Essays in Industrial Organization

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Abstract

This thesis contains three chapters on industrial organization, covering two distinct topics. Chapters 1 and 2 examine the impact of firm owner's corruption on the organization, while Chapter 3 analyzes the market dynamics of a two-sided newspaper market.

The first chapter investigate the agency conflict that arises when the principal (the owner) of a firm is involved in corruption. A corruptible principal has an incentive to conceal his or her illegal activities, while the agent (CEO or manager), due to their information advantage, is in a position to monitor this corruption, thus creating an agency conflict. We show that such corruption leads to increased bureaucracy within the firm as the principal reduces information flow, provides lower incentive wages, and limits delegation to the manager. Furthermore, we analyze additional inefficiencies caused by such corruption, including the principal's incentive to distort talent by hiring a corruptible manager and to expropriate from minority shareholders.

The second chapter investigates a screening mechanism through which a corruptible principal screens a manager when the manager's type (honest or corruptible) is private information at the time of hiring, and the corruptible manager can misappropriate funds from the firm. Our results show that if the potential for misappropriation by the manager is within an intermediate range, the principal can design a wage contract that ensures only a corrupt manager joins. This range increases if the reservation wage rises. We also demonstrate that the principal can completely offset the cost of the manager's misappropriation through a suitable wage contract, if these costs are not critically high.

The third chapter extends the vertical differential framework by incorporating the advertisement side to analyze the two-sided newspaper market. Newspaper markets are highly concentrated, with most being monopolies or duopolies within a service area. Existing literature attributes this market concentration to the network effect, which arises because newspaper readers derive positive utility from advertisements, especially classifieds. We demonstrate that newspaper markets can also be concentrated due to endogenous investment in quality, particularly when quality improvements involve fixed costs like newsroom size. This reason more closely aligns with empirical evidence, which shows that market concentration persists even when classified ad revenues declined significantly due to online platforms like Craigslist. We also show that several different types of market and product configurations emerge depending on advertisement levels.

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Nilesh Jain

To my teachers and parents.

Contents

Introduction

This thesis presents three essays on industrial organization, covering two distinct topics. Chapters 1 and 2 use the principal-agent model to explore the downstream impact of a firm owner's corruption on the firm's governance processes, managerial incentives, talent hiring, and overall performance. Chapter 3 analyzes the market dynamics of a two-sided newspaper market, examining how market configurations and product quality choices are determined by the advertisement levels.

Motivation

The existing corruption literature has established that firms often engage in corruption through collusion between the firm owner and bureaucrats or politicians (Agrawal and Knoeber, 2001; Fisman, 2001; Faccio, 2006). Such corruption can benefit firms by providing preferential access to scarce resources like financing and public resources (Li et al., 2008; Claessens et al., 2008; Khwaja and Mian, 2005). However, it also negatively impacts firm performance by:

- Increasing bureaucratic interference (Fisman and Svensson, 2007; Gaviria, 2002)
- Encouraging rent-seeking behavior by executives (Shleifer and Vishny, 1994)
- Distorting investments (Svensson, 2003), product choice (De Soto, 1989b), and governance practices (La rocca and Neha, 2017)

Despite these well-documented effects, the agency model examining the impact of an owner's corruption on firm governance remains under-explored. Corruption in agency theory model is commonly viewed as collusion between the supervisor and the agent it monitor;[1] however, the principal, who designs an optimal compensation contract to

[1] This supervisor could be an auditor colluding with management, a regulator colluding with firms, or tax inspector colluding with citizens.

reduce or alleviate such collusion is benevolent(Tirole, 1986; Laffont and Tirole, 1991; Mookherjee and Png, 1995).

Chapter 1 addresses this gap by developing a principal-agent model that analyzes how a firm owner's corruption affects firm's governance processes, such as incentive wages, delegation, transparency, and information flow. More specifically, this chapter studies the the agency conflict that occurs between a corruptible principal (or firm owner) and the manager (or CEO) whom the principal hires to run the firm. In our model, the principal benefits from corruption, while the manager indirectly monitors it. This creates a conflict because the manager's information about corruption poses a risk to the principal. This chapter also evaluates the impact of the principal's corruption on corporate governance, highlighting the resulting agency conflict between the principal (or promoter) and minority shareholders. Lastly, the chapter demonstrates that a principal with corruption motives benefits from hiring a corruptible manager who can collude with him to maintain secrecy and share the benefits, thereby distorting the firm's talent strategy. However, the principal's ability to efficiently screen for a corruptible manager is crucial in this context. Chapter 2 expands on this by discussing the screening mechanisms that the principal can use to design a wage contract that reveals the manager's type for collusion.

The second topic, discussed in Chapter 3, examines the newspaper market dynamics. Newspaper markets in OECD countries are highly concentrated, with most local markets being monopolies (Rosse, 1980; Dertouzos and Trautman, 1990). For example, 95% of U.S. cities have only one daily newspaper. In larger cities with two or more papers, these papers differ in format (tabloid vs. broadsheet) or political alignment (left- or right-leaning editorials). This pattern is also observed in developing countries. For example, metropolitan cities in India typically have one dominant English daily newspaper commanding over 60% market share.[2] Such market power for a leading firm is unique to print media and not observed in other types of media.

Extensive literature has sought to explain this concentration, particularly the prevalence of "one-newspaper cities." Most studies attribute this to network externality effects, which occurs when consumers derive positive utility from advertising (Furhoff, 1973; Bucklin et al., 1989; Gabszewicz et al., 2007; Chaudhri, 1998). Firms with greater circulation attract more advertising, which in turn attracts more readers, creating a positive feedback loop that can lead to a monopoly. Unlike radio and television, where consumers view ads as a nuisance, print media consumers might appreciate ads, such as classifieds. Empirical studies (Rosse, 1970; Dertouzos and Trautman, 1990; Thompson, 1989) support the view that consumers appreciate advertisements.

However, the theory relying solely on network effects fails to explain why such concen-

[2]Hindustan Times in New Delhi, Times of India in Mumbai and Bangalore, The Hindu in Chennai, and Deccan Chronicle in Hyderabad.

tration persists despite the significant decline in classified ads with the advent of Craigslist and Monster.com. Moreover, some studies suggest that readers' attitudes towards ads in newspapers differ across countries and regions (Sonnac, 2000; Gabszewicz et al., 2002). An alternative theory about endogenous investment in quality gains credence from empirical evidence provided by Berry and Waldfogel (2010). Using data from US metropolitan dailies, they showed that as market size increases, the number of newspapers changes relatively little, but the nature and quality of newspapers change dramatically. They found that as market size grows, newspapers invest in journalistic content (or quality), and this investment is fixed because it depends on the number of investigative reporters and journalists in the newsroom and the quality of staff, such as the number of Pulitzer awards. This corroborates the argument of Shaked and Sutton (1987) that if consumers have a higher willingness to pay for quality, as in a *vertically differentiated market*, and the burden of quality falls on fixed costs, at least one firm will have an incentive to invest in quality, undercut rivals, and attain significant market share. Angelucci and Cage (2019) provided additional evidence using the French dailies market, showing that when advertisement revenue declines, newspapers produce less journalistic-intensive content (or quality).

These findings clearly underscore the crucial role of quality in the newspaper market. While many papers discuss quality choices of players and the nature of competitions in vertically differentiated market, few incorporate the advertisement side to such market, which is vital for analyzing the two-sided newspaper market. Gabszewicz et al. (2012) is one such study that uses a vertical differentiation model to show the interaction between quality and advertisements, but their focus is mainly on explaining the rise of free daily newspapers. Chapter 3 adopts an approach similar to Gabszewicz et al. (2012) but under less restrictive assumptions and with broader perspectives to examine how product quality choices, consumer surplus, competitive dynamics, and market structure of newspaper market are determined by advertisement levels.

Chapter 1: Corruptible Principal: Owner corruption makes firm bureacratic

This chapter models the agency conflict that occurs between a corruptible firm owner (principal) and the CEO (or manager) whom the principal hire to run its business.

Brief overview of the agency problem and the principal's trade-off

The principal is corruptible and can engage in illegal activities in collusion with government officials to obtain private benefits. While the principal has perfect knowledge of when corruption is possible, he relies on the manager to exert costly but non-verifiable effort to acquire business signals that help the principal decide which project to execute. The project yields verifiable returns only when it is successful, which depends on the informativeness of the manager's business signal in conveying the correct state of the world. Each project can be implemented with or without corruption. If a corrupt project succeeds, the principal also gains private benefits. The principal compensates the manager with an incentive wage based on project success, and the manager is protected by limited liability.

The manager's information advantage also provides her with signals related to the principal's corruption opportunities, which can be leaked, posing a risk to the principal. This may occur because such signals can provide verifiable information that may trigger investigations by the media and/or regulatory authorities. The more effort the manager puts in, the more informative are both business and corruption signals. Therefore, higher effort increases the firm's profitability due to appropriate project selection, benefiting the principal, but it also poses a higher risk to the principal, thus creating an agency conflict. In this sense, the manager is indirectly monitoring the principal's corruption, which we refer to as *indirect monitoring*. This notion of employees monitoring executives' corruption has been documented and is consistent with empirical evidence (Dyck et al., 2010; Stiglitz, 1985; Dyck and Zingales, 2004).

Given the manager's access to corruption-related information, the corruptible principal faces a trade-off in organization design. He can either continue to give the manager the power to make informed decisions, choosing a *transparent (or no-signal-blocking) regime*, which risks exposing corruption, or opt for a *non-transparent (or signal-blocking) regime* that blocks the manager's access to information about corrupt projects, by putting firewalls around corrupt projects. If the manager does not receive the signal, projects are chosen randomly, reducing the probability of project success and consequently the firm's profitability, but lowering the principal's risk of exposure.

Key Results

Our results show that in a low corruption environment,[3] the principal chooses a transparent regime and may even forgo corruption opportunities if the manager obtains information about corruption. However, he reduces the manager's incentive wage, causing the manager to exert less effort, which makes the manager's signal about corruption less informative. The higher the potential benefit from corruption, the lower the incentive wage. In other words, the manager's indirect monitoring reduces corruption, but the manager receives a lower incentive wage, leading to less effort and a reduction in the

[3]The institutional corruption environment is determined by two factors: the prevalence of corruption opportunities and the magnitude of benefits derived from corrupt projects.

firm's verifiable profit.[4] In a high corruption environment, the principal opts for a non-transparent regime, blocking the manager's access to information about corrupt projects, by putting a firewall. The manager reduces effort further, significantly impacting the firm's verifiable profit, but the principal gains disproportionately higher private benefits.

Our result shows that corruption creates inefficiencies in a firm's governance processes, as it distorts the information flow and reduces the incentive wage. The principal gains at the expense of firm's verifiable profit. Corruption literature frequently highlights that the primary cost of corruption lies not in the act of giving or taking a bribe, but in the distortions caused by the illegal nature of corruption (Shleifer and Vishny, 1993; La rocca and Neha, 2017). Our model provides a theoretical basis for this argument by highlighting distortions within the firm.

Extensions

We extend our model to consider two additional aspects of the principal's corruption. First, we explore the scenario where the principal hires a corruptible manager who is willing to collude with him to destroy corruption evidence and keep corruption knowledge secret in return for sharing private benefits. In this scenario, we show that the firm gains efficiency as the principal need not adopt the costly *signal-blocking* regime, and the manager puts in higher effort, both of which increase the firm's verifiable profit. However, corruption levels increase with such collusion. This collusion can also have other side effects. It can distort talent within the firm if corruptibility is correlated with other undesirable attributes of the manager. For example, a corruptible manager might steal from the firm, leading the principal to incur additional monitoring costs. In this chapter, we assume that the principal can effectively and costlessly screen for a corruptible manager. However, the manager's type might be private at the time of hiring, and a corruptible manager could steal from the firm, requiring additional monitoring. The screening of a corruptible manager under such circumstances is discussed in Chapter 2.

Second, we consider the scenario when the principal does not have full ownership of the firm's cash flow but is still a controlling shareholder. Diluted ownership further distorts the principal's incentives, as the firm's verifiable profit is shared with the minority shareholders, whereas the private benefit fully accrues to the principal. As a result, the principal reduces the incentive wage and increases corruption further. Some firms that were previously using a no-signal-blocking regime would switch to a signal-blocking regime after going public or diluting cash flows, thus disproportionately increasing corruption at the cost of the firm's profitability. This creates an agency conflict between the principal and the minority shareholders. Shleifer and Vishny (1997) argue that this conflict, rather

[4] Profit that is reported in books and excludes the principal's private benefit

than the agency problem between the principal and management, is the fundamental corporate governance issue in most countries.

Apart from these two extensions, we also consider special cases to test the robustness of our model. These include scenarios where the principal does not hire a manager and where the manager is rewarded for reporting corruption by regulators, as seen in some industries.

Chapter 2: Corruptible Principal: Screening of manager for collusion

Chapter 1 demonstrated that a corruptible principal benefits from collusion with a corruptible manager. However, we assumed that the corruptible manager could be identified costlessly during hiring and did not impose any cost to the principal. In this chapter, we relax these assumptions and develop a mechanism for the principal to screen the manager. The manager's type (corruptible or honest) is private information at the time of hiring, and a corruptible manager can steal from the firm, imposing a cost.

Key aspects of the screening model

There are two productivity states, high or low. The corruption opportunity for the manager arises because the principal lacks information about the productivity states that determine the verifiable return. An informed manager can use this private information to misappropriate funds by reporting a low productivity state when the true state is high. The greater the difference in return across productivity states, the higher the potential for misappropriation by the manager.

The ability of a corruptible manager to engage in corruption also provides a screening mechanism. Because only a corruptible manager engages in corruption, the principal can detect such corruption through an imperfect audit that provides an ex-post verifiable signal about the state, revealing the manager's type.[5] Once the principal identifies that the manager has engaged in corruption and has information about the principal's corruption, the collusive side contract between them becomes mutually beneficial and self-enforceable. Since such a side contract is illegal in court, its self-enforceability is crucial. A side contract where both parties have incriminating evidence can be self-enforceable.

The principal's objective is to offer a wage contract that achieves the following: attracts the desired type of manager, provides incentives for the manager to exert the

[5]The corruptible manager has both a different strategy space and different utility than the honest manager, providing a desired screening condition.

desired level of effort, identifies whether the manager is corruptible to enter a side contract and reduce her exposure risk, and controls the cost of corruption if the manager is corruptible. Since the manager's type will only be revealed if the principal allows corruption to occur, the "collusion proofness" equivalence of Tirole (1986) does not hold in this context.[6] Avoiding the manager's corruption also means not revealing the manager's type, making a corruption-free contract potentially suboptimal.

Several studies have explored the beneficial aspects of allowing corruption or collusion between supervisors and agents to improve contracting (Kofman and Lawarree, 1996; Strausz, 1996; Tirole, 1992). These studies demonstrate that collusion avoidance may not always be ideal, as the process of collusion can sometimes provide valuable information.

This chapter applies a similar principle to identify manager types, enabling collusive side-contracts between the principal and the manager. To our knowledge, this is the only paper using such a mechanism for the principal-agent collusion.

Key results

Our results demonstrate three different types of outcome depending on the potential cost of corruption (C):

1. When C is not very high, the principal not only enters a collusive agreement with the corruptible manager but also mitigate the cost of the manager's corruption through a suitable wage contract. However, both types of managers participate, whereas the principal would prefer to hire only a corrupt manager.

2. When C is in the medium range, the principal reduces wage below the reservation wage to shut down the honest manager, hiring only a corruptible manager who colludes with the principal. This range increases if the reservation wage increases. Besley and McLaren (1993) terms such a wage strategy a "capitulation wage," so that only dishonest takes job.

3. When C is high, the manager's corruption becomes costly to the principal due to limited liability constraints that prevent wage adjustment. While the honest type is preferable, both types of managers participate. The corrupt manager's effort increases with C, but he also retains the surplus from his effort. In this scenario, the principal could invest in increasing audit effectiveness to reduce the cost of corruption.

[6] "Collusion proofness" in this context means no corruption by the manager. Tirole and most corruption models use a principal-supervisor-agent hierarchy where the agent and supervisor collude for corruption. In our model, the manager's corruption does not require agent-supervisor collusion, but the same principles apply. "Collusion proofness" posits that there is always an optimal contract that does not involve collusion.

The principal's contracting strategy also ensures that the honest manager's effort is not distorted due to asymmetric information. This contrasts with other adverse selection models, where the effort of the inefficient agent is typically distorted.

Chapter 3: Newspaper Market: Impact of advertisement on quality and market structure

This chapter extends the standard vertical differentiation model (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1983) to include advertisement side in order to analyze the two-sided newspaper market.

Adding the advertisement side creates tension between subscription and advertisement revenue. Firms set subscription prices and quality based on demand elasticities with respect to price and quality. However, setting a positive subscription price means some customers do not subscribe, causing firms to lose advertisement revenue. When potential advertisement revenue from non-subscribers is higher than subscription earnings, firms may switch to a corner solution with zero price and possibly minimal quality. Conversely, firms might reduce prices to boost circulation and gain more advertisement revenue, but such reductions might not be optimal for subscription revenue. This chapter shows how this tension leads to various types of market configurations.

Additionally, when advertisers prefer high-income consumers who are also willing to pay more for quality, competition for these consumers intensifies. This heightened competition makes the market extremely competitive at high advertisement levels, allowing the low-quality player to challenge the high-quality firm's leadership, thus creating a market outcome different from the pure vertical differentiation model.

Key aspects of model

We assume that newspapers are vertically differentiated, meaning we abstract from variety due to horizontal differentiation. In the newspaper market, horizontal differentiation occurs when products differ in format (such as tabloid vs. broadsheet), language (such as regional language vs. English), or content type (such as financial news vs. general dailies). Within each horizontal category, firms are vertically differentiated, and this is where quality-based competition becomes important. Given the focus of this chapter on quality choices and competition, we restrict our attention to a particular type of newspaper. Following are key characteristics of our model:

Consumer heterogeneity: Consumers are heterogeneous with respect to their income (Y), and their preference for quality content(v). Consumers' willingness to pay for quality depends on both income and preference for quality content, using a multiplicative form

vY. This formulation captures instances when some high-income consumers do not value reading newspapers as they get news from alternative sources, and when some low-income consumers have a higher willingness to pay for quality.

Advertisers prefer affluent customers: Empirical evidence strongly support this fact (Thompson, 1989; Dertouzos and Trautman, 1990).

Quality-dependent fixed cost: As demonstrated by Berry and Waldfogel (2010), the quality of a newspaper is primarily determined by the number of investigative journalists and reporters, which are part of the fixed costs. Therefore, we assume that this quality cost is convex in quality and does not depend on the number of subscribers.

Sequential entry of firms: Sequential entry allows us to consider strategic actions by a firm to deter entry.

Consumers can be ad-neutral, ad-lovers, or ad-haters: We show that our result is robust to consumer attitude about advertisements and does not solely depend on network externalities.

Key results

Our results show that high-quality firms have an advantage due to their investment in quality, allowing them to attain significant market share. This is in line with other studies on vertical differentiation (see Gabszewicz and Thisse (1979); Wauthy (1996)) and explains the market concentration observed in the newspaper industry. However, firms will not serve the lower end of the market unless consumer preference is homogeneous. Since the high-quality firm does not cater to the lower-end market, it creates an opportunity for the low-quality firm to fill the product gap and serve this segment, provided the advertisement level is not too low. Therefore, a natural monopoly occurs when the advertisement level is low and/or consumer preference is homogeneous. When the advertisement level is moderately high, the low-quality firm serves the lower end of the market as a free product with the lowest quality, while the high-quality firm behaves as a monopolist without competition, similar to what is suggested by Gabszewicz et al. (2012).

A novel and interesting finding in this paper is that as the advertisement level increases further, the low-quality player can challenge the high-quality firm's market leadership. This forces the high-quality firm to significantly raise its quality—much more than the monopolist level—to protect its customer base. In extreme cases, the high-quality firm might even drive out the competitor and deter further entry. Consequently, the high-quality firm offers a premium product with a lower price-to-quality ratio.[7] and both

[7] We use the price-to-quality ratio to effectively represent price because standalone price could be driven by changes in quality. A lower price-to-quality ratio more directly conveys higher consumer surplus.

the quality and the price-to-quality ratio improve with higher advertisements, benefiting consumers. This finding aligns with the empirical evidence provided by Angelucci and Cage (2019) and Pattabhiramaiah (2014). To our knowledge, this aspect of the impact of advertisements has not been considered in any other papers.

The potential entry of a third player increases market competition at high advertisement level, leading both existing players to raise their quality further and reduce their price-to-quality ratios. In fact, when advertisement levels are high, the profits of the top two players decline with increasing advertisement, which is the opposite of what happens in the duopoly model. This is consistent with Donnenfeld and Weber (1995)'s finding that under vertical differentiation, product competition among duopoly incumbents leads to entry deterrence. This provides a testable case for our model.

Chapter 1

Corruptible Principal: Owner corruption makes firms bureaucratic

1.1 Introduction

It is well established that in countries with a high level of corruption, firms engage in corruption through the firm owner's collusion with bureaucracy (with or without political involvement). Through such collusion, the owner of the firm receives preferential access to public resources, such as subsidized credit, regulatory licenses, government contracts, and favorable legislation, while the politician and/or bureaucrat extracts rent generated from these activities. Economists consider "crony capitalism" - the term used for such corruption - as a primary threat to capitalism and have shown that such corruption reduces competitiveness and economic growth. However, there is mixed empirical evidence on how such owner corruption impacts a firm's performance. On the one hand, the firm benefits from preferential treatment and access to scarce public resources. On the other hand, such corruption can breed inefficiency through higher bureaucratic interference and distortion in the choice of product, talent and technology.

This paper analyzes the downstream impact of firm owners' illegal corruption. We use the principal-agent model to demonstrate that firm owner's (principal's) corruption creates agency conflict with the company's CEO or manager (agent), who can monitor such corruption in the due course of business and thus posing a risk to the principal. This conflict causes the owner to provide a lower incentive wage and delegate less to the manager; in extreme cases, the owner may even block the manager's access to information by keeping relevant information hidden or by making him or her less involved in the business. This hinders informed decision making and reduces the firm's profitability, as the manager receives less information about the business environment and invests less effort into information gathering. In addition to reduced profitability, such distortions can lead to further governance inefficiencies: a) The firm becomes more bureaucratic because the owner delegates less and takes a more hands-on approach to replace the manager's information advantage. Lower wage not only reduces the managerial effort but also increases misalignment between the manager and the principal, which can lead to moral hazard in decisions that are under the manager's authority (Aghion and Tirole, 1997). b) The owner appropriates the private benefit at the expense of the firm's market value – we show that the higher the private benefit from corruption, the lower the firm's profitability. This creates conflict between the owner and the minority shareholders. c) Lower incentive wage, along with lower reliance on manager's information (or expertise), reduces the talent level in the firm. The owner usually appoints a family member who can be trusted to keep secret as the manager, even though that individual may not be the most competent person to run the business. d) The reduced transparency makes it difficult to raise outside finances (Lin et al., 2011). All of the above outcomes have been seen in the empirical literature on corruption (see 1.3).

Our model differs from the standard agency theory model of corruption where a benevolent principal designs an incentive to reduce or alleviate the impact of corruption that originates when a supervisor (or auditor) colludes with the agent it is supposed to monitor. In our model, the principal (owner) is the beneficiary of corruption, while the manager indirectly monitors corruption. Even though the manager may not have a direct incentive to monitor the principal's illegal business, his or her information advantage arising from running the business provides him or her with corruption-related information that can be leaked, posing a risk to the principal. In this sense, the manager indirectly monitors the principal, which creates agency conflict. Therefore, the principal, if corruptible, faces a trade-off in terms of organizational design: The principal can give the manager a high power incentive and let the manager make the informed decision, which entails the principal risking corruption exposure, given that the manager is likely to gain access to corruption-related information, or the principal can block the manager's information access, which reduces the firm's profitability but lowers the principal's risk of exposure. We model this trade-off and show that in a low corruption environment, the principal foregoes the corruption opportunities due to the manager's indirect monitoring, while in a high corruption environment, the principal tries to bypass this monitoring by blocking the manager's information access. This notion of employees monitoring executives' corruption has been documented and is consistent with the empirical evidence. Dyck et al. (2010) analyzed 216 cases of alleged corporate fraud and found that most corporate fraud is exposed by an employee (17%), non-financial market regulators (13%), or the media

(13%), all of whom have a weak incentive to expose fraud but superior information access. They argue for a strong whistleblower mechanism to deal with crony capitalism as it is more resistant to capture. Stiglitz (1985) and Dyck and Zingales (2004) have highlighted the important role that labor plays in monitoring controlling shareholders. Our model uses private benefits to represent the principal's gains from corruption, reflecting their concealed nature. This is consistent with the literature (Lin et al., 2011; La Porta et al., 2000), but we abstracts away from scenarios in which firms directly benefit from bureaucratic collusion, such as obtaining cheaper financing.

