A PROGRAMMING MODEL FOR IMPORT SUBSTITUTION IN INDIA*

By THOMAS E. WEISSKOPF

Indian Statistical Institute

1. Introduction

Within the past decade a considerable variety of interindustry programming models have appeared in the literature on economic growth and development.[1] These models have been concerned with the efficient allocation of resources between sectors —and sometimes over time—in the context of the planning of economic growth. Both because of the particularly pressing problems of growth in low-income regions of the world, and because of a widespread belief that the market mechanism tends to be less effective in relatively undeveloped economies, many of the programming models in the literature have been formulated and tested with data from developing countries. India, in particular, has been favoured in this respect : a long-standing commitment to economic planning, as well as a relatively plentiful supply of statistical data have made it especially attractive for model-builders.

Many of the programming models to date have been presented in 'demonstration' or 'pilot' studies, where the attention is focussed more on the theoretical formulation of computable models than on their numerical implementation. The models applied to India have shared this emphasis on technique as opposed to empirical detail. Numerical applications have been carried out on the basis of a relatively aggregated interindustry description of the economy, so that the results emerge in terms of broad sectoral aggregates and macroeconomic magnitudes.[2] While such studies and their results are undoubtedly of interest, there are a number of respects in which a more detailed quantitative analysis can yield significant dividends.

In the first place, it is useful to set up interindustry planning models on the basis of sectors or industries in terms of which the actual plans are formulated. Models which prescribe targets for the engineering sector as a whole are of little use to a planner who is interested in the future demand for railway wagons or diesel engines. Secondly, a detailed empirical study can adjust to the differing quality of statistical information available for different sectors of the economy. Depending on the availability and/or suitability of the data, some sectors can usefully be treated in great empirical detail

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[1] See, for example, Chenery and Kretschmer (1956), Sandee (1960), Chakravarty and Lefeber (1965), Bruno (1966) and Manne (1966).

[2] Two linear programming models have been applied to the Indian economy; Sandee's (1960) demonstration planning model involved 13 sectors, and the optimizing model developed by Chakravarty, Eckaus and Lefeber (1965) distinguished 11 sectors. Apart from these programming models, several models of the consistent requirements type have been based on a 30-sector interindustry classification : see Manne and Rudra (1965); Sabherwal, Saluja and Srinivasan (1965); and Bergsman and Manne (1966).

within the interindustry framework of a model, while others are best aggregated or made explicitly subject to exogenous assumptions. Finally, one of the crucial problems in the strategy of economic growth concerns the pattern of industrialisation and foreign trade. Many programming models involve a degree of sectoral choice between domestic production, imports and/or exports. If such choice elements are to be meaningfully approached in the context of a programming model, it is essential that the relevant sectors be defined in fairly precise terms rather than as broad aggregates.

With all of these considerations in mind the present study was formulated as a detailed empirical application of an interindustry programming model to the Indian economy, designed primarily to analyse the structure of imports and the scope for import substitution in Indian industry. The programming model is based on a highly disaggregated description of the economy—involving some 150 sectors—but it pays for the resulting costs of computation by being limited to a single period of time. All of the numerical magnitudes of the model relate to the target year 1975,[3] which is compared to the base year 1965. The interindustry detail is confined to the industrial part of the economy which forms the focus of the analysis, and in which the choice mechanism of the model is explicitly operative. The remaining parts of the economy —primarily the agricultural and service sectors—are treated exogenously. They enter the model only in so far as they affect the demand for industrial sector products, and as they are needed to compute the overall trade and expenditure aggregates.

The model is used to generate alternative patterns of domestic production and imports which satisfy a set of predetermined goals of final demand in the target year 1975.[4] It differs from a straightforward consistent requirements planning model in that it allows explicitly for choice between production and importing activities, according to comparative cost criteria in a linear programming framework. In contrast to the treatment of importing activities, estimates of exports are specified exogenously on the basis of independent projections for the target year 1975.[4] This asymmetry in the approach to foreign trade does not imply that export promotion is in any way less important than import substitution, but it reflects the more complicated nature of optimal choice among exporting activities.[4]

Given the final demand targets, and a set of basic assumptions about export prospects and noncompetitive import requirements, the model is programmed specifically to solve for a pattern of production and imports in the target year 1975 which minimizes a cost function made up of a weighted sum of domestic and foreign primary

[3] All references to calendar years are understood to apply actually to the Indian fiscal year which runs from April 1 to March 31; thus 1975 denotes the fiscal year 1975-76.

[4] Unless both the internal supply of, and the external demand for, each sectoral type of export can be regarded as perfectly elastic at a given price, the treatment of exporting activities calls for an explicitly non-linear formulation. Apart from some additional costs of computation, such an approach would require a much greater base of quantitative information on export possibilities in the target year 1975 than could be obtained for use in this study. In the absence of the data which would permit a realistic nonlinear formulation, it would appear just as meaningful to study the effect of alternative export possibilities by parametric variation of exogenously given export levels than by allowing for an arbitrary range of choice among exporting activities according to linear comparative cost criteria.

factor costs, measured respectively in rupees and dollars. By varying the weights—i.e., by altering the rate of exchange between rupees and foreign currency—alternative solutions are generated which satisfy the predetermined final demand goals with (inversely) varying requirements of internal and external resources. In this way, both the detailed sectoral and the overall macro-economic implications of alternative targets, assumptions and exchange rates, are explored in the optimizing framework of a linear programming model.

2. The empirical scope of the study

The industrial part of the Indian economy, which is treated endogenously within the scope of the interindustry framework of the model, is defined to include all mining, power and manufacturing industries, as well as road and rail transport. The remaining sectors of the economy—notably agriculture and all kinds of services—are treated as exogenous for the purposes of the analysis. The sectors of the economy which are here classified as endogenous account at present for about one-fourth of the net national product and employ only about one-sixth of the total labour force. These figures reflect the characteristic dependence upon agriculture of an economy as poor as India's. However, the endogenous sectors loom much larger in the analysis of the structure of imports with which this study is primarily concerned: they account for about four-fifths of current imports. In terms of gross domestic expenditure, one-half of total investment and one-third of total consumption is directed to the products of the endogeneous sectors.

The dividing line between the endogenous and exogenous sectors was determined primarily by two considerations : the relevance and stability of linear input-output coefficients and the nature of the 'make-or-buy' choice between domestic production and imports. The endogenous sectors of the model include those for which interindustry relations are readily quantifiable, and can be assumed to be relatively constant—or predictably changing—over time. It is much less meaningful in theory—and often impossible in practice—to deal with the exogenous sectors in terms of stable input-output and capital-output coefficients. Furthermore, the production and trade of the exogenous sector products depend very much on factors not usefully analysed by interindustry techniques. Agricultural production functions are notoriously nonlinear and are likely to be strongly affected by such non-material inputs as organization and education; the import of foodgrains is largely a matter of weather conditions and government policies. Services do not enter at all into foreign trade, and hence the question of import substitution does not even arise. In contrast, most of the products of the endogenous sectors can and do enter into foreign trade, and some of the important aspects of the comparative cost of producing and importing can be illuminated with an interindustry approach.

The mining, power, manufacturing, and road and rail transport industries of the economy are broken down into 147 sectors, which are classified into nine distinct groups in Table 1. The greatest degree of disaggregation has been carried out in

TABLE 1.

code	sector	code	sector	code	sector	code	sector
100	*mineral industries*	300	*light industries*	500	*metallurgical industries*	700	*electrical engineering industries*
111	iron ore	311	sugar	510	pig iron	711	thermal turbo-generators
112	manganese ore	312	tea	521	finished steel	712	hydro turbo-generators
113	chromite	313	vegetable oils	522	special steel	713	electric motors
114	bauxite	314	hydrogenated oils	531	ferro-manganese	714	transformers
115	copper ore	315	other food, beverages and tobacco	532	ferro-silicon	715	switchgear and controlgear
116	lead concentrate	320	jute textiles	533	other ferro-alloys	720	cables, wires and flexes
117	zinc concentrate	331	cotton textiles	541	aluminium	731	refrigerators
118	ilmenite	332	woollen textiles	542	copper	732	air conditioners
119	other metallic minerals	333	art silk fabrics	543	lead	733	water coolers
120	gold	334	other textile manufactures	544	zinc	734	electric fans
131	limestone	340	leather and products	545	tin	735	electric lamps
132	dolomite	350	rubber products	546	nickel	736	dry cells
133	china clay	361	wood products	547	other base metals	737	storage batteries
134	gypsum	362	glass	551	cast iron pipes	738	house service meters
135	salt	363	refractories	552	steel pipes and tubes	739	radio receivers
136	mica	364	other nonmetallic mineral products	553	iron castings	740	communications equipment
137	other nonmetallic minerals	370	cement	554	steel castings and forgings	750	other electrical eng. products
141	rock phosphates	381	paper and paperboard	555	heavy ferrous structurals		
142	sulphur	382	newsprint	560	light metal fabrication	800	*transport equipment industries*
143	asbestos					811	steam locomotives
144	cryolite and fluorspar	400	*chemical industries*	600	*mechanical engineering industries*	812	diesel locomotives
150	minor minerals	411	nitrogeneous fertilizers	611	machine tools	813	electric locomotives
160	crude oil	412	phosphatic fertilizers	612	boilers	821	railway wagons
		413	potassic fertilizers	613	diesel engines	822	railway coaching stock
200	*fuel and power industries*	421	sulphuric acid	614	pumps	831	automobiles
210	coal	422	soda ash	615	compressors	832	commercial vehicles
220	coke	423	caustic soda	616	refrigeration equipment	833	motorcycles and scooters
230	electricity	424	other inorganic chemicals	617	material handling equipment	840	bicycles
241	light distillates	430	organic chemicals	618	conveying and hoisting machinery	850	other transport equipment
242	kerosenes	441	dyestuffs	621	construction machinery		
243	diesel oils	442	plastic and synthetic resins	622	mining machinery	900	*transport services*
244	fuel oils	443	synthetic rubber	623	drilling machinery	910	rail goods transport
245	bitumens	444	synthetic fibres	624	agricultural machinery	920	rail passenger transport
246	other petroleum products	445	chemical pulp	631	textile machinery	930	road goods transport
250	lubricating oils	446	soaps	632	jute mill machinery	940	road passenger transport
		450	paints	633	sugar machinery	951	automobile transport
		460	drugs and pharmaceuticals	634	tea processing machinery	952	motorcycle and scooter transport
		470	other chemicals and products	635	paper mill machinery	953	bicycle transport
				636	cement machinery		
				637	chemical equipment		
				640	ball bearings		
				650	instruments		
				661	sewing machines		
				662	typwriters		
				663	watches and clocks		
				670	other mechanical engineering products		

the metallurgical and engineering sectors, and among certain chemical industries; it is in these sectors that many of the most crucial problems in regard to import substitution arise. Among the older manufacturing industries, a broader classification has been adopted, reflecting their relative self-sufficiency in the Indian economy. The final breakdown of industries corresponds closely to the kind of classification adopted by most of the Indian statistical and planning agencies. This is due both to considerations of data availability, and to the desirability of working with sectors that are meaningful from the point of view of the Indian planners. In each broad group of industries a residual sector (e.g. 'Miscellaneous chemicals and products', 'Other transport equipment') was formed to complete the coverage after individual industries, for which data were adequate, were distinguished.

To each of the 147 endogenous sectors there corresponds a distribution equation balancing supply (from domestic production or imports) with demand (from final consumption, investment, intermediate uses etc.). And, with few exceptions, to each of these sectors there corresponds a distinct domestic productive activity with which is associated a single sectoral output, a production function in the form of a vector in a current flow matrix, and an incremental fixed capital structure in the form of a vector in a capital matrix. The exceptions arise in the case of joint production, alternative techniques of production, and noncompetitive imports. While there are many instances of joint production in the economy, only in the case of petroleum refining does this study deal with more than a single major product. Seven varieties of petroleum products have been distinguished as sectors, but to these there correspond only two production activities; the basic refining process which yields light distillates, kerosenes, diesel oils, etc. in certain technologically determined fractions; and the further processing required for the production of lubricating oils. Alternative techniques of production were initially included for electricity generation, where the radically different hydro and thermal processes have to be separated; for rail transport, where coal, diesel and electric power are distinguished; and for motor goods transport, which uses petrol or diesel oil. The choice between the alternative techniques, however, is predetermined for the purposes of the analysis, since many of the considerations on which it depends could not be incorporated into the model. Finally, several sectors such as tin, sulphur, etc., are tied to raw materials unavailable in India: there can be no domestic production in these industries, whose products enter only as noncompetitive imports.

There are no overall distribution equations for the exogenous production activities of the economy; only their demand for the products of endogenous sectors and their impact on the balance of trade is considered. In Table 2, the exogenous part of the economy is classified into a number of sectors, each of which forms a source of demand for current and/or for capital account inputs from the endogenous sectors.[5] For each choice of an overall consumption goal in the target year, a corresponding

[5] The availability of data permitted a finer classification of exogenous sources of investment than of exogenous sources of current input demand. Although rail and road transport are included as endogenous sectors (910 to 940) in the model, their demand for construction capital inputs is treated exogenously.

set of demand vectors must be specified for the exogenous sectors. While no mechanical set of relations can be employed, one would naturally want to balance in a general way the ambitiousness of the exogenously determined consumption target with the extent of the input demand from the exogenous sectors. These input demand vectors can be treated as independent parameters, subject to variation according to different estimates of demand or different future goals.

TABLE 2. LIST OF EXOGENOUS SECTORS

A. *sources of demand for current inputs*
   1. agriculture and irrigation
   2. exogenous transport and other services

B. *sources of demand for capital inputs*
   1. agriculture
   2. major irrigation
   3. railway construction
   4. road construction
   5. other transport and communications
   6. social services
   7. private and commercial construction

Since the output of the exogenous part of the economy does not enter explicitly into the analysis, it is not included in the production and capital vectors of the endogenous industries. This means that the cost of agricultural and service inputs, other than rail and road transport, is omitted from the analysis. This omission results in an understatement of the domestic cost of production of sectors with significant inputs of agricultural commodities, notably the food and fibre industries, and it biases the model's choice mechanism against imports in these sectors. However, this bias is unlikely to affect the validity of the results since the case for such imports rather than domestic production is surely very weak. And since most of the output of these sectors is delivered directly to final consumption, the potential range of distortion through interindustry linkage is sufficiently limited to be negligible. The omitted cost of services—mostly trade and commerce—is also negligible, since it affects more or less equally the activities of production and importing.

Since the focus of the whole study is on import substitution, the structure of import requirements receives detailed empirical attention. Five separate sources of demand for imports are distinguished in the analysis.

First, there is the demand for the import of endogenous sector commodities as an alternative to domestic production. These are competitive imports which constitute separate activities of the programming model.[7] In general, import substitution is allowed full scope at the margin : that is, there is a free choice between importing or expanding domestic productive capacity to satisfy whatever demand is generated in the target year over and above that which can be satisfied by capacity existing in the base year. In a few sectors whose products are relatively heterogenous,[6] the scope for import substitution is exogenously restricted to reflect the fact that only some of the heterogenous products may actually be profitably substitutable. In these sectors there is hence a fraction of the total demand which must be satisfied by noncompetitive imports. In addition, there are a few endogenous sectors whose products simply cannot be produced in India (e.g. tin, sulphur, etc.); as noted earlier, the entire demand for these products must also be satisfied by noncompetitive imports.

