# INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination 2004-05

B. Stat. II Year
Analysis III

Date: 06.09.04   Maximum Marks : 40   Duration : $2\frac{1}{2}$ hrs.

1. Let $f: \mathbb{R}^n \to \mathbb{R}^m$ be a function. Define the notion of differentiability of f at any point $p \in \mathbb{R}^n$. Show that $f: \mathbb{R}^2 \to \mathbb{R}^3$, $f(x,y) = (\text{Sinx cosy}, \text{SinxSiny}, \text{Cos } x)$ is differentiable everywhere.

    [10]

2. Let $f(x,y) = e^{9x+2y}$ and $g(x,y) = \text{Sin}(4x+y)$.

    Let C be a differentiable Curve in $\mathbb{R}^2$ such that $c(0) = (0,0)$.

    Given $\dfrac{d}{dt}\left(f\left(c\left(t\right)\right)\right)\Big|_{t=0} = 2$ and

    $\dfrac{d}{dt}\left(g\left(c\left(t\right)\right)\right)\Big|_{t=0} = 1$. Find $c'(0)$.

    [6]

3. Let $U \subset \mathbb{R}^n$ be an open set and $p \in U$. Suppose $f = \left(f_1, \ldots, f_m\right) : U \to \mathbb{R}^m$ is such that all partial derivatives $D_i f_j$ exist and are bounded in an open ball $B(p, r) \subset U$. Prove that f is continuous at p.

    [10]

4. Let $f(x,y) = x^2 + y^3 + 3xy^2 - 2x$ and $p = (1, 0)$. Find the quadratic form associated to the critical point p and determine the nature of the critical point p.

    [8]

5. State inverse function theorem. For the function $f(x,y) = x^2 - xy + y^2 - 3$, determine if there exist a differentiable function φ defined in a neighbourhood of 1 and taking values in a neghbourhood of 2 such that $f\left(x, \varphi\left(x\right)\right) = 0$. If such a φ exists, determine $\varphi'(1)$.

    [6]
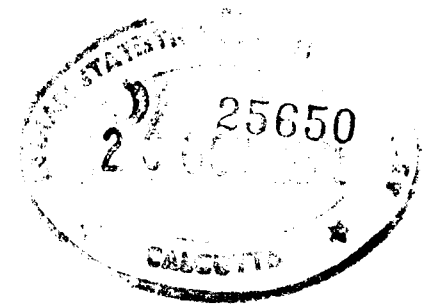
Physics-I

Group-B

Date- **9.9.04**   Maximum marks:15   Duration- 1 hour

Answer any two question.

1. Consider a system of N particles, $m_i(i = 1, 2, ...N)$ being the mass of the ith particle. Find the condition under which total linear momentum and angular momentum is conserved.

   Show that Gravitional force is conservative. Show also that in conservative force field total mechanical energy is conserved.

$$3 + 1\tfrac{1}{2} + 3$$

2. Consider a point unit mass moving with respect to origin under radial force. Show that motion will be in a plane and areal velocity of the particle is conserved.
   Show that if the force is inversely proportional to square of the distance from the origin and attractive, then the trajectory will be a conic section. Why planetary motion is elliptic?

$$2 + 4 + 1\tfrac{1}{2}$$

3. Show that in all inertial frames, the mechanical laws remain unchanged. Under what condition the pseudo-force arises? Find out the pseudo-forces arising in a rotating frame of reference, where the angular velocity of rotation is constant.

$$1 + 2 + 4\tfrac{1}{2}$$

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2004-2005)

B.Stat. (Hons.) II Year

PHYSICS I (GROUP A)

Date : **9**.9.2004   Maximum Marks : 15   Duration : 90 Minutes

Note : Answer any three of the following questions :

1. A hollow spherical shell carries charge density $\rho = \frac{k}{r^2}$ in the region $a \leq r \leq b$, where k is a constant.

   (a) Find the electric field in the three regions (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$.

   (b) Find the potential at the center, using infinity as your reference point.

   $$(3 + 2)$$

2. Find the electric field at a distance z above the center of a flat circular disk of radius R, which carries a uniform surface charge density σ. What happens to the electric field when $R \to \infty$ and $z >> R$?

   $$(4 + 1)$$

3. (a) Find the divergence of the function

   $$\vec{V} = s \left(2 + \sin^2 \phi\right) \hat{s} + s \sin \phi \cos \phi \hat{\phi} + 3z\hat{z}.$$

   (use $\nabla \cdot \vec{V} = \frac{1}{s} \frac{\partial}{\partial s} (sV_s) + \frac{1}{s} \frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$)

   (b) Test the divergence theorem for the above function, using the quarter-cylinder (radius 2, height 5).

   (c) What is the charge density of an electric dipole, consisting of a point charge $-q$ at the origin and a point charge $+q$ at $\vec{a}$?

   $$(1 + 3 + 1)$$

4. A sphere of radius R, centered at the origin, carries a charge density

   $$\rho(r, \theta) = \frac{AR}{r^2} (R - 2r) \sin \theta$$

   where A is a constant, and r, θ are the usual spherical coordinates. Find the approximate potential for points on the z-axis, far from the sphere.

   $$(5)$$

P.T.O

Full marks = 40; Answer any five; All questions carry equal marks.

1. Distinguish between the members of each pair:
   a) Plasma-membrane / Cell-wall
   b) Smooth endoplasmic reticulum / Rough endoplasmic reticulum
   c) Cell / Organelles
   d) Chromatin / Chromosome

2. What are the major differences between plant and animal cells with respect to structure and functions of various organelles? What is turgor pressure and what advantage do plant cells get out of it?

3. a) What are enzymes?
   b) Discuss the special features of enzyme-catalyzed reactions.

4. a) Proteins and nucleic acids are referred to as informational biomolecules, while homopolysaccharides, which consists of chains of a single sugar are not. Explain why?
   b) Give a brief account of the various kinds of movements exhibited by eukaryotic cells. Which of these movements are not observed in case of prokaryotic cells and why?

5. How proteins are digested in our system. "Amino acid composition of a protein determines its quality"- explain. Draw carbon and nitrogen cycles and mention their importance for human living.

6. What are aerobic and anaerobic metabolisms of glucose and mention the conditions when they undergo. How many ATPs are generated from each process and calculate the efficiencies of energy trapped in these processes.

Indian Statistical Institute
B-Stat. II (2004 –2005)
Mid–Semestral Examination
Economics–I

Date: 9. 9. 04   Full Marks: 100   Duration 3 hours

Answer <u>ALL</u> question

1.   (a) Define price elasticity of demand. Illustrate perfectly inelastic, inelastic, unit elastic, and perfectly elastic demand curves graphically.

(b) Show that an increase in the price of a good leads to an increase or a decrease in the total amount spent on purchase of the good according as the demand for the good is inelastic or elastic.

(c) What are 'necessary', 'inferior' and 'luxury' goods? Give examples.

(d) Examine graphically the effect of imposition of a tax on the sale and purchase of a commodity in terms of price paid by the consumer and that received by the producer.

[7+3+6+4=20]

2.   State clearly the axioms of consumer's choice. Briefly discuss the relevance of these axioms in the context of consumer's utility maximization problem.

[20]

3.   (a) Consider the indirect utility function

$$v\ (p_1,p_2,m) = \frac{m}{p_1 + p_2}$$

(i)   What are the Marshallian demand functions?

(ii)   What is the expenditure function?

(iii)   Comment on the relationship between the two goods and draw the indifference curve.

(b) Consider the direct utility function

$$u\ (x_1,x_2) = \min\ \{x_2 + 2x_1, x_1 + 2x_2\}$$

(i)   Draw the indifference curve for $u\ (x_1,x_2) = 20$.

Shade the area where $u\ (x_1,x_2) \geq 20$

P. T. O

(ii)   For what values of p_1/p_2 will the unique optimum be $x_1 = 0$?

(iii) For what values of p_1/p_2 will the unique optimum be $x_2 = 0$?

(iv)   If neither x_1 nor x_2 is equal to zero, and the optimum is unique, what must be the value of x_1/x_2 ?

[3+2+3+4×3=20]

4.   (a) Explain the idea of revealed preference (RP).

(b) Check whether the weak axiom of RP is satisfied in each of the following cases. Give reasons for you answer.

(i)   With income $m = 20$ and $(p_1, p_2) = (1,1)$ the choice is $(5, 15)$; with $m = 20$ and $(p_1, p_2) = (2, 0.50)$ the choice is $(8, 8)$.

(ii) With m = 5 and $(p_1, p_2) = (1, 2)$, the choice is $(1, 2)$; with m = 5 and $(p_1, p_2) = (2, 1)$ the choice is $(2, 1)$.

(iii) With m = 4 and $(p_1, p_2) = (2, 1)$ the choice is $(1, 2)$; with m = 4 and $(p_1, p_2) = (1, 2)$ the choice is $(2, 1)$.

[5+ 5×3= 20]

5.   (a) "If the production technology exhibits constant returns to scale, the output level of a profit maximizing firm, which faces constant output and input prices, remains indeterminate". Do you agree with this proposition? In any case, explain your answer.

(b) Show that if the production function is homogeneous of degree one, the cost function will be linear.

(c) Given the production function $y = x^a$, $0 < a < 1$, $x \in R_+$, derive the expression of the profit function, that is the maximum profit as function of input and output prices.

(d) Show that the CES production function $y = A[ax_1^{-\theta} + (1-a)x_2^{-\theta}]^{-\frac{1}{\theta}}$ tends to the Cobb-Douglas form $y = Ax_1^a x_2^{1-a}$ as $\theta \to 0$, where $A > 0, 0 < a < 1$ and $\theta \neq 0$.

[5 ×4 = 20]

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1. If (X,Y) has the joint density function given by $f(x,y) = \begin{cases} C(y - |x|)e^{-y} & \text{if } 0 < |x| < y \\ 0 & \text{otherwise} \end{cases}$, then find

(a) the probability $P[Y < 2|X|]$, and,

(b) the marginal density of X and the conditional density of X given Y.

(9 + 9)=[18]

2. If (X,Y,Z) is uniformly distributed on the region $(0,2) \times (0,1) \times (0,1)$, find the density of the random variable $U = \dfrac{X}{Y+Z}$.

[12]

3. Examine whether the function F on $\mathbb{R}^2$ defined as

$$F(x,y) = \begin{cases} 1 - (1+x)^{-1} - (1+y)^{-1} + (1+x+y)^{-1} & \text{if } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

is a bivariate c.d.f. or not.

[12]

4. If (X,Y) has a joint normal density with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$, then find constants a, b, c such that $U = a(X - \mu_1) + b(Y - \mu_2)$ and $V = c(Y - \mu_2)$ are i.i.d. $N(0,1)$ random variables and, by using this or otherwise, show that $\rho(X,Y) = \rho$.

(9 + 9)=[18]

5. Show that if $U_1, \dots, U_n$ are the order statistics of i.i.d. $U(0,1)$ random variables, then the random variables $Y_1 = U_1, Y_2 = U_2/U_1, \dots, Y_n = U_n/U_{n-1}$ are independent.

[12]

6. A rod of unit length is split into three pieces in the following way. First, it is split into two pieces by breaking it at a randomly chosen point and then the larger of the two pieces thus formed is split into two pieces by breaking it at a randomly chosen point. Find the probability that the three pieces thus obtained can form a triangle.

[12]

7. If X and Y are i.i.d. $Exp(\lambda)$ random variables and if $Z = X \wedge Y$, then find the conditional distribution of Y given Z. [It is not enough to merely say what the conditional distribution is. You need to prove it also.]

INDIAN STATISTICAL INSTITUTE
B. Stat. II year : 2004–2005
C & Data Structures
Mid Semester Examination

Date : 15. 09. 2004   Marks : 60   Time : 3 Hours

Answer any part of any question. The question is of 70 marks. The maximum marks you can get is 60. Please write all the part answers of a question at the same place.

