces of a country. The theoretical analysis generates a number of empirically testable propositions regarding the nature of extra-territorial influences on intra-territorial (or domestic) conflict.

Chapter 4: Conflict between class-divided communities with unequal sharing rules

This chapter examines how prior income inequality within a community combines with plunder sharing rules to affect decentralized individual efforts to expropriate another community, when the poor are dependent on the rich members of their community for the provision of public goods. We show that an individual's share of any rent accruing to a community, in consequence of expropriation of another community, may be a misleading proxy for her relative incentive to engage in inter-community conflict. Our findings provide micro-foundations for situations where one income class within a community may free-ride on another in such conflicts, despite members of the former class all standing to gain nominally more income from inter-community conflict, than those of the latter. These findings offer a broad theoretical perspective that helps to rationalize an empirical phenomenon often noted in historical and electoral studies – viz., the greater propensity of better-off segments within a society to support ethno-exclusivist, xenophobic or ultra-nationalist political programs, including fascism.[2] Our results also suggest that internally more equal communities, under certain conditions, may exhibit greater aggressiveness in inter-community conflict. This implication of our

2 See, among others, Hobsbawm 1987, 1992;Engineer 1995, Brustein 1998 and Dhattiwala and Biggs 2012 for discussions of the literature.

model stands in stark contrast to the central implication of the model developed by Esteban and Ray (2011), and suggests theoretical organizing principles for empirical research on the intensity of ethnic community. Lastly, our results point to a possible tension between the twin policy goals of reducing inequality, whether within or across ethnic groups in a society, and reducing the intensity of inter-ethnic conflict therein. Future research may delve deeper into the nature and robustness of this trade-off under alternative specifications of public good technologies and conflict success functions.

Possible Future Work

The first two chapters deal with inter-temporal and inter-spatial spill overs in the context of ethnic conflict. An interesting possible extension will be a generalization of the inter-spatial conflict as a conflict in a graph where the outcome of conflict in one node impacts the conflict in the neighbouring (of order 1) nodes, thus creating a ripple effect. Given this construct it will be interesting to observe the equilibrium(equilibria) resource allocations given that the contests are centralized or decentralized (both cases will yield different sets of equilibria). The comparative statics with respect to the nature of the graph will definitely yield interesting revelations. Moreover, investigating the correlation between the marginal impact of a particular node in terms of contributing to the overall conflict expenditure in the network and its degree (which is a measure of its connectivity) in the graph could be an interesting question as well.

As far as the last chapter is concerned the welfare implications (endogenizing the optimal choice of sharing rules) will be worth delving into. That is also a possible extension we are aiming for.

Chapter 2

A model of dynamic conflict in ethnocracies

(An extended version of this chapter has been published as:

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2.1. Introduction

The purpose of this chapter is to develop a simple analytical framework within which the dynamic interaction between ethnic conflict and ethnic control over institutions may be theoretically investigated. The essay addresses two main, policy-relevant, questions. First, how does an initial institutional settlement affect the joint evolution of conflict and horizontal equity, and, thus, social welfare? Second, how does the extent of institutional rigidity with regard to relative ethnic partisanship affect these variables? The first question has important implications for understanding how history, in the sense of policies of past agencies (such as colonialism, absolute monarchies or dictatorships) which governed more or less independently from popular pressures, may exert long-term influence on the joint evolution of conflict and inter-community resource distribution. It also provides useful insights into the long-term consequences of external interventions of contemporary vintage, such as those in Iraq, Afghanistan, Syria or Bosnia, that seek to establish particular forms of ethnic sharing of state institutions. The second question has evident implications for institution design and peace policy in ethnically divided societies. Together, the answers to these questions also permit one to explicate the welfare consequences of perfect institutional neutrality in societies exhibiting ethnocratic tendencies.

We set up a model of a contest, over the sharing of some exogenously given resource, between two combatants, interpreted as two ethnic groups. This resource is allocated by a composite state institution according to relative ethnic control; hence the ethnic groups contest the extent of institutional ethnic bias. This contest is repeated in every period over an infinite number of periods. The contest yields the per-period relative influence over state institutions, and thus, share of the contested resource, according to the standard Tullock (1980) contest success function. Relative influence achieved partly spills over into the next period.[3] Thus, most crucially, the efficiency with which an ethnic group's conflict inputs can be converted into success outcomes is partly endogenous in the model. It is given by the multiplicative combination of some initial share, interpreted as the degree of control over institutions provided to the ethnic group by a historical original settlement at the beginning of time, and the share acquired in the preceding period's contest. The relative weight on these two elements models the extent to which immediate mass ethnic political mobilizations intended to lobby/pressurize state institutions over short-term resource sharing has long-term effects on the ethnic bias thereof. A high weight on the former reflects a high degree of institutional rigidity, whereby the relative ethnic partisanship of institutions accords more closely to the original settlement. The combatants have a short time horizon: they discount the future completely. Relative cost of conflict inputs is assumed invariant over time, reflecting structural features of the economy as well as community organization, and one community has a persistent (though possibly minor) cost advantage.

The model departs from the literature on dynamic contests in its endogenization of the efficiency of contest effort. Baik and Lee (2000) and Lee (2003) set up a model of a two-period contest where efforts in the first period carry over *additively* into the second period, and players take

3. This may happen simply because, as part of the process of pressurizing institutions, ethnic groups manage to insert more of their representatives within institutions, some of whom then maintain their positions into the next period, and/or recruit individuals from their own group. Callander (2007) has shown how bandwagon and momentum might develop in sequential voting, driven by a combination of beliefs and the desire of voters to vote for the winning candidate. Some of these effects may also be present in our very different context, as opportunistic elites adjust their behaviour in line with revealed relative success (power) in mass ethnic conflict.

this spill-over into account in making their first period conflict allocation decisions. Schmitt et al. (2004) maintain this structure while extending it to a finite number (possibly more than two) of periods. Relative efficiency of contest effort of the players is exogenously given in these models. In this infinitely repeated contest, in contrast, *relative success* in the preceding period determines efficiency in the present period, and players completely discount the future. Maxwell and Reuveny (2005) consider conflict in a dynamic setting, where, as in my model, actors maximize their share of the contested output available in the current period, and this output is split deterministically among the groups based on their fighting effort. Thus, my model is similar to theirs in its assumptions of (a) short (i.e., current period) time horizon of combatants, and (b) conflict outcomes as deterministic shares of the prize. Reuveny, Maxwell, and Davis (2011) extend this framework to a succession of winner-take-all contests. In both these contributions, however, the spoils are used to increase the sizes of the groups, and the conflict repeats with these altered sizes. Thus, success in the preceding period affects outcomes in the current period *only* by relaxing the conflict resource constraint, assumed binding, therein. Relative conflict efficiency is exogenously given in these models. In contrast, We assume that conflict resource constraints are non-binding in every period, so that the mechanism through which past conflict affects present conflict in their model becomes inoperative in mine. Instead, past conflict outcomes directly determine relative conflict efficiency in the present in our model.[4]

We find that an initial settlement that does not negate the effort cost advantage of a community in conflict leads to secularly declining conflict, but increasing horizontal inequality. In these cases, a more advantageous initial settlement for the cost-advantaged community reduces conflict, but exacerbates inequality, in every period. This is true of greater institutional flexibility as well. More interestingly, initial ethnic settlements over state institutions that more than negate a

4 Distantly related is the literature on multi-stage races with farsighted players who compete in a finite sequence of simultaneous move component contests. Konrad and Kovenock (2009) study such a model, where the component contests are all-pay auctions with complete information. Players may win a prize both for winning each component contest and for winning the overall race. They characterize the unique subgame perfect equilibrium. Klump and Polborn (2006) and Irfanoglu, Mago, and Sheremeta (2014) provide related analyses of multi-stage electoral contests. Direct productivity carry-overs and prize sharing, which constitute our focus, do not feature in any of these contributions. On the other hand, unlike theirs, our analysis does not feature either an end state prize or farsighted players. See Konrad (2009, chap. 8) for a survey of the literature on dynamic contests.

community's cost advantage in the short run, but are not drastic enough to do so permanently, generate non-monotone evolution of both conflict and distribution. Subsequent to the settlement, conflict increases in initial periods, but declines in later period, asymptotically converging to a steady state. Conversely, inter-community resource inequality initially declines, and subsequently increases. Hence, a social welfare aggregation of inequality and efficiency (conflict reduction) exhibits non-monotone movement as well. In this zone, a marginal improvement in the low cost community's initial institutional settlement, and in the extent of institutional flexibility, both increase conflict in the initial periods (while reducing inequality), but reduce it in future periods (while increasing inequality). This situation appears to be most likely in real historical contexts. In these contexts, (a) welfare comparisons of alternative intervention packages require specification of both the normatively chosen trade-off between efficiency and equity and the time discount factor, and (b) such comparisons may be time inconsistent. Perfect institutional neutrality facilitates neither equity nor efficiency.

The results in this essay have important implications for the debate over 'liberal interventionism'. Such interventions by foreign entities are often sought to be politically justified by the jointly stated goal of conflict reduction and protection of weaker ethnic groups, within a context of growing ethnic conflict. It is natural to expect politicians and voters in countries which are in a position to intervene to have short time horizons: they are likely to prioritize immediate conflict reduction over long-term peace building. My results suggest that higher levels of short term peace and protection of weaker ethnic groups may be ensured by one-off interventions which impose a combination of higher institutional disadvantage and greater institutional rigidity on the ethnic group with a conflict cost advantage. However, such higher levels of short term conflict reduction are purchased at the cost of deeper conflict in the longer term. Thus, political short-termism in intervening countries is likely to bias peace-building interventions towards forms (specifically, greater institutional locking out of the stronger/dominant group post intervention and greater institutional rigidity with regard to emergent ethnic political pressure) that are likely to exacerbate conflict in the future. Such exacerbation in turn is likely to justify demands for continuing, deeper and more long-standing intervention, as for example has been the case in Iraq, Syria and Afghanistan, which would push the problem further into the future. Thus, my model suggests that external interventions, when effective in reducing current conflict and protecting weaker groups, may in fact end up sowing the seeds of greater future conflict.

Section 2.2 sets up the model. Section 2.3 discusses how greater institutional flexibility with regard to ethnic pressures affects the joint evolution of conflict and resource distribution. Section 2.4 extends the analysis to an explicit consideration of trade-offs between (horizontal) equity and efficiency (conflict reduction) by embedding it within an aggregative welfare framework. Section 2.5 discusses some implications for peace policy. Section 2.6 offers concluding remarks.

2.2. The model

Consider a scenario where two groups (ethnic communities), A and B, are engaged in contestation over some income-generating resource in every period $t \in \{1,2,3,\ldots\}$. The per-period monetary value of the resource being contested over, to community $i \in \{A, B\}$, is $p_i > 0$, and its population size is n_i. Thus, the item being contested over is simply something (e.g. agricultural land, natural (esp. mineral) resources, or foreign aid flows) which, if possessed by community i, would generate income p_i in every period for that community. Note that communities may (but need not) differ in their valuation of the good being contested.[5] The resource is allocated by a composite state institution (legislature, bureaucracy, armed forces and law courts). This composite state institution's decisions regarding the ethnic division of the resource is determined by a process of contestation (lobbying, political pressure, mass mobilization, direct street action, violence, etc.). Given any group $i \in \{A, B\}$, we shall denote the other group by $-i$. $M_{it} \geq 0$ is some non-contested, or productive, income accruing to each member of i in period t. For community $i \in \{A, B\}$ in period $t \in \{1,2,3,\ldots\}$, the proportion of the state

5 Differences in valuation of the item under contestation may reflect differential sales opportunities in global markets (as when international sanctions on oil or diamond sales from conflict zones differentially impact different ethnic militias) and/or possession of productivity enhancing complementary inputs such as technical personnel, ports and transport networks. Additionally, when one community has greater uncontested possession of a similar resource, say oilfields or mines, the contested resource may be more valuable to that community due to economies of scale and indivisibilities in commercial exploitation.

institution's decisions that act in its favour, and, thus, the *share* of the prize, is given by the following variant of the standard (Tullock, 1980) contest success function:

$$s_{it} = \frac{s_{i,0}^{1-\theta} {s_{i,t-1}}^{\theta} x_{it}}{s_{i,0}^{1-\theta} {s_{i,t-1}}^{\theta} x_{it} + s_{-i,0}^{1-\theta} {s_{-i,t-1}}^{\theta} x_{-i,t}} \text{ if } x_t > 0;$$

$$s_{it} = \frac{s_{i,0}^{1-\theta} {s_{i,t-1}}^{\theta}}{s_{i,0}^{1-\theta} {s_{i,t-1}}^{\theta} + s_{-i,0}^{1-\theta} {s_{-i,t-1}}^{\theta}} \text{ otherwise;} \qquad (2.1)$$

where x_{it}, x_{-it}, respectively, denote the total amounts of *conflict input* (or, *activist labour*) allocated by communities i and $-i$ in period *t*; $x_t \equiv x_{it} + x_{-i,t}$, $x_{it}, x_{-it} \geq 0$, and$\theta \in (0,1)$. From (2.1),

$$s_{it} = \frac{x_{it}}{x_{it} + {s_{-i,i,0}}^{1-\theta} {s_{-i,i,t-1}}^{\theta} x_{-i,t}} \text{ if } x_t \neq 0,$$

$$= \frac{1}{1 + {s_{-i,i,0}}^{1-\theta} {s_{-i,i,t-1}}^{\theta}} \text{ otherwise;} \qquad (2.2)$$

where $s_{-i,i,0} \equiv \frac{s_{-i,0}}{s_{i,0}}$; $s_{-i,i,t-1} \equiv \frac{s_{-i,t-1}}{s_{i,t-1}}$. The formulation in (2.2) clarifies the intuitive idea that the parameter θ captures the extent of *inter-temporal spill-overs* in conflict. When $\theta = 0$, the model reduces to the standard case of no spill-overs across battlefields (i.e., over time). Relative efficiency of conflict input for community B is given simply by its initial relative institutional share $s_{BA,0}$, and this remains invariant over time. When $\theta > 0$, positive spill-overs exist: success in one period acts, in effect, as a *force multiplier* in the next. To see this, suppose, without loss of generality, that $s_{BA,t-1} \equiv \frac{s_{B,t-1}}{s_{A,t-1}} > 1$, so that B is more successful in the period preceding t. Then, for all $\theta > 0$, ${s_{BA,t-1}}^{\theta} > 1$; furthermore, ${s_{BA,t-1}}^{\theta}$ is monotonically increasing in θ. Thus, greater success in the preceding period magnifies the effectiveness or productivity of one's conflict inputs in the current period, thereby translating into a higher success (i.e., share) in that period for any given deployment of conflict inputs therein by the two parties, and for any initial history $s_{BA,0}$. The higher the value of θ, the greater the extent of productivity spill-over from the immediate past to the present, and, correspondingly, the lower the importance of initial conditions in determining present conflict efficiency. In the limit $\theta = 1$, initial history ceases to matter completely in the determination of relative conflict efficiency. Conversely, spill-overs from the immediate past to the present act as a *force dampener* in the present for the combatant relatively less successful in the preceding period.

Community $i \in \{A, B\}$ in period t can purchase conflict inputs in that period, at a price q_{it}, which reflects the (broadly market determined) composite cost of training, equipping and retaining a unit of *activist* labour. Formally, we think of a unit of conflict input as one unit of activist labour, produced by the (cost-minimizing) combination of raw labour with specialized training and enabling equipment according to some linearly homogeneous production function, the latter being determined by the specific form of the conflict (e.g. whether lobbying, mass street mobilization or political violence). Given a linearly homogeneous production function, the conflict cost, q_{it}, can thus be thought of as the (constant) marginal cost of output (activist labour), determined in standard fashion by the cost-minimizing input combination weighted by the market determined prices of all inputs.[6]Among the multiple determinants of a group's conflict cost, perhaps the most important ones are: (i) the extent of the group's access to the international market for military goods, including mercenaries and military trainers, and (ii) the domestic (group-specific) return from productive labour.

Apart from training and equipment, however, the conflict efficiency of a unit of activist labour may be expected to be affected positively by support from elites who control the institutions of state power (the military, police, bureaucracy, and the judiciary). These institutions control access to items which, while not easy to procure from market purchase, can nonetheless complement activist

6 Intuitively, to participate effectively in conflict, each potential activist in a group needs to be first trained to some level to carry out activities pertaining to lobbying, propaganda, agitation, organization, violent confrontation, terrorism and/or civil war. Additionally, she has to be provided with some requisite level of material and human support equipment, say office space, transport and communication facilities, printing press, legal aid, safe houses, and/or armament. In both cases, costly material goods and human services, largely available at short notice through local or global markets, need to be purchased. Lastly, the activist has to be provided sufficient financial inducement to make her decision to participate in conflict activities individually rational: the magnitude of such payment would thus depend on her returns from productive activities. The human element in activism may be substituted to a large extent by better training or more material equipment: the cost minimizing combination will thus be determined by technology and relative prices. Arbatskaya and Mialon (2010) provide a formalization of this basic idea of multiple conflict inputs mapped (according to a Cobb-Douglas production function in their case) into an aggregate conflict effort variable.

labour in conflict situations. Sympathetic generals can provide access to existing stocks of heavy weaponry, supportive officials may easily concede demands from *i* but refuse to accede to political pressure from its rival, and partisan police officers and judges may refuse to act against activists from one community while clamping down heavily on those from the other. Thus, the relative efficiency, in mass political conflict to influence state institutions, of a unit of activist labour belonging to a community should be an increasing function of its relative control over elites who run institutions.

Mass ethnic political movements commonly seek to pressurize the state to simultaneously accede to its current resource demands and to increase its representation within state institutions which actually implement allocations at the ground level.[7] We therefore think of each contending community as using activist labour to pressurize those elite individuals who are not its natural sympathizers to act in its favour within institutions, as in the standard literature on lobbying and rent-seeking. Additionally, each contending community also uses activist labour to pressurize the state to increase its representation within state institutions. If institutions were completely subordinate to pressures from mass politics, their personnel composition should fully adjust to, or accommodate, the relative strength of the contending groups, as revealed by the relative conflict success ratio. Thus, A's relative control over institutions in period *t* would be identical to its relative conflict success in the preceding period ($\frac{s_{A,t-1}}{s_{B,t-1}}$). Recalling (2.2), this corresponds to our conflict success function when $\theta = 1$: intuitively, this corresponds to the idea of perfectly *flexible* institutions, i.e., institutions completely and accurately reflecting the prevailing ethnic balance of power in the mass political sphere.

Realistically, however, one would expect ethnic accommodation to be less than perfect. It is well-known in the political science literature that elites are self-perpetuating to a significant extent. Institutions such as the military, bureaucracy, police, and judiciary all have their internal hierarchies, specialized personnel requirements and relatively autonomous recruitment rules, which politicians representing mass movements find difficult to alter dramatically without risking elite resistance and

7 To illustrate, recent street demonstrations in US cities against police treatment of African-Americans demanded both changes in police practice and greater representation of African-Americans in the upper levels of the law-enforcement hierarchy.

consequent governance collapse in the short run. Thus, the ability of elites to resist political attempts at permanently restructuring their social composition or long-term ethnic alignment implies that institutions are likely to exhibit a certain rigidity: they are likely to exhibit a measure of relative community bias that lags behind the immediately revealed relative political strength. This consideration motivates and justifies the assumption $\theta \in (0,1)$. To see this more clearly, assume A is the weaker community in ethnic contestation in some period relative to the initial institutional settlement, i.e. $\frac{s_{A,t-1}}{s_{B,t-1}} < \frac{s_{A,0}}{s_{B,0}}$. Notice that, for $\theta \in (0,1)$, (i) $[\frac{s_{A,0}}{s_{B,0}} > (\frac{s_{A,0}}{s_{B,0}})^{1-\theta} \left(\frac{s_{A,t-1}}{s_{B,t-1}}\right)^{\theta} > \frac{s_{A,t-1}}{s_{B,t-1}}]$, and (ii) $\left[\frac{s_{A,0}}{s_{B,0}} - (\frac{s_{A,0}}{s_{B,0}})^{1-\theta} \left(\frac{s_{A,t-1}}{s_{B,t-1}}\right)^{\theta}\right]$ is increasing in θ. Thus, $\theta \in (0,1)$ implies that A's weakness in mass politics in the immediate past does translate into its relative weakness within institutions in the present, but only partially: institutional elites maintain independence from the immediately dominant community, B, to an extent. A lower value of θ implies a higher degree of elite conformity to the original balance of power ($s_{BA,0}$) while, conversely, a higher value of θ implies elites (and thereby institutions) more accommodative of immediate political strength. The polar case of $\theta = 0$ models perfect elite (and thereby, institutional) rigidity: the institutional balance of power rigidly conforms to the initial settlement regardless of the outcome in the subsequent ethnic conflicts. States such as Lebanon and Northern Ireland, where ethnic division of control over institutions currently follows relatively rigid rules codified in some past settlement (respectively, French colonialism imposed rules and the Good Friday Agreement), may, for modelling purposes, be thus ascribed values of θ close to 0. Conversely, states such as India, Israel, Belgium, South Africa and Iraq, where in recent years the relative presence of ethnic groups within institutions has changed significantly in response to mass political pressure, may be modelled as involving relatively high values of θ.[8]

8 In India, since Independence, recruitment quotas in the bureaucracy, police and the public sector have been progressively extended: initially to the so-called Scheduled Castes and Tribes, subsequently to Other Backward Castes, and, most recently, to sections of Muslims. The passing of the relevant legislation, and the extent of its subsequent implementation, have both reflected the fluctuating strength of mass ethnic political lobbies. Affirmative action in South Africa since the end of Apartheid, ethnic representation in Belgian institutions since the 1970s, minority ethnic rights in Israel since 1990, Sunni representation in state institutions in Iraq in the past decade, have all similarly evolved and changed significantly in response to mass ethnicizing political pressures. In these cases, the patterns of institutional ethnicization appear to be determined more by immediate and evolving ethnic politics than the terms of some original ethnic settlement.

*Remark 1.*Institutional rigidity (low θ) may be the *de facto* outcome even in the absence of *de jure* rules codified in past institutional settlements. Recall that institutional rigidity comes about when the ethnic composition of elites running state institutions is relatively impervious to change. Running institutions usually requires at least a minimal level of specialized, technocratic knowledge, training and experience. When education levels vary dramatically between ethnic groups, the better educated ethnic group may continue to dominate in state institutions simply because very few members of the other group achieve the minimal educational qualifications necessary to meet the entry requirements.[9] On the other hand, the less educated ethnic group is likely to have a lower opportunity cost of activist labour and thus a conflict cost advantage.

Following Maxwell and Reuveny (2005) and Reuveny, Maxwelland Davis (2011), We assume that individuals, and therefore communities, have a short time horizon: they completely discount future periods in their allocation decisions. As argued there, it is implausible to ascribe long time horizons to real-life political actors in situations of conflict stretching indefinitely into the future. Furthermore, this feature of the model serves to isolate, and thereby highlight, the role of *impersonal* inter-temporal spill-overs in influencing conflict outcomes over time. A third, pragmatic, defence is provided by the role it plays in enhancing the tractability of the analysis and drastically simplifying our exposition.

We assume that communities follow an equal surplus sharing rule, whereby each member of a community $i \in \{A, B\}$ receives an equal share of that community's net income (i.e., the amount $\frac{[s_{it}p_i - q_{it}x_{it} + n_i M_{it}]}{n_i}$) in each period. Thus, it is individually rational to maximize total community income. It is therefore justified to abstract from free-rider problems within a community to model each community as an individual combatant seeking to maximize its total income in each period,

9 For example, despite recruitment quotas in their favour since Independence, representation of the so-called Scheduled Castes and Tribes remains far short of both their quota levels and population shares in middle to upper level positions in the bureaucracy, police and the public sector in India. Reserved posts remain vacant due to the paucity of candidates meeting minimum requirements. See Varma (2012).

$[s_{it}p_i - q_{it}x_{it} + n_i M_{it}]$, subject to the conflict success function (2.1) above.[10] My formulation also abstracts from conflict financing issues, which is the focus of much of the literature on conflicts in multiple battlefields (e.g. Maxwell and Reuveny 2005; Kvasov 2007; and Reuveny, Maxwell, and Davis 2011; see Kovenock and Roberson [2012] for a recent survey).

Using (2.2), for period t, recalling that the *relative* conflict outcome share is denoted by: $s_{BA,t} \equiv \frac{s_{Bt}}{s_{At}}$, the first order conditions yield:

$$\frac{(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}x_{Bt}}{[x_{At}+(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}x_{Bt}]^2} = \frac{q_{At}}{p_A} \equiv C_{At}, \tag{2.3}$$

$$\frac{(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}x_{At}}{[x_{At}+(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}x_{Bt}]^2} = \frac{q_{Bt}}{p_B} \equiv C_{Bt}; \tag{2.4}$$

whereC_{it} is the ratio of the per unit conflict cost to benefit. Together, (2.3) and (2.4) yield:

$$\frac{x_{Bt}}{x_{At}} = \frac{q_{ABt}}{p_{AB}} \equiv C_{AB,t}; \tag{2.5}$$

where $q_{AB,t} \equiv \frac{q_{At}}{q_{Bt}}$ is the relative cost of deploying a unit of the conflict input, while $p_{AB} \equiv \frac{p_A}{p_B}$ is the relative benefit from acquiring the item under contestation, and $C_{AB,t} \equiv \frac{C_{At}}{C_{Bt}}$. Thus, the variable $C_{AB,t}$ measures the relative cost of conflict inputs for A, normalized by the relative benefit to A from acquiring the item under contestation. Using (2.3)-(2.5), we have individual conflict input deployments in *t* for any given value of the relative share in period $t-1$ and the initial balance of power:

$$x_{At} = \frac{(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}}{C_{Bt}[1+(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}C_{AB,t}]^2}; \tag{2.6}$$

$$x_{Bt} = \frac{(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}C_{AB,t}}{C_{Bt}[1+(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}C_{AB,t}]^2}. \tag{2.7}$$

10 Dasgupta and Kanbur (2011, 2007 and 2005a) have argued that mutual benefit from sharing of group-specific public goods, generated by decentralized voluntary contributions of group members, may play an important role in reducing welfare inequality, conflict and opportunistic behaviour within a group.

Together, (2.6)-(2.7) yield total conflict expenditure in battlefield (period) t:

$$E_t \equiv q_{At}x_{At} + q_{Bt}x_{Bt} = \frac{(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}}{[1+(s_{BA,0})^{1-\theta}(S_{BA,t-1})^{\theta}C_{AB,t}]^2}[\frac{q_{At}+C_{AB,t}q_{Bt}}{C_{Bt}}]. \qquad (2.8)$$

E_t provides a measure of both the intensity of conflict and the extent of social wastage thereby in a period; (2.6)-(2.7) yield the relative share in t as a function of that in the preceding and initial periods:

$$s_{BA,t} = (s_{BA,0})^{1-\theta}(s_{BA,t-1})^{\theta}\frac{x_{Bt}}{x_{At}} = (s_{BA,0})^{1-\theta}(s_{BA,t-1})^{\theta}C_{AB,t}. \qquad (2.9)$$

Using (2.9) and recalling that $s_{BA,t} \equiv \frac{s_{Bt}}{s_{At}}$, $s_{Bt} + s_{At} = 1$, we get:

$$s_{At} = [(s_{BA,0})^{1-\theta}(s_{BA,t-1})^{\theta}C_{AB,t} + 1]^{-1} = \frac{1}{s_{BA,t}+1},$$

$$s_{Bt} = \frac{(s_{BA,0})^{1-\theta}(s_{BA,t-1})^{\theta}C_{AB,t}}{[(s_{BA,0})^{1-\theta}(s_{BA,t-1})^{\theta}C_{AB,t}+1]} = \frac{s_{BA,t}}{s_{BA,t}+1}. \qquad (2.10)$$

Repeated use of (2.9) yields:

$$s_{BA,t} = s_{BA,0}[C_{AB,1}{}^{\theta^{t-1}}C_{AB,2}{}^{\theta^{t-2}} \dots C_{AB,t-1}{}^{\theta}C_{AB,t}]. \qquad (2.11)$$

The initial settlement, $s_{BA,0}$, and hence relative ethnic control over institutions in period 1, is a parameter in our model. The community-specific costs of conflict inputs are like-wise given, reflecting structural conditions in the economy (which determine the opportunity cost of converting productive labour to activist labour) and/or the extent of access to global arms and mercenary markets. Traditional forms of social organization, such as caste, clan or religious institutions, as well as norms of behaviour such as mandatory participation in collective annual festivals or pilgrimage, may reduce or enhance the transaction and search costs associated with recruiting and training activist labour, and organizing conflict activities in general. Variations in the functioning of such structures across communities may thus contribute to differences in conflict costs.[11]

11 The impact of population size on conflict cost appears intuitively ambiguous. Larger community size is likely to provide a larger pool of individuals better endowed with innate psychological and physical abilities ('aggression') that can be substituted for costly training in the production of activist labour. However,

Assumption 2.1 For all $t \in \{1,2,3,..,\}$, $[q_{AB,t} = \overline{q}_{AB}]$, with $\frac{\overline{q}_{AB}}{p_{AB}} > 1$.

Assumption 2.1 restricts attention to the case where the relative cost of military inputs is constant over time, and A has a (benefit-normalized) cost disadvantage. Intuitively, one community may have a persistent normalized cost advantage in conflict when its opportunity cost of deploying activist labour is lower (say, because it controls land that is less fertile or has a larger pool of unemployed), or when some external government/agency such as the US, NATO or the EU secularly subsidizes retention, training and equipment expenses, including arms, for its activist labour, or when it is better placed to financially exploit the contested item. One community may also have traditional internal institutions that are better at providing ready sites of activist recruitment and training.

