INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2008 - 2009)

B.Stat.-II yr.

PROBABILITY THEORY III

Date 01-09-08 Maximum Marks: 40 Duration: 10:30 -13:00

Note: Provide proper justifications by quoting theorems when used. Maximum you can score is 40 marks.

1. For a sample of size n from standard normal, show that the sample mean and sample variance are independent.

[7]

2. Each one in your class, independent of others, takes a sample of size n from uniform $(0,1)$ and calculates the largest order statistic. Let it be Z_i for the i-th student, where $1 \leq i \leq 9$. Lt X be the product of all the Z_i. Show that X has density

$$f(x) = \frac{n^9}{8!}\, x^{n-1} \, (-\log x)^8 \quad \text{for } 0 < x < 1.$$

[10]

3. Let X_n be the smallest order statistic from a sample of size n from uniform $(0,1)$. Show that $nX_n \Rightarrow \exp(1)$.

[6]

4. (a) Let X be a random variable with density

$$f(x) = 1 - |x| \quad \text{for} \quad -1 < x < 1.$$

Calculate the characteristic function of X.

(b) Let Y be a random variable with density

$$g(y) = \frac{1}{\pi}\,\frac{1-\cos x}{x^2} \quad \text{for} \quad -\infty < x < \infty.$$

P.T.O

You can assume that this is density function. Calculate the characteristic function of Y.

[8]

5. (a) Given a random variable Z, show that there is a number $M > 0$ such that $P(|Z| > M) < 0.01$.

   (b) Suppose $X_n \Rightarrow X$. Show that there is a number $M > 0$ such that for every n, $P(|X_n| > M) < 0.01$.

[2+4]

6. Let $(X_n; n \geq 1)$ and X be random variables, all taking values in the interval $[0, 1]$. Suppose that for every integer $k \geq 1$; $E(X_n^k) \to E(X^k)$. Show that $X_n \Rightarrow X$.

[8]

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INDIAN STATISTICAL INSTITUTE

Mid- Semester Examination: 2008-2009
B. Stat. II Year
Statistical Methods III

Date: 3.9.08 **Maximum Marks: 30** **Duration: 2 hours**

Answer as many as you can. The maximum you can score is 30.

1. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from Poisson (θ). Check whether the UMVUE of $g(\theta) = e^{-3\theta}$ attains the Cramer Rao Lower Bound.

[8]

2. If $X_1, X_2, \ldots\ldots, X_n$ are iid $U(\theta, \theta + |\theta|), \theta \in \mathbb{R} - \{0\}$, find out the maximum likelihood estimate for θ.

[4]

3. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from an exponential distribution with p.d.f.

$$f(x) = \begin{cases} \beta\, e^{-\beta(x-\alpha)} & x \geq \alpha \\ 0 & otherwise \end{cases}$$

Check whether $\left(X_{(1)}, S - X_{(1)}\right)$ is sufficient for (α, β). $[Here\ S = X_1 + X_2 + \ldots + X_n]$

If $\beta = 1$, assuming the completeness of $X_{(1)}$, show that

$$E\left[X_{(1)}\left(S - X_{(1)}\right)\right] = E\left(X_{(1)} E\left(S - X_{(1)}\right)\right) + (n-1) Var\left(X_{(1)}\right).$$

[3+5=8]

4. Suppose that $X_1, X_2, \ldots\ldots, X_n$ is a random sample from the following distribution

$$f_\theta(x) = \begin{cases} (1+\theta) x^\theta & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Find out (i) the maximum likelihood estimate and (ii) the method of moment estimate for $P\left(X_0 > \frac{1}{2}\right)$, where $x_0 \sim f_\theta$.

[4]

P.T.O.

5. Suppose that a novice shooter hits a target with probability $p(0 < p < 1)$. He started a practice session with 20 different targets of equal difficulty, and he stopped shooting at a target soon after hitting it. Assume that he was allowed to shoot at most 15 times at a target, and he succeeded to hit 10 out of these 20 targets. The number of shots required to hit these 10 targets were recorded as 7,8,3,3,5,2,4,6,3 and 9.

Write down the likelihood function and find the MLE of p. Also use the EM algorithm and show that it converges to the same solution.

[3+5=8]

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INDIAN STATISTICAL INSTITUTE
B. Stat. II year : 2008–2009
C & Data Structures
Mid Semester Examination

Date: 05. 09. 2008 | Marks: 60 | Time: 3 Hours

Answer any part of any question. The question is of 70 marks. The maximum marks you can get is 60. Please write all the part answers of a question at the same place.

1. (a) Write a function in C that calculates the square root of a floating point number upto 8 decimal digits.

   (b) Write a function in C which finds out the LCM of two unsigned integers and show how the function executes when the two integers are 1155 and 969.

   (c) Will your function, to calculate LCM, work properly for any pair of unsigned integers? Explain.

   6+8+6 = 20

2. (a) Write a C function using recursion to calculate the n-th Fibonacci number.

   (b) Solve the same problem using iterative method.

   (c) Explain with proper examples which one is more efficient.

   $3 + 3 + 4 = 10$

3. (a) Explain an efficient data structure to implement a Linear Feedback Shift Register (LFSR) of length ≤ 64 bits.

   (b) Write a C function to load the initial seed in the LFSR.

   $5 + 5 = 10$

4. (a) Clearly write down the program for heap sort in C.

   (b) Execute your program (explain with proper figures of binary tree) on the data set 25, 15, 11, 12, 3, 85, 37 (coming in this order).

   $5 + 5 = 10$

5. (a) Explain how a polynomial can be implemented using linked list.

   (b) Implement a C function that will take two polynomials as input and provide the sum as the output.

   $3 + 7 = 10$

6. (a) Briefly explain what the 'strtok' and 'strstr' functions do.

   (b) Implement the functions on your own in C programming language.

   $2 + 8 = 10$

Analysis-III: B. Stat II: Mid Semester Examination
September 9, 2008.

Maximum Marks 40 Maximum Time 2:30 hrs.

Answer all questions.

1. (a) Let (X, d) be a metric space and $s \in (0, 1)$. Show that (X, d^s) is also a metric space where d^s is defined by
$$d^s(x, y) = (d(x, y))^s, \text{ for } x, y \in X.$$
(b) Suppose that $X = \mathbb{R}^2$ and d is the usual metric on $\mathbb{R}^2$. Give a quick argument to prove that d^s defined in (a) is not induced by a norm. Show that $x_n \longrightarrow x$ in $(\mathbb{R}^2, d)$ if and only if $x_n \longrightarrow x$ in $(\mathbb{R}^2, d^s)$.
(c) Let U be an open set in $\mathbb{R}^n$ and $E_1 \supset E_2 \supset \ldots$ be a decreasing sequence of closed and bounded sets with $\cap_{n=1}^{\infty} E_n \subset U$. Show that $E_N \subset U$ for some N. 5+(2+4)+5=16

2. (a) Suppose $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ satisfies $|f(x)| \leq \|x\|^2$ for all $x \in \mathbb{R}^2$. Prove that f is differentiable at 0.
(b) Let $f : \mathbb{R}^n - \{0\} \longrightarrow \mathbb{R}$ be a continuous function such that for some fixed $d \geq 0$,
$$f(rx) = r^d f(x) \forall x \in \mathbb{R}^n - \{0\} \text{ and } r > 0.$$
Prove that if $d = 0$ then f is bounded on its domain and that f can be extended to 0 as a continuous function only if f is a constant.
(c) If f is as in (b) with $d > 1$ and f is also differentiable then show that $f'(rx) = r^{d-1}f'(x)$. 3+4+4=~~14~~ 11

3. Show that the equations: $x^2 - y\cos(uv) + z^2 = 0$,
$$x^2 + y^2 - \sin(uv) + 2z^2 = 2 \text{ and}$$
$$xy - \sin u \cos v + z = 0$$
implicitly define x, y, z as C^1 functions of (u, v) near $x = 1, y = 1, u = \pi/2, v = 0, z = 0$ and find $\frac{\partial x}{\partial u}$ and $\frac{\partial x}{\partial v}$ for the function $x(u, v)$. Is the function infinitely differentiable? 10

4. Consider the map $A \mapsto A^2$ from $M_n(\mathbb{R}) \longrightarrow M_n(\mathbb{R})$ as a map from $\mathbb{R}^{n^2} \longrightarrow \mathbb{R}^{n^2}$. Show that the map is invertible on some open set of the domain containing the identity matrix. 7

Indian Statistical Institute

Mid-Semester Examination (B. Stat-II, Biology I, 2008)

Full marks: 40, Answer any four questions, each question carry

Date: 11.9.08 **Time - 2hr.**

1. (a) In a population there is no "O" allele. One investigator id and 35 individuals had "A", "B" and "AB" blood groups re population. What are frequencies of "A" and "B" alleles in this p

   (b) Indicate whether each of the following statements, about the st stranded DNA, is true or false: (i) A+T=C+G; (ii) once the base strand is known, the base sequence of the other strand can be dec within each strand; (iv) when separated the two strands are ide sequence. (A, T, G and C are nucleotides in DNA)

2. (a) Oleic acid is an unsaturated fatty acid containing 18 carbon atc hydrogen atoms but a double bond between 8 and 9 carbon atoms. fatty acid be oxidized to generate ATP?

   (b) Explain why aerobic metabolism of glucose gener anaerobic metabolism of glucose in human.

3. Write notes on: Phenyl ketonuria and Ketone bodies.

4. α-ketoglutarate is an useful chemical for oxidation of both glucos of amino acids:- Show how it is used and re-generated in these rea

5. Write down five differences between DNA and RNA. Explain why DNA is used in one of the steps for polymerase chain reaction?

INDIAN STATISTICAL INSTITUTE
MID-SEMESTER EXAMINATION: 2008-09
B. STAT. II: 2008-09
ECONOMICS-I

Date: 12.9.08 | Maximum Marks: 40 | Duration: $2\frac{1}{2}$ Hours

Each question carries 7 marks. Answer any **six** questions. The maximum that you can score is 40.

1. (i) State and explain the law of demand. (ii) Consider the following information. Last year the price of rice was Rs 15 per kg and the sale was 1000 quintals. This year the corresponding price and sale are respectively Rs. 20 and 1500 quintals. Explain the situation.

2. (i) What do you mean by 'price support'? (ii) Consider the market for paddy with demand curve downward sloping and supply curve upward sloping, and the market price per kg is p_e. Now the government fixes the floor price at $\bar{p} > p_e$. The government has the following two options: either buy the excess paddy at price $\bar{p}$, or give subsidy per unit equal to $(\bar{p} - p_0)$ where p_0 is the price at which the firm sells all supply $S(\bar{p})$. Explain which option is less costly to the government.

3. Define marginal utility (MU) and marginal rate of commodity substitution (MRCS). Show that diminishing MU is neither necessary nor sufficient for diminishing MRCS.

4. State the Weak Axiom of Revealed Preference. Using it prove that the substitution effect is negative.

P.T.O.

5. Suppose a consumer mixes two sugar cubes with each cup of tea. (i) If x_1 denotes the number of cups of tea available, x_2 the number of sugar cubes and U the utility index representing the number of correctly sweetened cups of tea, what is the utility function? (ii) Derive the demand function for x_1. (iii) What can you say about its price elasticity of demand?

