TABLES OF THE NON-CENTRALITY PARAMETER OF F-TEST AS A FUNCTION OF POWER

By P. DASGUPTA

Indian Statistical Institute

SUMMARY. The non-centrality parameter of the power function of the analysis of variance test is tabulated as a function of the two degrees of freedom, the level of significance and the power of the test. Applications of these tables in the planning of statistical investigation involving one or more variables are also discussed.

1. Introduction

For given values of M, N, α and β; $M, N > 0$, $0 < \alpha, \beta < 1$, let $\partial = \partial(M, N, \alpha, \beta)$ be defined by the equation

$$P(\xi, M, N, \partial) = \beta \qquad \ldots \quad (1)$$

where $\xi = \xi(M, N, \alpha)$ in turn is defined by the equation

$$I\left(\xi, \frac{N}{2}, \frac{M}{2}\right) = \alpha \qquad \ldots \quad (2)$$

where $I(x, p, q)$ is the incomplete beta function ratio defined as

$$I(x, p, q) = \frac{\Gamma(p+q)}{\Gamma(p)\ \Gamma(q)} \int_0^x u^{p-1}(1-u)^{q-1}\, du, \ \ 0 < x < 1;\ p, q > 0$$

and $P(x, m, n, \theta)$ is the non-central beta distribution function with degrees of freedom M and N and non-centrality parameter θ.

$$P(x, M, N, \theta) = e^{-\theta} \sum_{r=0}^{\infty} \frac{\theta^r}{r!}\ I\left(x, \frac{N}{2}, \frac{M}{2}+r\right). \qquad \ldots \quad (3)$$

The function $\partial(M, N, \alpha, \beta)$ plays an important role in the theory of testing linear hypothesis (Kolodziejczyk, 1935, Tang, 1938) and in multivariate analysis (Rao, 1949). In this paper the values of $\partial(M, N, \alpha, \beta)$ are presented for $M = 1(1)\ 10$, $N = 10(5)\ 50(10)\ 100$, $\alpha = 0.01$, 0.05 and $\beta = 0.1(0.1)0.9$. The computational technique adopted in the preparation of the tables are described in Section 2. Applications of the tables in various statistical problems are discussed in Section 3.

Analogous tables of $\phi = \phi(f_1, f_2, \alpha, \beta)$ were prepared by Lehmer (1944) for $f_1 = 1(1)\ 10$, 12, 15, 20, 24, 30, 40, 60, 80, 120; $\alpha = 0.01$, .05; $\beta = 0.7$, 0.8 where $f_1 = M$, $f_2 = N$ and $\phi^2 = \dfrac{2\partial}{M+1}$ in our notation. In Lehmer's tables only $\beta = 0.7$ and 0.8 have been considered and the values of ϕ obtained are not all exact, because of some limiting expressions used by her. Values of ∂ as given in the present set of tables are presented to four places of decimals though the values are computed to more places.

2. Tables

2.1. *Algorithms used.* All computations were carried out on an IBM 1401-8K-4 tape electronic data processing machine system using programmes written in Fortran II language. Because of the limited storage capacity computations had to be done in several phases.

In the first phase equation (2) was solved by repeated polynomial interpolation using finer intervals. The algorithm used for evaluation of incomplete beta ratio is :

$$I(x, p, q) = \begin{cases} \dfrac{B(x, p, q)}{B(1, p, q)} & \text{if } x \leqslant \dfrac{1}{2} \\ 1 - \dfrac{B(1-x, p, q)}{B(1, p, q)} & \text{if } x > \dfrac{1}{2} \end{cases}$$

where

$$B(x, p, q) = \frac{x^p(1-x)^{q-1}}{p} + \frac{(q-1)}{p(p+1)} x^{p+1}(1-x)^{q-2} + \frac{(q-1)(q-2)}{p(p+1)(p+2)} x^{p+2}(1-x)^{q-3} + \ldots$$

$$+ \frac{(q-1)(q-2)\ldots(q-[q]+1)}{p(p+q)\ \ (p+[q]-1)} x^{p+[q]+1}(1-x)^{q-[q]} + \frac{(q-1)(q-2)\ldots(q-[q])}{p(p+1)\ldots(p+[q]-1)} [B(x, p+[q], q(q))]$$

where $[q]$ is the greatest integer not exceeding q. Writing $u = p+[q]$ and $1-f = q-[q]$, $B(x, u, 1-f)$ was computed from the expression

$$\frac{x^u}{u} + \frac{fx^{u+1}}{u+1} + \frac{f(f+1)}{2!} \frac{x^{u+2}}{u+2} + \ldots + \frac{f(f+1)\ldots(f+r-1)}{r!} \frac{x^{u+r}}{u+r}$$

where r is the smallest integer satisfying

$$\frac{(q-1)(q-2)\ldots(q-[q])}{p(p+1)\ldots(p+[q]-1)} \ \frac{f(f+1)\ldots(f+r)}{(r+1)!} \ \frac{x^{u+r+1}}{u+r+1} < 0.5 \times 10^{-10}.$$

This ensures in $B(x, p, q)$ an error not exceeding 0.5×10^{-10}.

In the second phase, using the value of ξ obtained from the first phase, $I\left(\xi, \frac{1}{2}N, \frac{1}{2}M+r\right)$ was computed for $r = 0, 1, 2, \ldots, K$; where K is the smallest integer satisfying $I\left(\xi, \frac{1}{2}N, \frac{1}{2}M+K\right) \geqslant 0.9999999$. This was done by using the relation $I\left(\xi, \frac{1}{2}N, \frac{1}{2}M\right) = \alpha$ and the recurrence formula

$$I\left(\xi, \frac{1}{2}N, \frac{1}{2}M+r+1\right) = I\left(\xi, \frac{1}{2}N, \frac{1}{2}M+r\right) + \frac{\xi^{(1/2)N}(1-\xi)^{(1/2)M+r}}{\left(\frac{1}{2}M+r\right)B\left(1, \frac{1}{2}N, \frac{1}{2}M+r\right)} \quad \cdots \quad (4)$$

In the third phase $P(\xi, M, N, \partial)$ was tabulated as a function of ∂, and ∂_0 the first approximation to $\partial(M, N, \alpha, \beta)$ was interpolated therefrom for $\beta = 0.1(0.1)0.9$ using Cubic polynomial approximation. An improved value of $\partial(M, N, \alpha, \beta)$ was next obtained by recomputing P at the points $\partial_0 - .05$, $\partial_0 - .025$, ∂_0, $\partial_0 + .025$ and $\partial_0 + .05$, and then using a five-point Lagrange's interpolation formula.