Let $\overline{C}_{AB} \equiv \frac{\overline{q}_{AB}}{p_{AB}}$ be the normalized relative cost. Given Assumption 2.1, (2.11) yields:

$$\text{for all } t \in \{0,1,2,3,..\},\ s_{BA,t} = s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1-\theta^t}{1-\theta}\right)}. \tag{2.12}$$

By (2.12), higher $s_{BA,0}$ implies greater $s_{BA,t}$ in every period *t*. Thus, an initial positive shock to a combatant's share gets transmitted over time: it causally generates greater share in every future period due to the productivity advantage it confers. Hence history, modelled as an initial division of control over institutions, exerts a permanent influence in our model. From (2.12),

$$\lim_{t \to \infty} s_{BA,t} = s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1}{1-\theta}\right)}. \tag{2.13}$$

Equation (2.13) shows that the degree of institutional flexibility with regard to ethnic influences, as measured by the parameter θ, continues to influence the relative share even in the limit. Initial conditions, as measured by the initial ethnicization ratio$s_{BA,0}$, do so as well. The relative share

as is commonly argued, larger communities may also find it more difficult to maintain community cohesion and prevent free-riding (see, for example, Olson 1965; Hardin 1982; and Esteban and Ray 2001).

asymptotically converges to the steady state level, $s_{BA,0}(\overline{C}_{AB})^{\frac{1}{1-\theta}}$. Lastly, since $\theta \in (0,1)$, (2.12) yields:

$$\text{for all } t \in \{1,2,3,\dots\},\ \frac{s_{BA,t+1}}{s_{BA,t}} = (\overline{C}_{AB})^{\theta^t}. \tag{2.14}$$

We can now characterize the movement of the relative share of the contested resource, and the consequent pay-offs, over time. Together, (2.12)-(2.14) immediately yield the following.

Proposition 2.1: Let Assumption 2.1 hold. Then $\mathrm{s}_{\mathrm{BA,t}}$ is monotonically increasing, at a decreasing rate, over time; $\mathrm{s}_{\mathrm{BA,t}}$ converges to $\mathrm{s}_{\mathrm{BA,0}}\ (\overline{\mathrm{C}}_{\mathrm{AB}})^{\frac{1}{1-\theta}}$.

$\overline{\mathrm{C}}_{\mathrm{AB}} > 1$ by Assumption 2.1. The relative share of B then increases over time, adjusting monotonically (and asymptotically) towards its steady state level ($s_{BA,0}(\overline{\mathrm{C}}_{\mathrm{AB}})^{\frac{1}{1-\theta}}$). Analogous results follow for the pay-offs to the two communities, as we proceed to summarize in Corollary 1 below.

From (2.6), (2.7), (2.9) and (2.10), ignoring the uncontested part $n_i M_{it}$, we get the contest pay-off for each community per period, net of conflict cost:

$$\text{for all } t \in \{1,2,3,\dots\};\ F_{A,t} = \frac{p_A}{[s_{BA,t}+1]^2}, F_{B,t} = \frac{p_B}{\left[1+\frac{1}{s_{BA,t}}\right]^2}. \tag{2.15}$$

Notice that, by (2.15), returns from engaging in conflict, net of costs, are positive: thus, conflict does not exhaust the entire prize. In light of (2.15), Proposition 2.1 yields the following.

Corollary 2.1: Let Assumption 1 hold. Then $F_{A,t}$ is monotonically decreasing and $F_{B,t}$ is monotonically increasing over time. $F_{A,t}$ converges to $\frac{p_A}{\left[s_{BA,0}\bar{C}_{AB}^{\frac{1}{1-\theta}}+1\right]^2}$, while $F_{B,t}$ converges to $\frac{p_B}{\left[1+\frac{1}{s_{BA,0}\bar{C}_{AB}^{\frac{1}{1-\theta}}}\right]^2}$.

Lastly, consider the behaviour of total expenditure on conflict by the two communities. Rearranging the expression in (2.8), and using (2.9), total expenditure on conflict is given by:

$$E_t = \frac{(p_A + p_B)}{\left[\frac{1}{S_{BA,t}^{\frac{1}{2}}}+(S_{BA,t})^{\frac{1}{2}}\right]^2}. \tag{2.16}$$

Recalling (2.13), (2.16) yields:

$$\lim_{t\to\infty} E_t = \frac{(p_A + p_B)}{\left[\frac{1}{s_{BA,0}^{\frac{1}{2}}(\bar{C}_{AB})^{\frac{1}{2(1-\theta)}}}+s_{BA,0}^{\frac{1}{2}}(\bar{C}_{AB})^{\frac{1}{2(1-\theta)}}\right]^2}. \tag{2.17}$$

Equation (2.16) implies that total conflict increases in $S_{BA,t}$ in the $(0,1)$ interval, attains its maximum at $S_{BA,t} = 1$, and subsequently declines in $S_{BA,t}$. Recalling that, by (2.12), $s_{BA,2} = s_{BA,0}\bar{C}_{AB}^{\,1+\theta}$, (2.16), (2.17) and Proposition 2.1(ii) immediately yield the following.

Corollary 2.2: Let Assumption 2.1 hold. Then total (per-period) expenditure on conflict:

(i) converges to $\frac{(p_A + p_B)}{\left[\frac{1}{s_{BA,0}^{\frac{1}{2}}(\bar{C}_{AB})^{\frac{1}{2(1-\theta)}}}+s_{BA,0}^{\frac{1}{2}}(\bar{C}_{AB})^{\frac{1}{2(1-\theta)}}\right]^2}$;

(ii) monotonically increases over time if $s_{BA,0}\bar{C}_{AB}^{\left(\frac{1}{1-\theta}\right)} \leq 1$;

(iii) monotonically decreases over time if $s_{BA,0}\overline{C}_{AB} \geq 1$; it first increases and subsequently decreases if $[s_{BA,0}\overline{C}_{AB}^{\left(\frac{1}{1-\theta}\right)} > 1$ and $s_{BA,0}\overline{C}_{AB}^{1+\theta} < 1]$. .

By Corollary 2.2(ii), social waste from conflict *increases* over time if the limiting value of $s_{BA,t}$ does not exceed unity, as the success ratio monotonically increases to approach its steady state (Proposition 2.1(ii)). By Assumption 2.1, B has a relative (normalized) cost advantage. Thus, Corollary 2.2(ii) implies that conflict increases secularly if the initial settlement puts the community with the lower conflict cost at a great disadvantage. Corollary 2.2(iii) implies that conflict decreases secularly if the period 1 relative share of B is not less than the conflict-maximizing value of unity. The most interesting case obtains if the initial (period 1) relative share of B is less than its conflict-maximizing value of unity, but its steady state value is greater than unity. Then conflict first increases (over $(s_{BA,1}, 1)$), eventually reaching its peak, and declines subsequently (over $(1, s_{AB,0}\overline{C}_{AB}^{\left(\frac{1}{1-\theta}\right)})$). The relative share of B increases asymptotically towards its steady state value of $s_{AB,0}\overline{C}_{AB}^{\left(\frac{1}{1-\theta}\right)}$ (Proposition 2.1(i)).

2.3 Conflict and institutional accommodation

We now proceed to specify how the extent of inter-temporal institutional accommodation of mass ethnic pressures affects the evolution of conflict and the distribution of contested income over time.

Proposition 2.2: Let Assumption 2.1 hold.

(i) For all $t \in \{2,3,...\}$, $\frac{\partial s_{BA,t}}{\partial \theta}, \frac{\partial F_{Bt}}{\partial \theta} > 0, \frac{\partial F_{At}}{\partial \theta} < 0$.

(ii) $\lim_{t\to\infty,\theta\to 1} s_{BA,t} = \infty$, $\lim_{t\to\infty,\theta\to 1} F_{B,t} = p_B$ and $\lim_{t\to\infty,\theta\to 1} F_{A,t} = 0$.

(iii) $\frac{\partial E_t}{\partial\theta} < 0$ for every period $t \in \{2,3,4,\ldots\}$ if $s_{BA,0}\overline{C}_{AB}^{\,1+\theta} > 1$; $\frac{\partial E_t}{\partial\theta} > 0$ for every period $t \in \{2,3,4,\ldots\}$ if $s_{BA,0}\overline{C}_{AB}^{\,\frac{1}{1-\theta}} \le 1$; if $[s_{BA,0}\overline{C}_{AB}^{\,1+\theta} < 1$ and $s_{BA,0}\overline{C}_{AB}^{\,\frac{1}{1-\theta}} > 1]$, then there exists $t^* \in \{3,4,5,\ldots\}$ such that $[\frac{\partial E_t}{\partial\theta} > 0$ if $2 \le t < t^*]$ and $[\frac{\partial E_t}{\partial\theta} < 0$ if $t > t^*]$.

(iv) $\lim_{t\to\infty,\theta\to 1} E_t = 0$.

Proof: Recall that $\overline{C}_{AB} > 1$ by Assumption 2.1, and note that $\left(\frac{1-\theta^t}{1-\theta}\right) = (1+\theta+\cdots+\theta^{t-1})$. Part (i) of Proposition 2.2 follows from (2.12) and (2.15). Part (ii) of Proposition 2.2 follows from (2.13) and Corollary 2.1. Now recall that, by (2.12), $s_{BA,1} = s_{BA,0}\overline{C}_{AB}, s_{BA,2} = s_{BA,0}(\overline{C}_{AB})^{(1+\theta)}$, and, by Proposition 2.1, $s_{BA,t}$ is monotonically increasing in time. Hence, if $s_{BA,0}\overline{C}_{AB}^{\,1+\theta} > 1$, $s_{BA,t} > 1$ for all $t \ge 2$. Conversely, recalling Proposition 2.1, if $s_{BA,0}\overline{C}_{AB}^{\,\frac{1}{1-\theta}} \le 1$ then $s_{BA,t} < 1$ for all $t \ge 1$. If $[s_{BA,0}\overline{C}_{AB}^{\,1+\theta} < 1$ *and* $s_{BA,0}\overline{C}_{AB}^{\,\frac{1}{1-\theta}} > 1]$, then there exists $t^* \in \{3,4,\ldots\}$ such that $s_{BA,t} < 1$ (resp. > 1) if t < (resp.>) t^*. From (2.16), $\frac{\partial E_t}{\partial s_{BA,t}} > 0$ if $s_{BA,t} \in (0,1)$, and $\frac{\partial E_t}{\partial s_{BA,t}} < 0$ if $s_{BA,t} > 1$. Recalling that, by (2.12), $\frac{\partial s_{BA,t}}{\partial\theta} > 0$, Proposition 2.2(iii) follows. Part (iv) of Proposition 2.2 follows from (2.17). ■

By Proposition 2.2(i), greater institutional flexibility magnifies B's cost advantage, increasing both B's success and pay-off in every period beyond the first. Proposition 2.2(ii) shows that an *infinitesimally small* cost advantage for B suffices to make that community's relative pay-off converge to an *arbitrarily high* value when the extent of institutional flexibility is sufficiently close to unity. Thus, the mediation of high institutional pliability translates even minor cost advantages into major pay-off gains over time. Parts (iii) and (iv) of Proposition 2.2 show that greater institutional accommodation has ambiguous (non-monotone) consequences for conflict. When one community has even a minimal cost advantage, a sufficiently high degree of institutional malleability serves to make conflict converge to an arbitrarily low level. If the initial settlement does not disadvantage the low cost combatant, B, too much (if at all) relative to its cost advantage, so that it receives the higher share in period 2 ($s_{BA,2} = s_{BA,0}\overline{C}_{AB}^{\,1+\theta} > 1$), any increase in institutional flexibility, by expanding B's dominance, reduces aggregate conflict. However, if the initial settlement counteracts B's cost advantage so much that B always receives the lower share, a marginal increase in institutional flexibility, by reducing A's dominance, increases conflict in every period. Now consider the intermediate case where the initial settlement counteracts B's cost advantage to a limited extent, so that B receives the lower share in initial periods but the higher share in later periods. Then, a marginal rise in institutional flexibility increases conflict initially, while reducing it in later periods.

Consider an initial settlement which *more than negates* the cost advantage of community B, so that it receives the lower share in period 1 ($s_{AB,0}\overline{C}_{AB} < 1$). Starting from an initial situation of complete institutional rigidity ($\theta = 0$), consider an increase in institutional flexibility (θ). For relatively low levels of institutional flexibility, the steady state relative share of B ($s_{AB,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}$) remains below 1. In this range, any increase in institutional flexibility increases B's share and pay-off, as well as aggregate conflict, while reducing A's share as well as pay-off, from period 2 onwards. Further increase in θ, while keeping the period 2 relative share of B ($s_{AB,0}\overline{C}_{AB}^{\,1+\theta}$) below unity, pushes the steady state relative share of B ($s_{AB,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}$) above 1. These two conditions will hold simultaneously for a range of θ. Increases in institutional flexibility in this range will increase conflict in initial periods while reducing it in later periods; B's share and pay-off will increase, and A's pay-off fall, throughout. If $s_{AB,0}\overline{C}_{AB}^{\,2} > 1$, then there must exist a range of values of θ close to 1 for which B receives the higher share in every period from 2 onwards. In this range, an increase in θ will reduce conflict in every period. Notice that such a range will not exist if the initial settlement sufficiently negates B's cost advantage. Thus, in sum, Proposition 2.2(iii) implies that greater institutional responsiveness to ethnicizing political pressures involves an explicit trade-off between efficiency (in the sense of reducing social wastage due to conflict) and equity across the communities; its impact on either of these two policy goals is however *non-monotone*.

2.4 Conflict and social welfare

We now clarify further the trade-off between equity and efficiency (conflict reduction) identified above, by embedding the analysis within a social welfare calculus. We also show how this trade-off, and thus social welfare, is affected by permanent changes in the relative cost advantage and the extent of institutional flexibility, as well as by short-term (i.e. one-off) conflict success shocks.

Let per period social welfare be given by a symmetric CES function of community pay-offs: $W_t = [{F_{At}}^{\sigma} + {F_{Bt}}^{\sigma}]^{\frac{1}{\sigma}}$; where $\sigma \in (-\infty, 0) \cup (0,1]$. Since $\frac{dF_{Bt}}{dF_{At}}|_{dW_t=0} = -(\frac{F_{Bt}}{F_{At}})^{1-\sigma}$, the parameter σ measures the normatively determined trade-off between efficiency (the level of total pay-off) and inter-community, or *horizontal*, equity (the distribution of pay-offs): a higher value of σ implies that a marginal reduction in income for the poorer community (say A) has to be attended by a lower increase in the income of the richer community to keep social welfare invariant. Hence, higher σ implies greater privileging of efficiency over horizontal equity. In the limiting case of $\sigma = 1$, the privileging of efficiency to the complete exclusion of equity considerations, maximizing social welfare simply entails maximizing total pay-off, and therefore, minimizing conflict. It is well known that $\lim_{\sigma \to 0} W_t = (F_{At}F_{Bt})^{\frac{1}{2}}$; thus, the geometric mean, or Cobb-Douglas, is generated as another limiting case. Furthermore, $\lim_{\sigma \to -\infty} W_t = \min\{F_{At}, F_{Bt}\}$, so that the Rawlsian maximin criterion, which privileges equity to the exclusion of efficiency considerations altogether, comprises a third limiting case. The elasticity of substitution is given by $\tau = \frac{1}{1-\sigma}$, so that $\tau \in (0,1) \cup (1, \infty]$, with greater elasticity intuitively implying a stronger emphasis on efficiency considerations, relative to equity.

To concentrate on the welfare properties of the conflict process and the distribution of *contested* income, and for ease of exposition, We shall ignore non-contestable resources, i.e., assume that, for all t, $M_{At} = M_{Bt} = 0$. Then, recalling (2.15), and assuming, for notational simplicity, that the two communities have identical valuations (i.e. $p_A = p_B \equiv p$),

$$\frac{W_t}{p} = \left(\frac{(s_{BA,t})^{2\sigma}+1}{[s_{BA,t}+1]^{2\sigma}}\right)^{\frac{1}{\sigma}}. \tag{2.18}$$

Since the relative share depends on the relative, not absolute, cost of conflict (recall (2.12) - (2.13)), social welfare is independent of *equi-proportionate changes* in conflict costs. Only changes in conflict costs that affect one community disproportionately, thereby altering the relative conflict cost, can affect social welfare. Now, noting Corollary 2.1, the steady state level of social welfare, W_S, is given by:

$$W_S = p\left(\frac{{s_{BA,0}}^{2\sigma}\overline{C}_{AB}^{\frac{2\sigma}{1-\theta}}+1}{\left[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1\right]^{2\sigma}}\right)^{\frac{1}{\sigma}}. \tag{2.19}$$

Using (2.19), we get the following.

Proposition 2.3: Let Assumption 2.1 hold, and suppose $\mathrm{p_A = p_B \equiv p}$. Then:

(i) steady state social welfare is decreasing (resp. increasing) in θ if [σ <(resp. >) $\frac{1}{2}$ and $\mathrm{s_{BA,0}\overline{C}_{AB}} \geq 1$]; it is initially increasing (resp. decreasing) and subsequently decreasing (resp. increasing) in θ over (0,1) if [σ <(resp. >) $\frac{1}{2}$ and $\mathrm{s_{BA,0}\overline{C}_{AB}} < 1$], the maximum (resp. minimum) being given by

$[\mathrm{s_{BA,0}\overline{C}_{AB}}^{\frac{1}{1-\theta}} = 1]$;

(ii) steady state social welfare is decreasing (resp. increasing) in $\mathrm{\overline{C}_{AB}}$ if [σ <(resp. >) $\frac{1}{2}$ and $\mathrm{s_{BA,0}} \geq 1$]; it is initially increasing (resp. decreasing) and subsequently decreasing (resp. increasing) in $\mathrm{\overline{C}_{AB}}$

over $(1,\infty)$ if [σ <(resp. >) $\frac{1}{2}$ and $s_{BA,0} < 1$], the maximum (resp. minimum) being given by $[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}} = 1]$;

(iii) steady state social welfare is initially increasing (resp. decreasing) and subsequently decreasing (resp. increasing) in $s_{BA,0}$ over $(0,\infty)$ if [σ <(resp. >) $\frac{1}{2}$], the maximum (resp. minimum) being given by $[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}} = 1]$.

Proof: Using (2.19), and taking a positive monotone transformation,

$$W_S = \left(\frac{1}{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1}\right)^{2\sigma} + \left(\frac{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}}{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1}\right)^{2\sigma},$$

so that:

$$\frac{\partial W_S}{\partial \theta} = \left(\frac{s_{BA,0}}{\left[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1\right]^2}\right)[\dddot{W}'\left(\frac{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}}{\left[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1\right]}\right) - \dddot{W}'\left(\frac{1}{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1}\right)]\frac{\partial\left(\overline{C}_{AB}^{\frac{1}{1-\theta}}\right)}{\partial\theta};$$

where $\dddot{W}(z) = z^{2\sigma}$. Clearly, if $\sigma <$ (resp. >) $\frac{1}{2}$, then $[\dddot{W}'(z_1) < \dddot{W}'(z_2)$ iff $z_1 >$ (resp. <) $z_2]$. Now, $s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}} > 1$ for all $\theta \in (0,1)$ if $s_{BA,0}\overline{C}_{AB} \geq 1$. Hence, if $[\sigma < \frac{1}{2}$ and $s_{BA,0}\overline{C}_{AB} \geq 1]$, then $[\dddot{W}'\left(\frac{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}}{\left[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1\right]}\right) - \dddot{W}'\left(\frac{1}{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1}\right)] < 0$ for all $\theta \in (0,1)$. Noting that $s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}} > 1$ for θ sufficiently close to 1, and that $\overline{C}_{AB}^{\frac{1}{1-\theta}}$ is monotonically increasing in θ (since $\overline{C}_{AB} > 1$ by Assumption 2.1), if $[\sigma < \frac{1}{2}$ and $s_{BA,0}\overline{C}_{AB} < 1]$, there must exist$\theta^* \in (0,1)$ such that $[\dddot{W}'\left(\frac{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}}{\left[s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1\right]}\right) - \dddot{W}'\left(\frac{1}{s_{BA,0}\overline{C}_{AB}^{\frac{1}{1-\theta}}+1}\right)]>$ (resp. <) 0 if $\theta <$ (resp. >) θ^*. Part (i) of Proposition 2.3 follows for $\sigma < \frac{1}{2}$. The argument is symmetric for $\sigma > \frac{1}{2}$.

Recalling that $\overline{C}_{AB} > 1$ by Assumption 2.1, parts (ii) and (iii) of Proposition 2.3 follow by arguments exactly analogous to that presented in the proof of part (i) of Proposition 2.3.

The central message of Proposition 2.3 is that steady state social welfare behaves in a non-monotone fashion, with regard to all three policy parameters in the model, if the initial settlement $s_{BA,0}$ more than negates a community's cost advantage. Starting from an original situation of perfect rigidity $(\theta = 0)$, and an initial settlement that more than negates B's structural cost advantage, greater institutional flexibility initially reduces steady state inequality (by improving B's relative share), but increases steady state conflict. Eventually, a point of equal shares is reached. Beyond this threshold, however, further increases in institutional flexibility increase inequality (as B achieves greater dominance), while reducing conflict. Suppose equity is privileged to a great extent, so that the elasticity of substitution is less than 2. Then social welfare initially rises and subsequently falls with increases in institutional flexibility. The opposite holds when efficiency is privileged more, so that the elasticity of substitution is greater than 2. Similar mechanisms operate for changes in the relative cost and the initial settlement. Notice that, when equity is privileged more, so that the elasticity of substitution is less than 2, the welfare maximizing level of any one of the three variables is lower, the higher the other two variables. Conversely, when efficiency is privileged more, the welfare minimizing level of any one of the three variables is lower, the higher the other two variables. It follows that, in the first case, setting high values of all three variables constitutes an incoherent policy package, but such generalized high values constitute a coherent policy package in the second case. Since social welfare converges to its steady state level (Proposition 2.1), results analogous to those in Proposition 2.3 must hold for *every period* subsequent to the passage of some finite number of periods.

Lastly, what happens to social welfare outside the steady state when the system receives a one-time shock to the relative share or a permanent shock to either the relative cost or the extent of institutional flexibility? How does social welfare adjust dynamically to such shocks?

Proposition 2.4: Let Assumption 2.1 hold, and let $p_A = p_B \equiv p$. Then:

(i) if $[s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1}{1-\theta}\right)} \leq 1]$ then W_t is monotonically decreasing (resp. increasing) over time, with $[\frac{\partial W_t}{\partial s_{BA,0}}, \frac{\partial W_t}{\partial \theta}, \frac{\partial W_t}{\partial \overline{C}_{AB}} <$ (resp. $>$) 0 for $t \geq 2]$, when $\sigma >$ (resp. $<$) ½;

(ii) if $[s_{BA,0}\overline{C}_{AB} \geq 1]$, then W_t is monotonically increasing (resp. decreasing) over time, with $[\frac{\partial W_t}{\partial s_{BA,0}}, \frac{\partial W_t}{\partial \theta}, \frac{\partial W_t}{\partial \overline{C}_{AB}} >$ (resp. $<$) 0 for $t \geq 2]$, when $\sigma >$ (resp. $<$) ½;

(iii) if $[s_{BA,0}\overline{C}_{AB}^{\;1+\theta} < 1, s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1}{1-\theta}\right)} > 1]$, then there exists $t^* \in \{3,4,5,\dots\}$ such that: (a)W_t decreases with time over $[1, t^*)$ and increases over (t^*, ∞), with $[\frac{\partial W_t}{\partial s_{BA,0}}, \frac{\partial W_t}{\partial \theta}, \frac{\partial W_t}{\partial \overline{C}_{AB}} <$ (resp. $>$) 0 if $t <$ (resp.$>$) t^* and $t \geq 2]$, when $\sigma >$½; and (b)W_t increases with time over $[1, t^*)$ and decreases over (t^*, ∞); with $[\frac{\partial W_t}{\partial s_{BA,0}}, \frac{\partial W_t}{\partial \theta}, \frac{\partial W_t}{\partial \overline{C}_{AB}} >$ (resp. $<$) 0 if $t <$ (resp.$>$) t^* and $t \geq 2]$ when $\sigma <$½.

Proof: From the expression for per period social welfare ((2.18) above), we have:

$$\frac{\partial W_t}{2p\partial s_{BA,t}} = \left(\frac{(s_{BA,t})^{2\sigma}+1}{[s_{BA,t}+1]^{2\sigma}}\right)^{\frac{1}{\sigma}-1} [\frac{(s_{BA,t})^{2\sigma-1}-1}{[s_{BA,t}+1]^{2\sigma+1}}]. \tag{2.20}$$

By (2.20), when $\sigma > \frac{1}{2}$, $[\frac{\partial W_t}{\partial s_{BA,t}} < 0$ if $s_{BA,t} \in (0,1)]$ and $[\frac{\partial W_t}{\partial s_{BA,t}} > 0$ if $s_{BA,t} > 1]$; when $\sigma < \frac{1}{2}$,

$$[\frac{\partial W_t}{\partial s_{BA,t}} > 0 \quad \text{if } s_{BA,t} \in (0,1)] \text{ and } [\frac{\partial W_t}{\partial s_{BA,t}} < 0 \text{ if } s_{BA,t} > 1]. \tag{2.21}$$

Furthermore, (2.20) yields:

$$\frac{\partial W_t}{2p\partial s_{BA,0}} = \left(\frac{(s_{BA,t})^{2\sigma}+1}{[s_{BA,t}+1]^{2\sigma}}\right)^{\frac{1}{\sigma}-1} [\frac{(s_{BA,t})^{2\sigma-1}-1}{[s_{BA,t}+1]^{2\sigma+1}}] \frac{\partial s_{BA,t}}{\partial s_{BA,0}}, \tag{2.22}$$

$$\frac{\partial W_t}{2p\partial \theta} = \left(\frac{(s_{BA,t})^{2\sigma}+1}{[s_{BA,t}+1]^{2\sigma}}\right)^{\frac{1}{\sigma}-1} [\frac{(s_{BA,t})^{2\sigma-1}-1}{[s_{BA,t}+1]^{2\sigma+1}}] \frac{\partial s_{BA,t}}{\partial \theta}, \tag{2.23}$$

$$\frac{\partial W_t}{2p\partial \overline{C}_{AB}} = \left(\frac{(s_{BA,t})^{2\sigma}+1}{[s_{BA,t}+1]^{2\sigma}}\right)^{\frac{1}{\sigma}-1} [\frac{(s_{BA,t})^{2\sigma-1}-1}{[s_{BA,t}+1]^{2\sigma+1}}] \frac{\partial s_{BA,t}}{\partial \overline{C}_{AB}}. \tag{2.24}$$

Recall that, by (2.12), $s_{BA,t} = s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1-\theta^t}{1-\theta}\right)}$. Thus, $\frac{\partial s_{BA,t}}{\partial s_{BA,0}}, \frac{\partial s_{BA,t}}{\partial \overline{C}_{AB}} > 0$. Furthermore, by Proposition 2.2(i), for all $t \in \{2,3,\ldots\}$, $\frac{\partial s_{BA,t}}{\partial \theta} > 0$. Now recall that, by (2.12), $s_{BA,t}$ is increasing over time and asymptotically converges to $s_{BA,0}(\overline{C}_{AB})^{\frac{1}{1-\theta}}$. Hence, if $[s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1}{1-\theta}\right)} \leq 1]$, then [for all $t \geq 1$, $s_{BA,t} < 1$]. Conversely, noting that by (2.12), $s_{BA,1} = s_{BA,0}\overline{C}_{AB}$, $[s_{BA,0}\overline{C}_{AB} \geq 1]$ implies [for all $t \geq 2$, $s_{BA,t} > 1$]. Lastly, noting that by (2.12), $s_{BA,2} = s_{BA,0}\overline{C}_{AB}^{\,1+\theta}$, if $[s_{BA,0}\overline{C}_{AB}^{\,1+\theta} < 1, s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1}{1-\theta}\right)} > 1]$, then there exists $t^* \in \{3,4,5,\ldots\}$ such that [$s_{BA,t} <$ (resp. >) 1 if t < (resp.>) t^*]. Using (2.20) - (2.24), Proposition 2.4 follows.

Note that that the claims made in Proposition 2.4 with regard to $\frac{\partial W_t}{\partial s_{BA,0}}, \frac{\partial W_t}{\partial \overline{C}_{AB}}$ actually hold for all $t \geq 1$. Suppose $\sigma >$ ½ (the elasticity of substitution exceeds 2), so that the social welfare function does not privilege equity too highly. By Assumption 2.1, B has a secular cost advantage. Consider an initial settlement that is large enough to counter-act B's cost-advantage completely, so that it suffices to depress B's share below that of A in the steady state, and therefore, in every period. Then B's share increases monotonically over time, as B's cost advantage partially, but increasingly, negates the adverse initial settlement. By Proposition 2.4(i), social welfare must then be above its steady state level in every period, monotonically declining to asymptotically converge to the latter. The lower the initial relative share of B, the lower the level of conflict (though the more unequal the distribution), hence the higher the social welfare in each period. Conversely, if the initial settlement only attenuates B's cost advantage without reversing it, so that B receives the higher share even in period 1, then, by Proposition 2.4(ii), social welfare must be below its steady state level in every period, monotonically increasing to asymptotically converge to the latter. The higher the initial share of B, the lower the level of conflict, thus the higher the social welfare in each period. Opposite conclusions follow if the social welfare function privileges equity to a great extent (the elasticity of substitution is less than 2). The case with shocks to either the relative cost or the degree of institutional flexibility is identical. Proposition 2.4(iii) considers the intermediate case, where the initial shock is large enough to move B's share below that of A in at least the first two periods, but is not enough to counteract B's cost advantage in later periods. Social welfare and the impact of the initial share on it both then move in a non-monotone fashion, as specified in Proposition 2.4(iii). The welfare impacts of shocks to either the relative cost or the degree of institutional flexibility are likewise non-monotone.

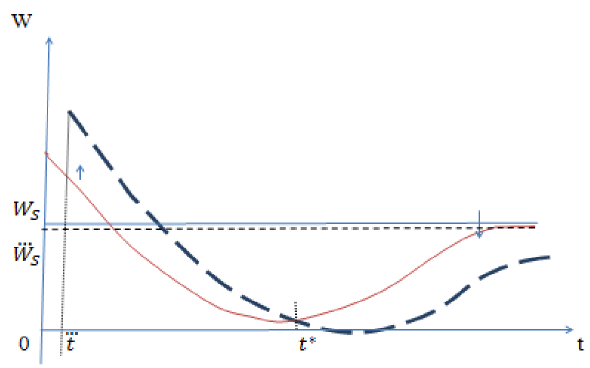
Since all relevant functions are continuous, results qualitatively identical to Propositions 2.3-2.4 hold even when the two communities differ in their valuation of the contested resource. The threshold value of the elasticity of substitution, with regard to which these results are articulated, now comes to differ from 2, with its magnitude depending on the exact relative valuation of the contested resource.