1. (a) Write a function in C without recursion to find out GCD of two positive integers and show how the function executes when the two integers are 3465 and 1938. Estimate the number of steps required by your method when one calculates the GCD of two positive integers m, n with $m > n$.

   (b) Implement functions to get the n-th fibonacci number using C in both recursive and iterative format. Explain which one is more efficient.

   (c) Write a function in C to calculate $a^x \bmod n$, where a, x, n are positive integers. Estimate the number of steps required by your method. Do not use the 'pow' function available in C.

   (d) Implement a function in C which takes the integer value n as the first arguement and then calculates the standard deviation of the following n real numbers available as arguements to the same function.

$$6 + 6 + 4 + 4 = 20$$

2. (a) Clearly write down the program for heap sort in C.

   (b) Execute your program (explain with proper figures of binary tree) on the data set $x_1, \ldots, x_{12}$ where $x_i = (a + ib) \bmod 29$, a is the last digit of your roll number and b is the last two digits of your roll number, $i = 1, \ldots, 12$.

   (c) Write down the program of bubble sort and execute it on the data set $x_1, \ldots, x_{12}$. Compare the result with the result of heap sort in terms of 'number of comparisons between two integers'.

$$8 + 4 + 8 = 20$$

3. (a) Describe the data structure of a binary search tree and write down the C program for insertion of a new data.

   (b) Execute your insertion function (explain with proper figures of binary tree) on the data set $x_1, \ldots, x_{12}$ as described in the previous question.

   (c) Implement the inorder traversal routine with and without recursion using C and run your routines on the binary tree generated in the last step.

   (d) Write the deletion algorithm (you may not write the C code) of a node from a binary search tree and explain with figures how you can delete the root element of the binary search tree created above.

$$8 + 4 + 10 + 8 = 30$$
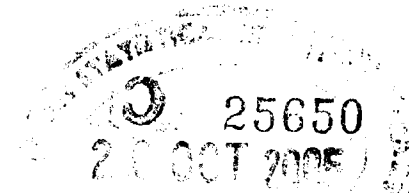
INDIAN STATISTICAL INSTITUTE
B-STAT (Hons.), Second Year, 1st Sememster
STATISTICAL METHODS – III
Mid-Semester Examination, September 17, 2004

Maximum Marks : 100   Time Allowed : 3 hours

(1). Consider i.i.d observations $X_1, X_2, \ldots, X_n$ with a common uniform distribution on $[0, \theta]$, where $\theta > 0$ is an unknown parameter. Obtain the maximum likelihood estimate of θ based on these i.i.d observations. Derive the mean squared error, the bias and the variance of the maximum likelihood estimate.

$$[5 + 5 + 5 + 5 = 20]$$

(2). Suppose that we have an observation X with a double exponential distribution having density $(1/2)\exp(-|x - \theta|)$. Here θ is an unknown parameter with a uniform prior distribution on $[-1, 1]$. Obtain the posterior distribution of θ given X and the Bayes estimate of θ.

$$[15]$$

(3). Let X be a binomial random variable with a known $n > 1$ and an unknown $0 < p < 1$. Obtain the best unbiased estimate for the variance of X (which is a function of the parameter p) based on the observation X. Justify your answer.

$$[20]$$

(4). Derive the joint density of the sample mean and the sample variance based on $n > 1$ i.i.d observations from $N(\mu, \sigma^2)$ distribution.

$$[15]$$

(5). Suppose that we have n independent data points $(X_1, Y_1), \ldots, (X_n, Y_n)$, where the conditional distribution of Y_i given X_i is Poisson with mean $= \lambda(\alpha + \beta X_i)$. Here λ is a smooth function on the real line with positive values, and the X_i's are non-random real constants. Describe the steps involved in Fisher's iterative method for computing maximum likelihood estimates for α and β. Derive sufficient conditions for invertability of the information matrix that is needed in Fisher's iterative method. For what form of the function λ, Fisher's iterative method will coincide with Newton-Raphson iteration. Justify your answer.

$$[6 + 7 + 7 = 20]$$

(6). Assignments $[10]$

Indian Statistical Institute

First Semestral Examination: 2004-2005; Biology-I,   B.Stat. II
Full marks: 60,   Attempt any Five,   Duration: 2hr 30min,   Date:2.12.2004
(Each question carries equal marks)

1. Explain mitosis and meiosis with examples. Two genotypes, A/A and A/a, express same phenotype; design an experiment to distinguish them. In a cross of "Aa Bb X Aa Bb", what fraction of the progeny will have recessive genotype at least at one locus ?

2. Mention three differences between DNA and RNA. What is the function of codons in protein synthesis. What do you mean by initiation and termination codons and how many of them should be present in an active gene? How many DNA sequences, of 15 nucleotides long, are possible with 5 different codons (such as ATG, TAA, AGG, GGG, CGC)? Calculate the number of sequences starting with ATG and ending with TAA.

3. (a) If blood groups of a mother and a father are 'A' and 'AB' respectively, what blood groups could be observed among their children?
   (b) If a man and a woman are heterozygous at a locus with two alleles and if they have three children, what is the chance that all three will also be heterozygous ?

4. a) The biomolecules of living organisms are ordered into a hierarchy of increasing molecular complexity. Depict this with appropriate examples.
   b) What features distinguish a prokaryotic cell from an eukaryotic cell?

5. a) What did Miller's experiment prove?
   b) Of DNA, RNA or proteins, which do you think could be the primitive life molecule? Give reasons.
   c) Why is the evolution of photosynthesis thought to have favoured the subsequent evolution of oxidative metabolism?
   d) Present-day cells use oxidative reactions, and not glycolysis, as their principal source of energy. Justify.

6. (a) Consider two populations M and N. Suppose estimates of allelic proportions at L loci, each with two alleles, are provided to you. Suggest an appropriate method to measure the genetic distance between the two populations.
   (b) Suppose similar estimates of allelic proportions are provided to you on four populations. Using the measure of genetic distance suggested by you in (a) above, how can you determine the genetic relationships among the four populations?

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2004-05
B. Stat. II Year
Analysis III

Date: 29.11.04   Maximum Marks: 60   Duration: 3 Hours

You are supposed to write the Statement of any result that you use.

Answer all Questions.

1.   Justify whether each of the following statements is true or false.

(i)   Let U be an open set in $\mathbb{R}^n$ and $f : U \to \mathbb{R}$ a function. Suppose $D_i f(a)$ exists for each $i, 1 \le i \le n$, where $a \in U$. Then f is necessarily continuous at a.

(ii)   There exists a C^1-function $f : \mathbb{R}^2 \to \mathbb{R}^2$ which is not one to one, but locally invertible at every point.

(iii)   Every critical point of the function

$$f(x, y) = x - x^3 y + y^2 \text{ is either a}$$

local maxima or a local minima.

(iv)   The function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) = \begin{cases} \left(x^2 + y^2\right) Sin \dfrac{1}{\sqrt{x^2 + y^2}} & , \quad (x, y) \ne (0,0) \\ \\ 0 & (x, y) = (0,0) \end{cases}$$

is a C^1-function.

[4+4+4+4=16]

2.(a)   State Fubini's Theorem.

(b)   Let A be the region in $\mathbb{R}^2$ in the first quadrant enclosed by the y-axis , the line $y = \dfrac{1}{2}$ and the

Circle $x^2 + y^2 = 1$. Use Fubini's theorem to Compute $\iint\limits_A xy$.

[2+6=8]

3.(a)   Let A be a region is $\mathbb{R}^3$ with volume k . Let $G : \mathbb{R}^3 \to \mathbb{R}^3$ be the map $G(x, y, z) = (ax, by, cz)$, where $a, b, c > 0$. What is the volume of $G(A)$? Justify your answer.

[P.T.O.]

(2)

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$ by change of variable formula.

[4+8=12

4.(a) Suppose u and v are C^1-scalar fields defined on an open set containing the circular disk 1 whose boundary is $x^2 + y^2 = 1$. Define two vector fields f and g by

$$f(x, y) = v(x, y)\, i + u(x, y)j, g(x, y) = \left(\frac{dy}{dx} - \frac{dy}{dy}\right)i + \left(\frac{dv}{dx} - \frac{dv}{dy}\right)j.$$

Find the value of $\iint_R f \cdot g \ dx \ dy$. It is given that on the boundary of R $u(x, y) = 1$ an $v(x, y) = y$.

(b) Suppose a function $f(x, y)$ satisfy the Laplace equation $\frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2} = 0$ on a region A whi is the interior of a C^1-Jordan curve c, with positive orientation. Prove that

$$\int_c \left(\frac{df}{dy}dx - \frac{df}{dx}dy\right) = 0$$

[8+4=1

5.(a) Let S denote the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$ and $F(x, y, z) = y^2 i + xyj + xzk$ a n be the unit normal with a non-negative z component. Use Stoke's theorem to compute

$$\iint_S (Curl \ F) \cdot n \ ds.$$

(b) Find the surface area of the paraboloid

$$Z = x^2 + y^2 \text{ with } 0 \leq z \leq 2.$$

[6+6=1

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Date : 08. 12. 2004   Marks : 100   Time : 3 Hours

Answer all questions. Please try to write all the part answers of a question at the same place.

1. (a) Write a C program that takes the preorder traversal data of a binary search tree as input and outputs the tree itself.

   (b) Explain what happens when the following codes are executed.