A third type of import which is distinguished in the analysis arises from the demand for agricultural raw materials which cannot be made available from domestic agricultural production. These products belong to the exogenous part of the economy, but they are used in endogenous industries. Imports of noncompetitive agricultural raw materials are related via fixed coefficients to the endogenous production activities which use them as inputs. Thus they are treated as a separate category of noncompetitive imports, distinguished from the imports of noncompetitive industrial products.

The fourth source of demand for imports is for the remaining commodities which belong to the exogenous part of the economy. These include primarily foodgrains for direct consumption, and military supplies for government use.[7] While the demand for a few minor exogenous categories of imports can be projected into the future, it is very difficult to forecast in advance the requirements of food and military imports. In any case, the supply of foreign exchange for such purposes is often quite independent of the supply available for other imports. Since the analysis of this study cannot meaningfully take these imports into consideration, all references to the balance of trade will be understood to exclude them. To the extent that foreign exchange will be necessary in the target year for the supply of food or tanks, it will represent an additional requirement over and above what is generated by the runs of the model.

The final source of import demand is of considerable significance in the Indian context and has therefore been incorporated in a separate way into the interindustry framework. The products of the engineering sectors (included in the groups 'mechanical engineering', 'electrical engineering' and 'transport equipment' in Table 1) differ from the products of almost all the other sectors in a basic way : they consist both of complete units and of parts. The demand for complete units arises either from fixed investment (in the case of capital equipment) or from direct consumption (in the case of consumer durables). The demand for parts, on the other hand, arises from two different sources : fabrication of new complete units, and maintenance of old units. The domestic production of complete units is typically also subject to quite different factors than the domestic production of parts. Like the production of most of the other endogenous sector products, the production of complete units

[6] These sectors include primarily the residual sectors defined earlier on page 251.

[7] Military supplies consist of industrial products, but they could not be included in any endogenous sector for lack of detailed statistics.

in the engineering sectors is limited primarily by the existing capacity of the capital stock, the availability of raw material inputs and the supply of primary factors such as skilled labour. The production of many parts and components, however, is most critically limited by factors, such as an uneconomic scale of demand or an inadequate technical knowledge, which could not be incorporated explicitly into the analysis.

The most satisfactory way of dealing with the important difference between complete units and parts would be to define a separate new sector for each type of component part. The existing availability of data, however, ruled out such an ambitious undertaking. The alternative adopted in this study was the following. Each of the non-residual engineering sectors listed in Table 1 is understood to represent complete units only. *All* engineering parts and components are divided into two groups: domestically produced and imported. Domestically produced parts are included in the output of the corresponding residual engineering sectors, while imported parts are treated as an additional category of noncompetitive imports.

Both the imports of complete units and the imports of parts are distinguished according to the classification of engineering sectors given in Table 1. The import of each type of complete unit enters into the corresponding distributional equation as an alternative source of supply, just as in the case of competitive imports in the other endogenous sectors. The import of each type of component part, on the other hand, is related via fixed coefficients to the production level of the corresponding domestic industry and to the existing stock of the corresponding type of equipment. These noncompetitive parts imports constitute the fifth and last source of import demand distinguished in the analysis.

3. The algebraic formulation of the model

The model on which this study is based is a single-period linear programming model which focusses on the structure of the economy in a future target year. The corresponding structure in a base year, for which the relevant economic data have already been made available, is used as a point of reference from which the future growth possibilities are charted.

The basic set of constraints of any interindustry model relate to the distribution of the supply of products from each endogenous sector among the alternative sources of demand. In its simplest form, the typical distribution constraint in the present model is formulated as follows :

$$d_i+m_i \geqslant t_i+v_i+c_i+e_i \qquad \dots \quad (3.1)$$

where d_i, m_i, c_i and e_i denote the level of domestic output, imports, consumption and exports of sector i products, respectively; and t_i and v_i denote the total level of current and capital account deliveries of sector i products throughout the economy.[8]

[8] All variables without a time super-script are understood to apply to the single target year of the model.

Constraint (3.1) simply requires that in the target year the total supply of each endogenous sector's output must be at least as great as the corresponding total demand. All of the sectoral supply and demand variables are expressed in terms of 1960 producers' prices.

In the formulation of the model, a distinction must be made between the products of the endogenous sectors and the domestic production activities which produce them. As discussed in Section 2, there is not a complete one-to-one correspondence between sectors and productive activities. The domestic output d_i of sector i products is related algebraically to the activity levels x_j of the productive activities j as follows :

$$d_i = \sum_j u_{ij} x_j \qquad \text{... (3.2)}$$

where U is a matrix with n rows corresponding to the n endogenous sectors and m columns corresponding to the m domestic production activities. U is equivalent to an identity matrix, with the following exceptions : (1) rows representing sectors whose products cannot be domestically produced have no corresponding columns, and (2) the rows representing the joint products of the petroleum industry have positive elements u_{ij} denoting the fractions in which they are produced by the refining activity j.

The target year level of production x_j of each domestic production activity j is made up of two components :

$$x_j = x_j^R + x_j^* \qquad \text{... (3.3)}$$

where x_j^R is defined as the output obtained from capacity existing already in the base year, and x_j^* represents the incremental output obtained from new capacity installed between the base year and the target year.[9] As a rather harmless simplification, it is assumed that in each sector the capacity remaining from the base year will be fully utilized in the target year; thus x_j^R applies both to the remaining capacity and to the corresponding production level in sector j. In the interval between the base and target years there will generally have been some retirement of the base year capital stock, so that x_j^R does not necessarily equal the base year productive capacity.

To all of the endogenous sectors whose products can be physically imported there corresponds an importing activity which provides an alternative source of supply to domestic production. Thus we may write for each such sector i :

$$m_i = \pi_i y_i \qquad \text{... (3.4)}$$

where y_i denotes the activity level (measured in c.i.f. dollars) of the activity for importing sector i products, and π_i is the equivalent domestic value (measured in 1960 producers' price rupees) of a dollar's worth of imports of sector i products. The export demand for sector i products is simply specified exogenously :

$$e_i = \bar{e}_i. \qquad \text{... (3.5)}$$

[9] Barred variables denote predetermined constants.

The total demand t_i for intermediate deliveries from each endogenous sector i is made up of three components :

$$t_i = t_i^R + t_i^* + \sum_k t_i^k. \qquad \dots \quad (3.6)$$

The first two terms account for the demand on current account from the endogenous productive activities, and the last term accounts for the demand from the exogenous part of the economy. The former are related to endogenous production levels as follows :

$$t_i^R = \sum_j a_{ij}^0 x_j^R \qquad \dots \quad (3.7)$$

$$t_i^* = \sum_j a_{ij}^* x_j^* \qquad \dots \quad (3.8)$$

Two separate current flow matrices are distinguished : A^0 is the base year matrix which reflects the input structure of production with 'old' capacity, and A^* is the corresponding incremental matrix which applies to production with the 'new' capacity installed between the base and the target year. The current input demand t_i^k for sector i products from each exogenous source k is related to an index T^k of total current input demand from exogenous source k by the following formula :

$$t_i^k = \tau_i^k T^k \qquad \dots \quad (3.9)$$

where τ_i^k is an estimated coefficient of demand for product i per unit value of the total demand index for exogenous source k.

The total demand for capital good deliveries from each endogenous sector i is also made up of several components :

$$v_i = v_i^F + v_i^{F'} + v_i^{I} + \sum_k v_i^k. \qquad \dots \quad (3.10)$$

The first term refers to the demand for fixed capital investment in the endogenous production activities, which is determined as follows :

$$v_i^F = \eta^F \sum_j b_{ij}^* k_j^*. \qquad \dots \quad (3.11)$$

The coefficient b_{ij}^* is an element of the incremental fixed capital structure matrix B^*, which gives the rate at which the products of sector i are required per unit increment in the value of capital stock installed in activity j. k_j^* includes both the expansion of the capital stock from its base year to its target year level and the replacement of part of the base year capital stock which is retired during the period. It is related to the incremental production variable x_j^* of equation (3.2) as follows :

$$k_j^* = \beta_j^* x_j^* \qquad \dots \quad (3.12)$$

where β_j^* is the incremental capital-capacity ratio defined in terms of value of capital stock per unit of productive capacity in activity j.[10]

[10] The use of an equality rather than an inequality constraint in equation (3.12) implies that target year capacities will be fully utilized in each activity. Since a single period optimizing model would surely not build additional capacity unless it intended to use it, this does not represent any restriction on the operation of the model. If historical experience suggests that capacity is unlikely to be fully utilized in a rapidly growing economy, the same effect can be incorporated simply by raising the values of the β_j^* in the proportion that new capacity is likely to be under-utilized.

The expression $\sum_j b_{ij}^* k_j^*$ in equation (3.11) represents the total amount of sector i products that must be added to fixed capital stock in the full period from the base year to the target year. To convert this stock variable into the flow variable required by the model—viz., the demand for investment goods in the single target year—the 'stock-flow' conversion factor η^F is applied. η^F approximates—for the endogenous activities as a whole—the ratio of target year fixed capital investment demand to the addition to fixed capital stock between the base and target years.[11]

The second term of equation (3.10) relates to the demand for inventory investment in the endogenous production activities, which is given by :

$$v_i^W = \eta^W \sum_j s_{ij}^*(x_j - x_j^0). \qquad \dots \quad (3.13)$$

Equation (3.13) is analogous to equation (3.11) : s_{ij}^* is an element of an incremental stock coefficient matrix S^* which is applied to the corresponding change in the level of domestic production in activity j between the base and the target year. η^W is a working capital stock-flow conversion factor, which approximates the ratio of target year inventory investment demand to the addition to inventory stock between the base and target years.[12]

The third term of equation (3.10) refers to a part of the demand for replacement investment which arises from the retirement of capital stock between the base and the target years. Because retirement rates differ as between different types of capital equipment, the residual productive capacity x_j^R for each activity j can be sustained with capital stock remaining in the target year only if some of the less durable types of capital are kept in the right proportions by partial replacement. This partial replacement investment must be evaluated exogenously according to the age structure and retirement rates of the various types of capital equipment existing in the base year:[13]

$$v_i^X = \bar{v}_i^X \qquad \dots \quad (3.14)$$

As noted above, the total demand for replacement investment in the endogenous production activities is not limited to the v_i^X terms but includes also a fraction of the v_i^F terms defined in equation (3.11).

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[11] The derivation and significance of the fixed capital stock-flow conversion factor η^F is discussed in Section 5 below.

[12] The derivation and significance of the working capital stock-flow conversion factor η^W is discussed in Section 5 below.

[13] It is assumed that none of the capital equipment installed between the base and the target year will have to be replaced before the target year.

The exogenous demand for endogenous capital inputs is summed in the last term of equation (3.10). Analogously to equation (3.9), the capital input demand v_i^k for sector i products from each exogenous source k is related to the total (market) value of investment V^k in exogenous sector k by the following equation :

$$v_i^k = \varphi_i^k V^k \qquad \text{... (3.15)}$$

where φ_i^k is a coefficient giving the capital input norm of the products of endogenous sector i per unit investment in exogenous sector k.

In the remaining equations relating to constraint (3.1), the final consumption demand c_i for the output of each individual endogenous sector i is related to the total (market) value of aggregate consumption C in the target year. The relationship is analogous to those of equations (3.9) and (3.15), but it is nonhomogeneous : incremental sectoral coefficients are introduced which differ from the corresponding base year ratios.

Each target year consumption demand c_i is expressed as the sum of two components :

$$c_i = c_i^{\beta} + c_i^{*}. \qquad \text{... (3.16)}$$

The first term c_i^{β} represents a per capita level of consumption equivalent to that of the base year :

$$c_i^{\beta} = \bar{c}_i^0 e^{nT} \qquad \text{... (3.17)}$$

where $\bar{c}_i^0$ is the base year level of consumption of sector i products, and n is the expected annual rate of growth of population.[1] The second term c_i^* includes that part of target year consumption which represents an increase above the base year per capita level :

$$c_i^* = \gamma_i^* \ (C - \overline{C}^0 e^{nT}). \qquad \text{... (3.18)}$$

The term in brackets represents the amount by which target year consumption expenditure exceeds the expenditure required to maintain the base year per capita consumption levels, and the coefficient γ_i^* denotes the proportion of this excess consumption expenditure which is spent on the products of sector i. The γ_i^* are thus equivalent to incremental per capita consumption coefficients; when divided by the corresponding average coefficients obtaining in the base year, they yield implied linear per capita expenditure elasticities.

This completes the presentation of the structural equations which underlie the initial set of distribution constraints (3.1). If all of the equations are directly substituted into the original constraints, these constraints can be expressed in reduced form in terms of the following independent variables :

- x_j^* : the endogenous incremental production activity levels;
- y_i : the endogenous importing activity levels;
- T^k : the indices of current input demand from each exogenous source;
- V^k : the market value of investment in each exogenous sector;
- C : the market value of aggregate consumption.

The reduced form P-1 of the typical distribution constraint is shown in Table 3.

TABLE 3. PRIMAL CONSTRAINTS

P-1	$\sum_j (u_{ij}-w_{ij})x_j^* + \pi_i y_i - \sum_k \tau_i^k T^k - \sum_k \phi_i^k V^k - \gamma_i^* C > \bar{q}_i \qquad (i = 1, \ldots, n)$	..
	where $\quad w_{ij} = a_{ij}^* + \eta^F b_{ij}^* \beta_j^* + \eta^W s_{ij}^*$	
	$\bar{q}_i = \sum_j (u_{ij} - a_{ij}^0) x_j^R + \bar{e}_i + \eta^W \sum_j s_{ij}^* (x_j^R - x_j^0) + (\bar{c}_i^0 - \gamma_i^* \bar{C}^0) e^{\alpha T}$	
P-2	$-x_j^* > x_j^R - x_j^M \qquad (j \epsilon XM)$	..
P-3	$-\sum_j \mu_i u_{ij} x_j^* + (1-\mu_i)\pi_i y_i > \sum_j \mu_i u_{ij} x_j^R \qquad (i \epsilon MM)$	..
P-4	$T^k > \bar{T}^k \qquad (k = 1, \ldots, l^T)$	..
P-5	$V^k > \bar{V}^k \qquad (k = 1, \ldots, l^V)$	..
P-6	$C > \bar{C}$	..
P-7	$L - \sum_j \lambda_j^X x_j^* > \sum_j \lambda_j^0 x_j^R$	..
	where $\quad \lambda_j^X = \lambda_j^* + \eta^F \lambda_j^\beta \beta_j^*$	
P-8	$M - \sum_j \mu_j^X x_j^* - \sum_i y_i - \sum_k \mu_k^V V^k - \mu^C C > \bar{q}^M$	..
	where $\quad \mu_j^X = \underset{(j \epsilon AM)}{\psi_j} + \underset{(j \epsilon PM)}{\xi_j} + \sum_{(i \epsilon PM)} (\xi_i / z) b_{ij}^* \beta_j^*$	
	$\mu_k^V = \sum_{(i \epsilon PM)} (\xi_i / z) \sigma_i \phi_i^k$	
	$\mu^C = \sum_{(i \epsilon PM)} (\xi_i / z) \sigma_i \gamma_i^*$	
	$\bar{q}^M = \sum_j \mu_j^X x_j^R + \sum_{i \epsilon PM} (\xi_i / z) \bar{s}_i^R + \sum_{i \epsilon PM} (\xi_i / z) \sigma_i (\bar{c}_i^0 - \gamma_i^* \bar{C}^0) e^{\alpha T}$	

The remaining constraints in the programming model are of three kinds. The first are inequalities which further constrain the basic activity variables x_j^* and y_i. The second are equalities which fix the exogenously specified values of the aggregate variables T^k, V^k and C. The third are equalities which define additional activities measuring the requirements of domestic and foreign primary resources.