2.5 Discussion

The analysis above carries important implications for understanding the connections between an initial institutional settlement brought about by historical shocks (colonialism, foreign intervention or an authoritarian regime), institutional ethnicization and the subsequent co-evolution of conflict and horizontal inequality within a society. Suppose a persistent and increasing conflict between ethnic groups over resource sharing is observed. The model would rationalize this either as a permanent phenomenon, in terms of the parametric configuration $[s_{BA,0}(\overline{C}_{AB})^{\left(\frac{1}{1-\theta}\right)} \leq 1]$ (Corollary 2.2(ii)) or as a transient one, in terms of $[s_{BA,0}\overline{C}_{AB}^{\left(\frac{1}{1-\theta}\right)} > 1$ and $s_{BA,0}\overline{C}_{AB}^{1+\theta} < 1]$ (Corollary 2.2(iii)). Suppose the policy objectives can be aggregated in terms of the normative adoption of an elasticity of substitution greater than 2. In the first case, Proposition 2.4(i) would imply that a one-time shock to the relative success ratio by an external agent such as the UN, NATO or other multi-lateral agencies, or some foreign government(s), that reduces B's share marginally below that prevailing on the adjustment path in some time period, say through a one-off subsidization of A's military expenses in that period, or through direct politico-military intervention, is likely to permanently reduce conflict. Institutional reforms that make institutions less responsive to immediate ethnic pressures (lower θ), say by permanently freezing the ethnic composition of large segments of the administrative apparatus at its current pattern, would further reduce conflict and improve social welfare. Both these interventions would however permanently worsen horizontal equity. Hence, they would be opposed by those who normatively privilege horizontal equity over conflict reduction, to the extent of, in effect, espousing an elasticity of substitution less than 2. For both groups, however, the policy stance would be *time invariant*, given their normative prior (i.e., elasticity of substitution). Thus, policy disagreements would hinge entirely on normative differences in the valuation of equity vis-à-vis efficiency, as formalized by differences in the elasticity of substitution chosen for policy valuation.

This policy clarity however disappears in the second, arguably empirically more common, case where the initial historical settlement imposed say by a colonial power is large enough to convert the dominant community (the one with a secular cost advantage) into the dominated one (in the sense of receiving the lower share) in the initial periods, but not large enough to do so permanently. By Proposition 2.4(iii), in the absence of further intervention, social welfare would evolve, over time, according to the unbroken schedule in Figure 1 below, given the normative privileging of efficiency over horizontal equity. As the relative share of B increases over time, reflecting this community's secular cost advantage, conflict first rises, reducing social welfare, as the relative share approaches unity. Beyond this, as B increases its share more and more over A, conflict falls, increasing social welfare. Thus, the time path of social welfare exhibits the U-shape portrayed in Figure 1 below.

FIGURE 1



Conflict increases over time, reducing social welfare, till t^*, and decreases subsequently, increasing welfare. For those prioritizing conflict reduction, a case for policy intervention may thus be perceived if the society is observed before t^*. Suppose this leads to an intervention in period $\ddot{t}$, which, as earlier, takes the form of a marginal reduction of B's share below its adjustment path and/or a reduction in the degree of institutional flexibility. Then, by Proposition 2.4(iii), the society permanently shifts to a new adjustment path, portrayed by the broken schedule in Figure 1. Conflict is lower (and thus social welfare higher) along the new adjustment path, relative to the old, for the initial periods (till t^*). However, conflict is higher (and thus social welfare lower) along the new adjustment path in every period after t^*. Steady state social welfare declines from W_S to $\ddot{W}_S$. The aggregate welfare consequence of the intervention is thus ambiguous: it depends both on the exact time path and the time discount factor adopted. When the elasticity of substitution is less than 2, the welfare schedule has an inverted U-shape, with the welfare implications of the intervention getting exactly reversed. Thus, one can no longer generate an unambiguous welfare ranking of policy interventions on the basis of their magnitude purely from the elasticity of substitution: this additionally requires explicit *inter-temporal trade-offs*. Furthermore, an element of time inconsistency gets built into the policy debate. Clearly, an *ex post* assessment of the intervention, in terms of its present and future consequences, made much after the event, may take a very different view than an *ex ante* assessment, even when they share the same normative presupposition (same elasticity) and the same time discount factor.

Interventions in conflict zones by foreign entities are often politically justified by the jointly stated goal of conflict reduction and protection of weaker ethnic groups, within a context of growing ethnic conflict. Based on a cross-national, time-series data analysis of 164 countries for the years 1981 to 2005, Choi and James (2014) find that, *ceteris paribus*, the US is likely to engage in military campaigns for humanitarian reasons rather than for its own security interests. Politicians and voters in liberal democracies which are in a position to intervene are likely to prioritize immediate conflict reduction over long-term peace building. As discussed above, higher levels of short term peace and protection of weaker ethnic groups may be ensured by one-off interventions which impose a combination of higher institutional disadvantage and greater institutional rigidity on the ethnic group with a conflict cost advantage. However, such higher levels of short term conflict reduction are purchased at the cost of deeper conflict in the longer term. Thus, political short-termism in intervening liberal democracies is likely to bias peace-building interventions towards forms (specifically, greater institutional locking out of the stronger/dominant group post intervention and greater institutional rigidity with regard to emergent ethnic political pressure) that are likely to exacerbate conflict in the future. Such exacerbation in turn is likely to justify demands for repeated and deeper intervention, which would push the problem even further into the future. Thus, the model suggests that external interventions, when effective in reducing current conflict and protecting weaker groups, may in fact end up sowing the seeds of greater future conflict and further external intervention. Interventions by liberal democracies in conflicted societies may therefore turn out to be self-reinforcing and self-replicating.

Ethnocratic settlements often form the key feature of peace deals to end civil wars, and are commonly imposed by colonial regimes, militarily superior external powers or multilateral agencies. As discussed in Section 2.1, recent settlements in Bosnia, Northern Ireland, Lebanon, Iraq and Afghanistan provide examples. In many other countries fragmented along ethnic lines, one may perceive such settlements developing gradually and internally as an outcome of domestic ethnic politics. Our analysis clarifies one reason for the popularity of such settlements: they may function as a commitment device. As shown in Propositions 2.1 and 2.2, relatively rigid institutions, combined with an initial settlement favourable to the group that is weaker (high cost) in conflict, may lead to that group's resource share granted to it by state institutions remaining relatively high indefinitely into the future. This might increase the weaker group's incentive to remain within the state system, i.e., engage with the institutionalized rules of conflict settlement and negotiation, instead of rejecting it altogether in favour of an all-out insurgency or civil war. However, the absence of all out civil war does not imply the absence of conflict: working within institutions also generates ethnic conflict over rent-seeking attempts to influence the functioning of those institutions. Such conflicts may take the form of litigation, lobbying, mass political mobilization, as well as selective use of violence.

The greater the weaker ethnic group's expected outcome in case of civil war and state breakdown, the higher the steady-state pay-off this group has to be offered to induce its members to accept a lasting institutional settlement: hence, the more biased the initial settlement has to be against the stronger (cost advantaged) ethnic group and the more rigid the consequent institutions (recall Corollary 2.1).[12] We are thus likely to observe a settlement which leads to *rising* rent-seeking conflict within a structure of formal ethnic peace and acceptance of civic institutions, at least in the initial periods subsequent to the settlement (Recall Proposition 2.4 parts (i) and (iii)). Indeed, inter-ethnic rent-seeking conflict would remain high even in the steady state. Thus, formal peace, when purchased through an ethnocratic settlement, runs the risk of locking the society into a permanent state of high rent-seeking conflict around rigid identitarian mobilizations. Furthermore, to the extent that high and persistent rent-seeking conflict has a tendency to develop a momentum and dynamic of its own, it may tip over into full-blown civil war and external intervention.

2.6 Concluding remarks

This chapter has developed a simple model of dynamic ethnic conflict when state institutions exhibit evolving ethnic bias. The main results involve the non-monotone nature of the relationship between conflict, distribution and social welfare on one hand, and key peace policy variables, such as the ethnic bias of an initial institutional division and the responsiveness of such bias to emerging political pressure from mass ethnic movements, on the other. These findings may be seen as highlighting the fundamental limitation of ethnocratic settlements as peace-building devices in conflict-ridden societies. The analysis suggests that, when successful in avoiding all-out civil war and state collapse, such ethnocratic settlements are likely to generate high levels of conflict over ethnic rent-seeking,

12 There is, for example, evidence that minority ethnic groups are more prone to insurrection if concentrated in rural areas, as opposed to being dispersed around the country (Jenne, Saideman, and Lowe 2007; Toft 2003). Ethnocratic settlements would have to offer such groups a large institutional share to induce them to accept state institutions, as typically happens in practice through regional autonomy and constitutional rules determining fiscal transfers.

within the framework of a fragile state, perpetually susceptible to external interference and intervention. This 'ethnocratic trap' (Howard 2012) can be avoided only by developing more inclusive, porous and flexible forms of identity formation and integrative institutions that cut across rigid ethnic divides (see Dasgupta and Kanbur 2005b) for a formal model that expands on this idea).

The analysis of course has a number of limitations and may be extended in various directions. We have abstracted from group size effects. A suitably amended version of the model with convex effort cost (as, for example, in Esteban and Ray 2001) would allow one to incorporate this aspect and explicitly investigate majority-minority issues. Furthermore, we have assumed an egalitarian surplus sharing rule within both communities. Different, and possibly asymmetric, sharing rules within the contending groups (e.g. Davis and Reilly 1999; Nitzan 1991) will in general generate different dynamic patterns of conflict and distribution. One may also explicitly model an additional contest within each community for sharing of the prize in every period, either simultaneously with the original inter-community contest (expanding the one-period structures in, for example, Dasgupta 2009 and Munster 2007) or subsequent to it (along the lines of Katz and Tokatlidu 1996).Acemoglu and Wolitzky (2014) have developed a model of conflict cycles generated by coordination problems due to incomplete information regarding the nature of groups different from one's own. An extension of our model can generate conflict cycles endogenously even in our complete information set-up. We have assumed that conflict input costs are constant over time. One may instead assume that each community invests a constant proportion of its net income in every period, and that the opportunity cost of activist labour increases with community-specific investment in the preceding period, due to a consequent increase in the community's labour productivity in income generating non-expropriatory activities. It is easy to see that the relative conflict cost may then exhibit cyclical movement, generating conflict cycles. All these extensions may be usefully addressed in future work.

Empirical research informed by the theoretical findings of this chapter would evidently be in order. Building on the large cross-country literature on civil wars, one may investigate whether a more rigid ethnocratic peace settlement increases the probability of recurrence of civil war. More generally, one may empirically examine the impact, of ethnocratic rigidity within institutions, on economic performance. The key issue here is to devise a suitable empirical measure of ethnocratic rigidity. This may perhaps be done at a crude level by estimating a log-linearized version of equation (2.9) above, with relative conflict success, past and present, proxied by the corresponding relative electoral performance of ethnic parties (or, more directly, the ethnic division of budgetary outlays, where available), and relative conflict cost proxied by the relative unemployment rate or the relative (average) wage rate. Alternative empirical strategies for estimating the extent of ethnocratic rigidity, and its connection with ethnic conflict, using cross-country data, may be explored as well.

Lastly, one may attempt to devise controlled experiments using our theoretical setting. Experiments on repeated multi-battle conflicts have been carried out by Zizzo (2002), Deck and Sheremeta (2012), and Mago, Sheremeta and Yates (2013); while experiments on multi-battle conflicts with spillovers have been done by Schmitt et al. (2004) and Sheremeta (2010) (see Dechenaux, Kovenock and Sheremeta (2014) for a survey). Experimental investigations of our theoretical framework that build on this literature may yield useful insights.

Chapter 3

Identity conflict with cross-border spillovers

(An extended version of this chapter is forthcoming as:

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3.1 Introduction

Given extra-territorial solidarity or identity of interest on the part of antagonistic domestic groups, the balance of power between them may be expected to be reflected in the 'foreign policy bias' of a country (say 1), as partisan military, logistical, organisational and propaganda support to favoured groups in another country (say 2), both official and private. Greater external support typically finds practical reflection in greater cross-border access to safe houses, sanctuaries, strategically important roads, mountains, pre-existing stocks of military hardware (especially aircraft, tanks and heavy artillery) that cannot be easily acquired from market purchase, military and political trainers, cross-border bases and training camps, diplomatic, media and propaganda support, international lobbying, etc. These diverse forms of external support from 1 to the group (say A) favoured by 1 in 2 are complementary to A's own mobilization of resources for conflict with other groups in 2: they augment the efficiency of such mobilization and thereby affect the conflict outcome in 2. They involve the enabling of foreign affiliates to use domestic territory and public goods such as existing social or security infrastructure and networks in 1: they do not typically require large-scale diversion of resources from domestic use therein. The conflict outcome in 2 in turn impacts on group conflict in 1. Thus, group conflicts are mutually determined under cross-territorial identification among contending groups. Since components of groups have conflicting as well as common interests, such mutual conditioning would involve the determination of intra-group conflict within the broader contending formations as well.

How does an increase in the ability, of the balance of power within one country to affect group conflict in another, affect conflict and inter-group distribution in either country? Since economic growth, greater commodity, capital and labour market integration, more extensive and porous common borders, can all be expected to expand such ability, the answer sheds light on the connection between economic growth, market openness and domestic conflict. Second, how do changes in demographic and economic fundamentals of a country, such as population size, population distribution between contending groups, their relative labour productivity and the strength of property rights protection, affect conflict in *another* country? The answers provide important insights into the workings of external drivers of domestic conflict. This chapter seeks to address these questions.

The revival of ethnic (especially religious) identities in recent decades, and the increasing salience of mass political conflict, both among rival ethnic identities and between religious and secular identities, over *extra-economic* aspects of life, lead us to focus on inter-group conflict over items of group-wide non-excludable benefit ('culture/religion') rather than private consumption ('income'). Building on Dasgupta and Kanbur (2011, 2007, 2005a), we visualize identity groups within a territory as held together by the common consumption of certain forms of group-specific public goods, which do not yield monetary benefits, but are intrinsically valuable. In accord with Dasgupta and Guha Neogi (2018), Dasgupta (2017) and Esteban and Ray (2011, 2008), we model such collective consumption as generating conflict between groups.

Societies with sharp ethnic divides also exhibit locational segregation: neighbourhoods are divided along ethnic lines, enterprises feature ethnic homogeneity in recruitment, and specific ethnic groups often cluster in particular occupations and market segments (see Bowles et al. 2014 and Schelling 1971, 1969 for discussions). Albeit to a lesser extent, this also holds for the secular-theocratic divide: secularizing or anti-clerical identities are often concentrated in cities, while religious or theocratic identities are more firmly rooted in the rural areas. Consequently, decentralized distributive conflicts *within* an identity group among its constituent clans/factions over expropriation of divisible consumption ('income') often acquire a greater immediacy and salience, compared to such conflicts across groups. We abstract from the latter to focus on the former, and think of such decentralized distributive conflict primarily in terms of local, neighbourhood or workplace level crime and extortion.

To fix ideas, then, we think of the following stylized scenario. Two ethnic groups live in different parts of a country. Individuals can only expropriate other individuals whom they can physically access. Thus, because of transport costs and locational segregation, individuals can only expropriate their ethnic kin.[13] However, one ethnic group wishes to pressurize the state to impose a common secular legal code regarding marriage and sexual behaviour over the entire country, while the other group wishes to impose religious (e.g. Sharia) law. Individual members of each group seek to lobby or pressurize the state to implement, in decentralized fashion, the legal code preferred by their group. The outcome is a composite legal code exhibiting both secular and religious features, with their shares (proportions) determined by the lobbying efforts deployed by the contending groups. However, individuals cannot pressurize the state to implement income transfers from the other group, say due to constitutional safeguards. As noted earlier, this abstraction is motivated by our focus on non-pecuniary sources of group conflict. The scenario is replicated in a neighboring country.

13 In ethnically segregated societies, both victims and perpetrators of street crime typically belong to the same ethnic group, and ethnic gangs operate in their respective ethnic neighbourhoods/areas. For example, during 1976-2005, 94% of black murder victims in the US were killed by black offenders, while 86% of white victims were killed by white offenders (http://www.thedailybeast.com/articles/2013/07/15/thetrayvon-martin-killing-and-the-myth-of-black-on-black-crime.html).

Within each country (or territory), two groups contest one another, in Tullock (1980) fashion, over sharing of one unit of a composite good (‘legal code’ for intuitive focus). Possession of this composite good leads to non-rival and non-excludable consumption benefits within a group, but is mutually exclusive between groups. Within each country, all members of each group also engage in Tullock contestation over the division of total group resources available for rival consumption (group ‘income’). Each member is endowed with one unit of effort, which she allocates among inter-group conflict, within group contestation, and productive (income-generating) activity. Each group in a country has an *affiliate* in the other country, interpreted alternatively as ethnic kin, or opponent of a common ethnic enemy. Success in inter-group conflict within a country depends on both internal effort mobilization and external support. The extent to which a given amount of inter-group conflict effort by a group (say A) in a country gets translated into conflict success (share) depends positively on the success (share) of A’s affiliate in the other country. All resource allocation happens simultaneously, so that conflict, production and distribution in the two countries are mutually determined. Thus, we contribute to both the theoretical literature on group contests over public goods (stemming from Katz et al. 1990)and that on production and expropriation(originating from Skaperdas 1992 and Hirshleifer 1991), by integrating the two and analysing spillovers across territories.[14]

We find that total effort allocated to within-group conflict and aggregate social output move together within a country, while total effort allocated to inter-group conflict moves in the opposite direction, as does total effort allocated to conflict of any kind. Under quite general restrictions, the

14 Recent contributions on simultaneous between and within group contests are Choi et al. (2016), Dasgupta (2009),Münster (2007)and Hausken (2005). These model conflicts solely over private goods, and cannot therefore address the non-pecuniary conflicts we highlight. Furthermore, they feature a single site of inter-group conflict. In contrast to the literature on conflict in multiple battlefields (see Kovenock and Roberson 2012 for a recent survey), the same agents do not confront one another in multiple battlefields (territories) in our model. Our agents do not maximize any aggregation of the payoffs in the two territories. They only maximize their payoffs generated in their own territory: the consequent outcome in one affects that in the other as a parametric change in the conflict environment. This captures the idea that while identity groups may feel tied to affiliates across borders because of certain shared features, cross-border differences in other features are sufficiently salient to preclude coordination to the extent that merits modelling in terms of a common aggregative group objective function across borders. Instead, we pursue the idea that decentralized groups pursue their objectives within their own territory, but greater success in doing so by a group advantages its cross-border affiliate. Partisan cross-border impact, and success in domestic inter-group conflict, are thus *joint products* in our model. Fu et al. (2015) study a distantly related problem of team contests with multiple pairwise battles, where team members choose in a decentralized fashion. Unlike their model, there is no overall team prize to be won as the aggregate consequence of outcomes in individual battlefields – only battlefield-specific prizes - in our set-up. Bayeet al. (2012) and Chowdhury and Sheremeta (2011) examine simultaneous-move single-battlefield two-player contests with complete information, where each player’s strategy has a spillovereffecton the other player’s payoff. In contrast, the spillover effects in our model take place across battlefields.

following hold. A unilateral increase in the ability of the inter-group balance of power in either country to influence conflict in the other country, i.e., in its 'spillover elasticity' (due, say, to faster economic growth in the former, or a unilateral relaxation of import restrictions, restrictions on private aid and capital flows and immigration controls in the latter) reduces effort allocations to external conflict in the former but increases them in the latter. Thus, greater *unilateral* economic integration by a country with another may increase conflict and reduce output in the integrating country. Greater spillover from a country benefits both the dominant (more successful) group in that country and its affiliate in the other. An equi-proportionate increase in spillover elasticities (due say to similar economic growth in the two territories, or an expansion in bilateral trade and labour market integration), interpreted as greater *bilateral* economic integration, affects group conflict in a non-monotone fashion. This initially increases group conflict in *both* countries; at intermediate levels it moves conflict in opposite directions across countries; at already high levels of integration it reduces conflict in both countries. It may affect the welfare of a group and its affiliate in opposite ways.

We also investigate how changes in demographic and economic fundamentals within a country affect conflict in another, incorporating a majority-minority divide. We find that population increase in a country that does not reduce the minority's share, an enlargement of the minority's share that does not reduce total population, weaker property rights protection across groups, and an increase in relative labour productivity of the majority, may all increase inter-group conflict in the *other* country, when the minority is dominated in both. However, when the minority dominates in one, but is dominated in another, such changes in one country may increase or decrease inter-group conflict in the other, depending on which country the changes occur in. These changes nonetheless always make the minority in the other territory better off, while making the majority therein worse off. Community neutral growth in labour productivity within a territory reduces both inter-group and aggregate conflict, but increases intra-group conflict and output within both groups in that territory.

Section 3.2 sets up the model, in the context of a single country. Section 3.3 embeds cross-border spillovers and examines how changes therein affect conflict and distribution. Section 3.4 discusses the impact of changes, in demographic and economic variables within a country, on conflict and distribution in the other country. Section 3.5 illustrates some possible applications of our findings and offers concluding comments.

3.2 The model

Consider a scenario where two identity *communities*, A and B, are spread across two territorial-cum political units (say, two countries or two provinces of the same country), 1 and 2. Cross-territorial identification within each community obtains due to the shared possession of some identity marker (religion, race, caste or language): territorial fragments of a community ('groups') *affiliate* with one another in this sense. For example, A and B may refer to different religions, so that membership of community $g \in \{A, B\}$ is defined by the common experience of practicing religion g irrespective of location: group g in 1 and group$g$ in 2 are affiliates. Given any$g \in \{A, B\}$, we shall denote the other community by –*g*; given any territory $j \in \{1,2\}$, we shall denote the other territory by –*j*.Let$C \equiv \{A, B\} \times \{1,2\}$. Thus, there are four groups, each defined formally bya pair $\langle g, j \rangle \in C$.

We first model the conflict process within each territory in isolation, postponing the discussion of cross-territory spillovers to Section 3. In each territory j, there are n_j individuals (alternatively, clans or factions)partitioned into the two communities, with group $\langle g, j \rangle$ being of population sizen_{gj} (so that $n_{Aj} + n_{Bj} \equiv n_j$); $n_{gj} \geq 2$. Each individual is endowed with one unit of effort. Individual $i \in \{1,2, \dots, n_{gj}\}$ in group $\langle g, j \rangle$ chooses productive effort$e_{ig,j}$, intra-group conflict effort $x_{ig,j}$, and inter-group conflict effort $y_{ig,j}$, subject to the non-negativity constraints$e_{ig,j} \geq 0$,$x_{ig,j} \geq 0$,$y_{ig,j} \geq 0$, and the budget constraint $[e_{ig,j} + x_{ig,j} + y_{ig,j} = 1]$. Total intra-group conflict effort for group $\langle g, j \rangle$ is $x_{gj} \equiv \sum_{i=1}^{n_{gj}} x_{ig,j}$, while total inter-group conflict effort allocated by that group is $y_{gj} \equiv \sum_{i=1}^{n_{gj}} y_{ig,j}$. There exists one unit of some composite group-specific public good in each territory, whose division is contested over by the two groups in that territory, with $\langle g, j \rangle$ getting the share p_{gj}; $p_{Aj} + p_{Bj} = 1$.The valuation of the group-specific public good by members of $\langle g, j \rangle$ is T_{gj}. Thus, the monetary equivalent of the benefit to each individual member of $\langle g, j \rangle$ from the group as a whole receiving the share p_{gj} of the public good is $p_{gj}T_{gj}$. Group $\langle g, j\rangle$ is endowed with some immovable and indivisible productive asset L_{gj} (intuitively identified with stock of human capital, infrastructure including transport and communication network, coastline, climate or land productivity); this asset is complementary to labour. For member i, of group $\langle g, j\rangle$, output is thus given by:

$$q_{ig,j} = L_{gj}(1 - x_{ig,j} - y_{ig,j}).$$

Group output is given by the sum of individual outputs:

$$q_{gj} = \sum_{i=1}^{n_{gj}} q_{ig,j} = L_{gj}(n_{gj} - x_{gj} - y_{gj}). \tag{3.1}$$

An individual member of the group can costlessly retain $(1 - \mu_{gj}) \in [0,1)$proportion of her output. The consequent pool of expropriable income/output, $\mu_{gj}q_{gj}$, is divided among the group members as the outcome of a process of decentralized intra-community distributive conflict, defined by the standard (Tullock1980) contest success function. The parameter μ_{gj} thus measures the extent to which individual property rights are protected within the group: a higher value of this parameter implies weaker protection of private property rights. This formulation permits the possibility that different groups protect private property rights of group members differentially. One group may provide stronger protection because its internal governance institutions (broadly interpreted to include the church, caste/clan/village councils etc.) are better at instilling property-preserving social norms within the group and/or censuring infringements. Thus, the net income of an individual in $\langle g, j\rangle$ is:

$$r_{ig,j} = \mu_{gj}\left(\frac{x_{ig,j}}{x_{gj}}\right)q_{gj} + (1 - \mu_{gj})L_{gj}[1 - x_{ig,j} - y_{ig,j}] \text{ if } x_{gj} \equiv \sum_{i=1}^{n_{gj}} x_{ig,j} > 0;$$

$$= \mu_{gj}\left(\frac{1}{n_{gj}}\right)q_{gj} + (1 - \mu_{gj})L_{gj}[1 - x_{ig,j} - y_{ig,j}] \text{ Otherwise} \tag{3.2}$$

A group member's aggregate utility, or payoff, is given by:

$$u_{ig,j} = p_{gj}T_{gj} + r_{ig,j}. \tag{3.3}$$

The outcome of the inter-group contest over sharing of the group-specific public good is defined by a Tullock contest success function, so that group $\langle A, j\rangle$ gets the fraction:

$$p_{Aj} = \frac{y_{Aj}}{y_{Aj}+z_j y_{Bj}} \text{ if } y_j \equiv y_{Aj} + y_{Bj} > 0;$$

$$= \frac{1}{1+z_j} \text{ otherwise;} \tag{3.4}$$

where $z_j > 0$ measures the relative efficiency of B's conflict effort in inter-community conflict in territory j.[15] All individuals in both territories simultaneously choose their inter and intra-community conflict effort allocations to maximize their payoff function (3.3), subject to(3.1), (3.2), (3.4) and the individual budget constraint. It can be checked that the payoff function (3.3) is strictly quasi-concave in$(x_{ig,j}, y_{ig,j})$. Hence, a unique solution exists to the maximization problem of the individual, given the conflict contributions by the rest of the society, z_j and the parameter μ_j.Note that, whilez_j is treated as an exogenous variable in the single territory (or partial equilibrium) analysis of this section, it will be endogenized via cross-territorial spillovers in Section 3.3 below.

Suppose an interior equilibrium exists. Then, the FOCs yield:

$$\mu_{gj}\left[\frac{x_{gj}-x_{ig,j}}{x_{gj}^2}\right] = \frac{[\mu_{gj}\left(\frac{x_{ig,j}}{x_{gj}}\right)+(1-\mu_{gj})]}{(n_{gj}-x_{gj}-y_{gj})}; \tag{3.5}$$

$$T_{gj}\frac{\partial p_{gj}}{\partial y_{ig,j}} = [\mu_{gj}\left(\frac{x_{ig,j}}{x_{gj}}\right) + (1-\mu_{gj})]L_{gj}. \tag{3.6}$$

Summing over all members of g in the territory, we get, from (3.5):

$$\left[\frac{n_{gj}-1}{x_{gj}}\right] = \frac{\mu_{gj}+(1-\mu_{gj})n_{gj}}{\mu_{gj}(n_{gj}-x_{gj}-y_{gj})}. \tag{3.7}$$

Hence,

$$x_{gj} = \frac{\mu_{gj}(n_{gj}-1)}{n_{gj}}(n_{gj}-y_{gj}). \tag{3.8}$$

15 The contest success function in (2) belongs to a class that has been axiomatized by Skaperdas (1996). The contest success function in (4) belongs to a class that has been axiomatized by Münster (2009).

Equation(3.5) implies equilibrium intra-community conflict allocation must be identical for all members; i.e., $\frac{x_{ig,j}}{x_{gj}} = \frac{1}{n_{gj}}$, and that $x_{gj} < n_{gj}$ (since $\frac{\mu_{gj}(n_{gj}-1)}{n_{gj}} < 1$) for any $y_{gj} \geq 0$. Furthermore, from (3.4),

$$\frac{\partial p_{gj}}{\partial y_{ig,j}} = \frac{z_j y_{-g,j}}{\left(y_{Aj}+z_j y_{Bj}\right)^2}. \tag{3.9}$$

Together, (3.6) and (3.9) yield:

$$\frac{z_j y_{-g,j}}{\left(y_{Aj}+z_j y_{Bj}\right)^2} = \frac{[\mu_{gj}\left(\frac{1}{n_{gj}}\right)+(1-\mu_{gj})]L_{gj}}{T_{gj}}. \tag{3.10}$$

Let

$$a_{gj} \equiv \frac{[\mu_{gj}\left(\frac{1}{n_{gj}}\right)+(1-\mu_{gj})]L_{gj}}{T_{gj}}. \tag{3.11}$$

Since $n_{gj} \geq 2$, a_{gj} is declining in the weakness of property rights protectionμ_{gj}. It is declining in group population n_{gj} as well. This variable is the opportunity cost of external conflict effortexpressed in units of the public good: the numerator is the income loss from shifting a unit of labour from production to external conflict, while the denominator is the monetary value of the public good. Define $\frac{a_{Aj}}{a_{Bj}} \equiv a_{AB,j}$. From (3.10), recalling that the exogenous variable a_{gj} is given by the parameters of the model according to (3.11), we have the equilibrium condition:

$$\frac{y_{Bj}}{y_{Aj}} = a_{AB,j}.$$

(3.12)

By (3.12), external conflict effort is inversely proportional to its opportunity cost, so that relative external conflict effort is simply the inverse of the relative opportunity cost of external conflict effort. Hence, using (3.10)-(3.12), resource wastage due to inter-community conflict in a territory is given by:

$$y_{Aj} = \frac{z_j}{a_{Bj}\left(1+z_j a_{AB,j}\right)^2},\ y_{Bj} = \frac{z_j a_{AB,j}}{a_{Bj}(1+z_j a_{AB,j})^2}. \tag{3.13}$$

Thus,$y_{gj} > 0$. By (3.13), the maximum value of y_{gj} is given by:

$$\overline{y}_{Aj} = \frac{1}{4a_{Aj}}, \overline{y}_{Bj} = \frac{1}{4a_{Bj}}. \tag{3.14}$$

Using (3.8), total conflict allocation (resource wastage) in a territory by a group is:

$$s_{gj} \equiv x_{gj} + y_{gj} = \mu_{gj}(n_{gj} - 1) + [\frac{n_{gj} - \mu_{gj}(n_{gj} - 1)}{n_{gj}}]y_{gj}. \tag{3.15}$$

Assumption 3.1: For every $\langle g, j \rangle \in C$, $[\frac{1}{4a_{gj}} < n_{gj}]$.

