INDIAN STATISTICAL INSTITUTE
Second Semester Examination : 2002-2003

B. Stat. II Year
Elements of Algebraic Structures

Date : 9.5.03 Maximum Marks : 100 Duration : 3 Hours

You may answer all questions. Max. marks you can score is 100.

I.(a) Let G be a group with identity e and a,b be elements of G, $a \neq e$ such that $a^5 = e, aba^{-1} = b^2$. Find the order of b.
[6]

(b) Let G be a finite group and A a subgroup. If all sets of the form Ax A have the same size for all x is G, show that A is normal in G.
[6]

II.(a) Let G be a finite group whose order is divisible by a prime p. Assume that $(ab)^p = a^p.b^p$ for all a,b in G. Show that the p-sylow Subgroup of G is normal in G.
[8]

(b) In a group of order 231, show that a 11-sylow subgroup is normal and lies in the center of G.
[8]

(c) Find the number of Abelian groups of order $2^4.3^3$.
[8]

III. Prove or disprove each of the following.

(a) If H, K are two subgroups of finite index in a group G then $H \cap K$ is a subgroup of finite index in G.
[Hint : Consider $Ha \cap Ka$ for a in G]
[8]

(b) A field of size 32 contains a subfield of size 8.
[8]

(c) I is a maximal ideal in a commutative ring R with multiplicative identity. For any two elements a, b of R, if the product ab belongs to I, then either a is in I or b is in I.
[8]

IV.(a) R is a ring with multiplicative identity 1 and R' is a commutative ring without zero divisions. If ϕ is a ring homomorphism of R into R' such that Kernel of ϕ is not equal to R, show that $\phi(1)$ is the multiplicative identity in R'.
[8]

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(b) If R is ring with unit element and its only right ideals are (0) and R itself, show that every nonzero element of R is invertible. [8]

(c) Let D be an integral domain, and a, b be elements of D. Let m, n be positive integers such that

$$a^m = b^m \ \ and \ \ a^n = b^n$$

If m, n are relatively prime, show that $a = b$. [8]

V.(a) Find the minimal polynomial of $\sqrt{2}+\sqrt{3}$ and the dimension of $Q(\sqrt{2}+\sqrt{3})$ over Q. Justify your answer.
(Q stands for the field of rational numbers) [8]

(b) Find the splitting field of $x^4 + x^2 + 1$ over Q. Find its dimension over Q. [8]

VI.(a) Explain how to construct a field of size p^n for a prime p and positive integer n. [4]

(b) Verify whether $x^4 + x + 1$ is irreducible over $\mathbb{Z}_2$. [3]

(c) Write down the elements of a field of size 16, indicating a generator of its multiplicative group. [8]

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INDIAN STATISTICAL INSTITUTE

B. Stat II: 2002 – 2003
Second Semester Examination
Economic Statistics and Official Statistics

Duration: 3 hours | Maximum marks: 100 | Date: 6 May 2003

*[Answer any **four** questions from Group A and any **one** from Group B. All questions carry equal marks.]*

Group A: Economic Statistics

1. (a) State Pareto law. Give your comments on the universality of Pareto law stating the evidences for and against this law. State and prove some properties of Pareto distribution.
   (b) Suppose X~Pareto(C,ν). Find the jth Moment distribution of X truncated at C+R from below where R is your roll number assuming that ν > j. Hence find the LR of the transformed distribution. [10+10=20]

2. Discuss the relative merits and demerits of the poverty indices proposed by (i) Sen and (ii) Chakravarty. [20]

3. Write down the criteria for a good measure of concentration proposed by Hall and Tideman. Examine whether $I = (I_{HH} + I_{HT})/2$ satisfies these criteria, where the symbols have usual meaning. [5+15=20]

4. (a) Describe how you will fit the following form of Engel curve
   $y = \alpha(x+\beta)/(x+\gamma), \quad \alpha>0, (x+\beta)>0 \ \& \ (x+\gamma)>0,$
   given observations on y and x. Hence explain how you will classify a commodity as necessary, luxury and inferior. Also draw rough sketch of the curve in each case. [20]

5. How do Prais and Houthakker introduce economy of scale parameters in Engel curve analysis to examine the effect of household size? How can one estimate these parameters? [10 + 10]

6. Write short notes on any two of the following:

   (i) Linear Expenditure System.
   (ii) Graphical test and methods of estimating three-parameter lognormal distribution.
   (iii)The problems of combining data and the choice of weights in the construction of Index Numbers. [10+10=20]

Group B: Official Statistics

1. Describe briefly the salient features of the Statistical System for official statistics in India. What these abbreviations stand for: CSO, SRS, NSSO, SDRD and GC? [20]
2. Give a brief account of household surveys conducted by NSSO. What types of consumer expenditure data are collected in the NSS rounds? How is absolute poverty in India measured and head count ratios are calculated utilizing such data? [20]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination : (2002 – 2003)
Course Name : B. Stat. II Year
Subject Name : Biology II
Date : 02.05.03 Maximum Marks : 100 Duration :Three hours

(Number of copies of the question paper required : Three)

1. What is Moisture Availability Index? Draw a suitable rice crop calendar with the following data

Week No.	Rainfall (mm) at 0.5 Prob	PET (mm)
22	0	58
23	0	62
24	12	51
25	23	38
26	45	32
27	56	27
28	12	34
29	62	24
30	123	21
31	156	17
32	98	23
33	63	25
34	34	26
35	22	28
36	12	32
37	7	33
38	2	36
39	0	37
40	0	38

15

2. Write in brief about different types of rice. Describe the different phases of growth and development of rice. Briefly describe the cultural practices associated with rainfed lowland rice cultivation.

15

3. Answer the following questions, in brief:
a. What are the defining properties of a stem cell? What is the major difference between a `unipotent' and a `pluripotent' stem cell?
b. What is Agrobacterium tumifaciens?
c. What is the `modular' construction of plants?
d. Briefly describe why Arabidopsis thaliana is considered as the model organism for plant development studies?
e. What are the major objectives of classical plant breeding?

5 x 3

4. a. Define soil.
b. Write short notes on any two of the followings :
i) Regolith ii) Soil profile iii) Weathering 2+8

OR

In what forms do the plant nutrients like N, P and K occur in the soil? What are the mechanisms of uptake of these nutrients by plants. 10

5. What are the differences between manure and fertilizers? 120 kg of N, 60 kg of P and 60 kg of K are to be given for 1 ha rice crop. 25% of the N should be applied through Compost. Calculate the required amount of compost, urea, single super phosphate and KCl. 2+8

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INDIAN STATISTICAL INSTITUTE
Second Semester Examination : 2002-2003

B.Stat.- II Year
EconomicsII

Date : - 02.05.03 | Maximum Marks : 60 | Duration : 3 hours

Answer as many question as you can. Maximum marks you can score is 60. Marks allotted to each question are shown at the bottom right hand corner of the question.

1. (a) Explain the concept of paradox of thrift in a simple Keynesian model

(b) Suppose in a simple Keynesian model (closed economy, no government) the saving function shifts parallely upward by 10 units . Following this, saving in the new equilibrium is found to decline by 10 units. Marginal propensity to consume (with respect to y) is given to be 0.8. It is given that the full employment output is 1200. What should be the level of autonomous expenditure so that the inflationary gap is zero in this model.

(20)

2. (a) Derive the stability condition in the simple Keynesian model. Express it in terms of relative slopes of the saving and investment schedules for a closed economy. Show the equilibrium in terms of saving and investment schedules in a diagram. Identify the stable and unstable cases.

(b) Extend the model of part (a) for an open economy. Suppose for this open economy the marginal propensity to save (with respect to y), $s = .2$, marginal propensity to invest (w. r. to y), $I_y = .4$. Is this equilibrium stable or unstable? Explain your answer.

(20)

3. (a) What is an inflationary gap ? Use the IS – LM diagram to show the existence of an inflationary gap in the system. How will you measure this inflationary gap in the model ?

(b) when an increase in govt. expenditure is financed by central bank loan it is said that the govt. is taking resort to deficit financing . There is a popular opinion against deficit financing as it is believed to fuel inflation. Do you think that this criticism is always justified ? Explain your answer in terms of IS-LM model .

(20)

4.(a) Consider a simple Keynesian economy (without government) where marginal propensity to consume (w.r.to y) is 0.8, marginal propensity to invest (w.r.to y) is 0.4 and the marginal propensity to import is 0.3. Autonomous expenditure in the aggregate demand function is 800. What is the equilibrium y ? Draw the expenditure function E(y) in a diagram and show the equilibrium y with a Keynesian cross.

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(b) Now suppose a restriction is imposed on import which makes Mmax = 3000 so that if import demand at any y is found to exceed 3000 the shortfall is met from domestic supply. What will be the expenditure function now ? draw the E(y) function in a diagram under this changed circumstances. Will there be any change in equilibrium ?

(7)

5. Suppose the equation of the IS curve is

$$y + 100r = 1700,$$

while the equation of the LM curve is

$$2y - 100r = 1000.$$

what are the states of the two markets at $y = 950, r = 6$

(3)

6. Suppose for a closed economy

$$C = 200 + \frac{3}{4}(y - T)$$

$$I = 200 - 25r$$

$$G = 100, T = 100.$$

Form the equation of the IS curve. Following an increase in money supply, C is found to increase by 50 units. Suppose in the initial equilibrium (before increase in money supply) $y = 900$. What will be the values of y and r in the new equilibrium ?

(5)

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INDIAN STATISTICAL INSTITUTE

B.Stat. II Year
Statistical Methods IV
Semestral Examination

Date: April 30, 2003 Maximum marks: 100 Maximum Time: $3\frac{1}{2}$ hours

This examination is closed book, closed notes. Statistical tables will be needed. The entire question paper is worth 105 marks, but the maximum score you can get is 100.