The additional inequality constraints are introduced as linear approximations to what are in fact likely to be nonlinear situations. For a few of the endogenous production activities of the model, upper bounds are imposed on the level of domestic production :

$$x_j \leqslant x_j^M \qquad (j \epsilon XM) \qquad \ldots \quad (3.19)$$

Such bounds are required when production is restricted in actuality by a factor which is not incorporated into the interindustry framework of the model. This is notably the case in several mining activities, where the scope for (profitable) production is sharply limited by the availability of mineral resources. A second set of inequality constraints is applied to some activities in order to limit the scope for import substitution afforded by the linear structure of the model. As discussed in Section 2, it is desirable to allow for the fact that in a few productive activities whose output is relatively heterogeneous some of the products may not be (profitably) substitutable. Thus the following type of constraint is introduced :

$$m_i \geqslant \mu_i(d_i+m_i) \qquad (i \varepsilon MM) \qquad \dots \quad (3.20)$$

where μ_i represents the minimum proportion of the supply of the products of sector i which must be imported.

The following three sets of constraints serve to introduce the target year goals, subject to the attainment of which the programming model minimizes costs. These goals are described by the aggregate variables T^k, V^k and C, for which values must be determined prior to each run of the model. Thus the constraints may be written as follows :

$$T^k = \overline{T}^k \qquad (k = 1, \dots, l^T) \qquad \dots \quad (3.21)$$

$$V^k = \overline{V}^k \qquad (k = 1, \dots, l^V) \qquad \dots \quad (3.22)$$

$$C = \overline{C} \qquad \dots \quad (3.23)$$

where the barred variables represent the pre-determined target year values.

The last pair of constraints in the model measure the endogenous use of the primary resources in the system : labour and foreign exchange. These constraints are required to define the two terms which enter the cost function, representing the domestic and foreign primary resources respectively.

Labour resources are required by each of the domestic production activities of the model. These labour requirements are measured in terms of their total wage cost rather than the size of the working force. If different categories of labour cannot be adequately distinguished and independently treated, it is more meaningful to deal with an aggregate based on wages than on numbers. The total labour cost in rupees incurred by the endogenous production activities[14] is given by :

$$L = \sum_j (\lambda_j^0 x_j^R + \lambda_j^* x_j^*) + \eta^F \sum_j (\lambda_j^B k_j^*). \qquad \dots \quad (3.24)$$

The first term measures the direct current costs of operating labour, where λ_j^0 denotes the labour cost per unit of residual capacity production, and λ_j^* the labour cost per unit of new capacity production, in 1975. The second term measures the indirect capital costs of construction labour required for the installation of the capital stock.[15] The coefficients λ_j^B may be regarded as the $(n+1)$-th row of the B^* matrix, giving the construction labour cost per unit increase in capacity for each activity j.

[14] Labour is of course also employed in the exogenous part of the economy, but this labour cost is extraneous to the model.

[15] Construction is not treated as a separate activity, so that both construction materials and labour inputs are related like machinery requirements directly to the capacity increases of the production activities.

The foreign exchange cost (i.e. the import requirements) generated by the model can be analysed in terms of the five separate sources of import demand distinguished in Section 2. Of these five, four are functionally related to the activities included in the model and are summed to yield the total value—measured in c.i.f. prices—of endogenous imports M :

$$M = \sum_{k=1}^{4} M^k. \quad \ldots \quad (3.25)$$

The first type are the competitive imports of endogenous sector products, whose total value is given by

$$M^1 = \sum_{i \in CM} y_i \quad \ldots \quad (3.26)$$

where CM is the set of endogenous sectors in which imports compete with domestic production. The second type are the noncompetitive imports of endogenous sector products, whose total value is given by

$$M^2 = \sum_{i \in NM} y_i \quad \ldots \quad (3.27)$$

where NM includes the set of sectors whose products cannot be produced domestically.[16] The third type are the imports of noncompetitive agricultural raw materials, with a total value of

$$M^3 = \sum_{j \in AM} \psi_j x_j \quad \ldots \quad (3.28)$$

where AM includes the set of endogenous production activities which use imported agricultural raw materials as inputs. To each activity j there corresponds at most one such input, which is required in the proportion ψ_j in terms of c.i.f. dollars per unit. A fourth category of imports was defined to cover the remaining imports of exogenous sector commodities, but since these are entirely exogenous to the model they are not included here.

The last source of imports was discussed in considerable detail in Section 2; it involves the demand for imported engineering parts and components both for further fabrication and for the maintenance of existing stock. The total value of such imports is expressed as follows :

$$M^4 = \sum_{j \in PM} \xi_j x_j + \sum_{i \in PM} (\xi_i / z) s_i \quad \ldots \quad (3.29)$$

where PM is the set of engineering activities (and corresponding sectors) in which parts are distinguished from complete units. The first and second terms of the equation cover the fabrication and the maintenance demands, respectively. ξ_j represents the total dollar value of imported parts required in the production of one unit of output x_j of activity j. The same coefficient ξ_i is applied to the existing stock s_i of equipment

[16] Strictly speaking, M^2 should include also the noncompetitive fraction of imports in sectors where a minimum proportion of imports is imposed by constraint (3.20).

of sectoral type i in order to determine the total embodied value of parts which can only be replaced by noncompetitive imports. Assuming that the average life of engineering parts is z years, a fraction $1/z$ will have to be replaced every year : this leads to the maintenance demand described by the second term in the equation.

The stock of equipment s_i can be expressed as follows :

$$s_i = s_i^R + s_i^* + \sum_k \sigma_i v_i^k + \sigma_i c_i. \qquad \text{... (3.30)}$$

The first two terms of the equation cover the stock of capital equipment in the endogenous productive activities of the economy. s_i^R is the residual in the target year which remains from the stock of type i existing in the base year, and s_i^* represents the addition to the endogenous stock of type i between the base and the target years given simply by

$$s_i^* = \sum_j b_{ij}^* k_j^*. \qquad \text{... (3.31)}$$

The third term of equation (3.30) represents the stock of sector i output which is held as capital equipment in the exogenous sectors, and the fourth term applies when the products of sector i can be held as consumer durables. Since these stocks are exogenous to the interindustry framework of the model, it is necessary to approximate them independently. σ_i is a rule-of-thumb conversion factor which relates the stock of durable equipment of sectoral type i to the corresponding exogenous investment and consumption flows v_i^k and c_i in the target year.

It remains now only to define the objective function which is to be minimized subject to the attainment of the targets prescribed in constraints (3.21), (3.22) and (3.23):

$$\Omega = O^L L + O^M M. \qquad \text{... (3.32)}$$

The function Ω consists of a weighted sum of the domestic (L) and foreign (M) primary resource costs. The relevant weights O^L and O^M must be pre-assigned for each run of the model; the corresponding weight ratio can be interpreted as the shadow rate of exchange between rupees and dollars, on the basis of which all other prices in the system are determined.

This completes the presentation of the primal constraints and objective function of the programming model. All of the constraints entering into the model can be expressed in reduced form in terms of the independent variables x_j^*, y_i, T^k, V^k and C (identified at the bottom of page 258) along with the following additional independent variables :

L : the total rupee c.i.f. dollar value of the wage bill in the endogenous production activities ;

M : the total c.i.f. dollar value of endogenous imports in the economy.

The full set of constraints in the model are shown in reduced form in Table 3. The constraints are arranged so that the independent variables and their coefficients appear on the left-hand side of the inequalities, and the constant terms appear on the right hand side.[17]

4. The price structure of the model

Corresponding to the primal form of the linear programming problem discussed in the previous section is the dual form of the problem, in which prices replace quantities as the independent variables. If we describe the primal form of the model in standard linear programming form as :

$$\text{minimize } \Sigma\, c_j x_j \text{ subject to } Ax \geqslant b, \quad x \geqslant 0$$

where x is the vector of primal activity variables, A the (rectangular) matrix of constraint coefficients, and b the constant right-hand side vector, we may set out the corresponding dual problem as follows :

$$\text{maximize } \Sigma\, b_i p_i \text{ subject to } pA \leqslant c, \quad p \geqslant 0$$

where p is the vector of dual variables, whose values emerge from each solution of the programming problem simultaneously with those of the primal variables x_j. The dual variables p_i measure the marginal reduction in the value of the cost function $(\sum_j c_j x_j)$ which can be achieved by relaxing the i-th constraint by one unit; thus they can be interpreted as the shadow prices associated with each constraint of the primal problem.[18]

The dual constraints of the present model can be spelled out most conveniently with reference to the constraint tableau shown in Table 4. Across the top of the table are listed the seven sets of independent activity variables in terms of which the primal constraints (in their reduced form) are expressed. All of these activity variables taken together constitute the x vector of the standard linear programming problem. Down the left side of the table are listed the dual price variables corresponding to each of the eight groups of primal constraints; these variables form the p vector of the standard problem. Within the table itself are given the constraint coefficients, taken from the reduced form of the primal constraints as given in Table 3 ; these coefficients constitute the matrix A of the standard problem. Finally, the corresponding elements of the right-hand side vector b and the cost function c are shown to the right and below the table, respectively.

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[17] Although some of the constraints are in fact equal'ties, they can be represented as inequalities in the appropriate direction, and they have been entered as such in Table 3.

[18] For a general theoretical treatment of linear programming theory and techniques, see Dantzig (1963), Gass (1958) or Hadley (1962); for discussions emphasizing the economic interpretation of linear programming problems, see Chenery and Clark (1959) and Dorfman, Samuelson and Solow (1958).

TABLE 4. CONSTRAINT TABLEAU

constraints	dual	D·1	D·2	D·3	D·4	D·5	D·6	D·7		right hand side
primal	activities	x_j^*	y_i	T^k	V^k	C	L	M		
P·1	p_i	$(u_{ij}-w_{ij})$	$\hat{\pi}_i$	$-\tau_i^k$	$-\phi_i^k$	$-\gamma_i^*$	0	0	>	$\bar{q}_i$
P·2	p_j^{XM}	-1	0	0	0	0	0	0	>	$x_j^R-x_j^M$
P·3	p_i^{MM}	$-\sum_i \mu_i u_{ij}$	$(1-\mu_i)\pi_i$	0	0	0	0	0	>	$\sum_j \mu_i u_{ij} x_j^R$
P·4	p_k^T	0	0	I	0	0	0	0	>	$\bar{T}^k$
P·5	p_k^V	0	0	0	I	0	0	0	>	$\bar{V}^k$
P·6	p^C	0	0	0	0	1	0	0	>	$\bar{C}$
P·7	p^L	$-\lambda_j^X$	0	0	0	0	1	0	>	$\sum_j \lambda_j^o x_j^R$
P·8	p^M	$-\mu_j^X$	-1	0	$-\mu_k^V$	$-\mu^C$	0	1	>	$\bar{q}^M$
		<	<	<	<	<	<	<		
cost function		0	0	0	0	0	θ^L	θ^M		

Just as the primal constraints can be read from each row of the tableau in the form :

$$\sum_i a_{ij} x_j \geqslant b_i$$

so the dual constraints can be read from each column in the form

$$\sum_j a_{ij} p_i \leqslant c_j .$$

With each of the seven groups of independent activity variables is associated a group of dual constraints, whose right-hand side constants are the corresponding elements of the primal cost function. In Table 5, the typical constraint of each of the seven groups of dual constraints is spelled out in the same way as the primal constraints in Table 3. Although the constraints are shown in the standard form as 'less than or equal to' inequalities, it can be immediately deduced that the last five groups of constraints are necessarily binding. This follows from the fact that the activity variables corresponding to constraint groups (D-3) through (D-7) (T^k, V^k, C, L, M) are all bound to appear with non-zero values in the solution to the model.[19]

[19] The theorem invoked here is simply the dual of the proposition that a primal constraint is binding if and only if the associated price variable takes on a nonzero value. See the references in footnote 18.

For the purpose of analyzing the price structure of the model, it is convenient to examine the dual constraints in the reverse of the order in which they appear in Table 5. Constraints D-7 and D-6 simply imply that the shadow prices p^M and p^L (associated with the primal constraints P-8 and P-7, defining the total use of the two

TABLE 5. DUAL CONSTRAINTS

D-1	$\sum_i (u_{ij}-w_{ij})p_i - p_j^{XM} - \sum_i \mu_i u_{ij}\, p^{MM} - \lambda_j\, p^L - \mu_j^X p^M < 0$ $(j \epsilon XM) \quad (i \epsilon MM)$	$(j = 1, \ldots m)$
D-2	$\pi_i\, p_i + (1-\mu_i)\pi_i\, p_i^{MM} - p^M < 0$ $(i \epsilon MM)$	$(i = 1, \ldots, n)$
D-3	$p_k^T - \sum_i \tau_i^k\, p_i < 0$	$(k = 1, \ldots, l^T)$
D-4	$p_k^V - \sum_i \phi_i^k\, p_i - \mu_k^V p^M < 0$	$(k = 1, \ldots, l^V)$
D-5	$p^C - \sum_i \gamma_i^*\, p_i - \mu^C\, p^M < 0$	
D-6	$p^L < \theta^L$	
D-7	$p^M < \theta^M$	

primary resources M and L) must be equal to the preassigned weights θ^M and θ^L in the minimand of the primal problem:

$$p^M = \theta^M \qquad \ldots \quad (4.1)$$

$$p^L = \theta^L. \qquad \ldots \quad (4.2)$$

Constraints D-6, D-5 and D-4 describe the determination of the shadow prices p^C, p_k^V and p_k^T (associated with the primal constraints P-6, P-5 and P-4, defining the target levels of the final demand variables C, V^k and T^k). Thus the shadow price of a (marginal) unit of consumption is given by :

$$p^c = \sum_i \gamma_i^* p_i + \mu^C \theta^M \qquad \ldots \quad (4.3)$$

where the first term represents the (marginal) cost of endogenous sector products—priced at the corresponding shadow prices p_i—and the second term covers the cost of noncompetitive parts and components imports—priced at the shadow price of imports equivalent to θ^M. The shadow prices associated with marginal units of investment V^k in each exogenous sector k, and the index of intermediate demand T^k from each exogenous source k, are given similarly by :

$$p_k^V = \sum_i \varphi_i^k p_i + \mu_k^V \theta^M \qquad k = 1, \ldots, l^V \qquad \ldots \quad (4.4)$$

$$p_k^T = \sum_i \tau_i^k p_i \qquad k = 1, \ldots, l^T \qquad \ldots \quad (4.5)$$

The dual constraints of the remaining two groups are *not* necessarily binding, for these constraints correspond not to purely definitional primal variables but to the basic choice variables of the model : the x_j^* and y_i. Only those constraints corresponding to the subset of x_j^* and y_i included in the basis of the optimal solution will turn out to hold as equalities; the remaining constraints will hold as inequalities.