Assumption 3.1 simply incorporates the intuitive idea that (external conflict benefit-normalized) labour productivity is high relative to the group population. Given any n_{gj}, Assumption 1 must necessarily hold if $\frac{L_{gj}}{T_{gj}}$ is sufficiently high (recall (3.11)), i.e., if the external conflict benefit-normalized labour productivity is sufficiently high. Assumption 3.1 is sufficient to ensure that the assumption of an interior solution to the individual's maximization problem neither violates the non-negativity constraints on effort allocation, nor the individual's budget constraint. More formally, recalling (3.8), (3.13), (3.14) and (3.15), Assumption 3.1 implies the following.

Lemma 3.1: Let Assumption 3.1 hold, and let $\langle x^*_{Aj}, x^*_{Bj}, y^*_{Aj}, y^*_{Bj} \rangle$ constitute the solution to the equation system (3.8) and (3.10). Then, for all $\langle g, j \rangle \in C$, $[x^*_{gj}, y^*_{gj} \geq 0]$ and $[n_{gj} > x^*_g + y^*_{gj}]$.

Using (3.8) and (3.13), we get the resource wastage due to intra-group conflict within a territory:

$$x_{Aj} = [\frac{\mu_{Aj}(n_{Aj} - 1)}{n_{Aj}}][n_{Aj} - \frac{z_j}{a_{Bj}(1 + z_j a_{AB,j})^2}],$$

$$x_{Bj} = [\frac{\mu_{Bj}(n_{Bj} - 1)}{n_{Bj}}][n_{Bj} - \frac{z_j a_{AB,j}}{a_{Bj}(1 + z_j a_{AB,j})^2}]. \tag{3.16}$$

Furthermore, using (3.4) and (3.12), we have the equilibrium shares in a territory:

$$p_{Aj} = \frac{1}{1 + z_j a_{AB,j}},\ p_{Bj} = \frac{z_j a_{AB,j}}{1 + z_j a_{AB,j}}. \tag{3.17}$$

In light of Lemma 3.1, we then immediately have the following.

Proposition 3.1: Let Assumption 3.1 hold. Then, for all $\langle g, j \rangle \in C$:

(i) equilibrium external conflict allocations are given by (3.13), internal conflict allocations by (3.16), and shares by (3.17);

(ii) y_{gj} and s_{gj} both increase as z_j increases over $(0, a_{BA,j})$, and decline as z_j increases over $(a_{BA,j}, \infty)$, with $\lim_{z_j \to \infty} y_{gj} = 0$;

and

(iii) x_{gj} declines as z_j increases over $(0, a_{BA,j})$, and increases as z_j increases over $(a_{BA,j}, \infty)$, with $\lim_{z_j \to \infty} x_{gj} = \mu_{Aj}(n_{Aj} - 1) > 0$.

By Proposition 3.1, the external conflict effort of either community in a territory (and hence aggregate external conflict in that territory) increases with the relative (external) conflict efficiency of B in that territory, z_j, till the latter reaches the relative opportunity cost of external conflict for B, and declines subsequently. Internal conflict behaves in the opposite fashion: the mirror image of external conflict. Nonetheless, total conflict effort by a community within a territory (and thus, overall conflict) follows the pattern of external conflict therein. Hence, output follows the pattern of internal conflict.

In light of Proposition 3.1, using (3.2)-(3.4), (3.13), (3.16) and (3.17), group payoffs are given by:

$$\pi_{Aj} = \frac{n_{Aj}T_{Aj}}{1+z_j a_{AB,j}} - L_{Aj}[\frac{n_{Aj}-\mu_{Aj}(n_{Aj}-1)}{n_{Aj}}]\frac{z_j}{a_{Bj}(1+z_j a_{AB,j})^2} + L_{Aj}\left(n_{Aj} - \mu_{Aj}(n_{Aj}-1)\right) \quad (3.18)$$

$$\pi_{Bj} = n_{Bj}T_{Bj}(\frac{z_j a_{AB,j}}{1+z_j a_{AB,j}}) - L_{Bj}[\frac{n_{Bj}-\mu_{Bj}(n_{Bj}-1)}{n_{Bj}}]\frac{z_j a_{AB,j}}{a_{Bj}(1+z_j a_{AB,j})^2} + L_{Bj}\left(n_{Bj} - \mu_{Bj}(n_{Bj}-1)\right). \quad (3.19)$$

Noting (3.18) and (3.19), Proposition 3.1 yields the following corollary.

Corollary 3.1: Let Assumption 3.1 hold. Then, for all j ∈ {1,2}, p_{Aj} declines monotonically as z_j increases, with $\lim_{z_j \to 0} p_{Aj} = 1$, $\lim_{z_j \to \infty} p_{Aj} = 0$; furthermore, the payoff of community A in j declines, and that of community B in j rises, monotonically in z_j.

Proof: The claim regarding p_{Aj} follows directly from (3.17). Using (3.18), and dropping the subscript j for notational simplicity,

$$a_B(1+za_{AB})^2\frac{\partial\pi_A}{\partial z} = -n_AT_Aa_A - L_A\left[\frac{n_A-\mu_A(n_A-1)}{n_A}\right] + 2L_A\left[\frac{n_A-\mu_A(n_A-1)}{n_A}\right][\frac{za_{AB}}{1+za_{AB}}]. \qquad (3.20)$$

Now, $\lim_{z\to\infty}\frac{za_{AB}}{1+za_{AB}} = 1$, and $\frac{za_{AB}}{1+za_{AB}}$ is increasing in z. Hence, using (3.11), for any $z \in (0,\infty)$, the RHS of (3.20) is less than $(n_A-1)(\mu_A\left[1-\frac{1}{n_A}\right]-1) < 0$. Thus, $\frac{\partial\pi_A}{\partial z} < 0$. Again, using (3.19),

$$a_B(1+za_{AB})^2\frac{\partial\pi_B}{\partial z} = n_BT_Ba_A - L_B\left[\frac{n_B-\mu_B(n_B-1)}{n_B}\right]a_{AB} + 2L_Ba_{AB}\left[\frac{n_B-\mu_B(n_B-1)}{n_B}\right][\frac{za_{AB}}{1+za_{AB}}]. \qquad (3.21)$$

The RHS of (3.21) is increasing in z. Suppose it is non-positive at $z=0$. Then $n_Ba_B \leq \frac{L_B}{T_B}\left[1-\frac{\mu_B(n_B-1)}{n_B}\right]$, or, using (3.11), $[1 \leq \frac{1}{n_B}]$, a contradiction, since $n_B \geq 2$. Hence $\frac{\partial\pi_B}{\partial z} > 0$.

By Corollary 3.1, the share of the public good received by A in a territory always declines with an increase in the relative conflict efficiency of B in that territory. Recall that, by Proposition 3.1(ii), A's output increases as the latter increases beyond $a_{BA,j}$. In this range, the first (share) effect reduces A's payoff, while the second (output) effect increases it. Corollary 3.1 implies that the share effect necessarily dominates. Conversely, for B, the two effects move in opposite directions when the relative conflict efficiency of B increases over $(0, a_{BA,j})$, with the share effect dominating.

3.3 Cross-territorial spillover

We now proceed to embed the idea of cross-territorial spillovers in our model. Define the relative share, or relative success, in territory j as: $\frac{p_{Bj}}{p_{Aj}} \equiv p_{BA,j}$. From (3.17), we have the within-territory equilibrium condition for this variable:

$$p_{BA,j} = z_ja_{AB,j}. \qquad (3.22)$$

Assumption 3.2: For all $j \in \{1,2\}$, $z_j = \left(z_{-j}a_{AB,-j}\right)^{\theta_{-j}}$, where $\theta_j \in (0,1)$.

Assumption 3.2 is the condition for cross-territorial equilibrium in our model. By (3.22), Assumption 3.2 implies that, in equilibrium, $z_j = p_{BA,-j}{}^{\theta_{-j}}$. The parameter $\theta_j$measures the extent to which dominance in domestic group conflict spills over into the *effective* bias of foreign policy. Since $\theta_{-j} = \frac{dz_j}{dp_{BA,-j}}(\frac{p_{BA,-j}}{z_j})$, it isthe *elasticity* of effective bias in foreign policy with respect to domestic group balance. When $\theta_1 = \theta_2 = 0$, the model reduces to the standard case of no spillover across territories and identical conflict efficiency ($z_1 = z_2 = 1$).When $\theta_j > 0$, positive spill-overs exist: success in one territory acts as a *force enhancer* in another. Suppose $p_{BA,2} > 1$, so that B is the dominant (more successful) group in 2. Then, greater success in 2 on part of B magnifies the relative productivity of its affiliate's external conflict effort in 1, thereby translating into a higher share in 1 for any given deployment of external conflict effort inputs therein by the two parties. The higher the value of θ_2, the greater the effective reflection of domestic group balance of power in the foreign policy of 2, and thus the greater the spillover from 2 to 1; hence the higher the relative productivity of external conflict effort deployed in 1 by the affiliate of the group which dominates in 2.[16]

We permit spillovers to be asymmetric: θ_1 need not be equal to θ_2. Territory 1 may be in a better position to influence group conflict in territory 2 than vice versa. This may reflect the greater responsiveness of foreign policy in 1 to a given domestic balance of ethnic power due to differences in domestic institutions. For example, this may obtain because foreign policy in 2 is determined to a greater extent by a self-replicating elite corps of professional soldiers and diplomats, with commensurately less intervention by politicians, or because 1 is an independent country while 2 is a small constituent province of a federal state, whose other constituents contain neither A nor B.

16 In real life, one would expect a finite upper limit on one country's ability to enhance the conflict efficiency of an affiliate group elsewhere. One may incorporate this by amending Assumption 2 to the following: for some $\underline{z_j}, \overline{z_j} > 0, \underline{z_j} < \overline{z_j}$, [$z_j = \underline{z_j}$ if $p_{BA,-j}{}^{\theta_{-j}} \leq \underline{z_j}$; $z_j = p_{BA,-j}{}^{\theta_{-j}}$ if $p_{BA,-j}{}^{\theta_{-j}} \in (\underline{z_j}, \overline{z_j})$, , and $z_j = \overline{z_j}$ if $p_{BA,-j}{}^{\theta_{-j}} \geq \overline{z_j}$]. Only the intermediate case $p_{BA,-j}{}^{\theta_{-j}} \in (\underline{z_j}, \overline{z_j})$ is of interest. Our substantive conclusions, presented in Propositions 2-4 below, will then apply under parametric configurations which satisfy: [$a_{AB,2}{}^{\frac{\theta_2}{1-\theta_1\theta_2}} a_{AB,1}{}^{\frac{\theta_1\theta_2}{1-\theta_1\theta_2}} \in (\underline{z_1}, \overline{z_1})$ and $a_{AB,1}{}^{\frac{\theta_1}{1-\theta_1\theta_2}} a_{AB,2}{}^{\frac{\theta_1\theta_2}{1-\theta_1\theta_2}} \in (\underline{z_2}, \overline{z_2})$] (notice (22)-(23)). The feature that the two groups are equally efficient in inter-group conflict spill-overs is for notational simplicity: it can be relaxed to capture cases where one group is inherently more efficient, say because of its traditional control over the military.

Alternatively, 1 may exhibit greater spillover than 2 simply because of the strategic nature of its terrain. Greater physical distance or less extensive/porous common borders may be expected in general to proportionately reduce both θ_1 and θ_2, while the richer territory may be expected to influence events in the poorer territory more effectively than the other way round, simply by virtue of its ability to deploy greater resources. By the same logic, across-the-board increase in wealth within both territories appears intuitively likely to increase spillovers in both directions. Increase in the wealth of a territory, j, may be modelled by an equi-proportionate increase in the labour productivity parameters L_{Aj}, L_{Bj}. It is evident from (3.11) that such an increase leaves the relative conflict cost $a_{AB,j}$ unchanged. Hence, noting (3.23) below, it follows that, given the spillover elasticities θ_1, θ_2, such a change has no effect on conflict in the other territory. Thus, the spillover effect of an equi-proportionate increase in labour productivity across communities within a territory, j, may be uniquely and parsimoniously modelled in terms of an increase in the spillover elasticity θ_j.

Using (3.20) and Assumption 2, we have:

$$p_{BA,1} = a_{AB,2}^{\frac{\theta_2}{1-\theta_1\theta_2}} a_{AB,1}^{\frac{1}{1-\theta_1\theta_2}}; \; p_{BA,2} = a_{AB,1}^{\frac{\theta_1}{1-\theta_1\theta_2}} a_{AB,2}^{\frac{1}{1-\theta_1\theta_2}}. \tag{3.23}$$

Together, (3.20)-(3.21) yield:

$$z_1 = \frac{p_{BA,1}}{a_{AB,1}} = a_{AB,2}^{\frac{\theta_2}{1-\theta_1\theta_2}} a_{AB,1}^{\frac{\theta_1\theta_2}{1-\theta_1\theta_2}}; \tag{3.24}$$

$$z_2 = \frac{p_{BA,2}}{a_{AB,2}} = a_{AB,1}^{\frac{\theta_1}{1-\theta_1\theta_2}} a_{AB,2}^{\frac{\theta_1\theta_2}{1-\theta_1\theta_2}}. \tag{3.25}$$

We wish to rule out the uninteresting and empirically unlikely possibility that A and B have exactly identical opportunity costs of external conflict effort on (geometric) average. Accordingly, we shall assume that, on (geometric) average, B has an advantage in the (parametrically given) opportunity cost of external conflict effort, and adopt the labelling convention that B's advantage in such opportunity cost is at least as high in territory 1 as in territory 2.Recall that $\frac{a_{Aj}}{a_{Bj}} \equiv a_{AB,j}$, so that $a_{AB,j}$ is explicitly derived as a function of the parameters of the model from (3.11).

Assumption 3.3: (i) $\bar{a}_{AB} \equiv \sqrt{a_{AB,1}a_{AB,2}} > 1$, (ii) $a_{AB,1} \geq a_{AB,2}$.

Let$v \equiv \frac{a_{AB,1}}{\bar{a}_{AB}}$.By Assumption 3(ii), $v \geq 1$, with the inequality holding strictly by Assumption 3(i) if $a_{AB,2} \leq 1$.

We first consider a *unilateral* increase in the ability of the inter-group balance of power in a territory to influence conflict in the other territory.

Proposition 3.2: Let Assumptions 3.1-3.3 hold, and let $\breve{\theta} \equiv \frac{\ln\left(\frac{v}{\bar{a}_{AB}}\right)}{\ln(\bar{a}_{AB}v)}$.

(i) If $a_{AB,2} \geq 1$, then external and total conflict fall, and internal conflict rises, in both territories as either θ_1 or θ_2 increases. B's share and payoff both rise, and those of A fall, in both territories.

(ii)If $a_{AB,2} < 1$, then, for all $j \in \{1,2\}$, external and total conflict fall, and internal conflict rises, in territory j as θ_j rises. B's share and payoff both rise, and those of A fall, in territory 1 if θ_1 rises. B's share and payoff both fall (resp. rise), and those of A rise (resp. fall), in territory 2 if θ_2 rises, when $\theta_1 <$ (resp. >) $\breve{\theta}$, with $\breve{\theta} \in (0,1)$.

(iii)If $a_{AB,2} < 1$, then, for all $j \in \{1,2\}$, external and total conflict rise (resp. fall), and internal conflict falls (resp. rises), in territory j as θ_{-j} rises when $\theta_1 <$ (resp. >) $\breve{\theta}$. B's share and payoff both rise, and those of A fall, in territory 2 if θ_1 rises. B's share and payoff both fall (resp. rise), and those of A rise (resp. fall), in territory 1 if θ_2 rises when $\theta_1 <$ (resp. >) $\breve{\theta}$, with $\breve{\theta} \in (0,1)$.

Proof: Equations (3.24) and (3.25) reduce respectively to:

$$z_1 = \bar{a}_{AB}^{\frac{\theta_2(1+\theta_1)}{1-\theta_1\theta_2}} v^{\frac{-(1-\theta_1)\theta_2}{1-\theta_1\theta_2}}; \tag{3.26}$$

$$z_2 = \bar{a}_{AB}^{\frac{\theta_1(1+\theta_2)}{1-\theta_1\theta_2}} v^{\frac{(1-\theta_2)\theta_1}{1-\theta_1\theta_2}}. \tag{3.27}$$

In turn, (3.26) and (3.27) yield, respectively:

$$\ln z_1 = \frac{\theta_2(1+\theta_1)}{1-\theta_1\theta_2}\ln \bar{a}_{AB} - \frac{(1-\theta_1)\theta_2}{1-\theta_1\theta_2}\ln v; \tag{3.28}$$

$$\ln z_2 = \frac{\theta_1(1+\theta_2)}{1-\theta_1\theta_2}\ln \overline{a}_{AB} + \frac{(1-\theta_2)\theta_1}{1-\theta_1\theta_2}\ln v. \tag{3.29}$$

From (3.28) and (3.29), we have:

$$\frac{1}{z_1}\frac{\partial z_1}{\partial \theta_1} = \frac{\theta_2}{(1-\theta_1\theta_2)^2}[(1+\theta_2)\ln \overline{a}_{AB} + (1-\theta_2)\ln v] = \frac{\theta_2}{(1-\theta_1\theta_2)^2}[\ln(\overline{a}_{AB}\, v)\left(\frac{\overline{a}_{AB}}{v}\right)^{\theta_2}]; \tag{3.30}$$

$$\frac{1}{z_1}\frac{\partial z_1}{\partial \theta_2} = \frac{1}{(1-\theta_1\theta_2)^2}[(1+\theta_1)\ln \overline{a}_{AB} - (1-\theta_1)\ln v] = \frac{1}{(1-\theta_1\theta_2)^2}[\ln\left(\frac{\overline{a}_{AB}}{v}\right)(\overline{a}_{AB}v)^{\theta_1}]; \tag{3.31}$$

$$\frac{1}{z_2}\frac{\partial z_2}{\partial \theta_1} = \frac{1}{(1-\theta_1\theta_2)^2}[(1+\theta_2)\ln \overline{a}_{AB} + (1-\theta_2)\ln v] = \frac{1}{(1-\theta_1\theta_2)^2}[\ln(\overline{a}_{AB}\, v)\left(\frac{\overline{a}_{AB}}{v}\right)^{\theta_2}]; \tag{3.32}$$

$$\frac{1}{z_2}\frac{\partial z_2}{\partial \theta_2} = \frac{\theta_1}{(1-\theta_1\theta_2)^2}[(1+\theta_1)\ln \overline{a}_{AB} - (1-\theta_1)\ln v] = \frac{\theta_1}{(1-\theta_1\theta_2)^2}[\ln\left(\frac{\overline{a}_{AB}}{v}\right)(\overline{a}_{AB}v)^{\theta_1}]. \tag{3.33}$$

We first establish the following lemma.

Lemma 3.2: Given Assumptions 1-3, the following must hold.

(i) For all $j \in \{1,2\}$**:** $\frac{\partial z_j}{\partial \theta_1} > 0$; $\frac{\partial z_j}{\partial \theta_2} > 0$ if $\frac{\overline{a}_{AB}}{v} \geq 1$, and when $\frac{\overline{a}_{AB}}{v} < 1$, [[$\frac{\partial z_j}{\partial \theta_2} < 0$ if $\theta_1 < \breve{\theta}$, and $\frac{\partial z_j}{\partial \theta_2} > 0$ if $\theta_1 > \breve{\theta}$], where $\breve{\theta} = \frac{\ln\left(\frac{v}{\overline{a}_{AB}}\right)}{\ln(\overline{a}_{AB}v)} \in (0,1)$].

(ii) $\lim_{\theta_1\to 0} z_1 = (\frac{\overline{a}_{AB}}{v})^{\theta_2}$, $\lim_{\theta_2\to 0} z_1 = 1$, $\lim_{\theta_1\to 0} z_2 = 1$, $\lim_{\theta_2\to 0} z_2 = (\overline{a}_{AB}v)^{\theta_1}$, $\lim_{\theta_1\to 1} z_1 = \overline{a}_{AB}{}^{\frac{2\theta_2}{1-\theta_2}}$ $\lim_{\theta_1\to 1} z_2 = \overline{a}_{AB}{}^{\frac{1+\theta_2}{1-\theta_2}}v$, $\lim_{\theta_2\to 1} z_1 = \overline{a}_{AB}{}^{\frac{(1+\theta_1)}{1-\theta_1}}v^{-1}$, $\lim_{\theta_2\to 1} z_2 = \overline{a}_{AB}{}^{\frac{2\theta_1}{1-\theta_1}}$.

Proof of Lemma 3.2: Since, by Assumption 3, $\overline{a}_{AB} > 1, v \geq 1$, noting that $\overline{a}_{AB} > 1$ implies $\frac{v}{\overline{a}_{AB}} < \overline{a}_{AB}v$, and that, if $\frac{\overline{a}_{AB}}{v} < 1$, $\ln\left(\frac{v}{\overline{a}_{AB}}\right) > 0$, part (i) of Lemma 2 follows from (3.30)-(3.33). Part (ii) follows from (3.26)-(3.27). ■

We now continue with the proof of Proposition 3.2.

(i) In this case, $\frac{\overline{a}_{AB}}{v} \geq 1$; hence, by Assumption 3.3, $a_{BA,1} \leq a_{BA,2} \leq 1$. Since, by Lemma 3.2(i), for all $j \in \{1,2\}$, $\frac{\partial z_j}{\partial \theta_1}, \frac{\partial z_j}{\partial \theta_2} > 0$; and by Lemma 3.2(ii) $\lim_{\theta_1\to 0} z_1 = (\frac{\overline{a}_{AB}}{v})^{\theta_2} \geq 1$, $\lim_{\theta_2\to 0} z_1 = 1$, $\lim_{\theta_1\to 0} z_2 = 1$, $\lim_{\theta_2\to 0} z_2 = (\overline{a}_{AB}v)^{\theta_1} > 1$, we have $z_1, z_2 > 1$. The claim follows from Proposition 3.1 (parts (ii) and (iii)) and Corollary 3.1.

(ii) In this case $a_{BA,1} = \frac{1}{\overline{a}_{AB}v} < 1, a_{BA,2} = \frac{v}{\overline{a}_{AB}} > 1$. By Lemma 3.2(i), $\frac{\partial z_1}{\partial \theta_1} > 0$, and by Lemma 3.2(ii), $\lim_{\theta_1 \to 0} z_1 = (\frac{\overline{a}_{AB}}{v})^{\theta_2} \in \left(\frac{\overline{a}_{AB}}{v}, 1\right)$. Now, if $\frac{1}{\overline{a}_{AB}v} > \frac{\overline{a}_{AB}}{v}$, then $\overline{a}_{AB} < 1$, a violation of Assumption 3.3. Hence, since $\frac{\partial z_1}{\partial \theta_1} > 0$, $z_1 > a_{BA,1}$. Recalling $\frac{\partial z_1}{\partial \theta_1} > 0$, the claim with regard to the impact of an increase in θ_1 within territory 1 follows from Proposition 3.1 (parts (ii) and (iii)) and Corollary 3.1.

Now notice that, by Lemma 3.2(i), [$\frac{\partial z_2}{\partial \theta_2} < 0$ if $\theta_1 < \breve{\theta}$, and $\frac{\partial z_2}{\partial \theta_2} > 0$ if $\theta_1 > \breve{\theta}$], where $\breve{\theta} = \frac{\ln\left(\frac{v}{\overline{a}_{AB}}\right)}{\ln(\overline{a}_{AB}v)}$; furthermore, by Lemma 3.2(ii), $\lim_{\theta_2 \to 0} z_2 = (\overline{a}_{AB}v)^{\theta_1}$. Recall that $a_{BA,2} = \frac{v}{\overline{a}_{AB}} > 1$. Suppose $(\overline{a}_{AB}v)^{\theta_1} < \frac{v}{\overline{a}_{AB}}$. Then $\overline{a}_{AB}{}^{\theta_1+1} < v^{1-\theta_1}$. Since, by Assumption 3.3, $\overline{a}_{AB} > 1, v > 1$, we must therefore have $\theta_1 < \breve{\theta} \equiv \frac{\ln\left(\frac{v}{\overline{a}_{AB}}\right)}{\ln(\overline{a}_{AB}v)}$. Thus, [if $\theta_1 < \breve{\theta}$, then $\lim_{\theta_2 \to 0} z_2 < a_{BA,2}$ and $\frac{\partial z_2}{\partial \theta_2} < 0$] and [if $\theta_1 > \breve{\theta}$, then $\lim_{\theta_2 \to 0} z_2 > a_{BA,2}$ and $\frac{\partial z_2}{\partial \theta_2} > 0$]. Recalling that, by Lemma 3.2(i), when $a_{AB,2} < 1$, $\breve{\theta} \in (0,1)$, the claim with regard to the impact of an increase in θ_2 within territory 2 follows from Proposition 3.1 (parts (ii) and (iii)) and Corollary 3.1.

(iii) In this case $a_{BA,1} = \frac{1}{\overline{a}_{AB}v} < 1, a_{BA,2} = \frac{v}{\overline{a}_{AB}} > 1$. By Lemma 3.2(i), [$\frac{\partial z_1}{\partial \theta_2} < 0$ if $\theta_1 < \breve{\theta}$, and $\frac{\partial z_1}{\partial \theta_2} > 0$ if $\theta_1 > \breve{\theta}$], where $\breve{\theta} = \frac{\ln\left(\frac{v}{\overline{a}_{AB}}\right)}{\ln(\overline{a}_{AB}v)} \in (0,1)$. Furthermore, by Lemma 3.2(ii), $\lim_{\theta_2 \to 0} z_1 = 1$, $\lim_{\theta_2 \to 1} z_1 = \overline{a}_{AB}{}^{\frac{(1+\theta_1)}{1-\theta_1}} v^{-1} \in (\frac{\overline{a}_{AB}}{v}, \infty)$. Now, notice that, by Assumption 3.3, $\frac{\overline{a}_{AB}}{v} > \frac{1}{\overline{a}_{AB}v}$. The claim with regard to the impact of an increase in θ_2 within territory 1 follows from Proposition 3.1 (parts (ii) and (iii)) and Corollary 3.1. By Lemma 3.2(i), $\frac{\partial z_2}{\partial \theta_1} > 0$, and by Lemma 3.2(ii), $\lim_{\theta_1 \to 0} z_2 = 1$, $\lim_{\theta_1 \to 1} z_2 = \overline{a}_{AB}{}^{\frac{1+\theta_2}{1-\theta_2}} v \in (\overline{a}_{AB}v, \infty)$. If $\frac{v}{\overline{a}_{AB}} > \overline{a}_{AB}v$, then $\overline{a}_{AB} < 1$, a contradiction. Thus, $\lim_{\theta_1 \to 0} z_2 = 1 < a_{BA,2} < \lim_{\theta_1 \to 1} z_2$, and $\frac{\partial z_2}{\partial \theta_1} > 0$. Now, putting $z_2 = \overline{a}_{AB}{}^{\frac{\theta_1(1+\theta_2)}{1-\theta_1\theta_2}} v^{\frac{(1-\theta_2)\theta_1}{1-\theta_1\theta_2}} = \frac{v}{\overline{a}_{AB}}$, we get: $\overline{a}_{AB}{}^{1+\theta_1} = v^{1-\theta_1}$, so that $z_2 <$ (resp. $>$) $a_{BA,2}$ if $\theta_1 <$ (resp. $>$) $\breve{\theta}$. Recalling that, by Lemma 3.2(i), when $a_{AB,2} < 1$, $\breve{\theta} \in (0,1)$ the claim with regard to the impact of an increase in θ_1 within territory 2 follows from Proposition 3.1 (parts (ii) and (iii)) and Corollary 3.1.

By Proposition 3.2(i), if one community, say B, has an external conflict cost advantage in at least one territory, and no disadvantage in either territory, then external conflict falls monotonically in *both* territories if either acquires greater ability to influence conflict in the other. Total conflict falls as well in both territories, while internal conflict and output both rise. The dominant group B unambiguously benefits in both territories. Thus, in this case, stronger cross-territorial spillover reduces conflict (thereby increasing output) overall, but increases inter-community inequality in both territories.