For $N = \alpha$, $P(M, N, \alpha, \partial)$ was computed by $\sum_{r=1}^{\infty} \frac{e^{-\theta}\theta^r}{r!} \times G\left(\xi, \frac{M}{2}+r\right)$ where $G(y, p) = \frac{1}{\Gamma(p)} \int_y^{\infty} e^{-t} t^{p-1} dt$ and ξ is the solution of $G\left(\xi, \frac{M}{2}\right) = \alpha$. The recurrence relation corresponding to (4) in this case is $G\left(\xi, \frac{M}{2}+r+1\right) = G\left(\xi, \frac{M}{2}+r\right) + \frac{e^{-\xi}\, \xi^{M/2+r}}{\Gamma\left(\frac{M}{2}+r+1\right)}$.

2.2. The accuracy of the tables was tested by recomputing P at the interpolated value of ∂. It was found that in all cases the computed P agrees with the required power to at least five places of decimals.

2.3. For single entry interpolation in N, we recommend using $1/N$ as argument and linear interpolation for $N > 50$ and quadratic interpolation for $N < 50$. This would result in an error of at most one unit in the third place of decimal.

3. Applications

3.1. *Determination of size of sample in one-way analysis of variance.* Consider the problem of testing, on the basis of samples of size n drawn from each of k normal populations with a common variance, the hypothesis that the mean values are equal. The power function of the standard one-way analysis of variance test procedure at level of significance α is $P(M, N, \alpha, \partial)$ where $M=k-1$, $N=k(n-1)$, and $\partial = \frac{1}{2} nk\psi$ where ψ is the ratio of the variance between the means of the population to the common variance, that is $\psi = \frac{1}{k} \sum_{i=1}^{k} (\mu_i-\bar{\mu})^2/\sigma^2$, where σ^2 is the common variance μ_i the mean of the i-th population and $\bar{\mu}=\frac{1}{k} \sum_{i=1}^{k} \mu_i$. For given values of k and ψ, the size of sample n required to ensure a preassigned power β is then obtained from the equation $n = 2\partial(M, N, \alpha, \beta)/k\psi$. The following table gives the approximate values of n to ensure $\beta = 0.50$ when $\alpha = 0.05$ for $k = 2(1)\ 10$, and $\psi = .005$, 01, .02, .04, .10 and .25.

TABLE 1

K/ψ	.005	.01	.02	.04	.10	.25
2	385	193	97	49	20	9
3	332	166	84	42	18	8
4	289	145	73	37	16	7
5	258	130	66	34	14	7
6	233	118	60	31	13	6
7	215	108	55	28	12	6
8	199	100	51	26	11	5
9	186	94	48	25	10	5
10	176	88	45	23	10	5

We may note in passing that when ψ is unknown, an unbiased estimate for it is provided by

$$\hat{\psi} = \frac{[(n-1)k-2]\left[1+\frac{k-1}{(n-1)k}F\right]-nk+3}{nk}$$

where F is the usual variance ratio statistic with $k-1$, $k(n-1)$ degrees of freedom.

3.2. *Determination of size of sample for comparison of the means of two multivariate normal populations.* Consider two samples, each of size n, from two p-variate normal populations with same dispersion matrix Σ. Let

m = difference in mean vectors of the two populations,

d = difference in mean vectors of the two samples,

$s = \frac{1}{2n-2}(S_1+S_2)$ where S_1 and S_2 are the corrected sum of products matrices for the two samples and $C = \frac{n}{2}$. An appropriate method for testing $H_0 : (m = 0)$

is provided by the Hotelling procedure : reject H_0 if $X = \dfrac{1}{1+\dfrac{CD^2}{2n-2}} > X_0$; accept H_0 otherwise; where $D^2 = d's^{-1}d$ is the Mahalanobis distance statistic, and X_0 is such that Prob $(X \leqslant X_0 | m = 0) = \alpha$, the level of significance. The power function of the above test procedure is given by $P(M, N, \alpha, \partial)$, where $M = p$, $N = 2n-p-1$ and $\partial = \dfrac{n(p+1)\psi}{4}$ where $\psi = \dfrac{m'\Sigma^{-1}m}{p+1}$. For given value of ψ the sample size n required to ensure a preassigned power β is obtained from the equation $n = 4\partial(M, N, \alpha, \beta)/(p+1)\psi$.

3.3. *Use of additional variables in the problem of discrimination between two multivariate normal population.* The power function of the Hotelling procedure for testing equality of the mean vectors of two multivariate normal populations with a common dispersion matrix is $P(M, N, \alpha, \partial)$ where $M = p$, $N = n_1+n_2-p-1$, $\partial = \dfrac{n_1 n_2}{2(n_1+n_2)} \Delta_p^2$ where p is the number of variables, n_1, n_2 the two sample sizes and Δ_p^2 the Mahalanobis distance based on the p-variables, α being the level of significance. If q extra variables are included, while the sample sizes remain fixed, the power is changed to $P(M^*, N^*, \alpha, \partial^*)$ where $M^* = M+q$, $N^* = N-q$ and $\partial^* = \partial\Delta_{p+q}^2/\Delta_p^2$. It is known that $\Delta_{p+q}^2 \geqslant \Delta_p^2$ so that $\partial^* \geqslant \partial$, and of course $M^* > M$, $N^* < N$. The power function $P(M, N, \alpha, \partial)$ increases with N and ∂, but decreases with M. Thus the addition of q extra characters might even decrease the power if the increment in ∂ is not sufficient to offset the loss caused by the changes in M and N. This was first noted by Rao (1949).

From the tables presented in this paper, it is possible to compute the minimum increment required in the distance, so that the power of the test is not reduced due to the inclusion of q extra variables. The following short table gives the values of ∂ to ensure a power of 0.50 when $\alpha = 0.05$ and the total sample size $n = n_1+n_2 = 30$, 50 and 70.