The dual constraints D-2 and D-1 can be rewritten as follows :

$$p_i+(1-\mu_i)p_i^{MM} \leqslant (1/\pi_i)0^M \qquad i = 1, ..., n \qquad \dots (4.6)$$
$$(i \epsilon MM)$$

$$\sum_i u_{ij}p_i - p_j^{XM} - \sum_i \mu_i u_{ij} p_i^{MM}$$
$$(j \epsilon XM) \qquad (i \epsilon MM)$$
$$\leqslant \sum_i w_{ij}p_i + \lambda_j^X 0^L + \mu_j^X 0^M \qquad j = 1, ..., m \qquad \dots (4.7)$$

where p_j^{XM} and p_i^{MM} are the shadow prices associated with the primal constraints P-2 and P-3, respectively.

For sectors i and activities j which are not affected by primal constraints P-2 and P-3, and for which there is a one-to-one correspondence between sector i and domestic production activity j, we can ignore the variables p_j^{XM} and p_i^{MM}, treat U as an identity, and set $i = j$. Constraints 4.6 and 4.7 can then be simplified to the following :

$$p_j \leqslant (1/\pi_j)0^M \qquad \dots (4.8)$$

$$p_j \leqslant \sum_i w_{ij}p_i + \lambda_j^X 0^L + \mu_j^X 0^M \qquad \dots (4.9)$$

The right-hand side of (4.8) is simply the cost of importing a domestic unit of sector j output : $(1/\pi_j)$ represents the c.i.f. dollar cost of a rupee's worth of sector j output (at 1960 producers' prices), which is multiplied by the shadow price of a dollar 0^M. The right-hand side of (4.9) is the cost of producing a domestic unit of sector j output in production activity j : the sum of the unit cost of endogenous sector inputs w_{ij}—evaluated at the shadow prices p_i—and the unit cost of domestic and foreign primary inputs λ_j^X and μ_j^X—evaluated at the pre-determined shadow prices 0^L and 0^M.

Constraints (4.8) and (4.9) reflect the operation of the choice mechanism of the model. According to the two constraints, the shadow price associated with each sectoral product must be less than or equal to both the unit cost of importing and the unit cost of producing the product. Clearly, the shadow price will be determined by the lower of the two bounds, and the single activity which provides the output at minimum cost will be included in the optimal solution to the programming run. For the included activity, the shadow price of the product just equals the corresponding cost; for the excluded activities, the cost exceeds the shadow price.

The determination of the shadow prices—and hence the operation of the choice mechanism of the model—becomes slightly more complicated when the simplifying conditions leading to constraints (4.8) and (4.9) do not hold. If, for some activity $j \epsilon XM$, a primal constraint of group P-2 is active, then the shadow price p_j^{XM} will assume a positive value attributable to the upper bound placed on the level at which activity j can be operated. p_j^{XM} can be interpreted as the shadow price of the scarce factor whose limited availability called for the upper bound on activity j. From constraint (4.7) it can be seen that a positive value for p_j^{XM} allows the shadow price of the output of activity j to *exceed* the corresponding cost of domestic production. Because of the upper bound on production activity j, the marginal demand for the corresponding sectoral output i must be satisfied by the alternative—less efficient—importing activity, whose unit cost determines the shadow price p_i. Thus p_j^{XM} measures the *extra* unit cost incurred when the (marginal) output from activity j must be provided by an activity more costly than the domestic production activity j.

If, for some sector $i \in MM$, a primal constraint of group P-3 is active, then the shadow price p_i^{MM} will assume a positive value attributable to the requirement that a fraction of the sectoral output be imported rather than produced domestically. p_i^{MM} can be regarded as the shadow price of a specific factor whose scarcity prevents complete import substitution in sector i. It is clear that a constraint P-3 can be binding only if the cost of importing the corresponding product exceeds the cost of domestic production, for there would otherwise be no penalty in having to import. From constraints (4.6) and (4.7) it can be deduced that the shadow price p_i of the sectoral output would be between the (lower) cost of domestic production and the (higher) cost of importing. In particular, p_i^{MM} measures the extra cost of importing vis-a-vis domestic production, and the import fraction μ_i—multiplied by p_i^{MM}—determines the extent by which the shadow price p_i exceeds the domestic cost of production of sector i products.

Up to this point we have assumed a one-to-one correspondence between sectors and activities, so that the left-hand side of constraint (4.7) could be expressed in terms of shadow prices associated with a single sector (and activity j). In cases of joint production[20], the single activity j produces output belonging to several sectors i; to each of these sectors there correspond a nonzero u_{ij} value, whose column sum equals unity. In such cases it is necessary to re-interpret the first term of constraint (4.7) as the *sum* of the shadow prices of each product produced by activity j; neglecting the second and third terms[21], the constraint requires that this sum does not exceed the corresponding unit cost of domestic production. In general—because of the fixed output proportions—it is likely that for only one of the joint products will demand be matched exactly by domestic supply; and the shadow price p_i of this product alone will equal the unit cost of domestic production. For the remaining joint products, the shadow prices p_i will be equal either to zero (when domestic supply exceeds the demand) or —via constraint (4.6)—to the unit cost of importing (when domestic supply falls short of the demand). In cases of alternative techniques of production,[22] no re-interpretation

[20] Only one case of joint production was actually included in the empirical application of the model: the petroleum refining activity (see the discussion in Section 2).

[21] These two terms do not apply to the single case of joint production noted in footnote 20.

[22] No cases of alternative techniques of domestic production were included in the final empirical application of the model (see the discussion in Section 2).

of constraint (4.7) would be called for, since for each activity j producing the sectoral output i there would be a single coefficient u_{ij} with a value of unity. The only difference this would make to the operation of the model is that the scope for choice among alternative supply activities would be enlarged; the shadow price p_i would of course still be determined by the cheapest source of supply of sector i products.

5. The stock-flow conversion factors

Discussion of the two stock-flow conversion factors η^F and η^W has been delayed until this section for two reasons. First, the estimation of the values to be attributed to η^F and η^W is a problem of some complexity, which can most conveniently be examined independently of the algebraic formulation of the primal and dual constraints of the programming model. Secondly, the interpretations given to η^F and η^W in the primal and dual forms of the model are quite different, and it is necessary to resolve the inconsistency before the model can be properly programmed.

As observed in Section 3, the stock-flow conversion factor η^F approximates —in the primal formulation of the model—the ratio of target year fixed capital investment demand to the addition to fixed capital stock between the base and target years. Denoting by V the level of fixed capital investment in the endogenous activities, and by K the corresponding capital stock, we may write

$$\eta^F = \frac{V^T}{K^T - K^0} \qquad \dots \quad (5.1)$$

where the superscripts '0' and 'T' refer to the base and target years of the model, respectively. A stock-flow conversion factor used for precisely this purpose was introduced by Manne[23]; his derivation of the numerical value to be given to the factor was based on the assumption of a constant exponential rate of growth of investment activity between the base and the target year. This rate of growth r had to be guessed at in advance of each programming run, but Manne showed that the numerical value of the stock-flow conversion factor was relatively insensitive to variation— within a 'reasonable' range—of r.[24]

For the purposes of the present model, in its primal form, a slightly different method for estimating η^F is suggested. The difference lies primarily in the fact that investment activity in the target year is related explicitly to the growth of output *beyond* the target year, as—in principle—it should be. Assuming an annual rate of growth of capital stock of r^T after the target year, and an average lag of θ years between the production of investment goods and the corresponding increase in productive capacity, we may express V^T as

$$V^T = K^{T+\theta+\frac{1}{2}} - K^{T+\theta-\frac{1}{2}} = r^T K^T (1+r^T)^{\theta-\frac{1}{2}}. \qquad \dots \quad (5.2)$$

[23] See Manne (1966); the same type of stock-flow conversion factor was later used also in Manne and Rudra (1965) and Sabherwal, Saluja and Srinivasan (1965).

[24] For an algebraic derivation of Manne's stock-flow conversion factor—and a demonstration that it is relatively insensitive to variation in r—see Manne (1966) or Manne and Rudra (1965).

If we define the average annual rate of growth of capital stock between the base and target years as r^0, we may write

$$K^T = K^0(1+r^0)^T \qquad \dots \quad (5.3)$$

where T is the corresponding length of the period in years. Substituting equations (5.2) and (5.3) into equation (5.1), we get :

$$\eta^F = \frac{r^T(1+r^T)^{\theta-t}}{1-(1+r^0)^{-T}}. \qquad \dots \quad (5.4)$$

Thus η^F is a function of four variables : $\eta^F(T, \theta, r^0, r^T)$. T is determined by the formulation of the problem; θ can be estimated from empirical data; r^T must be specified in advance as one of the target parameters of the model (like the final demand variables C, T^k and V^k); and r^0 must be estimated prior to each programming run[25] (like Manne's rate of growth of investment activity r). The working capital stock-flow conversion factor η^W is estimated in exactly the same way as η^F, with the single exception that the average lag θ is assumed to equal zero; thus

$$\eta^W(T, r^0, r^T) = \eta^F(T, 0, r^0, r^T) \qquad \dots \quad (5.5)$$

Each of the stock-flow conversion factors η^F and η^W is applied uniformly to investment demand from all of the endogenous activities of the model. In principle, it would be more accurate to apply distinct stock-flow conversion factors to each of the domestic production activities j, since the variables on which the values of η^F and η^W depend (see equation (5.4)), are likely to differ as between different activities. On the other hand, since the values of η^F and η^W are relatively insensitive to changes in the growth rates r^0 and r^T, and since the estimation of r^0 is in any case only approximative, the additional complexity would not appear to be justified.[26] One might further suggest that the value of η^F be distinguished according to the sector of *origin* of the capital goods—since the gestation lag θ may well differ as between different types of goods i—but the lack of sufficiently detailed information, and the relatively small effect of changes of this kind, dictated the simplest course of a common η^F for use in the present study.

The appropriate dual interpretation to be given to the parameters η^F and η^W emerges from a careful examination of the typical dual constraint of group D-1, as reproduced in equation (4.7). The right-hand side of equation (4.7) represents the unit cost of domestic production in activity j, and the first term $\sum_i w_{ij}p_i$ covers the unit cost of the endogenous sector inputs. The coefficient w_{ij} is made up of three parts (as shown in Table 3):

$$w_{ij} = a^*_{ij}+\eta^F b^*_{ij}\beta^*_j+\eta^W s^*_{ij}. \qquad \dots \quad (5.6)$$

[25] If the initial estimate of r^0 proves to be inconsistent with the results of the programming run, it is always possible to revise it for a second run and thus proceed by iteration to a consistent solution. In view of the fact, however, that the growth of final demand is pre-determined for each run, it should not be difficult to make a good initial estimate of the rate of growth r^0 of endogenous sector capital stock as a whole.

[26] For a similar defence of the use of a single stock-flow conversion factor, see the references given in footnote 24.

a^*_{ij} denotes the requirement on current account of inputs from sector i per unit of (incremental) output in production activity j; $b^*_{ij}\beta^*_j$ and s^*_{ij} represent the corresponding requirements of fixed and working capital stock per unit of output in activity j. While the full cost of current account inputs is naturally charged to production costs in the given target year, only a fraction of the value of the capital stock associated with the single year's production can properly be charged to production costs. This fraction must reflect both interest on the tied-up capital, and—in the case of fixed capital stock—depreciation of the equipment. Hence we can interpret η^F as the rate of interest plus depreciation applied to the stock of fixed capital, and η^W as the rate of interest applied to the stock of working capital.

While the rate of interest is necessarily an economy-wide constant, the rate of depreciation on fixed capital stock may well differ as between the different endogenous activities of the model. Thus—as in the case of the primal interpretation, but for a different reason—it would in principle be more accurate in the dual interpretation to distinguish between stock-flow conversion factors η^F_j applied to fixed capital stock in different production activities j. Even greater accuracy would call for a distinction between factors η^F_{ij}, taking into account also the differences in depreciation rates applying to capital goods of different sectors of origin i. Such refinements, however, would seem to be of secondary importance, since most of the numerical value of η^F will in any case be due to the common rate of interest, and possible variations in the rate of depreciation are not likely to have a significant effect on this value.

The problem remains, nevertheless, of reconciling the two different interpretations—and the correspondingly different numerical values—given to the stock-flow conversion factors η^F and η^W in the primal and dual forms of the model. From the point of view of optimal choice among alternative activities, it is essential that the values given to η^F and to η^W properly reflect the *cost* structure of the activities; thus it is the *dual* interpretation which is appropriate. On the other hand, from the point of view of projection of output and import requirements, it is essential that the values given to η^F and to η^W properly reflect the *demands* raised by the growing economy; here it is the *primal* interpretation which is appropriate.

The resolution of this dilemma lies in the particular structure of the programming model used in the present study. The structure is basically a very simple one, with the crucial characteristic that the (non-) substitution theorem[27] is applicable to it with only a few minor qualifications. Apart from the scarce factors implied by primal constraints P-2 and P-3, the only primary factors in the model are labour and foreign exchange; when these are combined with fixed weights in the cost function, the result is a single homogeneous factor which underlies all of the pricing in the system. According to the substitution theorem, optimal choice among alternative activities in a well-behaved open Leontief system is independent of the levels of final demand

[27] For the initial, independent statements of the theorem, see Samuelson (1951) and Georgescu-Roegen (1951). For formal proofs of the theorem, see also Koopmans (1951) and Arrow (1951).

when there is only a single basic scarce factor. The model of the present study is well-behaved—except for the one petroleum refining activity with joint products—and it has a single scarce factor—except for those implied by constraints P-2 and P-3. Neglecting possible minor errors on account of these exceptions, the substitution theorem should hold for all practical purposes of the study.

The applicability of the substitution theorem allows for a clear-cut distinction to be made between the primal and the dual aspects of each programming run. Although for any given run the model is solved simultaneously for the primal and corresponding dual solution, its operation can be viewed conceptually as taking place in two distinct steps. First, on the basis of the primary factor costs given by the labour and foreign exchange weights in the minimand, the total direct and indirect cost of each producing and importing activity is evaluated. Wherever it has a choice, the model selects the cheaper of the two activities for inclusion in the optimal solution. Secondly, having chosen the activity which supplied each sectoral product, the model determines the levels of these activities required to sustain the target year final demands. The first step in the solution process evaluates the shadow prices for each sectoral product, and determines the qualitative pattern of choice between domestic production and importing activities. The second step provides a detailed quantitative description of the structure of the economy in 1975.

The independence of these two steps can be exploited to allow for the proper recognition of each of the two interpretations given to η^F and η^W. Let the model first be programmed with values of η^F and η^W appropriate to the *dual* interpretation, and record the resulting choice of activities. These are to be included in the basis of the optimal solution. According to the substitution theorem, this optimal basis should not be affected if the final demand levels in the primal problem are subsequently altered. Let the model then be programmed a second time with the η^F and η^W values appropriate to the *primal* interpretation, but constrained to choose the same activities that were included in the solution to the first run. Thus the second run is designed to provide not an optimal solution—on its own terms—but a *feasible* solution corresponding to the basis which was optimal in the initial run. This feasible solution will then embody both the optimal choice of activities according to the appropriate price structure of the model, and the sectoral output and import levels which are consistent with the target year pattern of demand.