   i. `char *p, *q; while (*p++ = *q++);`

   ii. ```
int i, k = 1, n = 5;
for (i = 0; k < n+1; i = k-i)
{ printf("%d\n", k); k = k+i; }
```

   (c) Write a function in C that finds a given substring in a circular string. As an example, the substring "abc" is absent in the string "cxyzsdfgab", but exists when the string is considered in a circular manner.

   7+6+7 = 20

2. (a) Briefly explain three hashing strategies with clear description of collision resolution.

   (b) Select a specific hashing strategy among the above three and implement a function in C programming language that can manage search and insertion in a hash table.

   (c) Provide an analysis of average search/insertion time complexity for double hashing.

   9+6+5 = 20

3. (a) Clearly explain the insertion algorithm in a balanced binary search tree.

   (b) Write down C routines for single and double rotations.

   (c) Provide specific examples to demonstrate single and double rotations while inserting a node in a balanced binary search tree.

   10+5+5 = 20

   P. T. O.

4. Given positive integers $a, b, n$, the integer $b$ is called the inverse of $a$ modulo $n$ if $ab - 1$ is divisible by $n$.

(a) Given $a, n$, write a function in C that finds the inverse of $a$ modulo $n$.

(b) Execute your function with (i) $a = 7, n = 51$ and (ii) $a = 9, n = 39$.

(c) Describe the RSA public key cryptosystem and highlight where exactly the inverse finding algorithm is required.

$$6+4+10 = 20$$

5. (a) Briefly describe Linear Feedback Shift Register (LFSR).

(b) What are its applications in cryptography?

(c) Describe how you can efficiently implement an LFSR in C language.

(d) Consider an LFSR having connection polynomial $x^{16} + x^{14} + x^{12} + x^{10} + x^7 + x^5 + x^4 + x^2 + 1$. Take the binary pattern of last two digits of your roll number and append zeros after that to get a 16-bit initial seed. Evolve the LFSR 32 times to produce 32 bits as output. Comment on the randomness of this 32-bit pattern.

$$3+3+4+10 = 20$$

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TOTAL MARKS : 100      TIME ALLOWED : 3 hours

*This is a closed book and closed notes examination. Answer all questions, and you may use calculator for numerical computations.*

(1). Given below are the systolic blood pressures of six adults measured before they took a pain killer and after one hour from the time of taking the pain killer : (113, 120), (120, 124), (117, 119), (120, 125), (114, 118), (125, 129). Use these observations to test at 5% level using two different testing procedures to determine whether there is significant evidence to conclude that the pain killer taken by these individuals had an effect of increasing their blood pressures. State all your assumptions and hypotheses clearly.

[ 20 points ]

(2). A xerox machine producing 200 copies produced 18 defective copies while another xerox machine producing 300 copies produced 32 defective copies. Is there enough evidence to believe that there is a difference in the performance of the two machines ? Test at 5% level stating your assumptions and hypotheses clearly.

[ 15 points ]

(3). Consider i.i.d observations $X_1, X_2, \ldots, X_n$ with a common uniform distribution on $[\theta, \phi]$, where $\theta$ and $\phi$ are unknown real valued parameter. Describe the maximum likelihood estimates of $\theta$ and $\phi$ based on these observations. Compute the mean squared errors for these maximum likelihood estimations. Are these estimates unbiased ? What happens to the mean squared errors of maximum likelihood estimates as $n$ grows to infinity? Justify all your answers.

[ 25 points ]

(4). Consider the time series $X(t)$ satisfying the model $X(t) = \mu(t) + \epsilon(t)$,

1

P. T. O

and the observations are taken at time points $t = i/n$ for $i = 1, 2, \ldots, n$. Describe an estimate of the signal function $\mu(t)$ for which the mean squared error tends to zero as $n$ grows to infinity under appropriate conditions. Justify your answer and state the conditions assumed.

[ 20 points ]

(5). Consider $n$ independent observations $(Y_1, X_1), (Y_2, X_2), \ldots, (Y_n, X_n)$, where the $Y$'s are non-negative integer valued with a conditional Poisson distribution having mean $\lambda(\alpha + \beta X)$ for a given $X$. Here $\lambda$ is a known twice continuously differentiable function on the entire real line. Describe clearly the computational steps for obtaining the maximum likelihood estimates for $\alpha$ and $\beta$ by Fisher's method of scoring. What conditions are needed for non-singularity of the information matrix ? For what choices of $\lambda$, Newton-Raphson iterations will coincide with Fisher's method of scoring ? Justify all your answers.

[ 20 points ]

2

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## INDIAN STATISTICAL INSTITUTE
## First Semester Back Paper Examination 2004-05

### B. STAT. II YEAR
Analysis III

Answer all questions.

Please give the Statement of any result that you use.

1.    Justify whether each of the following statements is true or false.

i)    Let $U$ be an open set in $\mathbb{R}^n$ and $f : U \to \mathbb{R}$ be a function such that at some point $a \in U$, $D_i f(a)$ exists for all $i, 1 \le i \le n$. Then the directional derivative of $f$ at $a$ in any arbitrary direction must exist.

ii)    The Vector field $F(x, y) = \left( -\dfrac{y}{x^2 + y^2}, \dfrac{x}{x^2 + y^2} \right)$, for $(x, y) \ne (0, 0)$ is a gradient of some scalar field $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$.

iii)    Every critical point of the function $f(x, y) = x - x^3 y + y^2$ is either a local maxima or a local minima.

iv)    The function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = \cos(x + y)$ is differentiable at every point.

**[4+4+4+4=16]**

2.    a)    State Fubini's Theorem.

b)    Use Fubini's Theorem to compute $\iint_A xy$ , where A is the region in the first quadrant bounded by the y axis, the line $y = \dfrac{1}{2}$ and the Circle $x^2 + y^2 = 1$.

**[2+6=8]**

3.a) Let G : $\mathbb{R}^3 \to \mathbb{R}^3$ be the map which sends Spherical Coordinate $(\theta, \phi, \rho)$ into Cylindrical Coordinate ($\theta$, r, 2). Write down the Jacobian matrix for this map. Write down the change of variable formula for this case.

b) Find the volume of the solid which is the image of a ball of radius a under the linear map represented by the matrix.

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 7 \end{pmatrix}$$

[6+6=12]

4.a) State Green's Theorem : Prove the Theorem for a region A, where

$A = \{ (x, y) \in \mathbb{R}^2 : a \le x \le b, \quad g_1(x) \le y \le g_2(x)\}$,

$g_1$ and $g_2$ being $C^1$ functions on $[a, b]$ with $g_1(x) \le g_2(x) \quad \forall x \in [a, b]$.

b) Use Green's Theorem to deduce the following :

Let B be a region which is the interior of a $C^1$ - Jordon Curve with positive orientation with respect to B. Then for any $C^1$ - vector field F on B $\iint_B (div \ F) dy \, dx = \int_c^d F.n \, dt$, where the curve is given by $t \to (x(t), y(t)), c \le t \le d$ and $n(t) = (y'(t), -x'(t))$ is the outward drawn normal vector field.

[2+6+4=12]

5. a) Compute the line integral

$\int_c F \, dC$, where C is the portion of the paraboloid $x = y^2$ betwen $(1, -1)$ and $(1, 1)$ and

$F(x, y) = (x^2, \ xy)$

b) A parametric Surface is described by
$r(u, v) = u \cos v \, i + u \sin v \, j + u^2 k$, $0 \le u \le 4$, $0 \le v \le 2\pi$. Compute the Surface area.

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[6+6=12]

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Note : This paper carries questions worth a total of 135 marks. Answer as much as you can. The maximum marks you can score is 120.

1. (a) Let $X_1, X_2, X_3$ be i.i.d. $Exp(1)$ random variables and let $S = X_1 + X_2 + X_3$. Find the conditional distribution of the pair $(X_1, X_2)$ given $S$.

(b) State Fisher-Cochran Theorem. If $\underline{X}$ is a $d$-dimensional random vector having a $N_d(\underline{\mu}, \Sigma)$ density, show that the random variable $Y = (\underline{X} - \underline{\mu})' \Sigma^{-1}(\underline{X} - \underline{\mu})$ has a $\chi^2$-distribution with $d$ degrees of freedom. [10+(5+10)]=[25]

2. Suppose $(X, Y)$ is a pair of random variables with $P[X = x] = pq^{x-1}$, $x = 1, 2, 3, \ldots$, where $0 < p < 1$, $q = 1 - p$, and, the conditional distribution of $Y$, given $X = x$, having density $h_x(y) = xy^{x-1}$, $0 < y < 1$. Find the conditional distribution of $X$, given $Y = y$, and, hence show that $E[X \mid Y] = (1 + qY)/(1 - qY)$. [15+5]=[20]

3. Let $X_1, \ldots, X_n$ be i.i.d. $U(a, b)$ random variables.
   (a) Assuming that $n$ is odd, find the expected value of the median.
   (b) What is the expected value of the point that is closest to the centre of the interval $(a, b)$? Justify your answer. [10+5]=[15]

*need an elegant argument.*

4. (a) Find the characteristic function of the random variable having density function given by $f(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$.
   (b) Using the known characteristic function of $N(0, 1)$ distribution, find the characteristic function of $XY$ where $X$ and $Y$ are i.i.d. $N(0, 1)$ random variables.
   (c) If $X_1, X_2, X_3, X_4$ are i.i.d. $N(0, 1)$ random variables, find the distribution of the random variable $X_1 X_2 - X_3 X_4$. [10+10+10]=[30]

5. (a) State the two Borel-Cantelli Lemmas.
   (b) Let $\{X_n, n \ge 1\}$ be a sequence of i.i.d. $Exp(\lambda)$ random variables. Show that $P[X_n > c \log n$ infinitely often $] = 0$ or 1 according as $c\lambda$ is $> 1$ or $\le 1$. Deduce that $\limsup_{n \to \infty} \dfrac{X_n}{\log n} = \lambda$ with probability 1. [6+(12+7)]=[25]

6. (a) Show directly from definition that $X_n \xrightarrow{P} X$ implies $X_n^2 \xrightarrow{P} X^2$.
   (b) If $U_1, U_2, \ldots$ are i.i.d. $U(0, 1)$ random variables, and, if $X_n = \max\{U_1, \cdots, U_n\}$, $n \ge 1$, then show that, $X_n \longrightarrow 1$ almost surely as $n \to \infty$, and that, $n(1 - X_n)$ converges in distribution. [10+10]=[20]

Answer Question No. 1 and any Four from the remaining questions

1. (a) In the ordinal utility framework consider the statement: "The law of diminishing marginal utility always holds". Is it true? If yes, prove it. If not, explain through an example.

(b) Use the utility function $u(x_1, x_2) = (x_1 - \alpha_1)^{\beta_1}(x_2 - \alpha_2)^{\beta_2}$ with $\beta_1 + \beta_2 = 1$ and the budget constraint $m = p_1 x_1 + p_2 x_2$ to derive: (i) the Marshallian demand function $x(p,m)$, (ii) the indirect utility function $v(p,m)$, (iii) the Hicksian demand function $h(p,u)$, and (iv) the expenditure function $e(p,u)$, where the notations have their usual meanings.                **[ 4+(7+3+3+3)=20]**

Or,

1. (a) A consumer's expenditure function is given by $e(p_1, p_2, u) = p_1^{\alpha} p_2^{1/2} u$. Using the appropriate properties of $e(.)$, calculate the value of $\alpha$.

(b) Let $x_i(p,m)$ be the consumer's demand function for good $i$. Show that if all income elasticities are constant and equal, they must be equal to 1.

(c) "For any values of prices and income it cannot be the case that for a consumer all goods are inferior". Is this statement true? Justify your answer.

(d) Define Giffen goods and inferior goods. Show that all Giffen goods are inferior, but the converse is not true.

(e) Can two indifference curves cross? Justify your answer.

(f) Show that for a consumer with strictly convex preferences, the utility maximizing bundle is unique.                **[3+3+3+5+3+3=20]**

2. (a) Assume that firms' technologies exhibit increasing returns to scale (decreasing average cost). Then show that a competitive equilibrium does not exist. If firms' technologies exhibit decreasing returns to scale, what can you say about the long run competitive equilibrium of a competitive industry?

(b) Let the market demand and market supply functions in a competitive market be linear. Assume that market equilibrium exists (so put suitable restrictions on the

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parameters of the model). Estimate social welfare defined as the sum of consumers' surplus and producers' surplus. **[(6+6)+8=20]**