6. A person consumes two goods, X and Y, but he is to decide whether to buy from market A or from market B. His utility function is $U(x, y) = x^{1/2}y^{1/2}$, where x and y are the amounts of consumption of X and Y respectively. Further assume that the price of Y is 1 in both markets; the price of X is higher in market A but purchase from B market involves a transport cost T. How do you solve his problem?

7. A consumer consumes two goods, 1 and 2. The demand function for the first commodity is $x_1 = 10 + m/(10p_1)$. (i) Given initially, $p_1 = 3$ and $m = 120$, if price of the first commodity decreases to $p_1' = 2$, what is the price effect? (ii) Decompose the price effect into income effect and substitution effect.

8. Consider the following model: $D_t = a + bP_t$, $S_t = \alpha + \beta P_t$ and $P_t = P_{t-1} - k(S_t - D_t)$, where D_t, S_t and P_t denote respectively the amount of demand, the amount of supply and the price of the product at time t; a, b, α, β are real numbers and $k > 0$. (i) Interpret each relation. (ii) Derive the time path of price. (iii) Under what conditions will the price be stable in the dynamic sense?

INDIAN STATISTICAL INSTITUTE

Analysis III: B. Stat-II: Semester Examination

Date 21.11. 08

Maximum Marks 60 Maximum Time 3:30 hrs.

For a function f, both Df and f' denote derivative of f.

1. (a) Find all stationary points of the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by the formula $f(x, y) = xy(1 - x^2 - y^2)$ in the region $-1 \leq x \leq 1, -1 \leq y \leq 1$. Determine the relative maxima, relative minima and the saddle points.

(b) Let A be a real symmetric $n \times n$ matrix. Let $Q(x) = \langle Ax, x\rangle$ where $\langle \cdot, \cdot \rangle$ is the canonical inner product on $\mathbb{R}^n$. Give arguments to establish that there are points $x \in \mathbb{R}^n$ with $\|x\| = 1$ such that $DQ(x) = 0$, where $\|x\|^2 = \langle x, x\rangle$. Show that any such x is an eigenvector of A.

(c) Let $\mathbb{R}^n_+ = \{(x_1, x_2, \ldots x_n) \mid x_i \geq 0\}$. Define $P : \mathbb{R}^n_+ \longrightarrow \mathbb{R}$ by $P(x_1, x_2, \ldots, x_n) = x_1 x_2 \ldots x_n$. Consider the function $\phi(x) = \log P(x)$. Show that $D^2\phi(x)(v, v) \leq 0$ for all $v \in \mathbb{R}^n$.

10+10+5=25

2. (a) For finite dimensional vector spaces V_1, V_2, V_3 let $B : V_1 \times V_2 \longrightarrow V_3$ be a bilinear map. Find $DB(x, y)(h, k)$ for $(x, y), (h, k) \in V_1 \times V_2$.

(b) Let V_1, V_2, V_3, V_4 be finite dimensional vector spaces. Consider the trilinear map $T : V_1 \times V_2 \times V_3 \longrightarrow V_4$. Use part (a) to compute $DT(x, y, z)(\alpha, \beta, \gamma)$ where $(x, y, z), (\alpha, \beta, \gamma) \in V_1 \times V_2 \times V_3$. Conjecture a formula for the derivative of a p-linear map.

(c) Let $f : M_n(\mathbb{R}) \longrightarrow \mathbb{R}$ be given by $f(A) = \det A$. Show that $Df(A)(A) = n \det A$.
(Hint: View a matrix A as an n-tuple $(x_1, x_2, \ldots, x_n)$ where each $x_i \in \mathbb{R}^n$ is the i-th column vector of the matrix. Use your conjecture in (b))

5+5+5=15

3. (a) Find the line integral

$$\int_C \frac{(x+y)dx - (x-y)dy}{x^2 + y^2}$$

where C is the trace of the circle $x^2 + y^2 = a^2$ traversed once in anticlockwise direction.

(b) Let S be an open connected subset of $\mathbb{R}^2$. Let $f, g : S \longrightarrow \mathbb{R}^2$ be C^1-functions. Show that

$$\oint_C f\nabla g \cdot d\alpha = -\oint_C g\nabla f \cdot d\alpha,$$

P.T.O.

for every piecewise smooth Jordan curve C in S. (Integrations are in counterclockwise direction.)

7+7=14

4. Let $\mathcal{R}$ be a connected and simply connected region in $\mathbb{R}^2$ bounded by a piecewise smooth Jordan curve Γ. Let $r : \mathcal{R} \longrightarrow \mathbb{R}^3$ be a C^2 function given by

$$r(u,v) = (X(u,v), Y(u,v), Z(u,v))$$

so that $S = r(\mathcal{R})$ be a smooth simple parametric surface. Let $P : S \longrightarrow \mathbb{R}$ be a C^1-function. Define $p(u,v) = P(r(u,v))$.

Show that

$$\int\int_{\mathcal{R}} \left(\frac{\partial p}{\partial Z} \frac{\partial(Z,X)}{\partial(u,v)} - \frac{\partial p}{\partial Y} \frac{\partial(X,Y)}{\partial(u,v)} \right) dudv = \int_{\Gamma} p\frac{\partial X}{\partial u} du + p\frac{\partial X}{\partial v} dv.$$

using the following steps:

(a) Show that

$$\frac{\partial}{\partial u}\left(p\frac{\partial X}{\partial v}\right) - \frac{\partial}{\partial v}\left(p\frac{\partial X}{\partial u}\right) = \frac{\partial p}{\partial u}\frac{\partial X}{\partial v} - \frac{\partial p}{\partial v}\frac{\partial X}{\partial u}.$$

(b) Find $\frac{\partial p}{\partial u}$ and $\frac{\partial p}{\partial v}$.

(c) Use (a) and (b) to show $\frac{\partial}{\partial u}\left(p\frac{\partial X}{\partial v}\right) - \frac{\partial}{\partial v}\left(p\frac{\partial X}{\partial u}\right) = \left(\frac{\partial p}{\partial Z}\frac{\partial(Z,X)}{\partial(u,v)} - \frac{\partial p}{\partial Y}\frac{\partial(X,Y)}{\partial(u,v)}\right)$.

(d) Apply a theorem.

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2008-09
B. Stat. II Year
Probability Theory III

Date: 25.11.08 **Maximum Marks: 60** **Duration: 3 Hours**

1. **State precisely the theorems you are using.**
2. **This paper is set for 70 Marks. Maximum you can score is 60.**

1. Y_i is a Gamma variable with parameter α_i, for $1 \le i \le k+1$. $Y_1, Y_2, .. Y_{k+1}$ are independent. Put $X_i = \frac{Y_i}{\sum_1^{k+1} Y_i}$ for $1 \le i \le k$. (a) Calculate the joint density of $(X_1, ..., X_k)$.

2. X_1, X_2, X_3, X_4 are independent standard normal.

(a) Calculate the characteristic function of $X_1 X_2$.

(b) Show that $X_1 X_2 - X_3 X_4$ is two sided exponential.

[7+3]

3. $X_0, X_1, X_2, ...$ are iid exp (λ) variables. Define an integer valued Random Variable N by : For $k \ge 1$, $N = k$ iff $X_k > X_0$ but $X_i \le X_0$ for $i < k$.
Define $Z = X_N$.

(a) Show that $P(N = n) = \frac{1}{n(n+1)}$ for $n \ge 1$.

(b) Are Z and N independent ?

(c) Calculate the density of Z.

[4+4+2]

4. (a) Suppose φ is characteristic function of a r.v.X. Suppose for some $t_0 \ne 0$; $|\varphi(t_0)| = 1$. Show that X is a discrete random variable.

(c) Suppose that $\left|\varphi\left(\frac{1}{p}\right)\right| = 1$ for every prime $p \ge 2$. Show that $P(X = a) = 1$, for some number a.

[4+4]

5. $X_1, X_2, ...$ are iid mean 0 and variance 1.

(a) Set $Y_n = X_n + X_{n+2} + X_{n+4}$ for $n \ge 1$.

Show that the sequence $(Y_n)_{n \ge 1}$ obeys WLLN.

(b) Set $Z_N = \frac{X_1 + X_2 + ... + X_n}{n^{0.6}}$. Show that $Z_n \to 0$ in probability.

P.T.O.

(c) Set $W_n = \sum_1^n X_i \Big/ \sqrt{\sum_1^n X_i^2}$. Show that $W_n \Rightarrow N(0,1)$.

[4+5+3]

6. $X_1,\ldots,X_n$ are iid standard normal. Let A be an $n \times n$ symmetric nonzero matrix. Show that $X'AX$ is χ^2 iff A is idempotent.

[10]

7. For each statement below, state whether it is true or false and then justify your answer.

(a) For a random variable X if $EX^4 < \infty$ then $EX^2 < \infty$.

(b) If $X_n \to X$ in probability then $X_n \to X$ almost everywhere.

(c) If $\varphi(t)$ is a characteristic function then $|\varphi(t)|^2$ is also a characteristic function.

[2+4+4]

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First Semester Examinations (2008-2009)
B Stat – 1year
Remedial English
100 marks
1½ hours

Date: 25.11.08

1. Write an essay on any one of the following topics. Five paragraphs are expected.
a. My country.
b. Smoking is a bad habit.
c. Electricity.

(60 marks)

2. Fill in the blanks with appropriate prepositions:

a. I am weak _____ mathematics.
b. Samudragupta was known _____ his skill _____ music and song.
c. Your car differs _____ mine _____ several respects.
d. He differs _____ me _____ this question.
e. He worked _____ his book _____ twelve years.
f. I was angry _____ her _____ lying to me.
g. He is confident _____ his success.
h. I met him _____ my way ____ the station.
i. He was afraid _____ tell the truth.
j. We ran _____ that we might arrive _____ time.
k. They were grateful _____ to him _____ his kindness.
l. I am glad that he has recovered _____ his illness.

(20 marks)

3. Fill in the blanks with appropriate words:

Up the River Hudson ____ North America _____ the Catskill Mountains. ______ a certain village _____ the foot _____ these mountains, _____ lived long ____ a man named Rip Van Winkle. He was a simple and good-natured _____, a _____ kind neighbour and a great favourite _____ all the good wives _____ the village. The children _____ the village _____ would shout _____ joy, whenever they saw him. He helped _____ their sports, made playthings _____ them, taught _____ to fly kites and shoot marbles and told them long stories of ghosts, witches and Indians. He _____ sit ____ a whole day on a wet rock and fish without a murmur, even _____ he did not catch a single fish.

(20 marks)

INDIAN STATISTICAL INSTITUTE
1st Semester Examination
B. Stat. II year : 2008-2009.
C & Data Structures

Date: 28.11. 2008 Marks: 100 Time: 3 Hours

Answer any part of any question. The maximum marks you can get is 100. Please try to write all the part answers of a question at the same place.

1. (a) Write a function in C to check whether a string contains at least 2 vowels.
   (b) Write a function in C to multiply two integer matrices.
   (c) Consider an unsigned integer having 32 bits. Write a C function to count the number of 1's in it.
   (d) Write a C program without using recursion for inorder traversal in a binary search tree.