1. Given vector samples $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_m$ from the distribution $N_q(\mu_1, \Sigma)$ and $\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_n$ from the distribution $N_q(\mu_2, \Sigma)$, describe the likelihood ratio test for the hypothesis $\mu_1 = \mu_2$, with unspecified Σ. Justify the null distribution of this statistic, after stating the requisite main result(s) without proof. [2+13=15]

2. The table below gives the men's olympic sprint times in seconds for the 100 m and 200 m categories. If the sprint time for any one category is linearly regressed on the year, one gets a straight line with negative slope. Test for the parallelity of the regression lines for the two categories. [20]

Year	100 m	200 m
1948	10.30	21.10
1952	10.40	20.70
1956	10.50	20.60
1960	10.20	20.50
1964	10.00	20.30
1968	9.95	19.83
1972	10.14	20.00
1976	10.06	20.23
1980	10.25	20.19
1984	9.99	19.80
1988	9.92	19.75

3. In order to test whether the *medium of instruction at the school level* (represented by the variable X which is equal to 1 for the vernacular medium and 2 for the English medium) has any effect on *success in the B.Stat. admission test* (represented by the variable Y which is equal to 1 for success and 2 for failure), the following sampling scheme was used. Thirty candidates were chosen randomly from the group of selected candidates and another 60 from the list of unsuccessful candidates. The outcome was led to the following contingency table.

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v) The net reproduction rate is a measure of the
   a) annual excess of births over deaths
   b) annual rate at which women are replacing themselves on the basis prevailing fertility and mortality, assuming no migration.
   c) decennial growth rate of the population
   d) per generation growth rate assuming current age specific fertility a mortality rates and no migration.

[1 x 5

B) Determine whether each of the following statemen is 'true' or 'false' :

i) The mid year population is always a good estimate of the person years liv in a given year.

ii) In a country where mortality rates have remained relatively constant many years, a generation life table and a period life table will be alm identical.

iii) It would be possible to construct an age standardised rate of natu increase.

iv) For all practical purposes, the gross reproduction rate can never be low than the net reproduction rate.

v) Fertility rates specific for live birth order can be constructed only as peri rates and not as cohort rates.

[1 x 5

2. a) Discuss the pitfalls of births and deaths registration in India.
   b) Describe the Sample Registration System in India as an alternative registrati system to estimate annual births and deaths. Critically evaluate the samp registration system.

[5+10+5=

3. a) Derive the expression for instantaneous growth rate of a population.
   b) Derive the formula for logistic curve of growth under the optimum population approach.
   c) State assumptions involved in fitting th logistic curve by Hoetelling's metho Estimate various parameters by fitting the logistic curve through Hoetalling method.

[4+6+2+8=

4. a) Does the Total Fertility Rate strictly conform to ideas of a measure of

reproduction ? In what way does it differ from Gross Reproduction Rate ?
   b) From the data given below, calculate the gross reproduction rate (GRR), the net reproduction rate (NRR) and the mean length of generation (MLG) when the sex – ratio at birth is 104 :

Age group	# of children born to 1,000 women in a year	Person years lived at age – interval by 1,00,000 women
15-19	96	490814
20-24	192	489889
25-29	182	488317
30-34	131	485954
35-39	77	482719
40-44	27	478262
45-49	3	472086

[4+2+4+5+5=20]

5. a) What do you mean by a life table ? What are the assumptions involved in the construction of a life table ?
   b) Fill in th blanks in the life table given below :

Age x	l_x	d_x	q_x	L_x	T_x	e^0_x
4	95000	500	?	?	4850350	?
5	?	400	?	?	?	?

   c) The number of persons dying at age 75 is 476 and the complete expectation of life at 75 years and 76 years are respectively 3.92 years and 3.66 years. Find the numbers living at age 75 and at age 76.

[2+3+8+7=20]

INDIAN STATISTICAL INSTITUTE
Mid Semester Examination 2002-2003
Course : B. Stat II (Hons.)

Subject : Demography

Date : 28.2.2003 Maximum Marks : 100 Duration : Three hours

Answer any FIVE question

1. a) Define a population Census
   b) Describe the various methods for conducting census. Which was the method adopted in census of India, 2001 ?
   c) Give a short account of the main findings of the population Census of India, 2001.

[2+10+8 =20]

2. a) Write briefly on why there is a need for evaluation and adjustment of basic demographic data.
   b) Discuss methods of evaluating age data given by single years of age.
   c) Comment about the quality of relative age reporting, when the value of Myers' Digit Preference Index for the three countries are as follows :

Country A: '0' , Country B: '90', Country C: '180'

[5+9+6 =20]

3. a) What are the defects in using crude death rate to compare the mortality rates of two places ? How is this eliminated ?
   b) Calculate the crude and standardised death rates for the local population and non-local population from the following data and compare them with the crude death rate of the standard population.

Age Group	Standard Population		Local Population		Non-Local Population	
	Size	Deaths	Size	Deaths	Size	Deaths
0-10	600	18	400	16	200	-
10-20	1000	5	1500	6	1200	-
20-60	3000	24	2400	24	3600	-
60+	400	20	700	25	600	-
Total	5000	67	5000	71	5600	76

[8+12 =20]

4. a) What do you mean by Infant Mortality rate ? Describe the different methods of computing the infant mortality rate. Comment on their merits and demerits.
   b) Describe the trends in infant mortality rate in India during the twentieth century .

[10+10=20]

(Turn Over)

5. a) What do you mean by a life table ?
   b) Distinguish between (i) 'period' and 'cohort' life tables, and (ii) 'complete' and 'abridged' life tables.
   c) What are the assumptions involved in the construction of a life table ?

   d) (i) Show that in a life table :

$$_nq_x = \frac{2n._n m_x}{2 + n._n m_x}$$

where symbols have their usual meanings.
(ii) Critically evaluate $_nq_x$ value as given in (i) above.

[4+4+4+8=20]

6. a) Derive: $P_t = P_0 e^{rt}$
where symbols have their usual meanings

b) Define general fertility rate and total fertility rate stating their merits and demerits

c) Describe net reproduction rate (NRR) and problems associated with NRR as measure of natural growth of population. Suggest a suitable method to measure natural growth of population instead of NRR and its advantages.

[4+6+10=20]

7. Write notes on any four of the following :
a) Zelnik's method of adjustment of age data.
b) Dual Recording system for estimation of births in a population
c) Estimation of population through Gompertz curve
d) Neonatal Mortality Rate
e) Mean Length of Generation

[5 x 4 = 20]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) 2nd year

Statistical methods IV

Mid-semestral Examination

Maximum time: 2 hours February 25, 2003 Maximum marks: 100

1. The elements of the random vectors $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are jointly distributed as
$$\begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right).$$
Derive the conditional distribution of $\boldsymbol{X}_1$ given $\boldsymbol{X}_2$. [15]

2. If $\boldsymbol{X}_1, \ldots \boldsymbol{X}_n$ are samples from the multivariate normal distribution $N(\boldsymbol{\mu}_0, \Sigma)$ where $\boldsymbol{\mu}_0$ is a *known* vector, derive the maximum likelihood estimator of Σ, and describe its distribution. [20]

3. If $(X_1, Y_1), \ldots, (X_n, Y_n)$ are samples from a bivariate normal distribution with completely unknown parameters, derive a suitable test for the hypothesis $E(X_1) = E(Y_1)$. [40]

4. The following table gives the literacy rate in the districts of West Bengal, according to the 2001 census. Use an appropriate statistical test to determine whether there is significant correlation between the male and female literacy rates, after stating the relevant assumptions. [25]

District	Male literacy % 2001	Female literacy % 2001
Darjiling	81	64
Jalpaiguri	74	53
Koch Bihar	77	57
Uttar Dinajpur	59	37
Dakshin Dinajpur	73	55
Maldah	59	42
Murshidabad	61	48
Birbhum	72	52
Barddhaman	79	62
Nadia	73	60
North Twenty Four Parganas	84	72
Hugli	83	68
Bankura	77	50
Puruliya	74	37
Medinipur	85	65
Haora	84	71
Kolkata	84	78
South Twenty Four Parganas	80	60

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2002 – 2003)

B . STAT . II Year

PHYSICS - II

Date :- 21.2.03 Maximum Marks :- **30** Duration :- **3** *Hrs.*

Answer all questions . Each question carries 10 marks .

QUESTIONS

1) State and explain the ***Zeroth*** law of thermodynamics in necessary details. Analytically justify that an empirical notion of temperature could be developed from this law . Does temperature have an ***absolute*** scale ? (3 + 5 + 2)

2) Give a proper account of "***internal energy***" of various physical substances (solids , liquids and gases) in the context of thermodynamics . Why is this so important in our understanding of the ***First*** law of thermodynamics? Using this law, obtain the ***adiabatic*** eqn. of state for a perfect gas . How can one experimentally determine the ***specific heat ratio*** involved in this eqn. ? (4 + 2 + 2 + 2)

3) Using the ideas of molecular kinetics , show analytically that the average kinetic energy of a perfect gas molecule is proportional to the absolute value of temperature of the gas . Next , deduce the ***Maxwellian*** velocity distribution formula for a perfect gas in a steady state temperature T K . (3 + 7)

INDIAN STATISTCAL INSTITUTE

Mid-semester Examination: (2002 - 2003)

B.Stat II Year

BIOLOGY II

Date..21.2.03.......Maximum number: 100 Duration: 3 hours

(Number of copies of question paper required: 6 (Six))

Group A: Answer any three.

1. Juatify the followings (any four) 4x5 = 20

a) $$r = \frac{2.303\,(\log_{10} W_2 - \log_{10} W_1)}{(t_2 - t_1)}$$

[Where, r = growth rate; W1 & W2 are initial & final dry weight; t1 & t2 are initial & final time]

b) Growth curve always follows a sigmoid nature.

c) Dark period is more essential than light for flowering initiation.

d) Enzyme activity is very much dependent on both enzyme and substrate concentration.

e) If once 'Devernalization' occurs, the plant remains vegetative.

2. Define Photoinduction. If red light is used as last exposure on plant, what will be the result? How and where 'Florigen' is synthesised in plants? What do you mean 'Critical Point' regarding light reuquirement of a plant? How many categories are there? Give examples. 10

3. Describe differential phasic development of a rice plant. 10

4. Describe the pathway of Gibberellin synthesis in plant cell. Write briefly about effect of Gibberellin on plant growth. 10

Group B

2.5 x 4 = 10

1.

a) What are the micronutrients present in the soil?

b) What are the factors affecting the availability of the macronutrient to plants?

c) Write in brief about how nutrients are absorbed by the plants.

d) How are pH depndent charges develop in soil colloids?