3. (a) Roses, once in full bloom, have to be picked up and sold on the same day. On any day the market demand for roses is given by $P = a - Q$ where $Q$ is the number of roses demanded at price $P$. It is also given that the cost of growing roses, having been incurred by any owner of a rose garden long ago, is not a choice variable for him now.

   (i) Suppose there is only one seller in the market and he finds 1000 roses in full bloom on a day. How many roses should he sell on that day and at what price?

   (ii) Now suppose, the market is served by a large number of price-taking sellers. However, the total availability of roses on a day remains unchanged at 1000 units. Find the competitive price and the total number of roses sold that day.

(b) Consider the following scenario. A manufacturer of a product can sell its products only to a retailer who finally sells the products to the final users. The market demand for the product is $P = 5 - Q$ where $Q$ is the amount of the good demanded at price $P$, and the cost of producing the good by the manufacturer is $c = 1$ per unit. Find the price to be charged by the manufacturer for his products to the retailer and the price to be charged by the retailer to the final users. **[(6+6)+8=20]**

4. (a) The Laffer effect is said to occur when the tax revenue declines as the tax rate goes up (beyond a certain level). Consider the following scenario. Let the labor supply curve $L(.)$ be upward sloping in wage rate $W$ and labor demand is perfectly elastic at a wage rate $\overline{W}$. There is a tax on labor at the rate $t > 0$ such that if the firm pays $\overline{W}$, the worker gets $W = (1-t)\overline{W}$. Show that the Laffer effect occurs when the elasticity of labor supply is greater than $(1-t)/t$.

(b) Suppose that a monopolist faces two markets with demand curves given by $D(P_1) = 100 - P_1$ and $D(P_2) = 100 - 2P_2$. Its marginal cost is constant at Rs 20 per unit. Assume that resale between the markets is not possible. Then derive the optimal price-quantity combination in each market. If resale were possible, what would be the corresponding values? **[6+(7+7)=20]**

5. (a) Make clear the concept of Nash equilibrium.
   (b) Construct examples to show the following results:
       (i) Nash Equilibrium (in pure strategies) does not exist.
       (ii) Nash equilibrium exists and is unique.
       (iii) Multiple Nash equilibria exist. **[5+(3X5)=20]**

6. Write short notes on the following:
   (a) The first theorem of welfare economics.
   (b) Linearly homogeneous production function.
   (c) Two-part tariff.
   (d) Economic rent. **[4X5=20]**

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Indian Statistical Institute
First Semestral Examination: 2004 – 2005
B. Stat. II: 2004-05
Economics I
Date: **6.1.05**      Maximum Marks: 100      Duration: 3 Hours

Answer ALL questions

1. (a) A consumer's income is Rs. 120/- and he spends it on two goods 1 and 2. His utility function is $u(x_1, x_2) = x_1 x_2$. Suppose $p_1 = $ Rs. 2/- and $p_2 = $ Rs. 3/-.
   (i) What are the utility maximizing choices of goods 1 and 2?
   (ii) If price of good 1 increases by 44% with no change in price of good 2, by how much his income would have to be increased for enabling him to maintain the original utility level?

(b) What are "necessary", "inferior", and "luxury" goods? Give examples.

(c) Define Giffen goods. Show that all Giffen goods are inferior, but the converse is not true. **[(4+5)+6+5=20]**

2. Characterize a monopolistic market. Explain long run equilibrium of a monopolistic industry. **[20]**

3. (a) The inverse demand curve is given by $P(x) = 10 - x$ and the monopolist has a fixed supply of 4 units of a good available. How much will it sell and what price will it set? What would be the price and output in a competitive market with these demand and supply characteristics? What would happen if the monopolist had 6 units of the good available?

(b) A revenue-maximizing monopolist requires a profit of at least 1500. His demand and cost functions are $P(q) = 304 - 2q$ and $C = 500 + 4q + 8q^2$. Determine his output level and price. **[(3X5)+5=20]**

4. Consider a two-person (1 and 2), two-commodity ($X$ and $Y$), pure exchange competitive economy. The consumers' utility functions are:
$$U_1(x_1, y_1) = x_1^2 y_1^3 \quad \text{and} \quad U_2(x_2, y_2) = x_2^3 y_2^2$$
Their endowment vectors are: $(\overline{x}_1, \overline{y}_1) = (10, 0)$ and $(\overline{x}_2, \overline{y}_2) = (0, 10)$.
   (i) Derive the equilibrium price ratio ($P_x/P_y$) for this economy.
   (ii) Is the equilibrium so obtained Pareto efficient? **[14+6=20]**

*P. T. O*

5. Write short notes on the following:
   (a) Returns to scale.
   (b) The grand utility possibility locus .
   (c) Nash Equilibrium.
   (d) The production possibility locus.                                    [4X5=20]

Note : This paper carries questions worth a **total** of **100 marks**. Answer as much as you can. The **maximum marks** you can score is **45**.

1. Suppose $X$ and $Y$ are independent random variables having gamma distributions with parameters $(\lambda, \alpha_1)$ and $(\lambda, \alpha_2)$ respectively. Find the joint density of the random variables $X + Y$ and $\dfrac{X}{Y}$. Examine if they are independent.          [20]

2. If $X_{(1)} < \cdots < X_{(n)}$ are the order statistics for $n$ i.i.d. $U(0,1)$ random variables, find the mean vector and the dispersion matrix of the vector $(X_{(1)}, \cdots, X_{(n)})'$. What is the density of $R = X_{(n)} - X_{(1)}$?          [20]

3. Let $\underset{\sim}{X} = (X_1, \cdots, X_d)'$ have a $d$- variate normal density with mean vector $\underset{\sim}{\mu}$ and dispersion matrix $\Sigma$.
   (a) For $1 \leq m < d$, find the conditional distribution of $(X_{m+1}, \cdots, X_d)'$ given $(X_1, \cdots, X_m)'$
   (b) Assume $\underset{\sim}{\mu} = \underset{\sim}{0}$ and $\Sigma = I$. State and prove a necessary and sufficient condition for the quadratic form $\underset{\sim}{X}' A \underset{\sim}{X}$ to have a $\chi^2$ distribution.          (10+10)=[20]

4. (a) Define Convergence in Probability and Almost Sure Convergence.
   (b) Show that $X_n \xrightarrow{P} X$ implies that, for some subsequence $\{n_k\}$, $X_{n_k} \longrightarrow X$ almost surely.
   (c) Show that if $X_n \xrightarrow{P} X$ and if all the random variables are defined on a discrete probability space, then $X_n \longrightarrow X$ almost surely.          (2+9+9)=[20]

5. (a) State the inversion formula for characteristic functions.
   (b) Show that if the characteristic function $\phi_X$ of a random variable $X$ is integrable, then the random variable $X$ is absolutely continuous with a bounded continuous density.
   (c) Show that the function $\phi(t) = \max\{(1-|t|), 0\}$ is a characteristic function. (2+9+9)=[20

# INDIAN STATISTICAL INSTITUTE

### First Semestral Examination : (2004-2005)

### B.Stat. (Hons.) II Year

## PHYSICS I

### Group A (Electrodynamics)

Date : 2.12.2004     Maximum Marks : 30     Duration : 1.5 Hours

**Note : Answer any six of the following :**

1. Find the net force that the southern hemisphere of a uniformly charged sphere exerts on the northern hemisphere. Express your answer in terms of the radius $R$ and the total charge $Q$.                                    (5)

2. Consider a square of side $s$. Corners 1 and 3 are diagonally opposite to each other and have charge of $-q$ each. A charge of $+q$ is located at corner 2.

   (a) How much work does it take to bring in another charge, $+q$, from far away and place it at corner 4 which is diagonally opposite to corner 2?

   (b) How much work does it take to assemble the whole configuration of four charges ?                                    (3+2)

3. Assume that the magnetic field $\vec{B}$ points in the $x$-direction and the electric field $\vec{E}$ in the $z$-direction. A particle is released from the origin with velocity $\vec{V}(0) = \frac{E}{B}\hat{y}$. Find the trajectory of the particle.        (5)

4. A large parallel-plate capacitor with uniform surface charge $\sigma$ on the upper plate and $-\sigma$ on the lower is moving with a constant speed $v$ towards the right.

   (a) Find the magnetic field between the plates and also above and below them.

   (b) Find the magnetic force per unit area on the upper plate, including its direction.

1

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(c) At what speed $v$ would the magnetic force balance the electrical force ? $(2 + 1\frac{1}{2} + 1\frac{1}{2})$

5. A circular loop of wire, with radius $R$, lies in the $xy$ plane, centered at the origin, and carries a current $I$ running counter clockwise as viewed from the positive $z$-axis.

   (a) What is its magnetic dipole moment ?

   (b) What is the approximate magnetic field $\vec{B}$ at points far from the origin ?

   (c) How does the expression for $\vec{B}$ change for points on the $z$-axis when $z \gg R$ ? $(1+2+2)$

6. A metal bar of mass $m$ slides frictionlessly on two parallel conducting rails a distance $l$ apart. A resistor $R$ is connected across the left ends of the rails and a uniform magnetic field $\vec{B}$, pointing into the page, fills the entire region.

   (a) If the bar moves to the right at speed $v$, what is the current in the resistor ? In what direction does it flow ?

   (b) What is the magnetic force on the bar ? In what direction ?

   (c) If the bar starts out with speed $v_0$ at time $t = 0$, and is left to slide, what is its speed at a later time $t$ ?

   (d) Find the total energy delivered to the resistor. $(1\frac{1}{2}+1+1+1\frac{1}{2})$

7. (a) Show that $\delta(kx) = \frac{1}{|k|}\delta(x)$, where $k$ is any (non zero) constant.

   (b) Compute $(\hat{r}.\nabla)\hat{r}$ where $\hat{r} = \dfrac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2 + y^2 + z^2}}$ is the position unit vector. $(2\frac{1}{2}+2\frac{1}{2})$

8. A square loop of wire of side $a$, lies midway between two long wires, $3a$ apart, and in the same plane. (Actually, the long wires are sides of a large rectangular loop, but the short ends are so far away that they can be neglected). A clock wise current $I$ in the square loop is gradually increasing $\frac{dI}{dt} = k$ ($a$ constant).

   (a) Find the $emf$ induced in the big loop.

   (b) Which way will the induced current flow? Give reasons for your answer. $(3\frac{1}{2}+1\frac{1}{2})$

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# Physics-I

# Group-B

Date- 2.12.04 Maximum marks:30     Duration- 90 minutes

**Answer any five question.**

1. State Kepler's laws of planetary motion. A particle describes the path given by the equation $r^n = a^n cos n\theta$, under the central force $P$. Find the law of force.

   $2 + 4$

2. Let a particle move in a plane. Find the expression of radial and transverse acceleration using Lagrangian formalism. Find the equation of motion under the potential $-K/r$ using Lagrangian equation of motion.

   $3 + 3$

3. Let a lift moves vertically upward with constant acceleration. Explain why one feels heavier in the lift using the argument of pseudo force. Explain why one feels weightless when he falls freely.

   $4 + 2$

4. State the principles of special relativity. Find the transformation equations between space and time of two frames having relative velocity $v$ along $x$-axis.

   $1 + 5$

5. Using Lorentz transformation equations, show that simultaneity is relative. Let a rod fixed in a frame has length $L_0$. Find the length of the rod from a frame having relative velocity $v$ along $x$-axis w.r.t. the first frame.

   $3 + 3$

6. Let a particle moves with velocity $u$ along x-axis in a reference frame. Find the velocity of the particle w.r.t. a frame moving along x-axis with velocity $v$ w.r.t. first frame.

   $6$

3

# SUBJECT: DEMOGRAPHY

**FULL MARKS: 100**     DATE OF EXAMINATION:21FEBRUARY 2005

## Answer any FIVE questions

1.     a) Write briefly why there is a need for evaluation and adjustment of basic demographic data.

       b) Discuss a method of evaluating age data given by single years of age.

(8+12=20)

2.     a) Distinguish between mean age at marriage and singulate mean age at marriage.

