INDIAN STATISTICAL INSTITUTE
FINAL EXAMINATION: SEMESTER II (2001-2002)
B.STAT (Hons.) III Year
ANTHROPOLOGY

Date: 26 April 2002   Full marks 100   Time 3 Hours

Note: Use separate answerscripts for GROUP A and GROUP B. Answer any **five** questions from each group. Answers should be *brief and precise*.

GROUP A

1(a) Why is man unique in the animal kingdom?   10

(b) Classify 'primate' at *family* and *genus* levels with examples

2. Describe the changes that took place in the anatomical characteristics of man due to the assumption of erect posture?   10

3. What are the bases of Darwin's Theory of evolution? Mention the limitations of this theory.   10

4. Mention the subfields of 'biological' and 'social/cultural' anthropology separately.   10

5. Discuss the theory of demographic transition.   10

6. Discuss the role of social factors in genetic and environmental variations.   10

7. Define with examples 'Age-specific fertility rate' and 'Completed fertility rate'   10

GROUP B

1. Define cell. Write briefly the concepts of cell theory. Describe the functions of Endoplasmic reticulum and Golgi complex.   2+4+4

2. What is karyotype? State the bases on which the classification of human chromosomes has been made. How many chromosomes in each group are normally found in a diploid set of human cell?   2+4+4

3. What are the techniques in human biology to identify the factors that determine sex in humans. Why does a Down's syndrome develop?   5+5

4. Describe Mendel's law of segregation. At the ABO blood-group locus, if the genotypes of the parents are $I^A I^O$ and $I^A I^A$, what is chance of their two children both being (i) $I^A I^A$, (ii) $I^A I^O$ and (iii) $I^O I^O$?   4+2+2+2

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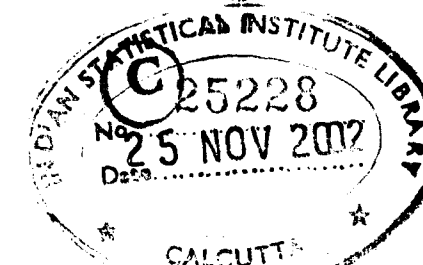
5. Describe the criteria of inheritance due to a single, completely dominant, rare autosomal gene. Illustrate with suitable examples. 5+5

6. State the Hardy-Weinberg principle. Suppose the estimated frequencies of alleles A, B and O in random sample of 1000 individuals from a population are 0.21, 0.34 and 0.45 respectively. If marriages take place at random in this population, what are the expected frequencies of the four blood groups in this population? 5+5

7. Write short notes on any *two* of the following: 5+5
   (a) Polygenic inheritance
   (b) Nature and nurture problem
   (c) Law of independent assortment
   (d) Significance of meiosis

1. (a) Let N be a Poisson random variable with expectation λ. Consider N independent (finite state space) Markov chains starting at the same state x_0 and having the same transition probability matrix P. Let $Z_k^{(n)}$ be the number among them after n steps that are found in state x_k. Find the distribution of $Z_k^{(n)}$.

(b) Let $\pi = (\pi_0, \pi_1, \cdots, \pi_m)$ be a stationary distribution for the Markov chain(s). For each j consider independent Poisson random variables N_j with mean π_j. Now suppose there are N_j independent Markov chains starting from $x_j, j = 0, 1, 2, \cdots, m$. Find the distribution of $Z_k^{(n)}$ (defined as the total number in the state x_k at the n-th stage coming from all the $N_0 + N_1 + \cdots + N_m$ Markov chains) in this set up. $7 + 7 = 14$ pts.

**2.** (a) Consider a Markov chain with state space $\{0, 1, \cdots, 5\}$ and the following transition matrix: 7 pts.

	0	1	2	3	4	5
0	1/2	1/2	0	0	0	0
1	1/3	2/3	0	0	0	0
2	0	0	1/8	0	7/8	0
3	1/4	1/4	0	0	1/4	1/4
4	0	0	3/4	0	1/4	0
5	0	1/5	0	1/5	1/5	2/5

Find $\lim_{n \to \infty} G_n(x, y)/n$ for $(x, y) = (0, 2), (2, 4), (3, 2)$.

(b) A fair coin is tossed repeatedly. From the observations $x_0 x_1 x_2 \cdots$, words of length 2 are read as $x_0 x_1, x_1 x_2, \cdots$. If the first two observations are HH, find the expected time to reach the word TH.

7 pts.

3. Of the two independent copies of an equipment, each acts as follows: the time to breakdown for each has distribution $\text{Exp}(\lambda)$, the time to repair for each has distribution $\text{Exp}(\mu)$. At $t = 0$ both the components are in working condition. Let X_t be the number of components in working condition at time t. Thus $X_0 = 2$ and let $P_i(t) = P(X_t = i), i = 0, 1, 2$. 5+4+5=14 pts.
(a) Find the differential equations satisfied by $P_i(t), i = 0, 1, 2$.
(b) Find the solutions to these equations.