TABLE 2

$n \backslash p$	2	3	4	5	6	7	8	9	10
30	2.771	3.343	3.861	4.360	4.854	5.352	5.865	6.401	6.960
50	2.642	3.134	3.561	3.952	4.321	4.677	5.025	5.368	5.709
70	2.592	3.055	3.450	3.806	4.136	4.449	4.749	5.040	5.324

Consider a test procedure with size = .05 and power = 0.50, based on a total sample of size 50, involving four characters. The above table shows that it will have its power undiminished if with the inclusion of one extra character ∂ is increased at least by $3.952-3.561 = 0.391$.

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NON-CENTRALITY PARAMETER OF *F* TEST AS A FUNCTION OF LEVEL OF SIGNIFICANCE, POWER AND THE TWO DEGREES OF FREEDOM

level of significance = 0.01

$N \backslash M$	1	2	3	4	5	6	7	8	9	10
					(power = 0.10)					
10	1.1748	1.8028	2.3516	2.8072	3.3048	3.8513	4.3304	4.8042	5.2743	5.7412
15	1.0479	1.5552	1.9833	2.3768	2.7510	3.1128	3.4602	3.8134	4.1580	4.4950
20	0.9901	1.4433	1.8170	2.1553	2.4735	2.7784	3.0742	3.3634	3.6473	3.9272
25	0.9572	1.3707	1.7220	2.0295	2.3150	2.5881	2.8509	3.1065	3.3560	3.6024
30	0.9350	1.3387	1.6617	1.9484	2.2138	2.4652	2.7066	2.9405	3.1685	3.3920
35	0.9210	1.3100	1.6192	1.8918	2.1428	2.3794	2.6058	2.8244	3.0370	3.2446
40	0.9101	1.2889	1.5879	1.8501	2.0904	2.3161	2.5314	2.7387	2.9397	3.1357
45	0.9010	1.2727	1.5630	1.8180	2.0502	2.2670	2.4742	2.6728	2.8649	3.0518
50	0.8940	1.2599	1.5449	1.7927	2.0183	2.2290	2.4289	2.6205	2.8055	2.9854
60	0.8850	1.2409	1.5167	1.7551	1.9711	2.1710	2.3616	2.5429	2.7174	2.8865
70	0.8780	1.2274	1.4968	1.7285	1.9377	2.1315	2.3141	2.4880	2.6551	2.8166
80	0.8728	1.2174	1.4820	1.7088	1.9120	2.1015	2.2787	2.4472	2.6087	2.7644
90	0.8687	1.2097	1.4705	1.6936	1.8937	2.0783	2.2514	2.4156	2.5728	2.7241
100	0.8655	1.2036	1.4614	1.6814	1.8785	2.0598	2.2296	2.3905	2.5442	2.6920
inf	0.8372	1.1495	1.3814	1.5747	1.7442	1.8971	2.0377	2.1686	2.2917	2.4081
					(power = 0.20)					
10	2.1246	3.1236	3.9905	4.8020	5.5829	6.3510	7.1041	7.8487	8.5872	9.3207
15	1.8877	2.6788	3.3402	3.9463	4.5216	5.0774	5.6198	6.1526	6.6780	7.1979
20	1.7813	2.4808	3.0519	3.5670	4.0503	4.5132	4.9617	5.3998	5.8299	6.2537
25	1.7210	2.3691	2.8890	3.3535	3.7850	4.1954	4.5908	4.9752	5.3511	5.7203
30	1.6822	2.2975	2.7855	3.2107	3.6150	3.9917	4.3529	4.7028	5.0436	5.3774
35	1.6551	2.2477	2.7132	3.1216	3.4969	3.8500	4.1875	4.5131	4.8295	5.1385
40	1.6352	2.2110	2.6600	3.0518	3.4100	3.7459	4.0658	4.3736	4.6719	4.9625
45	1.6199	2.1829	2.6192	2.9982	3.3434	3.6661	3.9724	4.2665	4.5509	4.8275
50	1.6078	2.1607	2.5870	2.9559	3.2908	3.6029	3.8986	4.1818	4.4552	4.7207
60	1.5898	2.1278	2.5393	2.8932	3.2129	3.5095	3.7893	4.0564	4.3134	4.5622
70	1.5771	2.1046	2.5058	2.8491	3.1579	3.4436	3.7123	3.9679	4.2133	4.4503
80	1.5677	2.0873	2.4808	2.8163	3.1172	3.3946	3.6550	3.9022	4.1389	4.3671
90	1.5604	2.0740	2.4615	2.7910	3.0857	3.3569	3.6107	3.8514	4.0814	4.3028
100	1.5546	2.0634	2.4462	2.7709	3.0607	3.3268	3.5756	3.8110	4.0357	4.2517
inf	1.5037	1.9706	2.3110	2.5944	2.8412	3.0632	3.2669	3.4561	3.6337	3.8015
					(power = 0.30)					
10	2.9878	4.2907	5.4190	6.4758	7.4943	8.4894	9.4688	10.4371	11.3975	12.3514
15	2.6473	3.6640	4.5117	5.2878	6.0241	6.7352	7.4291	8.1105	8.7820	9.4474
20	2.4957	3.3882	4.1144	4.7685	5.3819	5.9691	6.5380	7.0936	7.6389	8.1762
25	2.4101	3.2334	3.8920	4.4782	5.0230	5.5408	6.0397	6.5246	6.9986	7.4641
30	2.3552	3.1345	3.7500	4.2920	4.7940	5.2676	5.7216	6.1612	6.5894	7.0080
35	2.3170	3.0658	3.6515	4.1645	4.6352	5.0781	5.5011	5.9091	6.3054	6.6924
40	2.2880	3.0154	3.5792	4.0702	4.5188	4.9391	5.3392	5.7240	6.0968	6.4601
45	2.2673	2.9767	3.5239	3.9982	4.4297	4.8327	5.2152	5.5823	5.9371	6.2821
50	2.2502	2.9462	3.4802	3.9412	4.3593	4.7487	5.1174	5.4703	5.8109	6.1415
60	2.2250	2.9011	3.4156	3.8571	4.2553	4.6245	4.9726	5.3047	5.6241	5.9333
70	2.2072	2.8693	3.3702	3.7978	4.1821	4.5370	4.8708	5.1881	5.4926	5.7866
80	2.1939	2.8457	3.3364	3.7530	4.1278	4.4722	4.7951	5.1016	5.3949	5.6776
90	2.1837	2.8275	3.3104	3.7200	4.0859	4.4221	4.7367	5.0348	5.3190	5.5935
100	2.1756	2.8130	3.2897	3.6931	4.0526	4.3823	4.6904	4.9817	5.2597	5.5267
inf	2.1042	2.6802	3.1088	3.4576	3.7614	4.0344	4.2844	4.5166	4.7343	4.9400

