

*Synopsis of the thesis entitled
Local vs Global Incentive Compatibility in
Mechanism Design*

A DISSERTATION PRESENTED
BY
UJJWAL KUMAR
TO
THE ECONOMIC RESEARCH UNIT

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN THE SUBJECT OF
QUANTITATIVE ECONOMICS

INDIAN STATISTICAL INSTITUTE
KOLKATA, WEST BENGAL, INDIA

JUNE 2022

We study models in mechanism design theory where agents have private information (called a type) which has to be elicited by the mechanism designer. The cornerstone of the theory is the collection of incentive compatibility constraints which ensure that agents do not have incentives to misreport their types (or manipulate). The standard assumption in the theory is that the proposed mechanism must be immune to *all* possible misreports of agents. There is, however considerable experimental evidence that agents often choose to lie credibly by only misreporting to types that are “near” or “close to” their true types. Therefore, it is natural to consider models where an agent of a particular type can only misreport to an arbitrary set of pre-specified “local” types. We consider such models and provide answers to the following question: under what circumstances is immunity to misreporting via a “local” type (local incentive compatibility) equivalent to immunity to misreporting via an arbitrary type (incentive compatibility)? This thesis consists of five chapters. We provide a brief introduction of the chapters below.

Chapter 2 considers a voting model where each voter’s type is her strict preference. The type graph for a voter is a graph whose vertices are the possible types of the voter. Two vertices are connected by an edge in the graph if the associated types are “local”. Local-Global equivalence (LGE) is satisfied if local strategy-proofness implies strategy-proofness for deterministic social choice functions. This chapter identifies a condition on the graph that characterizes LGE. Our notion of “localness” is perfectly general - we use this feature of our model to identify notions of localness according to which various models of multi-dimensional voting satisfy LGE. Finally, we show that LGE for deterministic social choice functions does not imply LGE for random social choice functions.

Chapter 3 considers the same voting framework as in Chapter 2 except that each agent’s type is her weak preference, that is, preferences that can admit indifference. We provide a condition that is sufficient for LGE and another condition that is necessary. Moreover, the “gap” between the two conditions is small (in the sense that both conditions boil down to the single condition identified in Chapter 2 that characterizes LGE for the case of strict preferences). We use the sufficiency result to propose notions of localness according to which environments with the domain of single-plateaued preferences and the domain of all weak preferences, satisfies LGE.

Chapter 4 identifies a condition on preference domains that ensures that every locally strategy- proof and *unanimous* random social choice function is also strategy-proof. Furthermore every unanimous, locally strategy-proof deterministic social choice function is also group strategy-proof. The condition identified is significantly weaker than the characterization condition for local-global equivalence without unanimity in Chapter 2.

Chapter 5 considers standard mechanism design problem where a set of agents have valuations for each alternative in a finite set of alternatives. Based on these valuations, the planner has to select an alternative to be shared by all the agents and some payment for each agent. Such a decision scheme is called a

mechanism. Agents evaluate their net utilities by means of quasilinear utility functions. A mechanism is incentive compatible (IC) if no agent can increase his/her net utility by misreporting his/her type. We explore the equivalence of pointwise local incentive compatibility (PLIC) (as defined in Carroll [12]) and incentive compatibility (IC) in non-convex type-spaces. We provide a sufficient condition on a type-space called minimal richness for the said equivalence. Using this result, we show that PLIC and IC are equivalent on large class of non-convex type-spaces such as type-spaces perturbed by modularity and concave-modularity. The gross substitutes type-space and the generalized gross substitutes and complements type-space are important examples of type-spaces perturbed by modularity and concave-modularity, respectively. Finally, we provide a geometric property consisting of three conditions for the equivalence of PLIC and IC, and show that all the conditions are indispensable.

Chapter 6 studies standard mechanism design problems when agents have quasi linear utility function. We consider ordinal type-spaces, that is, possible valuations of each agent come from an underlying preference structure. This chapter explores the relation between different notions of local incentive compatibility (LIC) and incentive compatibility (IC) on ordinal type-spaces. In this context, we introduce the notion of ordinal local global equivalent (OLGE) and cardinal local global equivalent (CLGE) environments. First, we establish the equivalence between the two environments on strict ordinal type-spaces. Next, we consider ordinal type-spaces admitting indifference. We introduce the notion of almost everywhere IC and strong LIC, and provide a necessary and sufficient condition on ordinal type spaces for their equivalence. Finally, we provide results on how to (minimally) check the IC property of a given mechanism on any ordinal type-space and show that local types along with the boundary types form a minimal set of incentive constraints that imply full incentive compatibility.