From (3.23), given Assumption 3.3, and given $a_{AB,2} < 1$, $p_{BA,2} > 1$ iff $\theta_1 > \frac{\ln a_{BA,2}}{\ln a_{AB,1}}$. Since $a_{BA,2} \equiv \frac{v}{\overline{a}_{AB}}$ and $\ln a_{AB,1} = \overline{a}_{AB}v$, this threshold value of θ_1 can be written as $\frac{ln\left(\frac{v}{\overline{a}_{AB}}\right)}{ln(\overline{a}_{AB}v)}$. Parts (ii)-(iii) of Proposition 3.2 consider the case where B has a cost disadvantage in territory 2, which is at least compensated by a cost advantage in 1 (recall Assumption 3.3). Suppose that territory 1's ability to influence conflict in 2 is relatively low, so that B is the dominated (less successful) party in the inter-communal conflict in 2, though it is the dominant party in 1. This obtains when θ_1 is below the threshold $\frac{ln\left(\frac{v}{\overline{a}_{AB}}\right)}{ln(\overline{a}_{AB}v)}$. Then an increase in the ability of the group balance of power in either territory to influence conflict in the other territory reduces both external and total conflict in the former but increases them in the latter. Thus, greater spillover from a territory enhances inter-group peace (and thereby output) within, but aggravates inter-group conflict (and reduces output) outside, that territory. Greater spillover from a territory benefits the dominant community in that territory, in both territories. For example, greater spill-over from 2 benefits A in both territories. The threshold $\frac{ln\left(\frac{v}{\overline{a}_{AB}}\right)}{ln(\overline{a}_{AB}v)}$ is higher, the lower the mean relative cost $\overline{a}_{AB}$. Thus, the lower the cost advantage for B on average, the larger the interval for θ_1 over which these claims hold. In the limit, when neither community has a cost advantage on average, but each has a cost advantage in one territory, exactly neutralized by its cost *disadvantage* in the other territory, they must hold for all $\theta_1 \in (0,1)$. Note that, regardless of the extent of spillover from 2 to 1 (θ_2), the group with a cost advantage overall, B, is always the dominant group in 1 (see part (iii) in the proof of Proposition 3.2).

By Proposition 3.2, when each community has a cost advantage in one territory, a simultaneous increase in conflict spillover from both territories has contradictory effects. What is the net effect of such an increase? We now address this issue. We consider an equi-proportionate increase in conflict spillovers, which increases their geometric mean but not their relative proportion. We interpret it as capturing greater *bilateral* economic integration.

Proposition 3.3: Let Assumptions 3.1-3.3 hold, let $\theta \equiv \sqrt{\theta_1 \theta_2}$, $\theta_1 = w\theta, \theta_2 = w^{-1}\theta$, and suppose $a_{AB,2} < 1$. Define $\breve{\theta} \equiv \frac{\ln\left(\frac{v}{\overline{a}_{AB}}\right)}{\ln(\overline{a}_{AB} v)}$. Thenthere exists $\varepsilon \in (0,1)$ such that, for all $w > \varepsilon$, the following must hold.

(i) An increase in θ over $(0, \frac{\breve{\theta}}{w})$ increases external and total conflict in territory 2, while reducing internal conflict within each community in that territory; whereas an increase in θover $(\frac{\breve{\theta}}{w}, 1)$ reduces external and total conflict in territory 2, while increasing internal conflict within each community in that territory; with $\frac{\breve{\theta}}{w} \in (0,1)$. The share and payoff of community B both rise in 2, while the payoff of community A falls.

(ii) There exists $\theta^* \in (0, \frac{\breve{\theta}}{w})$ such that an increase in θ over $(0, \theta^*)$ increases external and total conflict in territory 1, while reducing internal conflict within each community in that territory; whereas an increase in θ over $(\theta^*, 1)$ reduces external and total conflict in territory 1, while increasing internal conflict within each community in that territory. An increase in θ over $(0, \theta^*)$ increases both the share and the payoff of community A, while reducing thepayoff of community B; an increase in θ over $(\theta^*, 1)$ reduces both the share and the payoff of community A, while increasing the payoff of community B.

(iii) $\frac{\partial\left(\frac{\breve{\theta}}{w}\right)}{\partial w}, \frac{\partial(\theta^*)}{\partial w}, \frac{\partial\left(\frac{\breve{\theta}}{w}\right)}{\partial \overline{a}_{AB}}, \frac{\partial(\theta^*)}{\partial \overline{a}_{AB}} < 0.$

Proof: In this case, (3.24) and (3.25) reduce respectively to:

$$z_1 = \overline{a}_{AB}^{\frac{\theta w^{-1}+\theta^2}{1-\theta^2}} v^{\frac{-(\theta w^{-1}-\theta^2)}{1-\theta^2}}; \tag{3.34}$$

$$z_2 = \overline{a}_{AB}^{\frac{\theta w+\theta^2}{1-\theta^2}} v^{\frac{(\theta w-\theta^2)}{1-\theta^2}}. \tag{3.35}$$

In turn, (3.34) and (3.35) yield:

$$\ln z_1 = (\tfrac{\theta w^{-1}+\theta^2}{1-\theta^2}) \ln \overline{a}_{AB} - \tfrac{(\theta w^{-1}-\theta^2)}{1-\theta^2} \ln v,$$

$$\ln z_2 = (\tfrac{\theta w+\theta^2}{1-\theta^2}) \ln \overline{a}_{AB} + \tfrac{(\theta w-\theta^2)}{1-\theta^2} \ln v;$$

using which we have:

$$\frac{1}{z_1}\frac{\partial z_1}{\partial \theta} = \frac{1}{(1-\theta^2)^2}\big[[w^{-1}(1+\theta^2)+2\theta]\ln \overline{a}_{AB} - [w^{-1}(1+\theta^2)-2\theta]\ln v\big]; \tag{3.36}$$

$$\frac{1}{z_2}\frac{\partial z_2}{\partial \theta} = \frac{1}{(1-\theta^2)^2}[[w(1+\theta^2)+2\theta]\ln \overline{a}_{AB} + [w(1+\theta^2)-2\theta]\ln v]. \tag{3.37}$$

Consider

$$H \equiv \big[[w^{-1}(1+\theta^2)+2\theta]\ln \overline{a}_{AB} - [w^{-1}(1+\theta^2)-2\theta]\ln v\big] = \ln\left(\frac{\overline{a}_{AB}}{v}\right)^{w^{-1}(1+\theta^2)} (\overline{a}_{AB}v)^{2\theta}. \tag{3.38}$$

Using (3.38), we have:

$$\frac{\partial H}{\partial \theta} \equiv 2\big[[\theta w^{-1}+1]\ln \overline{a}_{AB} - [\theta w^{-1}-1]\ln v\big] = 2[\ln\left(\frac{\overline{a}_{AB}}{v}\right)^{\theta w^{-1}} (\overline{a}_{AB}v)]. \tag{3.39}$$

Since $\overline{a}_{AB} > 1$, there must exist $\varepsilon_1 \in (0,1)$ such that $\left(\frac{\overline{a}_{AB}}{v}\right)^{\varepsilon_1^{-1}} (\overline{a}_{AB}v) = 1$. Then, since $\left(\frac{\overline{a}_{AB}}{v}\right) < 1$, by (3.39), for all $w > \varepsilon_1$, $\frac{\partial H}{\partial \theta} > 0$. Now, from (3.36), $\lim_{\theta\to 0} H < 0$, whereas

$\lim_{\theta\to 1} H = 2\big[[w^{-1}+1]\ln \overline{a}_{AB} - [w^{-1}-1]\ln v\big] = 2[\ln\left(\frac{\overline{a}_{AB}}{v}\right)^{w^{-1}} (\overline{a}_{AB}v)] > 0$ for all $w > \varepsilon_1$.

Hence, recalling (3.36), we have: given any $w > \varepsilon_1 \in (0,1)$, there exists $\theta^* \in (0,1)$ such that

$$\frac{\partial z_1}{\partial \theta} < \text{(resp. >) } 0 \text{ iff } \theta < \text{(resp. >) } \theta^*. \tag{3.40}$$

Recalling that $[\overline{a}_{AB} > 1, v \geq 1]$ by Assumption 3, and since $a_{AB,2} < 1$ implies $\frac{v}{\overline{a}_{AB}} > 1$, (3.37) yields: there exists $\varepsilon_2 \in (0,1)$ such that [if $w > \varepsilon_2$, then $\frac{\partial z_2}{\partial \theta} > 0$]. (3.41)

(i) Since $a_{AB,2} < 1$, $a_{BA,2} = v\bar{a}_{AB}{}^{-1} > 1$. Now let $\varepsilon = \max\{\varepsilon_1, \varepsilon_2\}$, and consider any $w > \varepsilon$. Then, from (3.41), $\frac{\partial z_2}{\partial \theta} > 0$ for all $\theta \in (0,1)$, and the minimum value of z_2 is 1. Hence, by parts (ii) and (iii) of Proposition 3.1, external and total conflict in 2 initially rise in θ, till z_2 reaches $a_{BA,2}$, and falls thereafter as z_2 keeps rising in θ if the threshold value of θ, at whichz_2= $a_{BA,2}$ (so that B's share is the same as that of the low cost combatant A) is less than 1; internal conflict behaves in the opposite fashion. Using (3.35), the threshold value of θ, $\hat{\theta}$, is given by$z_2 = \bar{a}_{AB}{}^{\frac{\theta w+\theta^2}{1-\theta^2}} v^{\frac{(\theta w-\theta^2)}{1-\theta^2}} = v\bar{a}_{AB}{}^{-1}$; so that $\bar{a}_{AB}{}^{(\theta w+1)} v^{(\theta w-1)} = 1$, implying $\hat{\theta} = \frac{\ln(a_{BA,2})}{w \ln(a_{AB,1})} = \frac{\breve{\theta}}{w}$. Since $a_{AB,1} = v\bar{a}_{AB} = (a_{BA,2})\bar{a}_{AB}{}^2$, and $\bar{a}_{AB} > 1$, we have $a_{AB,1} > a_{BA,2} > 1$, so that $\breve{\theta} \in (0,1)$. Since $w > \varepsilon_1$, we have $\left(\frac{\bar{a}_{AB}}{v}\right)^{w^{-1}} (\bar{a}_{AB} v) > 1$. Then $w > \breve{\theta}$, so that $\hat{\theta} \in (0,1)$. Since $\frac{\partial z_2}{\partial \theta} > 0$, the claims regarding shares and pay-offs follow from Corollary 3.1.

(ii) Let $\varepsilon = \max\{\varepsilon_1, \varepsilon_2\}$, and consider any $w > \varepsilon$. Since $a_{AB,2} < 1$, $1 < \bar{a}_{AB} < v$. Since (by Assumption 3) $a_{AB,1} > 1$, $a_{BA,1} = v^{-1}\bar{a}_{AB}{}^{-1} < 1$. Thus, recalling (3.30), and noting that $\lim_{\theta \to 0} z_1 = 1$, z_1 initially falls from with an increase θ from 0, reaching its minimum at $\theta = \theta^*$, andsubsequently rises. Recall that $a_{BA,1} = v^{-1}\bar{a}_{AB}{}^{-1} < 1$. Suppose $z_1(\theta^*) \leq a_{BA,1}$. Then, using (3.33), at $\theta = \theta^*$, $[\left(\frac{1-\theta w^{-1}}{1-\theta^2}\right) \ln v + \left(\frac{\theta w^{-1}+1}{1-\theta^2}\right) \ln \bar{a}_{AB} \leq 0]$, which implies $w \leq \frac{\ln(a_{BA,2})}{\ln(a_{AB,1})}\theta$, a contradiction, since$w > \varepsilon_1$ implies $\left(\frac{\bar{a}_{AB}}{v}\right)^{w^{-1}} (\bar{a}_{AB} v) > 1$, which in turn implies $w > \frac{\ln(a_{BA,2})}{\ln(a_{AB,1})}$.Hence we get:

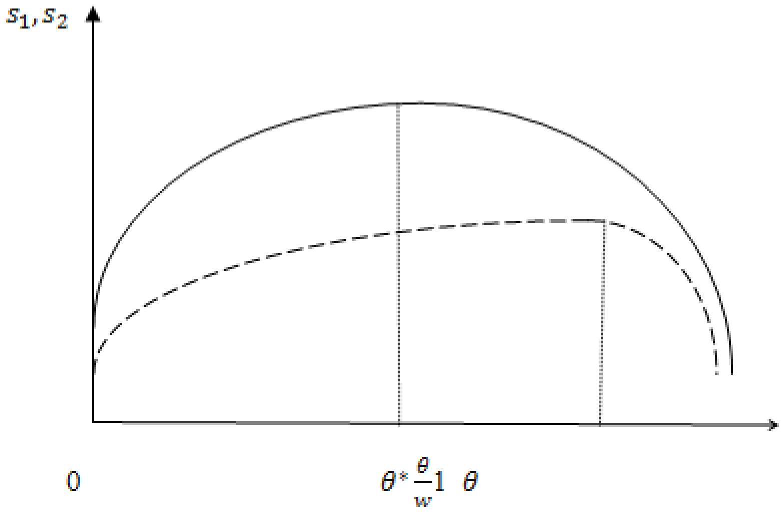
$$\text{for all } \theta \in (0,1),\ [z_1(\theta) > a_{BA,1}]. \qquad (3.42)$$

The claims regarding the effect of an increase in θ on the behaviour of internal, external and total conflict follow from (3.40), (3.42) and parts (ii) and (iii) of Proposition 3.1. Recalling (3.40), the claims regarding shares and pay-offs follow from Corollary 3.1. At $\theta = \frac{\breve{\theta}}{w}$, z_2= $a_{BA,2}$, so that $p_{BA,2} = 1$, implying $z_1 = 1$. Recall that, from (3.34), $\lim_{\theta \to 0} z_1 = 1$. In light of (3.40), it follows that $\theta^* < \frac{\breve{\theta}}{w}$.

(iii) Let $\varepsilon = \max\{\varepsilon_1, \varepsilon_2\}$, and consider any $w > \varepsilon$. Then, by (3.39), $\frac{\partial H}{\partial \theta} > 0$, and, by (3.38), $\frac{\partial H}{\partial \bar{a}_{AB}} > 0$. Since at $\theta = \theta^*, H = 0$, it follows that $\frac{\partial \theta^*}{\partial \bar{a}_{AB}} < 0$. Furthermore, since $\left(\frac{\bar{a}_{AB}}{v}\right) < 1$, $\frac{\partial H}{\partial w} > 0$, so that $\frac{\partial \theta^*}{\partial w} < 0$. Since $\frac{\breve{\theta}}{w} = \frac{\ln\left(\frac{v}{\bar{a}_{AB}}\right)}{w \ln(\bar{a}_{AB} v)}$, $\frac{\partial\left(\frac{\breve{\theta}}{w}\right)}{\partial w}, \frac{\partial\left(\frac{\breve{\theta}}{w}\right)}{\partial \bar{a}_{AB}} < 0$. ■

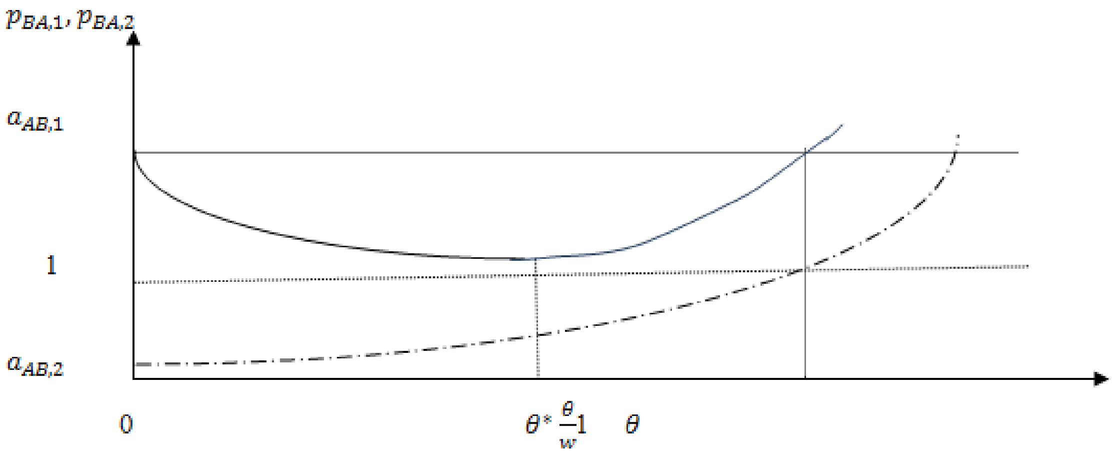
By Proposition 3.3, inter-group conflict initially rises in both territories as our measure of bilateral integration, the mean spillover elasticity θ, rises from 0 (intuitively, bilateral autarky), till it reaches some threshold value; it falls in both territories as θ rises further, beyond some other threshold value. In between, a rise in θ increases external conflict in one territory, but reduces it in the other. Total conflict behaves in the same fashion, while internal conflict and social output behave in the opposite fashion. Thus, in this intermediate case, greater bilateral integration has contradictory effects on conflict across territories. This is illustrated in Figure 1 below. In Figure 1, the unbroken inverted-U shaped schedule on top shows how inter-group (and aggregate) conflict behaves in territory 1, while the broken schedule below shows the behaviour of inter-group (or aggregate) conflict in 2. External and total conflict rise with the level of spillover in both territories over the interval $(0, \theta^*)$, while theyfall in both territories in response to greater bilateral integration over the interval $(\frac{\breve{\theta}}{w}, 1)$. Over the intermediate interval $(\theta^*, \frac{\breve{\theta}}{w})$, greater spillover increases external and total conflict in territory 2 (where B has a cost disadvantage), but reduces them in territory 1 (where B has a cost advantage).

Figure 1: Changes in levels of aggregate conflict with increased bilateral integration



Greater bilateral integration unambiguously benefits B in the territory where it has a cost disadvantage (2), but may possibly (though not necessarily) hurt that community in the territory where it has a cost advantage (1). Greater bilateral integration initially benefits the cost disadvantaged group in both territories. Thus, A in 1 and B in 2 both achieve greater success in their respective inter-group conflicts and achieve welfare gains. This is associated with greater external (and total) conflict in both territories. Despite its gains, A always remains the dominated (lower share) group in 1. Beyond a threshold level (θ^*), the identity of the beneficiary group gets reversed in 1. Further increases in bilateral integration come to benefit the cost advantaged (and dominant) group, B, in 1, while reducing inter-group (and aggregate) conflict in that territory. Such a reversal does not occur in 2. Increases in bilateral integration continue to benefit the cost disadvantaged group, B, in 2. Eventually, spillover effects from 1 come to outweigh B's cost disadvantage in 2, so that B becomes the dominant group in the external conflict in 2. Further increases in bilateral integration increase B's dominance (share) even more, while also reducing inter-group conflict in 2. Thus, so long as B's cost disadvantage in 2 is reflected in B being less successful than A in the inter-group conflict in 2, a marginal increase in bilateral integration increases inter-group and aggregate conflict in that territory. This is depicted in Figure 2. The unbroken U-shaped schedule illustrates the behavior of B's relative share in 1, while the broken U-shaped schedule below depicts the movement of B's relative share in 2.

Figure 2: Changes in relative shares with increased bilateral integration



By Proposition 3.3(iii), both the threshold values rise if either $\bar{a}_{AB}$ or w falls. Hence, since $\bar{a}_{AB} > 1$ by Assumption 3.3(i), the more the communities are similar on average in their conflict cost, the more likely it is that greater bilateral integration will increase conflict in *both* countries. The lower the relative spillover elasticity of territory 1 (where the community with overall cost advantage, B, dominates), the more likely that greater bilateral integration will increase conflict in both countries. The single-peak property of inter-group (and aggregate) conflict, stated in Proposition 3.3 and depicted in Figure 1, need not hold when w is sufficiently close to 0.

When the same community cost-dominates in both territories, an equi-proportionate increase in spillovers benefits the dominant community, magnifying its share and payoff in both territories. Since p_{Aj} tends to 0 as θ tends to 1, even an infinitesimally small advantage in external conflict cost translates into an arbitrarily large advantage in conflict success if the mean spillover elasticity $\theta \in (0,1)$ is sufficiently close to 1. Thus, strongerspilloversall around imply higher shares of the dominant community in this case. One may interpret this in the spirit of a 'knife-edge' result. Suppose, initially, the two communities faced identical costs of engaging in conflict, so that they shared the prize equally in the two territories. Now suppose, due to some change in the economic environment, the relative conflict cost shifts marginally in favour of B in just one territory. Even such a localized and marginal change can have arbitrarily large global consequences for conflict outcomes: it can increase the relative share of B in *both* territories to an *arbitrarily large* extent when cross-location identification leads to spillover effects that are sufficiently intense and pervasive.

3.4 Cost effect spill-over

How do changes in the fundamentals of a society such as population size, population composition, degree of private property rights protection and relative productivity within a territory affect conflict and distribution in the *other* territory? We now turn to this question.

Define, for all $g \in \{A, B\}$ and all $j \in \{1,2\}$, $l_{gj} \equiv \frac{L_{gj}}{T_{gj}}$, $l_{AB,j} \equiv \frac{l_{Aj}}{l_{Bj}}$, $\alpha_j \equiv \frac{n_{Aj}}{n_j}$. Thus, $l_{AB,j}$ is the relative productivity of community A in territory *j* and $\alpha_j \in (0,1)$ is the population share of community *A* in territory *j*. Using (3.11), we have the relative conflict cost of A in territory *j*:

$$a_{AB,j} \equiv \frac{[\mu_{Aj}+(1-\mu_{Aj})\alpha_j n_j]}{[\mu_{Bj}+(1-\mu_{Bj})(1-\alpha_j)n_j]}\left[l_{AB,j}\frac{(1-\alpha_j)}{\alpha_j}\right]. \tag{3.43}$$

By (3.43), the relative conflict cost in a territory depends on its total population, population composition, relative productivity, and property rights protection. These affect the balance of power (relative success) in domestic group conflict (recall (3.22)), which spills over into group conflict in the *other* territory, by affecting relative conflict efficiency in the latter (Assumption 3.2 and (3.23)).

Proposition 3.4: Let Assumptions 3.1-3.3 hold, let $\breve{\theta} \equiv \frac{\ln\left(\frac{v}{\bar{a}_{AB}}\right)}{\ln(\bar{a}_{AB}v)}$, and, for all $j \in \{1,2\}$,assume $[\mu_j = \mu_{A,j} = \mu_{B,j}$ and $\alpha_j \in (0,\frac{1}{2})]$. Define $Q_j \equiv \{l_{BA,j}, n_j, \alpha_j, (1-\mu_j)\}$. Then the following must hold.

(i) If $a_{AB,2} \geq 1$, then, for all $j \in \{1,2\}$, and for all $q_j \in Q_j$, a marginal increase in q_j increases external and total conflict in territory -j, while decreasing internal conflict and output within each community in that territory.

(ii) If $a_{AB,2} < 1$, then: (a) provided $\theta_1 \in (0,\breve{\theta})$, a marginal increase in any $q_1 \in Q_1$ decreases external and total conflict in territory 2, while increasing internal conflict and output within each community in that territory; and (b) provided $\theta_1 \in (\breve{\theta}, 1)$, a marginal increase in any $q_1 \in Q_1$ increases external and total conflict in territory 2, while decreasing internal conflict and output within each community in that territory.

(iii) If $a_{AB,2} < 1$, then a marginal increase in any $q_2 \in Q_2$ increases external and total conflict in territory 1, while decreasing internal conflict and output within each community in that territory.

(iv) For all $j \in \{1,2\}$, p_{Aj} rises as any $q_{-j} \in Q_{-j}$ increases; furthermore, the payoff of community A in j rises, and that of community B in j declines, monotonically in all $q_{-j} \in Q_{-j}$.

Proof: We prove Proposition 3.4 via the following two lemmas.

Lemma 3.3: Let Assumptions 3.1-3.3 hold, and let $\breve{\theta} \equiv \frac{\ln\left(\frac{v}{\bar{\bar{a}}_{AB}}\right)}{\ln(\bar{a}_{AB}v)}$.

(i) If $a_{AB,2} \geq 1$, then, for all $j \in \{A, B\}$, an increase in $a_{AB,-j}$ reduces external and total conflict in territory j, while increasinginternal conflict within each community in that territory.

(ii) If $a_{AB,2} < 1$, then: (a) provided $\theta_1 \in (0, \breve{\theta})$, an increase in $a_{AB,1}$ increases external and total conflict in territory 2, while reducing internal conflict within each community in that territory; and (b) provided $\theta_1 \in (\breve{\theta}, 1)$, an increase in $a_{AB,1}$ reduces external and total conflict in territory 2, while increasing internal conflict within each community in that territory.

(iii) If $a_{AB,2} < 1$, then a decrease in $a_{AB,2}$ increases external and total conflict in territory 1, while decreasing internal conflict within each community in that territory.

(iv) For all $j \in \{1,2\}$, the share of community A declines monotonically in territory j as $a_{AB,-j}$ increases, with $\lim_{a_{AB,-j}\to\infty} p_{Aj} = 0$; furthermore, the pay-off of A in j declines, and that of B in j rises, monotonically as $a_{AB,-j}$ rises.

*Lemma 3.4***:** Let Assumptions 3.1-3.3 hold. Then, for all $j \in \{1,2)$:

(i) $\frac{\partial a_{AB,j}}{\partial l_{AB,j}} > 0$; (ii) if $\mu_{Bj} >$ (resp. =) μ_{Aj}, then there exists $\gamma_j <$ (resp. =) $\frac{1}{2}$ such that $\frac{\partial a_{AB,j}}{\partial n_j} < 0$ for all $\alpha_j \in \left(0, \gamma_j\right)$; (iii) if $\mu_{Bj} = \mu_{Aj}$, then there exists $\rho_j \in (\frac{1}{2}, 1)$ such that, $[\frac{\partial a_{AB,j}}{\partial \alpha_j} < 0$ for all $\alpha_j \in \left(0, \rho_j\right)]$;

and (iv) if$\mu_A = \mu - e, \mu_B = \mu + e$ for some $e \geq 0$,then there exists $\sigma_j \in [\frac{1}{2}, 1)$ such that $[\frac{\partial a_{AB,j}}{\partial \mu_j} > 0$ for all $\alpha_j \in (0, \sigma_j)]$, with $\sigma_j > \frac{1}{2}$ iff $e > 0$.

Proof of Lemma 3.3

(i) From (3.24)-(3.25), respectively,

$$\ln z_1 = \left(\frac{\theta_2}{1-\theta_1\theta_2}\right) \ln a_{AB,2} + (\frac{\theta_1\theta_2}{1-\theta_1\theta_2}) \ln a_{AB,1}; \tag{3.44}$$

$$\ln z_2 = \left(\frac{\theta_1}{1-\theta_1\theta_2}\right) \ln a_{AB,1} + (\frac{\theta_1\theta_2}{1-\theta_1\theta_2}) \ln a_{AB,2}. \tag{3.45}$$

Since, by Assumption 3, $a_{AB,1} > 1$, (3.44) and (3.45) imply: for all $\theta_1, \theta_2 \in (0,1)$, $z_1, z_2 > 1$, and $\frac{\partial z_j}{\partial a_{AB,-j}} = \left(\frac{\theta_j}{1-\theta_1\theta_2}\right) > 0$. Since $a_{AB,1}, a_{AB,2} \geq 1$, we have $a_{BA,1}, a_{BA,2} \leq 1$. Part (i) of Proposition 4 follows from Proposition 1 (parts (ii) and (iii)) and Corollary 1.

(ii)-(iii) In this case $1 < \bar{a}_{AB} < v$, and $a_{BA,1} = \frac{1}{v\bar{a}_{AB}} < 1, a_{BA,2} = v\bar{a}_{AB}{}^{-1} > 1$. Then, using (3.26), $\frac{z_1}{a_{BA,1}} = \bar{a}_{AB}{}^{\frac{(1+\theta_2)}{1-\theta_1\theta_2}} v^{\frac{(1-\theta_2)}{1-\theta_1\theta_2}}$, implying $z_1 > a_{BA,1}$. From (3.27), $\frac{z_2}{a_{BA,2}} = \left(\frac{\bar{a}_{AB}{}^{(1+\theta_1)}}{v^{(1-\theta_1)}}\right)^{\frac{1}{1-\theta_1\theta_2}}$. This ratio is less than 1 at $\theta_1 = 0$, increasing in θ_1. It reaches 1 at $\theta_1 = \breve{\theta} \equiv \frac{\ln(v\bar{a}_{AB}{}^{-1})}{\ln(v\bar{a}_{AB})}$ and converges to ∞ as θ_1 converges to 1. Hence, $z_2 <$ (resp. >) $a_{BA,2}$ if $\theta_1 <$ (resp. >) $\breve{\theta}$. Recalling that, from (3.24)-(3.25),$\frac{\partial z_j}{\partial a_{AB,-j}} > 0$, parts (ii) and (iii) of Proposition 3.4 follows from Proposition 3.1 (parts (ii) and (iii)).

(iv) Recalling that, from (3.24)-(3.25), $[\frac{\partial z_j}{\partial a_{AB,-j}} > 0]$, and $\lim_{a_{AB,-j} \to \infty} z_j = \infty$, part (iv) of Proposition 3.4 follows from Corollary 3.1. ■

Proof of Lemma 3.4: In the proof of Lemma 3.4, we drop the territorial subscript *j* for notational simplicity.

(i) Part (i) follows immediately from (3.43).

(ii) From (3.43),

$$\left[l_{AB}\frac{(1-\alpha)}{\alpha}\right]^{-1} \frac{\partial a_{AB}}{\partial n} \equiv \left[\frac{(1-\mu_A)\alpha[\mu_B+(1-\mu_B)(1-\alpha)n]-(1-\mu_B)(1-\alpha)[\mu_A+(1-\mu_A)\alpha n]}{[\mu_B+(1-\mu_B)(1-\alpha)n]^2}\right]. \tag{3.46}$$

Let $\mu_B = \mu_A + 2\varepsilon$. The numerator on the RHS of (3.46) can then be written as:

$$Z = (1-\mu_A)\alpha[\mu_A + 2\varepsilon + (1-\mu_A-2\varepsilon)(1-\alpha)n] - (1-\mu_A-2\varepsilon)(1-\alpha)[\mu_A + (1-\mu_A)\alpha n]$$

$$= (1-\mu_A)(2\alpha-1)\mu_A + 2\varepsilon(\alpha - 2\alpha\mu_A + \mu_A).$$

Now, since $\alpha, \mu_A \in (0,1)$, $(\alpha - 2\alpha\mu_A + \mu_A) > 0$; if $\varepsilon \geq 0$, $\frac{\partial z}{\partial \alpha} = 2[(1-\mu_A-2\varepsilon)\mu_A + \varepsilon] > 0$, since $\mu_A + 2\varepsilon = \mu_B \in (0,1)$. At $\alpha = \frac{1}{2}$, [Z > 0 if $\varepsilon > 0$, and Z = 0 if $\varepsilon = 0$]. At $\alpha = 0$, $Z = -(1-\mu_A-2\varepsilon)\mu_A < 0$. Hence, given any $\varepsilon \geq 0$, there exists $\rho(\varepsilon) \leq \frac{1}{2}$ such that $\frac{\partial a_{AB}}{\partial n} < 0$ for all $\alpha \in \big(0, \rho(\varepsilon)\big)$., the inequality holding strictly if $\varepsilon > 0$.