6. Numerical applications of the model : some qualitative results

The programming model described in the previous sections was applied with the help of a detailed body of data on the present and future structure of the Indian economy. The sources and the methodology used to compile the required data are discussed in the Appendix. Given the basic structural coefficients, and the initial conditions of the economy in 1965, the model was programmed under a variety of parametric assumptions about the future in order to provide a wide spectrum of alternative (optimal) solutions for 1975. The key parameters include the following :

(1) the rate of growth of exports from 1965 to 1975; (2) the anticipated levels of a subset of non-competitive import coefficients (μ_i, ψ_i and ξ_i) which might reasonably be expected to decline by 1975; (3) the targetted rate of growth of aggregate consumption from 1965 to 1975; (4) the ratio of the weights (O^M/O^L) given to foreign and domestic costs in the objective function to be minimized.

Table 6 displays the alternative values assigned to these key parameters. The various cases can be divided into three groups according to the basic assumptions made about exports and noncompetitive imports. In group A, the rate of growth of exports was set equal to 5 percent per year, with an appropriate sectorwise breakdown, and the values of the relevant non-competitive import coefficients (applying mainly to machinery and parts imports) were assumed to fall to one half of their levels during the Third Plan period.[28] In group B, the rate of growth of exports was raised to 7 per cent per year; and in group C, the non-competitive import coefficients were lowered to one third of their Third Plan levels.

TABLE 6. IDENTIFICATION OF ALTERNATIVE CASES

case	rate of growth of consumption (percentage)	rate of growth of exports (percentage)	noncompetitive import coefficients*
A-1	7.5	5.0	½
A-2	6.0	5.0	½
A-3	4.5	5.0	½
B-1	7.5	7.0	½
B-2	6.0	7.0	½
B-3	4.5	7.0	½
C-1	7.5	5.0	⅓
C-2	6.0	5.0	⅓
C-3	4.5	5.0	⅓

*1975 values as compared with 1960-values, for a subset of noncompetitive imports

In each group of cases, the targetted annual rate of growth of aggregate consumption between 1965 and 1975 was fixed successively at 7.5 per cent, 6.0 per cent, and 4.5 per cent respectively. From the corresponding aggregate consumption levels in 1975, related sets of values were derived for the final consumption demand, and also for the associated exogenous sector demands on both current and capital account, for the output of each individual sector. Finally, alternative solutions were generated in every case by varying the ratio of weights on foreign and domestic costs in the minimand from 4.75 to infinity. Each weight ratio corresponds to an effective rate of exchange between rupees and dollars. When the ratio is equal to 4.75, it is assumed

[28] To the extent that noncompetitive import coefficients are reduced in any given case, the corresponding coefficients for inputs of domestically produced goods are increased.

that the official exchange rate[29] measures the relative scarcity of foreign exchange. As the ratio is raised above the initial level, a premium is placed upon foreign exchange, and when the ratio becomes infinite, foreign exchange costs alone enter into the minimand.

The solution to each programming run can be described in terms of both the primal and the dual variables of the linear programming problem. The primal variables are the basic choice activities of the model : the (incremental) domestic production activities, and the alternative importing activities. The dual variables are the shadow prices associated with each of the constraints of the system. These shadow prices are expressed in terms of the minimand; thus the shadow price of each sectoral commodity denotes the cost of that commodity in terms of the given weighted combination of primary factor costs.

The primal solution to each of the programming runs includes incremental output figures for 136 endogenous production activities, as well as import levels for all of the sectors which admit of imports. Because of the vast amount of data involved, these figures will not be tabulated here,[30] instead, the primal results will be summarised in the following section in terms of their macroeconomic implications. The discussion in this section will focus on the dual results, in so far as they determine the sectoral pattern of choice between domestic production and importing activities.

A similar qualitative pattern of choice characterized each set of solutions under the various assumptions considered. In the solutions for which the weight ratio $0^M/0^L$ was set equal to 4.75, there were—in addition to essential noncompetitive imports—also competitive imports in approximately 30 of the endogenous sectors. These sectors consisted mainly of modern engineering industries, but included also some base metals and heavy chemicals; they are listed in Table 7.[31] For the remaining hundred-odd producing sectors—of which about 80 faced competitive imports—domestic production was cheaper than importing at the pre-devaluation exchange rate, and was hence preferred for every run of the model. As the weight ratio was raised to reflect an increasing premium on foreign exchange, there was a progressive substitution of domestic production activities for competitive imports. The sectors involved are listed in Table 7 in the order in which the substitution took place under the initial set of basic assumptions. This ordering was relatively insensitive to the alternative

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[29] Since most of the work on this study was completed before the devaluation of the Indian rupee on June 5, 1966, the "official exchange rate" denotes the old rate of 4.75 rupees to the dollar

[30] A detailed presentation of primal sectoral results is given in Chapter VI and Appendix B of the author's doctoral thesis upon which this paper is based. The reader should note, however, that there have been some minor changes in the form of the model, in the data base for the programming runs, since the original work was carried out for the thesis.

[31] Because of the great number of coefficients required for the numerical applications of the model (there were approximately 5000 matrix entries in the final form of the linear programming problem), it is quite possible that isolated numerical errors may have crept in at various stages of the study. Hence the precise results at the sectoral level presented in Tables 7 and 8 should be regarded as preliminary and interpreted with caution.

assumptions considered. Finally, in all of the polar solutions for which foreign exchange costs alone were minimized, the model predictably replaced all competitive imports with domestic production activities and thereby reduced the import bill to the minimum of essential noncompetitive imports.[32]

TABLE 7. IMPORT SUBSTITUTION BY SECTOR

sl. no.	code no.	sector	sl. no.	code no.	sector
1.	738	house service meters	17.	731	refrigerators
2.	611	machine tools	18.	711	thermal turbo-generators
3.	662	typewriters	19.	544	zinc
4.	624	agricultural machinery	20.	542	copper
5.	612	boilers	21.	532	ferro-silicon
6.	613	diesel engines	22.	541	aluminium
7.	733	water coolers	23.	422	soda ash
8.	160	crude oil	24.	822	railway coaching stock
9.	622	mining machinery	25.	543	lead
10.	522	special steel	26.	623	motorcycles and scooters
11.	614	pumps	27.	421	sulphuric acid
12.	637	chemical equipment	28.	445	chemical pulp
13.	821	railway wagons	29.	423	caustic soda
14.	623	drilling machinery	30.	813	electric locomotives
15.	712	hydro turbo-generators	31.	812	diesel locomotives
16.	732	air conditioners	32.	640	ball bearings

In the solutions obtained by minimizing foreign exchange costs alone, the shadow prices for each sectoral distribution constraint reflect simply the (minimal) foreign exchange content of a unit of output from the corresponding domestic production activity. For each sector the ratio of the shadow price to the alternative import price then represents the relative foreign exchange content of domestic production vis-a-vis importing activities. The higher this ratio, the lower the net saving of foreign exchange afforded by import substitution. In Table 8, 41 endogenous production activities[33] are listed in the order of their relative foreign exchange content in 1975, as calculated from the shadow prices of an import-minimizing solution under the initial set of basic assumptions. Thre is naturally a fairly close correspondence between the rank orderings in Tables 7 and 8 : sectors near the top of Table 8 are found close to the bottom of Table 7. The sequential order of import substitution presented in Table 7 depends both on the relative foreign exchange contents shown in Table 8 and on the total domestic resource content of each production activity. Sectoral differences in the latter account for the differences in the ordering of the two tables : the higher the rupee content of a domestic production activity, the later it will substitute for imports as the premium on foreign exchange is increased.

[32] It is theoretically possible for the model to prefer imports to domestic production in foreign exchange minimizing solutions if the minimal foreign exchange content of domestic production actually by exceeds the corresponding import price. That this was not the case here can be verified from Table 8.

[33] The remaining domestic production activities that compete with imports all had percentages of loss than 30 per cent under the initial set of assumptions.

TABLE 8. RELATIVE FOREIGN EXCHANGE CONTENT OF DOMESTIC PRODUCTION ACTIVITIES

sl. no.	code	activity	groups A, B	group C
1.	640	ball bearings	95.8	81.4
2.	812	diesel locomotives	84.8	72.2
3.	421	sulphuric acid	81.8	77.7
4.	423	caustic soda	79.1	68.9
5.	445	chemical pulp	78.7	64.1
6.	813	electric locomotives	72.8	63.4
7.	833	motorcycles and scooters	58.6	50.6
8.	711	thermal turbo-generators	53.0	44.6
9.	541	aluminium	52.6	45.7
10.	712	hydro turbo-generators	52.0	44.6
11.	532	ferro-silicon	49.9	43.5
12.	443	synthetic rubber	47.1	38.3
13.	822	railway coaching stock	46.5	40.1
14.	522	special steel	46.1	43.1
15.	623	drilling machinery	46.0	39.2
16.	613	diesel engines	45.6	41.5
17.	422	soda ash	44.3	38.2
18.	732	air conditioners	42.9	37.9
19.	731	refrigerators	42.9	37.8
20.	637	chemical equipment	42.9	37.8
21.	714	transformers	41.3	38.5
22.	424	other inorganic chemicals	41.1	35.9
23.	614	pumps	40.8	35.7
24.	412	phosphatic fertilizers	39.5	37.9
25.	622	mining machinery	39.5	33.2
26.	442	plastics	39.4	32.3
27.	137	other nonmetallic minerals	38.2	32.1
28.	670	other mechanical engineering	36.3	30.6
29.	444	synthetic fibres	36.3	30.6
30.	750	other electrical engineering	33.6	30.6
31.	612	boilers	33.5	28.5
32.	430	organic chemicals	33.3	28.5
33.	611	machine tools	33.2	28.2
34.	821	railway wagons	32.9	28.9
35.	662	typewriters	32.5	27.7
36.	733	water coolers	32.1	27.8
37.	617	material handling equipment	31.6	26.8
38.	720	cables, wires and flexes	31.4	26.9
39.	382	newsprint	31.4	26.9
40.	624	agricultural machinery	31.3	27.3
41.	832	commercial vehicles	30.8	26.7

TABLE 9. MACRO-ECONOMIC RESULTS

(all figures in billion rupees)

g	θ^M/θ^L	Y	C	S	I	E	M	F
				group : *A*				
	4.75	388.7	329.8	58.9	68.3	13.2	22.6	+9.4
	6.00	392.7	329.8	62.9	70.4	13.2	20.7	+7.5
7.5 percent	7.50	396.2	329.8	66.4	72.5	13.2	19.3	+6.1
	10.00	397.0	329.8	67.2	73.0	13.2	19.0	+5.8
	15.00	400.5	329.8	70.7	75.5	13.2	18.0	+4.8
	∞	403.9	329.8	74.1	78.7	13.2	17.8	+4.6
	4.75	330.7	286.5	44.2	48.6	13.2	17.6	+4.4
	6.00	333.6	286.5	47.1	50.0	13.2	16.1	+2.9
6.0 percent	7.50	335.6	286.5	49.1	51.0	13.2	15.1	+1.9
	10.00	336.1	286.5	49.6	51.3	13.2	14.9	+1.7
	15.00	338.3	286.5	51.8	52.9	13.2	14.3	+1.1
	∞	340.3	286.5	53.8	54.6	13.2	14.0	+0.8
	4.75	283.1	248.5	34.6	34.6	13.2	13.2	−0.0
	6.00	285.1	248.5	36.6	35.4	13.2	12.0	−1.2
4.5 percent	7.50	285.3	248.5	36.8	35.5	13.2	11.9	−1.3
	10.00	285.9	248.5	37.4	35.8	13.2	11.6	−1.6
	15.00	286.8	248.5	38.3	36.4	13.2	11.3	−1.9
	∞	288.0	248.5	39.5	37.5	13.2	11.2	−2.0
				group : *B*				
	4.75	391.3	329.8	61.3	68.8	15.9	23.2	+7.3
	6.00	395.5	329.8	65.6	71.0	15.9	21.3	+5.4
7.5 percent	7.50	399.1	329.8	69.3	73.1	15.9	19.7	+3.8
	10.00	399.8	329.8	70.0	73.5	15.9	19.4	+3.5
	15.00	403.5	329.8	73.7	76.2	15.9	18.4	+2.5
	∞	407.1	329.8	77.3	79.6	15.9	18.2	+2.3
	4.75	333.0	286.5	46.5	48.9	15.9	18.3	+2.4
	6.00	336.2	286.5	49.7	50.4	15.9	16.6	+0.7
6.0 percent	7.50	338.3	286.5	51.8	51.5	15.9	15.6	−0.3
	10.00	338.8	286.5	52.3	51.8	15.9	15.4	−0.5
	15.00	341.0	286.5	54.5	53.3	15.9	14.7	−1.2
	∞	343.0	286.5	56.5	55.2	15.9	14.6	−1.3
	4.75	285.4	248.5	36.9	34.7	15.9	13.7	−2.2
	6.00	287.5	248.5	39.0	35.6	15.9	12.5	−3.4
4.5 percent	7.50	287.8	248.5	39.3	35.7	15.9	12.4	−3.5
	10.00	287.9	248.5	39.4	35.8	15.9	12.3	−3.6
	15.00	289.3	248.5	40.8	36.7	15.9	11.8	−4.1
	∞	290.6	248.5	42.1	37.7	15.9	11.5	−4.4
				group : *C*				
	4.75	391.3	329.8	61.5	69.1	13.2	20.8	+7.6
	6.00	396.6	329.8	66.8	71.9	13.2	18.3	+5.1
7.5 percent	7.50	399.6	329.8	69.8	73.6	13.2	17.0	+3.8
	10.00	400.8	329.8	71.0	74.4	13.2	16.6	+3.4
	15.00	404.7	329.8	74.9	77.3	13.2	15.6	+2.4
	∞	408.1	329.8	78.3	80.3	13.2	15.2	+2.0
	4.75	332.5	286.5	46.0	49.0	13.2	16.2	+3.0
	6.00	336.1	286.5	49.6	50.6	13.2	14.2	+1.0
6.0 percent	7.50	338.0	286.5	51.5	51.7	13.2	13.4	+0.2
	10.00	338.5	286.5	52.0	52.0	13.2	13.2	−0.0
	15.00	341.2	286.5	54.7	53.8	13.2	12.3	−0.9
	∞	343.2	286.5	56.7	55.6	13.2	12.1	−1.1
	4.75	284.6	248.5	36.1	34.8	13.2	11.9	−1.3
	6.00	286.8	248.5	38.3	35.7	13.2	10.6	−2.6
4.5 percent	7.50	287.0	248.5	38.5	35.8	13.2	10.5	−2.7
	10.00	287.6	248.5	39.1	36.1	13.2	10.2	−3.0
	15.00	289.0	248.5	40.5	37.0	13.2	9.7	−3.5
	∞	290.1	248.5	41.6	37.9	13.2	9.5	−3.7

7. Macroeconomic implications of the results

The nature of the detailed solutions to the programming runs described in the previous section can be further illuminated by examining the macroeconomic implications of the sectoral results for 1975. Aggregate consumption C (at market prices) appears as a variable in the model. Aggregate investment V (at market prices) can easily be obtained by summing : (1) the fixed and working capital investment generated by the model in the endogenous sectors; (2) the exogenously given replacement investment in these sectors; and (3) the exogenously specified investments V^k in the exogenous parts of the economy. The aggregate value of exports E (in dollars) is exogenously specified together with the corresponding sectoral export demands; and the aggregate value of imports M (in dollars) can be derived by supplementing the endogenously generated import total with an estimate of the total value of imports of exogenous sector products.[34] Given the values of C, V, E and M—and converting the dollar magnitudes into rupees at the official pre-devaluation exchange rate—the corresponding values of net foreign capital inflow (F),[35] gross savings (S) and gross national product (Y) can easily be calculated by means of the usual national income identities.