   5+5+5+10 = 25

2. (a) Write a function in C that takes the number of data n as the first argument and then accepts n many positive floating point numbers $a_1, \ldots, a_n$. The function should return the standard deviation of $a_1, \ldots, a_n$.
   (b) Explain the idea of function pointers with simple examples.
   (c) Briefly explain the idea of hashing.
   (d) Analyse the strategy of double hashing for collision resolution.

   5+5+4+6 = 20

3. (a) What is a balanced binary tree?
   (b) Derive the maximum height of a balanced binary tree having n many nodes?
   (c) Explain the insertion algorithm in a balanced binary tree.
   (d) Construct a balanced binary tree with the data set 89, 152, 53, 77, 91, 63, 88, a, 18, where a is your class roll number (last two digits). Consider that the data is being inserted in the tree in the order as given. Explain each step of insertion.

   2+8+7+8 = 25

4. (a) What additional properties need to be imposed on the definition of an m-way search tree to get a B-tree?
   (b) Derive the maximum height of a B-tree with degree m having n many nodes?
   (c) Explain the insertion algorithm for a B-tree.
   (d) Construct a 2-3 tree considering the data given in Question (3d).

   2+8+7+8 = 25

5. Describe in detail a probabilistic polynomial time algorithm to test whether a large integer is prime. 15

B.Stat - II yr.

Statistical Methods III

Date: 02.12.08

Time: 2 hours | First Semester Examination (2008-09) | Full Marks : 40

1. (a) Suppose that we have two independent observations X_1 and X_2 from a distribution with density function f, and we are interested in testing $H_0 : f(x) = 1;\ 0 \le x \le 1$ against $H_1 : f(x) = 4x^3;\ 0 \le x \le 1$. Show that the most powerful test of size α is given by

$$\begin{aligned} \phi(x_1, x_2) &= 1 \quad \text{if } log x_1 + log x_2 > -\nu/2 \\ &= 0 \quad \text{otherwise,} \end{aligned}$$

where $\int_0^{\nu} te^{-t/2}dt = 4\alpha$. Find out the power of this test. [4+4]

(b) Consider a class of probability density functions $f_\theta(x) = \theta x^{\theta-1};\ \ 0 < x < 1,\ \ 0 < \theta < \infty$. Can you use the result obtained in (a) to construct the UMP test for $H_0 : \theta = 1$ against $H_1 : \theta > 1$ based on a sample of size 2 ? Will it be the UMPU test for $H_0 : \theta = 1$ against $H_1 : \theta \ne 1$? [2+2]

2. Suppose that $X_1, X_2, \ldots, X_n$ are independent random variables and $X_k \sim N(k\theta, 1)$ for $k = 1, 2, , n$.

(a) Find out the minimal sufficient statistic for θ. [2]

(b) Check whether the minimal sufficient statistic is complete. [2]

(c) Is it possible to find the UMVUE for θ in this case? If you think it is possible, find out the variance of that UMVUE. If you think it is not possible, find out the locally minimum variance unbiased estimator of θ for $\theta = 0$. [4]

3. (a) If $X_1, X_2, \ldots, X_n$ are independent observations from $U(\theta, \theta+1)$, show that $X_{(1)}$ and $X_{(n)} - 1$ are consistent for θ. Hence show that the maximum likelihood estimator of θ is consistent. [2+1]

(b) Now consider a squared error loss function and assume that Θ has a density $\pi(\theta)$ on R. Show that the Bayes estimator is also consistent. [2]

(c) If we know that $\theta > \alpha$ and $\pi(\theta) = e^{-(\theta-\alpha)}I\{\theta > \alpha\}$, find out the Bayes estimator of θ when the loss function is given by $L(a, b) = I\{|a - b| > 0.1\}$. [3]

4. Prove or disprove the following statements :-

(a) Let $X_1, X_2, \ldots, X_n$ be n i.i.d. observations from $U(0, \theta)$. "For any positive integer $k < n$, $E(X_{(k)}/X_{(n)}) = E(X_{(k)})/E(X_{(n)})$."

(b) Let $\{f_\theta,\ \theta \in \Theta\}$ be a family of distributions which is not complete. "If we have n i.i.d. observations from this distribution, no sufficient statistic of θ can be complete."

(c) "Maximum likelihood estimator is always consistent."

(d) "Any admissible estimator with constant risk is minimax." [3 × 4]

First Semestral Examination (B. Stat-II, Biology I, 2008), Date: 9.1. 2009

Answer any five; All questions carry equal marks; Full marks = 50; Time = 2.5 hours

1. a). What evidence supports the theory that mitochondria arose during evolution from the invasion of small oxygen-using cells into large non-oxygen using cells? [2]

   b). The function of circulating white blood cells is to engulf invading bacteria in a process known as phagocytosis. Describe what happens to the bacteria after they are engulfed. [4]

   c). If pancreatic cells synthesizing digestive enzymes for export are supplied with radioactive amino acids, the pathway of proteins from synthesis to export can be followed. In what order does radioactivity appear in the organelles involved in this pathway? [4]

2. a). What are the major differences between plant and animal cells with respect to structure and functions of various organelles? [5]

   b). What is turgor pressure and what advantage do plant cells get out of it? [5]

3. a). How do the three elements of the cytoskeleton differ from each other in composition? [2]

   b). What are the functions of each type of cytoskeletal element? What is a reason for the lack of a cytoskeleton in prokaryotic cells? [4]

   c) How do eukaryotic and prokaryotic cells differ with respect to genome organization and reproduction? [4]

4. In comparison to "recessive alleles", "dominant alleles" for diseases are rare in the population "- explain. [10]

5. (a) A normal woman, whose father had hemophilia, marries a man. What is the chance that their first child will have hemophilia? (hemophilia is a X-linked disease) [5]

   (b) Can a reason be ascribed to a codon in the genetic code being a triplet? Why is the genetic code said to be comma-free? [2+3]

6. (a) DNA and RNA synthesis occur by very similar mechanisms. In what ways do they differ ? [5]

   (b) The codons UUA, UUG, CUU and CUC specify the amino acid "leucine". What could be the minimum number of tRNAs for translation of these codons and why? [5]

P.T.O

7. (a) The genotypes of an individual at four loci in four different chromosomes are *Aa*, *Bb*, *Cc* and *Dd*. What will be the possible four locus gametes produced by this individual? [5]

(b) A mutation in an essential human gene changes the 5' splice site of a large intron from *GT* to *CC*. Predict the phenotype of individuals who are homozygous or heterozygous for this mutation. [5]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination (2008-09)
B. Stat II year
Physics I

Date: 9.1.2009

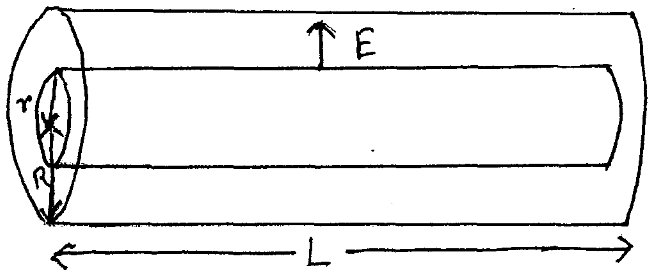
Maximum Marks 100 Duration 3 hours

Note: Use different Answer Sheets for different groups

Group A

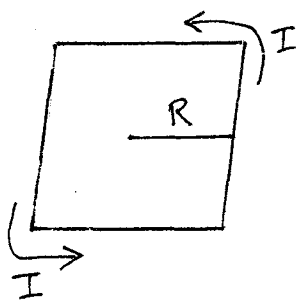
Maximum marks 50

1(a). Two long cylinders (radii r and R, as shown in the figure) are separated by a material of conductivuty σ. Find the current flowing from one end to the other in a length L, if the cylinders are maintained at a potential difference V.



(b) Find the magnetic field at the centre of a square loop, which carries a steady current I. See the figure below.

[6+6.5]



2.(a) State and explain the continuity equation. Write its form in the case of steady current.

(b)For which case, Ampere's law is not correct ? Explain how Maxwell fixed the correction to Ampere's law.

P. T. O

(c)Express the Maxwell's equations in matter in terms of free charges and currents. Using them show that

$$\mathbf{B} = \nabla \times A$$
$$\mathbf{E} = -grad\ \phi - \frac{\partial \mathbf{A}}{\partial t}$$

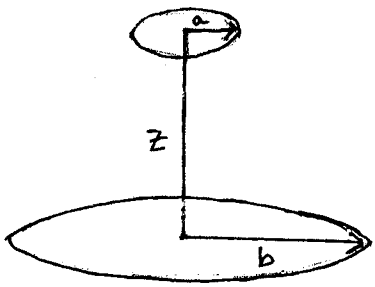
[4+4+4.5]

3. Suppose the density of magnetic charges is represented by ρ_m and the density of electric charges are denoted by ρ_e. Giving proper notations to the current of the magnetic charge and electric charge, express the new Maxwell's equations.
Show that these new Maxwell's equations are invariant under the duality transformation

$$\begin{aligned} \mathbf{E}' &= \mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha \\ c\mathbf{B}' &= c\mathbf{B}\cos\alpha - \mathbf{E}\sin\alpha \\ cq_e' &= cq_e\cos\alpha + q_m\sin\alpha \\ q_m' &= q_m\cos\alpha - cq_e\sin\alpha \end{aligned}$$

[4+8.5]

4. A small loop of wire (radius a) lies a distance z above the center of a large loop (radius b), as shown in the figure. The planes of the two loops are parallel, and perpendicular to the common axis.



(a) Suppose current I flows in the big loop. Find the flux through the little loop. (The little loop is so small that you may consider the field of the big loop to be constant.)

(b) Suppose current I flows in the little loop. Find the flux through the big loop.(The little loop is so small that you may treat it as a magnetic dipole)

(c) Find the mutual inductances, and confirm that $M_{12} = M_{21}$

[4+4.5+4]

5(a) Show that $\mathbf{A} = \frac{1}{2}(\mathbf{B} \times \mathbf{r})$ is a possible vector potential for a uniform magnetic field $\mathbf{B}$ where $\mathbf{r}$ denotes the position of the field point.
(b) Using Maxwell's equation (in source free space with $\epsilon_0 = \mu_0 = 1$) obtain

$$\frac{\partial}{\partial t}\left(\frac{E^2 + B^2}{2}\right) + \nabla \cdot (\mathbf{E} \times \mathbf{B}) = 0$$

[4+8.5]

Group B

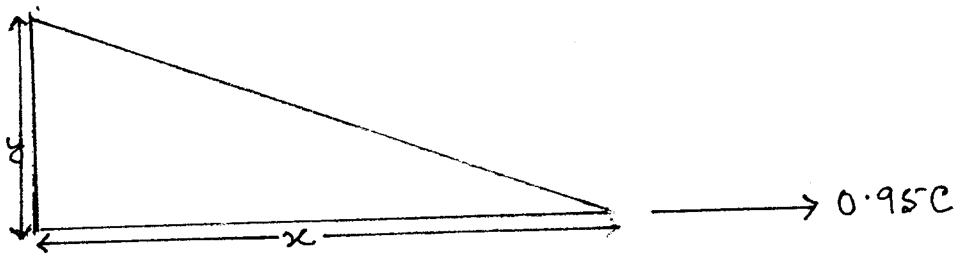
Answer all questions

In questions 1 and 2(a), c denotes the velocity of light in vacuum

1(a). A motorcycle rider is moving with a speed $0.80c$ past a stationary observer. If the rider tosses a ball in the forward direction with a speed of 0.70c, relative to himself, what is the speed of the ball as seen by the stationary observer?
1(b). Suppose that the motorcycle rider turns on a beam of light that moves away from him with a speed c in the forward direction. What does the stationary observer measure for the speed of light?