       b) Estimate singulate mean age at marriage. What are the data requirements for its computation ?

(6+10+4=20)

3.     a) Define crude and standardized death rates.

       b) In what way are the standardized rates are superior?

       c) Explain briefly the differences between the direct and indirect methods of standardizing death rates.

(6+4+10=20)

4.     a) Write a note on the Chiang's method of constructing an abridged life table.

       b) Fill in blanks in the life table given below:

| Age: $x$ | $l_x$ | $d_x$ | $q_x$ | $L_x$ | $T_x$ | $e^0_x$ |
|---|---|---|---|---|---|---|
| 4 | 95000 | 500 | ? | ? | 4350350 | ? |
| 5 | ? | 400 | ? | ? | ? | ? |

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c) For a certain population

$l_x = 10000 (121 - x)^{1/2}$

Find $q_x$ and the probability that a life aged zero will die between age 21 and 40.

(8+4+8=20)

5.    a) Discuss the properties of logistic curve.

b) Derive the logistic growth curve for a closed population whose births and deaths are respectively linear function of the size of a population. Estimate the initial and final population of this logistic growth curve.

(6+10+4=20)

6.    a) What are period and cohort measures of fertility? Explain them with examples.

b) Describe the curve of age patterns of fertility.

c) Define replacement index.

d) Derive mathematically relation between crude birth rate and total fertility rate as well as general fertility rate and total fertility rate.

(2+3+4+3+8=20)

7.    Write short notes:

a) Whipple's index

b) Census measures of fertility

c) Comparative mortality index

d) Greville's method

e) System of census

(4 x 5 = 20)

**INDIAN STATISTICAL INSTITUTE**
Mid-Semester Examination
B.Stat (Hons.) – II
Biology – II
Date...24..2..05........Maximum no. 40     Duration: 2.00 hrs.

1.  Write short notes on any five of the following                    3 x 5 = 15
    a)  Determine the growth rate (r) of a plant assuming the initial and final time
        is $t_1$ and $t_2$ respectively.
    b)  Low temperature effect of plant growth.
    c)  Photoperiodism.
    d)  Probable pathway of IAA synthesis.
    e)  Epical dominance of IAA.
    f)  Physiological responses of ABA.
    g)  Michaelis – Menten Constant (Km).

2.  What do you mean by 'Functional resistance'? Write in brief on structural
    protection mechanism of plants against diseases.              1 + 4 = 5

3. What are Phytoalexins? Write the salient features of Phytoalexins       1 + 4 = 5

OR

Write in brief on histological defense mechanism of a plant.              5

4. Write the name of different weather parameters related to crop growth. Also write
    the name of the devices to measure those parameters.              4

5. Write short notes on any three of the following                    3 X 2 = 6
    a)  Onset of monsoon
    b)  Withdrawal of monsoon
    c)  Moisture availability index
    d)  Evapotranspiration

### Firm-B

| Allocation | | | Receipts | | |
|---|---|---|---|---|---|
| ( Purchases from :- | | | ( Sales to :- | | |
| firm A | Rs. | 21,000 | households | Rs. | 45,000 |
| foreigners ) | Rs. | 19,000 | firm A | Rs. | 6,000 |
| wages | Rs.. | 22,000 | | | |
| rents | Rs. | 500 | foreigners) | Rs. | 21,000 |
| indirect taxes | Rs. | 1,500 | | | |
| taxes on profit | Rs. | 700 | (Addition to :- | | |
| depreciation | Rs. | 3,300 | | | |
| dividends | Rs. | 1,000 | | | |
| undistributed profit | Rs. | 2,000 | stocks ) | Rs.- | 1,000 |
| | **Rs. 71,000** | | | **Rs.71,000** | |

### Government

| Allocation | | | Receipts | | |
|---|---|---|---|---|---|
| interest on national debt | Rs. | 1,000. | Taxes:- | | |
| wages and salaries | Rs. | 6,000. | direct taxes on companies | Rs. | 2,700 |
| purchase from firm A | Rs. | 5,000. | direct taxes on persons | Rs. | 7,000 |
| (net decrease in indebtedness) | Rs. | 200. | indirect taxes | Rs. | 2,500 |
| | **Rs.12,200** | | | **Rs.12,200.** | |

From the above information find an estimate of GDP of the economy in question by the expenditure as well as income method.

(25)

2

a. Hindustan Lever Limited ,a joint stock company, purchased raw materials worth Rs. 1,000 and used half of it in production It paid Rs.1,000.in wages and salaries and spent Rs.100 on advertisement . Its sales exceeded its production by Rs.100 and it purchased a house with Rs.10,0 00 for one of its executives .In calculating GDP by spending approach which of the above information will you take into account? Explain.

P.T.O

**INDIAN STATISTICAL INSTITUTE**

Mid-semester Examination : (2004-2005)

B.Stat. II Year

Economics II

24.2.05...................Maximum Marks - 40................ Duration- 150 minutes

Answer all questions.

1 a. How do you define GDP? How can you arrive at a measure of GDP using three different methods, viz., income method, expenditure method and product method?

b. In a particular period an economy consisting of two firms (A and B) and a government sector records the data shown below :-

### Firm-A

| Allocation (Purchases from :- | | Receipts ( Sales to :- | |
|---|---|---|---|
| firm B | Rs. 6,000 | households | Rs. 10,000 |
| foreigners ) | Rs. 5,000 | firm B | Rs. 21,000 |
| wages | Rs.. 20,000 | government | Rs. 5,000 |
| rents | Rs. 1,000 | foreigners ) | Rs. 8,000 |
| indirect taxes | Rs. 1,000 | | |
| taxes on profit | Rs. 2,000 | ( Addition to :- | |
| depreciation | Rs. 5,000 | fixed capital | Rs. 6,000 |
| dividends | Rs. 7,000 | stocks) | Rs. 4,000 |
| undistributed profit | Rs. 7,000 | | |
| | **Rs. 54,000** | | **Rs.54,000** |

P.T.O.

b. A person lives in his own house. He is retired. He has kept all his savings in a fixed deposit savings scheme of the government with a post office. His explicit income consists solely of the interest income from this fixed deposit. Does he contribute to GDP or national income? From the above information can you discern any contribution made by the person to final expenditure?

c. Both households and the corporate sector keep their savings as deposits with a Commercial bank. The commercial bank earned in a given year Rs.10, 000. in interest income for loans it gave out and paid Rs.1,000. as interest to depositors one fourth of which went to the corporate houses. It spent Rs.2,000. in wages and salaries and rent. It purchased stationery of Rs.1,000. How much did the bank contribute to GDP and national income? In how many ways can you calculate the bank's contribution to GDP?

(5+5 +10).

# INDIAN STATISTICAL INSTITUTE

## Mid-Semester Examination:(2004-2005)

### B.STAT.(HONS.)II YEAR

## Physics-II

Date- **24.2.05** Maximum marks:40     Duration- 2 Hours 30 minutes

### Answer any five question.

1. What is a perfect black body? Discuss the limitations of the classical theory in explaining the distribution of radiation from a black body. State Planck's hypothesis and give a sketch of the derivation of Planck's law ofb distribution. Show that Rayleigh-Jean formula is a special case of Planck's law.

$$1 + 1 + 4 + 2$$

2. Explain how in the photo-electric effect, particle nature of light is manifested. Draw a graph between the frequency of light falling on a metallic surface and the maximum kinetic energy of photoelectrons emitted. How will this graph changes if i) the intensity of light is changed, ii) the metal is changed.

$$2 + 2 + 2 + 2$$

3. What is Compton effect? Derive the expression for the change in wavelength of scattered x-rays. Explain why Compton effect is not observed for visible light rays.

$$1 + 5 + 2$$

4. Let A be a self adjoint operator acting on a n-dimensional Hilbert space, having n different eigenvalues $a_1, a_2...a_n$ with corresponding eigenfunctions $|\psi_1 >, |\psi_2 > ...$etc. Show that $A = \sum a_i |\psi_i >< \psi_i|$. Also show that $\sum |\psi_i >< \psi_i| = I$, $I$ being unit operator on the Hilbert space. If a system is in the state $|\psi >= \frac{1}{\sqrt{5}}|\psi_3 > + \frac{2}{\sqrt{5}}|\psi_5 >$. Find the probability of the result being $a_3$ or $a_7$ if measurement of $A$ is performed on the system.

$$3 + 3 + 2$$

5. Consider a system with two dimensional complex Hilbert space. Let state of the system be $|\psi >= \frac{1}{\sqrt{10}}|0 > + \frac{3}{\sqrt{10}}|1 >$, $|0 >$ , $|1 >$ being eigenvector of $\sigma_z$ with eigenvalue $+1$ and $-1$ respectively. Write the density matrix representing the above state in terms of the operators $I, \sigma_x, \sigma_y, \sigma_z$. Show

1

that $\frac{1}{2}[I + \sigma.n]$ is a projection operator if $n$ is a unit vector. Show that collection of all operators of the form $\frac{1}{2}[I + a\sigma.n]$ with $0 \le a \le 1$ and $n$ being a unit vector, form a convex set.

$$4 + 2 + 2$$

6. Consider a two two-level systems in a state

$$|\psi_{12}> = a|0>|1> + b|1>|0>$$

where $|0>$ and $|1>$ are orthogonal unit vectors for the respective subsystem. Show that this state can not be written in the product form. Find the density matrix of the subsystem 1. Show that the following four states

$$\frac{1}{\sqrt{2}}[|0>|1> - |1>|0>]$$

$$\frac{1}{\sqrt{2}}[|0>|1> + |1>|0>]$$

$$\frac{1}{\sqrt{2}}[|0>|0> + |1>|1>]$$

$$\frac{1}{\sqrt{2}}[|0>|0> - |1>|1>]$$

form an orthogonal set.

$$3 + 2 + 3$$

7. Consider a unitary operator $H$ acting on two-level system in the following way,

$$H|0> = \frac{1}{\sqrt{2}}[|0> + |1>]$$

$$H|1> = \frac{1}{\sqrt{2}}[|0> - |1>]$$

Write the matrix form of $H$ in the computational basis. Consider a unitary operator acting on two two-level systems in the form

$$U|0>|0> = |0>|0> \quad U|0>|1> = |0>|1>$$

$$U|1>|0> = |1>|1> \quad U|1>|1> = |1>|0>$$

Write the matrix in the computational basis. Argue that this operator can not be written as tensor product of two unitary operators acting on each system. Show that two non-orthogonal states can not be cloned by a single machine.

$$2 + 2 + 1 + 3$$

2

This paper carries 60 marks. Maximum you can score is 50. You may use any theorem proved in the class.

1. $G$ is a group in which $(ab)^i = a^i b^i$ for 3 consecutive values of $i$. Show that $G$ is abelian. [10]

2. Let $M$, $N$ be normal subgroups of a group $G$ with $M \cap N = \{e\}$, $e$ being the identity element . If $m \in M$ and $n \in N$ show that $mn = nm$. [10]

3. $G$ is a group of order $pq$ where $p, q$ are distinct primes . If $G$ has a normal subgroup of order $p$ and another normal subgroup of order $q$ show that $G$ is cyclic . [10]

4. Let $n \ge 3$ , $S_n$ the group of permutations on $\{1, 2, \ldots, n\}$ and $A_n$ the subgroup of even permutations . Show that $A_n$ is the smallest subgroup containing all 3-cycles. [10]

5. $R$ is a ring in which $x^2 = x$ for all $x$. Show that $R$ is a commutative ring . [10]

6. The integers $m; n$ are relatively prime . $D$ is an integral domain and $a, b \in D$ satisfy $a^m = b^m$ and $a^n = b^n$ . Show that $a = b$ . [10]

INDIAN STATISTICAL INSTITUTE
B. Stat II: 2004 – 2005
Mid-semester Examination
Economic Statistics and Official Statistics
Duration: 3 hours          Maximum marks: 100          Date: 2.3.          2005

*[Answer Part I and Part II in separate answer scripts.]*

Part I: Economic Statistics
*[Answer question no. 1 and any three from the rest of the questions in Part I]*

1. Suppose the share of income of bottom 10% people in a community is 2.38% and their corresponding average income is Rs. 45.06/-. (i) Find the average income of all persons in the community and the Lorenz Ratio. (ii) Also find the share of income of bottom 97.62% people. (iii) What will be the overall mean income if the income of all the people if income of each person below the median income is doubled and income of each person above the median income is halved?          [11+1+11]

2. Describe different types of data that we face and the problems that we encounter with these data in econometric analysis.          [19]

3. Define Lorenz Curve (LC). State its properties. How will you compute Lorenz Ratio (LR) graphically from grouped data? Derive LC and LR for Pareto distribution.          [1+4+7+7=19]

4. Suppose $y_1, y_2, \ldots, y_n$ are incomes of n persons in a community. Describe how you will find Lorenz Ratio (LR) numerically. Prove that LR found by numerical method is equivalent to the LR found by the formula using Gini's Mean Difference (GMD).          [19]