NON-CENTRALITY PARAMETER OF F TEST AS A FUNCTION OF LEVEL OF SIGNIFICANCE, POWER AND THE TWO DEGREES OF FREEDOM

level of significance = 0.01

(power = 0.40)

N \ M	1	2	3	4	5	6	7	8	9	10
10	3.8458	5.4329	6.8007	8.0934	9.3337	10.5454	11.7382	12.9175	14.0872	15.2491
15	3.3902	4.6227	5.6416	6.5741	7.4587	8.3130	9.1467	9.9654	10.7728	11.5716
20	3.2020	4.2693	5.1304	5.9170	6.6489	7.3493	8.0280	8.6907	9.3411	9.9820
25	3.0911	4.0720	4.8550	5.5515	6.1987	6.8138	7.4062	7.9810	8.5447	9.0974
30	3.0201	3.9461	4.6759	5.3192	5.9126	6.4733	7.0108	7.5311	8.0379	8.5341
35	2.9707	3.8589	4.5520	5.1584	5.7147	6.2378	6.7374	7.2192	7.6871	8.1441
40	2.9344	3.7949	4.4611	5.0406	5.5697	6.0653	6.5370	6.9906	7.4301	7.8581
45	2.9066	3.7460	4.3916	4.9506	5.4589	5.9330	6.3839	6.8159	7.2334	7.6394
50	2.8846	3.7074	4.3368	4.8796	5.3716	5.8296	6.2631	6.6780	7.0783	7.4669
60	2.8521	3.6503	4.2559	4.7748	5.2426	5.6760	6.0846	6.4743	6.8490	7.2117
70	2.8292	3.6101	4.1990	4.7011	5.1519	5.5681	5.9592	6.3311	6.6878	7.0321
80	2.8122	3.5803	4.1568	4.6465	5.0846	5.4880	5.8661	6.2248	6.5681	6.8989
90	2.7990	3.5573	4.1242	4.6044	5.0328	5.4263	5.7944	6.1429	6.4759	6.7962
100	2.7886	3.5390	4.0984	4.5709	4.9916	5.3773	5.7374	6.0778	6.4026	6.7143
inf	2.6970	3.3792	3.8726	4.2788	4.6323	4.9495	5.2399	5.5093	5.7618	6.0001

(power = 0.50)

N \ M	1	2	3	4	5	6	7	8	9	10
10	4.7493	6.6231	8.2452	9.7650	11.2302	12.6619	14.0714	15.4651	16.8475	18.2208
15	4.1885	5.6107	6.8056	7.8938	8.9262	9.9233	10.8963	11.8520	12.7945	13.7269
20	3.9424	5.1812	6.1868	7.0920	7.9407	8.7530	9.5400	10.3085	11.0628	11.8061
25	3.8045	4.9390	5.8438	6.6483	7.3958	8.1061	8.7903	9.4552	10.1051	10.7434
30	3.7164	4.7850	5.6260	6.3670	7.0504	7.6962	8.3152	8.9143	9.4979	10.0693
35	3.6553	4.6784	5.4750	6.1727	6.8121	7.4133	7.9874	8.5410	9.0787	9.6037
40	3.6104	4.6003	5.3654	6.0307	6.6378	7.2065	7.7476	8.2679	8.7720	9.2630
45	3.5760	4.5406	5.2813	5.9222	6.5046	7.0486	7.5645	8.0594	8.5377	9.0028
50	3.5488	4.4935	5.2150	5.8367	6.4000	6.9242	7.4203	7.8951	8.3531	8.7977
60	3.5087	4.4240	5.1172	5.7106	6.2453	6.7406	7.2075	7.6526	8.0806	8.4948
70	3.4804	4.3751	5.0485	5.6220	6.1367	6.6117	7.0580	7.4823	7.8892	8.2820
80	3.4594	4.3389	4.9975	5.5564	6.0562	6.5162	6.9472	7.3561	7.7473	8.1243
90	3.4432	4.3109	4.9582	5.5059	5.9942	6.4427	6.8619	7.2588	7.6380	8.0020
100	3.4303	4.2887	4.9271	5.4657	5.9450	6.3843	6.7942	7.1816	7.5512	7.9060
inf	3.3175	4.0948	4.6553	5.1159	5.5164	5.8755	6.2040	6.5085	6.7939	7.0632

(power = 0.60)

N \ M	1	2	3	4	5	6	7	8	9	10
10	5.7521	7.9334	9.8227	11.6035	13.3013	14.9703	16.6137	18.2389	19.8510	21.4526
15	5.0015	6.7057	8.0740	9.3277	10.5168	11.6656	12.7867	13.8878	14.9740	16.0486
20	4.7605	6.1786	7.3295	8.3655	9.3369	10.2668	11.1678	12.0477	12.9114	13.7625
25	4.5925	5.8866	6.9181	7.8354	8.6870	9.4976	10.2777	11.0358	11.7770	12.5050
30	4.4853	5.7014	6.6578	7.5003	8.2774	9.0116	9.7154	10.3907	11.0603	11.7100
35	4.4110	5.5733	6.4782	7.2694	7.9940	8.6770	9.3284	9.9565	10.5666	11.1624
40	4.3565	5.4797	6.3469	7.1007	7.7886	8.4327	9.0457	9.6351	10.2061	10.7623
45	4.3148	5.4082	6.2468	6.9721	7.6313	8.2466	8.8302	9.3901	9.9312	10.4572
50	4.2819	5.3518	6.1679	6.8708	7.5075	8.0999	8.6606	9.1971	9.7146	10.2170
60	4.2332	5.2686	6.0516	6.7215	7.3249	7.8839	8.4105	8.9127	9.3955	9.8627
70	4.1990	5.2101	5.9700	6.6167	7.1969	7.7322	8.2352	8.7131	9.1716	9.6140
80	4.1736	5.1668	5.9094	6.5392	7.1021	7.6201	8.1053	8.5654	9.0057	9.4298
90	4.1540	5.1334	5.8626	6.4795	7.0291	7.5337	8.0053	8.4517	8.8780	9.2880
100	4.1384	5.1069	5.8256	6.4320	6.9712	7.4651	7.9260	8.3614	8.7767	9.1734
inf	4.0021	4.8758	5.5040	6.0198	6.4677	6.8692	7.2361	7.5763	7.8948	8.1953