Table 9 presents the 1975 values assumed by each of the above macroeconomic variables for a variety of different solutions to the programming model. A, B and C represent as before the three sets of basic assumptions about exports and noncompetitive imports; g denotes the targetted rate of growth of consumption; and $0^M/0^L$ is the effective rate of exchange (the rupee price of the dollar). For the purposes of the analysis, it is most interesting to compare the alternative values of S—as a measure of internal resources—and F—as a measure of external resources—required to sustain a given targetted rate of growth g of aggregate consumption. Figures 1-A, 1-B and 1-C display the values of S and F (in billions of rupees at 1960 prices) obtained under the alternative sets of basic assumptions A, B and C.

For each of the nine cases of Table 6, the set of alternative required combinations of internal and external resources is drawn on the appropriate diagram as a continuous contour.[36] For each group of basic assumptions, the three contours corresponding to the three different consumption targets can be interpreted as isoquants of an aggregative function relating the rate of growth of consumption to the inputs of savings and foreign capital. Additional isoquants of the same kind could be interpolated

[34] The exogenous imports included here involve a few miscellaneous agricultural and industrial products which could not be classified in any of the 147 endogenous sectors of the model. As noted in Section 2 food grains for direct consumption and military supplies for government use are exluded from this category.

[35] The net inflow of foreign capital is defined in this exercise simply as the balance of trade deficit on merchandise account, exluding the import of foodgrains and military supplies. To the extent that foreign exchange is required for the latter items, or for any net pyments under invisibles, an additional inflow of foreign capital would be called for.

[36] Because they actually represent a series of discrete steps, these lines should not really be continuous but piece-wise linear.

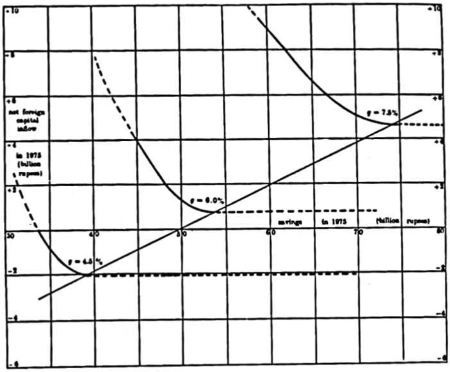
net foreign capital inflow in 1975 (billion rupees)
savings in 1975 (billion rupees)
g = 4.5 %
g = 6.0%
g = 7.5%

Fig. 1-A

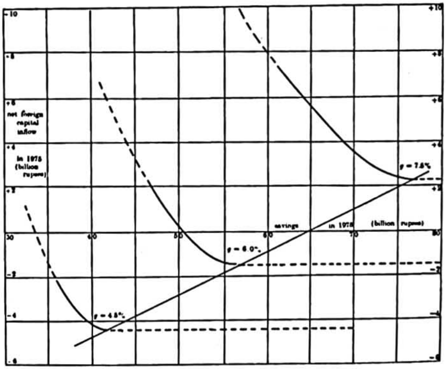
net foreign capital inflow in 1975 (billion rupees)
savings in 1975 (billion rupees)
g = 4.5%
g = 6.0%
g = 7.5%

Fig. 1-B

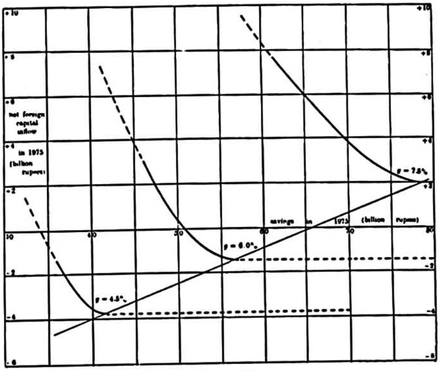
net foreign capital inflow in 1975 (billion rupees)
savings in 1975 (billion rupees)
g = 4.5%
g = 6.0%
g = 7.5%

Fig. 1.C

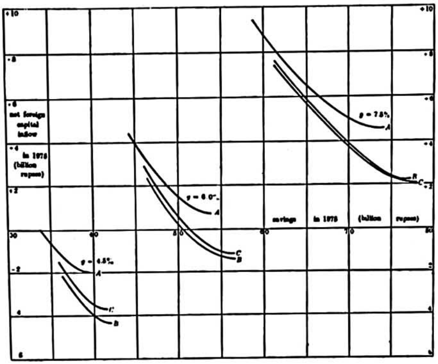
net foreign capital inflow in 1975 (billion rupees)
savings in 1975 (billion rupees)
g = 4.5%
g = 6.0%
g = 7.5%
A
B
C

Fig. 1.D

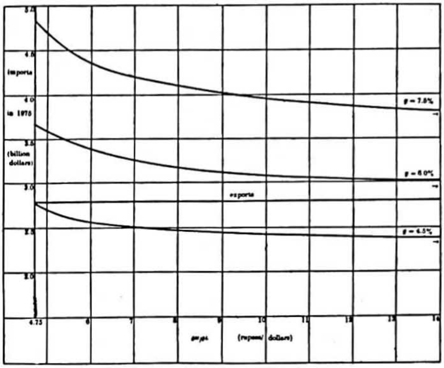
5.0
4.5
imports
4.0
in 1975
3.5
(billion
dollars)
3.0
2.5
2.0
4.75
6
7
8
9
10
11
12
13
14
(rupees/dollars)
exports
g = 7.5%
g = 6.0%
g = 4.5%

Fig. 2-A

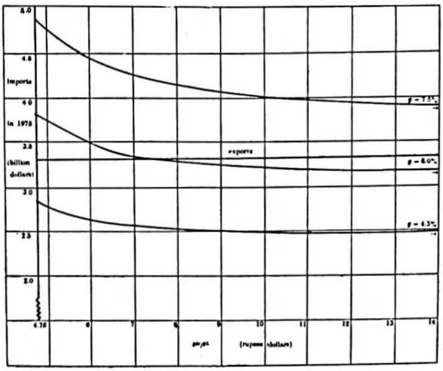
5.0
4.5
imports
4.0
in 1975
3.5
(billion
dollars)
3.0
2.5
2.0
4.75
6
7
8
9
10
11
12
13
14
(rupees/dollars)
exports
g = 7.5%
g = 6.0%
g = 4.5%

Fig. 2-B

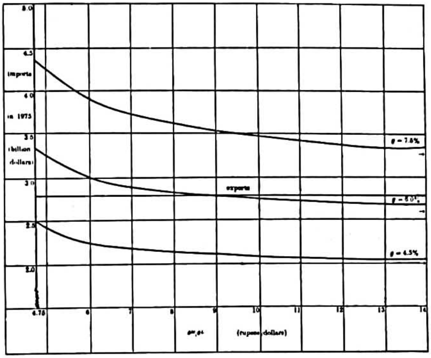

Fig. 2-C

TABLE 10. AGGREGATE COEFFICIENTS

		incremental capital output ratio 1965-1975			import-output ratio 1975* (percent)		
g	θ^M/θ^L	A	B	C	A	B	C
7.5 percent	4.75	2.03	2.03	2.04	5.82	5.95	5.34
	6.00	2.07	2.07	2.10	5.29	5.39	4.62
	7.50	2.12	2.11	2.13	4.87	4.94	4.27
	10.00	2.13	2.12	2.15	4.78	4.86	4.14
	15.00	2.18	2.18	2.21	4.51	4.57	3.85
	∞	2.25	2.26	2.28	4.41	4.46	3.73
6.0 percent	4.75	2.19	2.18	2.19	5.34	5.49	4.87
	6.00	2.23	2.22	2.24	4.82	4.94	4.25
	7.50	2.26	2.26	2.27	4.50	4.62	3.96
	10.00	2.27	2.27	2.29	4.44	4.56	3.90
	15.00	2.32	2.32	2.35	4.23	4.32	3.61
	∞	2.39	2.39	2.41	4.13	4.25	3.52
4.5 percent	4.75	2.50	2.47	2.48	4.67	4.82	4.18
	6.00	2.53	2.51	2.54	4.23	4.34	3.70
	7.50	2.54	2.52	2.55	4.18	4.30	3.67
	10.00	2.56	2.53	2.56	4.08	2.27	3.56
	15.00	2.59	2.58	2.62	3.96	4.07	3.37
	∞	2.66	2.64	2.67	3.89	3.97	3.28

*1965 value = 7.66

to represent different consumption targets. The left-hand end of the continuous part of each contour corresponds to the solution in which the weight ratio in the minimand conforms to the official pre-devaluation rate, while the right-hand end corresponds to the solution in which all the weight is placed on foreign exchange. The contours could also be extended further to the left (as indicated by the broken lines), where they would correspond to solutions based on weight ratios giving even greater emphasis to domestic vis-a-vis foreign costs.

Read from left to right, the isoquants of Figures 1-ABC reflect the substitution of domestic production activities for competitive imports that takes place as the premium on foreign exchange is raised. The marginal rate of substitution between savings and foreign capital inflow—given by the slope of the isoquants—shows considerable invariance under the alternative assumptions considered. Up to an effective exchange rate of about twice the official pre-devaluation rate, the isoquants are almost straight lines[37] and are also reasonably parallel as between cases. Thus for a wide range of combinations there is a more or less constant trade-off between domestic and foreign effort which equates one rupee of net foreign capital inflow with roughly two rupees of gross domestic savings.

The marginal rate of substitution between savings and foreign exchange increases rapidly as the foreign exchange minimizing solution is approached at the right hand end of each contour. This point defines the limit beyond which savings alone are of no avail in raising consumption possibilities. Further to the right, there is no more scope for import substitution, and the isoquants become straight lines parallel to the savings axis at a level representing the minimum net inflow of foreign capital required to sustain the given targetted rate of growth of consumption.

For each set of basic assumptions, a cut-off line joining the right-hand ends of the three different consumption isoquants divides the range of values where there are substitution possibilities (to the left) from the range of values where there is no further scope for import substitution (to the right). Each cut-off line can be used to determine the maximum amount of savings that can be translated into productive investment, and hence also the maximum sustainable rate of growth of consumption, corresponding to any given net inflow of foreign capital. Conversely, the cut-off line can be used to evaluate the minimum level of net foreign capital inflow consistent with any given rate of growth of consumption.

Under the initial set of basic assumptions, the maximum rate of growth of consumption that can be sustained without any net capital inflow appears from Figure 1-A to be approximately 5.5 per cent per year. This would call for gross savings of about 50 billion rupees in 1975, representing an average rate of saving of 15 per cent

[37] The isoquants cannot be perfectly straight lines, for at each successive import substituting step the marginal rate of substitution necessarily changes.

in 1975, and an implied marginal rate of saving between 1965 and 1975 of close to 19 per cent. To achieve a targetted rate of growth of consumption of 7.5 per cent per year, the minimum net capital inflow in 1975 would appear to be between 4 and 5 billion rupees. This in turn would require gross savings of close to 75 billion rupees in 1975, which implies an average rate of 18½ and a marginal rate of 23½ per cent. Alternative strategies with less emphasis on import substitution would allow the same consumption targets to be achieved with lower rates of saving and higher levels of foreign capital inflow.

To study the effect of changing the underlying assumptions about exports and noncompetitive imports, it is helpful to superimpose the isoquants of Figures 1-B and 1-C on those of Figure 1-A; the result is shown in Figure 1-D. As compared with the initial set of assumptions *A*, it will be observed that the more optimistic export projections of *B*, or the lower values for noncompetitive import coefficients of *C*, have the effect of displacing the isoquants downward. Thus they allow the same consumption targets to be satisfied with less savings and/or less foreign capital inflow, and they allow higher consumption levels to be attained with any given combination of internal and external resources. Furthermore, the isoquants—and hence the cut-off lines—under *B* and *C* are also shifted to the right, relative to their position under *A*. This means that a greater amount of savings can be translated into productive investment for any given level of net foreign capital inflow.

At a zero trade deficit, either the higher export projections or the lower noncompetitive import coefficients allow for a maximum (productive) level of gross savings in 1975 of approximately 65 billion rupees, which in turn will sustain a maximum rate of growth of consumption of the order of 6.5 per cent. As compared with the initial results, the more optimistic assumptions thus permit an increase of 1 per cent in the rate of growth of consumption without any additional foreign capital inflow. The corresponding average and marginal savings rates are 18 per cent and 23 per cent, respectively, representing increases of 3 per cent and 4 per cent over the requirements of the initial case. These differences serve to emphasize the critical importance of the basic assumptions underlying each particular solution to the programming model.

The macroeconomic results of the study can also be interpreted in terms of the aggregate demand for, and supply of, foreign exchange. Under any given set of basic assumptions, and for any given targetted rate of growth of consumption, the variation in the effective rupee price of the dollar (O^M/O^L) traces out an aggregate demand curve for foreign exchange.[38] As the price rises from the official pre-devaluation rate of 4.75 up to infinity, the successive solutions to the model provide for a progressive reduction in the total import bill and hence in the demand for foreign exchange. Figures 2-ABC illustrate this relationship for the alternative consumption growth

[38] For a similar interpretation, see Chenery (1955); Figure 2 in the cited article is analogous to Figures 2-ABC of the present study.

targets under each set of basic assumptions A, B and C. Each demand curve is shown as a smooth curve[39] in the range of prices from 4.75 to 14.00 rupees per dollar; a short arrow at the right-hand end of each curve defines the minimum value of the import bill attained as the price of foreign exchange goes to infinity.

Largely because of considerations of data availability, the scope for optimization in the model was limited to the choice between domestic production and importing activities, and no endogenous mechanism was permitted to choose among exporting activities.[40] As a result, the sectoral export levels were made independent of the variation in the weights given to domestic and foreign primary resource costs in the minimand, and the aggregate supply curve relating the supply of foreign exchange to its effective price becomes in each case a horizontal line. In Figures 2-A and 2-C, the aggregate supply curve is drawn at the constant total value of exports of 2.78 billion dollars; in Figure 2-B, exports are fixed at a level of 3.35 billion dollars.

The supply curves of Figures 2-ABC bring out clearly the implicit assumption of price inelasticity in the external demand for—or the internal supply of—each endogenous sectoral export. If in fact there is significant price elasticity in the sectoral demand for and supply of exports, the aggregate supply curves of foreign exchange should have positive slopes whose magnitude would depend on the numerical values of the relevant elasticities. To the extent that optimal sectoral export levels do vary with the effective price of foreign exchange, the programming model understates the possibilities of substitution between domestic and foreign resources in the economy, and the implications of any single solution to the model should be interpreted with caution.

8. Conclusion

It may be useful, in conclusion, to compare the qualitative nature of the macroeconomic results that emerge from the multisectoral model of this study with the results obtained from aggregate models of a similar kind. Chenery and Bruno (1962), McKinnon (1964), and Chenery and Strout (1966) have worked with aggregate models emphasizing the two independent constraints on growth imposed by savings, on the one hand, and by foreign exchange, on the other. The savings constraint is a familiar one : assuming a constant incremental capital-output ratio, the rate of growth of an economy is limited by the rate of investment which is equal to the sum of domestic savings and foreign savings (net capital inflow).

The phenomenon of an independent foreign exchange constraint has been more recently stressed in connection with the industrialization of underdeveloped economies.[41] When exports are limited exogenously (e.g., by stagnant world demand), and when noncompetitive imports are required in fixed proportions for domestic production and/or

[39] Like the isoquants of Figures 1-ABC, the demand curves of Figures 2-ABC actually cover a series of discrete steps and should be piece-wise linear.