5. Write short notes on any two of the following:
   (i) Law of Proportionate Effect.
   (ii) Three parameter lognormal distribution.
   (iii) Universality of Pareto Law.
   (iv) Estimation of two-parameter lognormal distribution.          $[9\frac{1}{2}+9\frac{1}{2}=19]$

Part II: Official Statistics
*[Answer all questions]*

6. Write a brief note on Index of Industrial Production. Indicate its uses.          [5]

7. Name the important statistical departments under Ministry of Statistics & Programme Implementation (MoS&PI), Government of India.          [5]

8. What are the national consumer price index numbers? What is your opinion about using WPI for measuring inflation in Indian economy?          [5+5]

### Group A : *ALL Questions are COMPULSORY.*

Q1. (a) Show that

$$I = \int ... \int dx_1 dx_2 ... dx_n$$

over $E_n = \{ u - \frac{1}{2} du \ < \ \sum a_i x_i \ < \ u + \frac{1}{2} du, \ v - \frac{1}{2} dv \ < \ \sum x_i^2 \ < \ v + \frac{1}{2} dv \}$
depends on the $a_i$'s only through $\sum a_i^2$.

(b) Let $g(S_{11}, S_{22}, S_{12})$ represent the joint density of $S_{11}, S_{22}, S_{12}$ defined over
$(0, \infty) \ x \ (0, \infty) \ x \ ( - \sqrt{(S_{11}S_{22})}, \ \sqrt{(S_{11}S_{22})})$.
Work out a formal representation for the joint density of $(s_1, s_2, r)$ in terms of g
where $s_1^2 = S_{11}, \ s_2^2 = S_{22}$ and $r s_1 s_2 = S_{12}$.

[10 + 10 = 20]

Q2. When $\sigma_1$ and $\sigma_2$ are known, suggest TWO exact tests for $\rho$, one using the
generalized variance and the other based on a quadratic form, given a random
sample of paired observations from a bivariate normal distribution with
parameters $m_1, m_2, \sigma_1, \sigma_2$ and $\rho$.
What would you do in case the population means are also known ?

[5 + 5 + 5 = 15]

Q3. Suppose you have a random sample of n pairs of observations on (X,Y) and these
are classified as follows :
Class I : # pairs for which X > 0, Y > 0;
Class II : # pairs for which X > 0, Y < 0;
Class III : # pairs for which X < 0, Y > 0;
Class IV : # pairs for which X < 0, Y < 0.
The frequency counts in the above classes are respectively $f_I, f_{II}, f_{III}$ and $f_{IV}$.
If (X,Y) follow bivariate normal distribution with means (0 ,0), unknown variances
and unknown correlation coefficient $\rho$, suggest a test for $H\_0 : \rho = 0.5$
based on the frequency counts alone.
Carry out the test for the case : n = 300, $f_I = 90$, $f_{II} = 95$, $f_{III} = 55$ and $f_{IV} = 60$.

[You may adopt 5 % level of significance.]

[8 + 7 = 15]

### Group B : Attempt BOTH the Questions

1.     A substance used in biological and medical research is shipped by airfreight
to users in cartons of 1000 ampoules. The data below, involving 10 shipments,
were collected on the number of times the carton was transferred from one
aircraft to another over shipment route (x) and the number of ampoules
broken upon arrival (y). Assume that a linear regression model is
appropriate here.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|----|
| $x_i$ | 1 | 0 | 2 | 0 | 3 | 1 | 0 | 1 | 2 | 0 |
| $y_i$ | 16 | 9 | 17 | 12 | 22 | 13 | 8 | 15 | 19 | 11 |

(a) Obtain the estimated regression function. Give a measure of goodness of fit and comment.

(b) Estimate $\beta_1$ with a 95% confidence interval and interpret the interval estimate.

(c) Conduct a $t$-test to decide whether or not there is a linear association between x and y. Use $\alpha = 0.05$. What is the P – value of the test?

(d) A consultant has suggested, based on previous experience, that the mean number of broken ampoules should not exceed 9 when no transfers are made. Conduct an appropriate test on the claim using $\alpha = 0.025$. Draw conclusions, and give the P – value of the test.

[8+8+7+7=30]

2.  (a) Consider the simple linear regression model with normal homoscedastic errors. Show that the least square estimators of the regression coefficients are actually their best linear unbiased estimators.

(b) Develop the analysis of variance for testing adequacy of the model fitted in 2(a) above.

[10+10=20]

# INDIAN STATISTICAL INSTITUTE
## Semestral II Examination 2004 – 05
## B. STAT II Year 2004 – 05
### Demography & SQC & OR

Date: 4.5.05          Full Marks: 100          Duration: 3 Hours

### Group A : Demography
Maximium Marks – 50

*Note : Symbols and notations have their usual meanings*

1.  a) How is the age of an individual recorded in Census ?

b) Define age of population and age heaping ?

c) How can you detect age heaping ?

d) Define Myer's Index for detection of errors in age returns and discuss its merits and demerits.

[2 + 4 + 2 + 7 = 15]

OR

e) Explain the methods for adjusting the conventional infant mortality rate.

f) Differentiate between crude death rate and specific death rate.

[8 + 7 = 15]

2.  a) What are the period and cohort measures of fertility ? Explain them with examples.

b) Describe the curve of age pattern of fertility.

c) Derive mathematically the relationship between TFR, CBR and GFR.

[4 + 4 + 7 = 15]

P.T.O.

OR

d) Distinguish between mean age at marriage and singulate mean age at marriage.

e) Derive Hajnal's method for computation of singulate mean age at marriage.

f) What are indirect methods of migration analysis ? Discuss any one of them.

$$[4 + 4 + 7 = 15]$$

3. a) Derive the formula for the logistic growth curve through optimum population approach.

b) If $\mu_x = A + BC^x$ , show that $l_x = K\, s^x g^{C^x}$

How will you estimate K, s, g & C, while fitting the above curve.

$$[7 + 5 + 8 = 20]$$

OR

c) Find $q_{50}$ given that $\mu_{50} = 0.01098$ and $\mu_{51} = 0.01173$.

d) In a certain life table,

$$\mu_x = 0.15 - 0.10\, x \quad \text{for } 0 \le x \le 1/2, \text{ and}$$

$$\mu_x = (0.01)^x \qquad \text{for } 1/2 \le x \le 1.$$

Find $l_1$ , assuming that $l_0 = 100{,}000$.

e) Prove that $m_x = -\dfrac{1}{L_x} \dfrac{d}{dx} L_x$

Using the result, prove that $m_x \doteqdot \mu_{x+\frac{1}{2}}$

$$[5 + 5 + (5 + 5) = 20]$$

Contd. (3)

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### Group B: SQC & OR
Maximum Marks: 50

1. This paper carries 60 marks. You may answer as much as you can, but the maximum you can score is 50.
2. Begin this group on a new answer-script.

1. State whether the following statements are True (T) or False (F). You need not copy the statements.

(a) A form, in either diagram on table format, that is prepared in advance for recording data is known as a flowchart.

(b) A process that is in statistical control will never have a point beyond a three-sigma limit on $\overline{X} - R$ control chart.

(c) In practice the proportion non-conforming coming from a manufacturing process that produces millions of items per day can be successfully monitored with a $p$ chart.

(d) Control charts were first devised by Dodge and Romig.

(e) Consumers risk of 10% means that the probability that a sampling plan will reject "good" material is 10%.

(f) From the standpoint of quality protection, the absolute size of a random sample is much more important than its relative size compared to the size of the lot.

(g) The OC-curve of plans with acceptance numbers greater than zero are superior to those of comparable plans with an acceptance number of zero.

(h) In a simplex tableau, if there is a tie for the departing variable, then the next BFS will be degenerate.

(i) If in any simplex iteration, the minimum ratio rule fails, then the LPP is infeasible.

(j) The simplex method may not move to an adjacent extreme point if the current iteration is degenerate.

$$[1 \times 10 = 10]$$

2. A process producing coaxial cables is being monitored by a "number of defects per unit" chart. The process average has been calculated as 0.10 defects per unit. Three-sigma control limits are employed and samples of size 200 are taken on a daily basis.

(a) Calculate the control limits for the chart.

(b) If the process mean were to suddenly shift to 0.15 per unit, what is the probability that this shift would be detected on the 5th subsequent day?

(c) What is the expected number of samples until an out-of-control signal is received?

$$[4+5+6=15]$$

P.T.O.

3. Consider a double sampling plan with the following parameters:

$$n_1 = 50 \qquad c_1 = 1 \qquad r_1 = 5 \qquad n_2 = 100 \qquad c_2 = 7.$$

Find the probability of acceptance of a lot that has fraction defective $p = 0.06$.

[10]

4.(a) A dairy has two plants each of which produces and supplies two products: Milk and Butter. Plants can each work upto 16 hours a day. In Plant-I, it takes 3 hours to prepare (from powder) and pack 1000 liters of milk and 1 hour to prepare and pack 100 kg of butter. In Plant-II, it takes 2 hours to prepare and pack 1000 liters of milk and 1.5 hours to prepare and pack 100 kg of butter.

In Plant-I, it costs Rs. 15,000/- to prepare and pack 1000 liters of milk and Rs. 28,000/- to prepare and pack 100 kg. of butter, whereas these costs are Rs. 18,000/- and Rs. 26,000/- respectively for Plant-II. The company is obliged to produce daily at least 10,000 liters of milk and 800 kg. of butter.

Formulate this as an LPP to find as to how should the dairy organize its production so that the required amount of the products be obtained at minimum cost.

(b) Consider the following LP:

$$\max x_0 = 4x_1 + x_2 + 3x_3 + 5x_4$$

st.
$$4x_1 - 6x_2 - 5x_3 + 4x_4 \geq -20$$
$$3x_1 - 2x_2 + 4x_3 + x_4 \leq 11$$
$$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 23$$
$$x_1, x_2, x_3, x_4 \geq 0$$

One of the simplex iteration Tableaus of the above LP is given below:-

|       | $y_0$ | $-x_1$ | $-x_3$ | $-x_6$ | $-x_7$ |
|-------|-------|--------|--------|--------|--------|
| $x_0$ | A     | 17     | -35    | -17    | H      |
| $x_5$ | B     | E      | 11     | 0      | 2      |
| $x_4$ | C     | F      | -6     | -3     | 2      |
| $x_2$ | D     | G      | -5     | -2     | I      |

Without performing the simplex iterations, find the missing entries $A, \ldots, I$. Show your calculations.

[10+15=25]

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INDIAN STATISTICAL INSTITUTE

B. STAT SECOND YEAR

Elements of Algebraic Structures

Date : 6.5.05    Semestral II Examination    Time : 3 hrs.

This paper carries 120 marks. Maximum you can score is 100. You may use any theorem proved in the class.

1. Let $M, N$ be normal subgroups of a group $G$.

   (a) Show that $MN$ is a subgroup of $G$, $M$ is normal in $MN$ and $M \cap N$ is normal in $N$. [7]

   (b) Show that $\frac{MN}{M}$ is isomorphic to $\frac{N}{M \cap N}$. [8]

2. Let $G$ be a group and $H, K$ be subgroups of $G$ of finite index . Prove that $H \cap K$ is of finite index in $G$. [15]

3. If $m, n, a_0, a_1, \cdots, a_r$ are integers such that $(m, n) = 1$ and $(x - m/n) | (a_0 + a_1 x + \cdots + a_r x^r)$ in $\mathbf{Q}[x]$, $\mathbf{Q}$ being the field of rationals , prove that $m | a_0$ and $n | a_r$ in the ring of integers . [15]

4. Let $J[i] = \{a + bi : a, b \text{ integers}\}$ . Let $p$ be an integer which is prime in the ring of integers. When is $p$ prime in $J[i]$ ? Justify your answer . [15]

5. Let $A$ be an ideal of a commutative ring $R$ . Let $N(A) = \{x : x^n \in A \text{ for some } n\}$ . Prove that $N(A)$ is an ideal containing $A$. [15]

6. Let $F = \mathbf{Z_{11}}$.

   (a) Show that $x^2 + 1$ and $x^2 + x + 4$ are irreducible over $F$ . [9]

   (b) Show that $\frac{F[x]}{x^2+1}$ and $\frac{F[x]}{x^2+x+4}$ are isomorphic . [6]

7. Find the degree of the splitting field of $x^3 + x + 1$ over the rationals. [15]

8. Let $F$ be a finite field. Show that $F - \{0\}$ is a cyclic group under multiplication . [15]

**B. STAT. II YEAR**
Economics - II

Date: 09.05.2005 — Maximum Marks : 60 — Duration : 200 Minutes

**Answer all questions.**

1. (a) Consider a simple Keynesian model with no government sector where the consumption function is given by $C = 100 + .7y$, y being real GDP and aggregate planned investment is 500 units.

    Compute the equilibrium level of income and

    Explain the following two statements with the example of this model.

    (i) aggregate saving is equal to aggregate planned investment only at equilibrium level of y.