We cover two additional aspects of principal's corruption. First, if the principal can effectively screen candidates and hire a corruptible manager with whom he or she can collude in keeping his or her corruption secret then inefficiencies in decision making are alleviated, but the corruption level remains high. In hierarchical organizations with multiple levels of management, this could have a cascading effect that corrupts the entire organization corrupt. This can also distort talent within the firm if corruptibility is correlated with other undesirable attributes of the manager. For example, the corruptible manager can steal from the firm which entails the principal to incur additional monitoring costs. Second, if the principal is not the full owner but rather a controlling majority stakeholder, then he or she will further increase corruption at the expense of lower reported profit, expropriating value from the minority stakeholder. The principal also has a strong incentive to reduce cash flow rights while maintaining the control rights through tools such as dual-class stocks, pyramids, and cross-holding. The literature highlights that higher private benefits, which is positively correlated with the difference between control and cash flow rights, facilitates expropriation of value by the controlling shareholder (Lin et al., 2012; La Porta et al., 2000). Thus, owner corruption is also a source of agency conflict between the owner and the minority stakeholder, and this has been frequently discussed in the corporate governance literature (Shleifer and Vishny, 1997; La Porta et al., 2002).

Some of the results of our model are similar to those of the traditional model. Both models predict that corruption reduces incentives and delegation, albeit due to different mechanisms. In our model, the principal reduces the incentive wage to remove the information content of the agent's signal, whereas in the traditional model it is done to reduce the agent's incentive to bribe. In the traditional model, renegotiation is done when the full commitment of ex-ante contract is not possible (Dewatripont, 1989; Strausz, 1996), whereas in our model, the principal screens a corruptible agent to make a commitment to renegotiate. Our model also offers insights into the agency conflict between the controlling and minority shareholders.

At the macro level, our results support the theory that corruption can be self-enforcing and that, depending on the historical path, two different equilibria can emerge. We show that as corruption opportunities increase due to the institutional environment, a firm's owner has a stronger incentive to aggressively block information at the expense of the firm's profitability. This results in disproportionately higher corruption by the firm, which creates a vicious cycle of increasing corruption as more firms resort to blocking information. Therefore, a high level of corruption can be self-reinforced. On the other hand, when corruption opportunities are few due to the institutional environment, the firm's owner will give the manager more incentive and may even forego corruption opportunities due to manager's indirect monitoring, which further reduces corruption. As more firms adopt the higher incentive approach and forego corruption opportunities, it creates a positive reinforcement cycle that can dampen corruption and eventually lead to a low corruption equilibrium at the macro level. In other words, past corruption becomes a determinant of future corruption, which leads to multiple equilibria based on historical paths.

1.2 Contribution to the literature

We make three contributions to the literature on corruption. First, we show how the secrecy imperative associated with illegal corruption impacts governance and information flow within the firm, which impacts its performance. It has been constantly highlighted that the primary social cost of corruption is not in the act of giving or taking a bribe, but in the distortion caused by the illegal nature of corruption (Shleifer and Vishny, 1993). However, no studies shows how such secrecy impacts the governance processes within the firm. Rajan and Zingales (2001), in a completely different context, investigated how the need to protect trade secrets can impact organizational structure. Many empirical studies that demonstrate that corruption causes poor firm performance conjecture that corruption creates distortion in talent, misallocates resources, and reduce productivity and innovation. La rocca and Neha (2017), empirically associated corrupt board members with poor firm performance to suggest that corrupt board members tend to destroy valuable information flow, communication, and coordination within the firm. This paper develops a principal-agent model to show how the principal's corruption impacts delegation and information flow within the firm and hence incentive wages and profits.

Second, we introduce the notion that the principal (residual owner) is corruptible. The canonical model, based on Tirole (1986), uses a principal-agent-supervisor setting in which corruption occurs due to collusion between the supervisor and the agent, while the principal, who designs the organization to reduce the impact of potential collusion, is benevolent. While corruption involving non-benevolent principals has been studied in many contexts (Shleifer and Vishny, 1998; De Soto, 1989a), it has not been investigated under the principal-agent model. In our model, corruption originates from the principal (owner), and corruption monitoring is performed by the agent (manager), as opposed to the principal or supervisor. This is helpful to investigate organizations inefficiencies arising from crony capitalism.

Finally, we provide a new mechanism, based on a firm's behavior, to support the self-enforcing multiple equilibria theory of corruption. These theories are based on the notion that past corruption determines current corruption. One such mechanism is that an individual perceives a higher benefit from corruption depending on how many other people are corrupt because expected punishment due to detection is lower when discovered by a corrupt rather than a noncorrupt superior or auditor (Lui, 1985; Cadot, 1987; Andvig and Moene, 1990). Another mechanism is that dishonest people prefer to work in a corrupt bureaucracy, and such selection reinforces corruption (Hanna and Wang, 2017). Murphy et al. (1991) and Acemoglu (1995) argue that rent-seeking exhibits naturally increasing returns and an increase in rent-seeking can make the rent-seeking more attractive, which will, in turn, attract more talent and investment from productive activities and can lead to multiple equilibria. Tirole (1996) discusses a different mechanism – "collective reputation"– that leads to a similar effect. He points out that it is not in an individual's interest to be honest if their group has a reputation for being corrupt. Our mechanism is based on the firm's behavior; an increase in corruption makes information distortion to engage in higher corruption more attractive, while a decrease in corruption makes increasing information flow and foregoing corruption opportunities more attractive.

1.3 Related literature

The literature on corruption by firms widely accepts that political connections of board members or executives are one of the primary mechanisms through which firms engage in corruption. Stigler (1971) and Krueger (1974) highlighted political connectedness of firms in their seminal paper, pointing out that regulations are passed for the benefit of large firms. Recent studies have provided further evidence of corruption through political connections across several countries, for example, Agrawal and Knoeber (2001) for USA, Fisman (2001) for Indonesia, Khwaja and Mian (2005) for Pakistan, Li et al. (2008) and Kang (2003) for China, Johnson and Milton (2003) for Malaysia, Collins et. al. (2009) for India, Domadenik et. al. (2016) for Slovenia, Dombrovsky (2008) for Latvia, and Vynoslavaska et al. (2005) for Russia and Ukraine. The evidence of the impact of corruption on firm performance is mixed, but there is more support for a negative impact.

The beneficial impact of corruption on firms comes through preferential access to scarce resources when firm management colludes with politicians. Claessens et al. (2008) show a positive correlation between Brazilian firms' campaign contributions and their future access to financing. Khwaja and Mian (2005) show that politically connected firms in Pakistan receive 45 percent larger loans, despite having a 50 percent higher default rate.

Similarly, Li et al. (2008), in China's context, show a positive relationship between firms' political connections and firm performance, as firms get easy access to loans from banks and state institutions. Agrawal and Knoeber (2001) suggested that political connections improve profitability, as connections can provide knowledge of government procedure and skills in terms of estimating government policy. Fisman (2001) estimated that Indonesian firms connected to Soeharto have a 23 percent higher valuation as a result. However, when Fisman et al. (2012) replicated a similar study in the United States to identify the value of the US Vice President Dick Cheney's connections, they found zero effect, which shows that political connectedness helps where the institutional environment is corrupt. Faccio (2006) confirmed this by examining the political connections of 20,202 publicly traded firms in 47 countries. He found that the political connection of board members is associated, on average, with a 2.29 percent increase in the firm's share value, but most of this effect exists in high-corruption countries.

While political connections bring firms some benefits, they also entail hidden costs. Shleifer and Vishny (1994) point out that connected members tend to extract rent in exchange for favors, and thus, the actual gain from political connections may be limited to a few executives. Desai and Olofsgard (2011) used the WBES, survey of 8000 firms in 40 countries, to show that politically connected firms are rewarded with an improved business environment, but these firms also provide benefits to politicians through bloated payrolls, political appointments, etc., and in the net, these firms under-perform compared to their counterparts. Zingales (2012) raised serious concern about political connections of firms (crony capitalism), even in Western countries, as managers and entrepreneurs look for favoritism and use illegal behaviors to overcome stiffer competition. He gives several examples where strong leaders with political connections have enriched themselves by siphoning money from the firm they manage. Dyck et al. (2013) used a natural experiment created by Arthur Anderson's demise to show that the probability of a firm engaging in corporate fraud in any given year is 14.5%, and such fraud destroys 20.4% of the firm's enterprise value. Dyck et al. (2010) analyzed 216 cases of corporate fraud and concluded that a firm's supervisory bodies (internal and external) fail to detect and punish large-scale corporate fraud, implying collusion between the board and management.

The negative impact of corruption on firms is seen through several mechanisms: a) Firms face more bureaucratic interference, which hinders their performance. Fisman and Svensson (2007) using data on Uganada firms, have shown that the negative effect of corruption on firm growth is three times that of normal taxation. Gaviria (2002) found evidence that corruption negatively influences sales and firm growth in Latin American firms and is likely due to bureaucratic interference. Athanasouli et al. (2012) found similar evidence for Greek firms, as did McArthur and Teal (2002) for African firms. b) Corrupt executives appropriate private benefits from corruption at the expense of firm performance (Zingales, 2012; Dyck et al., 2013). Dyck and Zingales (2004) show that countries with weaker legal institutions have higher private benefits of control. c) The controlling owner's corruption results in an agency conflict between the controlling owner and the minority shareholders, as the owner expropriate from minority shareholders, often using legal means. Shleifer and Vishny (1997) argue that this conflict, rather than the agency problem between investors and management, is the fundamental corporate governance problem in most countries. Such agency problems make it difficult for firms to raise outside finances. d) Corruption distorts a firm's choice of talent and technology. De Soto (1989a) found that corruption forces entrepreneurs to establish new firms underground and on a smaller scale. Svensson (2003) shows that bribe amount is negatively correlated with the degree of capital stock reversibility. Athanasouli and Goujard (2015) find that firms that are in more contract dependent (higher bureaucratic linkage) industries located in the more corrupt regions of Central and Eastern Europe tend to have lower management quality, more centralized decision-making processes, and lower R&D investment. e) Hiding illegal corruption distorts the internal governance process by reducing transparency and coordination, which lowers performance. La rocca and Neha (2017) analyzed 2,789 firms from 34 countries across Europe and found that firms with corrupt board members have lower financial performance. They emphasized that the secretive nature of corrupt board members or executive destroys valuable information flow, communication, and coordination within firms. The owner would prefer to promote family members to management positions to keep corruption within close-trusted groups, which limits firms growth. For instance, Bloom et. al. (2011) show that firm size in the Indian textile industry is limited by the number of male family members of the owner.

Looking at the theoretical literature on corruption, the most commonly used microeconomic model of corruption is based on agency theory with a principal-supervisor-agent setting where the potential for corruption is created when the supervisor colludes with the agent it is suppose to monitor. In this setting, the public or government (principal) empowers better informed public officials (supervisors) to make certain decisions, such as issuing licenses, inspecting pollution, appointing contractors, or processing tax returns, that affect third-parties like citizens, regulated firms, suppliers, or entrepreneurs. Corruption occurs when a public official (supervisor) colludes with a third party (agent) and makes the wrong decisions for personal gains. The model was first introduced in a seminal paper by Tirole (1986). In this model, a) the principal who designs the organization or institution to either remove corruption (collusion-proofness) or tolerate corruption, if it is less costly in the context, is benevolent; b) the principal has less information about the agent so he or she appoints a supervisor who can get more information through monitoring, which reduces the agent's information advantage; and c) the supervisor can collude with the agent through a side contract and refrain from sharing information about the agent with the principal, and thus, the agent maintains its information rent in lieu of making a side payment to the supervisor. The supervisor is risk averse or has limited liability protection, so that it is not optimal to give the supervisor residual claims; otherwise, the principal can avoid corruption without cost. The key question is whether it is optimal for the principal to avoid corruption (collusion proof) or tolerate it in some contexts. This model has been applied in many settings, for example, the government (principal), tax collector (supervisor), and the taxpayer citizen (agent), where the tax-payer has private information about the business's profit (Besley and McLaren, 1993), or the Congress (principal), regulator/inspector (supervisor), and the regulated firms/entrepreneurs (agent) having private information about their cost (Laffont and Tirole, 1991; Mookherjee and Png, 1995; Baron and Besanko, 1984). Similarly, within the firm the model is applied with the shareholder (principal), auditor (supervisor), and manager (agent) setting (Laffont and Martimort, 1998; Kofman and Lawarree, 1993; Khalil and Lawaree, 2006), or with the company director (principal), procurement manager (supervisor), and third-party supplier (agent) setting (Vafai, 2005).

Tirole (1986) established the "collusion proofness" principal which states that the optimal contract is equivalent to that which avoids collusion. Specifically, the optimal contract lowers (relative to the first best) the information rent to the agent to remove the agent's incentive to bribe the supervisor. However, this equivalence principle is not robust to situations in which there are multiple types of agents with different propensities for corruption (honest or dishonest). Kofman and Lawarree (1996) show that by allowing collusion to occur among dishonest supervisors, the principal will be able to screen among the types of agents, and such screening is optimal when the probability of a dishonest supervisor is low. The equivalence principal is also not robust to a situation in which the principal cannot make a long-term commitment to the contract; therefore, renegotiation is possible (Dewatripont, 1989). Strausz (1996) shows that when the principal contracts for a noisy signal to detect collusion, and if this signal is sufficiently informative, and the principal cannot make a full commitment, then it is optimal for the principal to allow collusion, as the gain from renegotiation outweighs the cost of collusion prevention. Khalil and Lawaree (1995) show that commitment to costly auditing is not credible unless the reputation and repetition play a role. Khalil and Lawaree (2006) shows that in the absence of audit commitment and in the presence of a collusive auditor, audits become non optimal and collusion occurs in equilibrium. Olsen and Torsvik (1998) show that corruption may be beneficial for the principal if only a limited long-term commitment is possible, as the corruption acts as a commitment device by relaxing dynamic information revelation constraints and hence creates long-term gain that can offset the short-term static cost of corruption. Tirole (1992) shows that collusion under certain circumstances may help complete the contract and increase overall efficiency when the principal is unable to use a complete contract.

Several studies have devised a mechanism to reduce the cost of collusion. Kofman and Lawarree (1993) show that having an additional external auditor may reduce the cost of collusion, which is allowed in equilibrium. Laffont and Martimort (1994) show that multiple auditors (or regulators) can limit the scope of collusion by inducing competition among auditors. Felli and Hortala-Vallve (2016) show that the principal can implement cost-free collusion prevention by using a whistleblow mechanism, if the supervisor and the agent have asymmetric information. Whistleblowing allows the principal to design a mechanism that compensates the uninformed party for breaching the side deal by reporting to the principal. Similarly, Celik (2009) shows that the principal can utilize the information asymmetry between the supervisor and the agent to weaken the coalition by manipulating the agent's opportunity cost at the time of collusion. Felli (1993) examined how the principal can use information exchanged during the side-contract negotiation to prevent collusion. Career design for supervisors, penalties and rewards, and other instruments that affect supervisors' ability to enter credible agreements with agents can also reduce collusion (Lambert-Mogiliansky et al., 2008; Buccirossi and Spagnolo, 2006; Dufwenberg and Spagnolo, 2015) (Lambert-Mogiliansky et al., 2008; Buccirossi and Spagnolo, 2006; Dufwenberg and Spagnolo, 2015.) On the contrary, when it becomes easier to collude, collusion can become more costly to avoid, and the optimal design will further reduce incentive and delegation. Martimort (1997) argues that firms become bureaucratic over time as the reciprocity norm develops between agents in an organization; consequently, it becomes harder to prevent collusion, and incentive schemes lose their flexibility.

How does corruption impact corporate governance and ownership structures? Owners generate private benefits through corruption, which creates a mechanism for the expropriation of minority shareholders. Grossman and Hart (1988) first established that the private benefit of control can make firms deviate from the optimal one share one vote policy, and there will be a control premium for voting rights. Burkart et al. (2003) discuss two benefits of management control by the majority owner. First, the majority owner provides a public good to minority shareholders by monitoring management, in which case the owner bears the extra cost, while all shareholders gain. Second, the majority owner works with management to share the loot, in which case, the owner obtains a private benefit at the expense of minority shareholders. Dyck and Zingales (2004) and Nenova (2003) find that control premiums are positive and higher in countries with weaker investor protection. Zingales (1995) and Bebchuk (1999) show that when private benefits are large, entrepreneurs are more likely to retain control when they go public. LaPorta et. al. (1997, 1999) demonstrate that strong investor protection laws curbs private benefits, therefore, more concentrated ownership is seen in countries with poor investor protection. Large firms in countries with poor investor protection are either state controlled or controlled by founding families. This results in poor capital market development, and entrepreneurs find it difficult to raise finances from outside investors (La Porta et al., 2000). Outside investors provide lower valuations or demand more cash flow ownership by controlling shareholders (La Porta et al., 2002). Lin et al. (2011, 2012) show that firms with higher private benefits face higher borrowing costs, and that banks structure syndicates for enhanced due diligence and monitoring in such cases. Most of these studies establish the relationship between investor protection and private benefit and not necessarily corruption, but the argument is often extended to corruption, as it is one source of private benefits, and lower investor protection is correlated with corruption.

This paper uses the principal-agent model to examine the impact of an owner's corruption on the firm's internal information flow, governance process and performance. Several studies have discussed this, but none have examined it in detail. In our model, the principal could be engaged in corruption, and monitoring is indirectly done by the agent (manager), as opposed to the principal. This is consistent with the empirical evidence that systemic corruption in firms originates from board members or executives, and that many instances of corporate fraud are detected by employees.

1.4 The Framework

Consider a firm owned by a risk-neutral principal (he), who runs it with the help of a risk-neutral manager (she).

1.4.1 States of the World, Projects and Payoffs

The state of the world is represented by a vector (y, z), where $y \in \{\tilde{a}, \tilde{b}\}$ and $z \in \{H, C\}$. There are four possible projects: $\mathcal{P} \in \{A, A^c, B, B^c\}$. Projects with the superscript c involves corruption and are referred to as *corrupt* projects. The variable y determines which project will be successful, as defined in 1.1.

Definition 1.1. *If $y = \tilde{a}$, then projects A and A^c are the* ***good projects****, meaning they will succeed. Similarly, if $y = \tilde{b}$, then projects B and B^c are the* ***good projects*** *that will succeed.*

The variable z indicates whether there is a corruption opportunity for the principal. If $z = C$, then a corrupt project may yield a private benefit. The variables y and z are independently distributed, with $Pr(y = \tilde{a}) = Pr(y = \tilde{b}) = \frac{1}{2}$ and $Pr(z = C) = p$, where $p \in (0, 1)$. A higher p corresponds to a greater likelihood of corruption opportunities.

Note: We assume two states for the variable y, as this is the minimum necessary to model the manager's information advantage. In section 1.7.6, we discuss whether and how our results change if there are more states, i.e., a more number of projects, or if y and z are not independent.

The payoffs from all four projects are represented by a vector $(\hat{v}, \hat{b})$, where $\hat{v} \in \{0, V\}$ is a verifiable return (with $V > 0$) that can be contracted upon, and $\hat{b} \in \{0, \gamma V\}$ is the principal's private benefit from corruption. A good project, as defined in 1.1, succeeds and yields a verifiable return V; otherwise, it fails and yields 0. If $z = C$ and a corrupt project succeeds, the principal obtains an additional private benefit of γV, where $\gamma > 0$ is a fixed proportion of V. Table 1.1 below summarizes how the project payoffs (verifiable profit and private benefit) depend on the state of the world.

	$(\tilde{a}, H)$	$(\tilde{b}, H)$	$(\tilde{a}, C)$	$(\tilde{b}, C)$
A	$(V, 0)$	$(0, 0)$	$(V, 0)$	$(0, 0)$
B	$(0, 0)$	$(V, 0)$	$(0, 0)$	$(V, 0)$
A^c	$(V, 0)$	$(0, 0)$	$(V, \gamma V)$	$(0, 0)$
B^c	$(0, 0)$	$(V, 0)$	$(0, 0)$	$(V, \gamma V)$

Table 1.1: Project payoffs in different states

p and γ determine the institutional environment for corruption. For example, high p and low γ signify many corruption opportunities with small benefits, indicating the prevalence of petty corruption, such as bribing low-level officials to remove bureaucratic hurdles. Conversely, low p and high γ signify few large-scale corruption opportunities, such as obtaining concessions for public resources (Rose-Ackerman, 2018).

We use private benefit to model the gains from corruption, consistent with the existing literature where the principal's gains are hidden (Burkart et al., 2003; La Porta et al., 2002; Lin et al., 2011). In doing so, we abstract out from cases where firms may benefit directly from the principal's collusion with bureaucrats such as getting cheaper financing (Li et al., 2008; Khwaja and Mian, 2005).

1.4.2 Information Structure

The principal gets a perfect signal regarding z so that he knows with certainty whether corruption is possible or not; however, he gets no signal regarding y. This creates a role for the manager. The manager can potentially get four types of signals α, α^c, β, and β^c. Let s denote a generic signal. The probability of getting any signal depends on the state of the world (y, z), as well as the 'quality' of signal $\lambda(e)$, where e denotes the (endogenous) effort level of the manager. These probabilities $Pr(s|(y, z), \lambda(e))$ are shown in Table 1.2:

	$(\tilde{a}, H)$	$(\tilde{b}, H)$	$(\tilde{a}, C)$	$(\tilde{b}, C)$
α	$\lambda(e)$	$\frac{1-\lambda(e)}{3}$	$\frac{1-\lambda(e)}{3}$	$\frac{1-\lambda(e)}{3}$
β	$\frac{1-\lambda(e)}{3}$	$\lambda(e)$	$\frac{1-\lambda(e)}{3}$	$\frac{1-\lambda(e)}{3}$
α^c	$\frac{1-\lambda(e)}{3}$	$\frac{1-\lambda(e)}{3}$	$\lambda(e)$	$\frac{1-\lambda(e)}{3}$
β^c	$\frac{1-\lambda(e)}{3}$	$\frac{1-\lambda(e)}{3}$	$\frac{1-\lambda(e)}{3}$	$\lambda(e)$

Table 1.2: Signal probabilities in different states

Note that $Pr(\alpha|(\tilde{a}, H), \lambda(e)) = \lambda(e)$, $Pr(\alpha|(\tilde{a}, C), \lambda(e)) = \frac{1-\lambda(e)}{3}$, etc. The manager's effort e is non-verifiable and has a cost of $C(e)$. $\lambda(e)$ and $C(e)$ are defined below:

Assumption 1.1.

(a) $\lambda(e)$ is increasing, strictly concave, twice differentiable, $\lambda(0) = \frac{1}{4}$, $\lim_{e\to\infty} \lambda(e) = 1$, $\lim_{e\to 0} \lambda'(e) = \infty$, and $\lim_{e\to\infty} \lambda'(e) = 0$.

(b) $C(e)$ is increasing, twice differentiable, and convex with $C(0) = 0$ and $\frac{C'(e)}{\lambda'(e)}$ is convex.

Note that for any $\lambda(e) > \frac{1}{4}$, α (resp. α^c) is an informative signal that the state of the world is $(\tilde{a}, H)$ (resp. $(\tilde{a}, C)$). Similarly for β and β^c. We shall call α and β the honest signals, and α^c and β^c the corrupt signals.

If the manager obtains a corrupt signal, say α^c (resp. β^c), and the principal implements a corrupt project A^c (resp. B^c), the principal is caught with probability $q > 0$. This may occur because a corrupt signal provides verifiable evidence of the principal's corruption motive, which could be inadvertently leaked by the manager or disclosed if the manager becomes a whistle-blower. Such information may trigger investigations by the media and/or regulatory authorities, or assist ongoing investigations. Examples of α^c or β^c from actual corruption cases include: a) financial entities or hidden accounts used to divert funds or conceal illegal activities; b) internal reports contradicting regulatory/quality norms or financial disclosures; c) evidence of conflict of interest in financial decisions, such as selecting a supplier or approving a loan; and d) evidence of collusive relationships between the principal and bureaucrats/politicians. Note that the mere presence of α^c or β^c is not enough to show that corruption has occurred, but this evidence can be used to punish the principal if corruption takes place and is investigated.

The q represents the strength of the legal and regulatory environment and will be lower in countries with weaker enforcement mechanisms. For ease of exposition, we assume $q = 1$. In Section 1.7, we consider the possibility that $0 < q < 1$. If the principal's corruption is exposed, he incurs a penalty of $P > \gamma V$. The magnitude of P is determined by legislation and is taken to be exogenous. The manager does not receive utility from exposing corruption and cannot provide false evidence of corrupt signals. Moreover, she cannot distinguish between A and A^c, or between B and B^c.