5 + 5 = 10

2. Write short notes on (anytwo):

a) Soil pH.

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b) Cation exchange capacity of the soil.
c) Isomorphic substitutions in clay lattice.

Group C

1. What is Agrobacterium tumifaciens? Briefly outline the procedure for making a transgenic plant. 2+3
2. Briefly describe why Arabidopsis thaliana is considered as model organism for plant development studies? What is the `modular' construction of plants? 3+3
3. Three broad classes of signaling events take place during mammalian development. Describe them briefly. 1+2
4. What are the defining properties of a stem cell? What is the major difference between a `unipotent' and a `pluripotent' stem cell? What is a morphogen gradient? 3+1+1
5. (I) What are the two major ways of plant genetic improvement?
(II) Write one major objective of classical plant breeding?
(III) Name three major practical methods of plant breeding?
(IV) What are the names of different plant selection methods?
(V) Write about one major difference in plant and animal development strategies.
(VI) What are the three classes of `segementation genes' in early Drosophila development?
(VII) Bithorax and antennapedia gene complexes in Drosophila are called homeotic selector genes- Is this statement correct?
(VIII) The developmental control genes of Drosophila have homologues in vertebrates. Name one and what they code for in vertebrates?
(IX) What is a half mouse embryo and a chimeric double embryo?
(X) What is a teratoma?
(XI) Is it possible to derive `transplantable cancers' from teratomas? Teratocarcinoma stem cells originate through mutations in genes responsible for the normal control of cell behavior- is this statement correct? 1 x 11

INDIAN STATISTICAL INSTITUTE
B-Stat Second year

Mid-semester Examination : (2002-2003)
Course Name- Economics (II)

Subject Name- Macrotheory

Full Marks-40 Date: 21.2.03 Time-2 hours
Answer any two questions. Each question carries equal marks.

1.(a) What is value added ? Show that aggregate expenditure on final goods and services equals the sum of the value added of all production units in an open economy.

(b) Consider the following income statement of a firm for a particular year.(All figures are in million rupees)

Sales		200
Less manufacturing costs of goods produced and sold :		
Raw materials from other firms	40	
Salaries	20	
Payment to labour contractor	30	
Depreciation on building and machinery	10	
	100	100
Gross margin		100
Less selling cost :		
Salaries to salesmen	10	
Purchase from advertising firm	5	
	15	15
Less business taxes (indirect)		10
Margin		75
Less interest paid on bonds		5
Less interest paid on bank loan		20
Profit before tax		50
Less corporate income taxes		20
Margin		30
Less dividend on stock		15
Addition to surplus		15

(i) Calculate the firms contribution to national income as :
(a) Wages and salaries

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(b) Interest
(c) Corporate profit
(ii) Calculate the firm's contribution to NNP

2. (a) In a closed economy without government in a certain year, production of new equipment was 5, new buildings produced was 10, production of consumption goods was 110, depreciation on existing building was 10,depreciation on existing equipment was 10, gross investment was 35. There did not take place any change in the inventory of intermediate inputs. Compute C, GDP and NDP for the economy.

(b) In an economy all consumption goods and intermediate inputs are supplied from domestic sources only. It is found that production of new equipment and buildings in a
certain year was 100 units while gross investment was 280 units. There was no change in inventory of consumption goods and intermediate inputs. It is given that the country's
net loan from abroad was 70, net factor income from abroad was –80,net transfer from abroad was –20. Find out the volume of exports of the country.

(c)A woman marries her butler but after the marriage the husband continues to wait on her and she continues to support him (but as a husband rather than as a wage earner).
What is the impact of this marriage on NI?

3. (a) Distinguish between gross and net investment. Consider the national income identity involving GDP on the one hand and consumption (C), investment (I), government expenditure (G) and trade balance (X – M) on the other hand. Is this investment gross or net ? Define personal saving and derive it starting with GDP. Specify the different components of national saving (NS). Derive the relationship between NS on the one hand and investment and current account surplus on the other. Is this investment gross or net ?

(b) The following information are given for an economy in a certain year :
NNP is 1,200; disposable income is 1,000; govt. budget deficit is 70; C is 850; net factor income from abroad is –20; the current account surplus is 20. (Assume net transfer from abroad is zero).
(i) S ?
(ii) I ?
(iii) G ?

INDIAN STATISTICAL INSTITUTE
B. Stat II: 2002 – 2003
Mid-semester Examination
Economic Statistics and Official Statistics

Duration: 3 hours Maximum marks: 100 Date: 19 February 2003

[Answer Qn. 1 or Qn. 2 and any two questions from the rest of the questions.]

1. (a) Suppose monthly per capita expenditures (x) of households in urban India follow Pareto law with inequality parameter $\nu = 2.0$ and the threshold parameter c = Rs. 10/-. Find
   (i) the average monthly per capita expenditure of all households,
   (ii) the average monthly per capita expenditure of households spending $\nless$ Rs 50/- per capita per month.

   (b) If households spending Rs. x per capita per month spend $x^{0.8}$ rupees per capita per month on a certain commodity then find (i) the average monthly per capita expenditures of households on the commodity. Also find (ii) the percentage increase on the consumption of the commodity if every household raises its per capita expenditure by 5%. [5+10+10+15]

OR

2. Suppose $y_1, y_2, \ldots, y_n$ are incomes of n persons in a community. Describe how you will find Lorenz ratio (LR) (i) graphically and (ii) numerically. Prove that LR found by numerial method is equivalent to the LR found by the formula using Gini's Mean Difference (GMD). If y_i = Rs. i /- and n = 100, then find LR. How will the value of the LR change if income of each person is increased by Rs. 50/-? What will happen if n becomes infinitely large? [5+5+16+7+5+2]

3. Describe Positive and Normative Measures of Inequalities. Write down the desirable properties of a measure of inequality. Examine Coefficient of Variation in the light of these properties. [8+5+17=30]

4. Define Lorenz Curve (LC). State its properties. Derive LCs and Lorenz Ratios for Pareto and Lognormal distributions. Also state and prove the properties of LCs in each case. [1+4+20+5=30]

5. Write short notes on any three of the following:

   (a) Law of Proportionate Effect.
   (b) Three parameter lognormal distribution.
   (c) Universality of Pareto Law.
   (d) Estimation of two-parameter lognormal distribution. [10+10+10=30]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination – 2002-03

B. Stat. II year

Elements of Algebraic Structures

Date : 17.02.03 | Maximum Marks : 100 | Duration : 3 Hours

You may answer all questions. Max. marks is 100

I.a) Find the transitive closure of the relation R whose matrix is given below

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

[10]

b) Show that if a function is invertible under function composition, then it is a bijection [10]

II.a) Show that a group of order 15 is Abelian. Is it cyclic [10]

b) In a finite group G, $(ab)^3=a^3b^3$ for all a,b in G. Show that every element of G can be written as y^3 for some y in G. · $3 \nmid o(G)$ [10]

III. Prove or disprove

a) A finite subset of a group, closed under multiplication is a Subgroup [10]

b) If f is a homomorphism of a group G into a group G' and $a \in G$, then $0(f(a)) = 0(a)$ [10]

c) If m is the least positive integer such that $a^m=e$ for an element a of a finite group G of n elements, then m divides n. [10]

d) The non zero rational numbers under multiplication is a cyclic group. [10]

IV.a) G is a group and $g \in G$

$$T_g : G \to G$$

$$T_g(x) = gxg^{-1}; x \in G$$

Let $I(G) = \{T_g / g \in G\}$

Show that I(G) is a group under function composition. [10]

b) If a group of order 28 has a normal Subgroup of order 4, show that it is Abelian. [10]

c) In a group G, let $X = \{a^{-1}b^{-1}ab | a, b \in G\}$ and let $G' = \langle X \rangle$, the Subgroup generated by X in G. Show that G' is normal in G and that G/G' is Abelian. [10]

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INDIAN STATISTICAL INSTITUTE
First Semestral Examination – 2002-2003
B. Stat. (Hons.) II Year
Analysis III

Date : 13.12.02 Maximum Marks : 70 Duration : 3 Hours

Answer five questions

1.(a) Let C be a closed and bounded subset and U an open subset of $\mathbb{R}^n$, $C \subseteq U$. Show that there exists $r > 0$ such that for all $\underset{\sim}{x} \in C$, $B(\underset{\sim}{x}, r) \subseteq U$. ($B(\underset{\sim}{x}, r)$ is the open ball with centre at $\underset{\sim}{x}$ and radius r.)

(b) Let C be a closed and bounded subset of $\mathbb{R}^3$ which is convex, that is, if $\underset{\sim}{x}, \underset{\sim}{y} \in C$, then the line segment $L\left(\underset{\sim}{x}, \underset{\sim}{y}\right) \subseteq C$. Let

$$C_{(x_1,x_2)} = \{x_3 \in \mathbb{R} : (x_1, x_2, x_3) \in C\},\ (x_1, x_2) \in \mathbb{R}^2$$

(i) Show that $C_{(x_1,x_2)}$ is a closed interval.

(ii) Let $l(x_1, x_2)$ denote the length of the interval $C_{(x_1,x_2)}$. (when $C_{(x_1,x_2)} = \phi$, $l(x_1,x_2)=0$.) Show that for any $\alpha \in \mathbb{R}$

$\{(x_1, x_2) : l(x_1, x_2) \geq \alpha\}$ is a closed subset of $\mathbb{R}^2$.

[6+8=14]

2.(a)(i) Let A be a closed nonempty subset of $\mathbb{R}^n$. Define

$$d\left(\underset{\sim}{x}, A\right) = \inf_{\underset{\sim}{y} \in A} \left|\underset{\sim}{x} - \underset{\sim}{y}\right|,\ \underset{\sim}{x} \in \mathbb{R}^n.$$

Show that $d\left(\underset{\sim}{x}, A\right)$ is a continuous function on $\mathbb{R}^n$.

(ii) If B and C are closed and disjoint nonempty subsets of $\mathbb{R}^n$, using the functions $d\left(\underset{\sim}{x}, B\right)$ and $d\left(\underset{\sim}{x}, C\right)$, find a continuous function f on $\mathbb{R}^n$ such that $f(\underset{\sim}{x}) = 0$ if $\underset{\sim}{x} \in B$ and $f(\underset{\sim}{x}) = 1$ if $\underset{\sim}{x} \in C$.