    (ii) aggregate saving is equal to actual aggregate investment at any level of y.

   (b) Consider a simple Keynesian model with government sector for a closed economy given by

   $$y^d = C + I + G, \quad y^d \text{ being planned aggregate demand.}$$

   $$C = C\,(YD) \quad , 0 < C' < 1$$

   $$YD = y - T \quad ; \ y \ \text{ is real GDP, T is total tax collection}$$

   $$T = tY \quad , \quad 0 < t < 1$$

   $$I = \hat{I}$$

   $$G = \overline{G}$$

   How will the equilibrium level of y change following an increase in t ? . How will the answer to this question change if instead, $G = ty$. Explain your answers.

   (c) Explain the paradox of thrift in a simple Keynesian model. Does it hold in an I S- L M model given by

   $$y^d = C + I$$

   $$C = C(y) \ , \ 0 < C' < 1 \text{ where y is real GDP.}$$

   $$I = I(r) , I' < 0 \quad \text{where r is real rate of interest and I is aggregate planned}$$

   investment demand.

   $\left(\dfrac{M^s}{P}\right)$ is the given supply of real balance,

   $L = L(y, r) \ ; \ L_y > 0 \ , \ L_r < 0$ is the demand for real balance. Explain your answer.

   **[7+7+8] = [22]**

2. Suppose there exists an inflationary gap in an economy. Explain, using the I S - L M model, how the economy, left on its own, adjusts to restore equilibrium under this situation. What happens to aggregate planned investment in the course of this adjustment ?

Suppose future growth of the economy is also a consideration of the government. Then how should the government intervene to eliminate this inflationary gap ?

Suppose the government has two targets to attain viz. output target and investment target. Explain using I S - L M framework what policies should the government adopt to achieve these two targets.

[22]

3. (a) The government of a country facing perfect capital mobility in a flexible exchange rate regime imposes tariff on import, distributions the tariff revenue back to the public inform of tax rebates. Find the effects of the tariff on the level of income and exchange rate using Mundeld-Fleming model. (Make the small country assumption) .

(b) You are given the following information about an economy practicing fractional reserve banking.

Currency = 50 billion local currency

Demand deposits = 160 billion local currency

Bank reserves = 20 billion local currency

(i) Define and calculate the following :

Monetary base ;

Money supply $(M_1)$ ; and

Money multiplier.

(ii) Suppose the central bank gains international reserves worth 10 billion local currency. Assuming that central bank credit remains unchanged what will be its impact on monetary base and money supply ?

(c) Suppose the central bank wants to keep the money supply unchanged despite this reserve gain. In what offsetting operation could it engage ? Explain your answer.

[10+6+6] =[22]

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# INDIAN STATISTICAL INSTITUTE

## Second Semestral Examination:(2004-2005)

## B.STAT.(HONS.)II YEAR

## Physics-II

Date- 9.5.05    Maximum marks: 60    Duration- 3 Hours

### Answer any six questions.

1. Stationary state wave function associated with a particle is given by

$$\begin{aligned}
\psi(x) &= 0 \; ; \; x < -a \\
&= A(1 + \cos\tfrac{\pi x}{a}) \; ; \; -a \le x \le a \\
&= 0 \; ; \; x > a
\end{aligned}$$

where $A$ and $a$ are real constants.
a) Is this a physically acceptable wave function ? explain.
b) Find $A$ so that $\psi(x)$ is normalized.
c) Find the classically - allowed region.
d) Find the average value of momentum for this state.

$$2 + 2 + 2 + 4$$

2. A particle is moving freely inside a 1-D infinite potential well with walls at $x = 0$ and $x = l$.
a) Using Schrodinger's equation show that the particle can have only discrete energy values.
b) If the particle is initially in the ground state of the box and if suddenly the length of the box is doubled, then what is the probability of finding the particle in first excited state of this new well (if one tries to find it out there).

$$5 + 5$$

3. State and prove the generalized uncertainty relation. What particular shape will it take for position and momentum observables.

$$8 + 2$$

4. Construct Schrodinger's equation for a particle moving in the potential:

$$\begin{aligned}
V(x) &= 0 \; ; \; x < 0 \\
&= V_0 \; ; \; x \ge 0
\end{aligned}$$

with $V_0 > 0$. Find the wave functions for this potential.

10

1

P.T.O

5. Consider two qubits in the singlet state

$$|\psi^- \rangle = \frac{1}{\sqrt{2}}[|0\rangle |1\rangle - |1\rangle |0\rangle]$$

Consider the same unitary operator $U$ on both the systems. Show that $U \otimes U |\psi^-\rangle = |\psi^-\rangle$ up to a global phase. Show that if spins are measured along vector $n_1$ for particle 1 and along $n_2$ for particle 2, then the probability that both the results show spin up is given by $\frac{1}{4}(1 - n_1.n_2)$. Using this result calculate the expectation value of the joint spin observable given by $n_1.\sigma \otimes n_2.\sigma$.

$$3 + 5 + 2$$

6. Argue why by sending one free qubit one can not send more than 1 cbit of information. Show that by sending one qubit one can send 2 cbits of information if the qubit is entangled with another qubit held by the receiver.

$$2 + 8$$

7. Show that for a quantum mechanical spin system associated with 2-dimensional Hilbert space, all quantum mechanical probabilities for spin measurements can be reproduced by Hidden Variable Theory. Show that the following state of three qubits

$$|\psi_{123} \rangle = \frac{1}{\sqrt{2}}[|0\rangle |0\rangle |0\rangle - |1\rangle |1\rangle |1\rangle]$$

is an eigen state of the following four observables $\sigma_x \otimes \sigma_y \otimes \sigma_y$, $\sigma_y \otimes \sigma_x \otimes \sigma_y$, $\sigma_y \otimes \sigma_y \otimes \sigma_x$ and $\sigma_x \otimes \sigma_x \otimes \sigma_x$. Using this result show that this state does not admit any local Hidden Variable theory.

$$3 + 3 + 4$$

8. Describe the process by which one can send a unknown state of a qubit to a distant receiver without sending the qubit physically. Check that if the sender performs some measurement on his side but does not send any classical information then reduced density matrix of the receiver's side does not change. Show that in this process there is no violation of the no-cloning principle.

$$6 + 3 + 1$$

2

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1. Define onset and cessation of southwest monsoon? Draw a suitable rice calendar with the following data.    2+8

| Week No. | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rainfall (mm) at 0.5 Prob. | 5 | 0 | 15 | 10 | 9 | 37 | 18 | 35 | 18 | 20 | 98 | 142 | 95 | 80 | 15 | 32 | 2 | 0 | 0 |
| PET (mm) | 45 | 42 | 33 | 37 | 33 | 25 | 22 | 20 | 22 | 20 | 19 | 17 | 19 | 20 | 25 | 28 | 31 | 33 | 34 |

2. What are the differences between Manures and Fertilizers? Calculate the quantity of VC, Urea, Single super pohosphate and Muriate of potash required for 1 ha. rice crop to meet the nutrient requirement of 120kg N, 60 kg $P_2O_5$ and 60 kg $K_2O$ per hectare. 25% of required N should be given through VC.    3+7

3. Describe the suitable agrotechniques for rice nursery bed preparation. Estimate the expected yield of rice grain in t/ha from the following data.
   i) Spacing - 20x10cm ii) Average no. of tillers/hill –10 iii) Average no. of effective tillers/hill – 8   iv) Average no. of grain/panicle –160 v) Average no. of unfilled grain/panicle –20 vi) Test weight -24 g.    4+6

4. Write short notes on any five of the following:    2 x 5
   a) Moisture availability index
   b) Soil pH
   c) Reproductive stages in rice
   d) Bulk density of soil
   e) Transgenic rice
   f) Capillary water
   g) Intercropping and Mixed cropping

5. Write in brief about any two of the following:    2X5
   a) Soil regolith.
   b) Soil texture.
   c) Cation exchange capacity

6. a) Determine the values of $a$ and $b$ so that $(0, 0)$ is a sink of

$$\frac{dx}{dt} = ax - by$$

$$\frac{dy}{dt} = bx + 2y$$

   b) Write down the Lotka – Volterra model for predator-prey interaction. Find the dynamic behavior of the model around the biologically feasible equilibria.    3+2+5=10

INDIAN STATISTICAL INSTITUTE
B. Stat II: 2004 – 2005
Second Semester Examination
Economic Statistics and Official Statistics
Duration: 3 hours          Maximum marks: 100          Date: //. 5. 2005

*[Answer Group A and Group B in separate answer scripts. Answer question no. 1
and any **three** from the rest of the questions of Group A and all questions of
Group B. Allotted marks are given in brackets[] at the end of each question.]*

## Group A: Economic Statistics

1.  (a) Suppose monthly per capita expenditures (x) of households in urban India follow
    Pareto law with inequality parameter $v = 2.0$ and the threshold parameter $c =$ Rs. 10/-.
    Find
      (i)  the average monthly per capita expenditure of all households,
      (ii) the average monthly per capita expenditure of households spending Rs 50/-
           per capita per month.
    (b) If households spending Rs. x per capita per month spend $x^{0.8}$ rupees per capita per
    month on -a certain commodity then find (i) the average monthly per capita
    expenditures of households on the commodity. Also find (ii) the percentage increase
    on the consumption of the commodity if every household raises its per capita
    expenditure by 5%.                                                      [10+16=26]

2.  (a) What do you mean by concentration in business and industry? Write down the
    criteria for a good measure of concentration in business and industry. Show how these
    criteria are satisfied by (i) Herfindahl – Hirschman index and (ii) Hall and Tideman
    index.
    (b) How is it different from a measure of inequality?                    [15+3=18]

3.  Write down the important steps in deriving Atkinson's measures of inequality based
    on the Social Welfare Function Approach. How can one interpret the unknown
    parameter in the measure?                                               [15+3=18]

4.  State the principle of Diminishing Transfer of Income. Examine CV, LR and RMD in
    the light of the above principle.                                       [18]

5.  Define Index Number. Why is the choice of base period and commodities so
    important while constructing a Consumer Price Index Number? Write down the
    Fisher's tests for an index number formula. Show that Fisher's Ideal Index Number
    formula satisfies these tests. Define chain base index numbers. What are the merits
    and demerits of chain base index number compared to the fixed base index number?
                                                                           [2+4+2+4+2+4=18]

6.  Describe in detail the economic and statistical criteria for choosing an Engel Curve.
                                                                           [18]

## Group B: Official Statistics

1.  Write briefly the procedure adopted by NSSO in surveying informal sectors.   [8]
                                      **or**
    Indicate various source sectors for constituting National Accounts Statistics.   [8]
2.  What are the classifications of land used in agricultural statistics?    [5]
3.  Write a short note on Annual Survey of Industries.                       [7]

**4(c).** Three independent samples from a bivariate normal population yield the following :

| sample size: | 25 | 30 | 35 |
|---|---|---|---|
| sample corr. coeff. : | 0.65 | 0.59 | 0.63 |

Suggest a combined estimate for the population corr. coeff., using z-transformation.