1.4.3 Actions and Strategies

The principal's role involves deciding on three key aspects. First, he must choose between two transparency regimes: (a) *no-signal-blocking (NSB)*, where the manager has unfettered access to all signals, and (b) *signal-blocking (SB)*, where the manager receives only α and β signals. The manager receives no signal (represented as $\emptyset$) under SB regime if the true signal is corrupt (either α^c or β^c). The principal can block corrupt signals by implementing firewalls around corrupt projects, either by handling these projects himself or by appointing a trusted family member. We denote a regime by R, with n denoting the NSB regime, and b denoting the SB regime, such that $R \in \{n, b\}$. The manager can observe whether the regime is NSB or SB, but the regime choice is non-verifiable. The set of signals available under the NSB regime is $\mathcal{S}^n$, where $\mathcal{S}^n = \{\alpha, \alpha^c, \beta, \beta^c\}$, and under the SB regime, it is $\mathcal{S}^b$, where $\mathcal{S}^b = \{\alpha, \beta, \emptyset\}$. With a slight abuse of notation, we say that the manager has the same set of report available, that is $\mathcal{S}^n$ under NSB and $\mathcal{S}^b$ under SB. When the manager reports α he makes a claim that his signal is α, and similarly for other signals. We let s denote the specific signal received by the manager and $\hat{r}$ denote a specific report.

Second, simultaneously with the regime choice, the principal also decides on managerial compensation. There is a limited liability constraint that managerial wages must be non-negative. Thus, the principal decides on the wage w that the manager obtains in the case of project success; wages in the case of project failure are zero.

Finally, the principal decides on what project to implement conditional on the manager's report ($\hat{r}$), and his signal z.

The manager's role is to decide her effort e based on the wage rate w and the regime R, i.e. $e : (w, R) \to [0, \infty)$. The manager also reports her signal $\hat{r} \in \mathcal{S}^R$.

1.4.4 The Timeline

Stage 1. The principal announces w and decides regime $R \in \{n, b\}$

Stage 2. The manager observes principal's regime choice and decides on her effort $e : (w, R) \to [0, \infty)$.

Stage 3. The principal receives $z \in \{C, H\}$ signal and the manager receives a signal $s \in \mathcal{S}^R$.

Stage 4. The manager submits a report $\hat{r} \in \mathcal{S}^R$, using the information (s, w, R).

Stage 5. The principal implements one of the project from the set $\{A, A^c, B, B^c\}$, using the information $(\hat{r}, z, w, R)$

Stage 6. Payoffs are realized. If the implemented project was A^c (resp. B^c), and the manager obtained the signal α^c (resp. β^c), then the principal is caught with probability q, and pays penalty P.

Both the principal and the manager are risk-neutral and maximize their expected income. The manager has a reservation utility of 0 and is protected by limited liability, that is, $w \geq 0$. We solve for perfect Bayesian equilibrium (henceforth PBE) of this game.

1.4.5 Game Tree

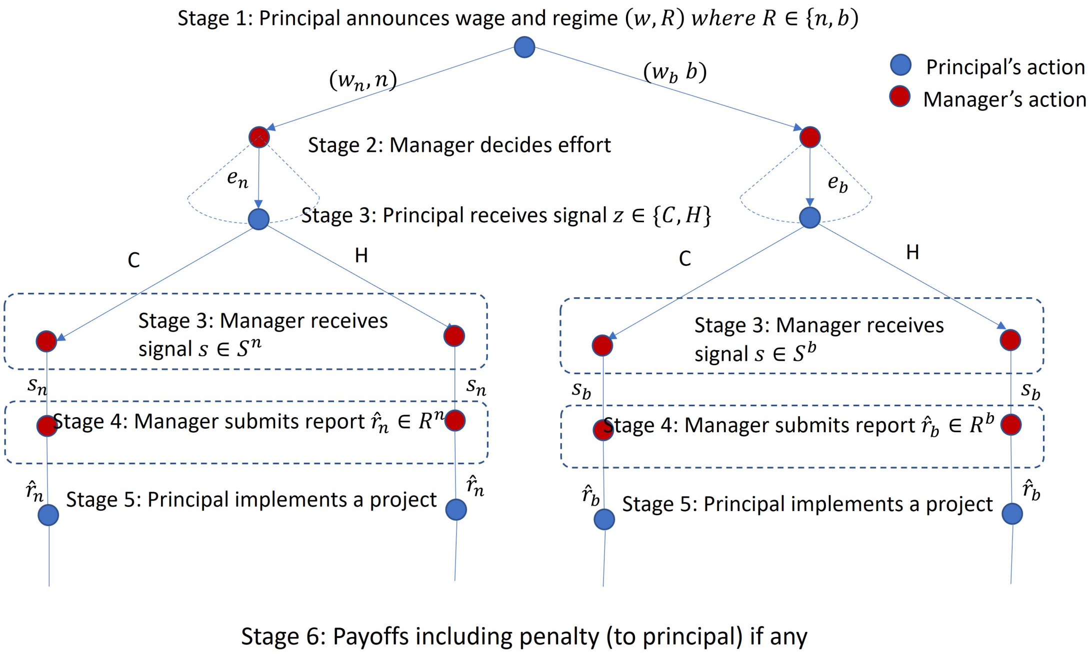


Figure 1.1: Game Tree

Figure 1.1 depicts the game tree. Please note that the principal's type — that he is corruptible — is common knowledge. Therefore, the principal's choice of regime, SB or NSB, does not reveal new information about his type, nor does it reveal the state z (whether a corruption opportunity exists) or y (which projects will succeed). However, the regime choice influences the information available to the manager, altering her beliefs about the principal's project selection strategy. This mechanism is analogous to the Bayesian persuasion framework (Kamenica and Gentzkow, 2011). Consequently, the manager will consider this updated belief when determining her effort.

1.5 Preliminary Analysis

We start by setting up a benchmark second best outcome. We then fix a transparency regime and solve for the PBE under the corresponding regime; first under NSB, and then under SB.

1.5.1 Benchmark: The Social Planner's Problem

Consider a social planner (he) who maximizes the expected *verifiable* project returns net of effort costs; thus he does not take the private benefits from corruption into account. The social planner does not know the state of the world (y, z), but can enforce any effort level e. Moreover, he knows the identity of all the projects, and can prevent the principal from implementing corrupt projects. We consider a scenario where he asks the worker to put in a certain effort level, observes the resultant signal and then asks the principal to implement A (resp. B) if the signal is either α, or α^c (resp. β, or β^c). Thus the social planner's optimization problem is given by:[1]

$$\max_{e} \quad \frac{2\lambda(e)+1}{3} V - C(e). \tag{SP}$$

Thus optimal effort e_{sp} chosen by the social planner is given by:

$$\frac{2}{3} V \lambda'(e_{sp}) = C'(e_{sp}) \tag{1.1}$$

Lemma 1.1. *There exist a well defined, strictly increasing and concave function* $E : \mathbb{R}^{++} \to \mathbb{R}^{++}$ *such that* $e_{sp} = E(V)$.

Proof. Define $f(e) \equiv \frac{3}{2}\frac{C'(e)}{\lambda'(e)}$. From Assumption 1, $f(e)$ is strictly increasing function in e, and hence f is invertible. Therefore $E = f^{-1}(e)$ is well defined and strictly increasing function. Assumption 1 ensures that $f(e)$ is a convex function and hence its inverse E is a concave function. Further, $e_{sp} > 0$ for all $V > 0$ because of Inada conditions in Assumption 1. □

1.5.2 No-signal-blocking (NSB) equilibrium

We first consider a scenario where the principal does not block any signal. Consider the ex-post probabilities of different states conditional on the manager's signal, i.e., $Pr(y|s \in \mathcal{S}^n)$ and $Pr(z|s \in \mathcal{S}^n)$, as shown in Table 1.2 (see Appendix 1.A.1 for derivations):

[1] $Pr(s \in \{\alpha, \alpha^c\} \mid y = \tilde{a}) = \lambda(e) + \frac{1-\lambda(e)}{3} = \frac{2\lambda(e)+1}{3}$; Therefore, the project A when $y = \tilde{a}$ will be chosen with probability $\frac{2\lambda(e)+1}{3}$. Same is true for project B when $y = \tilde{b}$.

	$y=\tilde{a}$	$y=\tilde{b}$	$z=C$	$z=H$
α	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
β	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
α^c	$\frac{3\lambda p+(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{(1-\lambda)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p+(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{2(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$
β^c	$\frac{(1-\lambda)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p+(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p+(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{2(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$

Table 1.3: Ex-post conditional probabilities of different states under NSB

Note: $Pr(y=\tilde{a}|s=\alpha)=\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$, $Pr(z=C|s=\alpha)=\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$, etc.

Let $\sigma(s)$ denote a mixed reporting strategy, and $\hat{r}(\sigma)$ denote any report which has positive support under $\sigma(s)$. For ease of exposition, we introduce few definitions.

Definition 1.2. *We say that the principal* respects the manager's report *if he implements either A or A^c following a report of either α or α^c, and either B or B^c following a report of either β or β^c.*

Definition 1.3. *We say that a mixed strategy σ* reports the state y information truthfully *if, $\hat{r}(\sigma(s))|_{s\in\{\alpha,\alpha^c\}}\in\{\alpha,\alpha^c\}$ and $\hat{r}(\sigma(s))|_{s\in\{\beta,\beta^c\}}\in\{\beta,\beta^c\}$,*

Lemma 1.2. *Fix an NSB regime and $w>0$, consider the continuation game in Stage 4:*

(a) Under any PBE, the manager reports the state y information truthfully and the principal respects the manager's report.

(b) There exists a continuum of PBEs where, upon receiving a signal of α^c (resp. β^c) the manager report randomizes over α and α^c (resp. β and β^c) .

Proof of Lemma 1.2. (a) Suppose not. Suppose $s\in\{\alpha,\alpha^c\}$ (wlog) but the manager reports $\hat{r}=\beta$ (or β^c) with probability $\delta>0$. Wlog we also assume $\delta\le\frac{1}{2}$.[2] For $w>0$ the manager will choose positive effort $e>0$ (from Inada conditions in Assumption 1.1). For $e>0$ we have $\lambda(e)>\frac{1}{4}$, and consequently $Pr(y=\tilde{a}|\alpha)>Pr(y=\tilde{b}|\alpha)$ and $Pr(y=\tilde{a}|\alpha^c)>Pr(y=\tilde{b}|\alpha^c)$ (Table 1.3). Therefore s is an informative signal of the state $y=\tilde{a}$ and hence the principal will respect the manager's report when $\delta\le\frac{1}{2}$. This results in the principal selecting the wrong project with probability δ. Since the manager receives wage $w>0$ only on the project success, her expected wage decreases with δ and hence the manager will choose $\delta=0$, which is a contradiction.

(b) From (a) the manager reports the state y information truthfully. However, let's assume that she mis-reports the corruption signal by randomizing over α and α^c (resp.

[2] $\delta>\frac{1}{2}$ is equivalent in outcome to the $\delta<\frac{1}{2}$ except that the principal will choose the project A or A^c when the manager reports β or β^c.

β and β^c) when the true signal is α^c (resp. β^c).[3] Such randomization only affects the principal's choice of project between A and A^c (resp. B and B^c) and therefore only impacts the principal's private benefit but does not impact the manager's expected income. Hence, all possible such randomization strategies can be supported as an equilibrium. □

Intuitively, Lemma 1.2 follows from the fact that the manager has no incentive to misreport state y since doing so reduces the chances of project success, and she receives a wage only when the project is successful. However, she can misreport the corruption signal using any possible randomization strategy, even though she does not benefit from doing so as she is indifferent between the principal's choice of a corrupt vs. honest project. However, her misreporting can increase the principal's chance of getting caught. If she were to incur an infinitesimally small cost when the principal is caught, she would report all signals truthfully. Since we are interested in analyzing the principal's problem when the manager does not have incentive to expose the principal's corruption, we make the following assumption under NSB.

Assumption 1.2. *The manager reports the corruption signal truthfully if doing so does not reduce her payoff.*

Lemma 1.3. *In any PBE, the manager reports all signal truthfully and the principal respects the manager's report, opting for a corrupt project iff $z = C$, and the manager reports a non-corrupt signal.*

Proof of Lemma 1.3. A truthful reporting follows from Lemma 1.2 and Assumption 1.2. If the manager reports a corrupt signal then the principal will never implement a corrupt project, since he will definitely be caught (recall $q = 1$) and the punishment penalty P exceeds the private benefit γV. However, if $z = C$ (which of course the principal gets to know), and the manager does not report a corrupt signal, the the principal will implement a corrupt project as $\gamma > 0$. □

Given Lemma 1.3, the principal's expected payoff function under NSB at Stage 1 can be written as:[4]

$$\Pi_n = \underbrace{\frac{2\lambda(e)+1}{3}(V-w)}_{Direct\ Effect} + \underbrace{\frac{1-\lambda(e)}{3}\, p\,\gamma\, V.}_{Indirect\ Effect} \tag{1.2}$$

Consider the trade-offs involved in eliciting a greater level of managerial effort e under NSB. The first term on the RHS, the direct effect, is increasing in e as a greater level

[3] The manager cannot mis-report α^c (resp. β^c) when she receives α (resp. β) as she cannot falsify the evidence of corruption.

[4] $\Pi_n(w) = Pr(\alpha)[Pr(\tilde{a}|\alpha)(V-w) + Pr[(\tilde{a},c)|\alpha]P] + Pr(\beta)[Pr(\tilde{b}|\beta)(V-w) + Pr[(\tilde{b},c)|\beta]P] + Pr(\alpha^c)Pr(\tilde{a}|\alpha^c)(V-w) + Pr(\beta^c).Pr(\tilde{b}|\beta^c)(V-w)$. See Table 1.3 for all the probabilities.

of e improves signal quality $\lambda(e)$, thereby increasing the chances of project success. The second term, the indirect effect however decreases with e as a higher level of e makes it more likely that the manager gets a signal of α^c or β^c in case $z = C$, so that the principal cannot invest in the corrupt project. We refer to this second effect as the manager's *indirect monitoring*. We use Lemma 1.3 to specify the manager's expected income (Π_n^m):

$$\Pi_n^m \quad = \quad \frac{2\lambda(e)+1}{3}\, w - C(e) \tag{1.3}$$

Thus the principal's optimization problem in Stage 1, let us call it the NSB problem (henceforth NSBP), can be written as:

$$\max_w \quad \frac{2\lambda(e)+1}{3}\,(V-w) + \frac{1-\lambda(e)}{3}\, p\,\gamma\, V \tag{NBSP}$$

subject to:

$$e = \arg\max_{e'} \; \frac{2\lambda(e')+1}{3}\, w - C(e') \tag{GIC}$$

$$\frac{2\lambda(e)+1}{3}\, w - C(e) \geq 0 \tag{IR}$$

$$w \geq 0 \tag{LL}$$

where, as is standard, GIC, IR and LL are the manager's (global) incentive compatibility, individual rationality and limited liability constraints respectively. Let (w_n, e_n) solve the NSBP.

Lemma 1.4.

(a) Given (LL) and (GIC), (IR) holds.

(b) If $w > 0$, one can replace the global incentive compatibility (GIC) by the the manager's first order condition (i.e. local incentive compatibility condition)

$$\frac{2w}{3}\,\lambda'(e_n) = C'(e_n). \tag{1.4}$$

(c) There exists a well defined, strictly increasing and concave function $E : \mathbb{R}^{++} \to \mathbb{R}^{++}$ such that

$$e_n = \begin{cases} E(w) & \text{if } w > 0, \\ 0, & \text{if } w = 0 \end{cases}$$

Proof of Lemma 1.4. (a) Given the limited liability constraint (LL), i.e. $w \geq 0$, the manager can always opt for $e = 0$, and ensure that she obtains a non-negative expected payoff. So that given (GIC), (IR) is necessarily satisfied.

(b) Given Assumption 1.1 and $w > 0$, the manager's objective function is strictly concave in e. Thus the first order condition will given a unique e_n for all $w > 0$ and (1.4) fully characterizes the (GIC) (see Bolton and Dewatripont (2005)).

(c) From (1.4) and Assumption 1.1, $e_n = E(w)$ where E is a well defined strictly increasing and concave function as defined in Lemma 1.1.[5] Inada conditions in Assumption 1.1, ensure that $E(w) > 0$ for all $w > 0$. If $w = 0$ then from GIC $e_n = 0$. Inada conditions also ensure that $e_n \to 0$ as $w \to 0$, making $e_n(w)$ a continuous function for all $w \geq 0$. □

Given Lemma 1.4, we can define $\lambda_n(w) \equiv \lambda(e_n(w))$ and re-write the principal's profit function as:

$$\frac{2\lambda_n(w)+1}{3}\,(V-w) + \frac{1-\lambda_n(w)}{3}\,p\,\gamma\,V. \tag{1.5}$$

The principal's problem simplifies to maximizing (1.5) subject to the (LL) constraint. Note that $\lambda_n(w)$ is strictly concave in w and the objective function (1.5) is strictly concave for $p\gamma < 2$.[6] Therefore, the first order condition (1.6) gives unique solution.[7] Let w_n^* solve (1.6).

$$\underbrace{\frac{2}{3}\,\lambda_n^{'}(w_n)\,(V-w_n)}_{Marginal\ benefit\ of\ w} = \underbrace{\frac{2\lambda_n(w_n)+1}{3}}_{Direct\ cost} + \underbrace{\frac{\lambda_n^{'}(w_n)}{3}\,p\,\gamma\,V}_{Indirect\ cost}, \tag{1.6}$$

.

Note that the LHS of (1.6) represents the marginal benefit of a higher wage, which induces more effort from the manager and hence provides a more informative signal, increasing the project's chance of success. The RHS represents the marginal cost of a higher wage, which includes a) the direct cost of the higher wage payment and b) the indirect cost of the foregone benefit of corruption due to the manager's monitoring effort.

Proposition 1.1. *Suppose the principal chooses the no-signal-blocking (NSB) regime.*

a) There exist a unique PBE where the principal respects the manager's report, opting for a corrupt project iff $z = C$, and the manager reports a non-corrupt signal.

b) In this PBE,

$$w_n = \begin{cases} 0, & \text{if } p\gamma \geq 2, \\ w_n^*, & \text{otherwise.} \end{cases}$$

Further, $\forall\ p\gamma < 2$, the managerial wage w_n is decreasing in p and γ, and increasing in V.

c) The equilibrium effort level e_n is decreasing in p and γ and is strictly less than the social planner effort e_{sp} for all p and γ.

[5] $E = f^{-1}(e)$ where $f(e) = \frac{3}{2}\frac{C'(e)}{\lambda'(e)}$.

[6] Since a strictly concave monotonic transformation of a concave function $e_n(w)$ is strictly concave.

[7] Second order condition is satisfied: $\frac{2}{3}\lambda^{''}(V - w_n^* - \frac{1}{2}\gamma V) - \frac{4}{3}\lambda^{'} < 0$

d) The principal's equilibrium profit increases monotonically with p, γ and V.

Proof of Proposition 1.1. (a) follows from Lemma 3, and strict concavity of both the principal's objective function (Equation (1.5)) and the manager's objective function (GIC).

(b) If $p\gamma \geq 2$, (1.5) is decreasing in w and hence the principal will choose wage $w_n = 0$ (LL binds). For $p\gamma < 2$, (1.5) is increasing and concave in w. Therefore, equilibrium wage will be the solution of (1.6) that is $w_n = w_n^*$. From implicit function theorem on (1.6), w_n is continuously decreasing in p and γ and increasing in V.[8]

(c) From Lemma 1.4, $e_n = E(w_n)$, where E is a strictly increasing function. Since w_n is decreasing in p and γ, e_n is decreasing in p and γ. From Lemma 1.1, $e_{sp} = E(V)$. Since, $w_n < V$ for all p and γ, $e_n < e_{sp}$.[9]

(d) The principal's payoff in equilibrium is represented by:

$$\Pi_n = \underbrace{\frac{2\lambda_n(w_n)+1}{3}(V - w_n)}_{Verifiable\ or\ reported\ profit} + \underbrace{\frac{1-\lambda_n(w_n)}{3}\, p\, \gamma\, V}_{Private\ benefit\ from\ corruption} \tag{1.7}$$

Using envelope theorem, $\frac{d\Pi_n}{dV} = \frac{d\Pi_n}{dp} = \frac{1-\lambda_n(w_n)}{3}\gamma\, V > 0$ for all $\gamma > 0$ and $\frac{d\Pi_n}{d\gamma} = \frac{1-\lambda_n(w_n)}{3}\, p\, V > 0$ for all $p \in (0,1)$. Therefore, the principal's profit in equilibrium increases with both p and γ (corruption). Profit is increasing in V because both terms of (1.7) is increasing in V. □

Proposition 1.1 implies that even without the loss of transparency under NSB, the manager invests less effort relative to the social planner effort. This occurs because the higher managerial effort reduces the principal's expected gains from corruption. We call this effect *manager's indirect monitoring*. The principal partially alleviates the effect of indirect monitoring by reducing the incentive wage which in turn reduces the managerial effort. Higher the potential for corruption (high p and γ), lower is the incentive wage and managerial effort. In the extreme, when corruption opportunity is sufficiently high ($p\gamma \geq 2$), the principal will reduce the incentive wage to zero, making the manager redundant.

We now evaluate how this result compares with the case when there was no indirect monitoring. To do so, we use a benchmark scenario in which the principal cannot be caught even if the manager has α^c or β^c, as the corruption signals do not provide verifiable evidence (equivalent to no enforcement or $q = 0$). In other words, the $q = 0$ case eliminates the effect of the manager's indirect monitoring.

[8] $\frac{dw_n}{dp} = \frac{\lambda_n'(w_n)\gamma V}{2\lambda_n''(w_n)[(V-w_n-\frac{1}{2}p\gamma V]-4\lambda_n'(w_n)} < 0$ and $\frac{dw_n}{d\gamma} = \frac{\lambda_n'(w_n)pV}{2\lambda_n''(w_n)[(V-w_n-\frac{1}{2}p\gamma V]-4\lambda_n'(w_n)} < 0$

[9] The principal will never set $w_n = V$ because he could reduce the wage by small $\epsilon > 0$ and increase the profit.

Proposition 1.2. *Indirect monitoring by the manager (a) reduces the equilibrium wage, (b) decreases the principal's benefit from corruption, and (c) increases the firm's verifiable profit if $p\gamma$ is below a critical threshold.*

See Appendix 1.A.2 for the proof. Proposition 1.2 follows from the fact that indirect monitoring limits the principal's corruption by making him forgo corruption opportunities when the manager has corruption signals. However, it is less intuitive why verifiable profit might increase with indirect monitoring. This occurs because, without indirect monitoring ($q = 0$), the principal sets an excessively high incentive wage compared to what is optimal without corruption. For $p\gamma$ below a critical threshold, this upward distortion has a greater negative impact on verifiable profit than the downward wage distortion caused by indirect monitoring.

Proposition 1.2 implies that indirect monitoring reduces corruption and can increase verifiable profit, both of which are socially desirable. This suggests that in low to moderate corruption environments, the natural monitoring by employees and managers can effectively control corruption. Institutionalizing and strengthening indirect monitoring through a whistleblower mechanism can not only help with fraud detection but also reduce the incidence of corruption. Many economists and corporate regulators advocate for a reward system for whistleblowers. Dyck et al. (2010) show that whistleblowers identify corporate fraud more effectively than the SEC and at a much lower cost due to their information advantage. Zingales (2012) argues that a whistleblower-based system is also resistant to capture, as any employee with information can be a whistleblower, making it difficult to buy them all off. However, the prerequisite for such a system is strong legal enforcement, i.e., high q.

The NSB equilibrium also implies that if the manager is corruptible, mutually beneficial collusion can occur between the manager and the principal since the principal has a strong incentive to overcome the indirect monitoring. We discuss the case of a corruptible manager in Section 1.7.2.

1.5.3 Signal-blocking (SB) equilibrium

We now consider a scenario where the principal *completely* blocks the manager's access to the corrupt signals α^c and β^c, possibly by putting firewalls around the corrupt projects.[10] We start by deriving the ex post probabilities of different states conditional on the manager's signal $s \in S^b$, $Pr(y|s)$ and $Pr(z|s)$:

[10] In Section 1.7, we allow partial signal blocking by the principal.

	$y=\tilde{a}$	$y=\tilde{b}$	$z=C$	$z=H$
α	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
β	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
$\emptyset$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{(2\lambda+1)p}{2(1-\lambda)(1-p)+(2\lambda+1)p}$	$\frac{2(1-\lambda)(1-p)}{2(1-\lambda)(1-p)+(2\lambda+1)p}$

Table 1.4: Ex-post probabilities of different states given the signals received by the manager under SB

Thus $Pr(y=\tilde{a}|\alpha)=\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$, $Pr(z=C|\emptyset)=\frac{(2\lambda+1)p}{2(1-\lambda)(1-p)+(2\lambda+1)p}$, etc. Table 1.4 shows that whenever managerial effort e is positive (so that $\lambda(e)>\frac{1}{4}$), α and β are informative signals for the states $y=\tilde{a}$ and $y=\tilde{b}$, respectively. $\emptyset$ signal however does not provide any information about the state y. Further, recall that the principal knows with certainty whether the state is corrupt or not.[11] The following lemma follows straightaway from these considerations.