TABLES OF THE NON-CENTRALITY PARAMETER OF F-TEST

NON-CENTRALITY PARAMETER OF F TEST AS A FUNCTION OF LEVEL OF SIGNIFICANCE, POWER AND THE TWO DEGREES OF FREEDOM

level of significance = 0.01

N \ M	1	2	3	4	5	6	7	8	9	10
					(power = 0.70)					
10	6.0301	9.4700	11.0005	13.7205	15.7138	17.0566	10.5698	21.4022	23.3395	25.2047
15	6.0880	7.9707	9.5402	10.9892	12.3562	13.6770	14.9663	16.2329	17.4823	18.7185
20	5.7222	7.3400	8.6543	9.8371	10.9406	12.0087	13.0381	14.0431	15.0304	10.0030
25	5.5181	6.9397	8.1625	9.2055	10.1748	11.0060	11.9835	12.8401	13.6894	14.5177
30	5.3983	6.7676	7.8520	8.8074	9.6886	10.5214	11.3190	12.0924	12.8452	13.5822
35	5.2984	6.6144	7.6383	8.5336	9.3546	10.1266	10.8637	11.5746	12.2652	12.9303
40	5.2320	6.5024	7.4823	8.3339	9.1110	9.8387	10.5312	11.1971	11.8423	12.4707
45	5.1822	6.4170	7.3634	8.1818	8.9255	9.6107	10.2781	10.9098	11.5203	12.1139
50	5.1424	6.3497	7.2690	8.0621	8.7797	9.4473	10.0701	10.6637	11.2670	11.8331
60	5.0838	6.2505	7.1320	7.8859	8.5648	9.1936	9.7861	10.3510	10.8941	11.4196
70	5.0425	6.1808	7.0353	7.7624	8.4143	9.0158	9.5809	10.1178	10.6328	11.1208
80	5.0119	6.1293	6.9637	7.6710	8.3030	8.8844	9.4289	9.9454	10.4394	10.9153
90	4.9883	6.0896	6.9086	7.6007	8.2173	8.7832	9.3121	9.8126	10.2907	10.7503
100	4.9695	6.0580	6.8649	7.5448	8.1493	8.7030	9.2194	9.7074	10.1727	10.6194
inf	4.8057	5.7834	6.4851	7.0601	7.5597	8.0060	8.4155	8.7940	9.1483	9.4826
					(power = 0.80)					
10	8.4093	11.4477	14.0327	16.4589	18.8005	21.0904	23.3450	25.5772	27.7911	29.9910
15	7.4144	9.6040	11.4295	13.1023	14.6910	16.2205	17.7258	19.1989	20.6522	22.0904
20	6.9813	8.8255	10.3393	11.7034	12.9832	14.2088	15.3967	16.5573	17.6967	18.8107
25	6.7102	8.3980	9.7435	10.9406	12.0533	13.1111	14.1305	15.1213	16.0902	17.0420
30	6.5508	8.1261	9.3684	10.4613	11.4690	12.4226	13.3363	14.2210	15.0829	15.9260
35	6.4406	7.9423	9.1108	10.1325	11.0695	11.9508	12.7924	13.6041	14.3927	15.1628
40	6.3600	7.8068	8.9229	9.8930	10.7783	11.6074	12.3965	13.1553	13.8906	14.6069
45	6.2984	7.7035	8.7800	9.7109	10.5569	11.3463	12.0956	12.8142	13.5088	14.1842
50	6.2498	7.6222	8.6676	9.5677	10.3829	11.1414	11.8592	12.5462	13.2089	13.8522
60	6.1781	7.5024	8.5022	9.3571	10.1270	10.8399	11.5117	12.1522	12.7680	13.3639
70	6.1278	7.4184	8.3864	9.2097	9.9470	10.6289	11.2686	11.8765	12.4594	13.0221
80	6.0905	7.3563	8.3006	9.1007	9.8155	10.4730	11.0888	11.6728	12.2314	12.7695
90	6.0617	7.3084	8.2347	9.0169	9.7137	10.3532	10.9507	11.5161	12.0561	12.5753
100	6.0388	7.2704	8.1823	8.9504	9.6330	10.2581	10.8412	11.3919	11.9170	12.4214
inf	5.8395	6.9403	7.7288	8.3746	8.9347	9.4359	9.8937	10.3176	10.7143	11.0883
					(power = 0.90)					
10	10.8025	14.5160	17.6931	20.6782	23.5612	26.3817	29.1600	31.9109	34.6398	37.3519
15	9.4730	12.1123	14.3166	16.3308	18.2576	20.1152	21.9207	23.7130	25.4728	27.2147
20	8.8834	11.1086	12.9181	14.5498	16.0818	17.5496	18.9730	20.3630	21.7298	23.0764
25	8.5578	10.5608	12.1590	13.5820	14.9055	16.1643	17.3770	18.5574	19.7114	20.8451
30	8.3519	10.2163	11.6832	12.9766	14.1705	15.2994	16.3820	17.4305	18.4522	19.4529
35	8.2098	9.9798	11.3574	12.5626	13.6683	14.7066	15.7024	16.6612	17.5927	18.5026
40	8.1061	9.8075	11.1203	12.2617	13.3037	14.2799	15.2091	16.1029	16.9690	17.8130
45	8.0269	9.6764	10.9402	12.0333	13.0260	13.9540	14.8348	15.6764	16.4950	17.2890
50	7.9645	9.5734	10.7988	11.8540	12.8098	13.6993	14.5413	15.3472	16.1248	16.8796
60	7.8725	9.4217	10.5908	11.5906	12.4909	13.3247	14.1100	14.8598	15.5803	16.2774
70	7.8080	9.3155	10.4455	11.4065	12.2681	13.0630	13.8097	14.5194	15.1990	15.8508
80	7.7602	9.2370	10.3380	11.2705	12.1037	12.8700	13.5870	14.2682	14.9193	15.5405
90	7.7233	9.1766	10.2554	11.1661	11.9773	12.7216	13.4171	14.0753	14.7038	15.3081
100	7.6941	9.1287	10.1899	11.0833	11.8772	12.6041	13.2821	13.9225	14.5331	15.1193
inf	7.4307	8.7133	9.6237	10.3685	11.0138	11.5900	12.1177	12.6053	13.0613	13.4913