1

P.T.O
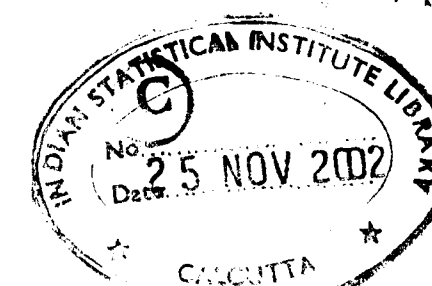
(c) Show that as $t \to \infty$ the solutions converge. Denote the limit by $p_i, i = 0, 1, 2$. Show that these p_i satisfy the equations obtained from (a) by setting the derivatives to 0 and replacing $P_i(t)$ by p_i.

4. Consider a birth and death process with $\lambda_n = \lambda$ and $\mu_n = n\mu$.

$$5+4+5 = 14 \text{ pts.}$$

(a) Write down the differential equations for $P_0(t), P_1(t), \cdots, P_n(t), \cdots$.

(b) Show that the generating function $P(s,t) = \sum_n P_n(t)s^n$ satisfies

$$\frac{\partial P}{\partial t} = (1-s)\{-\lambda P + \mu \frac{\partial P}{\partial s}\}.$$

(c) Verify that if $P_0(0) = 1$,

$$P(s,t) = e^{-\lambda(1-s)(1-e^{-\mu t})/\mu}$$

is a solution. Identify this distribution for fixed t and find the limiting distribution as $t \to \infty$.

5. Buses arrive in accordance with a Poisson process, the expected time between consecutive buses being $1/\alpha$. A passanger arrives at time t. His waiting time till the next bus is W_t. Find $P(W_t \leq x)$ for $x > 0$. (You may use the fact that the sum of n i.i.d. $\text{Exp}(\alpha)$ random variables has density

$$\frac{\alpha(\alpha x)^{n-1}}{(n-1)!}e^{-\alpha x}.)$$

8 pts.

2

Answer any five questions. All questions carry equal marks.

1. (a) State clearly the assumptions of the classical linear regression model. Discuss the relevance of each of the assumptions by mentioning briefly the nature of the problem(s) that arises in the least squares estimation of the parameters of the model when the assumption is violated.
   (b) Assuming that the disturbances are normally distributed, obtain the least squares and maximum likelihood estimators of the variance of the disturbance term in the linear regression model.

2. Describe how you would obtain least squares estimator of the regression coefficients in the classical linear regression model with exact linear restrictions on the regression coefficients. Examine if there is any gain in efficiency in taking into account the restriction compared to that of ignoring it. How would your results change when you are given unbiased estimates of some of the regression coefficients and you are estimating the other coefficients utilizing the extraneous information.

3. (a) What do you understand by the problem of heteroscedasticity in a linear regression relationship? Discuss, at least, two tests for detecting the presence of heteroscedasticity.
   (b) Show that the least squares estimator of the slope coefficient in the linear regression of Y on X from grouped observations is always less efficient than the corresponding estimator from ungrouped data.

4. What explanations are usually given for the existence of autocorrelation in the disturbances of a linear regression model? Discuss alternative methods of estimating the regression coefficients in a linear regression model when the disturbances follow a first-order autoregressive process with an unknown autocorrelation coefficient. What can you say about the properties of these estimators in   (i) small samples and (ii) large samples?

5. "Autocorrelated disturbances without lagged Y values do not produce biased estimators, even in small samples; lagged Y values with random disturbances will give ordinary least square (OLS) estimators which are consistent though biased in finite samples; the combination of the two problems, however, throws OLS off-beam and gives inconsistent estimators."
   Examine the above statement in connection with OLS estimation of the regression coefficients in a linear regression equation of Y_t on X_t and Y_{t-1}. Also, describe in this context the modification of the conventional Durbin-Watson test when lagged values of Y appear as explanatory variable(s) in the relationship.

6. Give an account of the errors-in-variables model. Show that the least squares estimator of the slope parameter in a two-variable linear regression model will be an underestimate no matter how large a sample is available. Discuss in this context the Instrumental Variables (IV) method of estimation and describe some of the instrumental variables that have been suggested for the errors-in-variable model.

Date : 03.12.2001   Maximum Marks : 100   Duration : 3 Hours

Answer Question No.6 and ANY THREE questions from the rest . Marks allotted to each question are given within the parentheses. Standard notations and symbols are used .