(b) State and prove Schwarz theorem for interchange of partial derivations.

[3+3+8=14]

3.(a) To maximise the quantity $x^a y^b z^c$ where a,b,c are positive constants, and x,y,z non negative variables subject to the constraint $x^k+y^k+z^k=1$ where k is a positive constant, apply Lagrange's method of multipliers to find stationary values.

[P.T.O.]

(b) Solve the maximisation problem in (a) giving adequate reasons.

(c) Show from (b) :

$$\left(\frac{u}{a}\right)^a\left(\frac{v}{b}\right)^b\left(\frac{w}{c}\right)^c \le \left(\frac{u+v+w}{a+b+c}\right)^{a+b+c}$$

for u, v, w, a, b, c > 0.

[7+4+3=14]

4.(a) Let $f=(f_1,\dots,f_n):U\to\mathbb{R}^n$ be continuously differentiable on the open subset U of $\mathbb{R}^n$. If the derivative matrix of f at a point $\underset{\sim}{x}^\circ \in U$ is nonsingular, show that f is one-one on a neighbourhood of $\underset{\sim}{x}^\circ$.

(b) Give an example to show that if in (a), the derivative matrix is nonsingular at all $\underset{\sim}{x} \in U$, it is not necessary that f is one-one on U.

[10+4=14]

5.(a) Show that if for a bounded nonnegative function f on a rectangle $B\subseteq\mathbb{R}^n$, $\int_B f=0$ and the set $A=\{x\in B: f(x)>0\}$ is Jordan measurable, then A has Jordan measure zero.

(b) Show that the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, where a, b, c > 0 is Jordan measurable.

(c) Find the volume of the solid bounded by the ellipsoid in (b).

[5+5+4=14]

6(a) Find the tangent space to the surface $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ at a point (x, y, z) and the outer unit normal.

(b) Verify Stokes's theorem for the 1-form $z^2dx-2xdy+y^3dz$ with respect to the surface $\{(x,y,z): x^2+y^2+z^2=1 \text{ and } z\ge 0\}$.

[7+7=14]

INDIAN STATISTICAL INSTITUTE
B.STAT-II (2002-2003)
Theory of Probability and its Applications - III
Semestral-I Examination
Maximum marks: 65. Time: 3 hours.

Date : 9 December, 2002.

Note: Answer as many questions as you wish.
The whole question paper carries 75 marks.
The maximum you can score is 65.

1.(a) Suppose that the random variables $X_1,X_2,\cdots,X_n$ follow the Dirichlet $D(r_1,r_2,\cdots r_n;r_{n+1})$ – distribution. Write down the bivariate density of $(X_1+X_2, X_3+X_4+\cdots+X_n)$.

(b) Let $X_1,X_2,\cdots,X_5$ be *iid* random variables each with uniform $U(0,1)$ – distribution. Let $Y_1<Y_2<\cdots<Y_5$ be the ordered X_i's. Let $U_1=Y_1$, $U_i=Y_i-Y_{i-1},\ i=2,3,\cdots,5$. Show that $(U_1,U_2,\cdots,U_5)$ has Dirichlet $D(1,1,1,1,1;1)$ – distribution. Using the properties of Dirichlet distribution write down the joint density of U_1+U_2, U_3+U_4. Hence find the bivariate joint density of (Y_2,Y_4).

[2+(3+2+3)]

2. Let $(X_1,X_2,\cdots,X_r)$ follow $r-$ variate Normal distribution with $E(X_i)=\mu, Var(X_i)=\sigma^2$ and $Cov(X_i,X_j)=\rho\sigma^2$, $i\ne j,\ i,j=1,2,\cdots,r$, where $-\frac{1}{r-1}<\rho<1$.

Let $\overline{X}=\frac{1}{r}\sum_1^r X_i$, $S^2=\sum_1^r (X_i-\overline{X})^2$.

(a) Show that $\overline{X}$ follows $N(\mu, \frac{1+(r-1)\rho}{r}\sigma^2)$.

(b) Show that $\frac{S^2}{(1-\rho)\sigma^2}$ follows χ^2_{r-1}.

(c) Show that $\overline{X}$ and S^2 are independent.

[10]

3. (a) Define convergence in probability. Show that X_n converges in probability to 0 if and only if every subsequence $\{X_{n_k}\}$ of $\{X_n\}$ has itself a further subsequence $\{X_{n_{k_r}}\}$ such that $X_{n_{k_r}}$ converges to 0 *a.s.*

(b) Show that if $X_n,Y_n, n=1,2,\cdots, X,Y$ are random variables defined on the same sample space such that $X_n\xrightarrow{P}X$ and $Y_n\xrightarrow{P}Y$ then $X_n+Y_n\xrightarrow{P}X+Y$.

(c) Define convergence in Distribution. Show that if $X_n,Y_n, n=1,2,\cdots$, are random variables defined on the same sample space and $X_n\xrightarrow{\mathcal{D}}X$ and $Y_n\xrightarrow{P}0$, then $X_n+Y_n\xrightarrow{\mathcal{D}}X$.

[(2+7)+4+(2+5)]

(Please turn overleaf)

4. (a) Let $X_n, n = 1, 2, \cdots$ be independent 0-1 valued random variables with

$$P(X_n = 1) = p_n = 1 - P(X_n = 0).$$

Show that $X_n \to 0$ $a.s$ if and only if $\sum_1^n p_n < \infty$.

(b) Let X_n, $n = 1, 2, \cdots$ be *iid* random variables such that $\frac{S_n}{n} \to a$ almost surely, where $S_n = \sum_1^n X_i$, and a is a real number. Show that $E(|X_1|) < \infty$.

(c) Let X_n, $n = 1, 2, \cdots$, be independent random variables such that

$$P(X_n = \epsilon_n) = P(X_n = -\epsilon_n) = \tfrac{1}{2} \text{ for all } n$$

where $\sum_1^\infty \epsilon_n^2 < \infty$. Show that $\sum_1^\infty X_n$ converges $a.s.$ State carefully any theorem that you may have to use to prove this.

[4+4+4]

5. (a) Prove that if X is a random variable with $E(|X|) < \infty$, then its characteristic function
is differentiable and

$$\phi'(t) = E(iXe^{itX}).$$

Also show that

$$\frac{|\phi(t) - 1 - itE(X)|}{|t|} \to 0 \text{ as } t \to 0.$$

(b) Let X_n, $n \geq 1$, be *iid* random variables with $E(|X_1|) < \infty$ and $E(X_1) = \mu$. Let $\psi_n(t)$ be the c.f. of $\frac{S_n}{n} - \mu$ where $S_n = \sum_1^n X_i$. Show that $|\psi_n(t) - 1|$ converges to 0 as $n \to \infty$ for all $t \in \mathbb{R}$. Use this to get a proof of $WLLN$ which asserts that $S_n/n \xrightarrow{P} \mu$.

[(5+5)+5]

6. Let $X_1, X_2, \cdots$, be *iid* random variables with X_1 having the exponential density $\alpha e^{-\alpha x} I_{[x>0]}$. Show that

$$\frac{\sqrt{n}}{\alpha}(\overline{X}_n^{-1} - \alpha) \xrightarrow{\mathcal{D}} N(0,1).$$

where $\overline{X}_n = \frac{1}{n}\sum_1^n X_i$.

[8]

INDIAN STATISTICAL INSTITUTE
Final Examination
B.Stat. (Hons.) II Year, I Semester
Statistical Methods III

Date: 5.12.02 Maximum time: 3 hours Maximum marks: 100

This test is closed book, closed notes. Tables of statistical distributions may be used. The entire set of questions is worth 105 marks.

1. Let X_{11}, X_{12}, X_{21} and X_{22} be *unobserved* samples from the uniform distribution over $[0, \theta]$, and $Y_i = \min\{X_{i1}, X_{i2}\}$, $i = 1, 2$ are observed.

   (a) Obtain the method of moments estimator of θ based on the observed data. Is this estimator unbiased?

   (b) Obtain the maximum likelihood estimator (MLE) of θ based on the observed data.

   (c) Is the MLE biased? If it is, determine whether the bias is positive or negative. If it is not, prove its unbiasedness. [6+7+7=20]

2. *Cedar-apple rust* is a fungus that affects apple trees. Red cedar trees are the immediate source of this fungus. If all cedar trees within the vicinity of the orchard are removed, the apple trees are not affected. In the first year of an experiment the number of affected leaves on 8 trees was counted (in summer); the following winter all red cedar trees within 100 meters of the orchard were removed and the following summer the same trees were examined for affected leaves. The results are given below.

tree number	number of rusted leaves: year 1	number of rusted leaves: year 2
1	38	32
2	10	16
3	84	57
4	36	28
5	50	55
6	35	12
7	73	61
8	48	29

The above data have to be analysed to test whether the infection reduced after removal of the cedar trees. Explain which test should be used for this purpose, after careful consideration. Calculate the p-value and clearly state your conclusions. [15]

3. The Survey of Study Habits and Attitudes (SSHA) test measures the motivation, attitude toward school, and study habits of students. Scores range from 0 to 200. A college gives the SSHA to a random sample

of both male and female first-year students. The scores for the women are as follows:

154	109	137	115	152	140	154	178	101
103	126	126	137	165	165	129	200	148

The scores of the male students are:

108	140	114	91	180	115	126	92	169	146
109	132	75	88	113	151	70	115	187	104

(a) Most studies have found that the mean SSHA score for men is lower than the mean score in a comparable group of women. Is this true for first-year students at this college? Carry out a test and give your conclusions.

(b) Give a 90% confidence interval for the mean difference between the SSHA scores of male and female first-year students at this college. [15+5=20]

4. A B.Stat. second year student begins the following daily exercise on 22 July, 2002: He records the maximum daily temperature at Kolkata, as reported by the meteorological office, and tries once in a month to fit a distribution to the data collected so far. He continues this study till 21 July, 2005. He uses a chi-squared goodness-of-fit test at level 0.95 to determine whether a particular distribution is appropriate for the data. On 21-08-02 he finds that several distributions described in his textbook are 'appropriate' for the data. However, on 21-07-05 none of these distributions appear to be appropriate for the data.