[40] See footnote 4 above.

[41] See McKinnon (1964) for a concise discussion with references to earlier work.

investment, there is always a point beyond which potential domestic savings cannot be put to use, and the growth of domestic output cannot be increased, for lack of foreign exchange to purchase specific complementary imports. At this point, a higher growth rate can be attained only by working directly on the foreign exchange constraint—by increasing exports, reducing noncompetitive imports, or receiving additional foreign aid (net capital inflow).

The implications of a simple aggregative model embodying these two constraints could also be portrayed in the form of the graphs in Figures 1-ABC. With a single aggregate capital-output ratio, and a single aggregate ratio of imports to total output,[42] the result would be consumption isoquants consisting of two straight lines meeting at a cut-off line of the same kind as shown in the figures. To the right of the cut-off line, the isoquants would be parallel to the savings axis, reflecting the fact that the foreign exchange constraint was binding and additional savings alone were of no use in raising consumption possibilities. To the left, the isoquants would be straight parallel lines, reflecting the constant trade-off between savings and foreign capital inflow that prevails when the savings constraint is binding. Since, under these circumstances, foreign capital inflow plays only the role of foreign savings, the slope of the lines would be 45 degrees in the case of *output* isoquants. In the case of consumption isoquants of the kind shown in Figures 1-ABC, the slope of the line would be less than 45 degrees because, unlike domestic savings, foreign savings add to the total supply of savings without subtracting from total consumption.

By contrast with the results of an aggregative model, the results of the multi-sectoral model of this study—involving a wide range of substitution possibilities between domestic production and imports—show a relatively smooth approach to the foreign exchange bottleneck. There is still, to be sure, a cut-off line beyond which no further possibilities for substitution arise; however, this cut-off line is reached only after all possibilities for import substitution have been exhausted. In the process, the overall import-output ratio in the economy gets depressed to a minimum level well below its base year value, and the overall capital-output ratio rises above what it would have been with less import substitution Thus the rigid implications of the aggregative model are tempered by the introduction of choice among linear activities at the sectoral level.

The element of choice in the programming model is brought into play by variation of the effective rate of exchange between the rupee and the dollar. The resulting re-allocation of domestic and foreign resources is reflected by changes in the values of the aggregate capital-output and import-output ratios. Table 10 presents

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[42] The assumptions of a single capital-output ratio and a single import coefficient could be relaxed to accomodate different coefficients associated with consumption and investment; the basic character of the aggregative model, as well as the conclusions, would remain unaffected.

the alternative values[43] for these ratios implied by alternative solutions to the programming model. A, B and C represent as before the three sets of basic assumptions about exports and noncompetitive imports; g denotes the targetted rate of growth of consumption; and $0^M/0^L$ is the effective rate of exchange (the rupee price of the dollar). It is clear from the table that the values of the two ratios vary not only with $0^M/0^L$, but also significantly with g and—in the case of the import ratio—with the alternative assumptions A, B and C.

The multisectoral linear programming model of this study provides a more flexible—and hence also a more realistic—representation of the economy than any simple aggregative, or less disaggregated, model could. In at least two important respects, however, a greater degree of realism could be achieved by widening the scope for choice in the model. First—as discussed at the end of Section 7—sectoral export levels could be made a function of the effective rate of exchange.[44] Secondly, the sectoral composition of consumption,—which was fixed (at the margin) for the present exercise—could also be allowed to adjust to some extent to the relative scarcity of domestic and foreign resources.[45]

Each of these extensions would have the effect of allowing for even greater flexibility in the model; for any given variation in the effective rate of exchange, an even greater degree of re-allocation of resources would be called for. The increased possibilities of substitution would further weaken the rigid conclusions of the simplest aggregative model, and push somewhat further back the spectre of a foreign exchange bottleneck. Whether this spectre can—in the Indian case—actually be wholly exorcised would depend on the extent to which export levels and consumption patterns are, or can be made, responsive to price changes. This is clearly a subject on which much more quantitative research will have to be carried out before any definitive judgments can be made.

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[43] The incremental capital-output ratios listed in Table 6 were calculated by relating the total cumulative gross investment requirements from 1963 to 1973 to the increase in aggregate production capacity from 1965 to 1975; thus a two-year average investment output lag was assumed. The actual figures obtained depend on an estimate of the extent to which overall capacity exceeded actual output in the Indian economy in 1965. Since any such estimate is necessarily very uncertain, the absolute values given in the table should be interpreted with some caution. More reliance can be placed on *relative* values, which are in any case the more significant from the point of view of the analysis.

[44] Work is now in progress on an extension of the model to incorporate choice of exporting activities with diminishing marginal earnings of foreign exchange.

[45] I am indebted to S. Chakravarty for emphasizing this point.

Appendix

DATA SOURCES AND METHODOLOGY

A vast body of data was required to apply the model formulated in Sections 3-5. The data requirements fall into three groups : (A) values for coefficients which describe the structural relationships between the economic variables of the model; (B) values for variables which depend only on the historical performance of the economy, and are thus (in principle) known in advance; and (C) values for parameters which relate to the specification of the targets of the model, and which must be exogenously assigned in advance. In the pages to follow, the sources and methodology used for obtaining each group of data requirements will be briefly discussed; a detailed presentation, together with a complete listing of the actual numerical values used in the programming runs, is given in Chapter V and Appendix A of the doctoral thesis upon which this paper is based.[46]

(A) The structural coefficients required for the implementation of the model can be divided into three groups consisting of matrices, vectors and scalars, respectively. The matrices U, A°, A^{*}, B^{*} and S^{*} describe the techno-economic interindustry relations between the 147 endogenous sectors (listed in Table 1) and the 136 domestic production activities. As discussed in section 2, a single production activity corresponds to each of the endogenous sectors with the exception of six sectors of noncompetitive imports (142, 144, 413, 545, 546, 547) and six sectors of petroleum products (241 through 246). The matrix U measures the output of sector i products which results from operating the production activity j at the level of a single unit; thus u_{ij} is equal to one wherever there is a one-to-one correspondence between sector i and activity j, and u_{ij} is otherwise nonzero only in the case of the six petroleum products produced by the single petroleum refining activity. The six corresponding coefficients were determined according to the average output proportions recorded in the production statistics for the Third Plan period.

Two current flow matrices were required for the application of the model : the matrix A°, which is applied to production from (old) capacity existing already in 1965, and the matrix A^{*}, which is applied to production from (new) capacity installed between 1965 and 1975. The matrix A° was constructed first, with coefficients applicable to production activities during the Third Plan period. Subsequently, an evaluation was made of all those coefficients a^{0}_{ij} for which information was available on changes in the future: this information was processed to yield incremental flow coefficients a^{*}_{ij} which were substituted into the initial matrix A° to yield the new matrix A^{*}. The use of these two distinct current flow matrices to deal with technological changes in input structure is preferable to the use of a single combined A matrix for 1975 insofar as new processes depend upon the introduction of new plant and equipment rather than on the mere passage of time. This is likely to be the case in most of the endogenous production activities of the model, except with respect to inputs of transport services. The shift which is anticipated from rail to road goods transport cannot be regarded as a function of the increases in industrial capacity between 1965 and 1975, and hence a single set of expected 1975 transport input coefficients were entered into both the A° and the A^{*} matrices.

[46] As noted in footnote 30, there have been some minor revisions of the data—based on more recently available figures—since the original work was carried out for the thesis.

The guiding principle in the construction of the current flow matrices was to rely as far as possible on technologically determined engineering data rather than on statistical surveys for any given year. Because of the attention focussed in this study upon import substituting industries, many of which in India were either non-existent or operating at very low levels in the past, historical data obtained from available surveys are often quite unsatisfactory. For such industries it is preferable—if not essential—to draw upon the technologies of other, more developed economies. In the form of norms used in the construction of 'material balances' for a variety of strategic commodities, much useful information about the present and future technological structure of the Indian economy has been gathered by the Perspective Planning Division (PPD) of the Planning Commission. In addition, a number of other research groups and organizations have prepared demand studies based upon the use of technological as well as historical norms. In this study an attempt was made to incorporate this information into the input-output matrices so that isolated material balances and demand projections could be analysed within a single, structurally interdependent framework. The extensive disaggregation of the industrial part of the economy into well-defined homogenous sectors, whose output can be measured in physical as well as value units, greatly facilitated the absorption of the technological norms.

The sources of data for the matrix coefficients consisted mainly of material balances, demand studies and utilization patterns for individual products as inputs. Where this type of row-wise data was not available, the more usual column-wise data provided by official industrial surveys was drawn upon to complete the coverage. In the industries for which small-scale production was significant, norms were evaluated separately for large-scale and small-scale enterprises and aggregated with weights reflecting their respective contributions to output in 1965. After the base year matrix A^0 was constructed, a consistency check was carried out by applying the coefficients to the actual output levels of the endogenous activities in 1965 and balancing the input-output accounts. In the few sectors where the discrepancy was too large to be explained by changes in stocks, some further readjustments were then applied to the coefficients of input into the residual sectors for which the data base was in any case the weakest.

The incremental matrix A^* differed from the base year matrix A^0 in a number of significant respects. All of the inputs of fuel and power were different. As noted earlier, the transport coefficients were adjusted in both the A^0 and A^* matrices to reflect the changed pattern of transport anticipated in 1975. The substitution of aluminium for copper in metal fabricating industries was explicitly recognized, and a number of other metal input coefficients were changed to reflect the economizing of non-ferrous metals. Changing input structures for steel, nitrogenous fertilizers and several other chemical industries were also taken into account. The very fact of a high level of disaggregation in the sector classification obviates the need to anticipate changes in the structural coefficients due to changing product mixes within more broadly defined sectoral aggregates.

The construction of the capital matrix B^* proceeded along two separate lines involving the inputs, respectively, of machinery and construction materials.[47] For each production activity, the ratio of the ex-works value of machinery and equipment to the total value

[47] Construction is not treated in the model as an endogenous industrial activity. Instead, the demand for construction materials from the endogenous activities is related directly to the expansion of the individual activities by including in the capital matrix coefficients for construction materials as well as for machinery and equipment.

of investment was first determined, and this ratio was further broken down into the three broad categories—nonelectrical machinery, electrical machinery, and transport equipment—on the basis of information drawn from studies carried out by the PPD. The remaining detailed breakdown into the machinery classification required for this study was then determined by using proportions based on a matrix of capital coefficients prepared for the United States economy.[48]

The general procedure for evaluating construction material coefficients was as follows. First, the economy was divided into fifteen exhaustive sectors for which the nature of the construction activity was materially different. Nine of these construction sectors encompassed the sectors classified as exogenous in this study; the remaining six—hydro electricity generation, thermal electricity generation, electricity transmission and distribution, mining and oil production, large-scale manufacturing industries and small-scale industries — covered the endogenous activities of the economy. For each of the fifteen construction sectors, the total investment outlay in the Third Five Year Plan, the proportion of the outlay intended for construction, and the estimated demand for the individual construction materials were tabulated on the basis of PPD and various other demand studies. Ratios of the demand for these construction materials to the total investment outlay were then calculated for each of the construction sectors. In the case of several minor construction materials, common input coefficients for all sectors were evaluated according to the ratio of the total availability of the material to the total construction outlay in 1965.

It would have been possible to include the fifteen types of construction sectors as additional activities within the endogenous interindustry structure. Since most of the construction sectors, however, are specific to particular exogenous or endogenous activities (or groups of activities), it was more convenient to relate the demand for construction materials directly to the corresponding capital formation. In the case of the nine exogenous sectors, the construction demand ratios discussed above were retained for use in calculating the exogenous investment demand coefficients for construction materials. In the case of the six endogenous construction sectors, the ratios were suitably weighted to derive for each of the endogenous mining, power and manufacturing activities in this study the relevant construction material coefficients in the capital structure matrix.

The capital structure of the transportation services poses special problems, and was not treated in the same way as in the other endogenous activities. Only transport equipment was related directly to increases in endogenous transport capacity. Railway construction and maintenance as well as the development of the national road system were treated as exogenous capital formation, and the demand on these accounts was included in the corresponding exogenous demand sectors. The requirements of the different kinds of transport equipment are related in the matrix to the total value of the equipment in the corresponding transport activity. Since it is incremental coefficients which are relevant for the treatment of investment in the model, and since there will be a marked change in the pattern of utilization of steam, diesel and electric locomotives, as well as in the efficiency of utilization of wagons, incremental stock coefficients—derived on the basis of information on future changes given in PPD studies—were substituted wherever necessary for base year coefficients obtained from official transport statistics.

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[48] See R. N. Grosse (1953).

As a rough consistency check on the capital-capacity ratios (discussed later) and the capital structure matrix, the vector β^* and the matrix B^* were applied to the estimated changes in capacity in the endogenous production activities between 1960 and 1965. The resultant endogenous demand for inputs into net fixed capital formation was then compared with total availability of capital equipment and construction materials during the five year period 1960-1965, after deducting for replacement demand and demand on capital account from the exogenous sectors. Wherever significant discrepancies arose, uniform adjustments were made in the input coefficients from the sector in question.

The last of the structural matrices required in the model is the (incremental) matrix of inventory coefficients S^*. In the absence of detailed information on the interindustry structure of working capital stock requirements, the simplifying assumption was made that a supply equivalent to three months' output of each storable commodity is held as inventory throughout the economy. Thus S^* becomes an almost diagonal matrix of the same form as U, except that zero s_{ij}^* entries correspond to u_{ij} entries for sectors whose products cannot be held in inventory (e.g. electricity, transport services). The value of each nonzero s_{ij}^* is then precisely equivalent to one quarter of the corresponding value u_{ij}.

The vectors of structural coefficients required by the model include the following : (1) the incremental capital-capacity ratios β_j^*; (2) the consumption and exogenous demand coefficients γ_i^*, τ_i^k and ϕ_i^k; (3) the noncompetitive import coefficients μ_i, ψ_j and ξ_j; (4) the durable good stock-flow ratios σ_i; (5) the import prices π_i; and (6) the labour cost coefficients λ_j^0, λ_j^* and λ_j^β.

The β_j^* were calculated primarily on the basis of figures collected by the PPD on expected additions to capacity, and the corresponding requirements of fixed investment, in the Fourth Plan period.

Both the γ_i^* and the τ_i^k were calculated indirectly by first projecting consistent sets of consumption and exogenous intermediate good sectoral demands to the year 1975, and then determining the γ_i^* and τ_i^k implicit in the projections. The consumption projections were made partly on the basis of information on consumer behaviour obtained from studies with National Sample Survey data, and partly on the basis of production levels for 1975 projected by the PPD. The resulting values thus take into account both the differing expenditure elasticities of consumer demand, and the differing priorities for consumption goods intended by the government planners. The projections of exogenous sector demand for endogenous intermediate goods were based on the view of the future contained in PPD studies.

The ϕ_i^k were determined directly according to sectoral norms used in studies of investment demand in the Third and Fourth Plan periods. The construction material norms were obtained in exactly the same way as the construction material coefficients for the endogenous activities in the matrix. Machinery norms were calculated by relating the total estimated availability of each relevant type of machinery to the corresponding investment outlays in the Third Plan period.