Lemma 1.5. *Fix an SB regime with $w>0$, and consider the continuation game in Stage 4. Under any PBE:*

(a) The manager reports the signal truthfully.

(b) It is optimal for the principal to respect the manager's report when she reports α or β, opting for a corrupt project iff $z=C$.

(c) If the manager reports $\emptyset$ signal, the principal chooses one of the corrupt projects at random if $z=C$, and one of the honest projects if $z=H$.

Proof of Lemma 1.5. For (a) and (b), the argument mimics that of Lemma 1.2. Moreover, since the manager does not receive corruption signals, the issue of hiding corruption signals does not arise. Therefore, the degenerate mixed strategy equilibria discussed in Lemma 1.4 are not possible under SB.

(c) follows since $\emptyset$ signal carries no information regarding y, and the principal knows the realization of z with certainty. □

Given Lemma 1.5, the principal's (Π_b) and the manager's expected payoff (Π_b^m) functions are:[12]

$$\Pi_b \;=\; (1-p)\,\frac{2\lambda(e)+1}{3}\,(V-w)+\frac{p}{2}\,(V-w+\gamma\, V). \tag{1.8}$$

$$\Pi_b^m \;=\; \left((1-p)\,\frac{2\lambda(e)+1}{3}+\frac{p}{2}\right)\,w-C(e) \tag{1.9}$$

[11]However, $\emptyset$ report could be informative about whether the state $z=C$ if $p\geq\frac{1}{2}$. Of course, such information is redundant for the principal.

[12]$\Pi_b=Pr(\alpha)[Pr(\tilde{a}|\alpha)(V-w)+Pr[(\tilde{a},c)|\alpha]P]+Pr(\beta)[Pr(\tilde{b}|\beta)(V-w)+Pr[(\tilde{b},C)|\beta]P]+\frac{1}{2}Pr(\emptyset)[(V-w)+Pr(z=C|\emptyset)P]$

Thus the principal's optimization problem (denoted as SBP), can be written as:

$$\max_{w} \; (1-p)\,\frac{2\lambda(e)+1}{3}\,(V-w)+\frac{p}{2}\,(V-w+\gamma\,V) \tag{SBP}$$

subject to:

$$e = \arg\max_{e} \left((1-p)\,\frac{2\lambda(e)+1}{3}+\frac{p}{2}\right)\,w-C(e) \tag{GIC}$$

$$\left((1-p)\,\frac{2\lambda(e)+1}{3}+\frac{p}{2}\right)\,w-C(e)\geq 0 \tag{IR}$$

$$w\geq 0 \tag{LL}$$

The solution of the SB problem (see Appendix 1.A.3) yields the following equations to characterize equilibrium wage (w_b), effort level (e_b), and the principal's profit (Π_b).

$$\underbrace{\frac{2}{3}\,\lambda_b'(w_b)\,(1-p)\,(V-w_b)}_{Marginal\ benefit\ of\ wage}=\underbrace{\frac{2\lambda_b(w_b)+1}{3}\,(1-p)+\frac{p}{2}}_{Marginal\ direct\ cost\ of\ wage} \tag{1.10}$$

$$e_b = E(w_b(1-p)) \tag{1.11}$$

$$\Pi_b=\underbrace{\left((1-p)\,\frac{2\lambda_b(w_b)+1}{3}+\frac{p}{2}\right)\,(V-w_b)}_{Verifiable\ Profit}+\underbrace{\frac{p}{2}\,\gamma\,V}_{Private\ benefit} \tag{1.12}$$

where E is a well defined strictly increasing function as defined in Lemma 1.1[13] and $\lambda_b(w,p)\equiv\lambda(E(w(1-p)))$. Proposition 1.3 summarizes the SB equilibrium.

Proposition 1.3. *Suppose the principal chooses signal-blocking.*

a) There exist a unique PBE where the principal respects the manager's report when she reports α or β, opting for a corrupt project if $z=C$. If the manager reports $\emptyset$ signal, the principal chooses one of the corrupt projects at random if $z=C$, and one of the honest projects at random if $z=H$.

b) In this equilibrium, the manager's wage w_b is independent of γ. If λ is not too concave, w_b decreases in p for all $p\in(0,1)$. Otherwise, w_b is non-monotonic in p, meaning w_b increases with p when p is below a critical threshold and decreases with p when p is above this threshold. Moreover, $w_b\to 0$ as $p\to 1$.

c) The equilibrium effort level e_b is decreasing in p, independent of γ, and is strictly less than the social planner effort e_{sp} for all p and γ.

[13] $E=f^{-1}(e)$ where $f(e)=\frac{3}{2}\frac{C'(e)}{\lambda'(e)}$

d) The principal's profit monotonically increases with V and γ but non-monotonic in p. There exist a critical threshold $\bar{\gamma}$ such that the principal profit decreases with p if $\gamma < \bar{\gamma}$, and increases with p otherwise.

The detailed proof is shown in the Appendix 1.A.3. Proposition 1.3 highlights the following points:

- As p increases, effort becomes less responsive to wages due to the factor $1-p$ in the wage-effort equation $e_b = E(w(1-p))$. The manager anticipates that her effort will be wasted in the corrupt state and therefore puts in less effort if p is higher. Under the NSB regime, the manager's effort does not change with p for a given wage.[14]

- The wage under SB (w_b) depends solely on p, whereas the wage under NSB (w_n) depends on the product $p\gamma$. This is because, under NSB, the principal tries to counteract the manager's indirect monitoring by reducing her wage, an effect that depends on the expected benefit from corruption ($p\gamma$). Under SB, the principal hides his corruption by blocking the signal in the corrupt state, which depends only on the probability of the corrupt state.

- w_b could increase with p for small values of p if λ_b is sufficiently concave.[15] If λ_b is sufficiently responsive to wage at $p = 0$, then as p increases from $p = 0$, the impact of reduced effort is compensated by an increase in wage. However, if λ is sufficiently flat at $p = 0$, the wage monotonically decreases with p for all $p \in (0, 1)$. Regardless of how the wage responds to p, managerial effort e_b monotonically decreases with p as stated in (c).

- The principal's profit decreases with p if and only if γ is below a critical threshold $\bar{\gamma}$. When p increases, three effects occur: a) an increase in expected private benefit from corruption, b) a loss of information in the corrupt state that is more likely, and c) lower effort by the manager. When $\gamma < \bar{\gamma}$, the latter two effects dominate, while at higher γ, the first effect dominates. This contrasts with the NSB regime, where the principal's profit monotonically increases with p for all γ. This implies that NSB will dominate SB if $\gamma < \bar{\gamma}$.

Since none of the results in subsequent sections qualitatively change whether w_b increases with p at small p or w_b monotonically decreases with p, we assume specific functional forms for $\lambda(e)$ and $C(e)$ such that λ_b meets the condition required for w_b to decrease monotonically with p. This reduces the number of cases and simplifies the analysis without much loss of generality.

[14] The manager effort indirectly decreases in NSB with p due to lower wage.

[15] As measured by the Arrow-Prat index $(-w\frac{\lambda_b''}{\lambda_b'})$

Assumption 1.3. *$\lambda(e) = \frac{e+\lambda(0)}{1+e}$, and $C'(e) = \mu(1+e)^{\kappa-1}$ where κ represents the degree of convexity of the cost function, with $\kappa = 1$ (resp. $\kappa > 1$) if the cost function is linear (resp. strictly convex).*[16]

The endogenous λ_b, under Assumption 1.3, takes the form $\lambda_b(w,p) \equiv 1-(w(1-p))^{-\frac{1}{\kappa+1}}$.

1.6 Principal's regime decision: NSB vs SB

We start by showing that the principal opts for signal-blocking whenever the gain from corruption, formalised as γ, is large.

Proposition 1.4. *There exists $\gamma_c(p) \in (\bar{\gamma}, 2)$ such that for all $\gamma > \gamma_c$ the principal will choose SB and for all $0 \leq \gamma \leq \gamma_c$ the principal will choose NSB. Further, $\gamma_c(p)$ is decreasing in p.*

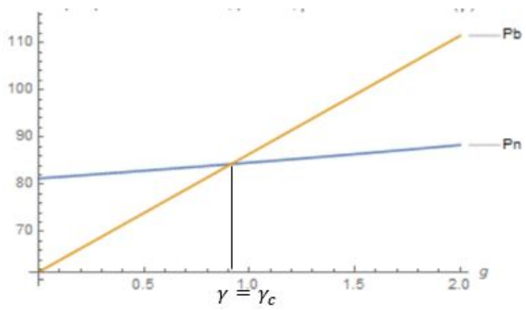


Figure 1.2: Principal's payoff in SB, P_b, and NSB, P_n, at different γ

Proof of Proposition 1.4. The principal's profit under NSB, Π_n, and that under SB, Π_b, is given by (1.7) and (1.12), respectively. We can decompose the difference between the two profits as follows:

$$\Pi_b - \Pi_n = \underbrace{p\,\gamma\,V\left(\frac{1}{2}-\frac{1-\lambda_n}{3}\right)}_{A:\,Corruption} - \underbrace{p\left(\frac{2\lambda_b+1}{3}-\frac{1}{2}\right)(V-w_b)}_{B:\,Loss\ of\ signal} + \underbrace{\frac{2\lambda_b+1}{3}(V-w_b)-\frac{2\lambda_n+1}{3}(V-w_n)}_{C:\,Wage\ incentive\ effect} \tag{1.13}$$

A) *Corruption* - this captures the difference in expected private benefit under SB and NSB; this component is increasing in γ.

B) *Loss of signal* - this captures the fact that under SB there is a loss of information as informative signals are blocked when $z = C$. This component is increasing in p but independent of γ.

[16] $\lambda(e)$ of this form is Tullock's success function. This form does not meet the Inada conditions, but this does not affect our results qualitatively. Also instead of parameterizing $C(e)$ using κ, one can parameterize the $\lambda(e)$ function, and derive equivalent comparative statics results in terms of the concavity of $\lambda(e)$.

C) *Wage incentive effect* - this captures the difference arising from equilibrium wage differences under the two regimes. This effect could be positive or negative depending on the parameters p and γ. It increases with γ because NSB wage and hence λ_n decreases with γ (Proposition 1.1), while λ_b is independent of γ (Proposition 1.3).

Note that $\Pi_b - \Pi_n$ in (1.13) is:

1. Increasing in γ. Terms A and C increase, while Term B is independent of γ.

2. Negative for $0 < \gamma < \bar{\gamma}(p)$ because $\Pi_b = \Pi_n$ for $p = 0$, Π_n is increasing in p (from Proposition 1.1), and Π_b is decreasing in p for $\gamma < \bar{\gamma}$ (from Proposition 1.3).

3. Positive if $\gamma = \frac{2}{p}$. From Proposition 1.1, if $p\gamma = 2$ then $w_n = 0$ and $\lambda_n = \frac{1}{4}$. Hence (1.13) becomes $\frac{V}{2} + (\frac{2\lambda_b+1}{3}(1-p) + \frac{p}{2})(V - w_b)$ which is greater than 0.

Therefore, from the Intermediate Value Theorem, there exists $\gamma_c > \bar{\gamma}(p)$ such that for all $\gamma > \gamma_c$ the principal will choose SB regime. Appendix 1.A.7 provides the remaining proof that $\gamma_c(p) \in (\bar{\gamma}(p), 2)$ and that $\gamma_c(p)$ is decreasing in p. □

Intuitively, as γ increases, the wage decreases in the NSB regime but has no impact on the wage in SB regime. The lower wage in the NSB regime reduces the quality of the signal and effort, making it less beneficial relative to the SB regime, albeit with a lower private benefit.

1.6.1 Comparison between NSB and SB regime

In this section, we compare the endogenous variables (effort,private benefit, wage, and verifiable profit) in two regimes. Comparison between SB (Equations (1.10)-(1.12)) and the NSB (Equations (1.6) - (1.7)) equilibrium shows three effects. The principal gains from signal-blocking by avoiding the manager's indirect monitoring (indirect cost in Equation (1.6)) and thus obtains higher private benefit from corruption. However, signal-blocking adds two costs: a) effort is less responsive to wage due to factor $1-p$ in Equation (1.11)) and b) no informative-signal in corrupt state due to factor $1-p$ in Equation (1.10)). Lemma 1.6 compares the wage effort relationship in SB regime with that in NSB.

Lemma 1.6. *Let $e_b(w)$ and $e_n(w)$ represent the manager's optimal effort for a given w under SB (b) and NSB (n). Define $\lambda_b(w) \equiv \lambda(e_b(w))$ and $\lambda_n(w) \equiv \lambda_n(e_n(w))$ then*

a) $e_b(w) < e_n(w)\ \ \forall w > 0$ *and* $p \in (0,1)$

b) $\frac{de_b}{dw} < \frac{de_w}{dw}\ \ \forall w > 0$ *and* $p \in (0,1)$

c) $\lambda_b(w) < \lambda_n(w)\ \ \forall w > 0$ *and* $p \in (0,1)$

d) $e_b(w) = e_n(w)$ *and* $\lambda_b(w) = \lambda_n(w)$ *if* $p = 0$

Proof. All the above statements follow from the following three statements:

- $e_b(w, p) = e_n(w(1 - p))$
- $e_n(w)$ and $e_b(w)$ are increasing in w.
- $\lambda(e)$ is increasing in e

Intuitively, Lemma 1.6 shows that the effort is less responsive to a given wage in the SB regime because the manager's effort only increases the success probability in the honest state. As $p \to 1$, effort stops responding to the wage. This is an inefficiency introduced by the SB regime, where the manager's expertise and information are less valued, leading to reduced effort.

Lemma 1.7. *The private benefit from corruption increases discontinuously when the principal chooses SB regime.*

Proof. The maximum possible private benefit from corruption in NSB is $\frac{1-\lambda_n}{3}\, p\, \gamma\, V \leq \frac{1}{4} p\, \gamma\, V$ as $(\lambda_n \geq \frac{1}{4})$. The private benefit from corruption in SB is $\frac{1}{2} p\, \gamma\, V$. Therefore, SB increases corruption benefit by at least twice that in NSB, for any p and γ. □

This disproportionate jump in corruption due to signal-blocking gives rise to vicious cycle of increasing corruption when the starting corruption level is high in the economy.

Proposition 1.5. *If* w_n *and* w_b *represent the equilibrium wages under NSB and SB regimes, respectively, then there exists a critical* $p_c(\gamma) \in [0, 1]$ *such that* $w_b > w_n$ *iff* $p < p_c$*. Further,* $p_c(\gamma)$ *is an increasing function of* γ *with* $p_c(2) = 1$*.*

See Appendix 1.A.4 for the proof. Figure 1.3 portrays the wages in the two regimes at different values of p and γ as described in Proposition 1.5.

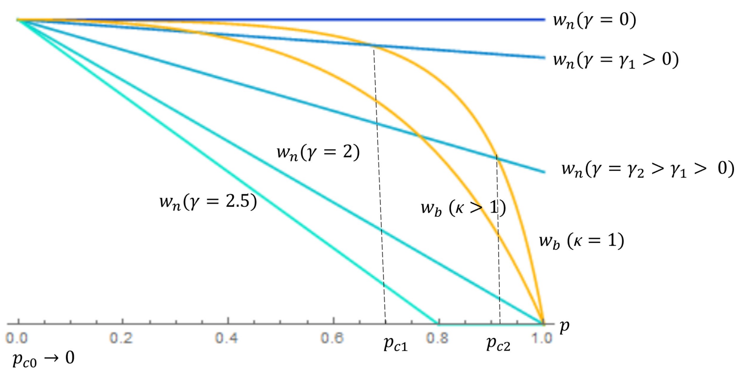


Figure 1.3: Relationship between wage and p under SBand NSB regimes

Proposition 1.5 states that wages in the SB regime are higher than in the NSB regime at a low p and a high γ, and vice versa. This is because the wage distortion (reduction relative to optimal) in SB increases with p, while the wage distortion in NSB increases with the product $p\,\gamma$. Therefore, for a low p, when the wage distortion in SB is small, but a high γ such that the product $p\,\gamma$ is sufficiently high, NSB experiences relatively higher distortion. Regardless of whether signals are blocked, corruption reduces the incentive wage, causing principals to reduce delegation and rely less on managers' efforts.

Remark 1.1. *Corruption reduces incentive wages, irrespective of the principal's use of signal-blocking technology. In the presence of signal blocking, the wage decreases with p only, while without signal blocking, the wage decreases with the product $p\,\gamma$.*

Remark 1.2. *A lower wage level can be a testable hypothesis for identifying high corruption environments, such as those found in regulated industries and politically connected firms.*

Even though signal blocking results in higher wages at lower p, it leads to lower verifiable profit for all $p \in (0,1)$ and $\gamma \leq 2$, as shown in Proposition 1.6.

Proposition 1.6. *Let VP_n and VP_b represent the firm's verifiable profit (excluding private benefits) in no-signal-blocking (NSB) and signal-blocking (SB) regime, respectively. Then:*

a) $VP_n = \frac{2\lambda_n+1}{3}\,(V - w_n)$

b) $VP_b = \left(\frac{2\lambda_b+1}{3}\,(1-p) + \frac{p}{2}\right)\,(V - w_b)$

and $VP_n > VP_b$ for all $\gamma \leq 2$ and $p \in (0,1)$. For $\gamma > 2$, there exists p' such that $VP_n > VP_b$ iff $p > p'$.

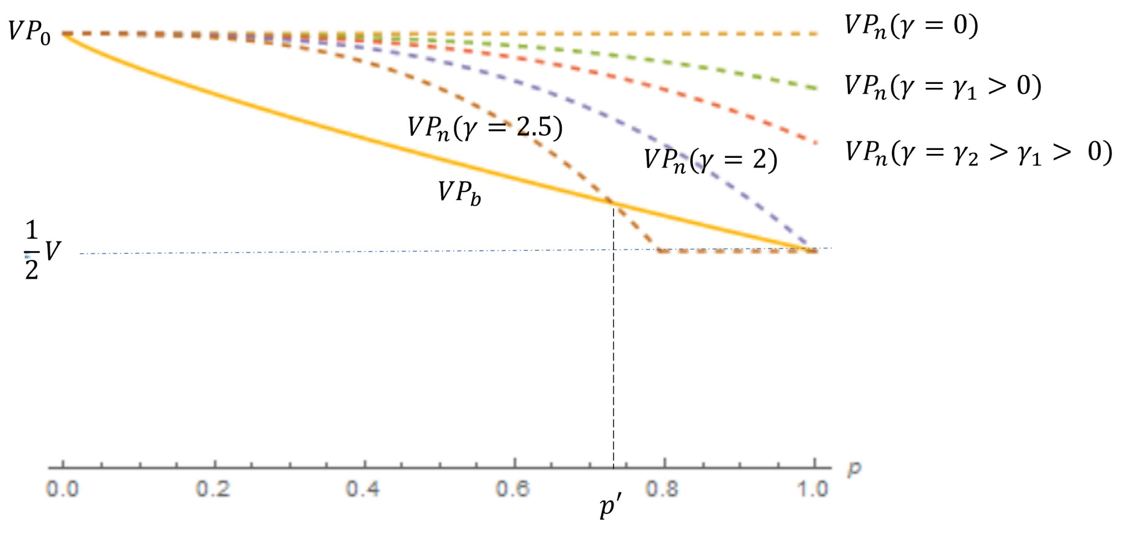


Figure 1.4: Verifiable profits in SB(b) and NSB(n) cases for different values of p and γ

The proof of Proposition 1.6 is given in Appendix 1.A.5. Figure 1.4 graphically portrays Proposition 1.6. Proposition 1.6 implies that except for a very high γ (> 2) and p ($p > p^{'}$) the verifiable profit is lower with SB than with NSB regime.

We saw in Proposition 1.5 that in a low p ($< p_c$) and high γ environment, the wage in SB is higher than in NSB. However, SB also involves less effort for a given wage, as discussed in Lemma 1.6. Therefore, the combined effect of signal loss and muted managerial effort results in lower verifiable profit in SB for all p and $\gamma \leq 2$.

Since the critical threshold for switching to SB, γ_c, is less than 2 (see Proposition 1.4), Proposition 1.6 implies that the principal's gains from corruption when switching to a SB regime come at the cost of lower verifiable profit, which reduces the firm's market value.

We also compare social surplus (Proposition 1.7), which is defined as the sum of verifiable profits to the firm and the surplus to the manager. Note that the manager receives surplus due to limited liability rent for her effort. We exclude the private benefit of corruption from the social surplus, as it is not the social planner's objective to increase corruption, even if it is welfare-enhancing for some individuals, because corruption entails many hidden costs.

Proposition 1.7. *If e_n and e_b represent the effort level for a given wage in NSB and SB regime, respectively, then the social surplus in the two cases are given by:*

a) $SS_n = \frac{2\lambda(e_n(w_n))+1}{3} V - C(e_n(w_n))$ *for no-signal-blocking*

b) $SS_b = \left(\frac{2\lambda(e_b(w_b))+1}{3}(1-p) + \frac{p}{2}\right) V - C(e_b(w_b))$ *for signal-blocking*

Further $SS_n > SS_b$ *for all* $0 < \gamma < 2$ *and* $p \in (0,1)$

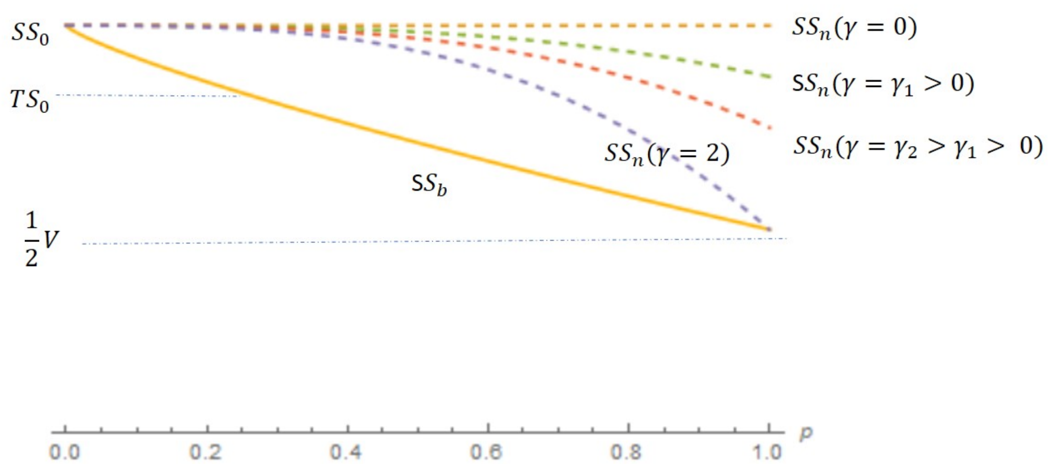


Figure 1.5: Social surplus in SB(b) and NSB(n) for different values of p and γ

Figure 1.5 shows the social surplus in the two cases for various p and γ values. The proof of Proposition 1.7 is given in Appendix 1.A.6.

So far, we have shown that signal-blocking results in higher corruption, lower verifiable profits, and less social surplus as long as $p\gamma \leq 2$.

Remark 1.3. *Corruption reduces both verifiable profit and social surplus (defined as verifiable profit + manager surplus) irrespective of signal blocking. However, signal-blocking results in a significantly larger reduction in verifiable profit and social surplus. This is due to the combined effect of information loss and reduced managerial effort.*

1.6.2 Impact of corruption on endogenous parameters

Proposition 1.4 showed that the principal uses signal-blocking if γ exceeds a critical threshold γ_c. Section 1.6.1 examined how corruption, wages, verifiable profit, and social surplus behave under both regimes. We now combine these findings to illustrate the impact of the exogenous corruption environment (p and γ) on these endogenous parameters.

Figure 1.6 depicts the effects of increasing the corruption parameter γ (with constant $p = \frac{1}{2}$) on the principal's profit, firm's reported (verifiable) profit, principal's private benefits, managerial wage, and managerial surplus. When the principal switches to signal-blocking, corruption increases and verifiable profit decreases discontinuously. For $p = \frac{1}{2}$, managerial wage increases upon switching to signal-blocking; however, the manager's surplus decreases as she exerts less effort and earns lower moral hazard rent. This non-monotonic increase in wage alongside a decrease in verifiable profit and corruption at higher γ can serve as a testable hypothesis.