   Is this a plausible story? Explain. [10]

5. Let $X_1, \ldots, X_{25}$ be samples from $N(\mu_x, 1)$ and $Y_1, \ldots, Y_{25}$ be samples from $N(\mu_y, 1)$, where μ_x and μ_y are unknown parameters. The two samples are independent of one another. Find a region of the real line such that another independent sample from the distribution with density

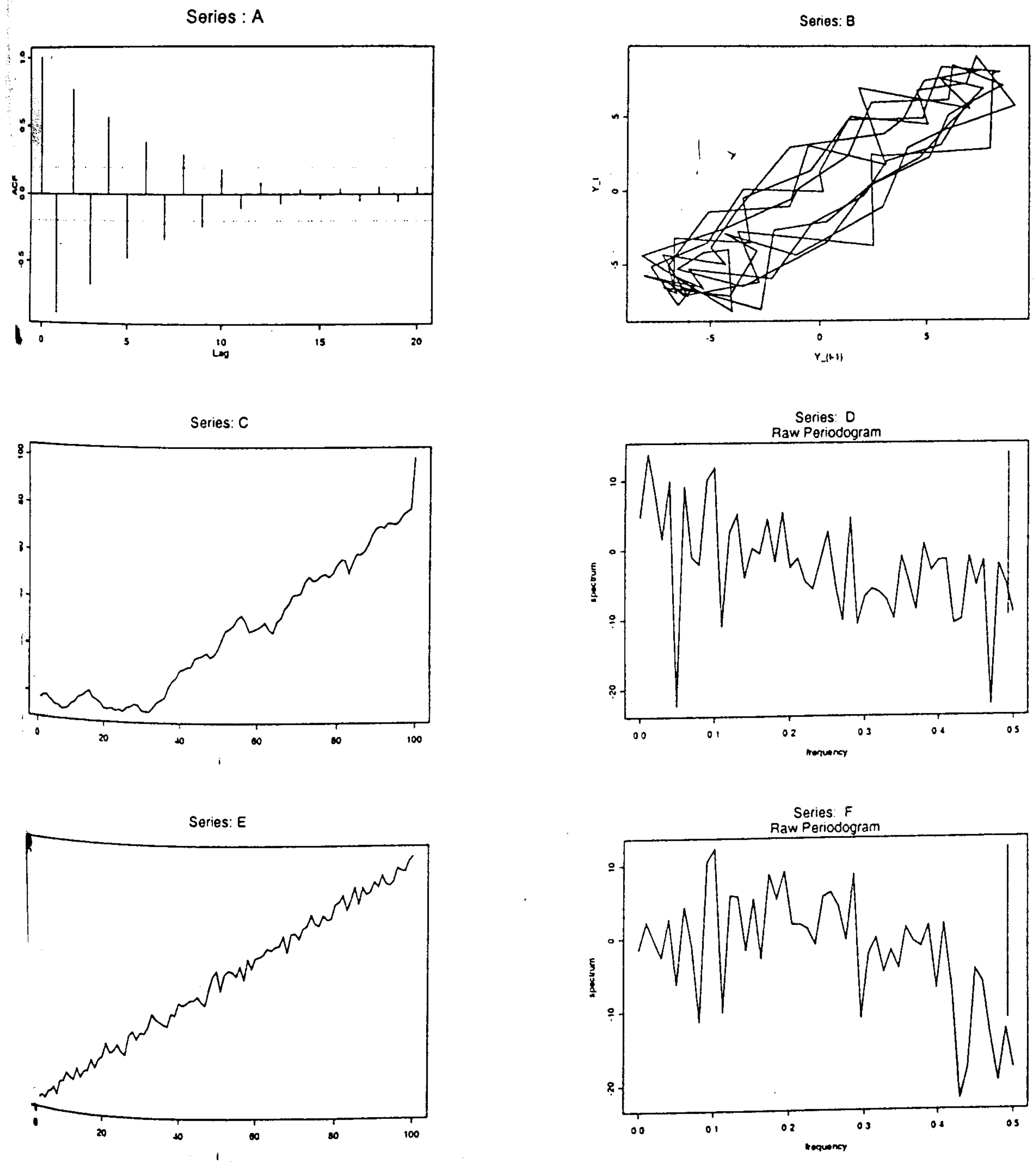
$$f(z) = \frac{p}{\sqrt{2\pi}} e^{-(z-\mu_x)^2/2} + \frac{1-p}{\sqrt{2\pi}} e^{-(z-\mu_y)^2/2}, \quad -\infty < z < \infty, \ 0 < p < 1,$$

   is contained in this interval with probability 0.95. The parameter p is unknown. Give a precise justification of your answer. [15]

6. A sample of size 100 was generated from each of the following models to produce the six plots given in the next page.

   (a) $Y_i = Z_i + 0.9Z_{i-1}$, Z_i white noise with zero mean.
   (b) $Y_i = 0.9Y_{i-1} + Z_i$, Z_i white noise with zero mean.
   (c) $Y_i = -0.9Y_{i-1} + Z_i$, Z_i white noise with zero mean.
   (d) $Y_i = i + 1 + Z_i$, Z_i white noise with zero mean.
   (e) $Y_i = i + 1 + Z_i$, Z_i random walk with zero mean.
   (f) $Y_i = \cos(0.2\pi i) + Z_i$, Z_i white noise with zero mean.

owever, the plots have been mixed-up. Determine which plot corresponds to which odel, and give clear reasons for your conclusions. [25]



Indian Statistical Institute, Kolkata

First Semestral Examination for B. Stat. II (2002-03)

Physics I

Answer as many as you can.

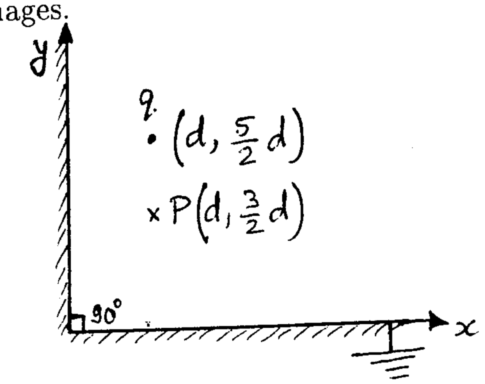
Date: December 2, 2002

Maximum Marks: 100 **Duration: Three hours**

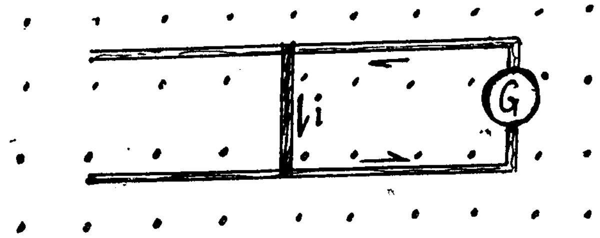
1. (a) How does a vector $\vec{A}$ transform under an inversion of co-ordinates ($x' \to -x$, $y' \to -y$ and $z' \to -z$)? How does the cross product $\vec{A} \times \vec{B}$ of two ordinary vectors $\vec{A}$ and $\vec{B}$ transform under inversion? [5 + 5]

   (b) Show that $\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A}$. [10]

2. A point charge q is placed near two grounded conducting plates at right angle to each other (see the figure). Find the electric potential at a point P as shown in the figure using the method of images. [20]



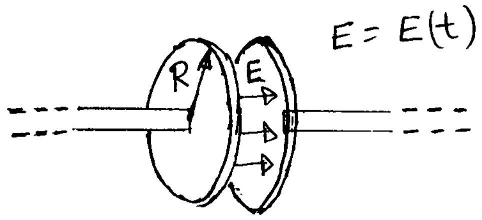
3. A conducting rod of mass m and length l slides without friction on two long horizontal rails. A uniform magnetic field B, pointing outward from the plane of the paper, fills the region in which the rod is free to move. The generator G supplies a constant current i that flows down one rail, across the rod and back to the generator along other rail. If the rod starts moving from its initial position of rest, show that the velocity of the rod at any time t is $v = ilBt/m$. [20]



P.T.O

4. (a) Why Maxwell's correction is required in Ampère's law? Write down the Ampère's law with Maxwell's correction in differential form. [5+5]

(b) A parallel plate capacitor with circular plates, each of radius R, is charged to create a time-dependent electric field $\vec{E}$ in the region between the plates. Find an expression for the induced magnetic field $\vec{B}$ at various distances r from the centre of the plates in the region between them. Consider both the cases when $r \leq R$ and $r \geq R$. [5 + 5]



5. (a) Consider the following hypothetical situation for an observer on the gound. An outlaw escapes on a straight highway in his getaway motorcycle at a speed equal to $\frac{3}{4}$ times the speed of light C. A cop following him in a car moving at a speed $C/2$ fires a bullet. The speed of the bullet with respect to the gun is $C/3$. Does the bullet hit the outlaw (i) according to Galilean Mechanics, (ii) according to special theory of relativity? [5 + 7]

(b) The average lifetime of a muon, a particle that disintegrates after its lifetime, at rest is 2×10^{-6}sec. How long, on the average, a muon last, if it is moving with a velocity $\frac{3}{5}C$. [8]

6. If $\mathcal{L}(q, \dot{q}, t)$ is Lagrangian of a system, find an expression for generalized momentum. Define Hamiltonian $\mathcal{H}$ of a system and derive Hamilton's canonical equations. Write down the Lagrangian of a particle of mass m and carrying a charge Q in a electromagnetic field $\vec{B}$ with scalar potential Φ and vector potential $\vec{A}$. Using this Lagrangian, derive the expression for the Hamilotonian of this system. [10+10]

Indian Statistical Institute

First Semestral Examination : (2002-2003)

Biology-I, B.Stat. II

Date: 2.12.02

Full marks: 55; *Answer any Five;* *Duration: 2hr 30min*

(Each question carries equal marks)

1. Consider an autosomal locus with two alleles 'A' and 'a'. Suppose the allele 'a' is recessive and determines a disease. Thus, individuals of genotype 'aa' are affected and individuals of 'AA' and 'Aa' are unaffected. What is the conditional probability that an individual is affected given that his/her parents are both unaffected? [You may assume that the underlying population is in Hardy-Weinberg equilibrium].