The values given to the noncompetitive import coefficients μ_i, ψ_j and ξ_j in the target year 1975 are crucial to the outcome of each run of the programming model. Estimates for all of these coefficients were first made on the basis of production and import data compiled for the Third Plan period, using the average ratios prevailing during the whole period.

Subsequently, an assumption had to be made about the extent to which the values of these coefficients could be reduced by 1975. In order to allow for analysis of the sensitivity of the results to variations in the coefficients, two different assumptions were made to derive two alternative sets of values. The initial assumption was that all of these noncompetitive import coefficients would be cut in half by 1975; the alternative—and more optimistic—assumption was that the values would be reduced to a third of their initial levels.

The coefficients ξ_j for imported parts and components were evaluated with the help of import data for the engineering sectors, where a distinction was made between the import of complete units and of parts. The total value of parts imports of each type of equipment in any given year had to be related to the sum of the domestic production and a part of the existing stock of that type of equipment. The fraction of the stock to be included represents that proportion for which imported parts have to be replaced every year : it can be estimated as the reciprocal of the average life of the relevant parts. For the purposes of the study, the average life of all imported parts—denoted in Section 3 by z—was assumed to be two years. The required production data were collected from standard sources and the required capital stock figures were estimated according to a formula analogous to equation (3.30) of Section 3.

In order to deal with the import of parts and components for maintenance of exogenous sector capital stock, values of the stock-flow conversion factor σ_i are required for each sector i producing durable equipment. If the output of sector i is assumed to grow at a constant exponential rate of growth r_i, and if the average life of equipment of type i is z_i, then the following formula can be derived[49] for σ_i :

$$\sigma_i = \frac{1-e^{-r_i z_i}}{r_i} \qquad \text{... (A.1)}$$

In the estimation of the σ_i, the rates of growth r_i were assumed to be uniformly equal to 10 per cent per year, and the z_i were set equal to 10 years for consumer durables, 30 years for transport equipment, and 20 years for all other types of machinery and equipment.

The sectoral import prices π_i are crucial to the operation of the choice mechanism of the model, since they determine the relative profitability of importing activities which compete with domestic production activities. The price π_i represents literally the rupee value (at 1960 producers' prices) of a dollar's worth of imports (at c.i.f. prices) of sector i products. For the majority of sectors of the model whose output is reasonably homogeneous and can be measured in consistent physical units, it was possible to evaluate the π_i by dividing the appropriate 1960 rupee producers' prices per physical unit by the corresponding dollar import prices obtained from foreign trade statistics.

For the remaining sectors (including all of the 'residual' sectors defined in Section 2, and most of the machinery sectors of the 600 and 700 groups) whose output encompasses a a wide variety of related products, this procedure could not be followed. The heterogeneity of the sectoral output, as well as the unquantifiable differences in quality that characterize even a single type of product, rendered futile any attempt to collect a representative set of comparable import and domestic producers' prices. In the face of such statistical difficulties, it was necessary to make an explicit (if somewhat arbitrary) assumption about the rupee value of a dollar's worth of imports.

[49]For a derivation of this formula, see footnote 10 to Chapter V of the doctoral thesis.

The approach adopted for the purposes of the model was to set the import price π_i for each heterogeneous sector equal to the official exchange rate[50] of 4.75 rupees to the dollar. This procedure implies that a rupee's worth of domestic production at *producers' prices* is equivalent to a rupee's worth of imports at *c.i.f. prices*. It is consistent with the assumption that the excess of the domestic market price over the c.i.f. import price of the same good can be accounted for entirely by the margin of indirect taxes, trade and transportation costs over the ex-works price of domestic output.[51]

Although three sets of labour coefficients ($\lambda_j^0, \lambda_j^*, \lambda_j^\beta$) occur in the algebraic formulation of the model, a lack of sufficient quantitative information made it impossible to distinguish between the operating labour cost of old (λ_j^0) and new (λ_j^*), productive capacity. Thus numerical values were estimated for only two sets of coefficients : λ_j, the total wage bill per unit of production in activity j, and λ_j^β, the total cost of construction labour per unit increase in capital stock in activity j. The λ_j were estimated by multiplying ratios of value added to producer's price (compiled by the PPD) by ratios of wage cost to value added (drawn from another inter-industry study of the Indian economy).[52] The λ_j^β were determined by multiplying coefficients of total construction cost per unit increase in each sector j by a fraction representing the wage proportion of total construction cost, which varied between activities according to the share of large-scale and small-scale enterprises; the same sources were used as in the estimation of the λ_j.

(B). The historically predetermined variables include the base-year production levels $\bar{x}_j^0$; the base-year consumption levels $\bar{c}_i^0$; the target year residual capacity levels $\bar{x}_j^R$; the target year residual stock levels $\bar{s}_i^R$; and the target year exogenous replacement investment demand $\bar{v}_i^r$. The $\bar{x}_j^0$ were determined from official sources of production statistics for organized industries, supplemented by information on small-scale industrial output given in various plan documents. For a majority of sectors, figures were obtained first in physical units, and subsequently converted into 1960 producers' price values on the basis of price data obtained from standard statistical sources. For most of the endogenous sectors whose products enter into consumption, the entire domestic availability of output (i.e. production plus imports minus exports minus additions to stock) could be attributed to consumption and used to estimate

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[50] See footnote 20.

[51] The use of the official exchange rate as a measure of the marginal rate of substitution between a dollar's worth of imports (at c.i.f. prices) and a rupee's worth of domestic output (at producers' prices) is open to the criticism that tariffs and/or quota restrictions on many of the goods concerned suggest a higher implicit exchange rate than 4.75 rupees to the dollar. In the absence of a detailed investigation into the structure of tariffs and quota restrictions (a task which was beyond the scope of this study), it might be desirable to experiment with implicit exchange rates for the heterogeneous sectors set uniformly higher than the (pre-devaluation) official rate—e.g., at the post-devaluation rate of 7.5 rupees to the dollar. It should be noted, however, that any such change would call not only for revision of the π_i, but also for adjustments in all of the input coefficients involving the corresponding rows i. This is because the coefficients were originally estimated on the basis of flows involving both domestically produced and imported goods, which were aggregated (according to standard practice) at the prevailing exchange rate of 4.75 rupee to the dollar. In order to achieve consistency with a different set of implicit exchange rates, it would be necessary to re-compute those coefficients using the new rates for aggregating the domestic and foreign component of each input flow.

[52] See Manne and Rudra (1965).

the corresponding $\bar{c}_i^0$. In the remaining sectors whose output is only partly destined for consumption, the $\bar{c}_i^0$ were estimated as row residuals from a set of input-output accounts constructed with the aid of the base-year current flow matrix A^0.

In the absence of specific data for the individual productive activities regarding the residual capital stock that will be available in 1975, it was necessary to estimate the x_j^R and $\bar{s}_i^R$ by applying depreciation factors to the actual levels existing in the base year 1965. If it is assumed that the stock of capital equipment in each activity j has been growing at an exponential rate r_j, and that each type of capital equipment i is used for exactly z_i years before it is retired, the following formula can be derived[53] for the fraction δ_{ij} of base year capital equipment of type i in activity j which will have been retired within T years:

$$\delta_{ij} = \frac{e^{r_j T}-1}{e^{r_j z_i}-1} \qquad \text{... (A.2)}$$

Thus if $\bar{k}_{ij}^0$ and $\bar{k}_{ij}^R$ denote base year and target year residual capital equipment of type i in activity j, we have

$$\bar{k}_{ij}^R = (1-\delta_{ij})\bar{k}_{ij}^0. \qquad \text{... (A.3)}$$

Now it is clear that $\bar{s}_i^R$ can be expressed very simply as

$$\bar{s}_i^R = \sum_j \bar{k}_{ij}^R = \sum_j (1-\delta_{ij})\bar{k}_{ij}^0. \qquad \text{... (A.4)}$$

In order to determine the depreciation factor applicable to base-year productive capacity of each activity j, it is necessary to allow for the different average lives z_i which characterize different types of capital equipment i. By convention, this depreciation factor δ_j was determined according to the rate implied by the type of capital equipment with the longest life (and hence the slowest rate of depreciation). Thus

$$\delta_j = \min_i \delta_{ij} \qquad \text{... (A.5)}$$

and the target year residual $\bar{x}_{ij}^R$ was hence expressed as

$$x_j^R = (1-\delta_j)\bar{p}_{ij}^0 \qquad \text{... (A.6)}$$

where $\bar{p}_j^0$ denotes the base year capacity level in activity j. It follows that, for all other types of capital equipment, a capital replacement term k_{ij}^* must be calculated to make up the difference between the stock of equipment of type i actually remaining in the target year and the stock required to sustain the target year residual productive capacity in activity j. This replacement term is given by

$$k_{ij}^* = (\delta_{ij}-\delta_j)\bar{k}_{ij}^0. \qquad \text{... (A.7)}$$

The total amount of capital of type i that is required for such replacement in the endogenous activities between the base and the target years is thus equal to $\sum_i k_{ij}^*$, and this generates the exogenous replacement investment demand $\bar{v}_i^T$ in the target year:

$$\bar{v}_i^T = \eta^P \sum_i k_{ij}^* = \eta^P \sum_i (\delta_{ij}-\delta_j)\bar{k}_{ij}^0. \qquad \text{... (A.8)}$$

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[53] For a derivation of this formula, see footnote 4 to Chapter V of the doctoral thesis.

Thus the historically predetermined variables $\bar{x}_j^R$, $\bar{s}_i^R$ and $\bar{v}_i^F$ were estimated[54] (according to equations (A.4), (A.6) and (A.8)) in terms of the 1965 capacity and capital stock levels $\bar{p}_j^0$ and $\bar{k}_{ij}^0$, the stock-flow conversion factor η^F and the depreciation factors δ_{ij}. The δ_{ij} are in turn functions of the rates of growth r_j prior to 1965, the average lives z_i of capital equipment and the ten-year time interval T between 1965 and 1975. The values for $\bar{p}_j^0$ were drawn from plan documents. In the absence of more accurate information, values for $\bar{k}_{ij}^0$ were calculated according to the following equation :

$$\bar{k}_{ij}^0 = b_{ij}^* \beta_j^* \bar{p}_j^0 \qquad \ldots \quad (A.9)$$

where b_{ij}^* and β_j^* are the capital coefficients and incremental capital-capacity ratios for production activity j. The rates of growth r_j were calculated on the basis of available time series on industrial output. The average life of all types of machinery in the industrial production activities was assumed to be 20 years, and the average life of construction materials and structures was assumed to be 40 years; hence there were just two δ_{ij} to be calculated for each activity j.

(C). The exogenously specified parameters of the programming model include : (1) the aggregate value of endogenous consumption $\overline{C}$ in the target year 1975; (2) the corresponding value of $\bar{T}^k$ and $\bar{\Gamma}^k$ for each of the exogenous sources of demand in 1975; (3) the expected rate of growth of population n between 1965 and 1975; (4) the vector of export demands $\bar{e}_i$ from each endogenous sector in 1975; (5) the set of upper bounds $\bar{x}_j^U$ on production levels for crude oil and a limited number of mining activities in 1975; and (6) the stock-flow conversion factors η^F and η^{IF} in both their primal and their dual interpretations.

Three alternative sets of assumptions were made for the target year variables $\overline{C}$, $\bar{T}^k$ and $\bar{\Gamma}^k$, corresponding to annual rates of growth of total consumption between 1965 and 1975 of 7.5 per cent, 6.0 per cent, and 4.5 per cent, respectively. Given each rate of growth, as well as an estimate of 1.0 for the elasticity of endogenous sector consumption with respect to total consumption, the known 1965 level of endogenous consumption $\overline{C}^0 = \sum_i \bar{c}_i^0$ was projected to the target year 1975 to determine the corresponding value of $\overline{C}$. Estimates for the associated target year values of $\bar{T}^k$ and $\bar{\Gamma}^k$ were first made for the case in which total consumption was assumed to grow at an annual rate of 7.5 per cent. Since this leads to 1975 figures which are close to those set out in the perspective plan of the Planning Commission,[55] the estimates for $\bar{T}^k$ and $\bar{\Gamma}^k$ were based on figures drawn from related plan documents. The corresponding estimates for the remaining cases of 6.0 per cent and 4.5 per cent rates of growth of total consumption were derived by reducing somewhat arbitrarily the initial estimates for the 7.5 per cent case.

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[54] Because the nature of the capital equipment required for replacement and expansion of capacity in several of the transport activities was known to differ considerably from the nature of the stock existing in 1965, a slightly different procedure was used to calculate the $\bar{x}_j^R$, $\bar{s}_i^R$ and $\bar{v}_i^F$ in these activities.

[55] See Government of India (1964).

The rate of growth of population n was set equal to 2.5 per cent per year in every case, so that the implied rates of growth of *per capita* consumption were 5.0 per cent, 3.5 per cent and 2.0 percent respectively.

Two alternative sets of projections were made for the 1975 endogenous sectoral export demand levels $\bar{e}_i$; and in each case the corresponding dollar values of total exports and exogenous imports[56] in the target year were also estimated in order to permit the determination of the overall balance of trade. The first set of export demand projections—reflecting an annual rate of growth of total exports of 5 per cent from 1965 to 1975—was based on figures drawn from the Ministry of Commerce,[57] and the second set—reflecting an annual rate of growth of 7 per cent—was based primarily on the perspective plan of the Planning Commission.[58]

The figures used for the production bounds $\bar{x}_j^M$ were based on a mixture of guess work and forecasts in various plan documents.

According to equations (5.4) and (5.5) of Section 5, the primal values to be assigned to the stock-flow conversion factors η^F and η^W can be derived as a function of the four parameters : T(the time interval from the base to the target year), θ (the average investment gestation lag), r^0 (the average annual rate of growth of capital stock between the base and the target year), and r^T (the average annual rate of growth of capital stock after the target year). Thus, to determine the appropriate primal values for η^F and η^W it suffices to specify the corresponding values of these four parameters. T is of course equal to 10 years; θ was assumed to be 2 years. The value assigned to r^0 must be related to the target year final demand goals; the higher the planned rate of growth of consumption and other exogenous sources of final demand, the higher the rate of growth of domestic productive capital stock. As a first approximation, r^0 was in every case set equal to the corresponding rate of growth of consumption (7.5 per cent, 6.0 per cent or 4.5 per cent) between 1965 and 1975.[59] The value assigned to r^T is in principle independent of the target year final demand goals, since the planned rate of growth beyond the target year need not be tied to the planned rate of growth up to the target year. However, in the interest of continuity, r^T was also set equal to the specified rate of growth of consumption between the base and target years.

Finally, the dual values to be assigned to the stock-flow conversion factors η^F and η^W were shown in Section 5 to depend upon the assumed rate of interest and the average rate of depreciation in the economy : η^F is equal to the sum of the two rates, and η^W is equal to the rate of interest alone. For the purposes of the study, the rate of interest was fixed at 15 per cent, while the average rate of depreciation was estimated at 3 per cent per year.

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[56] 'Exogenous imports' includes imports of the few miscellaneous agricultural and industrial products which could not be classified in any of the 147 endogenous sectors of the model. See also footnote 34.

[57] See Government of India (1965).

[58] See Government of India (1964).

[59] As observed in footnote 25, the first approximation to r^0 can be revised for a second run so as to be consistent with the results of the first run, and this process can be continued by interation up to any desired degree of accuracy. In practice, however, it was found in every case that the first approximation satisfactory for the purposes of the analysis.

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