NON-CENTRALITY PARAMETER OF F TEST AS A FUNCTION OF LEVEL OF SIGNIFICANCE, POWER AND THE TWO DEGREES OF FREEDOM

level of significance = 0.05

N \ M	1	2	3	4	5	6	7	8	9	10
					(power = 0.10)					
10	0.2585	0.4105	0.5580	0.6919	0.8213	0.9480	1.0728	1.1062	1.3186	1.4403
15	0.2422	0.3792	0.4981	0.6081	0.7130	0.8144	0.9135	1.0109	1.1069	1.2019
20	0.2344	0.3615	0.4695	0.5681	0.6610	0.7500	0.8364	0.9208	1.0037	1.0853
25	0.2299	0.3512	0.4528	0.5446	0.6304	0.7121	0.7909	0.8676	0.9125	1.0160
30	0.2269	0.3444	0.4419	0.5292	0.6103	0.6872	0.7609	0.8324	0.9020	0.9701
35	0.2248	0.3396	0.4342	0.5183	0.5961	0.6694	0.7396	0.8074	0.8732	0.9374
40	0.2233	0.3361	0.4284	0.5102	0.5855	0.6562	0.7237	0.7887	0.8516	0.9130
45	0.2221	0.3333	0.4240	0.5039	0.5773	0.6460	0.7114	0.7742	0.8349	0.8940
50	0.2211	0.3312	0.4204	0.4990	0.5707	0.6379	0.7016	0.7626	0.8215	0.8788
60	0.2197	0.3279	0.4152	0.4915	0.5610	0.6257	0.6869	0.7453	0.8016	0.8560
70	0.2187	0.3256	0.4114	0.4862	0.5541	0.6170	0.6764	0.7330	0.7873	0.8398
80	0.2180	0.3239	0.4086	0.4823	0.5489	0.6106	0.6686	0.7238	0.7766	0.8276
90	0.2174	0.3226	0.4065	0.4792	0.5449	0.6055	0.6625	0.7166	0.7683	0.8181
100	0.2169	0.3215	0.4047	0.4768	0.5416	0.6015	0.6577	0.7109	0.7617	0.8106
inf	0.2128	0.3121	0.3894	0.4550	0.5130	0.5657	0.6142	0.6595	0.7022	0.7425
					(power = 0.20)					
10	0.7507	1.1615	1.5141	1.8440	2.1612	2.4707	2.7749	3.0754	3.3731	3.6687
15	0.7077	1.0542	1.3457	1.6121	1.8643	2.1074	2.3442	2.5764	2.8052	3.0314
20	0.6847	1.0040	1.2666	1.5029	1.7241	1.9353	2.1397	2.3389	2.5341	2.7264
25	0.6713	0.9748	1.2206	1.4395	1.6425	1.8350	2.0201	2.1998	2.3751	2.5472
30	0.6626	0.9558	1.1907	1.3980	1.5891	1.7693	1.9417	2.1084	2.2706	2.4291
35	0.6565	0.9424	1.1696	1.3688	1.5514	1.7229	1.8864	2.0438	2.1965	2.3455
40	0.6519	0.9325	1.1539	1.3471	1.5234	1.6884	1.8451	1.9957	2.1414	2.2831
45	0.6484	0.9248	1.1418	1.3304	1.5018	1.6617	1.8133	1.9585	2.0987	2.2348
50	0.6456	0.9187	1.1322	1.3170	1.4846	1.6405	1.7879	1.9288	2.0646	2.1963
60	0.6414	0.9096	1.1179	1.2972	1.4590	1.6088	1.7500	1.8845	2.0137	2.1386
70	0.6385	0.9032	1.1078	1.2831	1.4408	1.5864	1.7231	1.8530	1.9775	2.0976
80	0.6363	0.8984	1.1002	1.2726	1.4272	1.5696	1.7030	1.8295	1.9504	2.0669
90	0.6346	0.8947	1.0943	1.2645	1.4167	1.5566	1.6874	1.8112	1.9294	2.0431
100	0.6332	0.8917	1.0897	1.2580	1.4083	1.5462	1.6749	1.7966	1.9126	2.0240
inf	0.6211	0.8655	1.0482	1.2004	1.3336	1.4537	1.5639	1.6664	1.7626	1.8536
					(power = 0.30)					
10	1.2562	1.8698	2.3986	2.8913	3.3644	3.8255	4.2784	4.7256	5.1686	5.6083
15	1.1735	1.6935	2.1251	2.5180	2.8891	3.2464	3.5941	3.9350	4.2706	4.6023
20	1.1340	1.6114	1.9980	2.3442	2.6673	2.9755	3.2733	3.5635	3.8478	4.1276
25	1.1125	1.5641	1.9245	2.2437	2.5389	2.8185	3.0871	3.3473	3.6016	3.8507
30	1.0980	1.5332	1.8767	2.1782	2.4553	2.7161	2.9655	3.2063	3.4404	3.6693
35	1.0877	1.5116	1.8432	2.1322	2.3964	2.6440	2.8798	3.1067	3.3267	3.5411
40	1.0801	1.4956	1.8183	2.0981	2.3527	2.5905	2.8161	3.0327	3.2420	3.4457
45	1.0743	1.4832	1.7991	2.0718	2.3190	2.5492	2.7670	2.9755	3.1766	3.3719
50	1.0696	1.4734	1.7839	2.0509	2.2923	2.5163	2.7279	2.9300	3.1246	3.3131
60	1.0627	1.4587	1.7612	2.0199	2.2524	2.4674	2.6696	2.8621	3.0469	3.2254
70	1.0578	1.4484	1.7451	1.9978	2.2242	2.4327	2.6283	2.8139	2.9917	3.1631
80	1.0541	1.4407	1.7332	1.9814	2.2031	2.4068	2.5974	2.7780	2.9505	3.1164
90	1.0513	1.4347	1.7239	1.9687	2.1868	2.3868	2.5735	2.7501	2.9185	3.0803
100	1.0490	1.4299	1.7165	1.9585	2.1738	2.3708	2.5544	2.7278	2.8930	3.0514
inf	1.0290	1.3878	1.6510	1.8686	2.0583	2.2288	2.3848	2.5297	2.6655	2.7938