2. (a) If three individuals have the blood groups A, B, and O, what could be the possible genotypes for blood groups of their parents? (b) Hemophilia is caused by an X-linked recessive allele. In a particular population the frequency of males with hemophilia is 1:4000. What is the expected frequency of females with hemophilia in that population?

3. How many different codons are found in our cells? What should be the codons at the beginning and end of a gene? How many of initiation and termination codons should be present in a gene? How many genes are possible with 10 codons, each codon comprising a triplet of nucleotides (A, T, G or C)? What is the function of codons in protein synthesis?

4. a) What is the basis of the classification of living cells as photosynthetic or heterotrophic?
   b) How does ATP function in the transformation of energy by living cells?
   c) The enzyme catalyzed linked reactions or consecutive sequences make the regulation of metabolism possible. Explain.

5. The biomolecules of living organisms are ordered into a hierarchy of increasing molecular complexity. Depict this with appropriate examples.

6. What do you think, could be the primitive life molecule, DNA, RNA or protein? Give reasons.

INDIAN STATISTICAL INSTITUTE
FIRST SEMESTER EXAMINATION: 2002 – 2003
COURSE NAME: B. STAT. II
SUBJECT NAME: ECONOMICS-I

Date: 2/12/2002 | Maximum Marks: 60 | Duration: 3 Hours

Answer any THREE questions [3x20]

1. (a) State and explain the Weak Axiom of Revealed Preference. [6]

(b) A consumer is observed to purchase $x_1 = 22$, $x_2 = 9$ at prices $p_1 = 2$, $p_2 = 6$. He is also observed to purchase $x_1 = 16$, $x_2 = 10$ at prices $p_1 = 3$, $p_2 = 5$. Given this data, examine whether his behavior is consistent with the axioms of the revealed preference theory. [8]

(c) Using the theory of revealed preference prove that the substitution effect is negative. [6]

2. (a) Assume that production involves two inputs, capital (K) and labor (L). The unit prices of capital and labor are respectively r and w. Derive cost functions for the following production functions:

(i) $q = 20K^{\frac{1}{2}}L^{\frac{1}{3}}$, (ii) $q = \left\{\min\left(\frac{K}{2}, \frac{L}{3}\right)\right\}^{\frac{1}{2}}$. [6+6]

(b) Marginal products of capital and labor are respectively given by the following functions. Derive the expression for the production function. $MP_K = L - bK$, $MP_L = K - cL$; $b, c > 0$. [8]

3. (a) A monopolist uses one input, X, which he purchases at the fixed price $r = 5$ to produce his output, Q. His demand and production functions are respectively $P = 85 - 3q$ and $q = 2\sqrt{x}$. Determine the values of p, q *and* x at which the monopolist maximizes his profit. [8]

(b) A monopolist has two plants to produce a product. The corresponding cost functions are $C_1(q_1) = \frac{q_1^2}{2}$ *and* $C_2(q_2) = q_2$ respectively. What is the cost of producing an amount of output q? Suppose the monopolist has got an order of supplying 5 units of output. How will he allocate his production between these two plants? [5+3]

(c) Show that a price-discriminating monopolist will charge a higher price in a relatively inelastic market. [4]

4. Assume that there are 100 competitive firms supplying a commodity Q. Out of them 50 firms are at location A and the other 50 firms are at location B. It costs Rs 6 to transport to the market a unit of Q from location A and Rs 10 from location B. All firms possess the same production technology. The production cost of each firm is given by the function $C(q) = 0.5q^2$. (a) Derive the supply curve of each firm and the supply curve for the industry as a whole. (b) Let the market demand for the product be $D = 1600 - 20p$. Find the equilibrium market price and the corresponding supply of each firm in equilibrium. [10+10]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2002-2003)

B.Stat. (Hons.) II Year

C & Data Stuctures

Date : 29.11.02 Maximum Marks : 100 Duration : 3 hours

Note : You may attempt any part of any question. Maximum you can score is 100. You may give a flowchart instead of algorithm wherever algorithm is required.

1.a) What are the two ways in which a C-subprogram can access non-local data ?
b) Write a C-function which generates garbage and a dangling pointer.
c) For the following C-subprogram what will be printed on a call of

```
    int k=4; print(&k,&k);
C-subprogram :
void print(int *x, int *y)
{ *x=*x+1;
  *y=*y+1;
  printf("%d %d \n", *x,*y);
}
```

d) Draw and describe the general layout of an activation record for C.
(2 + 4 + 4 + 5 =15)

2. a) Define a Macro in C and explain how it will work ?
b) What are the contents of a #include file ?
How will you include a file of your own named myfile in a C-program you are writing ?
c) What are extern, static and automatic identifiers ?
d) Give an example of operator overloading in C-language.
(2+(3+2)+(2+2+2)+2 =15)

3. A file contains the following information about students residing in the hostel :

			Number-of-meals-taken during the month		
Roll No.	Name	Course	Breakfast	Lunch	Dinner

Write an algorithm and a program in C which will print the mess-bill for each course Roll number-wise. The charges for breakfast , lunch and dinner is to be input through keyboard.
(Use dynamic memory allocation for this problem.)
(5 + 10 =15)

4. Write an algorithm and a C-program to merge two linked lists such that elements of the two lists alternate in the new list. Do not create a new list but implement using pointer changes in the old lists. (5+10=15)

5. Write an algorithm and a C-program to search for an integer i from a binary-search tree. (5+10 = 15)

6. Write an algorithm and a C-program to compare two binary search trees using recursion. (5+10 = 15)

7. Write an algorithm and a C-program to a create a hash table using division X mod M hash function and open addressing using linear probing for strings read from a file in stream.
(5 + 10 = 15)

8. Write an algorithm and a C-program to implement queue as a linked list. (5+10 = 15)

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination : 2002-03
B. Stat. (Hons.) II Year
C & Data Structures

Date : 27.09.02 Maximum Marks : 100 Duration : 3 Hours

Answer as many questions as you can. The maximum obtainable marks is 100.

Q1.(a) Write an algorithm for the Insertion Sort.

(b) Analysis your algorithm for following Cases :

(i) Best Case
(ii) Worst Case, and
(iii) Average Case

[5+3+3+4=15]

Q2. Prove the memory location for the element $A(i_1, i_2, \ldots, i_n)$ in an array

$$A(l_1 : u_1, l_2 : u_2, \ldots\ldots, l_n : u_n)$$

is
$$a_0 + \sum_{j=1}^{n} (i_j - l_j) a_j$$

where
$$a_j = \begin{cases} 1, & j = n \\ \prod_{k=j+1}^{n} (u_k - l_k + 1), & \textit{otherwise} \end{cases}$$

and a_0 = location for the element $A(l_1, l_2, \ldots, l_n)$.

[9]

Q3. Write down the generalized list structure for the following expression

$$x^{11}y^3z^3 + x^4yz^3 - x^{10}y^3z^2 + 2x^8y^3z^2 + 3x^8y^2z^2 + x^4y^4z + 16x^3y^4z - 2yz.$$

Please explain the node structure that you use.

[5]

Q4. Construct the binary search tree for the following Key Sequence :

23, 4, 17, 52, 43, 25, 12, 64, 44, 55, 19, 38, 7, 99.

What will be the output Sequence if you traverse this in (a) Inorder, (b) pre-order, and (c) post-order ?

[4+3x3=13]

P.T.O.

Q5.(a) Define (i) height balance binary tree, and
(ii) the balance factor.

(b) Diagraminatically show all possible types of rebalancing rotations.

(c) Construct a height balance binary tree for the following key sequence.

34, 38, 40, 9, 6, 22, 14, 26, 18, 31. should indicate the balance factors and

For each intermediate step, you applied rebalancing type (if necessary).

[3+8+11=22]

Q6.(a) Construct a heap with all intermediate outputs for the following key sequence.

17, 23, 25, 18, 12, 6.

(b) Use the above heap to print the keys in the Sorted Order. (should indicate all intermediate heaps.)

(c) How will you use this heap for a priority Queue operations.

[4+4+2=10]

Q7. Define B-tree.

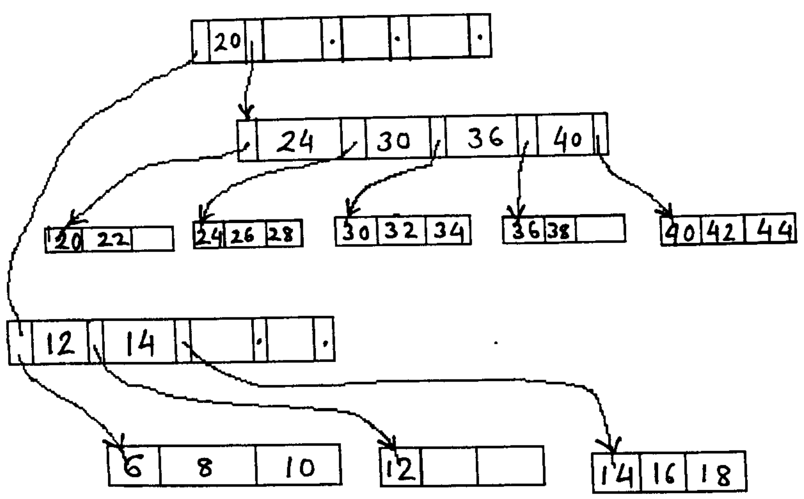


Fig. 1 : B-tree of Order 5

Consider a B-tree (Fig.1) of order 5 where each physical block contains almost 3 records.
Perform the following operations on Fig.1 and show the modified B-tree in each case.

(a) Insert a Key 23 in B-tree of Fig.1.
(b) Insert a Key 19 in B-tree of Fig.1.
(c) Insert a Key 25 in B-tree of Fig.1, and then delete a Key 12 from the modified B-tree.

[3+2+3+6=14]

Q8.(a) Construct a separate chaining hash table when the size of the address region is 9 (number 0 through 8) for the Keys a hash value given in Table 1.

Key	1_A	1_B	3_A	1_C	3_B	8_A	5_A	1_D	1_E
hash value	1	1	3	1	3	8	5	1	1

Table 1 : Key & its hash value

Compute the average number of probes for successful and unsuccessful searches for this hash table.