[7+2]

2(a). A spaceship in the form of a triangle flies past an observer with a speed of $0.95c$. When the ship is at rest relative to the observer, the distances x and y are measured to be $52m$ and $25m$ respectively. Will there be any change in the shape of the spaceship as seen by an observer at rest when the spaceship is in motion along the direction as shown in the figure? If the answer is in the affirmative, calculate the change.



2(b). If the transformation equations between two sets of coordinates are

$$P = 2(1 + q^{\frac{1}{2}} cos\ p)q^{\frac{1}{2}} sin\ p, \qquad Q = log(1 + q^{\frac{1}{2}} cos\ p)$$

then show that
(i) the transformation is canonical.
(ii) the generating function of this transformation is $F_3 = -(e^Q - 1)^2 tan\ p$.

[6+(5+5)]

3(a). Apply Hamilton's principle to derive Lagrange's equations from the Lagrangian $L(q_k, \dot{q}_k), (k = 1, 2 \cdots n)$ of a given physical system where q_k are the generalized coordinates and $\dot{q}_k$ are the generalized velocities.
3(b). Show that for a conservative system, if the potential energy is velocity independent, then the Hamiltonian $H(p_k, q_k), (k = 1, 2 \cdots n)$ represents the total energy of the system. Here q_k denotes the generalized velocities and p_k denotes canonical momenta.
3(c). Under what condition is the Hamiltonian a constant of motion?

[5+5+1]

4. Obtain the Hamiltonian of a non-relativistic charged particle of mass m and charge e starting from the Lagrangian

$$L = \frac{1}{2}mv^2 + \frac{e}{c}\vec{v}.\vec{A} - e\phi$$

Here $\vec{A}(\vec{r}, t)$, $\phi(\vec{r}, t)$ are the vector and scalar potentials of the electromagnetic field respectively and $\vec{v}$ is the velocity of the particle.

[6]

P.T.O

5. A particle of mass m falls a given distance z_0 in time $t_0 = \sqrt{\dfrac{2z_0}{g}}$ and the distance travelled in time t is given by $z = at + bt^2$, where the constants a and b are such that the time t_0 is always the same. Show that the integral $\int_0^{t_0} L dt$ is an extremum for real values of the coefficients only when $a = 0$ and $b = \dfrac{g}{2}$. Here L denotes the Lagrangian of the system and g is the acceleration due to gravity.

[8]

Indian Statistical Institute

First Semestral Examination: (2008–2009)

B.Stat.(Hons.) – II year

Economics I

Date: **09.1. 2009** *Maximum Marks –60* *Duration: **3 hours***

*Answer any **five** questions.*

1. (a) A consumer consumes two goods, 1 and 2. While good 1 is infinitely divisible, good 2 is available only in integer quantities. Draw the map of indifference curves for this consumer and explain.

(b) Suppose that the prices of these goods are $p_1 = 5$ and $p_2 = 10$, income of the consumer is 100, and the consumer's preference is given by the utility function $U(x_1, x_2) = x_1 x_2$. Find the equilibrium purchase of the consumer.

[6+6=12]

2. A good Q is produced using only input x. Let P be the price of the good Q. The industry for Q consists of 100 identical competitive firms each of which has the production function $q = \sqrt{x}$. Each firm behaves as if the price of x (denote it by P_x) were constant. Derive the industry's supply curve in each of the following cases. Also comment on the slope of the industry's supply curve.

(a) Suppose the industry as a whole faces a horizontal supply curve for input x such that P_x = Rs. 10 at any level of supply.

(b) Suppose the industry as a whole faces a vertical supply curve for input x such that its supply is 100 units irrespective of the price of x.

[6+6=12]

3. On a tropical island there are 100 boat builders, numbered 1 through 100. Each builder can build up to 12 boats a year and each builder maximizes profit given the market price. Let y denote the number of boats built per year by a particular builder, and for each i, from 1 to 100, boat builder has a cost function $C(y) = 11 + iy$. Assume that in the cost

P.T.O

function the fixed cost, 11, is a quasi-fixed cost, that is, it is paid only if the firm produces a positive level of output. If the price of a boat is 25, how many builders will choose to produce a positive amount of output and how many boats will be built per year in total?

[12]

4. A monopolist sells its product in two separate markets. The inverse demand function in market 1 is given by $q_1 = 10 - p_1$, and the inverse demand function in market 2 is given by $q_2 = a - p_2$, where $10 \leq a < 20$. The monopolist's cost function is $C(q) = 5q$, where q is aggregate output.

(a) Suppose the monopolist must set the same price in both markets. What is the optimal price? What is the reason behind the restriction that $a \leq 20$?

(b) Suppose the monopolist can charge different prices in two markets. Compute the prices it will set in the two markets.

(c) Compute consumers' surplus in case (a) and (b). Determine who benefits from differential pricing and who does not, relative to the case where the same price is charged in both markets.

[5+3+4=12]

5. Consider an industry with three firms, each having average cost equal to 0. The inverse demand curve facing this industry is $p = 120 - q$, where q is aggregate output.

(a) If each firm behaves as in the Cournot model, what is firm 1's optimal output choice as a function of its beliefs about other firms' output choices?

(b) What output do the firms produce in equilibrium?

(c) Firms 2 and 3 decide to merge and form a single firm with average cost still equal to 0. What output do the two firms produce in equilibrium? Is firm 1 better off as a result? Are firms 2 and 3 better off post-merger? Would it be better for all the firms to form a cartel instead? Give explanation in each case.

[3+3+6=12]

6. Consider a firm with production function $F(x_1, x_2) = \min\{2x_1, x_1 + x_2\}$ where x_1 and x_2 are amounts of factors 1 and 2.

(a) Draw an isoquant for output level 10.

(b) Show that the production function exhibits constant returns to scale.

(c) Suppose that the firm faces input prices $w_1 = w_2 = 1$. What is the firm's cost function?

[5+3+4=12]

7. (a) There is a cake of size 1 to be divided between two persons, 1 and 2. Person 1 is going to cut the cake into two pieces, but person 2 will select one of the two pieces for himself first. The remaining piece will go to person 1. What is the optimal cutting decision for player 1? Justify your answer.

(b) Prove the following (any two).

(i) Conditional factor demand functions are downward sloping.

(ii) A monopolist's price is an increasing function of its marginal cost of production.

(iii) Marginal productivity of a factor is falling when its average productivity is maximum.

[6+2x3=12]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: 2008-2009

B.Stat. (Hons.) 2nd Year. 2nd Semester

Statistical Methods IV

Date: February 23, 2009 **Maximum Marks: 60** **Duration: 3 hours**

• This question paper carries 70 points. Answer as much as you can. However, the maximum you can score is 60.

• You must state clearly any result stated (and proved) in class you may need in order to answer a particular question.

---

1. Let $\boldsymbol{X} \sim \mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Let

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{pmatrix}, \quad \boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix},$$

where $\boldsymbol{X}_i, \boldsymbol{\mu}_i \, (i = 1, 2)$ are $p_i \times 1$, and $\boldsymbol{\Sigma}$ is a $p \times p$ positive definite matrix. Find the distribution of $Q \stackrel{def}{=} (\boldsymbol{X} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{X} - \boldsymbol{\mu}) - (\boldsymbol{X}_1 - \boldsymbol{\mu}_1)^{\mathrm{T}} \boldsymbol{\Sigma}_{11}^{-1} (\boldsymbol{X}_1 - \boldsymbol{\mu}_1)$. [9]

2. Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be i.i.d. $\mathrm{N}_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $n > 2$, where both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are unknown, $\boldsymbol{\mu} \in \mathbb{R}^2$ and $\boldsymbol{\Sigma}$ is positive definite. Let ρ and r denote the population and sample correlation coefficients, respectively. It is known that a suitable test-statistic for testing $\mathrm{H}_0 : \rho = 0$ against $\mathrm{H}_1 : \rho \neq 0$ is given by $T \stackrel{def}{=} \sqrt{n-2}\, r / \sqrt{1 - r^2}$, large values of $|\, T \,|$ being significant. Find the null distribution of T, either using appropriate results about Wishart distribution, or from first principle. [12]

3. Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be i.i.d. $\mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $n > p$. Let $\bar{\boldsymbol{X}} \stackrel{def}{=} \sum_{i=1}^n \boldsymbol{X}_i / n$, $\mathbf{S} \stackrel{def}{=} \sum_{i=1}^n (\boldsymbol{X}_i - \bar{\boldsymbol{X}})(\boldsymbol{X}_i - \bar{\boldsymbol{X}})^{\mathrm{T}}$. Suppose $T \stackrel{def}{=} (\boldsymbol{X}_1 - \bar{\boldsymbol{X}})^{\mathrm{T}} \mathbf{S}^{-1} (\boldsymbol{X}_1 - \bar{\boldsymbol{X}})$. Find the distribution of T. [14]

4. Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be i.i.d. $\mathrm{N}_p(\mu \mathbf{1}_p, \boldsymbol{\Sigma})$, where $\mathbf{1}_d$ is the $d \times 1$ vector all of whose entries are 1, and $\boldsymbol{\Sigma} = \sigma^2[(1-\rho)\boldsymbol{I}_p + \rho \boldsymbol{J}_p]$; $\mu \in \mathbb{R}, \sigma > 0, -1/(p-1) < \rho < 1$. Find the MLE of (μ, σ^2, ρ), based on $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$. [14]

5. Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be i.i.d. $\mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $n > p$, where both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are unknown, $\boldsymbol{\mu} \in \mathbb{R}^p$ and $\boldsymbol{\Sigma}$ is positive definite. Based on $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$, we wish to test the hypothesis $\mathrm{H}_0 : \boldsymbol{\mu} = k\mathbf{1}_p$ for some unknown $k \in \mathbb{R}$ against $\mathrm{H}_1 : \mathrm{H}_0$ is false. Obtain the LRT for this testing problem. Obtain the null distribution of the LRT or of a suitable equivalent of this. [14+7 = 21]

****** *Best of Luck!* ******

Indian Statistical Institute

Periodical Examinations, (2008-2009)

Bachelor of Statistics, Second Year

Elements of Algebraic Structures

Date: February 27, 2009 Maximum Marks: 60 Time: 2 Hours

This is an open-book test. However, materials other than textbooks are not allowed.