1.From an SRSWOR sample of n units a random sub-sample of m units are duplicated and added to the original sample . Show that the mean based on (n+m) units is an unbiased estimator of the corresponding population mean and its variance is greater than the variance of the mean based on n units by the approximate factor

$$F = \frac{n(n+3m)}{(n+m)^2}$$

(10+15) = [25]

2. (a) Find the bias in $\hat{Y}_R = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{x_i} .X$ as an estimator of the population total Y of the study variable y under SRSWOR and also find an unbiased estimator of Y utilizing the information on the auxiliary variable where X is the population total of the auxiliary variable x and (x_1, y_1) , ... , (x_N, y_N) are the values of the auxiliary variable and the variable under study .

(b) Explain why it is not generally possible to estimate unbiasedly the sampling variance of the estimated mean based on a single systematic sample . What do you mean by an interpenetrating network of sub-samples ? Explain how this technique can be utilized in estimating unbiasedly the sampling variance of the estimated mean in case of circular systematic sampling .

(13+12) = [25]

3. Describe the Rao-Hartley-Cochran's (RHC) sampling strategy for estimating the population total of a finite population of size N . Derive an expression for the variance of the RHC estimator and also find an expression for an unbiased estimator of the variance . Obtain the optimal choices of the sizes of the random groups to be formed under the RHC scheme by considering both the situation when N is a multiple of the sample size n and when it is not so . In each case compare the minimum value of the variance of the RHC estimator with that of the Hansen-Hurwitz estimator based on PPSWR sampling involving n draws .

(5+10+5+5) = [25}

4. For a given sampling design , let π_i^c and π_{ij}^c denote respectively the probability that a unit i and a pair of units i and j will not be included in a sample .
(a) Show that for any fixed effective-size (n) design

(i) $\sum_{i=1}^{N} \pi_i^c = N-n$ P.T.O.

(ii) $\quad \sum_{j=1\neq i}^{N} \pi_{ij}^c = (N-n-1)\ \pi_i^c$

(iii) $\quad \sum_{i\neq j=1}^{N}\sum \pi_{ij}^c = (N-n)(N-n-1)$ where N is the population size .

(b) For Lahiri-Midzuno-Sen sampling scheme , prove that

$$\pi_i^c\ \pi_j^c - \pi_{ij}^c \geq 0\ \forall\ 1\leq i\neq j\leq N\ .$$

$$(5+5+5+10) = [25]$$

5. (a) What do you mean by a πps scheme ? Describe Brewer's πps sampling scheme to select a sample of n units . Calculate the first order inclusion probability π_i for this scheme and justify that it is really a πps scheme .

(b) Show that the Horvitz-Thompson estimator based on Brewer's πps sampling scheme is more efficient than the Hansen-Hurwitz estimator based on PPSWR sampling scheme involving the same number of draws.

$$(10+15) = [25]$$

6. In a demographic survey , it is proposed to have stratified sampling using the districts in a region as strata . The relevant data are given in the following table .

District Sl. No.	No. of Villages	Population per village	Standard deviation
1.	1953	487	564
2.	1664	829	931
3.	1381	822	996
4.	1174	1083	1167
5.	531	1956	1940
6.	1391	664	625
7.	1996	456	779
8.	1951	372	556
9.	3369	339	591

(a) Assuming the cost of enumeration and tabulation per person is $\frac{1}{4}$ th of a rupee and the overhead cost to be Rs.10,000 , determine the optimum values of n_h's that would minimize the sampling variance of the estimator of the overall population mean for a given expected total cost of Rs .80,000 when villages are selected with SRSWR from each stratum .

(b) For the same value of the total sample size n obtained in (a) find the values of n_h's when the allocation is made in proportion to $N_h\sigma_h$ and obtain the cost-efficiency of the procedure as compared to that of (a) .

$$(10+15) = [25]$$

Date : 05.12.2001　　　　Maximum Marks : 100　　　　Time : 4 hrs.

This paper carries 125 marks. Answer all questions. Maximum you can score is 100. Precisely justify all your steps.

1. Find the orthogonal trajectories of the family of all circles through the origin with centres on the x-axis. [10]

2. Find the general solution of

$$y'' + 4y = \sin x$$

by the annihilator method. [10]

3. (a) Find the general solution of the equation

$$(x^2 + x)y'' + (2 - x^2)y' - (2 + x)y = x(x + 1)^2$$

by rewriting the left hand expression as $x^2(y'' - y') + x(y'' - y) + 2(y' - y)$, and consecutively solving two first order equations.