(b) Verify these the successful $(C_N(M))$ and unsuccessful search $(C'_N(M))$ performance (using definition and formula) for seperate chaining method are as follows :

$$C_N(M) = 1 + \frac{1}{2}\frac{N-1}{M}, \ and$$

$$C'_N(M) = \frac{N}{M} + \left(1 - \frac{1}{N}\right)^N$$

where , N = Number of inserted records in the hash table.
M = Size of address region of the hash table

(c) Use Table 1 to construct a hash table, so that it perform as separate chaining and also preserve the order of appairance of these key.

(d) You are required to design a hash table for an Organization where the personnel manager decided to store information of all the employees in a Table, so that the access time of the most recent record of employees will be relatively faster than the record of retired employees.

[(3+4)+5+5+5=22]

Mid- Semestral Examination of Biology-I, B.Stat. II, 2002-03

Date : 25.9.02 *Full marks: 45; Answer any Five .* Time : 2 hrs.

1. "Quality of an edible protein depends on its amino acid composition"- explain. Why do you think some amino acids are "essential" in terms of food ingredients? How can the quality of proteins be improved? Describe the importance of carbon and nitrogen cycles in human living.

2. Explain mitosis and meiosis with examples. In a cross of "Aa Bb X Aa Bb", what fraction of the progeny will have recessive genotype for at least one gene ?

3. What are aerobic and anaerobic metabolisms of glucose ? How many ATPs will be generated from each processes and calculate the efficiencies of energy trapped in these processes.

4. What are the major differences between plant and animal cells with respect to structure and functions of various organelles? What is turgor pressure and what advantage do plant cells get out of it?

5. a) Distinguish between photosynthetic and heterotrophic cells.
   b) How does ATP function in the transformation of energy by living cells?

6. Distinguish between the members of each pair:
   a) Plasma-membrane / Cell-wall
   b) Smooth endoplasmic reticulum / Rough endoplasmic reticulum
   c) Cell / Organelles
   d) Chromatin / Chromosome

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : 2002-03
B.Stat. (Hons.) II Year
Physics I

Date : 25.09.2002 **Maximum Marks : 100** **Duration : 2 hrs. 30 minutes**

All questions carry equal marks. Answer any five.

Q1.(a) Show that $\vec{a}\times(\vec{\nabla}\times\vec{b})-(\vec{a}\times\vec{\nabla})\times\vec{b}=\vec{a}(\vec{\nabla}.\vec{b})-(\vec{a}.\vec{\nabla})\vec{b}$

(b) If $\vec{r}=\vec{a}\sin wt+\vec{b}\cos wt$, where $\vec{a}$, $\vec{b}$ and w are independent of time t, show that

(i) $\dfrac{d^2\vec{r}}{dt^2}=-w^2\vec{r}$

(ii) $\vec{r}\times\dfrac{d\vec{r}}{dt}=-w(\vec{a}\times\vec{b})$

(c) Can any one of the two vectors given below represent an electrostatic field ?

(i) $\vec{A}=\alpha[xy\,\hat{i}+4\,yz\,\hat{j}+6zx\,\hat{k}]$,

(ii) $\vec{B}=\alpha\left[y^2\hat{i}+(2xy+z^2)\hat{j}+2y\hat{k}\right]$,

where α is a constant $(\neq 0)$ with appropriate units.

[8+6+6]

Q2.(a) Show that Kronecker-Delta symbol $\delta^i_j\,(i,j=1,2,3)$ transforms like a mixed tensor of second rank.

(b) Evaluate $\delta^i_j\delta^j_k\delta^k_i\;(i,j,k=1,2,3)$.

(c) Find $F^{\alpha\beta}G_{\alpha\beta}$, if

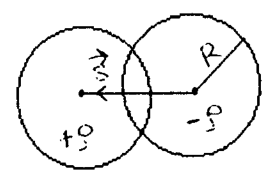
$$F^{\alpha\beta}=\begin{pmatrix}0 & \frac{E_1}{C} & \frac{E_2}{C} & \frac{E_3}{C}\\ \frac{-E_1}{C} & 0 & B_3 & -B_2\\ \frac{-E_2}{C} & -B_3 & 0 & B_1\\ \frac{-E_3}{C} & B_2 & -B_1 & 0\end{pmatrix}\quad and\quad G_{\alpha\beta}=\begin{pmatrix}0 & B_1 & B_2 & B_3\\ B_1 & 0 & \frac{-E_3}{C} & \frac{E_2}{C}\\ B_2 & \frac{E_3}{C} & 0 & \frac{-E_1}{C}\\ B_3 & \frac{-E_2}{C} & \frac{E_1}{C} & 0\end{pmatrix}.$$

$\vec{E}=(E_1,E_2,E_3)$, $\vec{B}=(B_1,B_2,B_3)$ and C constant.

[4+3+13]

P.T.O.

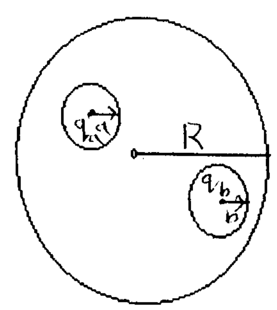
Q3.(a) Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Find the field in the region of overlap, if $\vec{s}$ is a vector from the negative centre to the positive centre.



(b) Two spherical cavities of radii a and b in the interior of a neutral conducting sphere contain point charges q_a and q_b respectively.

(i) Find the surface charges σ_a, σ_b and σ_R

(ii) What is the electrostatic fields within each cavity and outside the conductor.



[12+8]

Q4. Consider a system of N particles with mass m_i for the ith particle. Define the centre of mass. Show that if the external force on the system is zero, the total linear momentum is conserved. Prove that in a conservative force field, the total mechanical energy is conserved. Discuss how the pseudo force arise in a rotating frame of reference.

[1+4+5+10]

Q5. Show that if a particle moves in a central force field, it's motion will be confined in a plane and also show that it will cover equal area in equal time. Derive the equation of motion of a particle when the nature of the central force is gravitational. Discuss the nature of the orbit in that case.

[8+8+4]

Q6. Derive Lagrange's equation of motion from D'Alembert's principle. Define Lagrangian of a particle in a conservative force field. Write down the Lagrangian for a particle of mass m moving in a straight line under the force proportional to x and acting towards the origin. Get the equation of motion from Lagrangian.

[8+4+8]

INDIAN STATISTICAL INSTITUTE
MID-SEMESTER EXAMINATION: 2002 – 2003
COURSE NAME: B. STAT. II
SUBJECT NAME: ECONOMICS-I

Date: 25/09/02 Maximum Marks: 40 Duration: 2 Hours

You may answer all the questions, but your maximum score cannot exceed 40

1. (a) Explain the concept of stable equilibrium price.

(b) Consider the following lagged market model. The market demand is $D_t = ap_t + b$, market supply is $S_t = Ap_t + B$, and the price adjustment equation is $p_t = p_{t-1} + k(D_{t-1} - S_{t-1})$, $k > 0$.

(i) Derive the expression for the time path of price.

(ii) If $a = -0.5$, $b = 100$, $A = -0.1, B = 50$ *and* k = 6, explain the nature of the time path of price?

[3+6+4]

2. The demand for a product in Indian market is given by the function $D = 90 - P$. Its supply comes from both home firms and foreign firms. The home supply curve is $S_h = P$. The foreign supply (S_f) curve is exactly same as the home supply curve.

(a) Find the aggregate supply curve and the equilibrium market price of the product.

(b) Now suppose that the domestic industry lobbies for protection, and as a result the local government agrees to put a tariff of Rs 3 per unit on foreign products.

(i) What will be the price to be paid by the consumers in the new situation?

(ii) Also find the corresponding supply of goods by the domestic firms and by the foreign firms.

(iii) Draw a diagram to show your results.

[3+5+2+3]

3. Let the utility function be $U(x, y) = \min\{y + 2x, x + 2y\}$.

(i) Draw the indifference curve for $U(x, y) = \overline{U}$ and shade the area where $U(x, y) \geq \overline{U}$.

(ii) For what values of $\frac{p_x}{p_y}$ will the unique optimum be $x = 0$?

(iii) For what values of $\frac{p_x}{p_y}$ will the unique optimum be $y = 0$?

(iv) Hence draw the demand curve for X.

[6+2+2+3]

4. Consider a consumer who has the utility function $U(x, y)$ with MRCS constant and equal to 1 along the indifference curve. Prices of the products are $p_x = 5$ and $p_y = 10$. Derive the income consumption curve and estimate the income elasticity of demand for X.

[3+3]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination
B.Stat. (Hons.) II Year, I Semester
Statistical Methods III

Date: 23-09-02 Maximum time: 3 hours Maximum marks: 100

This test is closed book, closed notes. RMMR tables may be used. The entire set of questions is worth 110 marks.

1. Consider the *shifted exponential* distribution, having density

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)} & \text{if } x \geq \theta, \\ 0 & \text{if } x < \theta, \end{cases} \qquad -\infty < \theta < \infty.$$

   Find the 'Maximum Likelihood' and 'Method of Moments' estimators of θ based on n independent samples from this distribution. Is either of these estimators unbiased? Which estimator has smaller mean squared error? [4+3+3+2+8=20]

2. Let $X_1, X_2, \ldots, X_n$ be samples from a distribution which has the density

$$f(x;\sigma_1^2,\sigma_2^2,\eta) = \frac{\eta}{\sqrt{2\pi\sigma_1^2}} e^{-x^2/(2\sigma_1^2)} + \frac{1-\eta}{\sqrt{2\pi\sigma_2^2}} e^{-x^2/(2\sigma_2^2)}, \quad \sigma_1^2 > \sigma_2^2 > 0,\ 0 \leq \eta \leq 1.$$

   Describe a procedure to obtain the Maximum Likelihood Estimator of the three parameters, clearly showing all the necessary steps. [15]

3. A very large data set, divided into twenty modules of equal size, has been scrutinized twice for 'obvious' errors (such as out-of-range data, inconsistent dates, etc.), and the number of errors detected in the process have been recorded. Every error is corrected as soon as it is detected. Let N_{ij} be the number of errors detected in the ith module during the jth round of scrutiny ($i = 1, 2, \ldots, 20$, $j = 1, 2$). Derive a confidence interval for the average number of remaining 'obvious' errors per module, with confidence coefficient $1 - \alpha$, describing the assumptions clearly. Can you check the validity of any of these assumptions? [15+5=20]