Attempt all questions. *The paper carries a total of 70 marks. Maximum you can score is 60. Figures in the right margin indicate the marks on the different parts of a question. All notations used are as defined in the class.*

1. Let p be a prime number of the form $2^n + 1$. Consider the group Z^p, of integers modulo p, under product.
   a) What is the order [2] in Z^p?
   b) Using a), prove that n must be a power of 2. [7+3=10]

2. Using only Lagrange's Theorem, prove that every group of order 9 is abelian. [10]

3. Let G be a group and N a normal subgroup so that G/N is abelian. Show that $xyx^{-1}y^{-1}$ is in N for all x,y in G. [6]

4. Let G be the group of all formal symbols x^iy^j, i=0, 1, j=0, 1, ..., n-1, where $x^2 = e$, $y^n = e$ and $xy = y^{-1}x$. Let $N=\{e, y, y^2, ..., y^{n-1}\}$. Prove that N is normal in G and $G/N \approx W$, where W is the group {1, -1} under multiplication. [3+5=8]

5. A subgroup H of a group G is said to be characterstic subgroup if $f(H)$ is a subset of H, for all f in AUT(G). Prove that any characterstic subgroup of G is normal in G. [9]

6. Let $p=(a_1, a_2, ..., a_k)$ be a permutation in S_n, where k is an odd natural number. Prove there exist t is in S_n such that $t^2 = p$. [6]

7. Let k such that $1<k<n$. Find the number of element x in S_n such that cycle containing 1 in the cylcle decomposition of x has length k. [6]

8. If G is a group and $|G| = r\,p^m$ and p does not divide r, then
   a) All p-Sylow subgroups of G are conjugates, *i.e.*, if H and J are two p-Sylow subgroups, then there exists g in G, such that $J = gJg^{-1}$.
   b) If n is the number of distinct p-Sylow subgroups of G, then n = 1 mod p, and n divides r. [7+8=15]

INDIAN STATISTICAL INSTITUTE
MID-SEM Examination: (2008 – 2009)
B. Stat II Year
Biology II

Date: 2. 3. 09 | **Maximum Marks: 30** | **Duration: 2:00 hours**

(Attempt any Three questions)

1. Name the different meteorological variables that are related to crop production. Write the names of apparatus used for this purpose. Write in brief about the consumptive use of water. Define MAI. 2+3+2+3

2. Write in brief about different types of rice. Briefly describe the cultural practices associated with rainfed lowland rice cultivation. 3+7

3. What are the differences between Manures and Fertilizers? Calculate the quantity of VC, Urea, SSP and KCL required for 1 ha. potato crop to supply the nutrient requirement of 200 kg N, 100 kg P_2O_5 and 100 kg K_2O per hectare. 50% of required N should be given through VC. 3+7

4. Describe the suitable agrotechniques for rice nursery bed preparation. Estimate the expected yield of rice grain in t/ha from the following data.
   i) Spacing - 20x10cm ii) Average no. of tillers/hill –9 iii) Average no. of effective tillers/hill –7 iv) Average no. of grain/panicle –145 v) Average no. of unfilled grain/panicle –30 vi) Test weight -20 g. 6+4

5. Write short notes on any five of the following: 2 x 5
   a) Monsoon onset
   b) Potential Evapotranspiration
   c) Cup counter anemometer
   d) Reproductive stages in rice
   e) Microclimate
   f) Field capacity
   g) Capillary water

INDIAN STATISTICAL INSTITUTE

Mid-Sem Examination : (2008-2009)

B. Stat. II Year

Physics II

Duration 2 hrs

[Use separate answer script for each group]

Group A (Thermodynamics)

Date: 02.03.2009 Maximum Marks: 15

Attempt any five questions.

1. Calculate the work done when 1 mole of a van der Waal gas expands quasi-statically and isothermally from a volume of 10 to 20 litres at 0^0C. Given that $a = 0.14$ Nm^{4}mole^{-2} and $b = 4.1 \times 10^{-5}$m^{3}mole^{-1}.

2. Air in a cylinder is compressed to $\frac{1}{15}$ of its initial volume. If the initial pressure is 1.01×10^5 Pa and the initial temperature is 27^0C, calculate the final pressure and temperature after compression. Treat air as an ideal gas with $\gamma = 1.40$.

3. Show that no engine can be more efficient than a reversible engine operating between the same two reservoirs.

4. An ideal gas is allowed to expand freely into vacuum from volume V to $4V$. What is the change in its internal energy? Also calculate the change in its entropy.

5. Derive Maxwell's first thermodynamic relation

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

for a gaseous system. Here T, V, P, S have their usual meaning.

6. Find the heat transferred when the pressure on 100 g of water at 0^0C is increased reversibly and isothermally from 0 to 10^8 Pa. Is heat absorbed or rejected? The expansivity of water at 0^0C is $-67 \times 10^{-6}K^{-1}$.

P. T. O

Group B (Quantum Mechanics)

Total Marks 15

Attempt any five questions

1. Show that in a Bohr atom if the electron is considered as a wave travelling along the circular path. then the nth orbit will contain n complete waves.

2. A ray of ultraviolet light of wavelength 3000A falling on the surface of a meterial whose work function is 2.28eV, ejects an electron. What will be the velocity of the emitted electron?

3. A proton is confined to a nucleus of radius 5×10^{-15}m. Calculate the minimum uncertainty in its momentum. Also calculate the minimum kinetic energy the proton should have. Given that mass of proton is 1.67×10^{-27}kg.

4. An excited state of a certain nucleus has a lifetime of $5 \times 10^{-18} s$. Find the minimum possible uncertainty in its energy.

5. An incident photon of freequency ν_0 gets sacttered through an angle θ from a free stationary electron. Find the scattered freequency when $\theta = 90^o$.

6. Using the uncertainty principle, estimate the size and ground state energy of the hydrogen atom.

Indian Statistical Institute

Mid Semestral Examination: (2008–2009)

B.Stat.(Hons.) – II year

Economics and Official Statistics

Date: 4. 3. 09 *Maximum Marks –* ***50*** *Duration:* ***2 hours***

Answer any **three** questions. The maximum you can score is **50**.

1. (a) State the axioms that a price index number must satisfy.

(b) Examine the suitability of the following function as a price index number.

$$G(q_0, p_0, q_1, p_1) = \prod_{i=1}^{n} \left(\frac{p_{i1}}{p_{i0}}\right)^{\alpha_i}, \ \alpha_i > 0 \ \forall \ i \neq n, \ \alpha_n < 0 \text{ are real constants,}$$

and $\sum_{j=1}^{n} \alpha_j = 1$.

(c) In 1980, a Statistical Bureau started constructing an index number series with 1980 as base year.

Year	1980 (=100)	1986	1990
Index	100	140	200

In 1991 the Bureau reconstructed the index number series on a plan with base year 1990.

Year	1990 (=100)	1995	2000
Index	100	150	210

In 2001 the Bureau again reconstructed the series on yet another plan with base year 2000.

Year	2000 (=100)	2005	2008
Index	100	180	240

Obtain a continuous series with base year 1980.

[8+5+5=18]

2. (a) Show that Laspeyres' price index is always greater than Paasche's price index.

(b) What are 'Time reversal' and 'Factor reversal' tests? Show that Fisher's price index satisfies both tests.

P. T. O

(c) Describe a procedure of estimating the standard error of an estimated Laspeyres' price index number.

[6+6+6=18]

3. (a) What is Pareto's law?

(b) Derive the Pareto distribution from Pareto's law. Find the mean and variance of this distribution.

(c) Explain the procedure of fitting a Pareto distribution to data on distribution of persons by income class.

[2+9+7=18]

4. (a) Define Lorenz curve (LC) and Lorenz ratio (LR) for a continuous size distribution of income. When is LR=0? When is LR=1? Explain.

(b) Derive the LC and LR for Lognormal distribution $\Lambda(\mu,\sigma^2)$.

[9+9=18]

INDIAN STATISTICAL INSTITUTE
B. Stat II Year 2008-2009
Mid-Semestral Examination

Subject : Demography & SQC & OR

Date : 06.03.2009 | Full Marks : 100 | Duration : 3 hrs.

Instruction : ***Begin each group on a separate answer-script.***

Group A : SQC & OR (Maximum Marks : 50)

Note : This group carries 55 marks. You may answer as much as you can, but the maximum you can score is 50.

1. The system $Ax = b, x \geq 0$ is given by

$$\begin{aligned} x_1 + x_2 - 8x_3 + 3x_4 &= 2 \\ -x_1 + x_2 + x_3 - 2x_4 &= 2 \end{aligned}$$

Find

a) a nonbasic feasible solution

b) a basic solution which is not feasible

c) a BFS which corresponds to more than one basis matrix. Write all the corresponding basis matrices.

d) a solution which is neither basis nor feasible.

[2+2+4+2=10]

2. Consider the problem P.

$$\begin{aligned} &\text{P : Max } z = cx \\ &\text{st} \quad Ax = b \\ &\quad x \geq 0,\ b \geq 0 \end{aligned}$$

How is the optimal solution affected:

a) when the cost vector c is replaced by $\lambda c, \lambda > 0$?

b) When the cost vector c is replaced by $(c + \lambda), \lambda \in \mathbb{R}$.

[5+5=10]

P.T.O

3. Consider the following problem P.

P : Minimize $z = 2x_1 + 3x_2 + 5x_3 + 2x_4 + 3x_5$

$$\text{St} \quad \begin{aligned} x_1 + x_2 + 2x_3 + x_4 + 3x_5 &\geq 4 \\ 2x_1 - 2x_2 + 3x_3 + x_4 + x_5 &\geq 3 \end{aligned}$$

$$x_i \geq 0 \qquad i = 1 \cdots 5$$

Solve P using the results of Duality Theory. Mention clearly the results that you use.

[15]

4. A small company makes three different products P1, P2, P3. Each product requires works on two different processes A, followed by B. Process A can be done by either of the machines type A_1 or A_2. Process B can be done by any one of the three machines B_1, B_2 or B_3.

Product P1 can be undergo process A on machine type A_1 or A_2; while process B can be done on B_1, B_2 or B_3.

Product P2 can undergo process A on machine type A_1 or A_2; while process B can be done on machine type B_1 only.

Finally, Product P3 can undergo process A on machine type A_2 while process B can be done on machine type B_2 only.

The time in minutes required by one unit of each product on each variety of machine is given in Table 1; along with the total minutes of machine time available per week and the cost per week of running the machine at full capacity. The cost of raw materials for one unit of each product and its selling price are given in the last two rows of the table.

The marketing department feels that the demands for the three products P1, P2 and P3 are 800, 900 and 1,000 respectively.

How should the production department plan its activities so as to maximize profit for the company.

[20]

Table : Data for Question No. 4

	Product			**Total Time available per week (minutes)**	**Cost at full capacity**
	P1	P2	P3		
A_1	5	10	-	6,000	300
A_2	7	9	12	10,000	321
B_1	6	8	-	4,000	250
B_2	4	-	11	7,000	783
B_3	7	-	-	4,000	200
Material Cost	0.25	0.35	0.50		
Selling Price	1.25	2.00	2.80		

P.T.O

Group B: Demography

Maximum Marks: 50

Note: (i) Desk calculators are allowed in the exam, (ii) Symbols and notations have their usual meaning.