**5(a).** Obtain an expression for the variance of the sample 100pth percentile in a random sample from a continuous population with density f(.).

**5(b).** Obtain an expression for the large sample variance of the sample median in a sample of odd size from a Cauchy population. Hence suggest a test for the parameter of Cauchy population based on the sample median.

**5(c).** Carry out the test in (b) when the hypothesized value of the parameter is 3.5, the sample size n equals 25 and sample median is 4.0.

**6(a).** Consider the usual multiple linear regression model with normal homoscedsatic errors. Show that the least square estimators of the regression coefficients are actually their best linear unbiased estimators.

**6(b).** Suppose $y$ has the distribution $N_p(0, \sigma^2 I)$. State and prove a necessary and sufficient condition for the quadratic form $y'Ay$ to have the chi-square distribution.

**7(a).** Define the frequency chi-square for testing goodness of fit and derive its sampling distribution for large samples.

**7(b).** A statistical package that generates random numbers is to be tested for its randomness. The package is used to generate 100 single digit integers between 0 and 9, and the frequencies of the integers are summarized below.

| Integer | : 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Observed Frequency | : 7 | 10 | 11 | 8 | 9 | 10 | 8 | 14 | 6 | 17 |

At 5% level of significance test whether there is any reason to suspect the unbisedness (or randomness) of the package.

**8.** Suppose we have data on two variables x and y from three cities A, B and C with $n_1$, $n_2$ and $n_3$ as the number of observations respectively. It is known that a linear regression of y on x is a good fit at the three cities and that the errors at A, B and C are of similar magnitude. We do not however know how these three lines compare to each other. In order to explore them we would like to test the following hypotheses following the suggestions of the subject matter specialists.

(i)    All the three lines are identical.

(ii)   Slopes of the lines at B and C are same.

(iii)  Intercepts of the lines at A, B and C are same.

Develop the ANOVA test statistics for the above three hypotheses, explaining clearly how the various sums of squares are to be computed.

## INDIAN STATISTICAL INSTITUTE
## 203 B. T. ROAD
## KOLKATA 700108

Semestral Examination     Stat. Methods IV     B. Stat. II / Semester II of 2004-05

Date : 13.5.05     Time : 3½ hours     Full Marks : 100

### Note : Answer any six questions. Each question carries 17 marks.

1.   Starting from the joint distribution (no derivation required) of sample sum of squares and products in a random sample from bivariate normal distribution, indicate the steps involved to derive the sampling distribution of the sample correlation coefficient. Hence, or otherwise, deduce the form of the density of the same in the null case.

2.(a). Show that the sampling distribution of the sample correlation coefficient in samples from a bivariate normal population depends only on the population correlation coefficient and on no other parameters.

2(b). The joint score distribution in Mathematics & Statistics is a bivariate normal with known means but all other parameters unknown. In a random sample of size 30, it is found that only for 10 paired observations, the x-value exceeds its mean and corresponding y-value exceeds its mean. From this, suggest an estimate of $\rho$, the population correlation coeff.
Also using large sample approximation (no derivation of the result required), test the null hypothesis that $\rho$ is equal to 0.35.

3.   X and Y are independent normal variates with the same unknown mean and unequal unknown variances. You are given a random sample of size m from X-population and another sample of size n from Y-population.

(a) Obtain an expression for P = Prob. [ $|X - Y| < d$], d > 0 (given).
(b) Suggest a plug-in estimate for P based on the available data.
(c) Obtain a large sample approximation for the estimate of P proposed in (b) above.
(d) Using (b) and (c), carry out a test for the null hypothesis that P is equal to 0.95, given d = 3 and sample variances are 0.85 (m=50) and 1.12 (n=70) respectively for X and Y.

4(a). Obtain a general expression for the variance-stabilizing transformation of a statistic T whose mean is $\theta$ and variance is $g(\theta)$.

4(b). Deduce the z-transformation of the sample correlation coefficient.

*P.T.O*

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## INDIAN STATISTICAL INSTITUTE

Semestral II Examination : 2004-05

B.Stat II Year : 2004-05

Group A : Demography

Date : 22.7.05     Maximum Marks 50     Duration : 1 hr. 30 min.

Note: Symbols and notations have their usual meanings

1. a) What is vital registration system ?
    b) Give an account of Chandrasekaran and Deming's method for completeness of data from the vital registration system.
    c) Describe Zelnik's method for smoothing age data by considering 6-year age interval.

[ 2+7+6= 15 ]

OR

    d) Describe the different columns of a life table.
    e) Explain why the mortality situation of two places can not usually be compared. Describe in this connection, the construction and usefulness of standardized death rates.

[ 6+4+5=15 ]

2. a) Define census measures of fertility and its merits and demerits.
    b) Differentiate between gross and net reproduction rate.
    c) Discuss some standardised measures of fertility.

[ 5+6+4=15 ]

OR

    d) Differentiate between neonatal mortality rate and perinatal mortality rate.
    e) Differentiate between comparative mortality figure and comparative mortality index.
    f) Define replacement index. Why is it considered better than net reproduction rate?

[ 5+5+5=15 ]

*turn over*

1

3.
a) Derive the formula

$$\int_0^x v(t)\,dt = -\log S(x) \ ,$$

where $v(x)$ is the force of nuptiality at age $x$ and $S(x)$ is the proportion of persons single at age $x$.

b) Show that

$$_nNM_x^r = \frac{1}{_nS_x}\,_nNM_x^f \ ,$$

where symbols have their usual meanings.

c) Prove that

$$q_x = \frac{2m_x}{2 + m_x}$$

$$\left[8+7+5=20\right]$$

OR

d) For a certain life table :

$$l_x = 20000 - 80x - x^2,$$

(i)    What is the ultimate age in life table?
(ii)   Find $l_x$ and $q_x$
(iii)  Find $_{10}P_{20}$.

e) Given

$$\mu_x = \frac{1}{(a_0 + a_1 x)(b_0 + b_1 x)}$$

Find an expression for $l_x$.

$$\left[4+(3+3)+4+6=20\right]$$

## Group B : SQC & OR

Maximum Marks : 50        Duration : $1\frac{1}{2}$ Hours

Answer Group B in a separate answer-script.

The specifications on the pressure set-point on an our compressor air regulator is $100 \pm 3$ psi. The most recent $\overline{X}$ - R chart for the regulators is running at $\overline{X} = 99$ with UCL = 99.75 and LCL = 98.25. A subgroup size of $n = 4$ is used.

Assuming that the process has a normal distribution with mean $\mu = 99$ and process standard deviation $\sigma = 0.25$, what is the probability that two out of three consecutive points fall beyond 2 sigma limits ?

What is the probability that an increase in the process main by $2\sigma$ is detected by the second sample ?

What is the average number of samples needed until this shift is detected ?

$$[5+5+5=15]$$

Suppose that a single sampling acceptance rectification plan $n = 150$ $c = 1$ is being used for receiving inspection where the vendor ships the products in lots of size $N = 3000$. Find the AOQL for this plan.

Describe the operation of a double sampling plan.

$$[10+5=15]$$

A bank is in the process of formulating its loan policy for the next quarter. A total of Rs. 12 crores is allocated for that purpose. The following table provides the types of loan, the interest rate charged by the bank and the possibility of bad debts as estimated from past experience :

| Type of Loan | Interest Rate | Probability of Bad Debt |
|---|---|---|
| Personal | 0.140 | 0.10 |
| Car | 0.130 | 0.07 |
| Housing | 0.120 | 0.03 |
| Farm | 0.125 | 0.05 |
| Commercial | 0.100 | 0.02 |

Bad debts are assumed unrecoverable and hence produce no interest revenue.

Competition with other financial institutions in the area requires that the bank allocate at least 40% of the total funds to farm and commercial loans ; home loans must equal at least 50% of the personal, car and home loans. The bank has a stated policy specifying that the overall ratio for bad debts on all loans must not exceed 0.04.

Formulate the bank's objective of maximizing its net return as an LPP.

b) Maximize    $X_0 = 2x_1 + x_2$
st.       $x_1 + x_2 \leq 5$
          $-x_1 + x_2 \leq 0$
          $6x_1 + 2x_2 \leq 21$
          $x_1, x_2 \geq 0$

# INDIAN STATISTICAL INSTITUTE
## 203 B. T. ROAD
## KOLKATA 700108

Backpaper Examination     Stat. Methods IV     B. Stat. II / Semester II of 2004-05

Date : 22.7.05     Time : 3 hours     Full Marks : 100

**Note : Answer all the five questions. Each question carries 20 marks.**

1(a). In a random sample from a bivariate normal distribution, show that the sample means are distributed independently of the sample variances and covariance.

1(b). Indicate the density of the difference of the sample means.

1(c). Suggest a test for testing the difference between the two population means in case all the five parameters are unknown.

2. In a random sample from an exponential population, obtain the distribution of the sample range and derive an explicit expression for its mean in terms of the population mean and the sample size.

3(a). What is a variance-stabilizing transformation? Indicate its use in statistical inference and illustrate the same with an example.

3(b). Obtain an expression for the large sample efficiency of the sample median [as against the sample mean] for estimating the mean of a normal population with unknown variance.

4. A substance used in biological and medical research is shipped by airfreight to users in cartons of 1000 ampoules. The data below, involving 10 shipments, were collected on the number of times the carton was transferred from one aircraft to another over shipment route (x) and the number of ampoules broken upon arrival (y). Assume that a linear regression model is appropriate here.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|----|
| $x_i$ | 1 | 0 | 2 | 0 | 3 | 1 | 0 | 1 | 2 | 0 |
| $y_i$ | 16 | 9 | 17 | 12 | 22 | 13 | 8 | 15 | 19 | 11 |

(a) Obtain the estimated regression function. Give a measure of goodness of fit, test its significance, and comment.

(b) Estimate the slope parameter with a 95% confidence interval and interpret the interval estimate.

(c) Conduct an ANOVA test to see whether or not there is a significant linear association between x and y. Use $\alpha = 0.05$. What is the P – value of this test?

(d) A consultant has suggested, based on previous experience, that the mean number of broken ampoules should not exceed 9 when no transfers are made. Conduct an appropriate test on the claim using $\alpha$ = 0.025. Draw conclusions, and give the P – value of the test.

5(a). Consider the usual multiple linear regression model with normal homoscedsatic errors : $y = X\beta + e$ etc. Prove that $a'y$ is the best linear unbiased estimator of its expectation if and only if $a$ lies in the column space of X.

5(b). Give an unbiased estimator of $\sigma^2$, and obtain its sampling distribution.

5(c). Develop the analysis of variance for testing the adequacy of the model fitted at 5(a) above.

Answer all questions.

1. Explain income determination in a Simple Keynesian Model. Show the equilibrium in a diagram using the Keynesian cross as well as the saving-investment schedule, taking investment to be an increasing function of income.

Derive and interpret the autonomous expenditure multiplier. Given other things the same, compare the value of the multiplier in two cases (i) closed economy (ii) open economy . Explain your answer.

[25]

2. Discuss the income determination in an IS - LM model. Explain the impact of an increase in (i) government expenditure and (ii) money supply, on equilibrium output and interest rate. In each case, show the path of the system from initial to new equilibrium in a diagram, making suitable assumptions about the adjustment of y and r.

If the interest sensitivity of money demand is not significant enough then show that monetary policy will be more effective than fiscal policy in eliminating deflationary gap in the system.

[25]

3. Consider a small open economy facing perfect capital mobility under a flexible exchange rate regime. Discuss the effectiveness of fiscal and monetary policies in this set up using the Mundell-Fleming model.

[25]

4. Explain in detail the precise mechanism involved in the process by which a balance of payment surplus, surfaces as a money supply increase in an economy under fixed exchange rate.

[25]

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