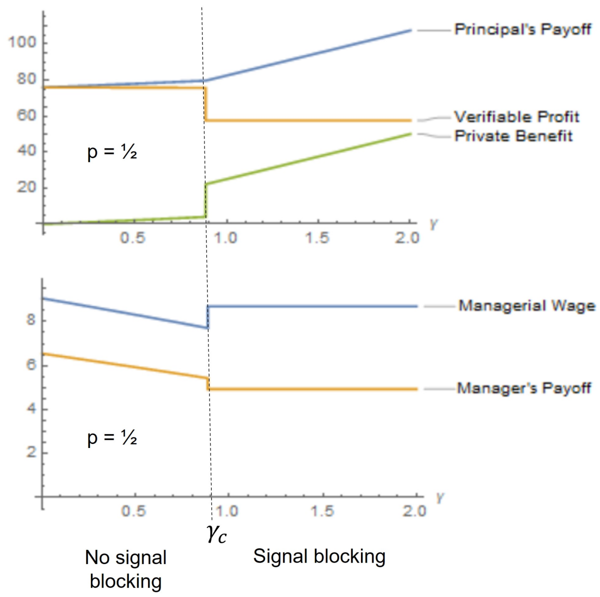


Figure 1.6: Principal's payoff, verifiable profit, private benefit, managerial wage and surplus after considering the principal's endogenous decision to choose transparency regime.

Remark 1.4. *A high corruption environment breeds more corruption. Specifically, a high corruption environment as measured by higher γ drives more firms to block signals that disproportionately increases corruption. This vicious circle of increasing corruption can lead the economy to a sustained high corruption equilibrium.*

Remark 1.5. *Discontinuous reduction in reported profit and non-monotonic increase in wage can be a testable hypothesis for the institutional environment of corruption.*

1.6.3 Implications

The results discussed thus far support following arguments that are consistent with empirical evidence. First, corruption creates inefficiencies in a firm's governance process. The need to hide illegal corruption creates agency conflict between the principal and the manager which distorts the information flow and decision making. Corruption literature frequently highlights that the primary cost of corruption lies not in the act of giving or taking a bribe, but in the distortions caused by the illegal nature of corruption (Shleifer and Vishny, 1993). Our model provides a theoretical basis for this argument by highlighting distortions within the firm.

Second, The principal gains at the firm's expense, as the firm's verifiable profit decreases with corruption, regardless of the transparency regime chosen. The literature documents that corrupt executives appropriate private benefits from corruption at the expense of firm performance (Zingales, 2012; Dyck et al., 2013; La rocca and Neha, 2017).

Third, corruption lowers the incentive wage, which results in lower delegation to the CEO. The owner reduces the manager's information advantage which entails taking more hands-on approach to managing the organization, thus creating a more bureaucratic organization. Athanasouli and Goujard (2015) provides evidence that corruption results in bureaucratic and centralized decision making. Lower wages and less delegation can also reduce talent inside the firm, as the manager's expertise is less valued, more so if the principal uses signal-blocking. Very often the owner appoints a trusted family member, who can be trusted to keep secret, as the manager, even though that individual may not be the most competent person (Morck et al., 2000; Bertrand and Schoar, 2006). In addition, the lower incentive wage reduces alignment between the manager and the principal, which worsens the agency problem in other dimensions as well. For example, when taking decisions where she has authority, the manager may propose or execute projects that are not in the principal's interest, motivating the principal to further reduce delegation (Aghion and Tirole, 1997).

Fourth, indirect monitoring by the manager can control corruption in a low-corruption environment, but it might be ineffective in a high-corruption environment, as the principal reduces transparency. This supports the argument by Zingales (2012) that strengthening whistleblower mechanisms can effectively address corporate corruption, but with qualification that the corruption level should not be so high (γ) that the principal resort to pernicious signal-blocking.

Fifth, a high corruption environment breeds even more corruption. In such an environment, the firm owner may switch to a signal-blocking regime, increasing corruption. This can create a vicious cycle where more and more firms use signal blocking, causing high corruption to persist in the economy. Conversely, in a low-corruption environment, the principal forgoes corruption opportunities due to indirect monitoring by the manager, positively reinforcing and dampening corruption. This supports the multiple equilibria theory of corruption (Lui, 1985; Cadot, 1987; Andvig and Moene, 1990; Hanna and Wang, 2017), with our model predicting this based on firm behavior.

Finally, we can infer that the firm owner adopts different level of transparency and incentive wage based on the type of corruption opportunities. Signal-blocking can result in a higher wage than no-signal-blocking at a low p and a high γ, a situation that can be considered as a few large cases of corruption. Rose-Ackerman (2018) terms this category of corruption as grand corruption that typically involves securing government contracts and obtaining concessions for public resources. In such cases, the wage distortion with signal blocking is small. However, the no-signal-blocking wage could be distorted significantly due to the combined effect of $p\gamma$; therefore, the principal may find signal-blocking optimal in grand corruption. In contrast, the situation of a high p and a low γ can be liken to what Rose-Ackerman (2018) defines as petty corruption. Such corruption occurs frequently in day-to-day business activities such as offering bribe to low-level public officials to remove bureaucratic hurdles. In this situation, signal-blocking is very costly, as it shuts down information about a large proportion of projects. Hence, the principal may find no-signal-blocking optimal in which case he will have to forego some corruption opportunities. Therefore, petty corruption can be controlled by whistleblower mechanism.

1.7 Extensions

1.7.1 Imperfect enforcement

We relax the assumption $q = 1$ and consider that the principal will be caught with some exogenous probability $q < 1$ when the manager has an inkling of corruption through the signal α^c (or β^c) and the principal implements project A^c (or B^c). q signifies the strength of the legal and regulatory environments. A country with weak legal enforcement will have a low q.

This will have no impact on the SB regime as the manager never receives signals α^c and β^c. However, the dynamics change in the NSB regime. The manager will still communicate the signal truthfully i.e. $\hat{r} = s$ because her incentive does not change. However, when the manager reports a corrupt signal, i.e, $\hat{r} \in \{\alpha^c, \beta^c\}$, and the principal observes $z = C$, the principal will implement the A^c (or B^c) project instead of A (or B) iff $\gamma V > qP$ which implies $q < \frac{\gamma V}{P}$. Let us call this threshold $q_c \equiv \frac{\gamma V}{P}$. The principal's project selection strategy is depicted in the table below:

$\hat{r}\mid z$	$z = C$	$z = H$
α	A^c	A
α^c	A^c if $q < q_c$ otherwise A	A
β	B^c	B
β^c	B^c if $q < q_c$ otherwise B	B

If $q \geq q_c$ then nothing change from the NSB equilibrium descrbed in Section 1.5.2. If $q < q_c$, then the expected payoff from implementing a corrupt project when the manager reports α^c (or β^c) is given by $\gamma V - qP = (1 - \frac{q}{q_c})\gamma V$. Therefore, the principal's expected payoff is given by:

$$\Pi_n = \frac{2\lambda + 1}{3}\,(V - w_n) + \frac{1-\lambda}{3}\,p\,\gamma\,V + \boldsymbol{\lambda\,p\,(1 - \frac{q}{q_c})\,\gamma\,V}\;\mathbb{I}_{\boldsymbol{q<q_c}} \tag{1.14}$$

$$\text{where } \mathbb{I}_{q<q_c} = 1 \text{ if } (q < q_c) \text{ else } 0$$

The term highlighted in bold shows the difference from NSB equation (1.7). Since the manager's incentive does not change, the wage effort equation is given by (1.4). The first order condition for the equilibrium wage is given by:

$$\frac{2}{3}\lambda_n^{'}(w_n)\,(V - w_n) + \boldsymbol{\lambda_n^{'}(w_n)\,p\,(1 - \frac{q}{q_c})\,\gamma\,V}\;\mathbb{I}_{\boldsymbol{q<q_c}} = \frac{2\lambda_n(w_n) + 1}{3} + \frac{1}{3}\,\lambda_n^{'}(w_n)\,p\,\gamma\,V \tag{1.15}$$

By comparing Equation (1.15) to Equation (1.6), we observe that an increase in wage has an additional positive effect (highlighted in bold) because the principal can obtain a higher private benefit in the corrupt state if $q < q_c$. Therefore, the effect of the manager's indirect monitoring (the last term on the RHS) is partially or fully offset. If $q = \frac{2}{3}q_c$, both terms cancel each other, resulting in no wage distortion due to corruption. This implies that the verifiable profit is optimal even in the presence of corruption if $q = \frac{2}{3}q_c$. The verifiable profit decreases on both sides of $q = \frac{2}{3}q_c$ (see Figure 1.7) because if $0 \leq q < \frac{2}{3}q_c$, the wage is higher than optimal, and if $\frac{2}{3}q_c < q < q_c$, the wage is lower than optimal. However, from (1.15) and (1.14), corruption and the manager's wage increase monotonically as q decreases because the effect of indirect monitoring is reduced.

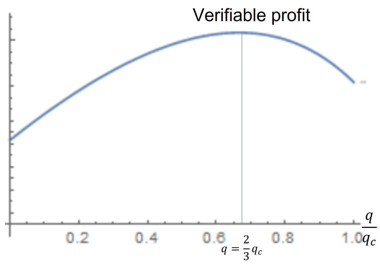


Figure 1.7: Verifiable profit at different levels of q

Let's now examine how the principal's decision to block signals changes. Let's assume that q_c remains constant when we increase γ.[17] We can observe from (1.14) that $\frac{d\Pi_n}{d\gamma}\Big|_{q<q_c} > \frac{d\Pi_n}{d\gamma}\Big|_{q\geq q_c}$. This implies that $\gamma_c\big|_{q<q_c} > \gamma_c\big|_{q\geq q_c}$ (see Figure 1.8). In other words, the critical γ_c threshold for using SB increases as q decreases. If q becomes much smaller ($q < \frac{2}{3} - \frac{1}{6\lambda_n}$), then the principal will not block signals for all $\gamma > 0$.[18] In conclusion, the principal will prefer NSB over SB for a wider range of γ with weaker enforcement (lower q).

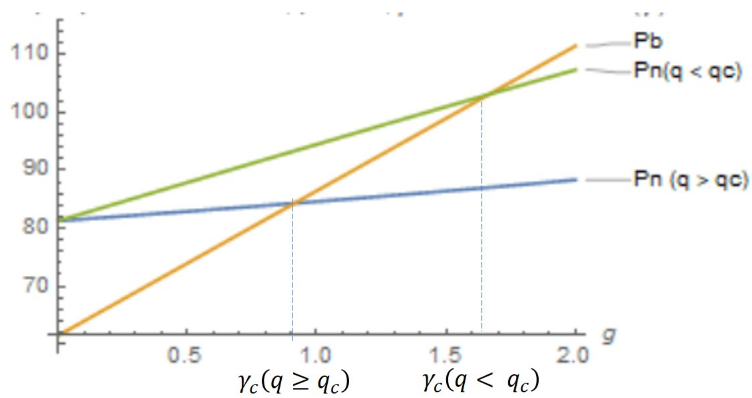


Figure 1.8: Principal's pay-off in SB (P_b) and NSB (P_n) at different levels of q and γ

We have shown that weaker enforcement (lower q) increases corruption, but the firm's efficiency may also increase if q is not too small, as the corruptible principal resorts to less information distortion to hide corruption. However, higher corruption caused by weaker enforcement can have many long-term negative effects on the firm and the economy, which we have not modeled. For example, weaker legal enforcement could also result in less investor protection, making it difficult for firms to raise capital (La Porta et al., 2000). Empirical evidence shows that firms with higher levels of corruption face higher bureaucratic hurdles, which could lower verifiable returns V (Fisman and Svensson, 2007; Gaviria, 2002). The economy-wide impact of distortions caused by higher corruption is widely discussed in the literature (see section 1.3).

[17] i.e., the ratio of the penalty on getting caught P to the benefit from corruption γV remains constant.

[18] This is q at which the slope of $P_n\big|_{q<q_c}$ becomes equal to the slope of P_b in the Figure 1.8.

1.7.2 Collusion with the corruptible manager

As discussed in Section 1.5.2, a mutually beneficial trade arises between the manager and the principal if the manager is corruptible and willing to collude with the principal. This creates the possibility of a side-contract between the principal and a corruptible manager after the manager receives a signal indicating the principal's corruption. We extend our model to incorporate the negotiation of such a side-contract, wherein the manager destroys corruption evidence in exchange for side payments. For this extension, we assume that the manager is corruptible and that the principal is aware of the manager's corruptibility at the time of hiring. The principal uses this information into wage and organizational decisions made in Stage 1. A more realistic assumption, however, could be that the manager's type is private information, which we discuss in Chapter 2. We modify the timeline as illustrated below, which differs from the timeline in Section 1.4 in Stage 0 and Stage 5, as highlighted.

Stage 0. The principal hires a manager who is known to be corruptible.

Stage 1. The principal announces w and decides regime $R \in \{n, b\}$

Stage 2. The manager observes principal's regime choice and decides on her effort $e : (w, R) \to [0, \infty)$.

Stage 3. The principal receives $z \in \{C, H\}$ signal and the manager receives a signal $s \in \mathcal{S}^R$.

Stage 4. The manager submits a report $\hat{r} \in \mathcal{S}^R$, using the information (s, w, R).

Stage 5. The principal and the manager negotiate a side contract if the manager reports a corrupt signal and the principal observes $z = C$. The principal implements a project from the set $\{A, A^c, B, B^c\}$.

Stage 6. Payoffs are realized.

This side contract will not come into play under the SB regime, as the manager never receives signal α^c (or β^c). However, the NSB case will change as follows:

Suppose the manager reports a corrupt signal and the principal observes $z = C$. The principal could implement A^c (or B^c) instead of $A (or B)$ if he successfully negotiate this side-contract, giving him higher payoff. Let us assume that the principal and the manager negotiate using Nash bargaining with weight δ for the principal. The principal's and the manager's outside options are $V - w$ and w, respectively, if they do not negotiate, as the principal will implement A (or B) to avoid getting caught. However, if they negotiate, there is additional surplus of γV.[19] Therefore, the manager's payoff from side-contract

[19]This assumes that $q \geq q_c$. If $q < q_c$, then the additional surplus will be qP. The rest of the mechanism and insights remain the same.

will be $w + (1 - \delta)\gamma V$, and the principal's payoff will be $V - w + \delta\gamma V$, and both will agree to the side-contract. Given this possibility of a side-contract, in which the manager benefits, the manager will report all signals truthfully in Stage 4. The principal's project choice at Stage 5 will be:

$\hat{r}\vert\ z$	$z = C$	$z = H$
α	A^c	A
α^c	A^c and side-contract	A
β	B^c	B
β^c	B^c and side-contract	B

We use the subscript 'rn' to denote this scenario of side-contract renegotiation in order to distinguish from "n", NSB. The principal's and manager's payoff equation will be modified as follows (changes from NSB is highlighted in bold):

$$\Pi_{rn} = \tfrac{2\lambda+1}{3}\,(V - w) + \tfrac{(1-\lambda)\,p}{3}\,\gamma\,V + \boldsymbol{\lambda\,p\,\delta\,\gamma\,V}$$

$$\Pi^m_{rn} = \tfrac{2\lambda+1}{3}\,w + \boldsymbol{\lambda\,(1-\delta)\,p\,\gamma\,V} - C(e)$$

FOC for the manager's optimal effort:

$$\left[\frac{2w}{3} + \boldsymbol{(1-\delta)\,p\,\gamma\,V}\right]\ \lambda^{'}(e_n) = C^{'}(e_n) \tag{1.16}$$

By comparing Equations (1.16) and (1.4), we can observe that the marginal benefit of the effort (LHS) is higher for the manager under collusion, as she also has a share in the private benefits from corruption (bold term in (1.16)). Hence, for a given wage, the manager will invest more effort under collusion than without collusion. Equation (1.16) also shows that the equilibrium wage could be zero (limited liability binding) with positive effort if the manager's share $(1 - \delta)$ is sufficiently high. Also note that the e_n will be an increasing function of w, p, γ and a decreasing function of δ. Consequently, the same will be true for the $\lambda_{rn}(e_{rn})$. The first order condition for the equilibrium wage is given by:

$$\frac{2}{3}\,\lambda^{'}_{rn}(w_{rn})\,(V - w_{rn}) + \boldsymbol{\lambda^{'}_{rn}(w_{rn})\,\delta\,p\,\gamma\,V} = \frac{2\lambda_{rn}(w_{rn}) + 1}{3} + \frac{1}{3}\,\lambda^{'}_{rn}(w_{rn})\,p\,\gamma\,V \tag{1.17}$$

From Equations (1.16) and (1.17), we can infer that collusive side-contract has two effects that reinforce each other to the principal's benefit. First, the manager puts higher effort for a given wage and second the principal gets higher benefit from corruption.

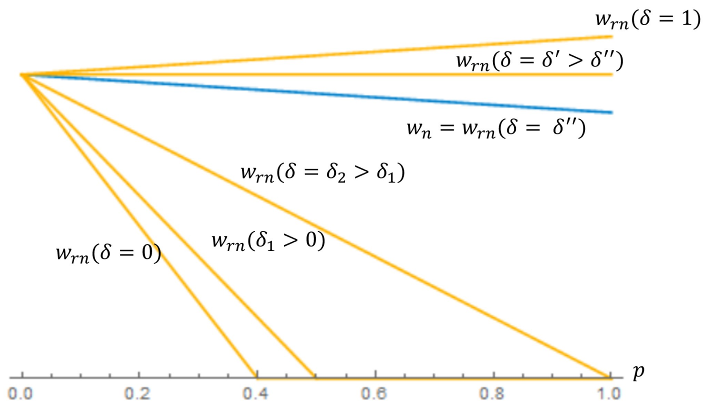


Figure 1.9: Wage with collusive side-contract (w_{rn}) and without collusion (w_n) under NSB for different values of δ

Figure 1.9 and Lemma 1.8 describe how wage changes with p when a corruptible manager colludes with the principal. The picture with γ will be similar, as the wage under NSB depends on the product $p\gamma$.

Lemma 1.8. *Consider a scenario where the principal hires a corruptible manager who is willing to negotiate a side-contract to destroy the evidence of α^c (or β^c). Let δ represent the principal's bargaining weight, and w_{rn} the equilibrium wage under such collusion. There exists a threshold $\delta^{'}$ such that, for all $0 < \delta < \delta^{'}$, w_{rn} decreases with p, and increases with p otherwise. Further, there exists another threshold $\delta^{''}$ such that for all $0 < \delta < \delta^{''}$ and $p \in (0, 1)$, $w_{rn} < w_n$, and $w_{rn} > w_n$ otherwise.*

The proof is provided in Appendix 1.A.8. Since the manager shares the private benefit and the principal take manager's type into consideration while choosing wage, the principal adjusts the wage downward. The higher the manager's share (high $1 - \delta$ or low δ) and the greater the corruption potential $p\gamma$, the more significant the wage reduction relative to that without collusion. In other words, the manager's incentive is more aligned with the principal, and she need not be given a high incentive wage to induce effort. This also means that the limited liability constraint (LLC) binds at a much lower level than $p\gamma = 2$. However, when the manager's share is very low (as δ approaches 1), indicating lower alignment, the wage increases with corruption. In this case, the principal pays a higher wage to induce greater effort in order to capture more of the private benefit.

Proposition 1.8. *Consider a scenario in which the principal hires a corruptible manager who is willing to negotiate a side-contract to destroy the evidence of α^c (or β^c). If δ is the principal's bargaining weight in such collusive side-contract then:*

a) The principal chooses NSB over a wider range of γ.

b) Verifiable profit is increasing in corruption benefit ($p\,\gamma\,V$) and is strictly higher than that without collusion for all $p\gamma > 0$.

Further, if δ is such that the limited liability constraint is not binding then:

c) NSB strictly dominates SB for all γ.

d) Manager's expected payoff may decrease with higher corruption.

e) Higher bargaining power may not be always beneficial to the principal as his expected payoff is non-monotonic in his share δ.

The proof is provided in Appendix 1.A.8. Intuitively, when the principal colludes with a corruptible manager, two effects occur. First, the manager's indirect monitoring is removed, which benefits the principal as he gets a higher benefit from corruption. Second, the manager is paid a lower wage, as she is compensated with a share of private benefits, which aligns the manager's and principal's objectives. In fact, the manager puts in higher effort with a lower wage. We term the second effect the *incentive alignment effect*. Note that the second effect arises because the manager makes the effort decision (stage 2) before the side-contract (stage 5). These two effects reinforce each other, obviating the need for signal-blocking (c) or making signal-blocking less desirable (a). The firm's verifiable profit also increases (b) due to a lower wage bill and a higher probability of success due to higher managerial effort and no loss of information.

If corruption results in benefits to both the firm and the principal, then what are the costs? One cost is the hidden cost of corruption, as discussed earlier. Another impact is that the manager may lose surplus (d). When the manager shares the private benefit, her incentive is aligned with that of the principal, similar to getting a partial residual right. Consequently, the principal, by reducing the incentive wage, extracts the moral hazard rent the manager was earning. This is only possible when $w_{rn} > 0$. Once the limited liability constraint binds, i.e., $w_{rn} = 0$, the manager surplus increases due to limited liability rent.[20] Finally, of these two effects, the first effect increases with δ, while the manager's incentive alignment effect decreases with δ. (e) shows that the second effect dominates the first until $w_{rn} > 0$ i.e., the principal's profit decreases with δ. Once $w_{rn} = 0$, the second effect loses its teeth, and the principal's profit increases with δ. This is a paradoxical situation in which higher bargaining power is harmful to the principal. Besley and McLaren (1993) refers to such a situation as the "persuasion paradox." The principal could alleviate such a situation by making additional transfers to the manager.

[20] There are two sources of limited liability rent to the manager. One is because he cannot be paid negative wages when the project is not successful, and the second is because his wage cannot be reduced below zero when he earns from private benefit.

Since the principal screens the corruptible manager at the time of hiring, can he sign an ex-ante contract with a corruptible manager instead of side-contract after the manager receives a corrupt signal? One alternative could be that the principal ex-ante sells the rights of earning verifiable profit and obtaining private benefit to a corruptible manager and extract all the rent through a fixed upfront payment, which is equivalent to selling the firm. This avoid moral hazard rent to the manager. However, such contract for illegal corruption is hard to write and enforce. For example, bureaucrats may not trust a manager to collude in corruption. Another alternative is that the principal signs an ex-ante contract with a corruptible manager, where the manager, in addition to receiving a wage, also gets a share of the private benefit if she destroy corruption evidence, which principal can verify. If the bargaining weight remains the same, this contract will yield outcomes equivalent to those described above. This is because, in our model, both the manager and the principal anticipate the opportunity to collude while deciding on wage and effort. However, the bargaining weight could differ based on two different timings of negotiation, potentially altering the outcomes.

Remark 1.6. *The principal has a strong incentive to hire a corruptible manager if he can screen such manager efficiently.*

Remark 1.7. *If the principal can screen and hire a corruptible manager, the principal and the manager will collude to increase corruption. However, the firm will also gain efficiency for two reasons: (a) the principal will avoid costly signal-blocking, and (b) there will be a better alignment of interests between the principal and the manager, resulting in the manager investing greater effort at a lower wage.*

Does this mean that the principal will always hire a corruptible manager if he can screen efficiently? This is not necessary because a corruptible manager can not only collude with the principal but also engage in stealing from the firm which is costly to the principal. The principal would then need to implement a more elaborate and costly monitoring mechanism. In addition, a corruptible manager can distort the talent within the firm if corruptibility is correlated with other undesirable attributes. Therefore, there is a trade-off in hiring a corruptible manager. This trade-off is modeled in Chapter 2.

1.7.3 Firm has minority shareholders

Let us assume that the principal owns η fraction of the firm's cash flow but is still a controlling shareholder because he owns a majority voting right.[21] Therefore, the principal will receive only the η fraction of the verifiable profit $(V - w)$ when the project is successful, but the private benefit from corruption fully accrues to the principal.

[21] The controlling shareholder of many firms owns a much smaller cash flow rights (smaller than 50%) but use dual class stocks, pyramid structure, or cross holding of shares to own majority of voting rights. See La Porta et al. (2000).

The manager's payoff function does not change based on the ownership structure, so the effort wage equation remains unchanged, as in Equation (1.4) for NSB and Equation (1.11) for SB. The principal's payoff function in the two cases can be written as:

$$\Pi_{n'} = \eta \, \tfrac{2\lambda_n(w)+1}{3} \, (V - w) + \tfrac{1-\lambda_n(w)}{3} \, p \, \gamma \, V = \eta \, \left[\tfrac{2\lambda_n(w)+1}{3} \, (V - w) + \tfrac{1-\lambda_n(w)}{3} \, p \, (\tfrac{\gamma}{\eta}) \, V \right] \quad (1')$$

$$\Pi_{b'} = \eta \, \left(\tfrac{2\lambda_n(w)+1}{3} + \tfrac{p}{2} \right) \, (V - w) + \tfrac{p}{2} \, \gamma \, V = \eta \, \left[\left(\tfrac{2\lambda_n(w)+1}{3} + \tfrac{p}{2} \right) \, (V - w) + \tfrac{p}{2} \, (\tfrac{\gamma}{\eta}) \, V \right] \quad (6')$$

The above equations are equivalent to (1.5) (NSB) and (1.12) (SB) except that the γ is scaled up by the factor $\frac{1}{\eta}$, and the principal receives η fraction of the total payoff. The first order condition for optimal wage remains the same, except γ is replaced by $\gamma' = \frac{\gamma}{\eta}$.