Answer the following questions

1. a) Given an autonomous differential equation $f(N) = \frac{dN}{dt}$, where N is the population density. Show that $n(t) = n(0)exp\left\{\frac{df(N)}{dN}/N^*\right\}t$, where $n(t)$, $n(0)$ are perturbations at time t and at time 0, N^* is limit for the perturbation.
b) Define the Lyapunov stability for the population of size N which has a stable equilibrium N^*.

[12, 3]

2. Suppose $l_x = s(x).l_0$ and $s(x) = \frac{1}{10}(100 - x)^{1/2}$, where $s(x)$ is survival function and $l_0 = 1000$. Then construct the life table columns d_x, q_x, L_x, T_x and e_x^0 columns of the life table for ages $x = 0, 1, 2, ..., 5$. [15]

3. Suppose $k(x)dx$ is number of persons of age between x and $x + dx$, ${}_nK_x^{(0)}$ is number of persons of age between x and $x+n$ at time 0, then prove that persons who will be alive at the end of t years among ${}_nK_x^{(t)}$ i.e. ${}_nK_{x+t}^{(t)}$ is $K_x^{(0)}\frac{{}_nL_{x+t}}{{}_nL_x}$. Derive the survival matrix of population projection by the method of difference equations. [5, 10]

4. Using the definition of *the force of mortality*, derive an expression for ${}_np_x$ and subsequently show that ${}_{n/m}q_x = \int_n^{n+m} {}_tp_x\mu_{x+t}dt$ (by taking *uniform assumption* on deaths).

[10]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2008–2009

B.Stat. (Hons.) 2nd Year. 2nd Semester

Statistical Methods IV

Date: April 30, 2009 **Maximum Marks: 90** **Duration: 4 hours**

- Answer all questions.
- You should provide as much details as possible while answering a question. You must state clearly any result stated (and proved) in class you may need in order to answer a particular question.

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1. Suppose $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ are i.i.d. $\mathrm{N}_p(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{\mu} \in \mathbb{R}^p$ and Σ is a $p \times p$ positive definite matrix. Let $\boldsymbol{\mu}$ be unknown. We wish to test the hypothesis $\mathrm{H}_0 : \Sigma$ *is diagonal* against $\mathrm{H}_1 : \mathrm{H}_0$ *is false*. Denote by $\mathbf{R}$, the sample correlation matrix. Let λ denote the likelihood ratio test statistic (LRT) for testing H_0 against H_1.

(a) Show that $-2 \log \lambda = -n \log(\det \mathbf{R})$.

(b) Argue that under H_0, $-2\log\lambda \xrightarrow{d} \chi^2_{p(p-1)/2}$ as $n \to \infty$. [6+5 = 11]

2. Suppose $\boldsymbol{X} \stackrel{def}{=} (X_1, \ldots, X_p)^{\mathrm{T}}$ is a random vector with a positive definite dispersion matrix Σ.

(a) Denote the multiple correlation coefficient between X_1 and $(X_2, \ldots, X_p)^{\mathrm{T}}$ by $\bar{R}$. Derive an expression for $\bar{R}$ in terms of the entries of Σ.

(b) Suppose now that $\boldsymbol{X} \sim \mathrm{N}_p(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{\mu} \in \mathbb{R}^p$ and Σ are both unknown. Let $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$ be i.i.d. realizations of $\boldsymbol{X}$. Suppose, moreover, that we wish to test the hypothesis $\mathrm{H}_0 : \bar{R} = 0$ against $\mathrm{H}_1 : \bar{R} > 0$. Denote by R, the sample multiple correlation coefficient. Let λ denote the LRT for testing H_0 against H_1.

(1) Show that $\lambda = (1 - R^2)^{n/2}$.

(2) Show that under H_0, $[R^2/(1-R^2)] \cdot [(n-p)/(p-1)] \sim F_{p-1,n-p}$.

[8+(8+8) = 24]

3. Suppose $\boldsymbol{y} \sim \mathrm{N}_n(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \mathbf{I}_n)$, where $\mathbf{X}$ is an $n \times (p+1)$ matrix with fixed (non-random) entries and all entries in its first column are 1, $\boldsymbol{\beta} \equiv (\beta_0, \beta_1, \beta_2, \ldots, \beta_p)^{\mathrm{T}} \in \mathbb{R}^{p+1}$, $\sigma > 0$. Suppose we wish to test the hypothesis $\mathrm{H}_0 : \beta_1 = \cdots = \beta_p = 0$ against $\mathrm{H}_1 : \mathrm{H}_0$ *is false*. Let $\mathbf{J}_n$ denote the $n \times n$ matrix with all entries equal to 1, and let $\mathbf{H} := \mathbf{X}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}$.

[P.T.O.]

(a) Show that a suitable test for testing H_0 against H_1 is the following: reject H_0 if F is large, where

$$F \stackrel{def}{=} \frac{\boldsymbol{y}^{\mathrm{T}}(\mathbf{H} - n^{-1}\mathbf{J}_n)\boldsymbol{y}/p}{\boldsymbol{y}^{\mathrm{T}}(\mathbf{I}_n - \mathbf{H})\boldsymbol{y}/(n-p-1)}.$$

(b) Find the non-null distribution of F. [8+9 = 17]

4. Suppose $X_1, X_2, \ldots$ are i.i.d. $\mathrm{N}(\theta, 1)$ variables, where $\theta \in \mathbb{R}$. Consider the problem of estimating the parametric function $\psi(\theta) := P_\theta(X_1 \leq c_0)$, where c_0 is a known constant.

(a) Find the MLE of $\psi(\theta)$, denoted by $\hat{\psi}_n$, based on $X_1, \ldots, X_n$.

(b) Show that for suitable $\mu \in \mathbb{R}$ and $\sigma > 0$, to be obtained by you, the asymptotic distribution of $n^{1/2}(\hat{\psi}_n - \mu)$, as $n \to \infty$, is normal with mean zero and variance σ^2.

[3+7 = 10]

5. Consider n i.i.d random variables $X_1, \ldots, X_n$, having exponential distribution with scale parameter θ, $\theta > 0$. Denote the corresponding order statistics by $X_{1:n} \leq \cdots \leq X_{n:n}$. Suppose that we have a Type II censored sample where only the first r order statistics $X_{1:n} \leq \cdots \leq X_{r:n}$ are observed. Suppose, moreover, that we wish to test the hypothesis $\mathrm{H}_0 : \theta = \theta_0$ against $\mathrm{H}_1 : \theta < \theta_0$.

(a) Find the uniformly most powerful (UMP) test for testing H_0 against H_1, based on $X_{1:n} \leq \cdots \leq X_{r:n}$.

(b) Explain how you will obtain r such that the UMP test obtained in (a) above has a desired size α and approximately a desired power $1 - \beta$ $(0 < \alpha, \beta < 0.10)$ at a specified point $\theta_1 < \theta_0$. [8+9 = 17]

6. Suppose $X_1, X_2, \ldots$ are i.i.d. $\mathrm{U}(\theta - \sigma, \theta + \sigma)$ variables, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown. Let $0 < p_1 < 0.5$. Denote by $X_{1:n} \leq \cdots \leq X_{n:n}$, the order statistics corresponding to $X_1, \ldots, X_n$. Let $r_n = [np_1] + 1, s_n = n - r_n + 1$. Suppose that we have the symmetrically doubly (Type II) censored sample, $X_{r_n:n}, \ldots, X_{s_n:n}$. Let $W_n := X_{s_n:n} - X_{r_n:n}$ be the quasi-range based on these observations. Obtain, with reasons, a consistent estimate of σ which is a multiple of W_n. You have to establish consistency of your estimate. [11]

****** *Best of Luck!* ******

Indian Statistical Institute

Semestral Examinations, (2008-2009)

Bachelor of Statistics (Hons.)

Second Year, Second Semester

Elements of Algebraic Structures

Date: May 4, 2009 — Maximum Marks: 80 — Time: 4 Hours

This is an open-book test. However, materials other than textbooks are not allowed.

Attempt all questions. *The paper carries a total of 101 marks. Maximum you can score is 80. Figures in the right margin indicate the marks on the different parts of a question. All notations used are as defined in the class.*

1. Can there be an injective group homomorphism from ZxZxZ → ZxZ? Justify your answer. [8]

2. Let G be a group, a is an element of G. Let H be a normal subgroup of G such that gcd(o(a),o(G)/o(H)) = 1. Prove that a is in H. [10]

3. Give an example of a ring R with an ideal I and an element b such that b is not a unit in R but $(b+I)$ is a unit in R/I. [8]

4. Let $f(x)$ be an irreducible cubic polynomial in $Q[x]$, such that the derivative of $f(x)$ is positive for all x. Prove that the degree of its splitting field is 6. [8]

5. Find a real number a, such that $Q(2^{½}, 5^{1/3}) = Q(a)$. [15]

6. Let u be transcendental over the field F. E is an extension of F ($F \neq E$), which is contained in $F(u)$. Show that u is algebraic over E. [10]

7. Let v be a primitive fifth root of unity and w a primitive fifth root of 2. Show that $Q(v) \cap Q(w) = Q$. [15]

8. Let K/F be an algebraic extension and let R be a ring contained in K and containing F. Show that R is a subfield of K containing F. [15]

9. (a) Find all the ideals of $Z/8Z$.
   (b) Construct an example of a ring with 851 ideals. [4+8=12]

Indian Statistical Institute

Semestral Examination: (2008–2009)

B.Stat.(Hons.) – II year

Economic Statistics and Official Statistics

Date: 08.5.09 *Maximum Marks –**100*** *Duration: 3 hours*

Answer any **three** questions from **GROUP A** and ALL questions from **GROUP B**.

Use **separate answer scripts** for the two groups.

GROUP A

(This part carries 75 Marks. The maximum you can score is 70.)

1. (a) State if each of the following statements is true or false. Justify your answer.

   (i) For comparing living standards between two countries one should use "Official Exchange Rate" rather than "Purchasing Power Parity".

   (ii) The Lorenz Ratio and Gini coefficient of income inequality are equal for a continuous income distribution.

   (iii) The Head Count Ratio, which measures the intensity of poverty, is an additively decomposable measure.

   (iv) The Durbin-Watson statistic is one of the measures of autocorrelation in time series data.

   (v) If tax is proportional to the value of a consumer item, then taxing a luxury item is progressive.

[4+6+4+6+5=25]

2. (a) Describe the different types of data that are used in the analysis of consumer behaviour.

   (b) Define 'income elasticity of demand'. How are the commodities classified based on this elasticity? Sketch the Engel curves corresponding to these classifications. Justify your answer.

   (c) Discuss the problems of 'identification' and 'least squares bias' likely to arise in estimation of demand functions from time series data.

[5+10+10=25]

P. T. O

3. (a) Define the Specific Concentration Curve (SCC) for a particular commodity. What does the point (0.4, 0.3) on a SCC signify?

   (b) Discuss the properties of SCC and derive its relationship with the Lorenz Curve (LC). When does the SCC reduce to LC?

   (c) Assuming that income follows a Lognormal distribution and the Engel curve of an item is of constant elasticity form, describe methods of estimating the Engel elasticity using the SCC.