(iii) From (3.43),

$$\frac{1}{l_{AB}}\frac{\partial a_{AB}}{\partial \alpha} = \frac{[\mu_A+(1-\mu_A)\alpha n]}{[\mu_B+(1-\mu_B)(1-\alpha)n]}\left[\frac{-1}{\alpha^2}\right] + \frac{[(1-\mu_A)n]}{[\mu_B+(1-\mu_B)(1-\alpha)n]} + \frac{[\mu_A+(1-\mu_A)\alpha n](1-\mu_B)n}{[\mu_B+(1-\mu_B)(1-\alpha)n]^2}. \tag{3.47}$$

Then, assuming $\mu = \mu_A = \mu_B$, (3.47) yields:

$$\frac{1}{l_{AB}}\frac{\partial a_{AB}}{\partial \alpha} = \frac{[\mu+(1-\mu)\alpha n][\mu+(1-\mu)(1-\alpha)n]}{[\mu+(1-\mu)(1-\alpha)n]^2}\left[\frac{-1}{\alpha^2}\right] + \frac{(1-\mu)n[2\mu+(1-\mu)n]}{[\mu+(1-\mu)(1-\alpha)n]^2}$$

$$= \frac{(1-\mu)n[2\mu+(1-\mu)n] - [\frac{\mu}{\alpha}+(1-\mu)n][\frac{\mu}{\alpha}+(1-\mu)\left(\frac{1-\alpha}{\alpha}\right)n]}{[\mu+(1-\mu)(1-\alpha)n]^2}. \tag{3.48}$$

The numerator on the RHS of (3.48) increases as α increases. At $\alpha = \frac{1}{2}$, the numerator is $d \equiv [2\mu + (1-\mu)n][-2\mu] < 0$, implying $\frac{\partial a_{AB}}{\partial \alpha} < 0$. At $\alpha = 1$, the numerator is positive if μ is sufficiently close to 0. Part (iii) of Lemma 3.2 follows by continuity.

(iv) From (3.41), assuming $\mu_A = \mu - e, \mu_B = \mu + e$ for some $e \geq 0$,

$$(l_{AB}\frac{(1-\alpha)}{\alpha})^{-1}\frac{\partial a_{AB}}{\partial \mu} = [\frac{[1-\alpha n][\mu_B+(1-\mu_B)(1-\alpha)n] - [1-(1-\alpha)n][\mu_A+(1-\mu_A)\alpha n]}{[\mu_B+(1-\mu_B)(1-\alpha)n]^2}]. \tag{3.49}$$

The numerator on the RHS of (3.49) tends to $\left[(\mu_B - \mu_A)(1-\frac{n}{2})^2\right]$ as α tends to $\frac{1}{2}$; $\left[(\mu_B - \mu_A)(1-\frac{n}{2})^2\right] \geq 0$ if $\mu_B \geq \mu_A$, with the inequality holding strictly iff $\mu_B > \mu_A$. Let

$q = [1-\alpha n][\mu_B + (1-\mu_B)(1-\alpha)n] - [1-(1-\alpha)n][\mu_A + (1-\mu_A)\alpha n].$

Then $\frac{\partial q}{\partial \alpha} < 0$. Hence, the numerator in (3.49) is positive for all $\alpha \in (0, \frac{1}{2})$ if $\mu_B \geq \mu_A$, and it is positive at $\alpha = \frac{1}{2}$ iff *e*> 0. Part (iv) of Lemma 3.2 follows by continuity. ■

Proposition 3.4 follows immediately from Lemmas 3.3-3.4. ■

Given Assumptions 3.1-3.3, suppose A, the community with a conflict cost disadvantage on average, is the minority in both territories. Suppose further property rights protection is the same across communities within a territory (though it may vary across territories). Then, by Proposition 3.4(i), provided A does not have a cost advantage in either territory, a share-preserving population increase in any territory increases both external and total conflict in the *other* territory. The same effect obtains if, in any territory, either the minority's population share increases (without reducing total population), or property rights protection weakens uniformly across the territory, or the majority community achieves an increase in its relative labour productivity. By Proposition 3.4(iii), if the minority has a conflict cost advantage in 2, then such changes in 2 must increase both external and total conflict in 1. By Proposition 3.4(ii), given that the minority has a conflict cost advantage in 2, but a disadvantage on average, such changes in 1 will increase both external and total conflict in 2 when 1's ability to influence conflict in 2 (i.e., θ_1) is sufficiently high, so that the minority is the dominated (less successful) community in 2 (as well as in 1). The higher the value of average relative cost of conflict for the minority ($\overline{a}_{AB}$), i.e. the farther apart the two communities are on average in terms of their conflict cost, the lower the threshold value $\breve{\theta}$: hence the larger the range of spillover values from 1 to 2 over which this holds. Thus, when the majority community has a large cost advantage on average, changes in the parameters under focus have the same effect on conflict in the other territory, irrespective of the location of such changes.

In sum, therefore, population increase in a territory that does not reduce the minority's share, an enlargement of the minority that does not reduce total population, weaker property rights protection across communities, and an increase in the relative labour productivity of the majority community, all causally increase both inter-group and aggregate conflict in the *other* territory, when the minority community is the dominated one in both territories. However, when the minority community dominates in one territory, but is dominated in another, such changes in one territory may increase or decrease inter-group and aggregate conflict in the other territory, depending on which territory the changes occur in. These changes nonetheless always make the minority in the other territory better off, while making the majority therein worse off (Proposition 3.4(iv)).

The mechanism driving these results is as follows. Given our assumptions, within a territory, j, growth of the minority population at a rate at least as high as the growth rate of the majority (with non-decreasing total population), weaker property rights protection across communities, and higher relative labour productivity of the majority community, all reduce $a_{AB,j}$ - the relative conflict cost of the minority, A, in *j* (Lemma 3.4). A is more successful in consequence in its conflict with B in *j*. Such success spills over into the other territory, -*j*, increasing the relative efficiency of the minority's conflict effort in that territory, thereby making A more successful there as well. Cross-territorial spillovers make greater success of A in the two territories mutually reinforcing, so that, overall, the minority is more successful in its conflict with the majority in both territories. In consequence, A is better off in -*j*, while B is worse off therein. An increase in its relative conflict efficiency in -*j*, due to the spillover effect of greater success on its part in *j*, induces the minority to invest greater effort against the majority in –*j*. If the majority dominates in –*j*, then it responds by shifting effort from internal to external conflict. Both aggregate and external conflict levels increase in –*j* in consequence, while output and internal conflict fall. The impacts on conflict and group payoffs within *j* are, however, ambiguous in general, since such changes affect conflict and payoffs within *j* through multiple channels.

The assumption that property rights protection is at least as strong within the minority as within the majority community ($\mu_{Bj} \geq \mu_{Aj}$) actually suffices for the claim regarding the effects of a cross-community improvement in property rights protection within a territory. If $\mu_{Bj} > \mu_{Aj}$, the claim continues to hold if A is the minority in both territories, and may hold even if A constitutes the majority in one, or both, territories. If $\mu_{Bj} > \mu_{Aj}$, the claim regarding the effect of a share-preserving population increase continues to hold if A constitutes a sufficiently small minority in both territories.

Given identical property rights protection across communities, the claim regarding the effect of an increase in the minority's population share (with constant total population) may hold even if A constitutes the majority in one, or both, territories. The claim regarding the effect of an increase in relative productivity holds independently of any assumption, regarding property rights protection across communities or the population share of the cost disadvantaged community, A, whatsoever.

3.5 Concluding Remarks

This chapter has developed a parsimonious model of simultaneous between-group ('ethnic') conflict over public goods with group-specific non-monetary benefits (state 'culture' or 'religion'), decentralized intra-group distributive conflict over private consumption ('income'), and production, with conflict spillovers across politico-administrative territories such as countries or provinces of a country. The theoretical analysis generates a number of empirically testable propositions regarding the nature of extra-territorial influences on intra-territorial (or domestic) conflict.

It is typically difficult, if not impossible, to prevent partisan political aid, both financial and material, from being routed through standard aid, investment and business transaction channels, just as it is difficult to prevent activists from utilizing entry procedures intended for economic migrants and refugees. Greater economic integration also makes the integrating country more susceptible to external pressures in the form of trade and investment sanctions. The Proposition 3.2 thus implies that greater unilateral economic integration by a country with another country, through unilateral relaxation of import restrictions, restrictions on private aid and capital flows and immigration controls, may increase ethnic conflict and reduce output in the integrating country. This happens when the dominant group in one country is affiliated to the dominated group in another. The country whose influence on the integrating country expands may thus be able to 'export out' its own ethnic conflict, i.e., reduce such conflict within. Current anxieties over the absorption of Arab refugees in Europe and North America, and their political fall-out, may be understood in this light. Faster growth in a country, by increasing its ability to influence conflict within its neighbours, may likewise reduce domestic ethnic conflict, while increasing it within its neighbours. When the dominant group in one country is affiliated to the dominant group in another, greater unilateral integration by either country reduces ethnic conflict in both. Conversely, in this case, a diminution in such integration, or, more generally, in the ability of either country to influence events in the other country, due to domestic economic stress or political developments caused by defeats in wars elsewhere, increases ethnic conflict in both countries.

Proposition 3.3 shows that, when the external affiliate of the dominant group in one country would be dominated without external support, greater bilateral integration initially increases ethnic conflict in both countries. Greater bilateral integration may reduce ethnic conflict in both countries at very high levels of such integration, when one becomes a political 'dependency' of another, in that the dominant group in the former owes that status solely to support from the latter. For conflict reduction, there may thus be an ‘all or nothing’ aspect to bilateral integration.

Proposition 3.4 reveals causal connections between ethnic conflict within a country and key characteristics of its neighbours, or, more generally, countries with which it has strong trade and immigration linkages. Being situated next to a more populous neighbour may make a country more conflict-prone. Greater ethnic polarization within its neighbor*s* may causally imply greater ethnic conflict inside a country. Ethnic occupational specialization and locational segregation imply that trade and labour market deregulation, and privatization of public sector units, may affect different ethnic groups differently, as may environmental degradation. If such deregulation and privatization (or environmental degradation) in a neighbouring country or territory affect the economic opportunities of majority and minority ethnic groups therein differently, they may influence group conflict within a country, in ways identified by Proposition 3.4. A country-wide weakening or collapse of the state machinery (and thus of property rights protection), due to civil war and external intervention or due to a sustained economic crisis, may turn neighbouring countries more fragile and conflict-ridden. Lastly, the direct effect of a community-neutral increase in labour productivity within a territory reduces both inter-group and aggregate conflict, but increases intra-group conflict and group output within both communities in that territory. These domestic effects are all strengthened if the consequent economic growth indirectly expands that territory's ability to influence ethnic conflict in the other territory (recall Proposition 3.2).

Our theoretical conclusions may be usefully subjected to econometric investigation, especially in a cross-country context. Contest-theoretic models have been subjected to experimental investigation (see Dechenaux et al.(2015) for a survey). In particular, Chowdhury, Jeon and Ramalingam (2016) have recently developed an interesting experimental analysis of identity-based group conflict. Our model may also be usefully subjected to experimental scrutiny. At a theoretical level, four extensions are of particular interest. First, one may examine conflict spillovers using axiomatized contest success functions other than the Tullock formulation used in this paper (see Münster (2009); and Skaperdas 1996). Second, one may use alternatives to our summative specification for each community's aggregate group conflict effort, such as a constant elasticity of substitution aggregation (e.g. Kolmar and Rommeswinkel (2013)). The 'best-shot' and 'weakest-link' specifications are also of interest in this regard (e.g.Chowdhury and Topolyan (2016); Chowdhury et al. (2016) ; Barbieri et al. (2014); Chowdhury et al. (2013) and Lee (2012)). Third, the effect of asymmetries within a group, in terms of subgroups of different sizes and different intra-subgroup sharing rules, may be explored. Deploying a utility function more general than the linear formulation adopted in this paper, Dasgupta and GuhaNeogi (2018) have examined how internal fragmentation of groups into sub-groups of different sizes affects the outcome of between-group conflict over the division of public goods. However, neither within-group conflict nor cross-territorial spillover figures in their analysis. Embedding these two features of the present paper into their framework may yield useful insights. Fourth, there may be contexts where greater success (share) in inter-group conflict for a community, A, in territory 1reduces its share in territory 2. This is the case of negative spillovers. This may happen because greater success for A in 1 polarizes territory 2 more sharply along identity divides: sections of the other community, B, in 2, develop greater antagonism towards A in 2, and perceive A as more of a threat, in response. `This may also happen because large numbers of B individuals in 1 shift effort to the group conflict in 2, physically migrating there, and/or large numbers of A individuals in 2 migrate to 1. The first mechanism involves the idea of an 'identity backlash' and a consequent feedback loop. The second involves increasing returns to scale in group conflict, momentum and bandwagon effects, as well as the desire to benefit from a larger existing share of the group-specific public good. It is easy to see that both these mechanisms, which may operate simultaneously, counteract the tendencies highlighted in our analysis, and, when stronger than the latter, are likely to generate equilibria that involve the effective resolution of ethnic conflict, through the creation of what are, in effect, mono-ethnic territories, with large-scale ethnic cleansing and exchange of population when the second mechanism operates, and cultural/religious assimilation of the weaker group to the stronger one otherwise. This is in contrast to our focus on a situation of persistent ethnic conflict in equilibrium, which presupposes a stable and non-trivial division of the population within a territory into competing ethnic groups. Both cases therefore require a theoretical apparatus different from the one developed in this paper. In real world conflict contexts, persistent ethnic conflict may and often does tip over into ethnic cleansing and population exchange, and thus analysis of these cases would complement the analysis in this paper. We look forward to developments along the above-discussed lines.

Chapter 4

Conflict between class-divided communities with unequal sharing rules

4.1 Introduction

How do class divisions, in the broad sense of income or wealth inequality, within a community affect the nature of its conflict over divisible resource-sharing with another community? The purpose of this essay is to shed theoretical light on this issue. We consider a scenario where two communities, each internally class-differentiated into rich and poor segments, contest one another for the division of some exogenously given resource, or rent. Any share of the resource accruing to a community is distributed internally between its constituent classes according to a given sharing rule. All members of an income class within a community receive equal shares, though rich and poor individuals may receive different shares. Each member of either community has some given uncontested income and first decides how much resource ('income') to allocate to the rent-seeking contest with the other community. They subsequently decide how to allocate their total income (their uncontested income net of their conflict expenditure, plus their individual share of the rent) between private consumption and contribution to a community-specific pure public good (whose consumption is non-rival and non-excludable within the community). We find that, when communities are sufficiently large (in that they have sufficiently many members of both classes), only rich individuals participate in (i.e., contribute to) conflict with the other community, regardless of share of the rent accruing to the rich as a class. Indeed, the rich may be the only participants in the inter-group contest even if each poor individual is assured a higher share of any rent accruing thereby to her community, than any rich individual. The community which offers a larger rental share to its rich as a class is more successful in the rent-seeking contest, even if each rich individual receives a higher share in the other community. Thus, what determines rent-seeking success is the relative class share of the rich, not the relative individual shares. Given its dispersion, any increase in the geometric mean of the class share of the rich across the two communities increases conflict (measured as the total resource wasted in rent-seeking). Given the geometric mean, any increase in its dispersion reduces conflict. Marginal exogenous redistributions of non-contestable income, within or across communities, are however conflict-neutral. When only the poor of one community (say *M*) contest the rich of another community (say *H*), a marginal exogenous redistribution of non-contestable income to the poor within *M* (whether from *H* or from the rich in *M*), increases both *M*'s conflict success and overall conflict. Such redistributions from the rich in *M* to either class in *H* have the same effect. Marginal intra-*H* redistributions of non-contestable income are however conflict-neutral. These results stand in sharp contrast to the case typically analyzed in the literature on inter-group conflict with exogenous sharing rules, where group members are not connected to one another through common consumption of a group-specific public good.

In modeling communities, interpreted broadly as ethno-linguistic or religious non-class identity groups, as collections of individuals held together by voluntary contributions to a group-specific public good, we follow the lead of Dasgupta and Kanbur (2011, 2007, 2005a, 2005b).[17] We extend this literature by explicitly modeling a process of inter-community resource conflict and highlight how voluntary public good provision conditions the relationship between class-specific sharing rules and individual incentives to participate in such conflict. A second related strand of the recent literature explicitly models conflict between identity communities as occurring over the division of group-specific public goods (e.g., Bakshi and Dasgupta (2020, 2018); Dasgupta and Guha Neogi (2018); Dasgupta (2017); Esteban and Ray (2011)). Our analysis, in contrast, belongs firmly within a different strand, which focuses on inter-community conflict over the division of a private divisible resource ('income'), which is distributed within a community according to some exogenous

17 Standard examples of such group-specific public goods include ethno-linguistic or religious institutions, practices and festivals. When communities are residentially segregated, local public goods such as law enforcement, roads, parks, museums, public libraries, art galleries, village wells, sports clubs etc., all constitute examples as well. See in particular Dasgupta and Kanbur (2007) for an extended discussion.

sharing rule.[18] In doing so, we extend this third strand of the literature (e.g., Nitzan (1991); Katz and Tokatlidu (1996); Warneryd (1998); Davis and Reilly (1999); Balart et al (2018))[19] by analyzing how:(a) the decentralized voluntary provision of community-specific public goods, and (b) prior income inequality within a community, affect individual incentives to expropriate another community in such a setting. We thus integrate the literature on resource allocation within an unequal community characterized by decentralized voluntary provision of public goods with that on inter-group conflict over private resources with exogenous within-group sharing rules.

In its express focus on the impact of intra-community income inequality on inter-community conflict, our investigation is close in spirit to that of Esteban and Ray (2011). However, as already noted, while the object of inter-community conflict is a public good in their model, it is a private good in our model. Second, rich members of a community purchase 'activist labor' from poor members in their model, which is then deployed against the other community. There is no intra-community market for activist labor in our model. Instead, it is the provision of a community-specific public good by the rich which connects them, indirectly, to the poor of their community, in our model. These fundamental differences in the model structure between their contributions and ours lead to major differences in the conflict consequences of intra-community inequality. In real-life conflict contexts, the aspects highlighted in their contribution are likely to often co-exist with those we focus on. We therefore view our analysis as complementary in spirit to that of Esteban and Ray (2011). Huber and Mayoral (2019) use data from 89 countries to empirically prove that high inequality within an ethnic group can make inter-ethnic conflict *more violent*. The reason cited is that inequality decreases the opportunity cost to poor group members of fighting, and also decreases the opportunity

18 This also demarcates the present contribution from the general theoretical literature on rent-seeking over public-goods (e.g., Kolmar and Rommeswinkel 2013; Chowdhury et al. 2013; Lee 2012; Epstein and Mealem 2009; Baik 2008; Riaz et al. 1995; 1993; Katz et al. 1990 and Ursprung 1990).

19 Nitzan (1991) analyses the equilibrium in a rent seeking contest where the sharing rules of the groups are different linear combinations of two sharing rules, one based on an equal-division of the prize, and the other on each member's relative effort. Davis and Reilly (1999) extend the analysis of Nitzan (1991). Katz and Tokatlidu (1996), and Warneryd (1998) deal with equilibrium outcomes for different sharing rules in nested contests. Balart et al (2018) endogenizes the choice of sharing rules in a rent seeking contest between groups of different sizes.

cost to rich group members of funding the conflict. This chapter argues in terms of the public good contribution channel. Since the rich are the only contributory members of the group-specific public good, when the poor population is large enough, only the poor engage in the inter-community rent seeking contest (though the rich get a share of it), because a part of the share that goes to the rich translates into the community specific public good which provides utility to the poor.

The intuitive structure of the problem we pose for ourselves is the following. In a single-good world, when all consumption within a group is private, each member's benefit from contributing resources ('money') to the expropriation of another group's wealth depends uniquely on her own personal share of the spoils. Then, given identical opportunity cost of contributing, those members who are assured a higher share should contribute more. Since money is invested to acquire more money in this formulation, the extent of prior inequality in the distribution of non-alienable income/wealth within the group should have no bearing on how much any member is willing to invest in the expropriation of another group. Nor should the magnitude of income inequality across groups make any difference.

This simple individual cost-benefit calculus however becomes considerably more complicated when a group also happens to be a *community*, i.e., when members' welfare levels are inter-connected through common consumption of a group-specific public good, produced by voluntary contributions of income.[20]A large prior income gap between rich and poor members of a group may lead to the poor all free-riding on the rich for public good provision. A higher personal share of any income gain from expropriating another group then has the direct consequence of increasing a poor member's marginal gain from contributing to such expropriation, as earlier. However, a higher share for all poor members, by correspondingly reducing the shares of rich individuals, reduces the amount of the public good produced within the group in consequence of such expropriation. This reduces the welfare of each poor person. A priori, it is not clear which effect should dominate. Thus, it seems

20 Dasgupta and Kanbur (2007) argue that, intuitively, it is precisely the presence of some group-specific public good(s) that makes an identity community out of a mere group (i.e., an arbitrary collection) of individuals.

quite possible that an increase in the class share of the poor in the spoils of conflict may actually reduce the incentive of every poor individual to participate in (i.e., contribute to) conflict. Suppose now that the rich have a higher marginal valuation of the public good than the poor. The marginal overall gain to each rich individual from contributing to expropriation may then be higher than that for any poor individual, even if the latter personally receives a higher share of any additional income accruing to her community through expropriation than the former. Thus, it is no longer evident that the agents who receive higher shares should contribute more to expropriation, nor can one a priori rule out the possibility that higher shares for the poor would reduce their contribution. Lastly, the prior distribution of uncontested income, both within and across groups, by affecting both the aggregate level and individual valuations of the public good within each community, may now come to affect individual incentives to contribute to expropriating the other community. The outcome of the inter-group resource conflict may thereby get influenced, along with its aggregate intensity.

Remark 1: Given that what is contested over is an economic resource, why isn't the conflict happening along the class lines instead? The answer lies in the paper by Esteban and Ray (2008). They suggest that in the presence of economic inequality, there is a systemic bias toward ethnic conflict. And this happens due to two forces acting simultaneously. First, unlike class alliances, ethnic groups possess within-group income heterogeneity. It is true that such heterogeneity may weaken within-group coordination and hence reduce the level of collective action. But the paper argues that this effect is eclipsed by the within-group specialization that such heterogeneity provides. The elite contribute financial resources, while the masses contribute conflict labour. This is the synergy that drives ethnic alliances. Thus our assumption of ethnic conflicts taking place in presence of sufficient intra-group income inequality, which pre-supposes the existence of ethnic alliance as opposed to class alliance is substantiated by this result. Our unique proposition is the public good contribution of the rich, being the channel to incentivize the poor to fight. In a community with a large population of poor, only the poor contribute to the inter- community conflict and the rich free ride, whereas when it comes to public good contribution, the poor free-ride on the rich.

Remark 2: Another question which might be asked is that why isn't the rent necessarily channeled as the public good solely. That is tantamount to two communities fighting over a public good. Well in that case it would contradict the findings of Collier and Hoeffler (2004) who claim that greed or economic viability or opportunity is the dominant motivation for conflict. In this chapter the rent sought after, is allocated for private consumption and public good provision endogenously. Mayoral and Ray (2019) on the other hand studied a model of social conflict, in which the conflict may be over a public or a private good and empirically concluded that conflict is more likely in the presence of a private prize when the group is small, and it is more likely in the presence of a public prize when the group is large. Chapter 4 assumes large enough groups though what is being contested over (S) is not 'public' per se. Does it contradict the findings of Mayoral and Ray (2019)? No, since the group specific public good contributions in the second stage implicitly weaves in the notion of a 'public prize' in the model. Thus the endogenization of allocation to private consumption and public good makes the model conform with the fundamental ideas of both the papers.

Remark 3: Using Expected payoff and share, interchangeably is a common practice in the contest theory literature. Nitzan (1991), Katz and Tokatlidu (1996), Warneryd (1998) substantiate the remark.

Our analysis is informed and motivated broadly by the lively debate among historians and political scientists as to which classes constitute the core support base for aggressive identity politics, including fascism. For example, Hobsbawm (1987, chapter 6) has highlighted the essentially middle and lower-middle class basis of ethno-linguistic nationalism in Europe in the 1875–1914 period. Hobsbawm (1987, p. 160) also provides an interesting illustration of class-specific differences in military participation from Britain. Volunteer enlistment of working-class soldiers during the South African War (1899–1902) rose and fell with unemployment. This was however not the case for volunteer recruitment from lower-middle and white-collar classes. This phenomenon may be interpreted as signaling greater susceptibility of these classes to the ideology of aggressive nationalism. The extent and sectoral composition of working class support for the Nazi party in Germany has been debated extensively by historians (see, for example, Brustein (1998) for an overview).The traditional view of Hindu-Muslim violence in post-Independence India is that it is "an urban phenomenon rooted among the petty bourgeoisie" (Engineer 1995, p. 106). This perspective has however been challenged recently by scholarly analysis of the 2002 riots in the Indian state of Gujarat. In these riots, substantial numbers of individuals belonging to poor and socially marginalized sections of the Hindu community were observed to have participated in violence against Muslims, both in urban and rural areas (Dhattiwala and Biggs (2012)). Our theoretical analysis offers a prism through which these debates can be schematically organized and interpreted in part.

Section 4.2 sets up the model. Our comparative static results are presented in sections 4.3 and 4.4. Section 4.5 concludes.

4.2 The model

Consider a society consisting of two communities, H and M. Each community $c \in \{H, M\}$is internally divided into income classes of rich (R) and poor (P) members. An income class $g \in \{R, P\}$ in community c contains n_c^g individuals; $n_c^g \geq 1$.Thus, total population of a community is $n_c \equiv n_c^R + n_c^P$. Notice that the two communities H and M need not be of equal population size, nor do they need have identical population shares for the two income classes. All members of a class $g \in \{R, P\}$ belonging to community $c \in \{H, M\}$ have identical exogenous (non-contestable) incomeI_c^g; $I_c^R > I_c^P > 0$. Note thatI_H^gneed not be the same as I_M^g– the non-contestable income of an individual member of either class may vary across communities, though all members of an income class within a community must be equally wealthy.

All individuals live for two periods. In period 1, the two communities engage in a Tullock (1980) contest over the division of a given amount of resource (rent), S. All individuals simultaneously decide how much resource to invest in the rent-seeking contest. Let x_{kc}^g denote the rent-seeking expenditure by individual k belonging to class g in community c; $k \in \{1, 2, \ldots, n_c^g\}$. The amount of rent going to community c is given as:

$$S_c = \left(\frac{X_c}{X}\right) S \text{ if } X > 0,$$

$= \frac{S}{2}$ otherwise; (4.1)

where X_c is the total expenditure on rent-seeking by community c, $X_c = \sum_{k=1}^{n_c^P} x_{kc}^P + \sum_{k=1}^{n_c^R} x_{kc}^R$; so that $X \equiv X_H + X_M$. In line with standard practice, we shall use the total expenditure on rent-seeking, X, as the measure of aggregate conflict in society.

The rent received by community c, S_c, is divided between its constituent classes according to an exogenous sharing rule, so that the class g in$c$ receives the share γ_c^g; $\gamma_c^g \in [0,1]$ and $[\gamma_c^R + \gamma_c^P = 1]$.Note that the classes*R* and *P* within any given community need not get equal shares, and that the *R* class in community *H* may receive a different share of any rent accruing to that community than the *R* class in *M*. Within each class $g$in community c, the rent is equally divided, so that each member of that class receives $\frac{S_c\gamma_c^g}{n_c^g}$. The net income accruing to any member, *k*, of class *g* in community *c* at the end of period 1 is thus:

$$i_{kc}^g \equiv I_c^g + \frac{S_c\gamma_c^g}{n_c^g} - x_{kc}^g. \tag{4.2}$$

All consumption occurs in period 2. In period 2, all individuals simultaneously allocate their respective period 2 net incomes, i_{kc}^g, between private consumption (V_{kc}^g) and a community-specific pure public good. All prices are set at unity for notational simplicity. Let B_{kc}^g denote the spending by member *k* of class *g* in community *c* on that community's public good. The total amount of the public good generated within a community, B_c, is given simply by the total spending on that good by all members of the community: $B_c = \sum_{k=1}^{n_c^P} B_{kc}^P + \sum_{k=1}^{n_c^R} B_{kc}^R$. We shall denote by $B_c^g$the total spending on the community's public good by its class *g*, byB_c^{-g} that by the class other than *g*, and by $B_{-k,c}^g$the total public good spending by all members of class *g* in community *c* except *k*. The representative individual's problem is:

$$\underset{V_{kc}^g,\, B_c}{Max} \left[a_c(V_{kc}^g)^{\rho_c} + (1-a_c)(B_c)^{\rho_c}\right],$$

where $a_c, \rho_c \in (0,1)$subject to the budget constraint:

$$V_{kc}^{g} + B_c = i_{kc}^{g} + B_c^{-g} + B_{-k,c}^{g}; \tag{4.3}$$

and the additional constraint:

$$B_c \geq B_c^{-g} + B_{-k,c}^{g}. \tag{4.4}$$

Thus, preferences exhibit the CES form within both communities and are identical within a community, but may vary across communities. The second constraint simply incorporates the assumption that individuals cannot divert other community members' public good contributions to their own private consumption. Then the demand functions for private consumption, derived as the solution to the period 2 individual optimization problem above, subject to the budget constraint (4.3) alone, must satisfy:

$$V_{kc}^{g} = \beta_c B_c, \tag{4.5}$$

where$\beta_c \equiv \left(\frac{a_c}{1-a_c}\right)^{\frac{1}{1-\rho_c}}$. These unrestricted demand functions are given as the solution to (4.3) and (4.5).

Condition C1.

(i) For every $k \in \{1,2,\dots,n_c^P\}$, $[i_{kc}^{P} < \beta_c \left(\frac{n_c^R I_c^R + S_c \gamma_c^R - X_c^R}{n_c^R \beta_c + 1}\right)]$.