For the NSB case, a higher effective γ lowers the wage (Proposition 1.1). This implies that if η reduces, the principal will further distort wages downward to reduce the cost of indirect monitoring, resulting in lower verifiable profit and higher private benefit. Intuitively, the principal shares the verifiable profit with non-controlling shareholders, while he fully captures the private benefit. This gives him a perverse incentive to increase private benefit at the expense of verifiable profit.

For the SB regime, the wage does not depend on γ. Therefore, if the principal was already choosing SB regime, there would be no impact on wages, verifiable profits, and corruption. However, the incentive to switch from NSB to SB increases because the minority shareholders share the cost of signal loss, while the principal fully captures the higher private benefit. Therefore, the critical threshold (γ_c) for switching from NSB to SB decreases as η decreases. It can be shown that $\frac{d\gamma_c}{d\eta} = \frac{\gamma_c}{\eta}$ which means that if the principal dilutes his share by 50%, the threshold γ_c is reduced by 50%.

If the manager is corruptible then there is no indirect monitoring, so the principal do not get higher private benefit from reducing wage. However, he increases wage more than optimal, which induces higher managerial effort and higher project success leading to higher private benefit, while the cost of sub-optimal higher wage is shared by the minority shareholders. The net impact depends on whether limited liability (LL) is binding before η reduction. If LL is not binding then managerial wage and effort increases while verifiable profit decreases as η reduces. If LL is binding, there is no impact except for those marginal firms that move from LL binding to non-binding.

Proposition 1.9. *Suppose the principal owns a fraction $\eta \in (0, 1)$ of the firm's cash flow but retains control due to majority voting right. If η decreases then*

a) If the manager is non-corruptible, the critical γ_c for switching to SB reduces in proportion to reduction in η i.e. $\frac{d\gamma_c}{d\eta} = \frac{\gamma_c}{\eta}$. The verifiable profit reduces for those firms that continue to choose NSB or switch to SB after cash flow dilution. The managerial wage reduces for those firm that continue to choose NSB.

b) *If the manager is corruptible and limited liability (LL) is not binding, the managerial wage increase but verifiable profit decreases. No impact if LL is binding.*

This implies that after going public or diluting cash flow rights, the principals of firms that are not choosing SB regime will engage in more corruption at the cost of the firms' profitability. In other words, the principals of such firms realize higher private benefits by expropriating minority shareholders. For this reason, Shleifer and Vishny (1997) cite this conflict between controlling and minority shareholders as the main agency problem in many countries. Lin et al. (2011) show that firms' borrowing costs increase due to this risk, while Lin et al. (2012) demonstrate that banks respond by increasing their monitoring efforts. Does this mean that principals prefer diluting shares to the maximum extent possible? There are two limitations. First, going public also increases scrutiny of the firm and requires firms to display more transparency (even in countries with weaker investor protection). This may hinder the principal's ability to block signals effectively and can therefore discourage the principal from going public. It has been documented that firms in high-corruption countries resist regulations demanding higher transparency and public disclosures (La Porta et al., 2000). Second, anticipating that the principal can expropriate from investors, new investors may discount the firm valuation or demand that the controlling shareholder maintain high cash flow rights to reduce the incentive to expropriate (La Porta et al., 2002). If the investors perfectly anticipate after dilution profit then dilution may not be beneficial for the principal. However, if the investors cannot perfectly anticipate as the principal's type and expected corruption opportunities are private information then the principal has an incentive to dilute. The empirical evidence suggest that entrepreneurs use complex non-transparent mechanism to dilute shares in countries with weaker investor protection (Lin et al., 2011; La Porta et al., 2000).

Remark 1.8. *A corruptible principal may increase corruption by using signal-blocking after the dilution of his cash flow rights. This results in lower verifiable profit, and thus, the principal gains from corruption at the expense of minority shareholders.*

Remark 1.9. *The corruptible owner of a company has a strong incentive to dilute its cash flow rights while maintaining the controlling voting rights in order to increase the value of the private benefit. However, this incentive will be limited if going public increases the transparency requirement or if the outside investors fully anticipate increase in corruption and therefore discount valuation and/or demand more shareholding from the owner.*

1.7.4 Does the principal always hire a manager?

In the base model, we assumed that the principal always hire a manager. We now relax this assumption and allow the principal to decide whether to hire a manager or not at the stage 1. If the principal do not hire a manager then he randomly chooses between project A and B if $z = H$ and between A^c and B^c if $z = C$. He will not be exposed if he implements a corrupt project. Therefore, the principal's expected payoff is $\frac{1}{2}\,V + \frac{1}{2}\,p\,\gamma\,V$.

Proposition 1.10 highlights that hiring a manager is preferable under SB regime for all γ and $p \in (0, 1)$ because SB allows the full private benefit (equal to that without a manager), and the manager adds a strictly positive value in the honest state (since $p < 1$). Therefore, the principal will always hire a manager. If the principal chooses NSB, then for a critically high γ, he will opt not to hire a manager. This is because, when $p\gamma$ is high, the managerial wage is low enough that the manager's effort and value-add is small relative to the loss in private benefit.

Proposition 1.10. *Suppose principal can choose to not hire a manager then*

a) There exists $\gamma_o(p)$ such that for all $\gamma > \gamma_o(p)$, the principal will not hire a manager under NSB.

b) The principal will always hire a manager under SB.

c) The critical threshold for switching to SB $\gamma_c < \gamma_o$ for all $p \in (0, 1)$. Therefore, the principal will hire a manager for all γ and $p \in (0, 1)$.

Proof is shown in Appendix 1.A.9. Proposition 1.10 implies that the principal will always hire a manager.

1.7.5 The manager is rewarded for reporting corruption

Consider a scenario in which the manager receives a reward from regulators for whistle-blowing by the regulators.[22] Since the manager can receive a reward only when the principal is exposed, the manager may not truthfully report the α^c (or β^c) signal she receives. Therefore, the principal will have to offer the manager information rent (an additional incentive compatible constraint) to make the manager truthfully reveal the signal. This means that the benefit of corruption decreases for the corruptible principal, if he does not block signals. If the corruption benefit is small, this will further dampen corruption. However, if the corruption benefit is high, some marginal firms may switch from NSB to SB and corruption will increase. Therefore, the reward mechanism will be beneficial only if the corruption level is low in the economy.

Remark 1.10. *Whistleblower rewards can be more effective when the corruption level is low in the economy, but if the corruption level is high, they can have the unintended consequence of increasing corruption.*

[22] Many government agencies such as FDA, SEC and IRS provide rewards to whistle-blower. Even in developing economies such reward for whistleblowers is prevalent.

1.7.6 What if there are more than four projects

The number of projects is determined by two assumptions: a) the number of possible values of the state variable y, and b) y and z are independent so that every state has both a corrupt and an honest project. We have assumed y takes two values $\{\tilde{a}, \tilde{b}\}$, which is the minimum necessary to model the manager's information advantage. If we have more states, say $n(> 2)$, then the manager's positive effort will still provide the informative signal about the state, and all decisions remain the same. Specifically, the manager will report truthfully, and the principal will respect the manager's report. This will imply that all propositions will hold.

However, the relative difference in principal's payoff between the NSB and SB regimes changes because random selection becomes more costly. Under the SB regime and two states, when the principal receives $z = C$ he selects the project randomly which has probability of success $\frac{1}{2}$. With n states it becomes $\frac{1}{n}$. As n increases, the randomly selected project has a lower chance of success, so the SB regime becomes less attractive, and the critical γ_c for switching to the SB regime increases. If the number of states $n \to \infty$, the randomly selected project will have almost zero chance of success and hence will also not yield private benefit to the principal. This implies that SB will be dominated for all γ when n is large.

Consider another situation where y and z are not independent. Some states do not have a corrupt project, because the project is monitored closely by the government auditors. While some states only have a corrupt project, as necessary approval requires bribing bureaucrats. For illustration, consider that the state $y = \tilde{a}$ only has one good project A, while both B and B^c are the right project in state $y = \tilde{b}$. In such a situation, if the manager receives α and truthfully reports it, the principal may ignore her report and take a risky bet of implementing B^c when the γ is sufficiently high. Knowing that the principal is likely to ignore her, the manager will reduce her effort. Therefore, our results will change. Thus, differential opportunities across states can create additional distortions in managerial effort and the firm's profit. However, it is important to note that this distortion arises from the variation in opportunities across states, not from the manager's indirect monitoring, which was the focus of our paper.

1.7.7 Principal chooses the level of transparency

Let us consider a more general model where the principal can, instead of making a binary decision between either full signal-blocking or no signal-blocking, choose the level of blocking (or transparency) $b \in [0, 1]$. Here, we slightly abuse the notation that b represents degree of signal-blocking. At stage 1, the principal announces the wage and chooses the level of blocking. $b = 0$ is equivalent to no-signal-blocking, and $b = 1$ is equivalent to signal-blocking of the base model. We also assume that the level of blocking is perfectly observable by the manager but not verifiable in the court. We use the subscript "b" to represent the equilibrium endogenous variables (similar to full blocking).

The principal's and the manager's profit function:

$$\Pi_b = \left[(1-b\,p)\,\tfrac{2\lambda(w)+1}{3} + \tfrac{b\,p}{2}\right]\,(V-w) + \left(\tfrac{1-\lambda(w)}{3} + b\,\tfrac{2\lambda(w)+1}{6}\right)\,p\,\gamma\,V$$

$$\Pi_b^m = \left[(1-b\,p)\,\tfrac{2\lambda(e)+1}{3} + \tfrac{b\,p}{2}\right]\,w - C(e)$$

The manager's wage effort equation is given by:

$$(1-b\,p)\,\frac{2w}{3}\,\lambda^{'}(e_b) = C^{'}(e_b) \tag{1.18}$$

$e_b(b,p,w)$ is an increasing function of w and a decreasing function of b and p. With a higher b, the wage effect is muted. The first order conditions for equilibrium wage w_b and b are as follows:

$$\frac{\partial \Pi_b}{\partial w} = 0 \Rightarrow V - w_b - \frac{1}{2}\,\frac{1-b}{1-bp}\,p\,\gamma\,V = \frac{\lambda_b(w_b) + \frac{1}{2} + \frac{3}{4}\frac{bp}{1-bp}}{\lambda_b^{'}} \tag{1.19}$$

$$\frac{\partial \Pi_b}{\partial b} = 0 \Rightarrow$$

$$-\left[p\,\frac{4\lambda_b(w_b)-1}{6} - \frac{2}{3}\,(1-b\,p)\,\frac{\partial \lambda_b}{\partial b}\right]\,(V-w_b) + \left[\frac{2\lambda_b(w_b)+1}{6} - \frac{1}{3}\,(1-b)\,\frac{\partial \lambda_b}{\partial b}\right]\,p\,\gamma\,V = 0 \tag{1.20}$$

It can be shown that for an interior solution ($w_b > 0$ and $b \in (0,1)$), the second order condition is satisfied so that the FOC is sufficient and provides a unique solution.

Proposition 1.11. *If the principal can choose the level of blocking (b), then there exists $\gamma^{'}(p), \gamma^{''}(p)$ such that for all $\gamma < \gamma^{'}(p)$, the optimal $b = 0$, and for all $\gamma > \gamma^{''}(p)$ the optimal $b = 1$; otherwise optimal $b \in (0,1)$ and increases with γ. Further, $\gamma^{'}(p)$ and $\gamma^{''}(p)$ is decreasing in p.*

Proof of Proposition 1.11 is shown in Appendix 1.A.10. We use computer simulation to identify the range of γ for which b has an interior solution. The result is shown in Figure 1.10.[23] Apart from the small range of γ where optimal $b \in (0,1)$, the solution is very similar to the binary decision discussed in Section 1.6. Therefore, all our results and implications still holds with the more general model.

This model strengthens and supports our result on multiple equilibria. We see two extremes, $b = 1$ and $b = 0$, even when we allow partial blocking. In a high corruption environment, high $p\gamma$, firms resort to full blocking, $b = 1$, resulting in much higher

[23] We use the same value of exogenous parameters as in Section 1.6.2.

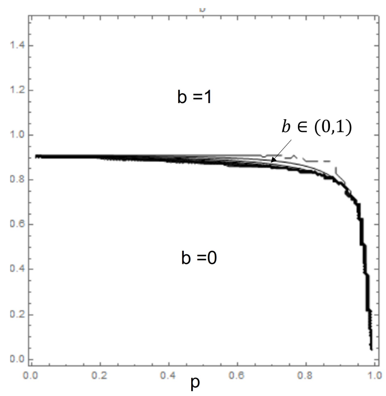


Figure 1.10: Optimal b with different p(x-axis) and γ(y-axis) values

corruption at the expense of reduced efficiency. In a low corruption environment, low $p\gamma$, firms refrain from blocking signals, reducing corruption due to indirect monitoring.

1.8 Concluding Remarks

This paper's contribution lies in analyzing the impact of the principal's (owner's) corruption on a firm's governance. While corruption by owners, board members, or executives has been widely documented and discussed in the literature, the existing literature does not analyze the agency problem that arises within an organization when the principal engages in corruption. We deviate from the traditional agency theory model of corruption and introduce the notion of a corruptible principal; the agency problem arises because the agent can detect the principal's corruption due to its information advantage.

We find that the principal's corruption makes the firm more bureaucratic by reducing transparency and/or managerial incentive. The principal's gain from corruption comes at the expense of the firm's verifiable profit and managerial income. In addition, such corruption can exacerbate conflicts between the firm's controlling shareholder and minority shareholders and distort the talent within the firm.

We also find that the principal's corruption can distort the firm's behavior such that a higher corruption environment breeds more corruption, while corruption is reduced in a lower corruption environment due to managers' indirect monitoring. Therefore, it can give rise to two different types of equilibrium at the economy level.

1.A Appendices

1.A.1 Ex-post probabilities of different state given signal received by the manager

Ex-ante unconditional probability of occurrence of different states:

$Pr(\tilde{a}, H) = Pr(\tilde{b}, H) = \frac{1-p}{2}$ and $Pr(\tilde{a}, C) = Pr(\tilde{b}, C) = \frac{p}{2}$

Unconditional probabilities of the manager receiving different signals under NSB:[24]

$$\begin{aligned} Pr(\alpha) &= Pr(\beta) = \frac{1}{6}[3\lambda(1-p) + (1-\lambda)(1+p)], \\ Pr(\alpha^c) &= Pr(\beta^c) = \frac{1}{6}[3\lambda p + (1-\lambda)(2-p)]. \end{aligned}$$

Under NSB, ex-post probability of true state given signal $s \in S^{NSB}$ received by the manager i.e. $Pr[(y, z)|s]$:

$Pr[(\tilde{a}, H)|\alpha] = Pr[(\tilde{b}, H)|\beta] = \frac{3\lambda(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$

$Pr[(\tilde{b}, H)|\alpha] = Pr[(\tilde{a}, H)|\beta] = \frac{(1-\lambda)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$

$Pr[(\tilde{a}, C)|\alpha] = Pr[(\tilde{b}, C)|\alpha] = Pr[(\tilde{a}, C)|\beta] = Pr[(\tilde{b}, C)|\beta] = \frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$

$Pr[(\tilde{a}, C)|\alpha^c] = Pr[(\tilde{b}, C)|\beta^c] = \frac{3\lambda p}{3\lambda p+(1-\lambda)(2-p)}$

$Pr[(\tilde{a}, C)|\beta^c] = Pr[(\tilde{b}, C)|\alpha^c] = \frac{(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$

$Pr[(\tilde{a}, H)|\alpha^c] = Pr[(\tilde{b}, H)|\beta^c] = Pr[(\tilde{a}, H)|\beta^c] = Pr[(\tilde{b}, H)|\alpha^c] = \frac{(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$

The following table summarizes the ex-post probabilities.

	$(\tilde{a}, H)$	$(\tilde{b}, H)$	$(\tilde{a}, C)$	$(\tilde{b}, C)$
α	$\frac{3\lambda(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$
β	$\frac{(1-\lambda)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{3\lambda(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$
α^c	$\frac{(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$
β^C	$\frac{(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p}{3\lambda p+(1-\lambda)(2-p)}$

From the above table we can derive the following probabilities:

[24] $Pr(\alpha) = Pr[\alpha|(\tilde{a}, H)].Pr[(\tilde{a}, H)] + Pr[\alpha|(\tilde{b}, H)].Pr[(\tilde{b}, H)] + Pr[\alpha|(\tilde{a}, C)].Pr[(\tilde{a}, C)] + Pr[\alpha|(\tilde{b}, C)].Pr[(\tilde{b}, C)]$

	$y=\tilde{a}$	$y=\tilde{b}$	$z=C$	$z=H$
α	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
β	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
α^c	$\frac{3\lambda p+(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{(1-\lambda)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p+(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{2(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$
β^C	$\frac{(1-\lambda)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p+(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{3\lambda p+(1-\lambda)p}{3\lambda p+(1-\lambda)(2-p)}$	$\frac{2(1-\lambda)(1-p)}{3\lambda p+(1-\lambda)(2-p)}$

Table 1.5: Ex-post probabilities of different states given signals received by the agent

Under SB, when α^c and β^c signals are blocked, the ex-post probabilities if $s=\emptyset$:
$Pr[(\tilde{a},H)|\emptyset]=Pr[(\tilde{b},H)|\emptyset]=\frac{(1-\lambda)(1-p)}{2(1-\lambda)(1-p)+(2\lambda+1)p}$
$Pr[(\tilde{a},C)|\emptyset]=Pr[(\tilde{b},c)|\emptyset]=\frac{\frac{1}{2}(2\lambda+1)p}{2(1-\lambda)(1-p)+(2\lambda+1)p}$
Unconditional probabilities of the manager receiving different signals are given by:

$$Pr(\alpha)=Pr(\beta)=\frac{1}{6}[3\lambda(1-p)+(1-\lambda)(1+p)]$$
$$Pr(\emptyset)=\frac{1}{3}[(2\lambda+1)p+2(1-\lambda)(1-p)]$$

	$(\tilde{a},H)$	$(\tilde{b},H)$	$(\tilde{a},C)$	$(\tilde{b},C)$
α	$\frac{3\lambda(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$
β	$\frac{(1-\lambda)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{3\lambda(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$
$\emptyset$	$\frac{(1-\lambda)(1-p)}{2(1-\lambda)(1-p)+(2\lambda+1)p}$	$\frac{(1-\lambda)(1-p)}{2(1-\lambda)(1-p)+(2\lambda+1)p}$	$\frac{\frac{1}{2}(2\lambda+1)p}{2(1-\lambda)(1-p)+(2\lambda+1)p}$	$\frac{\frac{1}{2}(2\lambda+1)p}{2(1-\lambda)(1-p)+(2\lambda+1)p}$

	$y=\tilde{a}$	$y=\tilde{b}$	$z=C$	$z=H$
α	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
β	$\frac{(1-\lambda)}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{3\lambda(1-p)+(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{2(1-\lambda)p}{3\lambda(1-p)+(1-\lambda)(1+p)}$	$\frac{(2\lambda+1)(1-p)}{3\lambda(1-p)+(1-\lambda)(1+p)}$
$\emptyset$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{(2\lambda+1)p}{2(1-\lambda)(1-p)+(2\lambda+1)p}$	$\frac{2(1-\lambda)(1-p)}{2(1-\lambda)(1-p)+(2\lambda+1)p}$

Table 1.6: Ex-post probabilities of different states given signals received by the manager when α^c and β^c are blocked

1.A.2 Proof of proposition 1.2

Proof. The absence of indirect monitoring ($q=0$) implies that the principal will not be caught, therefore the principal will always implement corrupt project when $z=C$.

The manager's effort wage equation will remain the same as Equation (1.4), as the marginal cost and benefit of her effort do not change because she does not benefit from corruption. However, the equilibrium wage and the principal's payoff in equilibrium will now be given by Equations (6') and (8'), respectively:

$$\frac{2\lambda_n^{'}(w_n^{'})+1}{3}(V-w_n^{'}+p\gamma V)=\frac{2\lambda_n(w_n^{'}+1)}{3} \tag{6'}$$

$$\Pi_{n'}=\frac{2\lambda_n(w_n^{'})+1}{3}(V-w_n^{'}+p\gamma V) \tag{8'}$$

where $w_n^{'}$ represents the optimal wage without indirect monitoring.

Comparing Equation (1.6) and (6), we can see that without indirect monitoring the wage $w_n^{'}$ increases with $p\gamma$, while the wage w_n with indirect monitoring decreases with $p\gamma$. Therefore, the equilibrium wage is lower with indirect monitoring $w_n < w_n^{'}$. In addition, indirect monitoring significantly reduces corruption. The private benefit from corruption due to indirect monitoring is lower by: $(\frac{2\lambda_n(w_n^{'})+1}{3}-\frac{1-\lambda_n(w_n)}{3})p\gamma V>0$.

Now we see how verifiable (reported) profit changes due to indirect monitoring. Let VP and $VP^{'}$ represent the verifiable profit with and without indirect monitoring, respectively. We already know that w_n decreases while $w_n^{'}$ increases with the expected benefit from corruption $p\gamma$. We can evaluate how VP and $VP^{'}$ changes with p using implicit function theorem as shown below:

$$\frac{dVP}{dp}=\frac{\frac{1}{3}p(\lambda_n^{'}(w_n)\gamma V)^2}{\lambda_n^{''}(w_n)(V-w_n-\frac{1}{2}p\gamma V)-2\lambda_n^{'}(w_n)}=\frac{\frac{1}{3}p(\lambda_n^{'}(w_n)\gamma V)^2}{\lambda_n^{''}(w_n)(\frac{\lambda_n(w_n)+\frac{1}{2}}{\lambda_n^{'}(w_n)})-2\lambda_n^{'}(w_n)}<0$$

$$\frac{dVP^{'}}{dp}=\frac{\frac{2}{3}p(\lambda_n^{'}(w_n^{'})\gamma V)^2}{\lambda_n^{''}(w_n^{'})(V-w_n^{'}+p\gamma V)-2\lambda_n^{'}(w_n^{'})}=\frac{\frac{2}{3}p(\lambda_n^{'}(w_n^{'})\gamma V)^2}{\lambda_n^{''}(w_n^{'})(\frac{\lambda_n(w_n^{'})+\frac{1}{2}}{\lambda_n^{'}(w_n^{'})})-2\lambda_n^{'}(w_n^{'})}<0$$

We can easily show that the function $F(w)=|\frac{(\lambda_n^{'}(w))^2}{\lambda_n^{''}(w)(\frac{\lambda_n(w)+\frac{1}{2}}{\lambda_n^{'}(w)})-2\lambda_n^{'}(w)}|$ decreases with w

Both VP and $VP^{'}$ are decreasing in p. However, at the lower value of p when both w_n and $w_n^{'}$ are closer and have less distortion. $VP^{'}$ decreases at faster rate than VP due to factor 2. However, as p increases $w_n^{'}$ becomes much higher relative to w_n and the impact of decreasing F(w) overwhelms the factor 2. We can show that VP and $VP^{'}$ behaves exactly the similar way for γ.

Therefore, below a critical value of $p\gamma$ verifiable profit is higher with monitoring than without monitoring and the opposite is true if $p\gamma$ above the critical value. □

1.A.3 Solution of SB Problem

SBP problem:

$$\max_w \ (1-p)\,\frac{2\lambda(e)+1}{3}\,(V-w)+\frac{p}{2}\,(V-w+\gamma V)$$

subject to:

$$e=\arg\max_e\,((1-p)\,\frac{2\lambda(e)+1}{3}+\frac{p}{2})\,w-C(e) \tag{GIC}$$

$$((1-p)\,\frac{2\lambda(e)+1}{3}+\frac{p}{2})\,w-C(e)\geq 0 \tag{IR}$$

$$w\geq 0 \tag{LL}$$

Lemma 1.9.

(a) *Given (LL) and (GIC), (IR) holds.*

(b) *One can replace the global incentive compatibility condition (GIC) by the manager's first order condition:*

$$\frac{2}{3}(1-p)w\lambda^{'}(e_b) = C'(e_b). \tag{1.21}$$

where e_b is a unique effort that satisfies (1.21).

(c) *There exists a well defined, strictly increasing and concave function $E : \mathbb{R}^{++} \rightarrow \mathbb{R}^{++}$ such that*

$$e_b = \begin{cases} E(w(1-p)) & \text{if } w > 0, \\ 0, & \text{if } w = 0 \end{cases}$$

Proof of Lemma 1.9. (a) Given the limited liability constraint (LL), i.e. $w \geq 0$, the manager can always opt for $e = 0$, and ensure that she obtains a non-negative expected payoff. So that given (GIC), (IR) is necessarily satisfied.