   (b) Using the solution of (a) above (or otherwise) get two linearly independent solutions of the homogeneous equation in (a) and find the general solution of the equation by the method of variation of parameter.

$$[20 + 10 \text{ (or } 10 + 20) = 30]$$

4. (a) By making a change of variable $t = 1/x$, show that $x = \infty$ is a regular singular point of the Legendre's equation

$$(1 - x^2)y'' - 2xy' + p(p + 1)y = 0.$$

   What are the indices? [10]

   (b) Find Frobenius series solution(s) of the transformed equation above for $p \geq 0$. [20]

P. T. O

1. Write all the following expressions choosing the most appropriate terms provided in parenthesis ---10X2=20
a. The science that deals with the study of the succession of rocks is known as [lithostratigraphy/ biostratigraphy/ chronosrtatigraphy].
b. Fossils generally provide the [absolute age/ relative age/ radiometric age] of the host rock.
c. According to the theory of organic evolution proposed by Charles E. Darwin, the organic world started with primitive forms and through geologic time ultimately becomes [complex/ more simple/ extraordinary simple].
d. The above-mentioned change of the organic world, according to Darwin, was [sudden/ stochastic/ gradual].
e. In cladistics the [shared derived character states/ primitive character states/ all possible character states] are considered to build the phylogeny.
f. The [sphericity/ roundness/ size] of a detrital grains indicates how angular its corners are.
g. [Quartz/ Calcite/ Muscovite] has three sets of cleavages.
h. The majority of the rock-forming minerals are [borates/ molybdates/ silicates].
i. P-waves can pass through [solid/ liquid/ gas].
j. We get an idea about the core of the earth from the [moon/ sun/ meteorites].

2. What is a mineral? Name four rock forming minerals and the mineral group in which they belong. 3X5=15

3. What is a fossil? State the conditions required for fossilization ---10
OR
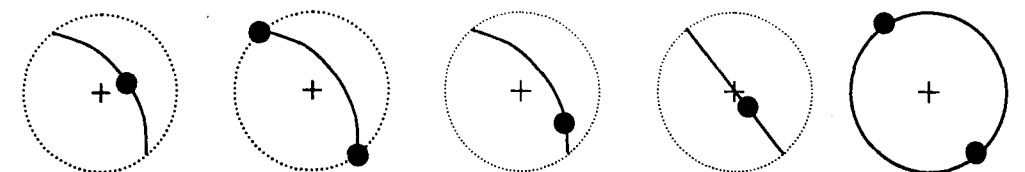What is lava? How do lavas form in the Earth? Do lavas form in the Moon? Justify your answer. ---10

4. Why do earthquakes occur? Does it occur in the Moon? Can you suggest any way of predicting earthquake? ---15

5. Write short notes (with illustrations) on the primary sedimentary structures. ---15

6. Name the major agents that can transport clastic sediments and describe the characteristics of the deposits formed in each case. ---15

7. a. Describe and illustrate different types of faults. ---5
b. Name the folds depicted in the following setereograms showing the axial planes (soild curves) and fold axes (dots): ---5

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5. (a) The Legendre's polynomial of degree n is given by the *Rodrigue's formula*

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n.$$

Show that P_n satisfies the differential equation

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$

[Hint : Let $u = (x^2 - 1)^n$. Verify directly that for any $k \geq 1$,

$$(x^2 - 1)u^{(k+2)} - 2(n - k - 1)xu^{(k+1)} - (2n - k)(k + 1)u^{(k)} = 0]$$ [10]

(b) Prove that the Legendre's polynomials $\{P_n\}$ form a sequence of orthogonal functions on $[-1, 1]$. [10]

(c) Evaluate $\displaystyle\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx$. [10]

6. Solve the original isoperimetric problem in polar coordinates. That is, choose the origin to be a point on the curve and the polar axis to be the tangent line at that point; then maximize

$$\frac{1}{2}\int_0^\pi r^2 d\theta$$

with the side condition that

$$\int_0^\pi \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$$

must be constant. [15]

[Hint : The curvature of a polar curve is given by $\dfrac{|2r'^2 - rr'' + r^2|}{(r'^2 + r^2)^{3/2}}$.]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2001-2002)
B-Stat III
Economics III

te : 7.12.01

Answer any five questions   Full Marks: 100

All questions carry equal marks   Time: $3\frac{1}{2}$ Hours

1. Discuss the two existing views that seek to explain the severe BOP crisis that gripped Indian economy in the late eighties.

2. Critically evaluate the new trade and exchange rate policies of the Government of India.

3. Is there any justification for resorting to a system like SLR for financing infrastructural investment in an economy like India?

4. Discuss the problem of sustainability of public debt in the context of the ongoing recession in India.

5. Why is monetised deficit considered bad? To what extent Government's obsession with the monetised deficit is justified in the context of the current economic situation in India?

6. Discuss the genesis of the currency crisis of the ASEAN economies with special emphasis on the role played by capital account convertibility in triggering off the crisis. What lessons can India learn from their experience?

7. Why was WTO formed? Discuss the implications of the inclusion of new issues under WTO for developing countries like India.