Keywords: Local incentive constraints, strategy-proofness, mechanism design, strategic voting, unanimity, pointwise local incentive compatibility, (global) incentive compatibility, non-convex type-spaces, minimally rich type-spaces, gross substitutes type-space, generalized gross substitutes and complements type-space, adjusted local incentive compatibility, uniform local incentive compatibility, ordinal type-spaces

References

- [1] Aaron Archer and Robert Kleinberg. Truthful germs are contagious: a local-to-global characterization of truthfulness. *Games and Economic Behavior*, 86:340–366, 2014.
- [2] Mark Armstrong. Optimal multi-object auctions. *The Review of Economic Studies*, 67(3):455–481, 2000.
- [3] Lawrence M Ausubel and Paul R Milgrom. Ascending auctions with package bidding. *Advances in Theoretical Economics*, 1(1), 2002.
- [4] Salvador Barberà. Indifferences and domain restrictions. *Analyse & Kritik*, 29(2):146–162, 2007.
- [5] Salvador Barberà, Hugo Sonnenschein, and Lin Zhou. Voting by committees. *Econometrica*, 59(3): 595–609, 1991.
- [6] Salvador Barberà, Faruk Gul, and Ennio Stacchetti. Generalized median voter schemes and committees. *Journal of Economic Theory*, 61(2):262–289, 1993.
- [7] Salvador Barberà, Dolors Berga, and Bernardo Moreno. Individual versus group strategy-proofness: when do they coincide? *Journal of Economic Theory*, 145(5):1648–1674, 2010.
- [8] Yoram Barzel. Private schools and public school finance. *Journal of Political Economy*, 81(1): 174–186, 1973.
- [9] Peter Barse, Gerhard Glomm, and Eckhard Janeba. Composition of government budget, non-single peakedness, and majority voting. *Journal of Public Economic Theory*, 3(4):471–481, 2001.
- [10] Dolors Berga. Strategy-proofness and single-plateaued preferences. *Mathematical Social Sciences*, 35(2):105–120, 1998.
- [11] Juan Carlos Carbajal and Rudolf Müller. Implementability under monotonic transformations in differences. *Journal of Economic Theory*, 160:114–131, 2015.
- [12] Gabriel Carroll. When are local incentive constraints sufficient? *Econometrica*, 80(2):661–686, 2012.
- [13] Siddharth Chatterjee. Computational complexity of local-global equivalence. Available at <https://drive.google.com/file/d/1XdHYBqxzc3CEXt2ahJP1giMSUoKQFOPp/view>, 2020.

- [14] Shurojit Chatterji and Arunava Sen. Tops-only domains. *Economic Theory*, 46(2):255–282, 2011.
- [15] Shurojit Chatterji and Huaxia Zeng. On random social choice functions with the tops-only property. *Games and Economic Behavior*, 109:413–435, 2018.
- [16] Shurojit Chatterji and Huaxia Zeng. Random mechanism design on multidimensional domains. *Journal of Economic Theory*, 182:25–105, 2019.
- [17] Shurojit Chatterji, Souvik Roy, Sadhukhan Soumyarup, Arunava Sen, and Huaxia Zeng. Probabilistic fixed ballot rules and hybrid domains. *Journal of Mathematical Economics*, 100:102656, 2022.
- [18] Wonki Jo Cho. Incentive properties for ordinal mechanisms. *Games and Economic Behavior*, 95:168–177, 2016.
- [19] T.H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms (2nd. Edition)*. MIT Press, Boston, 2001.
- [20] Gabrielle Demange. Single-peaked orders on a tree. *Mathematical Social Sciences*, 3(4):389–396, 1982.
- [21] Dennis Epple and Richard E Romano. Public provision of private goods. *Journal of political Economy*, 104(1):57–84, 1996.
- [22] Urs Fischbacher and Franziska Föllmi-Heusi. Lies in disguise - an experimental study on cheating. *Journal of the European Economic Association*, pages 525–547, 2013.
- [23] Drew Fudenberg and Jean Tirole. *Game theory*. MIT Press, Cambridge, 1991.
- [24] Satoru Fujishige and Zaifu Yang. A note on kelso and crawford’s gross substitutes condition. *Mathematics of Operations Research*, 28(3):463–469, 2003.
- [25] Allan Gibbard. Manipulation of voting schemes: a general result. *Econometrica*, pages 587–601, 1973.
- [26] Allan Gibbard. Manipulation of schemes that mix voting with chance. *Econometrica*, 45:665–681, 1977.
- [27] Faruk Gul and Ennio Stacchetti. Walrasian equilibrium with gross substitutes. *Journal of Economic theory*, 87(1):95–124, 1999.
- [28] Miho Hong and Semin Kim. Unanimity and local incentive compatability in sparsely connected domains.
https://www.dropbox.com/s/2zsqs1sobefsd9mi/ULIC_Full_200925.pdf?dl=0, 2020.
 unpublished manuscript.
- [29] Norman J Ireland. The mix of social and private provision of goods and services. *Journal of Public Economics*, 43(2):201–219, 1990.