(ii) For every $k \in \{1,2,\dots,n_c^R\}$, $[i_{kc}^{R} = I_c^R + \left(\frac{S_c \gamma_c^R - X_c^R}{n_c^R}\right)]$.

Lemma 4.1: Suppose Condition C1 holds in period 2 for some $c \in \{H, M\}$.Then there exists a unique Nash equilibrium in the period 2 sub-game of voluntary contributions to the community-specific public good within that community c. This Nash equilibrium is characterized as follows:

$$\text{for every } k \in \{1,2,\dots,n_c^P\}, \left[V_{kc}^{P} = i_{kc}^{P}\right], \tag{4.6}$$

for every $k \in \{1,2,\dots,n_c^R\}$, $\left[V_{kc}^R = \beta_c\left(\frac{n_c^R I_c^R + S_c\gamma_c^R - X_c^R}{n_c^R\beta_c + 1}\right)\right]$, (4.7)

$$B_c = \left(\frac{n_c^R I_c^R + S_c\gamma_c^R - X_c^R}{n_c^R\beta_c + 1}\right) > \frac{i_{kc}^P}{\beta_c}. \quad (4.8)$$

Condition C1 refers to a situation where (a) all rich individuals within a community have identical period 2 incomes (because they have made identical rent-seeking investments in period 1), and (b) the period 2 income gap between the rich and the poor is large enough to make the poor free-ride on the rich of their community for public good provision. Lemma 4.1, whose proof is obvious and therefore omitted, characterizes the individual consumption bundles within any community which obtain in the Nash equilibrium when Condition C1 holds for that community.

Suppose now that Condition C1 holds, so that only rich individuals contribute to the public good within a community. Then the utility of each member of that community comes to depend on the period 2 money income of other community members, as well as that of herself, in the period 2 Nash equilibrium. The period 2 money incomes of other rich individuals, by affecting their public good contributions, affect the equilibrium consumption bundle of any rich individual. The total class income of the rich affects the consumption bundle of every rich individual for the same reason. Thus, period 2 personal incomes (i_{kc}^g) become inadequate as money-metric measures of individual welfare, and have to be augmented by incorporating the monetary equivalent of the benefit from accessing others' public good contributions.

Dasgupta and Kanbur (2007) advanced 'real', or 'equivalent', income as the money-metric measure of an individual's benefit from having access to the public good contributions of all other members of her community. It is simply that level of income which would give her the same utility as what she achieves in the Nash equilibrium characterized by Lemma 4.1, if, somehow, she was to lose access to the combined public good contribution of all other members of her community. More formally, the real income of an individual in the Nash equilibrium of the public goods game, M_{kc}^g, is given as the solution to:

$$D\left(M_{kc}^{g}\right) = U(V_{kc}^{g}, B_c),$$

where $D(.)$ is her indirect utility function, $U(.)$ is her direct utility function, and (V_{kc}^{g}, B_c) is her equilibrium consumption bundle. Evidently, the real income is invariant with respect to any positive monotone transformation of the direct utility function. Furthermore, an individual is better off in one Nash equilibrium, as compared to another, if and only if her real income is higher in the former. Thus, the real income provides a consistent money-metric measure of an individual's welfare in any equilibrium.

Suppose Condition C1 holds for some community c. Then, by Lemma 4.1 (equations (4.7) and (4.8)), the real income of an R individual in that community in period 2 is given by:

$$M_{kc}^{R} = V_{kc}^{R} + B_c = \left(\frac{\beta_c+1}{n_c^R\beta_c+1}\right)(n_c^R I_c^R + S_c\gamma_c^R - X_c^R). \tag{4.9}$$

Furthermore, by Lemma 4.1 (equations (4.6) and (4.8)), the real income of a P individual in community c in period 2 can be written as:

$$M_{kc}^{P} = i_{kc}^{P} + B_c - L(i_{kc}^{P}, B_c); \tag{4.10}$$

where i_{kc}^{P} is defined by (4.2), B_c is defined by (4.8) and $L(i_{kc}^{P}, B_c)$ is a *loss* function.

Condition C1 ensures that all rich individuals within the community contribute positive amounts for public good provision. Thus, they are all in interior equilibria. Hence, the aggregate public good contribution by other members of their community has the same effect on their welfare as an equivalent cash transfer. The real income of any rich individual is therefore the sum of her period 2 net income i_{kc}^{P} and the total public good spending by all other community members. Clearly, this is nothing but the total monetary cost of her equilibrium consumption bundle. This however does not hold for a poor individual, who, by Condition C1, must be at a corner equilibrium. Such a person would be better off if she could convert some of the public good spending by others to her own private consumption. For such a person, the in-kind, rather than cash, nature of public good transfers imposes a welfare loss, so that her real income is less than the value of her consumption bundle. The size of this loss is captured by the loss function L.

What are the properties of the loss function, L? Recall that the utility function has the form $\left[a_c(V_{kc}^g)^{\rho_c} + (1-a_c)(B_c)^{\rho_c}\right]$, and that, by (4.5), the unrestricted demand functions must satisfy: $V_{kc}^g = \beta_c B_c$, where$\beta_c \equiv \left(\frac{a_c}{1-a_c}\right)^{\frac{1}{1-\rho_c}}$(so that $a_c = \frac{\beta_c{}^{1-\rho_c}}{(1+\beta_c{}^{1-\rho_c})}$ and $1-a_c = \left(\frac{1}{1+\beta_c{}^{1-\rho_c}}\right)$). Hence:

$$M_{kc}^P = \left(\frac{1}{1+\beta_c}\right)^{\frac{1-\rho_c}{\rho_c}} \left[\beta_c\left(\frac{i_{kc}^P}{\beta_c}\right)^{\rho_c} + (B_c)^{\rho_c}\right]^{\frac{1}{\rho_c}}. \tag{4.11}$$

From (4.11), we get:

$$\frac{\partial M_{kc}^P}{\partial i_{kc}^P} = \left(\frac{\beta_c{}^{\rho_c}}{1+\beta_c}\right)^{\frac{1-\rho_c}{\rho_c}} \left[\beta_c\left(\frac{i_{kc}^P}{\beta_c}\right)^{\rho_c} + (B_c)^{\rho_c}\right]^{\frac{1-\rho_c}{\rho_c}} (i_{kc}^P)^{\rho_c-1} = \left[\left(\frac{1}{1+\beta_c}\right)\left(\beta_c + \left(\frac{\beta_c B_c}{i_{kc}^P}\right)^{\rho_c}\right)\right]^{\frac{1-\rho_c}{\rho_c}}. \tag{4.12}$$

$$\frac{\partial M_{kc}^P}{\partial B_c} = \left(\frac{1}{1+\beta_c}\right)^{\frac{1-\rho_c}{\rho_c}} \left[\beta_c\left(\frac{i_{kc}^P}{\beta_c}\right)^{\rho_c} + (B_c)^{\rho_c}\right]^{\frac{1-\rho_c}{\rho_c}} (B_c)^{\rho_c-1} = \left[\left(\frac{1}{1+\beta_c}\right)\left(1 + \beta_c\left(\frac{i_{kc}^P}{\beta_c B_c}\right)^{\rho_c}\right)\right]^{\frac{1-\rho_c}{\rho_c}}. \tag{4.13}$$

Recall that, by Lemma 4.1 (equation (4.8)), $\frac{\beta_c B_c}{i_{kc}^P} > 1$. Together, equations (4.2), (4.8), (4.10), (4.11), (4.12) and (4.13) immediately yield the following characterization of the real income function for a poor individual.

Lemma 4.2: Suppose Condition C1 holds for some $c \in \{H, M\}$. Then, given anyi_{kc}^P, B_c>0, (i)$L(i_{kc}^P, B_c) \in (0, B_c)$, (ii) $\frac{\partial L(i_{kc}^P,B_c)}{\partial B_c} \in (0,1)$, (iii)$\frac{\partial^2 L(i_{kc}^P,B_c)}{\partial B_c{}^2} > 0$, (iv) $\frac{\partial^2 L(i_{kc}^P,B_c)}{\partial B_c \partial i_{kc}^P} < 0$, (v) $\frac{\partial L(i_{kc}^P,B_c)}{\partial i_{kc}^P} < 0$, (vi) $\lim_{i_{kc}^P \to 0} \frac{\partial L(i_{kc}^P,B_c)}{\partial i_{kc}^P} = -\infty$, (vii) $\frac{\partial^2 L(i_{kc}^P,B_c)}{\partial i_{kc}^P{}^2} > 0$, (viii) $\lim_{i_{kc}^P \to \beta_c B_c} \frac{\partial L(i_{kc}^P,B_c)}{\partial i_{kc}^P} = \frac{\partial L(i_{kc}^P,B_c)}{\partial B_c} = 0$, (ix) $\lim_{B_c \to \infty} \frac{\partial L(i_{kc}^P,B_c)}{\partial i_{kc}^P} = -\infty$.

By Lemma 4.2, the equilibrium real income of a non-contributing poor person is less than the money value of her consumption bundle. A marginal increase in public good provision increases her real income, but by less than that increase. The real income benefit of a marginal increase in public good provision is higher, the higher the poor person's period 2 nominal income. The real income benefit of a marginal increase in public good provision falls as the amount of the public good increases. The monetary value of a given amount of the public good rises for a poor person as her nominal income rises, but at a decreasing rate. Her real income approaches the total money value (or cost) of a poor person's consumption bundle as her nominal income reaches the threshold level, beyond which she would start contributing to the public good.

In period 1, all individuals simultaneously choose their conflict (or rent-seeking) allocations x_{kc}^{g} so as to maximize their respective period 2 real incomes, subject to the contest success function specified by (4.1). Suppose now that the following parametric restriction holds.

A1. For every $c \in \{H, M\}$, $\left[\frac{(n_c^R I_c^R - S)\beta_c}{n_c^R \beta_c + 1} \geq (I_c^P + S)\right]$.

Notice that, provided $(I_c^R - I_c^P > S)$, A1 must be satisfied if n_c^R is sufficiently large, i.e., if there are sufficiently many rich individuals in the community. Thus, intuitively, A1 is the combination of two ideas – (a) the contested rent is larger than the rich-poor income gap, and (b) the rich population is large, so that each rich person spends a high proportion of her period 2 nominal income on her private consumption.

It is easy to see that, given any $X_{-c} > 0$, the solution to the period 1 optimization problem within community *c* must yield $X_c < S$. Furthermore, any such X_c can be sustained as a symmetric equilibrium within the community, in that all members of any class g in *c* make identical conflict contributions $\frac{X_c^g}{n_c^g}$. Thus, A1 implies that, given any $c \in \{H, M\}$ and any $X_{-c} > 0$, every symmetric solution to community *c*'s conflict allocation problem in period 1 must generate a period 2 nominal income distribution within that community which satisfies Condition C1.Given A1, in light of Lemma 4.1, the real incomes in the intra-community equilibrium must then be given by (4.9) - (4.11). The marginal net real income gains are accordingly given by:

$$\frac{dM_{kc}^R}{dx_{kc}^R} = \left(\frac{\beta_c + 1}{n_c^R \beta_c + 1}\right)\left[S\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R - 1\right]; \tag{4.14}$$

$$\frac{dM_{kc}^P}{dx_{kc}^P} = \frac{\partial M_{kc}^P(i_{kc}^P, B_c)}{\partial i_{kc}^P}\left[S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) - 1\right] + \frac{\partial M_{kc}^P(i_{kc}^P, B_c)}{\partial B_c}\left(\frac{1}{n_c^R \beta_c + 1}\right) S\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R. \tag{4.15}$$

In any intra-community equilibrium, we must have $\frac{dM_{kc}^R}{dx_{kc}^R}, \frac{dM_{kc}^P}{dx_{kc}^P} \leq 0$, with the inequality holding strictly for at most one income class if $X_c > 0$ in that equilibrium. It can be seen from (4.14)-(4.15) that, given the satisfaction of Condition C1 in period 2, the real income optimization problem in period 1 must, in turn, yield a unique X_c as the equilibrium response by community c to any given $X_{-c} > 0$.

Thus, the symmetric solution to the decentralized optimization problem within a community yields that community's reaction function, which specifies its conflict allocation, given the conflict allocation by the other community. We shall denote by R^c the equilibrium conflict response of community *c*; $R^c = R^c(X_{-c}, \gamma_c^P, n_c^P, n_c^R, I_c^P, I_c^R, \beta_c)$.

Notice that Lemma 4.1, Lemma 4.2, (4.7), (4.8) and (4.14) imply that the intra-community equilibrium outcome would remain invariant if, in each community, the entire *R* class was replaced by a single individual with wealth $\widehat{I_c^R} \equiv n_c^R I_c^R$ and preferences such that the unrestricted demand functions were given by $V_c^R = n_c^R \beta_c B_c$, instead of (4.5). Thus, given any size of the *R* population n_c^R, that class can be aggregated as a single individual with uncontested income $n_c^R I_c^R$ and a private to public consumption ratio of $n_c^R \beta_c$.[21] It follows that the efficient level of conflict investment from the community's perspective, i.e., the level which maximizes the community's share of the rent net of conflict expenditure, given the aggregate conflict allocation by its opponent, is generated by putting $\gamma_c^R = 1$, i.e., by allocating all income from rent-seeking to the *R* class alone. Furthermore, $\gamma_c^R = 1$ constitutes the unique conflict efficient class share for a community whenever that community has more than one poor member. Evidently, $\gamma_c^R = 1$would also maximize the (symmetric) equilibrium real income, and thus the welfare, of every rich member of *c*. However, putting $\gamma_c^R = 1$ would not, in general, maximize the welfare of poor members of the community. For example, when $n_c^R = n_c^P = 1$, recalling Lemma 4.1 and Lemma 4.2, it is evident that, given any $X_{-c} > 0$, the

21 This is a straightforward application of the well-known neutrality property of games involving voluntary contributions to a pure public good (Bergstrom *et al.* 1986).

equilibrium real income of the poor member of community c is higher under $\gamma_c^R = 0$ than under $\gamma_c^R = 1$. Hence, a trade-off exists, in general, between the objectives of conflict efficiency and intra-community equity, making the choice of class shares a non-trivial problem for any community leader.

4.3 Community reaction functions

How does the conflict allocation of a community change in response to changes in its income distribution, population structure, or inter-class sharing rule? We now address these questions. First notice that $\gamma_c^R \leq \left(\frac{\gamma_c^P}{n_c^P}\right)$ (i.e., the per capita share of the poor is at least as high as the class share of the rich) iff $\gamma_c^P \geq \frac{n_c^P}{(1+n_c^P)}$. We then have the following conclusion.

Proposition 4.1: Let A1 hold, let $R^c(.)$ denote the reaction function of community $c \in \{H.M\}$, let $X_{-c} = a > 0$, and let $\ddot{\gamma}_c^P \in [\frac{n_c^P}{(1+n_c^P)}, 1)$. Suppose that $R^c(a, \ddot{\gamma}_c^P, \dots) > 0$. Then, for every $\gamma_c^P \in [\ddot{\gamma}_c^P, 1]$, $R^c(a, \gamma_c^P, \dots) = X_c^P > 0$, with: (i) $\frac{\partial R^c}{\partial \gamma_c^P} > 0$ when $\left[\beta_c \geq \frac{n_c^P - 1}{n_c^R}\right]$, (ii) $\frac{\partial R^c}{\partial I_c^P} > 0$, and (iii) $\frac{\partial R^c}{\partial I_c^R} < 0$.

Proof of Proposition 4.1: Let A1 hold. Given some $X_{-c} > 0$, suppose that the best response value of X_c is positive at some $\ddot{\gamma}_c^P \in [\frac{n_c^P}{(1+n_c^P)}, 1)$. Suppose further that R individuals in community c make positive contributions to the inter-group contest in equilibrium at $\gamma_c^P \geq \ddot{\gamma}_c^P$. Then, by (14), we must have:

$$\left[S\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R - 1\right] = 0,$$

which, since $\left[\ddot{\gamma}_c^P \geq \frac{n_c^P}{(1+n_c^P)}\right]$, which is equivalent to the condition $\left[\gamma_c^R \leq \left(\frac{\gamma_c^P}{n_c^P}\right)\right]$, yields:

$$\left[S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) - 1\right] \geq 0.$$

Then, since, by Lemma 4.2, $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}, \frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c} > 0$, (4.15) implies: $\frac{dM_{kc}^P}{dx_{kc}^P} > 0$. Now recall that, in any equilibrium, we must have $\frac{dM_{kc}^R}{dx_{kc}^R}, \frac{dM_{kc}^P}{dx_{kc}^P} \leq 0$. Hence, $X_c^R > 0$ cannot constitute an equilibrium if $\gamma_c^P \in [\ddot{\gamma}_c^P,1]$, implying $X_c^P = X_c$. Since $X_c^P = X_c > 0$ in equilibrium at $\gamma_c^P = \ddot{\gamma}_c^P$, by (4.15), we must therefore have, at that value of γ_c^P,

$$\frac{dM_{kc}^P}{dx_{kc}^P} = \frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}\left[S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) - 1\right] + \frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}\left(\frac{1}{n_c^R\beta_c+1}\right)S\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R = 0. \quad (4.16)$$

Now, from any initial equilibrium situation satisfying (4.16), consider the impact, on $\frac{dM_{kc}^P}{dx_{kc}^P}$, of a marginal increase in γ_c^P, holding X_c and X_{-c} constant. This affects $\frac{dM_{kc}^P}{dx_{kc}^P}$through three channels. First, given $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}$ and $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}$, it directly increases $\frac{dM_{kc}^P}{dx_{kc}^P}$ when $[\left(\frac{1}{n_c^P}\right) \geq \left(\frac{1}{n_c^R\beta_c+1}\right)]$, since $\gamma_c^R = 1 - \gamma_c^P$, and (by Lemma 4.2) $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P} > 1$, $0 < \frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c} < 1$. Second, it affects $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}$ and $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}$by reducing B_c. By Lemma 4.2, this effect increases $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}$ and reduces $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}$. Since, in the initial equilibrium, $\frac{dM_{kc}^P}{dx_{kc}^P} = 0$, implying $\left[S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) - 1\right] < 0$, both marginal effects must increase $\frac{dM_{kc}^P}{dx_{kc}^P}$. Third, a marginal increase in γ_c^P, holding X_c and X_{-c} constant, must increase i_{kc}^P and therefore (by Lemma 4.2) reduce $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}$. By the same argument as before, this effect must increase $\frac{dM_{kc}^P}{dx_{kc}^P}$ as well. Since all three effects are positive, we must therefore have: $\frac{d^2M_{kc}^P}{dx_{kc}^P d\gamma_c^P} > 0$ at the equilibrium when $\gamma_c^P \in \left(\frac{n_c^P}{(1+n_c^P)}, 1\right)$. Hence, by (4.16), X_c must rise in response a marginal rise in γ_c^P to restore the equilibrium requirement $\frac{dM_{kc}^P}{dx_{kc}^P} = 0$. Therefore, the equilibrium (or best response) value of X_c, if positive at $\ddot{\gamma}_c^P$, must be monotonically increasing inγ_c^P over $[\ddot{\gamma}_c^P, 1]$.

Suppose that $R^c(a, \ddot{\gamma}_c^P, \dots) > 0$. Then, as established in the preceding paragraph, we must have $R^c(a, \gamma_c^P, \dots) > 0$ for every $\gamma_c^P \geq \ddot{\gamma}_c^P$. Hence, for every $\gamma_c^P \geq \ddot{\gamma}_c^P$, we must have$\frac{dM_{kc}^P}{dx_{kc}^P} = 0$, implying $\left[S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right)-1\right]<0$.By Lemma 4.2, $\frac{\partial^2 M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c \partial i_{kc}^P}>0$ and $\frac{\partial^2 M_{kc}^P(i_{kc}^P,B_c)}{\partial {i_{kc}^P}^2}<0$. It follows that $\frac{d^2 M_{kc}^P}{dx_{kc}^P di_{kc}^P}>0$ at the equilibrium. Hence, by (4.16),$\frac{\partial R^c}{\partial I_c^P}>0$ at [any $\gamma_c^P \geq \ddot{\gamma}_c^P$ and any I_c^P satisfying A1], given the other parameters of the model.

From an initial parametric configuration satisfying A1, any increase in I_c^R increases B_c, which, by Lemma 4.2, decreases $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}$ and increases $\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}$. By an argument exactly analogous to that deployed in the preceding paragraph, the equilibrium value of X_c must fall in consequence. ■

Suppose the per capita share of the poor is at least as high as the class share of the rich. Then only the poor in a community will participate in, i.e., contribute to, the rent-seeking conflict with the other community, even as they free-ride on the rich for public good provision. Conversely, the rich will provide the public good of the community, but free-ride on the poor in the inter-community conflict. Thus, a clear class-based division of labor emerges within the community in this case. Provided the private to public consumption ratio of rich individuals is sufficiently high, a larger class share for the poor, by increasing the conflict gains of poor individuals, incentivizes them to contribute more against the other community. Conversely, a rise in the per capita uncontested income of the rich, by increasing the amount of the public good, reduces the marginal real income gain to a poor person of allocating money to conflict. The poor therefore contribute less to inter-community conflict. A fall in the per capita uncontested income of the poor also reduces the marginal real income gain to a poor person of allocating money to conflict, thereby reducing aggregate conflict allocation. Thus, given a community's population distribution and its per capita income, any increase in the community's (uncontested) income gap $(I_c^R - I_c^P)$ must reduce its conflict allocation. Put another way, if only the poor of a community participate in inter-community conflict, greater internal class differentiation in uncontested incomes would make the community externally *less aggressive*.

*Remark 1***.**The egalitarian *individual* sharing rule, whereby all members of a community receive equal shares ($\frac{1}{n_c}$) regardless of their class position, satisfies the condition $\left[\gamma_c^P \geq \frac{n_c^P}{(1+n_c^P)}\right]$ if, and only if, $n_c^R = 1$. The equal *class* shares rule ($\gamma_c^P = \frac{1}{2}$) satisfies $\left[\gamma_c^P \geq \frac{n_c^P}{(1+n_c^P)}\right]$ if and only if $n_c^P = 1$. Thus, egalitarian sharing, whether across individuals or across classes, does not ensure the participation of the poor in communal conflict in general. In the special case $n_c^R = n_c^P = 1$, where the two forms of equal sharing are equivalent, Proposition 1 implies that only the poor member of the community will contribute to conflict. By a continuity argument, it follows that, in this case, the *R* individual in *c* may free-ride on the *P* individual of her community in the rent-seeking contest even if she is offered a larger share of the rent than the latter. In marked contrast, if all consumption within a community, say *H*, is assumed to be private, then, with equal individual shares $\frac{1}{n_H}$ (or equal class shares and equal-sized classes) there must exist a symmetric equilibrium within *H*, where all members of *H* contribute equally to the conflict with *M* (regardless of their class location and irrespective of the population size of *H*). Furthermore, if all consumption is private, rich individuals can never free-ride on their poor community members for conflict contributions, if their individual shares are higher than those of the poor.

What happens if the per capita rental share of the poor is less than the class share of the rich within a community, i.e., if $\gamma_c^R > \left(\frac{\gamma_c^P}{n_c^P}\right)$? This will, for example, necessarily obtain under the egalitarian individual sharing rule whenever $n_c^R > 1$, and under the equal class shares rule whenever $n_c^P > 1$. Indeed, this may hold even if the class share of the poor is higher than that of the rich (i.e., even if $\gamma_c^P > \frac{1}{2}$). First notice that this must be the case if $\left[\left(\frac{\gamma_c^P}{n_c^P}\right) + \left(\frac{1}{n_c^R\beta_c+1}\right)\gamma_c^R \leq \gamma_c^R\right]$. By (4.8), the term $\left(\frac{1}{n_c^R\beta_c+1}\right)\gamma_c^R$ is simply the proportion of rental income accruing to their community that is spent on public consumption by the rich as a class. Hence, $\left(\frac{n_c^R\beta_c}{n_c^R\beta_c+1}\right)\gamma_c^R$ is the proportion of rental income accruing to their community that is spent on private consumption by the rich as a class. Noting that

$\left[\left(\frac{\gamma_c^P}{n_c^P}\right) \le \left(\frac{n_c^R\beta_c}{n_c^R\beta_c+1}\right)\gamma_c^R\right]$ is equivalent to $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$, we then have the following conclusion.

Proposition 4.2: Let A1 hold, let $R^c(.)$ denote the reaction function of community $c \in \{H.M\}$, let $X_{-c} = a > 0$, and let $\ddot{\gamma}_c^P \in (0, \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)]$. Suppose that $R^c(a, \ddot{\gamma}_c^P, \ldots) > 0$. Then, for all $\gamma_c^P \in [0, \ddot{\gamma}_c^P]$, $R^c(a, \gamma_c^P, \ldots) = X_c^R > 0$, with: (i) $\frac{\partial R^c}{\partial \gamma_c^P} < 0$, (ii) $\frac{\partial R^c}{\partial I_c^P} = 0$, and (iii) $\frac{\partial R^c}{\partial I_c^R} = 0$.

Proof of Proposition 4.2: Let A1 hold, and suppose that, given some $X_{-c} > 0$, the equilibrium aggregate response of community *c*, X_c, is positive at some $\gamma_c^P = \ddot{\gamma}_c^P \in (0, \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)]$. First notice that the condition $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$ is equivalent to: $\left[\left(\frac{1}{\gamma_c^R}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) \le \left(\frac{n_c^R\beta_c}{n_c^R\beta_c+1}\right)\right]$. If *R* individuals in *c* participate in conflict at any $\gamma_c^P \le \ddot{\gamma}_c^P$, then, from (4.14), $\left[\left[S\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R - 1\right] = 0\right]$; hence $\left[S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) = \left(\frac{1}{\gamma_c^R}\right)\left(\frac{\gamma_c^P}{n_c^P}\right)\right]$. From (4.15), then, recalling that, by Lemma 4.2, $\frac{\partial M_{kc}^P(i_{kc}^P, B_c)}{\partial i_{kc}^P} > 1, \frac{\partial M_{kc}^P(i_{kc}^P, B_c)}{\partial B_c} < 1$, we get:

if $[\left[\left(\frac{1}{\gamma_c^R}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) + \left(\frac{1}{n_c^R\beta_c+1}\right) - 1\right] \le 0]$, then $\frac{dM_{kc}^P}{dx_{kc}^P} < 0$.

Notice now that: the condition $[\left[\left(\frac{1}{\gamma_c^R}\right)\left(\frac{\gamma_c^P}{n_c^P}\right) + \left(\frac{1}{n_c^R\beta_c+1}\right) - 1\right] \le 0]$ is equivalent to $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$. Thus, we have:

if $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$, then *P* individuals in *c* cannot participate in rent-seeking when *R* individuals in *c* do. (4.17)

Now if *P* individuals in *c* do participate in conflict, then, from (4.15),

$$S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right)\left[\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}+\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}\left(\frac{1}{n_c^R\beta_c+1}\right)\left(\frac{\gamma_c^R n_c^P}{\gamma_c^P}\right)\right]=\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}.$$

Denoting $\bar{\Delta}\equiv \frac{\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial B_c}}{\frac{\partial M_{kc}^P(i_{kc}^P,B_c)}{\partial i_{kc}^P}}$, we thus have:

$$S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right)=\frac{1}{\left[1+\bar{\Delta}\left(\frac{1}{n_c^R\beta_c+1}\right)\left(\frac{\gamma_c^R n_c^P}{\gamma_c^P}\right)\right]}. \tag{4.18}$$

Since, by Lemma 2, $\bar{\Delta}< 1$, (4.18) implies:

$$S\left(\frac{X_{-c}}{X^2}\right)\left(\frac{\gamma_c^P}{n_c^P}\right)>\frac{1}{\left[1+\left(\frac{1}{n_c^R\beta_c+1}\right)\left(\frac{\gamma_c^R n_c^P}{\gamma_c^P}\right)\right]}. \tag{4.19}$$

Now recall that the condition $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$ is equivalent to $\left[\left(\frac{1}{\gamma_c^R}\right)\left(\frac{\gamma_c^P}{n_c^P}\right)\le\left(\frac{n_c^R\beta_c}{n_c^R\beta_c+1}\right)\right]$, which can be rewritten as: $\left[\left(\frac{\gamma_c^R n_c^P}{\gamma_c^P}\right)(1-\left(\frac{1}{n_c^R\beta_c+1}\right))\ge 1\right]$, or $\left[\left(\frac{\gamma_c^R n_c^P}{\gamma_c^P}\right)\ge 1+\left(\frac{1}{n_c^R\beta_c+1}\right)\left(\frac{\gamma_c^R n_c^P}{\gamma_c^P}\right)\right]$. Then, from (4.19), we get:

$$\text{if } \left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right], \text{ then } \left[S\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R > 1\right]. \tag{4.20}$$

But (4.20), together with (4.14), implies that if $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$, then $\frac{dM_{kc}^R}{dx_{kc}^R}>0$, implying that *R* individuals in *c* must participate in conflict if *P* individuals in *c* do so. In light of (4.17), therefore, we have a contradiction, which establishes the claim that only *R* individuals in *H* will participate in conflict under the parametric configuration $\left[\gamma_c^P \le \left(\frac{n_c^R\beta_c n_c^P}{n_c^R\beta_c+1+n_c^R\beta_c n_c^P}\right)\right]$, if $X_c>0$.

Since only *R* individuals in *c* participate in the rent-seeking contest at $\gamma_c^P=\ddot{\gamma}_c^P$, where $X_c>0$ by assumption, the equilibrium at that γ_c^P is defined, from (14), by the following condition:

$$\frac{dM_{kc}^R}{dx_{kc}^R}=\left(\frac{\beta_c+1}{n_c^R\beta_c+1}\right)\left[S\left(\frac{X_{-c}}{X^2}\right)(1-\gamma_c^P)-1\right]=0.$$

The claim that, given any $X_{-c} > 0$, the equilibrium level of X_c declines as γ_c^P rises within $[0, \dddot{\gamma}_c^P]$, but is invariant with respect to changes in I_c^P and I_c^R which maintain A1, then follows immediately. ■

By Proposition 4.2, if the per capita rental share of the poor is lower than the proportion of the community's rental income spent on private consumption by the rich as a class, then the poor become dependent on the rich in terms of both public good provision and conflict participation. Any increase in the rental share of the rich within a community then increases total conflict allocation of that community. However, changes in income distribution within a community has no effect on either conflict participation or conflict allocation, provided such changes do not induce the poor to start contributing to the public good. Note that, by Proposition 4.2, poor individuals may free-ride on the conflict contribution of their community's rich individuals even when each poor individual stands to receive a larger share of her community's rental income than any rich individual. The following example illustrates this point.