[5+14+6=25]

4. Consider a production function with two inputs K and L.

   (a) Define 'Marginal Rate of Technical Substitution' (MRTS) between K and L. Show that the MRTS is equal to the slope of the isoquant and also equal to the ratio of input prices in equilibrium.

   (b) Define 'Elasticity of Substitution' (ES) for the production function. Draw the isoquants where the ES between the two inputs is (i) zero and (ii) infinity.

   (c) Obtain the ES for the CES production function.

   (d) Prove or disprove the following:
   The Cobb-Douglas production function must show increasing returns to scale for a determinate solution of the problem of profit maximization under perfect competition.

[7+6+5+7=25]

GROUP B
OFFICIAL STATISTICS

Total Marks : 30

1. Describe the function of National Sample Survey Organisation 8
2. Write the names of two important official statistics compiled and released by CSO 4
3. What are the 9-fold classification of land used in Agricultural Statistics ? 6
4. Which organisation is responsible for Foreign Trade Statistics ? 2
5. Write a brief note of IIP 4
6. Briefly assess the role of Wholesale Price Index number in measuring inflation 6

INDIAN STATISTICAL INSTITUTE

B. STAT II YEAR 2008-2009

2nd Semester Examination
Subject: Demography & SQC & OR

Date: May 12, 2009 **Maximum Marks: 100** **Dur: 3 hrs 15 min**

Group A: Demography (Maximum Marks: 50)

Instructions: (i) Begin each group on a seperate answer-script (ii) Desk calculators are allowed in the exam, (iii) Symbols and notations have their usual meaning.

Answer the following questions

1. Given a differential equation of population dynamics $P' + 2tP = \sin t$ passing through initial point $(0, P_0)$, where P is the size of the population and t is time. Then show that the solution of the equation, ϕ, on $(t,P) - axis$ is $P_0 + \int_0^t e^{s^2} \sin s ds$ for $-\infty < t < \infty$. [6]

2. For a Malthusian type of population, prove that $\int_0^{\omega} B(t) \exp(-ra) s(a) da = 1$. Here $B(t)$ is birth rate at time t, r is natural growth rate and $s(a)$ is survival function at age a. [8]

3. What are the three basic assumptions of population dynamics model. In the Lotka's theory of stable population, show that the characteristic equation is $\int_0^{\infty} \lambda(a) \exp[-ca - \int_0^a \mu(s) ds] da = 1$. Here $\lambda(a)$ and $\mu(a)$ are fertility and mortality rates at age a, and c is a constant. [15]

4. Let ${}_nK_x^{(t)}$ and ${}_nK_{x+t}^{(t)}$ be the number of persons of age between x and $x+n$, $x+t$ and $x+t+n$ at time t respectively, for $t = 0, 1, 2, \ldots$. Prove that ${}_nK_{x+t}^{(t)} = {}_n K_x^{(0)} \frac{{}_nL_{x+t}}{{}_nL_x}$. Here ${}_nL_x$ and ${}_nL_{x+t}$ are life table populations for the ages between x and $x+n$ and $x+t$ and $x+t+n$. [10]

5.Find the solution ϕ for the population dynmics equation $P'' + 2P' = t$ on the $(t,P) - axis$. (Initial conditions are $\phi(0) = 1, \phi'(0) = 0$). [10]

6 (a). The following values of q_x have been derived from a mortality experience: $q_0 = 0.011$, $q_1 = 0.005$ and $q_2 = 0.003$. Construct the corresponding columns l_x, d_x, and $q_x$using a radix of 10,000.

(b). Obtain an expression for μ_x if $l_x = ks^x b^{x^2} g^{c^x}$, where k, s, b, g, c are constants. [5, 5]

P. T.O

Group B : SQC & OR (Maximum Marks : 50)

Note : This group carries 55 marks. You may answer as much as you can, but the maximum you can score is 50.

1. Two alloys A and B are made from four different metals I, II, III and IV, according to the following specifications:-

Alloy	Specification	Selling Price ($)/ton
A	At most 80% of I At most 30% of II At most 50% of IV	200
B	Between 40% & 60% of II At least 30% of III At least 70% of IV	300

The four metals, in turn, are extracted from three different ores with the following data:

Ore	Max Quantity(tons)	Constitutents (%) I	II	III	IV	others	Purchase Price ($)/ton
1	1000	20	10	30	30	10	30
2	2000	10	20	30	30	10	40
3	3000	5	5	70	20	0	50

The company has to decide how much of each alloy should be produced to maximize the profit. Formulate this company's problem as a LP model. [15]

2. Subgroups of $n = 6$ items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After 50 subgroups have been analyzed, we have

$$\sum_{j=1}^{50} \bar{x}_j = 1000 \quad \text{and} \quad \sum_{j=1}^{50} S_j = 75$$

a. Compute the control limits for the $\bar{x}$ and S control charts.

b. Assume that all points on both the charts lie within the control limits. What are the natural tolerance limits for the process?

c. Suppose the specification limits are 19 ± 4. Assuming that if an item exceeds the upper specification limit it can be reworked, while if it falls below the lower specification limit, it must be scrapped, what percent scrap and rework is the process now producing?

d. If the process were centered at $\mu = 19.0$, what would be the effect on percent scrap and rework? [4+2+5+4=15]

3. a. Consider a double sampling acceptance rejection plan with the following parameters:-

$$n_1 = 50 \quad c_1 = 1 \quad n_2 = 100 \quad c_2 = 3$$

Find the probability of accepting a lot that has fraction defective p=0.05, in the second sample.

b. Suppose that a single sampling acceptance rectification plan n=150, c=1 is being used for receiving inspection where the vendor ships the product in lots of size N=3000. Find the AOQL for this plan. [5+10=15]

4. State whether the following statements are True (T) or False (F). You need not copy the statements.

a) A form, in either diagram or table format, that is prepared in advance for recording data is known as a flowchart.

b) A process that is in statistical control will never have a point beyond a three-sigma limit on $\bar{X} - R$ control chart.

c) In practice the proportion of non-conforming items coming from a manufacturing process that produces millions of items per day can be successfully monitored with a p- chart.

d) Control charts were first devised by Dodge and Romig.

e) Consumers risk of 10% means that the probability that a sampling plan will reject "good" material is 10%.

f) From the standpoint of quality protection, the absolute size of a random sample is much more important than its relative size compared to the size of the lot.

g) The OC-curve of plans with acceptance number greater than zero are superior to those of comparable plans with an acceptance number of zero.

h) In a simplex tableau, if there is a tie for the departing variable, then the next BFS will be degenerate..

i) If in any simplex iteration, the minimum ratio rule fails, then the LPP is infeasible.

j) The simplex method may not move to an adjacent extreme point if the current iteration is degenerate.

P.T.O

k) The principal function of a control chart is a tool for process adjustment.

l) The chance of getting a false out-of-control signal above the 3σ limit is the same for both an $\overline{X}$ and an R-chart.

m) Sampling plans with the same per cent samples give the same quality protection.

n) Quality characteristic falling within specification limits implies that the process is in statistical control.

o) Fraction defective chart requires only the recording of number of defectives and therefore are more economic than $\overline{X}$ - charts.

p) In an LP problem if the solution space is unbounded, the objective value always will be unbounded.

q) In real-life problems, the variables of an LP model are always necessarily non-negative.

r) Ishikawa is associated with the PDCA wheel.

s) Quality is directly proportional to variability.

t) When measurements show a lack of statistical control, the standard error of the average has no meaning.

[0.5X20

2nd Draft

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2008-09
Course Name: B. Stat.-II yr.
Subject Name: Biology II
Date: 14.5.09 Maximum Marks: 80 marks Duration: 3 hours

Please answer any ten questions from Part A and all questions from Part B.

Part A

Short questions (For many questions, there are no right or wrong answers since experiments are not yet conclusive worldwide; therefore the point of view put forward by you will be assessed for logical explanation): (10 x 3)

1. What are the basic differences between terminally differentiated cells (e.g. skin cells) and stem cells (e.g. bone marrow cells)?
2. Let us assume, my bone marrow stem cells usually produce 10^5 macrophage (Mϕ) and 10^3 dendritic cells (DC) per day as they constitute a large part of my `innate immunity' defense system. Due to a recent kidney transplant in my body (organ donated by my own brother), I have to take `Chloramphenicol', a general suppressor of protein synthesis. This would no longer allow bone marrow stem cells to produce Mϕ and DC at the normal rate. On the other hand, if I do not take Chloramphenicol, my immune system would reject the precious transplanted kidney. Due to such massive immuno-suppression (epigenetic changes) for a long period, one day, plasticity develops in my bone marrow stem cells so that bone marrow cells would never be able to supply 10^5 Mϕ and 10^3 DC cells per day, even though demand for Mϕ and DC are the same as earlier.
   (a) Is there a chance that I might contract plastic anemia (elastic is reversible; plastic is irreversible) leading to blood cancer or nothing will happen- simply my body would regain the normal status after sometime?
   (b) Epigenetic changes (as my innate immunity genes interacts with environment of Chloramphenicol) in my sperm producing cells might be a route to carry over plastic anemia to my sons and daughters. Chloramphenicol being a chemical can enter all kinds of cells in my body with varying degree and might cause errors in protein synthesis in my gonadal cells. What will be the frequency (low, medium and high) of occurrence of such mutations? Take into account the fact that humans produce lesser number of progenies in their lifetime compared to mouse?

2. (a) In the aforesaid example, it also appears that my body has a sensor and feedback system in place, which calculates daily demands of Mϕ and DC cells in my circulating blood and delivers accordingly. Do you think that this is similar to BPOs (business processing outsourcing) operating in India for better management of the production oriented factories located far away (e.g., in USA). What is the evolutionary advantage of keeping `supply on demand' model in my bone marrow stem cells, compared to ` knowledge based productions (guesses, modeling and statistics etc.)'? Do you think that my body saves lots of ATP via adopting this BPO kind of model?
(b) Weissman had cut tails of mouse for ~15 generations and this is cited to describe limitations of Lamarckism (or Lamarckian evolution theory), put forward by French Biologist, Jean-Baptiste Lamarck. Lamarckism is often described as heritability of acquired characteristics or soft inheritance. At the advanced level of biology after Biology II second semester, we know now that, Lamarckism is necessary for acquiring mutation in my innate immunity genes when they are exposed to chloramphenicol or for Giraffes to acquire a longer neck or plants to become day light insensitive.

P.T.O

(i) Do you think that Weissman should have cut the tails of mouse for ~100 million generations and then use appropriate statistical modeling techniques to find out if one of mouse progenies would have short tail after random mating for 100 million generations. Do you think that this kind of experiments is physically possible anywhere in the world?