- [30] Alexander S Kelso and Vincent P Crawford. Job matching, coalition formation, and gross substitutes. *Econometrica: Journal of the Econometric Society*, pages 1483–1504, 1982.
- [31] Ujjwal Kumar and Souvik Roy. Local incentive compatibility in non-convex type-spaces. https://mpra.ub.uni-muenchen.de/110872/1/MPRA_paper_110872.pdf, 2021. unpublished manuscript.
- [32] Ujjwal Kumar, Souvik Roy, Arunava Sen, Sonal Yadav, and Huaxia Zeng. Local global equivalence in voting models: A characterization and applications. *Theoretical Economics*, 16(4):1195–1220, 2021.
- [33] Ujjwal Kumar, Souvik Roy, Arunava Sen, Sonal Yadav, and Huaxia Zeng. Local global equivalence for unanimous social choice functions. *Games and Economic Behavior*, 130:299–308, 2021.
- [34] Alexey I Kushnir and Lev V Lokutsievskiy. When is a monotone function cyclically monotone? *Theoretical Economics*, 16(3):853–879, 2021.
- [35] Ron Lavi, Ahuva Mualem, and Noam Nisan. Two simplified proofs for Roberts’ theorem. *Social Choice and Welfare*, 32(3):407–423, 2009.
- [36] Michael Le Breton and Arunava Sen. Separable preferences, strategyproofness and decomposability. *Econometrica*, 67:605–628, 1999.
- [37] Michel Le Breton and Vera Zaporozhets. On the equivalence of coalitional and individual strategy-proofness properties. *Social Choice and Welfare*, 33(2):287–309, 2009.
- [38] Debasis Mishra, Anup Pramanik, and Souvik Roy. Multidimensional mechanism design in single peaked type spaces. *Journal of Economic Theory*, 153:103–116, 2014.
- [39] Debasis Mishra, Anup Pramanik, and Souvik Roy. Local incentive compatibility with transfers. *Games and Economic Behavior*, 100:149–165, 2016.
- [40] Shuhei Morimoto. Group strategy-proof probabilistic voting with single-peaked preferences. https://www.biz.tmu.ac.jp/wp-content/uploads/sites/9/2020/05/RP-22_Morimoto.pdf, 2020. unpublished manuscript.
- [41] Hervé Moulin. On strategy-proofness and single peakedness. *Public Choice*, 35(4):437–455, 1980.
- [42] Kazuo Murota. Discrete convex analysis: A tool for economics and game theory. *Journal of Mechanism and Institution Design*, 1(1):151–273, 2016.
- [43] Renato Paes Leme. Gross substitutability: An algorithmic survey. *Games and Economic Behavior*, 106:294–316, 2017.
- [44] Hans Reijniere, Anita van Gellekom, and Jos AM Potters. Verifying gross substitutability. *Economic Theory*, 20(4):767–776, 2002.
- [45] Shin Sato. Strategy-proofness and the reluctance to make large lies: the case of weak orders. *Social Choice and Welfare*, 40(2):479–494, 2013.

- [46] Shin Sato. A sufficient condition for the equivalence of strategy-proofness and nonmanipulability by preferences adjacent to the sincere one. *Journal of Economic Theory*, 148(1):259–278, 2013.
- [47] Mark Allen Satterthwaite. Strategy-proofness and arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217, 1975.
- [48] Akiyoshi Shioura and Zaifu Yang. Equilibrium, auction, and generalized gross substitutes and complements. *Journal of the Operations Research Society of Japan*, 58(4):410–435, 2015.
- [49] JE Stiglitz. The demand for education in public and private school systems. *Journal of Public Economics*, 3(4):349–385, 1974.
- [50] Rakesh V Vohra. *Mechanism design: a linear programming approach*, volume 47. Cambridge University Press, 2011.