Example 1. Suppose $a_H = \frac{1}{2}$, so that $\beta_H = 1$. Suppose further that $n_H^R = 2, n_H^P = 1, \gamma_H^P = 0.38$. Then, each *R* individual in *H* receives a 0.31 share of any rental income accruing to *H*. Yet, by Proposition 2, the *P* individual in *H* will not participate in the rent-seeking contest with *M* in any equilibrium, even though she stands to receive a higher share of the rental income accruing to *H* than either of the *R* individuals in her community. Clearly, there exists an equilibrium where both *R* individuals participate in the rent-seeking contest and contribute identical amounts. In a private consumption economy, the participation pattern is reversed. These rental shares would then lead to the *P* individual in *H* alone participating in the conflict with *M* – no *R* member of *H* would engage in the inter-community contest in any equilibrium. Evidently, in this community, the *P* individual will not participate in rent-seeking under the equal individual shares rule ($\gamma_H^R = \frac{2}{3}$) either (recall Remark 1). ■

Propositions 4.1 and 4.2 characterize properties of the response function for a community *c*, given any positive conflict allocation by the other community, for any $\gamma_c^P \in \left[0, \left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)\right] \cup$

$\left[\left(\frac{n_c^P}{1+n_c^P}\right), 1\right]$, provided the best response of c is positive. The second limitation is not substantive, since$X_c = 0$ cannot constitute a Nash equilibrium allocation in the rent-seeking game. What about the restriction on the range of γ_c^P, necessitated jointly by Propositions 4.1 and 4.2? We now proceed to address this issue.

Remark 2 Propositions 1 and 2 imply that, when$\gamma_c^P \in \left[0, \left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)\right] \cup \left[\left(\frac{n_c^P}{1+n_c^P}\right), 1\right]$, changes in income levels I_c^R, I_c^P within a community do not change the class composition of the set of participants in the rent-seeking conflict – depending on γ_c^P, either only poor individuals participate, or only the rich do. Notice that$\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right) < \left(\frac{n_c^P}{1+n_c^P}\right)$; hencethe interval $(\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right), \left(\frac{n_c^P}{1+n_c^P}\right))$ is non-empty. When γ_c^P falls in this interval, changes in income levels I_c^R, I_c^P may change the class composition of the set of contributors to rent-seeking in c. For example, an increase in I_c^R, by increasing the amount of the public good in c, will reduce $\frac{\partial M_{kc}^P(i_{kc}^P, B_c)}{\partial B_c}$ and increase $\frac{\partial M_{kc}^P(i_{kc}^P, B_c)}{\partial i_{kc}^P}$ (recall Lemma 4.2). This effect may possibly induce P individuals in c to stop participating in the rent-seeking contest. Since $\frac{\partial^2 M_{kc}^P(i_{kc}^P, B_c)}{\partial {i_{kc}^P}^2} < 0$ and $\frac{\partial^2 M_{kc}^P(i_{kc}^P, B_c)}{\partial B_c \partial i_{kc}^P} > 0$, a decrease in I_c^P may have a similar effect. Thus, the class composition of the set of conflict participants is, in general, ambiguous in this case. Notice however that the term$\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)$ is increasing in $n_c^R \beta_c$, with $\lim_{n_c^R \beta_c \to \infty} \left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right) = \left(\frac{n_c^P}{1+n_c^P}\right)$. The parametric interval $(\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right), \left(\frac{n_c^P}{1+n_c^P}\right))$ can therefore be made arbitrarily small by suitably increasing $n_c^R \beta_c$.Thus, the larger the size of the rich population within a community, and the higher the private to public consumption ratio of rich individuals, the smaller the range of conflict shares where intra-community income distribution affects the class identity of conflict participants. Furthermore, the larger the size of the rich population within a community (and the higher their private to public consumption ratio), the larger the parametric range for the conflict share, within which only rich individuals participate in inter-community conflict independently of the intra-community income distribution. Now define the population share of the rich within a community as $r_c \equiv \frac{n_c^R}{n_c}$. Then the term $\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)$ reduces to $\left(\frac{1}{\frac{1}{(1-r_c)n_c} + 1 + \frac{1}{n_c{}^2 r_c \beta_c (1-r_c)}}\right)$, which increases as n_c rises, with $\lim_{n_c \to \infty} \left(\frac{1}{\frac{1}{(1-r_c)n_c} + 1 + \frac{1}{n_c{}^2 r_c \beta_c (1-r_c)}}\right) = 1$. Similarly, $\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)$ is increasing in n_c^P, with $\lim_{n_c^P \to \infty} \left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right) = 1$. Hence, given any intra-community population share of the rich and any sharing rule, only the rich will participate in communal conflict whenever the community is sufficiently large. Given any population size of the rich and any sharing rule, the same would hold if the poor are sufficiently numerous. The following corollary formalizes these observations (notice that $X_c = X_c^R$ holds trivially if $X_c = 0$).

Corollary 4.1: Let A1 hold.

(i) Given any $\ddot{\gamma}_c^P \in [0, \frac{n_c^P}{(1+n_c^P)})$, if $n_c^R \beta_c > \left[\frac{\ddot{\gamma}_c^P}{n_c^P - \ddot{\gamma}_c^P (1+n_c^P)}\right]$, then, for every $\gamma_c^P \in [0, \ddot{\gamma}_c^P]$ and every $X_{-c} > 0$, $R^c(X_{-c}, \gamma_c^P) = X_c^R$.

(ii) Define $r_c \equiv \frac{n_c^R}{n_c}$. Given any $r_c \in (0,1)$, and any $\ddot{\gamma}_c^P \in (0,1)$, if$n_c > \overline{n}_c$, then, for every $\gamma_c^P \in [0, \ddot{\gamma}_c^P]$ and every $X_{-c} > 0$, $R^c(X_{-c}, \gamma_c^P) = X_c^R$; $\overline{n}_c$being defined as the solution to $\left[\left(\frac{1}{\frac{1}{(1-r_c)\overline{n}_c} + 1 + \frac{1}{\overline{n}_c{}^2 r_c \beta_c (1-r_c)}}\right) = \ddot{\gamma}_c^P\right]$.[22]

(iii) Given any $n_c^R \beta_c > 0$, and any $\breve{\gamma}_c^P \in (0,1)$, if $n_c^P > \ddot{n}_c^P$, then, for every $\gamma_c^P \in [0, \breve{\gamma}_c^P]$ and every $X_{-c} > 0$, $R^c(X_{-c}, \gamma_c^P) = X_c^R$; $\ddot{n}_c^P$ being defined as the solution to $\left[\left(\frac{n_c^R \beta_c \ddot{n}_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c \ddot{n}_c^P}\right) = \breve{\gamma}_c^P\right]$.

22 Recall that the term $\left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)$ reduces to $\left(\frac{1}{\frac{1}{(1-r_c)n_c} + 1 + \frac{1}{n_c{}^2 r_c \beta_c (1-r_c)}}\right)$. It can be checked that, given total population (n_c), the second term first increases, and subsequently declines, as the proportion of rich individuals in the population (r_c) increases over (0,1). Hence, the range of class shares of the poor (i.e., of γ_c^P) over which only rich individuals engage in conflict (i.e., the interval $[0, \left(\frac{n_c^R \beta_c n_c^P}{n_c^R \beta_c + 1 + n_c^R \beta_c n_c^P}\right)]$) first expands, and then contracts, as the population share of the rich increases, given total population of the community.

Corollary 4.1(i) is the formal articulation of the claim that the larger the size of the rich population within a community, and the higher the private to public consumption ratio of rich individuals, the smaller the range of conflict shares where intra-community income distribution affects the class identity of conflict participants. Corollary 4.1(ii) implies that, given any population composition, and any class sharing rule, only the rich within a community will engage in conflict with another community whenever that community is of sufficiently large population size. The larger the community, the larger their class share has to be to induce the poor to engage in inter-communal conflict. Thus, intuitively, Corollary 4.1(ii) suggests that conflict with the other community is more likely to be the preoccupation of the rich in more numerous communities. Given the numerical size of the rich within their community, Corollary 4.1(iii) suggests that the poor of a community are more likely to stay aloof from conflict with another community when they are more numerous. The larger the population of the poor within a community, the larger the class share they need to be offered to induce them to engage in inter-communal conflict. Together, Corollary 4.1(i) and 4.1(iii) suggest that, when the two communities are identical in terms of the size of any one class, the community that is larger overall is more likely to see only its rich participate in conflict. Thus, the broad intuitive thrust of Corollary 4.1 is that smaller communities are more likely to see their poor members engage in conflict with another community. Thus, a small minority is more likely to be represented only by its poor members in communal conflict, while a large minority or a majority community is more likely to be represented solely by its rich.

Remark 3 Notice that, whenever $n_c^P > 1$, Corollary 1 implies that one can construct communities where only the rich participate in rent-seeking under the equal class shares rule ($\gamma_c^R = \frac{1}{2}$; recall Remark 1).

4.4 Conflict, class shares and income distribution

How do the properties of the community reaction functions characterized in Section 3 combine to affect inter-community conflict? Specifically, how do changes in the rental share and the income distribution within a community affect (a) the equilibrium division of the contested resource between the communities, and (b) total resource allocation to inter-community conflict (i.e., its aggregate intensity)? How do changes in cross-community income distribution impact on these variables? We now turn to these questions.

We first consider the case where only the R class in either community engages in inter-community conflict. As discussed earlier (Corollary 4.1), this intuitively applies more closely to a scenario where two large communities contest one another.

Proposition 4.3: Let A1 hold, let $G^R \equiv \sqrt{\gamma_H^R \gamma_M^R}$,let $\Delta \equiv min\left\{\frac{\gamma_H^R}{G^R}, \frac{\gamma_M^R}{G^R}\right\}$, and let $\underline{\gamma} \equiv 1 - \min\{\left(\frac{n_H^P}{1+\left(n_H^R \beta_H\right)^{-1}+n_H^P}\right), \left(\frac{n_M^P}{1+\left(n_M^R \beta_M\right)^{-1}+n_M^P}\right)\}$. Suppose $\gamma_H^R, \gamma_M^R \geq \underline{\gamma}$. Then the following hold in equilibrium.

(i) $[S_H > S_M]$iff $[\gamma_H^R > \gamma_M^R]$, and S_H is increasing in $\left(\frac{\gamma_H^R}{\gamma_M^R}\right)$ over $\left[\underline{\gamma}, \frac{1}{\underline{\gamma}}\right]$.

(ii) X is increasing in both γ_H^R and γ_M^R over $[\underline{\gamma}, 1]$.

(iii) X is increasing in Δ over $\left[\sqrt{\underline{\gamma}}, 1\right]$, given G^R.

(iv) Changes in I_M^R, I_M^P, $I_H^R$and I_H^P affect neither equilibrium community shares, nor aggregate conflict expenditure.

Proof of Proposition 4.3

Suppose A1 holds, and let $\gamma_H^R, \gamma_M^R \geq \underline{\gamma}$. Then, by Proposition 2, $X_c = X_c^R$ for all $c \in \{H, M\}$. Hence, in equilibrium, from (4.14),

$$\left(\frac{X_{-c}}{X^2}\right) = \frac{1}{\gamma_c^R S}, \tag{4.21}$$

so that:

$$\left(\frac{X_{-c}}{X_c}\right) = \frac{\gamma_{-c}^R}{\gamma_c^R}. \tag{4.22}$$

Since $\gamma_H^R, \gamma_M^R \geq \underline{\gamma}$, $\left(\frac{\gamma_H^R}{\gamma_M^R}\right) \in \left[\underline{\gamma}, \frac{1}{\underline{\gamma}}\right]$. By (4.22), noting (4.1), $[S_H > S_M]$ iff $[\gamma_H^R > \gamma_M^R]$, and S_H is increasing in $\left(\frac{\gamma_H^R}{\gamma_M^R}\right)$ over $\left[\underline{\gamma}, \frac{1}{\underline{\gamma}}\right]$.

Now note that, from (4.21):

$$X = S\left(\frac{\gamma_H^R \gamma_M^R}{\gamma_H^R + \gamma_M^R}\right) = \frac{SG^R}{\Delta + \frac{1}{\Delta}}. \tag{4.23}$$

Since $\gamma_H^R, \gamma_M^R \geq \underline{\gamma}$, and by construction $\Delta = min\left\{\sqrt{\frac{\gamma_H^R}{\gamma_M^R}}, \sqrt{\frac{\gamma_M^R}{\gamma_H^R}}\right\}$, we must have $\Delta \in \left[\sqrt{\underline{\gamma}}, 1\right]$. Parts (ii) and (iii) of Proposition 4.3 follow immediately from (4.23). Part (iv) of Proposition 4.3 follows directly from Proposition 4.2. ■

Proposition 4.3 refers to the case where both communities are large enough to ensure that only the rich participate in rent-seeking (recall Corollary 4.1). Then the community which offers a larger rental share to its rich as a class is more successful in the rent-seeking contest, even if each rich individual receives a higher share in the other community (because it has fewer rich individuals). Thus, what determines rent-seeking success is the relative class share of the rich, not the relative individual shares. An increase in the class share of the rich within either community increases conflict ((measured as the total resource wasted in rent-seeking). Given the geometric mean of the class share of the rich across the two communities, any increase in its dispersion reduces conflict. Changes in the distribution of uncontested incomes, whether within or between communities, affect neither equilibrium community shares, nor aggregate conflict expenditure, provided they do not induce the poor in either community to start contributing to their community's public good.

What happens when the poor members of one community, say *M*, engage in inter-community conflict with the rich of another community, say *H*? Intuitively, in light of Corollary 1, we think of this as broadly capturing a situation where, *H* comprises of large numbers of both poor and rich individuals, but the size of the rich class is minuscule within M (in consequence of which that class can capture only an insignificant share of any rent). Our next set of results characterize this scenario.

Let $\Im_M \equiv (\gamma_M^P, n_M^P, n_M^R, I_M^P, I_M^R, \beta_M)$, $\Im_H \equiv (\gamma_H^P, n_H^P, n_H^R, I_H^P, I_H^R, \beta_H)$. Define the functions: $X_M^*(\Im_M, \Im_H)$, $X_H^*(\Im_M, \Im_H)$, such that $\langle X_M^*(.), X_H^*(.)\rangle$ constitutes a solution to the equation system:

$$X_M = R^M(X_H, \Im_M);$$

$$X_H = R^H(X_M, \Im_H);$$

where $R^M(X_H, \Im_M)$ and $R^H(X_M, \Im_H)$ are the community reaction functions. We shall call $\langle X_M^*(\Im_M, \Im_H), X_H^*(\Im_M, \Im_H)\rangle$ an *equilibrium conflict allocation function*. Define $X^*(\Im_M, \Im_H) \equiv X_M^*(\Im_M, \Im_H) + X_H^*(\Im_M, \Im_H)$. We shall call $X^*(\Im_M, \Im_H)$ an equilibrium *total conflict function*. Clearly, $X_M^*(\Im_M, \Im_H), X_H^*(\Im_M, \Im_H) > 0$.

Proposition 4.4: Let $\langle X_M^*(\Im_M, \Im_H), X_H^*(\Im_M, \Im_H)\rangle$ constitute an equilibrium conflict allocation function. Suppose, for some $(\widehat{\Im}_M, \widehat{\Im}_H)$ which satisfies A1 and [$\gamma_M^P > \frac{n_M^P}{(1+n_M^P)}$ and $\gamma_H^P < \left(\frac{n_c^P}{1+(n_c^R\beta_c)^{-1}+n_c^P}\right)$], $\langle X_M^*(\widehat{\Im}_M, \widehat{\Im}_H), X_H^*(\widehat{\Im}_M, \widehat{\Im}_H)\rangle$ constitutes a locally stable conflict equilibrium. Assume further that $\left[\frac{\partial R^M(X_H^*(\widehat{\Im}_M,\widehat{\Im}_H),\widehat{\Im}_M)}{\partial X_H} < \frac{X_M^*(\widehat{\Im}_M,\widehat{\Im}_H)}{X_H^*(\widehat{\Im}_M,\widehat{\Im}_H)}\right]$. Then:

(i) $\frac{d\left(\frac{X_H^*(\widehat{\Im}_M,\widehat{\Im}_H)}{X^*(\widehat{\Im}_M,\widehat{\Im}_H)}\right)}{d\gamma_H^R} > 0$; (ii) $\frac{d\left(\frac{X_M^*(\widehat{\Im}_M,\widehat{\Im}_H)}{X^*(\widehat{\Im}_M,\widehat{\Im}_H)}\right)}{dI_M^P} > 0$, $\frac{dX^*(\widehat{\Im}_M,\widehat{\Im}_H)}{dI_M^P} > 0$; (iii) $\frac{d\left(\frac{X_M^*(\widehat{\Im}_M,\widehat{\Im}_H)}{X^*(\widehat{\Im}_M,\widehat{\Im}_H)}\right)}{dI_M^R} < 0$, $\frac{dX^*(\widehat{\Im}_M,\widehat{\Im}_H)}{dI_M^R} < 0$; and

(iv) provided $\left[\beta_c \geq \frac{n_c^P-1}{n_c^R}\right]$, $\frac{d\left(\frac{X_M^*(\widehat{\Im}_M,\widehat{\Im}_H)}{X^*(\widehat{\Im}_M,\widehat{\Im}_H)}\right)}{d\gamma_M^P} > 0$, $\frac{dX^*(\widehat{\Im}_M,\widehat{\Im}_H)}{d\gamma_M^P} > 0$.

We shall prove Proposition 4.4 via the following lemma.

Lemma 4.3: Let $\langle X_M^*(\Im_M, \Im_H), X_H^*(\Im_M, \Im_H)\rangle$ constitute an equilibrium conflict allocation function. Suppose, for some $(\widehat{\Im}_M, \widehat{\Im}_H)$ which satisfies A1, $\langle X_M^*(\widehat{\Im}_M, \widehat{\Im}_H), X_H^*(\widehat{\Im}_M, \widehat{\Im}_H)\rangle$ constitutes a locally stable conflict equilibrium. Suppose further that $\frac{\partial R^{c}(X^{*}_{-c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H),\widehat{\mathfrak{I}}_c)}{\partial A_c} > 0$ for some $A_c \in \mathfrak{I}_c$. Then: (i) $\frac{dX^{*}_{c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}{dA_c} > 0$; (ii) $\frac{dX^{*}_{-c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}{dA_c} > 0$ (resp. < 0); if $\frac{\partial R^{-c}(X^{*}_{c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H),\widehat{\mathfrak{I}}_{-c})}{\partial X_c} > 0$ (resp. < 0); (iii) $\frac{dX^{*}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}{dA_c} > 0$ (resp. < 0) if $\frac{\partial R^{-c}(X^{*}_{c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H),\widehat{\mathfrak{I}}_{-c})}{\partial X_c} > -1$ (resp. < -1); and (iv) [$\frac{d\left(\frac{X^{*}_{c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}{X^{*}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}\right)}{dA_c} > 0$ when $\frac{\partial R^{-c}(X^{*}_{c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H),\widehat{\mathfrak{I}}_{-c})}{\partial X_c} < \frac{X^{*}_{-c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}{X^{*}_{c}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)}$.

Proof of Lemma 4.3

Suppose, for some $(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)$ which satisfies A1, $\langle X^{*}_{M}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H), X^{*}_{H}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)\rangle$ constitutes a locally stable conflict equilibrium. Let $X^{*}_{M} = X^{*}_{M}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)$, $X^{*}_{H} = X^{*}_{H}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H)$. Then, we must have:

$$X^{*}_{H}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H) = R^{H}(X^{*}_{M},\widehat{\mathfrak{I}}_H);$$

$$X^{*}_{M}(\widehat{\mathfrak{I}}_M,\widehat{\mathfrak{I}}_H) = R^{M}(X^{*}_{H},\widehat{\mathfrak{I}}_M).$$

Hence, without loss of generality, for any $A_H \in \mathfrak{I}_H$,

$$\frac{dX^{*}_{H}}{dA_H} = \left(\frac{\partial R^{H}}{\partial X_M}\right)\left(\frac{dX^{*}_{M}}{dA_H}\right) + \frac{\partial R^{H}}{\partial A_H};$$

$$\frac{dX^{*}_{M}}{dA_H} = \left(\frac{\partial R^{M}}{\partial X_H}\right)\left(\frac{dX^{*}_{H}}{dA_H}\right).$$

Thus,

$$\frac{dX^{*}_{H}}{dA_H} = \frac{\frac{\partial R^{H}}{\partial A_H}}{[1-\left(\frac{\partial R^{H}}{\partial X_M}\right)\left(\frac{\partial R^{M}}{\partial X_H}\right)]}; \tag{4.24}$$

$$\frac{dX^{*}_{M}}{dA_H} = \frac{\left(\frac{\partial R^{H}}{\partial A_H}\right)\left(\frac{\partial R^{M}}{\partial X_H}\right)}{[1-\left(\frac{\partial R^{H}}{\partial X_M}\right)\left(\frac{\partial R^{M}}{\partial X_H}\right)]}; \tag{4.25}$$

$$\frac{dX^{*}}{dA_H} = \frac{\frac{\partial R^{H}}{\partial A_H}(1+\frac{\partial R^{M}}{\partial X_H})}{[1-\left(\frac{\partial R^{H}}{\partial X_M}\right)\left(\frac{\partial R^{M}}{\partial X_H}\right)]}. \tag{4.26}$$

It is easy to check that an equilibrium $\langle X_M^*(\widehat{\mathfrak{I}}_M, \widehat{\mathfrak{I}}_H), X_H^*(\widehat{\mathfrak{I}}_M, \widehat{\mathfrak{I}}_H)\rangle$ is locally stable if and only if, at that pair of community conflict allocations, $\left(\frac{\partial R^H}{\partial X_M}\right)\left(\frac{\partial R^M}{\partial X_H}\right) < 1$. Parts (i), (ii) and (iii) of Lemma 3 then follow immediately from (4.24), (4.25) and (4.26), respectively.

Now notice that, since $\langle X_M^*(\widehat{\mathfrak{I}}_M, \widehat{\mathfrak{I}}_H), X_H^*(\widehat{\mathfrak{I}}_M, \widehat{\mathfrak{I}}_H)\rangle$ constitutes a conflict equilibrium, $X_M^*(\widehat{\mathfrak{I}}_M, \widehat{\mathfrak{I}}_H), X_H^*(\widehat{\mathfrak{I}}_M, \widehat{\mathfrak{I}}_H) > 0$. Then $\frac{X_H^*}{X^*} = \frac{1}{1+\frac{X_M^*}{X_H^*}}$, so that:$\frac{d\left(\frac{X_H^*}{X^*}\right)}{dA_H} > 0$ iff $\frac{d\left(\frac{X_H^*}{X_M^*}\right)}{dA_H} > 0$. We have:

$$\frac{d\left(\frac{X_H^*}{X_M^*}\right)}{dA_H} = \left(\frac{\left(\frac{dX_H^*}{dA_H}\right) - \left(\frac{X_H^*}{X_M^*}\right)\left(\frac{dX_M^*}{dA_H}\right)}{X_M^*}\right).$$

Since $\left(\frac{dX_H^*}{dA_H}\right) > 0$ by Lemma 4.3(i), we therefore have:

$$\frac{d\left(\frac{X_H^*}{X_M^*}\right)}{dA_H} > 0 \text{ iff } \left(\frac{\frac{dX_M^*}{dA_H}}{\frac{dX_H^*}{dA_H}}\right) < \left(\frac{X_M^*}{X_H^*}\right).$$

Now, using (4.24) and (4.25), we get:

$$\left(\frac{\frac{dX_M^*}{dA_H}}{\frac{dX_H^*}{dA_H}}\right) = \left(\frac{\partial R^M}{\partial X_H}\right).$$

Part (iv) of Lemma 4.3 follows. ■

Proof of Proposition 4.4

We shall first establish that, given any $\widehat{\mathfrak{I}}_H$ which satisfies A1 and $[\gamma_H^P < \left(\frac{n_c^P}{1+(n_c^R\beta_c)^{-1}+n_c^P}\right)]$,

for every $X_M > 0$, if $R^H(X_M, \widehat{\mathfrak{I}}_H) > 0$, then $\left[\frac{R^H(X_M,\widehat{\mathfrak{I}}_H)}{X_M} > \frac{\partial R^H(X_M,\widehat{\mathfrak{I}}_H)}{\partial X_M} > -1\right]$.(4.27)

Since $[\gamma_H^P < \left(\frac{n_c^P}{1+(n_c^R\beta_c)^{-1}+n_c^P}\right)]$, by Proposition 2, $R^H(X_M, \widehat{\mathfrak{I}}_H) = X_H^R$. Hence, if $R^H(X_M, \widehat{\mathfrak{I}}_H) > 0$, using (4.14), $R^H(X_M, \widehat{\mathfrak{I}}_H)$ must constitute the solution to:

$$X_M \left(1 + \frac{X_H}{X_M}\right)^2 = S\gamma_H^R. \tag{4.28}$$

From (4.28), we get:

$$\left(1 + \frac{X_H}{X_M}\right) + 2\left(\frac{X_H}{X_M}\right)\left(\left(\frac{X_M}{X_H}\right)\frac{\partial R^H}{\partial X_M} - 1\right) = 0; \tag{4.29}$$

which implies $\left(\left(\frac{X_M}{X_H}\right)\frac{\partial R^H}{\partial X_M} - 1\right) < 0$, i.e., $\frac{\partial R^H}{\partial X_M} < \frac{X_H}{X_M}$. Furthermore, (4.29) yields:

$$\frac{\partial R^H}{\partial X_M} = \left(\frac{1}{2}\right)\left(\frac{X_H}{X_M} - 1\right);$$

which implies $\frac{\partial R^H}{\partial X_M} > -1$. This establishes (4.27).

Now note that, if $\langle X_M^*(\widehat{\Im}_M, \widehat{\Im}_H), X_H^*(\widehat{\Im}_M, \widehat{\Im}_H)\rangle$ constitutes a conflict equilibrium, then $X_M^*(\widehat{\Im}_M, \widehat{\Im}_H), X_H^*(\widehat{\Im}_M, \widehat{\Im}_H) > 0$. Proposition 4.4 then follows immediately from Proposition 4.1, Proposition 4.2, Lemma 4.3 and (4.27). ■

Suppose the rich in *H* engage in rent-seeking contestation against the poor in *M*, in some initial locally stable equilibrium. Suppose further that the community reaction function of *M* is strictly concave in some neighborhood of that initial equilibrium. Then a marginal increase in the rental share of the rich in *H* makes that community more successful in the rent-seeking contest (Proposition 4.4(i)). The impact on aggregate conflict, measured as the total rent-seeking expenditure incurred by the two communities together, is however indeterminate without additional assumptions regarding the slope of the reaction function of *M*. Marginal changes in the distribution of non-contestable income within *H* have no effect on conflict outcomes (recall Proposition 4.2). Analogously, provided the private to public consumption ratio of rich individuals in *M* is not too low, a marginal increase in the rental share of the poor in *M* increases that community's share of the total rent in the new equilibrium (recall Proposition 4.1). However, in this case, aggregate equilibrium conflict expenditure must increase as well (Proposition 4.4(iv)). A marginal increase in the per capita non-contestable income of the poor in M has the same effects (Proposition 4.4(ii)), as does a marginal decline in the per capita non-contestable income of the rich in that community (Proposition 4.4(iii)). Thus, when the poor of a community engage in rent-seeking conflict with the rich of another community, a marginal redistribution of non-contestable income from the rich to the poor within the former community increases its conflict success, as well as overall conflict, even though such redistributions within the latter community are conflict-neutral. Notice that a marginal exogenous redistribution of non-contestable income from H to the poor in M makes M more aggressive as well, increasing its conflict success, as well as overall conflict. A marginal exogenous redistribution of non-contestable income from the rich in M to either class in H has the same effect.

Lastly, what happens when the poor of one community contests against the poor of another community in a locally stable initial equilibrium? In light of Proposition 4.1 and Proposition 4.4(parts (ii)-(iv)), it is easy to see that, provided both community reaction functions are strictly concave in some neighborhood of the initial equilibrium, a marginal increase in the rental class share of the poor within either community must increase its conflict success. The same holds for a marginal increase in the non-contestable income of the poor or a marginal decline in the non-contestable income of the rich within either community. However, in all three cases, the impact on aggregate conflict is indeterminate without additional assumptions regarding the slopes of the community reaction functions.

4.5 Concluding remarks

This chapter has examined how prior income inequality within a community combines with plunder sharing rules to affect decentralized individual efforts to expropriate another community, when the poor are dependent on the rich members of their community for the provision of public goods. We have shown that an individual's share of any rent accruing to a community, in consequence of expropriation of another community, may be a misleading proxy for her relative incentive to engage in inter-community conflict. Our findings provide micro-foundations for situations where one income class within a community may free-ride on another in such conflicts, despite members of the former class all standing to gain nominally more income from inter-community conflict, than those of the latter. These findings offer a broad theoretical perspective that helps to rationalize an empirical phenomenon often noted in historical andelectoral studies – viz., the greater propensity of better-off segments within a society to support ethno-exclusivist, xenophobic or ultra-nationalist political programs, including fascism.[23] Our results also suggest that internally more equal communities, under certain conditions, may exhibit greater aggressiveness in inter-community conflict. This implication of our model stands in stark contrast to the central implication of the model developed by Esteban and Ray (2011), and suggests theoretical organizing principles for empirical research on the intensity of ethnic community. Lastly, our results point to a possible tension between the twin policy goals of reducing inequality, whether within or across ethnic groups in a society, and reducing the intensity of inter-ethnic conflict therein. Future research may delve deeper into the nature and robustness of this trade-off under alternative specifications of public good technologies and conflict success functions.

23 See, among others, Hobsbawm 1987, 1992;Engineer 1995, Brustein 1998 and Dhattiwala and Biggs 2012 for discussions of the literature.

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