(b) Given Assumption 1.1 and $w > 0$, the manager's objective function is strictly concave in e. Thus the first order condition will given a unique e_b for all $w > 0$ and (1.21) fully characterizes the (GIC).

(c) From (1.21) and Assumption 1.1, $e_b = E(w(1-p))$ where E is a well defined strictly increasing and concave function as defined in Lemma 3.1 (recall $E = f^{-1}(e)$ where $f(e) = \frac{3}{2}\frac{C'(e)}{\lambda'(e)}$). Inada conditions in Assumption 1.1, ensure that $E(w) > 0$ for all $w > 0$. If $w = 0$ then from GIC $e_b = 0$. □

Let us define $\lambda_b(w,p) \equiv \lambda(e_b)$. Thus we can re-write the principal's objective function as:

$$\Pi_b = (1-p)\frac{2\lambda_b(w)+1}{3}(V-w) + \frac{p}{2}(V - w + \gamma V). \tag{1.22}$$

which is strictly concave in w.[25] Thus the principal's problem simplifies to maximizing (1.22) subject to the (LL). The first order condition of the principal's problem, ignoring the (LL) is:

$$\underbrace{\frac{2}{3}\lambda_b^{'}(w_b)(1-p)(V-w_b)}_{Marginal\ benefit\ of\ wage} = \underbrace{\frac{2\lambda_b(w_b)+1}{3}(1-p) + \frac{p}{2}}_{Marginal\ direct\ cost\ of\ wage} \tag{1.10}$$

Note that the FOC cannot be satisfied for any $w \leq 0$.[26] Thus the (LL) is necessarily satisfied, and (1.10) yields the unique optimal w_b.

[25] $\frac{d^2\Pi_b}{dw^2} = (1-p)[\frac{2}{3}\lambda_b^{''}(w_b)(V-w_b) - \frac{4}{3}\lambda_b^{'}(w_b)] < 0$ for all $p \in (0,1)$. Note:$\lambda_b^{''}(w_b) < 0$ as $\lambda_b(w)$ is a strictly concave function: monotonic concave transformation of a strictly concave function $e_b(w)$.

[26] From Assumption 1.1, $\lambda^{'}(w) \rightarrow \infty$ when $w \rightarrow 0$ and hence FOC is not satisfied for $w \leq 0$

Characterization of wage w_b in SB

Lemma 1.10. *(a) w_b is independent of γ.*

(b) w_b is non-monotonic in p if λ_b is sufficiently responsive to effort when $p = 0$, which means w_b increases with p if p is below a certain threshold and decreases with p above this threshold. For all other cases, w_b is monotonically decreasing in p. Further, $w_b \to 0$ as $p \to 1$.

(c) w_b is strictly concave in p and $-\frac{w_b}{1-p} < \frac{dw_b}{dp} < \frac{w_b}{1-p}$

Proof of Lemma 1.10: (a) Since the wage equation (1.10) does not depend on γ directly or indirectly through e_b, the w_b is independent of γ.

(b) We can rewrite (1.10) for $p < 1$ as:

$$\frac{2}{3}\lambda_b'(w_b)(V - w_b) = \frac{2\lambda_b(w_b) + 1}{3} + \frac{p}{2(1-p)} \tag{1.23}$$

Notice that when $p \to 1$ the RHS approaches ∞, and from Assumption 1.1 LHE approaches ∞ when $w_b \to 0$. Using implicit function theorem on (1.10):

$$\frac{dw_b}{dp} = -\frac{\frac{2}{3}\frac{\partial^2\lambda_b}{\partial p \partial w_b}(V-w_b) - \frac{2}{3}\frac{\partial \lambda_b}{\partial p} - \frac{1}{2(1-p)^2}}{\frac{2}{3}\lambda_b''(w_b)(V-w_b) - \frac{4}{3}\lambda_b'} \text{ where } \lambda_b' = \frac{\partial \lambda_b}{\partial w_b} \text{ and } \lambda_b'' = \frac{\partial^2 \lambda_b}{\partial w_b^2}$$

From the definition of $\lambda_b(w_b, p) \equiv \lambda(E(w(1-p)))$ we can derive the following terms:

$$(1-p)\frac{\partial^2 \lambda_b}{\partial w_b \partial p} = -\lambda_b'' w_b - \lambda_b' \tag{1.24}$$

$$(1-p)\frac{\partial \lambda_b}{\partial p} = -\lambda_b' w_b \tag{1.25}$$

Substituting above terms,

$$\frac{dw_b}{dp} = \frac{\frac{1}{1-p}\left[\frac{2}{3}(\lambda_b'' w_b + \lambda_b')(V - w_b) - \frac{2}{3}\lambda_b' w_b + \frac{1}{2(1-p)}\right]}{\frac{2}{3}\lambda_b''(w_b)(V - w_b) - \frac{4}{3}\lambda_b'} \tag{1.26}$$

The denominator of the above expression is negative for all p, but the numerator's sign and hence the sigh of $\frac{dw_b}{dp}$ depends on the concavity of function λ.

- If $\lambda_b(w, p) = ln(w(1-p))$ (i.e. $\frac{\partial^2 \lambda_b}{\partial w_b \partial p} = 0$), then the numerator is negative for $p < \frac{1}{4}$ and hence w_n is non-monotonic in p; increasing in p if $p < \frac{1}{4}$ and decreasing in p otherwise.

- If $\lambda_b(w, p)$ is a linear or power function as $\lambda_b(w, p) = (w(1-p))^\alpha$ where $\alpha \leq 1$ (i.e. $\frac{\partial^2 \lambda_b}{\partial w_b \partial p} < 0$) then numerator is positive for all p and hence w_b is monotonically decreasing with p.

- If λ is of the form such that $\lambda_b(p, w) = 1 - (w(1-p))^{-\alpha}$ where $\alpha > 0$ (i.e $\frac{\partial^2 \lambda_b}{\partial w_b \partial p} > 0$) then the numerator is positive for all p if $\alpha \leq \frac{1}{2}$.

If we use functional representation specified in Assumption 1.3, we can derive the following terms:

$$\lambda_b(w, p) = 1 - (w(1-p))^{-\frac{1}{\kappa+1}}, \frac{\partial^2 \lambda_b}{\partial w_b \partial p} = \frac{\lambda_b'(w_b)}{(\kappa+1)(1-p)} \text{ and } \frac{\partial \lambda_b}{\partial p} = -\frac{1 - \lambda_b(w_b)}{(\kappa+1)(1-p)} \tag{1.27}$$

Substituting above terms, $\frac{dw_b}{dp} = -\frac{\frac{2}{3}\frac{1}{(1-p)(\kappa+1)}\lambda_b'(w_b)(V-w_b)+\frac{2}{3}\frac{1-\lambda_b}{(1-p)(\kappa+1)}-\frac{1}{2(1-p)^2}}{\frac{2}{3}\lambda_b''(w_b)(V-w_b)-\frac{4}{3}\lambda_b'}$

Replacing $\frac{2}{3}\lambda_b'(w_b)(V - w_b)$ from FOC, $\frac{dw_b}{dp} = -\frac{\frac{1}{(1-p)(\kappa+1)}\frac{2\lambda_b+1}{3}+\frac{p}{2(1-p)})+\frac{2}{3}\frac{1-\lambda_b}{(1-p)(\kappa+1)}-\frac{1}{2(1-p)^2}}{\frac{2}{3}\lambda_b''(w_b)(V-w_b)-\frac{4}{3}\lambda_b'}$

Simplifying we get,

$$\frac{dw_b}{dp} = \frac{\frac{1}{2(1-p)^2}(1 - \frac{2-p}{\kappa+1})}{\frac{2}{3}\lambda_b''(w_b)(V - w_b) - \frac{4}{3}\lambda_b'(w_b)} \tag{1.28}$$

Since the numerator is positive for $\kappa \geq 1$, and the denominator is negative for increasing and strictly concave $\lambda_b(w)$, we have $\frac{dw_b}{dp} < 0$. This implies that for the chosen functional form w_b is a continuously decreasing function of p.

(c) We first show that $\lim_{p\to 1} \frac{dw_b}{dp} = -\infty$.

We can see from FOC that $\lim_{p\to 1} \lambda'(1-p) = +\frac{3}{4V}$. Using this we can derive: $\lim_{p\to 1} \frac{dw_b}{dp} = -\frac{w_b}{1-p}\delta$ where δ is a constant that does not depend on p. As $1-p$ which is linear reaches zero faster than w_b due to concavity of λ, we get $\lim_{p\to 1} \frac{dw_b}{dp} = -\infty$.

Since $\frac{dw_b}{dp}\Big|_{p\to 1} - \frac{dw_b}{dp}\Big|_{p=0} < 0$, w_b is a strictly concave function in p. Note that this is irrespective of any functional form.

From concavity of w_b and $w_b \to 0$ as $p \to 1$, we get $\frac{dw_b}{dp} > -\frac{w_b}{1-p}$. The equation (1.26) can be written as $\frac{dw_b}{dp} = \frac{w_b}{1-p} + \frac{\frac{2}{3}\lambda_b' V + \frac{1}{2(1-p)}}{\frac{2}{3}\lambda_b''(V-w_b)-\frac{4}{3}\lambda_b'} \cdot \frac{1}{1-p}$ which implies $\frac{dw_b}{dp} < \frac{w_b}{1-p}$ as the second term is negative. Hence $-\frac{w_b}{1-p} < \frac{dw_b}{dp} < \frac{w_b}{1-p}$ □

Proof of Proposition 1.3

(a) Follows from Lemma 1.5, and strict concavity of the principal's objective function and the manager's objective function (GIC).

(b) Follows from Lemma 1.10.

(c) From Lemma 1.9, the equilibrium effort is given by $e_b = E(w_b(1-p))$. e_b doe not depend on the γ as w_b is independent of γ. Also note that $e_b = E(w_b(1-p)) < E(V) = e_{sp}$ as $w_b < V$ and $(1-p) < 0$.

Since, $\frac{de_b}{dp} = E'.((1-p)\frac{dw_b}{dp} - w_b) < 0$ (From Lemma 1.10c), therefore, e_b decreasing in p for all $p \in (0, 1)$. Note that $\frac{de_b}{dp} <$ even if $\frac{dw_b}{dp} > 0$.

(d) The equilibrium payoff of the principal is given by:

$$\Pi_b = \underbrace{\frac{2\lambda_b(w)+1}{3}(1-p)(V-w_b) + \frac{p}{2}(V-w_b)}_{Verifiable\ profit} + \underbrace{\frac{p}{2}\gamma V}_{Private\ benefit} \quad (1.12)$$

From the envelope theorem,

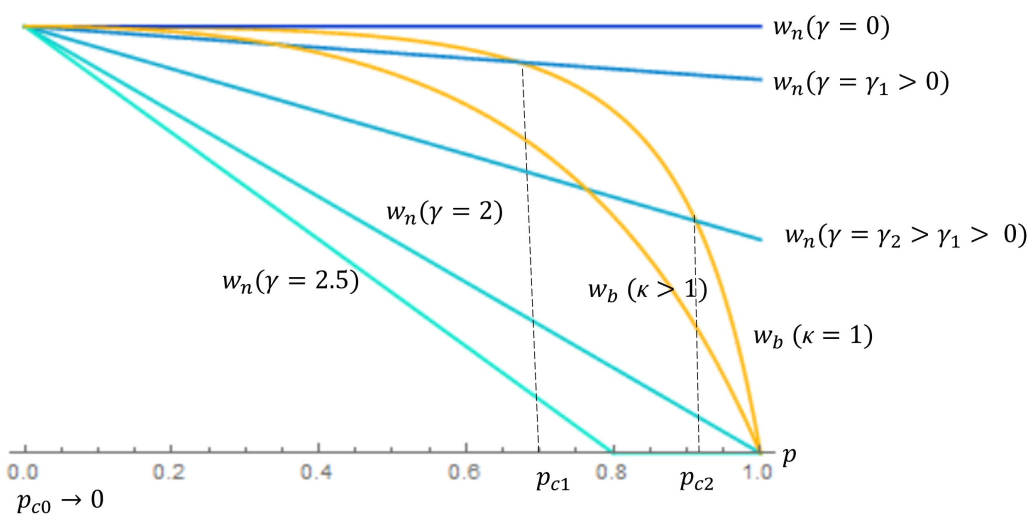
$$\frac{d\Pi_b}{dp} = \underbrace{\frac{1}{2}\gamma V}_{Private\ benefits} - \underbrace{(\frac{2\lambda_b(w_b)+1}{3} - \frac{1}{2})(V-w_b)}_{Information\ loss} + \underbrace{\frac{2}{3}(1-p)\frac{\partial\lambda_b}{\partial p}(V-w_b)}_{Lower\ managerial\ effort} \quad (1.29)$$

$\frac{d\Pi_b}{dp} < 0$ iff $\frac{1}{2}\gamma V < (\frac{2\lambda_b(w_b)+1}{3} - \frac{1}{2} - \frac{2}{3}(1-p)\frac{\partial\lambda_b}{\partial p})(V-w_b)$. Since the left hand side of the above inequality is independent of γ, we can derive the critical threshold $\bar{\gamma} > 0$ so that:

$$\frac{d\Pi_b}{dp} < 0 \text{ iff } \gamma < \bar{\gamma}(p) \text{ where } \bar{\gamma}(p) = (\frac{4\lambda_b(w_b)-1}{3} - \frac{4}{3}(1-p)\frac{\partial\lambda_b}{\partial p})(1-\frac{w_b}{V}) \quad (1.30)$$

Similarly using envelope theorem, $\frac{d\Pi_b}{dV} > 0$ and $\frac{d\Pi_b}{d\gamma} = \frac{p}{2}V > 0$.

1.A.4 Proof of Proposition 1.5



Proof. Equation (1.6) is a first order condition for w_n

$$\frac{2}{3}\lambda_n'(w_n)(V-w_n) = \frac{2\lambda_n(w_n)+1}{3} + \frac{\lambda_n'(w)}{3}p\gamma V \quad (1.6)$$

Equation (1.23) is a first order condition for w_b

$$\frac{2}{3}\lambda_b'(w_b)(V - w_b) = \frac{2\lambda_b(w_b) + 1}{3} + \frac{p}{2(1-p)} \tag{1.23}$$

For $p = 0$, $w_b = w_n = w_0 > 0$ (from Lemma 1.6) as both the above wage equation converges. Let's denote $\lambda_n(w_0) = \lambda_b(w_0) = \lambda(w_0)$.

From implicit function theorem (See Proposition 1 for $\frac{dw_n}{dp}$ and Lemma 1.10 for $\frac{dw_b}{dp}$) $|\frac{dw_n}{dp}|_{p=0} = |\frac{\lambda_n'(w_0)\gamma V}{2\lambda_n''(w_0)(V-w_0)-4\lambda_n'(w_0)}| > |\frac{\frac{3}{2}\frac{\kappa-1}{\kappa+1}}{2\lambda_n''(w_0)(V-w_0)-4\lambda_n'(w_0)}| = |\frac{dw_b}{dp}|_{p=0}$ iff $\gamma > \frac{3}{2V\lambda_n'(w_0)}\frac{\kappa-1}{\kappa+1} = \gamma'$ where $\kappa \geq 1$ represents the convexity of $C(e)$ or concavity of $\lambda(e)$ as discussed in Section 1.4.2.

Therefore, if $\gamma > \gamma'$, w_n is below w_b near $p = 0$, otherwise w_n is above w_b.

Now let's look how w_n and w_b changes with p. From Proposition 1.1, w_n is decreasing in p for all $\gamma > 0$ and $w_n = 0$ iff $\gamma \geq \frac{2}{p}$. Therefore, $w_n > 0$ for all $p \in (0, 1)$ if $\gamma < 2$. While, from Lemma 3, w_b is decreasing in p and $w_b \to 0$ as $p \to 1$. Also $\lim_{p\to1} |\frac{dw_n}{dp}| < \lim_{p\to1} |\frac{dw_b}{dp}| = \infty$. Therefore w_b will cut w_n from the top if $\gamma' < \gamma < 2$. w_n will remain above w_b for all $p \in (0, 1)$ if $\gamma < \gamma'$. If $\gamma > 2$, w_n will become zero for $p < 1$ and w_b will approach zero at $p = 1$ and therefore w_n will remain below w_b for all $p \in (0, 1)$.

To summarize for $\gamma < 2$:

- $w_n = w_b = w_0$ at $p = 0$
- w_n and w_b is continuously decreasing in p for all $\gamma > 0$ and $p \in (0, 1)$
- if $\gamma < \gamma' = \frac{3}{2V\lambda_n'(w_0)}\frac{\kappa-1}{\kappa+1}$ then w_n is above w_b for all p.
- if $\gamma > \gamma'$ then w_n is below w_b near $p = 0$ and w_b cuts w_n from above at $p_c \in (0, 1)$

For $\gamma > 2$

- $w_n = w_b = w_0$ at $p = 0$
- w_n and w_b is continuously decreasing in p for all $p \in (0, 1)$
- w_n is below w_b near $p = 0$ and w_n approaches zero at $p = \frac{2}{\gamma} < 1$. Therefore, w_n remains below w_b for all $p \in (0, 1)$.

Therefore, for all $\gamma' < \gamma < 2$, there exists $p_c(\gamma) \in (0, 1)$ such that $w_n < w_b$ if $p < p_c$ and $w_n > w_b$ if $p > p_c$. If $\gamma < \gamma'$ then $w_n > w_b$ for all $p \in (0, 1)$ and hence $p_c(\gamma) = 0$. If $\gamma > 2$ then $w_n > w_b$ for all $p \in (0, 1)$ and hence $p_c(\gamma) = 1$.

In addition, $p_c(\gamma)$ is increasing function of γ because the slope of w_n decreases with γ while that of w_b does not change with γ.

□

1.A.5 Proof of Proposition 1.6

VP_n and VP_b can be derived from Equation (1.7) and (1.22) after removing the private benefits:

$$VP_n = \frac{2\lambda_n(w_n)+1}{3}(V - w_n)$$
$$VP_b = [(1-p)\frac{2\lambda_b(w_b)+1}{3} + \frac{p}{2}](V - w_b)$$

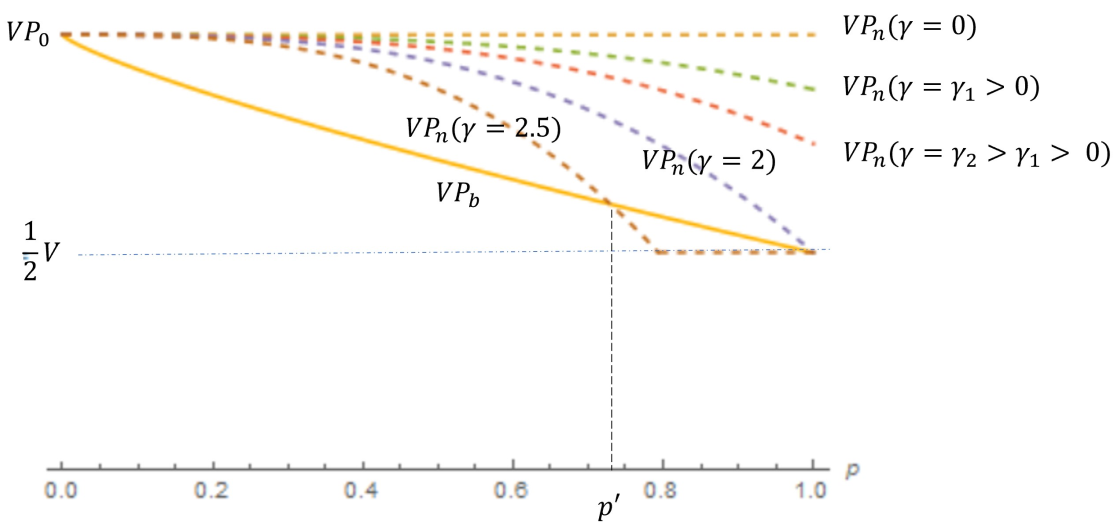


Figure 1.11: Verifiable profit in two cases

Figure 1.11 shows the verifiable profit in the two cases for different values of p and γ. First, we prove the following lemma.

Lemma 1.11. *a) VP_n is continuously decreasing and strictly concave in p if $p\gamma < 2$ and b) VP_b is continuously decreasing and strictly convex in p for all $\gamma > 0$*

Proof of Lemma 1.11

(a) $\frac{dVP_n}{dp} = \frac{\partial VP_n}{\partial w_n}\frac{dw_n}{dp} + \frac{\partial VP_n}{\partial p} = \frac{\partial VP_n}{\partial w_n}\frac{dw_n}{dp}$. No direct effect as the VP_n does not depend on p. However, there is an indirect effect through wage which decreases with p.

Hence, $\frac{dVP_n}{dp} = \frac{1}{3}\lambda_n{}'(w_n)p\gamma V \, . \, \frac{dw_n}{dp} < 0$ for all $p \in (0,1)$ and $p\gamma < 2$ and $\frac{dVP_n}{dp} - 0$ when $p\gamma \geq 2$. Hence, VP_n is continuous and strictly decreasing in p if $p\gamma < 2$.

For $p\gamma < 2$, $\frac{d^2VP_n}{dp^2} = \frac{1}{3}\lambda_n''(w_n)p\gamma V(\frac{dw_n}{dp})^2 + \frac{1}{3}\lambda_n'(w_n)\gamma V\frac{dw_n}{dp} + \frac{1}{3}\lambda_n'(w_n)p\gamma V\frac{d^2w_n}{dp^2} < 0$ (as all three terms are -ve) because λ_n is increasing and strictly concave, and w_n is concave w.r.t. p

(b) $\frac{dVP_b}{dp} = \frac{\partial VP_b}{\partial w_b}\frac{dw_b}{dp}(= 0 \text{ from FOC }) + \frac{\partial VP_b}{\partial p} = \frac{\partial VP_b}{\partial p}$ (only direct effect).

$\frac{dVP_b}{dp} = [\frac{1}{2} - \frac{2\lambda_b(w_b)+1}{3} + \frac{2}{3}(1-p)\frac{\partial\lambda_b}{\partial p}](V - w_b) = [\frac{1}{2} - \frac{2\lambda_b(w_b)+1}{3} - \frac{2}{3}\lambda_b' w_b](V - w_b)$ ($\frac{\partial\lambda_b}{\partial p}$ from (1.25))

Since $\frac{1}{4} < \lambda_b$, $\frac{dVP_b}{dp} < 0$ for all $p \in (0,1)$ and hence VP_b is strictly decreasing in p.

We can replace $\lambda_b^{'}$ from FOC (1.23) in the above expression to get: $\frac{dVP_b}{dp} = (\frac{1}{2} - \frac{2\lambda_b+1}{3})V - \frac{w_b}{2(1-p)}$.

$\frac{d^2VP_b}{dp^2} = \frac{\partial^2 VP_b}{\partial w_b \partial p}\frac{dw_b}{dp} + \frac{\partial^2 VP_b}{dp^2}$

$\frac{d^2VP_b}{dp^2} = (-\frac{2}{3}\frac{\partial \lambda_b}{\partial p}V - \frac{w_b}{2(1-p)^2}) - (\frac{2}{3}\lambda_b{}'V + \frac{1}{2(1-p)})\frac{dw_b}{dp}$ (replacing $\frac{\partial \lambda_b}{\partial p}$ using (1.25))

$\frac{d^2VP_b}{dp^2} = \frac{w_b}{(1-p)}(\frac{2}{3}\lambda_b^{'}V - \frac{1}{2(1-p)}) - (\frac{2}{3}\lambda_b{}'V + \frac{1}{2(1-p)})\frac{dw_b}{dp}$

Since $\frac{dw_b}{dp} < 0$ and $\frac{2}{3}\lambda_b{}'(1-p)V \geq \frac{1}{2}$, $\frac{d^2VP_b}{dp^2} > 0$ and hence VP_b is strictly convex.

□

Proof of Proposition 1.6

We know that when $p = 0$, NSB and SB are equivalent. Let's say that at $p = 0$, $w_n = w_b = w_0 > 0$, $\lambda_n(w_0) = \lambda_b(w_0) = \lambda_0 > \frac{1}{4}$ and $VP_n = VP_b = VP_0 = \frac{2\lambda_0+1}{3}(V - w_0) > \frac{1}{2}V$

First consider the case when $\gamma < 2$.