NON-CENTRALITY PARAMETER OF F TEST AS A FUNCTION OF LEVEL OF SIGNIFICANCE, POWER AND THE TWO DEGREES OF FREEDOM

level of significance = 0.05

N \ M	1	2	3	4	5	0	7	8	9	10
					(power = 0.40)					
10	1.7804	2.5007	3.2852	3.9312	4.5512	5.1551	5.7483	6.3339	6.9139	7.4897
15	1.0015	2.3418	2.0030	3.4120	3.8934	4.3561	4.8003	5.2475	5.6819	6.1111
20	1.0062	2.2209	2.7267	3.1731	3.5894	3.9861	4.3693	4.7420	5.1082	5.4679
25	1.5743	2.1608	2.0253	3.0354	3.4143	3.7728	4.1170	4.4506	4.7761	5.0951
30	1.5536	2.1179	2.5595	2.0459	3.3005	3.0340	3.9527	4.2004	4.5504	4.8510
35	1.5300	2.0877	2.5133	2.8831	3.2205	3.5365	3.8373	4.1265	4.4060	4.6801
40	1.5282	2.0655	2.4791	2.8367	3.1614	3.4643	3.7516	4.0272	4.2930	4.5526
45	1.5199	2.0483	2.4528	2.8009	3.1158	3.4086	3.6856	3.9506	4.2062	4.4542
50	1.5133	2.0347	2.4319	2.7724	3.0796	3.3644	3.6331	3.8897	4.1367	3.3759
60	1.5035	2.0144	2.4009	2.7302	3.0257	3.2986	3.5550	3.7990	4.0331	4.2592
70	1.4965	2.0000	2.3789	2.7003	2.9876	3.2520	3.4997	3.7347	3.9596	4.1763
80	1.4913	1.9894	2.3625	2.6780	2.9592	3.2172	3.4584	3.6867	3.9047	4.1144
90	1.4873	1.9811	2.3498	2.6607	2.9372	3.1903	3.4264	3.6495	3.8622	4.0664
100	1.4841	1.9745	2.3397	2.6470	2.9196	3.1688	3.4009	3.6198	3.8283	4.0282
inf	1.4557	1.9162	2.2504	2.5253	2.7643	2.9787	3.1747	3.3564	3.5267	3.6874
					(power = 0.50)					
10	2.3529	3.3616	4.2235	5.0249	5.7937	6.5426	7.2781	8.0043	8.7236	9.4376
15	2.1636	3.0332	3.7231	4.3489	4.9391	5.5070	6.0594	6.6007	7.1337	7.6603
20	2.1199	2.8826	3.4939	4.0393	4.5474	5.0316	5.4992	5.9545	6.4000	6.8394
25	2.0775	2.7962	3.3626	3.8619	4.3228	4.7588	5.1773	5.5828	5.9783	6.3660
30	2.0500	2.7402	3.2775	3.7469	4.1772	4.5818	4.9683	5.3413	5.7038	6.0579
35	2.0307	2.7010	3.2180	3.6664	4.0752	4.4578	4.8217	5.1717	5.5109	5.8414
40	2.0164	2.6720	3.1739	3.6069	3.9997	4.3650	4.7132	5.0461	5.3679	5.6807
45	2.0053	2.6497	3.1400	3.5611	3.9416	4.2953	4.6296	4.9493	5.2577	5.5568
50	1.9966	2.6319	3.1132	3.5248	3.8955	4.2392	4.5632	4.8725	5.1701	5.4584
60	1.9836	2.6056	3.0732	3.4708	3.8271	4.1558	4.4645	4.7581	5.0398	5.3118
70	1.9744	2.5870	3.0450	3.4326	3.7786	4.0967	4.3946	4.6771	4.9474	5.2078
80	1.9675	2.5731	3.0239	3.4041	3.7426	4.0527	4.3425	4.6167	4.8785	5.1303
90	1.9622	2.5624	3.0077	3.3821	3.7146	4.0187	4.3022	4.5700	4.8252	5.0702
100	1.9580	2.5539	2.9947	3.3646	3.6923	3.9916	4.2701	4.5327	4.7826	5.0222
inf	1.9205	2.4784	2.8802	3.2097	3.4956	3.7517	3.9856	4.2023	4.4052	4.5966
					(power = 0.60)					
10	3.0060	4.2268	5.2686	6.2370	7.1602	8.0713	8.9604	9.8382	10.7077	11.5709
15	2.7996	3.8072	4.6332	5.3820	6.0883	6.7676	7.4280	8.0763	8.7141	9.3441
20	2.7047	3.6159	4.3442	4.9034	5.5981	6.1742	6.7306	7.2724	7.8031	8.3252
25	2.6502	3.5065	4.1792	4.7716	5.3182	5.8352	6.3313	6.8121	7.2810	7.7406
30	2.6140	3.4358	4.0725	4.6262	5.1373	5.6159	6.0729	6.5140	6.9420	7.3614
35	2.5901	3.3863	3.9980	4.5279	5.0107	5.4624	5.8920	6.3051	6.7053	7.0953
40	2.5718	3.3497	3.9420	4.4539	4.9172	5.3490	5.7583	6.1506	6.5298	6.8984
45	2.5577	3.3216	3.9000	4.3970	4.8453	5.2618	5.6553	6.0317	6.3946	6.7467
50	2.5465	3.2993	3.8670	4.3519	4.7883	5.1926	5.5737	5.9374	6.2873	6.6262
60	2.5298	3.2662	3.8172	4.2849	4.7037	5.0898	5.4524	5.7972	6.1278	6.4470
70	2.5181	3.2427	3.7820	4.2375	4.6439	5.0172	5.3667	5.6979	6.0149	6.3201
80	2.5093	3.2253	3.7558	4.2023	4.5993	4.9631	5.3028	5.6240	5.9307	6.2255
90	2.5025	3.2118	3.7355	4.1750	4.5649	4.9213	5.2533	5.5668	5.8655	6.1523
100	2.4971	3.2011	3.7194	4.1533	4.5374	4.8879	5.2139	5.5212	5.8136	6.0939
inf	2.4493	3.1064	3.5770	3.9619	4.2954	4.5937	4.8660	5.1181	5.3540	5.5764