P. T. O

4. Out of a sample of size 200 from an unknown (continuous) distribution, 80 happen to have positive values. Another set of 200 samples will be drawn subsequently. Can you provide two numbers a and b such that the interval (a, b) includes the fraction of positive values in the second sample with probability 0.95? Explain. [15]

5. *Brother Dairy* buys milk from several local milkmen. There is a suspicion that a particular milkman is adding water to the supplied milk. The freezing temperature of natural milk varies normally, with mean $\mu = -.545°C$ and standard deviation $\sigma = .008°C$. Added water raises the freezing temperature towards 0°C, the freezing point of water. *Brother Dairy* measured the freezing point of samples of the milk supplied by the suspected adulterer for five consecutive days. The mean measurement is $-.537°C$. Is this good evidence that the milkman is adding water to the milk? State the hypotheses, carry out the test, give the p-value, and state your conclusion. [20]

6. Given 10 independent samples from the distribution of Problem 1, describe how you can test the null hypothesis $\mathcal{H}_0 : \theta = 0$ against the alternative hypothesis $\mathcal{H}_1 : \theta > 0$, at the level $\alpha = e^{-4}$. What is the probability of Type II error for this test, if the 'true' value of θ happens to be .35? What happens to this error probability if the 'true' value of θ is .5? Explain. [10+4+4+2=20]

INDIAN STATISTICAL INSTITUTE
B.STAT-II (2002-03)
Probability - III (Mid Semememstral Examinations)
Max. marks:35 Time:2 hours 30 minutes.
Date: 19 September, 2002

Note: ***Answer as much as you can. The whole paper carries 50 marks. The maximum you can score is 35.***

1. Let $X_1, X_2, \cdots, X_n$ be *iid* random variables and let $a_1, a_2, \cdots, a_n$ be real numbers such that $\sum_1^n (a_i - \overline{a})^2 > 0$. Let

$$R = \frac{\sum_1^n (X_i - \overline{X})(a_i - \overline{a})}{S_X S_a}$$

where $\overline{X} = \frac{1}{n}\sum_1^n X_i$, $\overline{a} = \frac{1}{n}\sum_1^n a_i$, $S_X = (\sum_1^n (X_i - \overline{X})^2)^{\frac{1}{2}}$ and $S_a = (\sum_1^n (a_i - \overline{a})^2)^{\frac{1}{2}}$.

Complete the following exercises to find the probability density of R.

(a) Show that it is possible to choose an orthogonal matrix A such that its first two rows are $\alpha = (\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}, \cdots, \frac{1}{\sqrt{n}})$ and $\beta = (\frac{(a_1-\overline{a})}{S_a}, \frac{(a_2-\overline{a})}{S_a}, \cdots, \frac{(a_n-\overline{a})}{S_a})$.

(b) Let $\underset{\sim}{Y} = A\underset{\sim}{X}$, where $\underset{\sim}{X}^T = (X_1, X_2, \cdots, X_n)$. Show that $Y_1 \sim N(\sqrt{n}\mu, \sigma^2)$ and $Y_j \sim N(0, \sigma^2)$, $j = 2, 3, \cdots, n$ and $Y_1, Y_2, \cdots, Y_n$ are mutually independent.

(c) Verify that

$$T \stackrel{\text{def}}{=} \sqrt{n-2}\frac{R}{\sqrt{(1-R^2)}} = \frac{Y_2/\sigma}{\sqrt{\frac{U}{n-2}}}$$

where $U = \frac{1}{\sigma^2}\sum_{i=3}^n Y_i^2$. Hence conclude that T follows Student's t – distribution with $(n-2)$ degrees of freedom.

(d) Find the density function of the random variable R.

[2+3+5+5]

2. $X_1, X_2, \cdots,$ are *iid* random variables with exponential $\mathcal{E}(\alpha)$ – distribution ($\alpha > 0$). Let $S_0 = 0$ and $S_n = \sum_1^n X_i$, $n \geq 1$. Let $t > 0$ be a fixed real number. Let us define a positive integer-valued random variable as follows :

$$N = k \text{ if and only if } S_{k-1} < t \leq S_k,\ k = 1, 2, \cdots.$$

(a) Find $P(N = k, X_k \leq a)$, for $k = 1, 2, \cdots$ and $a > 0$.

(b) Show that the random variable X_N has the density :

$$f(x) = \begin{cases} \alpha^2 x e^{-\alpha x} & \text{for } 0 < x \leq t \\ \alpha(1+\alpha t)e^{-\alpha x} & \text{for } \quad x > t. \end{cases}$$

[3+7]

(Please turn overleaf)

3. (a) Define convergence in probability.

(b) Show that if $X_n \xrightarrow{P} 0$, then given any $\epsilon > 0$, there exists a real number $M > 0$ such that $P(|X_n| \leq M) \geq 1 - \epsilon$ for all $n \geq 1$.

[2+5]

4. Let $X_n, Y_n,\ n = 1, 2, \cdots$ be random variables such that $\sum_{n=1}^{\infty} P(X_n \neq Y_n) < \infty$.

(a) Show that $P(\liminf A_n) = 1$, where $A_n = \{\omega : X_n(\omega) = Y_n(\omega)\}$

(b) Let $S_n = X_1 + X_2 + \cdots + X_n$ and $S_n' = Y_1 + Y_2 + \cdots + Y_n$. Use (a) to show that $\frac{1}{n} S_n \to 0$ almost surely if and only if $\frac{1}{n} S_n' \to 0$ almost surely.

[3+7]

5. Let $X_n,\ n = 1, 2, 3, \cdots$ be iid random variables such that $\frac{1}{n} S_n$ converges to μ $a.s.$ where $S_n = X_1 + X_2 + \cdots + X_n$ and μ is a constant..

(a) Show that $\frac{1}{n} X_n$ converges to 0 $a.s.$

(b) Use 2nd Borel-Cantelli lemma to show that for every $\epsilon > 0$,

$$\sum_{1}^{\infty} P(|X_1| > n\epsilon) < \infty.$$

[2+6]

INDIAN STATISTICAL INSTITUTE

B. Stat (Second year)
Analysis III
Instructor: S. C. Bagchi
Maximum Marks 100

Year 2002-2003
Mid-semester Examination
Date: September 16, 2002.
Maximum Time 3 hrs.

Answer any five of the questions.

1. (a) Let S be a nonempty closed subset of $\mathbb{R}^n$, not necessarily bounded and $\tilde{a} \notin S$. show that there exists $\tilde{b} \in S$ such that

$$|\tilde{a} - \tilde{b}| = \inf_{\tilde{x} \in S} |\tilde{a} - \tilde{x}|.$$

(b) Assume the Heine-Borel theorem for a closed bounded rectangle in $\mathbb{R}^n$. Prove that any open cover of a closed and bounded subset $S \subset \mathbb{R}^n$ admits a finite subcover.

[10+10=20]

2. (a) Decide if the following limits exist:

$$(i) \quad \lim_{(x,y) \longrightarrow (0,0)} \frac{\sin(x^4 + y^4)}{x^2 + y^2}$$

$$(ii) \quad \lim_{(x,y) \longrightarrow (0,0)} f(x, y)$$

where

$$f(x,y) = \begin{cases} \frac{xy}{x^2 - y^2} & \text{if } x \neq \pm y \\ 0 & \text{otherwise} \end{cases}$$

(b) Let f be a continuous function on $\mathbb{R}^n$. If S is a closed and bounded subset of $\mathbb{R}^n$, show that $f(S)$ is a closed subset of $\mathbb{R}$.

[$5 \times 2 + 10 = 20$]

3. (a) Find the set of points $\tilde{x} \in \mathbb{R}^n$ where the function $\rho(\tilde{x}) = |\tilde{x}| = (x_1^2 + \cdots + x_n^2)^{1/2}$ is infinitely differentiable.

(b) Let f be a infinitely differentiable function on $\mathbb{R}$ such that

$$f(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 1 \\ 0 & \text{if } |x| \leq 2 \end{cases}$$

and $0 \leq f(x) \leq 1$ at all $x \in \mathbb{R}$. Find the set of points $\tilde{x} \in \mathbb{R}^n$, where the function $f(|\tilde{x}|)$ is infinitely differentiable.

[12+8=20]

P. T. O

4. (a) Show that if f has continuous partial derivatives on an open subset $U \subset \mathbb{R}^n$, then f is differentiable at each $x \in U$.
(b)Does there exist a function f on $\mathbb{R}^2$ such that

$$D_1 f(x,y) = x + y$$

and

$$D_2 f(x,y) = -x + y$$

for all $(x,y) \in \mathbb{R}^2$?

[$12 + 8 = 20$]

5. Let $\phi(t) = (\phi_1(t), \phi_2(t))$, $t \in (a,b)$ be a differentiable curve in $\mathbb{R}^2$. For $t_0 \in (a,b)$, we call the vector and $(\phi_1'(t_0), \phi_2'(t_0))$ the tangent to ϕ at the point $\phi(t_0)$.

Let now ϕ and ψ be two differentiable curves passing through a point $P = (x_0, y_0) \in \mathbb{R}^2$, $P \neq (0,0)$ and let θ be the map

$$\theta_1(x,y) = \frac{x}{x^2+y^2}, \quad \theta_2(x,y) = \frac{y}{x^2+y^2} \qquad (x,y) \in \mathbb{R}^2.$$

Show that the angle between the tangents to ϕ and ψ at P is the same as the angle between $\theta(\phi)$ and $\theta(\psi)$ at $\theta(P)$.

[20]

6. (a) Calculate $\frac{d^n}{dt^n}F(t)$ where

$$F(t) = f(x + ht, y + kt), \quad x, y, h, k, t \in \mathbb{R}$$

and f is a function on $\mathbb{R}^2$ with continuous partial deriatives of all orders (i.e. a C^∞-function).
(b) Show that if f is continuously differentiable on $\mathbb{R}^n$ and $\tilde{a}, \tilde{b} \in \mathbb{R}^n$, then there exists $\tilde{x} \in L(\tilde{a}, \tilde{b})$ such that

$$\nabla f(\tilde{x}) \perp \tilde{b} - \tilde{a}.$$

[10+10=20]

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