3. (a) Following aforesaid logic, suppose that a researcher has demonstrated in a research article (by genomic sequence analysis, since it is very difficult to simulate evolution in the laboratory or non-availability of human DNA who lived in even ~130,000 B.C.) that 12 genes which play important role in `innate immunity' are evolutionarily conserved and do not allow any deleterious mutations. As a statistician, which of the following statements you will keep in mind while reading this paper-

i) Whether the author have included samples simulating humans of ~130,000 B.C.,

ii) Modern humans have not traversed enough time for doing these experiments or simply the basic premise of such experiments are wrong (Neanderthal characteristics disappeared and modern human features appeared only ~130,000 years ago).

iii) If a human generation is ~65 years (including overlaps), how many human generations could be approximately covered in 130,000 years?

iv) Restricted gene flow, due to marriage pattern, geographical locations, choice of mate on linguistics etc. might reduce probability of occurrence of such deleterious mutations further more- what is your opinion?

v) If I make a statement that it is easy to talk about genetic evolution using theoretical biology tools, but it is too difficult to prove them in the laboratory- am I right or my entire proposition described above is full of flaws, ignorance and bias against the aforesaid logic of research- if so, why?

4. What is a morphogen gradient? Define a plant module with a sketch.

5. Self-pollinated and cross-pollinated crops follow strictly rules of self- and cross-fertilization. In nature, is it true or false?

6. Why is Mendel called "blind man's stick" when you work as breeder? Why is statistics important to analyze plant and `non-human' animal breeding data? As per classical Mendelian genetics, one should observe 9:3:3:1 in F2 for a di-hybrid cross, when the genes responsible for two pairs of alternating phenotypes segregate and assort independently. Now suppose in the field, you make a dihybrid cross and find a ratio of 9.5:2.5:3.5:0.5 in F2. Do you need to apply Statistics for deciphering the reasons behind such kind of observation? Which of following test statistics or principles could be used for such situations- t-test, z-test, χ^2-test, $p^2+q^2+2pq=1$ (Hardy–Weinberg)?

6. Humans cannot be bred by the breeders due to ethical reasons (e.g., possibility of pursuit of Eugenics)- Then, how can one assess genetic recombination taking place in the different geographical populations?

7. Write about four distinct parameters that might restrict random gene flow in the human population compared to when you breed a crop plant? For example, tightly linked genes hardly segregate and then assort independently in parental and subsequent filial generations (progenies).

8. There are certain physical and physiological parameters in crop plants which might restrict random gene flow- write about any four of these. For example, pollen sterility in a self pollinated plant does not allow self pollination and thus prevents inbreeding at some genetic loci.

9. Severe diseases (including some parasitic diseases like cerebral malaria) play important roles in selection of a particular plant or human population (host) in the evolutionary scale. Diseased plant and animals are expected to lose the Darwinian `struggle for existence' and are expected to disappear from the earth. Can you think of a situation when such disease carrying gene(s) would be allowed- what kind of advantage they can offer to a population?

10. Plant germinates and human is born with a set of defense mechanisms and they are called `innate immunity' system. Most of these mechanisms are non-specific in nature. When a plant and animal grow in nature, they interact with environment and learn to acquire a set of mechanisms, called `acquired immunity'- mostly specific for specific organisms. Now- do you think `innate immunity' mechanisms are static in nature and do not change over evolution? If you think that they do not change- suppose there is a

natural calamity which can shut off all the evolutionarily conserved innate immunity mechanisms. Then how some organisms survive and others die- what helps them perform better than others?

11. Suppose, in a simplistic situation, my body utilizes 30000 genes in all possible permutations and combinations to function normally. Also suppose, out of these 30000, 25000 genes do house-keeping functions and therefore, it will be difficult to live without them. Then there are 5000 more genes apparently lying idle in my genome. Can you imagine any possible function of these apparently idle genes (Tips: nursery beds for creating novel genes to adapt in new environment)?

12. Many anthropologists consider the great expansion of agricultural yield, invention of agricultural practices, plant and animal breeding techniques using Genetics to be the singular achievement of 'human brain' skill in the modern era (Farb, 1978). If I say that it appears that the brain has evolved faster than other body organs, then brain should face less selection pressure. Glial cells in brain perform similar functions to that of Mϕ and DC in circulating blood tissue, i.e., phagocytose foreign bodies. Do you think, glial cells are more evolved than Mϕ and DC in terms of their role in innate immunity?

Part B

(5 x 10)

1. What are the basic objectives of plant and animal breeding? What are the basic mechanisms of plant propagation? What are the methods of plant and animal improvement? Describe in brief how one can surpass regular sexual processes for plant propagation and carry out gene recombination using genetic engineering. (3+2+2+3)

2. What is pureline and mass selection? Describe plant hybridization and mutation breeding. Write a comparative statement on different methods of plant improvement in self and cross pollinated crops. (4+4+2)

3. Define properties of stem cell with examples from different animal tissues. What is a totipotent cell and how are they used for plant tissue culture? What is segment polarity, even skipped and pair rule genes in *Drosophila melanogaster* development? (5+2+3)

4. Define all the parameters of environment and gene interactions. (10)

5. Write about the basic strategies, similarities and differences of plant and animal development. (10)

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2008-2009)

B. Stat. II Year

Physics II

Duration: 3 hrs

Date: 15.05.09

[Use separate answer script for each group]

Group A (Statistical Mechanics)

Maximum Marks: 30

Attempt any four questions.

1(a) 1 Kg of water at 100^0C is placed in thermal contact with 1 Kg of water at 0^0C. Calculate the total change in entropy.

[Assume that the specific heat of water is constant at 4190 J/Kg.K over this temperature range].

(b) A Carnot engine with the sink at 10^0C has an efficiency of 30%. By how much must the temperature of the source be changed to increase its efficiency to 50%?

(c) One mole of a van der Waal gas undergoes a reversible isothermal volume change. Using the Tds relations, find the heat flow in the process. $3\frac{1}{2} + 1\frac{1}{2} + 2\frac{1}{2}$

2. Consider a particle of mass m enclosed in a volume V and having momentum $p \leq P$, where $P = \sqrt{2mE}$, E being the energy.

(a) Determine the (asymptotic) number of microstates lying in the range p and $p + dp$.

(b) If the energy varies from 0 to ∞, obtain the partition function of the system. $3 + 4\frac{1}{2}$

3(a) State Liouville's Theorem in the context of ensemble theory and use it for the classification of stationary ensembles.

(b) Starting from $S = -k\sum_r P_r ln P_r$, (where P_r is the probability of the system to be in the E_r energy state), deduce $S = k ln\Omega$ for a microcanonical ensemble, Ω being the number of microstates.

(c) Let the system in part (b) be in the ground state. What is the value of S when the ground state is (i) degenerate, (ii) non-degenerate. $3\frac{1}{2} + 2 + 2$

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4. The expression for entropy S of an ideal gas having N particles, each of mass m, contained in a volume V at temperature T is $S = Nkln(V) + \frac{3}{2}Nk[1 + ln(\frac{2\pi mkT}{h^2})]$, where k is the Boltzmann constant.

(a) If two samples of the same ideal gas at same temperature T and same particle density (N/V) are mixed together, find the change in the entropy of the system before and after the mixing.

(b) Modify the given expression for S in such a way that the process described in part (a) becomes explicitly reversible. $3\frac{1}{2} + 4$

5(a) Obtain the grand partition function for a classical system of N one dimensional particles for the cases when the particles are (i) distinguishable. (i.e.. harmonic oscillators) (ii) indistinguishable.

(b) Find the relative root-mean-square fluctuation in the number of particles N. When is the fluctuation negligible? $4 + 3\frac{1}{2}$

Group B. Quantum Mechanics

Total Marks: 30

Attempt any three questions. All questions carry equal marks.

1. (a) A photon of energy E is scattered from a free electron of rest mass m and observed at scattering angle greater than $\pi/3$. Show that the energy of the scattered photon is less than $2mc^2$ however large E may be.

(b) Consider the bound states of two particles having the same mass m and interacting via a potential $V(r) = kr$ where k is a constant. Using the Bohr model find the speed, the radius and the energy of the system in the case of circular orbits.

2. (a) It is given that for a free particle of mass m, $< p >_{t=0}= p_0$ and $< x >_{t=0}= x_0$. Find $< p >$ and $< x >$.

(b) The wave function of a particle in an infinite potential well $(0 < x < L)$ is given by $\psi(x,t) = \frac{1}{\sqrt{2}}[u_2(x)exp(-iE_2t/\hbar) + u_3(x)exp(-iE_3t/\hbar)]$. Calculate the probability that the electron is in the range $(0, L/2)$.

3. Define raising ($a^\dagger$) and lowering (a) operators for the harmonic oscillator and find (i) $[a, a^\dagger]$, (ii) $[H, a]$ and (iii) $[H, a^\dagger]$ where H denotes the harmonic oscillator Hamiltonian.

4. (a) Calculate $(\Delta x)^2(\Delta p)^2$ for the harmonic oscillator ground state where for an operator A, $(\Delta A)^2 =< A^2 > - < A >^2$. What can you infer from the result?

(b) An electron of kinetic energy 8 eV is moving from left to right in a potential given by

$$V(x) = 6\ eV, \quad x > 0$$
$$= 0, \quad x < 0$$

What is the probability that (i) it will continue along its original direction after reaching the step (ii) it will be reflected back from the potential step?

5. (a) In the case of hydrogen atom the normalized wave function for $1s$ electron is given by $\psi_{1s} = (\pi a_0^3)^{-1/2}\ e^{-r/a_0}$. Find the probability of finding this electron in a sphere of radius a_0.

(b) Calculate $< L^2 >$ and $< L_z >$ for the states ψ_{322}, ψ_{200} of the hydrogen atom, where L^2 and L_z denote respectively the square and z component of the angular momentum operator.

INDIAN STATISTICAL INSTITUTE

Second Semester Examination (2008-2009)

Course - B Stat II Year

Subject : Economics II (Macroeconomics)

Date: 15.5.09 Maximum marks-60 Duration 2.5 hours

Answer all questions.

1.In a simple Keynesian model for a closed economy without government, there are two groups of income earners. Group 1 earns 800, while the income of group 2 is Y – 800, where Y denotes NDP. Average consumption propensities of Group 1 and Group 2 are 0.6 and 0.5 respectively. Investment function is given by $I = 400 + 0.1Y$.**(i)** Derive the aggregate saving function and the equilibrium amount of saving. **(ii)**Now suppose there takes place a transfer of income of 100 units from Group 1 to Group 2. How will it affect the aggregate saving function? Do you observe paradox of thrift here? Explain (8+14 = 22)

2. (i) 2(i).Explain how a shift in the demand for real balance function is likely to impact
the equilibrium in the IS-LM model.
(ii) Following an increase in autonomous demand for commodity in the above model the government decides to implement an accommodating monetary policy. Subsequently, y is found to increase by 2400 units. Compute the increase in real balance supply undertaken by the government, if an exogenous increase in demand for real balance by 1 unit is found to shift the LM curve horizontally by -4 units.

(10+12 = 22)

3(i) Suppose that the government takes an additional loan of Rs.100 from the central bank. Explain how this will lead to an increase in the stock of high-powered money by the same amount. Assuming CRR to be unity, show how people will come to hold an additional amount of money of the same amount at the end of the operation of the money multiplier process when high-powered money goes up by Rs.100.

(ii) Show that in the Mundell Fleming model the monetary policy is completely ineffective in a fixed exchange rate regime under perfect capital mobility.

(12+10=22)