NON-CENTRALITY PARAMETER OF *F* TEST AS A FUNCTION OF LEVEL OF SIGNIFICANCE, POWER AND THE TWO DEGREES OF FREEDOM

level of significance = 0.05

N \ M	1	2	3	4	5	6	7	8	9	10
					(power = 0.70)					
10	3.7945	5.2577	6.5057	7.6662	8.7798	9.8640	10.9300	11.9836	13.0264	14.0618
15	3.5302	4.7260	5.7065	6.5947	7.4324	8.2383	9.0224	9.7910	10.5478	11.2954
20	3.4094	4.4804	5.3450	6.1114	6.8246	7.5041	8.1603	8.7994	9.4255	10.0414
25	3.3401	4.3493	5.1404	5.8366	6.4790	7.0865	7.6696	8.2345	8.7856	9.3257
30	3.2953	4.2609	5.0080	5.6595	6.2563	6.8172	7.3529	7.8698	8.3721	8.8629
35	3.2639	4.1991	4.9156	5.5359	6.1007	6.6291	7.1310	7.6148	8.0820	8.5390
40	3.2407	4.1534	4.8474	5.4447	5.9859	6.4903	6.9682	7.4264	7.8692	8.2996
45	3.2229	4.1184	4.7951	5.3746	5.8978	6.3836	6.8427	7.2817	7.7049	8.1154
50	3.2087	4.0906	4.7536	5.3192	5.8280	6.2991	6.7433	7.1669	7.5746	7.9694
60	3.1877	4.0494	4.6921	5.2369	5.7244	6.1738	6.5956	6.9966	7.3811	7.7524
70	3.1728	4.0202	4.6486	5.1787	5.6512	6.0852	6.4913	6.8761	7.2443	7.5988
80	3.1617	3.9986	4.6163	5.1355	5.5968	6.0193	6.4136	6.7865	7.1424	7.4844
90	3.1531	3.9818	4.5913	5.1021	5.5547	5.9683	6.3536	6.7172	7.0636	7.3960
100	3.1463	3.9685	4.5714	5.0755	5.5212	5.9278	6.3058	6.6619	7.0008	7.3255
inf	3.0860	3.8500	4.3902	4.8412	5.2263	5.5703	5.8842	6.1747	6.4462	6.7022
					(power = 0.80)					
10	4.8360	6.6032	8.1111	9.5141	10.8611	12.1740	13.4643	14.7386	16.0012	17.2549
15	4.4936	5.9236	7.0938	8.1551	9.1504	10.1109	11.0577	11.9770	12.8823	13.7768
20	4.3379	5.6179	6.6381	7.5473	8.3942	9.2014	9.9809	10.7403	11.4843	12.2163
25	4.2490	5.4413	6.3800	7.2033	7.9631	8.6816	9.3712	10.0396	10.6916	11.3307
30	4.1916	5.3326	6.2140	6.9823	7.6859	8.3474	8.9791	9.5887	10.1812	10.7601
35	4.1514	5.2546	6.0983	6.8282	7.4928	8.1145	8.7057	9.2743	9.8251	10.3616
40	4.1217	5.1971	6.0130	6.7148	7.3500	7.9429	8.5043	9.0425	9.5625	10.0681
45	4.0989	5.1530	5.9476	6.6277	7.2414	7.8113	8.3497	8.8645	9.3609	9.8424
50	4.0808	5.1180	5.8958	6.5588	7.1551	7.7071	8.2273	8.7236	9.2012	9.6636
60	4.0539	5.0662	5.8191	6.4567	7.0271	7.5526	8.0459	8.5147	8.9642	9.3983
70	4.0349	5.0296	5.7650	6.3847	6.9368	7.4436	7.9178	8.3672	8.7969	9.2108
80	4.0208	5.0224	5.7247	6.3312	6.8696	7.3626	7.8226	8.2574	8.6724	9.0713
90	4.0099	4.9814	5.6936	6.2898	6.8178	7.3000	7.7490	8.1726	8.5762	8.9635
100	4.0012	4.9646	5.6689	6.2569	6.7765	7.2501	7.6904	8.1052	8.4996	8.8776
inf	3.9244	4.8173	5.4513	5.9676	6.4138	6.8121	7.1752	7.5111	7.8249	8.1205
					(power = 0.90)					
10	6.4935	8.7194	10.6217	12.3937	14.0965	15.7570	17.3896	19.0026	20.6011	22.1887
15	6.0235	7.7088	9.2525	10.5721	11.8170	13.0176	14.1855	15.3308	16.4590	16.5741
20	5.8114	7.3885	8.6454	9.7662	10.8109	11.8069	12.7692	13.7068	14.6257	15.5300
25	5.6908	7.1567	8.3035	9.3129	10.2447	11.1263	11.9727	12.7932	13.5937	14.3787
30	5.6131	7.0070	8.0843	9.0226	9.8822	10.6905	11.4626	12.2070	12.9324	13.6404
35	5.5588	6.9043	7.9319	8.8209	9.6304	10.3877	11.1082	11.8010	12.4724	13.1267
40	5.5188	6.8281	7.8199	8.6726	9.4453	10.1652	10.8476	11.5018	12.1341	12.7488
45	5.4880	6.7696	7.7340	8.5590	9.3035	9.9947	10.6479	11.2725	11.8748	12.4591
50	5.4636	6.7233	7.6661	8.4692	9.1914	9.8590	10.4901	11.0612	11.6697	12.2200
60	5.4275	6.6548	7.5656	8.3363	9.0255	9.6605	10.2564	10.8228	11.3660	11.8904
70	5.4020	6.6065	7.4947	8.2427	8.9080	9.5109	10.0918	10.6337	11.1510	11.6509
80	5.3830	6.5706	7.4421	8.1731	8.8210	9.4156	9.9605	10.4931	10.9928	11.4730
90	5.3683	6.5428	7.4015	8.1194	8.7540	9.3350	9.8751	10.3847	10.8700	11.3356
100	5.3566	6.5208	7.3692	8.0767	8.7016	9.2710	9.8001	10.2984	10.7723	11.2263
inf	5.2537	6.3270	7.0857	7.7025	8.2347	8.7064	9.1418	9.5413	9.9146	10.2660