- At $p = 0$, $VP_n = VP_b = VP_0$
- As $p \to 1$, $VP_n > \frac{V}{2}$ and $VP_b = \frac{V}{2}$ because $w_n > 0$ if $p\gamma < 2$(proposition 1.1) and $w_b \to 0$ (proposition 1.3)
- VP_b is continuously decreasing and strictly convex in p (Lemma 1.11)
- VP_n is continuously decreasing and strictly concave in p (Lemma 1.11)

The above 4 statements implies that $VP_n > VP_b$ for all p when $\gamma < 2$ (see Figure 1.11)

For $\gamma > 2$

- At $p = 0$, $VP_n = VP_b = VP_0$
- $VP_n = \frac{V}{2}$ for $p \in [\frac{2}{\gamma}, 1]$ because $w_n = 0$ for all $p\gamma \geq 2$ (proposition 1.1)
- $VP_b > \frac{V}{2}$ for all $p < 1$
- VP_n cuts VP_b from above as VP_n is concave and decreasing and VP_b is convex and decreasing

The above 4 statements implies that there exist $p^{'}$ such that $VP_b < VP_n$ if $p < p^{'}$ and $VP_b > VP_n$ if $p > p^{'}$

□

1.A.6 Proof of Proposition 1.7

Proof. Proof is similar to Proposition 1.6. SS_n and SS_b can be seen from equation (5) and (11) by removing the wage payment and instead subtracting the cost of effort of the manager.

Step 1: For $\gamma = 2$, the end points at $p = 0$ and $p = 1$ are same for both the the curve SS_n and SS_b

From Lemma 1.6, when $p = 0$, $w_n = w_b = w_0$, $e_n(w_0) = e_b(w_0) = e(w_0)$, $\lambda(e_n(w_0)) = \lambda(e_b(w_0)) = \lambda_0$.

Therefore, $SS_n = SS_b = SS_0 = \frac{2\lambda_0+1}{3}V - C(e(w_0))$

Note: Social surplus is higher than firm verifiable surplus at $p = 0$ i.e. $SS_0 > VP_0$. This is because manager is getting positive surplus due to limited liability rent i.e. $\frac{2\lambda_0+1}{3}w_0 > C(e(w_0))$.

As $p \to 1$, $w_b \to 0$, $e_b(w_b) \to 0$ and $SS_b \to \frac{V}{2}$.

We also know that $w_n = 0$ for $\gamma \geq \frac{2}{p}$. Therefore, if $\gamma = 2$ and $p \to 1$ then $w_n \to 0$ and $SS_n \to \frac{V}{2}$. So $SS_n = SS_b = \frac{V}{2}$ at $p = 1$ and $\gamma = 2$.

Step 2: SS_n is decreasing and strictly concave and SS_b is decreasing and strictly convex in p.

$\frac{dSS_n}{dp} = \frac{\partial SS_n}{\partial w_n}\frac{dw_n}{dp} + \frac{\partial SS_n}{\partial p} = \frac{\partial VP_n}{\partial w_n}\frac{dw_n}{dp}$ (no direct effect)

$\frac{dSS_n}{dp} = [\frac{2}{3}V\lambda'(e_n(w_n))e'(w_n) - C'(e_n(w_n))e'(w_n)]\frac{dw_n}{dp}$

$= [\frac{2}{3}V\lambda'(e_n(w_n))e'(w_n) - \frac{2}{3}w_n\lambda'(e_n(w_n)e_n'(w_n)]\frac{dw_n}{dp}$ Replacing $C'(e)$ from wage effort equilibrium

$= [\frac{2}{3}V\lambda_n'(w_n) - \frac{2}{3}w_n\lambda_n'(w_n)]\frac{dw_n}{dp}$ from definition of $\lambda_n'(w_n) = \lambda'(e_n).\frac{de_n}{dw_n}$

$= [\frac{2}{3}\lambda_n'(w_n)(V - w_n)]\frac{dw_n}{dp} < 0$ as $\frac{dw_n}{dp} < 0$ for $\gamma < 2$

$\frac{d^2SS_n}{dp^2} = \frac{2}{3}[\lambda_n''(w_n)(V - w_n) - \lambda_n'(w_n)](\frac{dw_n}{dp})^2 + [\frac{2}{3}\lambda_n'(w_n)(V - w_n)]\frac{d^2w_n}{dp^2} < 0$ because both terms are -ve as $\lambda_n(w)$ is increasing and strictly concave and w_n is concave w.r.t. p. Therefore SS_n is decreasing and strictly concave w.r.t. p

$\frac{dSS_b}{dp} = \frac{\partial SS_b}{\partial w_b}\frac{dw_b}{dp} + \frac{\partial SS_b}{\partial p}$

$= [\frac{2}{3}(1-p)V\lambda'(w_b) - C'(e_b(w_b))\frac{de_b}{dw_b}]\frac{dw_b}{dp} + [\frac{1}{2} - \frac{2\lambda(e_b(w_b))+1}{3} + \frac{2}{3}(1-p)\frac{\partial\lambda}{\partial p}]V - C'(e_b)\frac{\partial e_b}{\partial p}$

Replacing $C'(e_b)$ from wage effort equation

$\frac{dSS_b}{dp} = [\frac{2}{3}(1-p)V\lambda'(w_b) - \frac{2}{3}w_b(1-p)\lambda'(e_b(w_b))e'(w_b)]\frac{dw_b}{dp} + [\frac{1}{2} - \frac{2\lambda(e_b(w_b))+1}{3} + \frac{2}{3}(1-p)\frac{\partial\lambda}{\partial p}]V - \frac{2}{3}w(1-p)\lambda'(e_b)\frac{\partial e_b}{\partial p}$

Replacing $\lambda'(e_b)\frac{de_b}{dw_b} = \lambda_b'(w_b)$ and $\lambda'(e_b)\frac{\partial e_b}{\partial p} = \frac{\partial\lambda_b}{\partial p}$ $\frac{dSS_b}{dp} = \frac{2}{3}(1-p)\lambda_b'(w_b)(V - w_b)\frac{dw_b}{dp} + [\frac{1}{2} - \frac{2\lambda(w_b)+1}{3}]V + \frac{2}{3}(1-p)\frac{\partial\lambda_b}{\partial p}(V - w_b)$ $\frac{dSS_b}{dp} = (\frac{2\lambda_b+1}{3}(1-p) + \frac{p}{2})\frac{dw_b}{dp} + [\frac{1}{2} - \frac{2\lambda(w_b)+1}{3}]V + \frac{2}{3}(1-p)\frac{\partial\lambda_b}{\partial p}(V - w_b)$ (from FOC)

$\frac{dSS_b}{dp} < 0$ for all $p \in (0, 1)$ as all three terms are negative because $\frac{dw_b}{dp} < 0$, $\lambda_b(w_b) > \frac{1}{4}$ and $\frac{\partial\lambda_b}{\partial p} < 0$. Therefore, SS_b is continuously decreasing in p.

To show that SS_b is strictly convex we check the inequality: $SS_b\big|_{p=1} - SS_b\big|_{p=0} > \frac{dSS_b}{dp}\big|_{p=0} \Rightarrow \frac{V}{2} - (\frac{2\lambda_0+1}{3}V - C(e(w_0)) > \frac{2\lambda_0+1}{3}\frac{dw_b}{dp}\big|_{p=0} + (\frac{1}{2} - \frac{2\lambda_0+1}{3})V + \frac{2}{3}(1-p)\frac{\partial\lambda}{\partial p}\big|_{p=0}(V-w_0) \Rightarrow$
$C(e(w_0)) > \frac{2\lambda_0+1}{3}\frac{dw_b}{dp}\big|_{p=0} + \frac{2}{3}(1-p)\frac{\partial\lambda}{\partial p}\big|_{p=0}(V-w_0)$

LHS in the above inequality is positive as $w_0 > 0$ and the RHS is negative (first term is ≤ 0 and second term is negative) so this inequality is always satisfied. Hence, SS_b is strictly convex.

Step 3: From step 1 and step 2, it follows that $SS_n > SS_b$ for $\gamma = 2$

- The end points (p = 0 and p =1) of SS_n and SS_b are same
- SS_n is continuously decreasing and strictly concave
- S_b is continuously decreasing and strictly convex

Therefore, $SS_n > SS_b$ for all $p \in (0,1)$ and $\gamma = 2$

Step 4: SS_n is decreasing in γ and SS_b is independent of γ, therefore if $SS_n > SS_b$ for all p and $\gamma = 2$ then $SS_n > SS_b$ for all $p \in (0,1)$ and $\gamma < 2$

□

1.A.7 Proof of Proposition 1.4

Proof. Main section shows that $\gamma_c > \bar{\gamma}(p)$. We can further refine by showing that $\gamma_c < 2$. If we substitute $\gamma = 2$ in equation (1.13) and use VP_n and VP_b as defined in Proposition 1.6.

$\Pi_b - \Pi_n = 2pV(\frac{1}{2} - \frac{1-\lambda_n}{3}) + VP_b\big|_{\gamma=2} - VP_n\big|_{\gamma=2}$
$\geq \frac{1}{2}pV + VP_b - VP_n\big|_{\gamma=2}$ as $\frac{1}{4} \leq \lambda_n < 1$ and VP_b does not depend on γ
$> \frac{1}{2}pV + VP_b - VP_0$ as $VP_n < VP_0\ \forall p \in (0,1)$ and $\gamma > 0$ (Proposition 1.6)
$> \frac{1}{2}pV + p\frac{dVP_b}{dp}\big|_{p=0}$ as VP_b is strictly convex w.r.t p (Lemma 1.11)
$= \frac{1}{2}pV + p[\frac{1}{2} - \frac{2\lambda_0+1}{3} - \frac{2}{3}\frac{1-\lambda_0}{\kappa+1}](V-w_0)$ $\frac{dVP_b}{dp}$ from Proposition 1.6 and $(1-p)\frac{\partial\lambda_b}{\partial p}$ from (1.27)
$> \frac{1}{2}pV - \frac{p}{2}(V-w_0)$ as $\lambda_0 < 1$ and $\kappa \geq 1$
> 0 as $w_0 > 0$

Sine $\Pi_b - \Pi_n$ is increasing in γ, positive at $\gamma = 2$ and negative at $\gamma = \bar{\gamma}(p)$, from Intermediate value theorem there exists $\gamma_c \in (\bar{\gamma}(p), 2)$ such that for all $\gamma > \gamma_c$ signal-blocking dominates.

Let's denote that $\gamma = \gamma_c(p)$ for a given p, then profit difference in two cases is zero.

$$F(p,\gamma_c) = [p\gamma_c V(\frac{1}{2} - \frac{1-\lambda_n}{3})] - [p(\frac{2\lambda_b+1}{3} - \frac{1}{2})(V-w_b)] + [\frac{2\lambda_b+1}{3}(V-w_b) - \frac{2\lambda_n+1}{3}(V-w_n)] = 0 \tag{1.31}$$

Apply implicit function theorem to the above equation we have $\frac{d\gamma_c}{dp} = \frac{\frac{2\lambda_b+1}{3}(V-w_b)-\frac{2\lambda_n+1}{3}(V-w_n)}{p^2V(\frac{1}{2}-\frac{1-\lambda_n}{3})}$. The denominator is always positive, so the sign of $\frac{d\gamma_c}{dp}$ is the sign of the term $\frac{2\lambda_b+1}{3}(V - w_b) - \frac{2\lambda_n+1}{3}(V - w_n)$. We call this term "Wage incentive effect" of signal-blocking. Lemma 1.12 below shows that this term is always negative for $\gamma < 2$, hence $\frac{d\gamma_c}{dp} < 0$ i.e. $\gamma_c(p)$ is decreasing in p.

□

When signal is blocked, two effect happens simultaneously: a) wage effect is muted as the impact is only on the honest project and b) signal loss results in lower chance of success. Both the effect reduces the verifiable profit. In the following lemma we are only considering the impact of the first effect assuming no signal loss.

Lemma 1.12. *Define a function wage incentive effect on project surplus as* $WE = \frac{2\lambda(w)+1}{3}(V-w)$. *Corresponding function in two cases are:*

1. $WE_n = \frac{2\lambda_n(w_n)+1}{3}(V - w_n)$ *for no-signal-blocking*

2. $WE_b = \frac{2\lambda_b(w_b)+1}{3}(V - w_b)$ *for "signal blocking".*

Then $WE_n > WE_b$ *for all* $0 < \gamma < 2$ *and* $p \in (0, 1)$

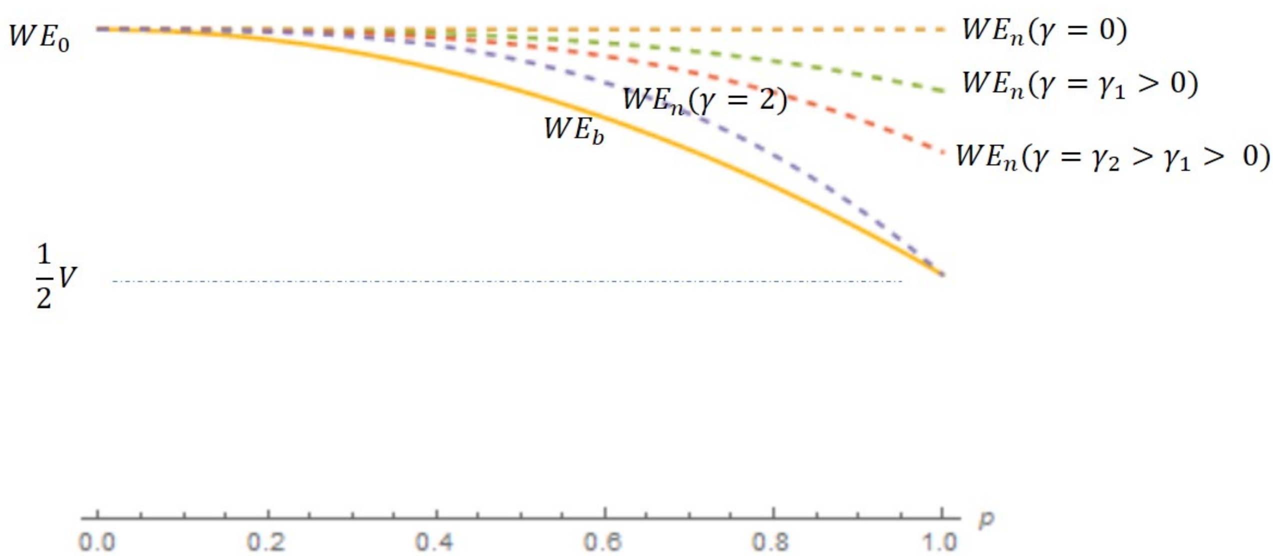


Figure 1.12: Wage effect on verifiable profit in "blocking" WE_b and "no-blocking" WE_n at different γ

Proof. When $p = 0$ both the cases are similar and $w_n = w_b = w_0$, $\lambda_n = \lambda_b = \lambda_0$. Hence $WE_n = WE_b = WE_0$.

Since the wage in "blocking" case w_b does not depend on γ (Lemma 3), the WE_b does not depend on γ. Whereas, WE_n ("no blocking" case) reduces with higher γ as the principal tries to reduce the indirect monitoring impact of the manager. Therefore, to show that $WE_n > WE_b$ for all $0 < \gamma < 2$, it is suffice to show that $WE_n > WE_b$ for $\gamma = 2$. The figure 12, shows how WE_n and WE_b changes with p and γ. As we have shown earlier as $p \to 1$, both $w_b \to 0$ and $w_n(\gamma = 2) \to 0$. Therefore for $\gamma = 2$ as $p \to 1$, $WE_n = WE_b = \frac{1}{2}V$.

$\frac{dWE_n}{dp} = \frac{1}{3}\lambda_n^{'}(w_n)p\gamma V\frac{dw_n}{dp} < 0$. Therefore, WE_n is a continuously decreasing function in p. Also note that $\frac{dWE_n}{dp}\Big|_{p=0} = 0$.

$WE_n\Big|_{p=1} - WE_n\Big|_{p=0} = \frac{V}{2} - \frac{2\lambda_0+1}{3}(V - w_0) < 0 = \frac{dWE_n}{dp}\Big|_{p=0}$. Therefore, WE_n is strictly concave in p.

$\frac{dWE_b}{dp} = \frac{p}{2(1-p)}\frac{dw_b}{dp} + \frac{2}{3}\frac{\partial\lambda_b}{\partial p}(V - w_b) < 0$. As both $\frac{dw_b}{dp} < 0$ and $\frac{\partial\lambda_b}{\partial p} < 0$. Therefore WE_b is decreasing function. Also $\frac{dWE_b}{dp}\Big|_{p=0} = -\frac{2}{3}\frac{1-\lambda_0}{\kappa+1}(V - w_0) < 0 = \frac{dWE_n}{dp}\Big|_{p=0}$

$WE_n > WE_b$ for $\gamma = 2$ and all $p \in (0, 1)$ follows from:

- Both WE_n and WE_b has same value at the end points $p = 0$ and $p = 1$.
- Both WE_n and WE_b are continuously decreasing function in p.
- WE_n is a strictly concave function in p. WE_b could be a concave function or a convex function w.r.t. p but has lower slope at $p = 0$ i.e. $\frac{dWE_b}{dp}\Big|_{p=0} < \frac{dWE_b}{dp}\Big|_{p=0}$. This means that WE_n has higher concavity than WE_b w.r.t. p.

Since $WE_n > WE_b$ for $\gamma = 2$, $WE_n > WE_b$ for all $0 < \gamma < 2$ and $p \in (0, 1)$.

□

1.A.8 Corruptible manager - proof of Lemma 1.8 and Proposition 1.8

Proof of Lemma 1.8

Proof. We can use implicit function theorem to derive

$$\frac{dw_{rn}}{dp} = -\frac{\lambda_{rn}^{'}\gamma V(3\delta-2)-\frac{3}{2}(1-\delta)\frac{\kappa+2}{\kappa+1}\lambda_{rn}^{'}\gamma V(\frac{V-w_{rn}+\frac{3}{2}(\delta-\frac{1}{3})p\gamma V}{w_{rn}+\frac{3}{2}(1-\delta)p\gamma V})}{\lambda_{rn}^{''}(\frac{\lambda_{rn}+\frac{1}{2}}{\lambda_{rn}^{'}})-2\lambda_{rn}^{'}} \text{ and we know } \frac{dw_n}{dp} = \frac{\frac{1}{2}\lambda_n^{'}(w_n)\gamma V}{\lambda_n^{''}(\frac{\lambda_n+\frac{1}{2}}{\lambda_n^{'}})-\lambda_n^{'}}.$$

We can see that $\frac{dw_{rn}}{dp}$ is increasing function of δ and > 0 for $\delta = 1$ and < 0 for $\delta = 0$. Therefore, from intermediate value theorem, there exists $\delta^{'}$ such that $\frac{dw_{rn}}{dp} > 0$ for all $\delta^{'} < \delta < 1$.

At $p = 0$, w_{rn} and w_n converge. We can apply the intermediate value theorem to $\frac{dw_{rn}}{dp}\Big|_{p=0}$ to prove that there exists $\delta^{''}$ such that if $0 < \delta < \delta^{''}$ then $\frac{dw_{rn}}{dp}\Big|_{p=0} < \frac{dw_n}{dp}\Big|_{p=0}$ and $\frac{dw_{rn}}{dp}\Big|_{p=0} > \frac{dw_n}{dp}\Big|_{p=0}$ otherwise.

We can then show that if $\frac{dw_{rn}}{dp} < \frac{dw_n}{dp}$ for $p = 0$ then w_{rn} is below w_n for all $p \in (0, 1)$. This is because as δ reduces $p\gamma$ at which w_{rn} hits zero line reduces. □

Proof of Proposition 1.8

Proof. (a) and (c) - Suppose the limited liability constraint is not binding i.e. $w_{rn} > 0$. Using envelop theorem:

$\frac{d\Pi_{rn}}{d\gamma} = \frac{(1-\lambda_{rn})pV}{3} + \lambda_n p\delta V + (\frac{2}{3}(V-w) + p\gamma V(\delta - \frac{1}{3}))\frac{\partial \lambda_{rn}}{\partial \gamma}$ while $\frac{d\Pi_b}{d\gamma} = \frac{pV}{2}$

We know that $\Pi_b|_{\gamma=0} < \Pi_{rn}|_{\gamma=0}$ due to loss of signal without private benefit (proposition 5). Therefore, no-signal-blocking will strictly dominate signal-blocking iff $\frac{d\Pi_{rn}}{d\gamma} > \frac{d\Pi_b}{d\gamma}$ for all γ i.e.

$\frac{(1-\lambda_{rn})pV}{3} + \lambda_{rn} p\delta V + (\frac{2}{3}(V-w) + p\gamma V(\delta - \frac{1}{3}))\frac{\partial \lambda_{rn}}{\partial \gamma} > \frac{1}{2}pV$

$\frac{(1-\lambda_{rn})pV}{3} + \lambda_{rn} p\delta V + \frac{2\lambda_{rn}+1}{3\lambda'_{rn}}\frac{\partial \lambda_{rn}}{\partial \gamma} > \frac{1}{2}pV$ (from FOC, Equation (1.17))

$\frac{(1-\lambda_{rn})pV}{3} + \lambda_{rn} p\delta V + \frac{2\lambda_{rn}+1}{3\lambda'_{rn}}\frac{3}{2}(1-\delta)pV\lambda'_{rn} > \frac{1}{2}pV$ (note: $\frac{\partial \lambda_{rn}}{\partial \gamma} = \frac{\partial \lambda_{rn}}{\partial w}(\frac{3}{2}(1-\delta)p)$

$\frac{1-\lambda_{rn}}{3} + \lambda_{rn}\delta + (\lambda_{rn} + \frac{1}{2})(1-\delta) > \frac{1}{2}$

$\frac{\delta}{2} < \frac{2\lambda_{rn}+1}{3}$ which implies $\delta < 1$ (as $\lambda_{rn} \geq \frac{1}{4}$).

Therefore no-signal-blocking dominates for all $\delta < 1$ and $w_{rn} > 0$. (c)

Now suppose the δ is low enough that limited liability constraint is binding i.e. $w_{rn} = 0$. Then principal's payoff function: $\Pi_{rn} = \frac{2\lambda_{rn}(\gamma)+1}{3}V + (\frac{1}{3} + \lambda_{rn}(\gamma)(\delta - \frac{1}{3}))p\gamma V$.

$\frac{d\Pi_{rn}}{d\gamma} = \frac{(1-\lambda_{rn})pV}{3} + \lambda_{rn}\delta pV + (\frac{2}{3}V + p\gamma V(\delta - \frac{1}{3}))\frac{\partial \lambda_{rn}}{\partial \gamma}$

$= \frac{(1-\lambda_{rn})pV}{3} + \lambda_{rn}\delta pV + (\frac{2}{3}V + p\gamma V(\delta - \frac{1}{3}))(\frac{1-\lambda_{rn}}{\kappa+1}(1-\delta)pV)$

where κ is the degree of convexity of C(e) and/or concavity of $\lambda(e)$.

Note that profit function slope is greater with renegotiation than without renegotiation under no-signal-blocking i.e. $\frac{d\Pi_{rn}}{d\gamma} > \frac{d\Pi_n}{d\gamma}$ for all $p\gamma < 2$ because of the positive second and the third term. However, this slope can be lower than that in signal-blocking $\frac{1}{2}pV$ when $p\gamma > 2$ as the third term becomes negative. Therefore, with $w_{rn} = 0$, signal-blocking may dominate for high value of γ but the critical threshold for switching to signal-blocking is higher with renegotiation than that without renegotiation. Intuitively, with corruptible manager "no-signal-blocking" is less costly.

(b) When $(p = 0)$ (no corruption opportunity), renegotiation does not matter and hence both cases converge. We can show that

$$\frac{dVP_{rn}}{dp} = \begin{cases} \gamma V\lambda'_{rn}[p(\delta - \frac{1}{3})\frac{dw_{rn}}{dp} + (1-\delta)(V - w_{rn})] & \text{if } w_{rn} > 0 \\ \frac{2}{3}V\frac{\partial \lambda_{rn}}{\partial p} & \text{if } w_{rn} = 0 \end{cases}$$

If $w_{rn} = 0$ then $\frac{dVP_{rn}}{dp} > 0$ as $\frac{\partial \lambda_{rn}}{\partial p} >$ for all $p \in (0,1)$

If $w_{rn} > 0$ and

i) $\delta > \delta'$ then $\frac{dVP_{rn}}{dp} > 0$ as $\frac{dw_{rn}}{dp} > 0$ (lemma 5)

ii) if $\delta \leq \frac{1}{3}$ then $\frac{dVP_{rn}}{dp} > 0$ as $(\delta - \frac{1}{3})\frac{dw_{rn}}{dp} \geq 0$

iii) if $\delta \in (\frac{1}{3}, \delta')$ then $\frac{dVP_{rn}}{dp} > \gamma V\lambda_{rn}'[(\delta - \frac{1}{3})(w_0 - w_{rn}) + (1-\delta)(V - w_{rn})]$ as w_{rn} is decreasing and concave w.r.t p and w_0 is wage at $p = 0$

$\rightarrow \frac{dVP_{rn}}{dp} > \gamma V\lambda_r n'[(\delta - \frac{1}{3})w_0 + \frac{1}{3}w_{rn} + (1-\delta)V] > 0$

Therefore $\frac{dVP_{rn}}{dp} > 0$ for all $p \in (0,1)$ while $\frac{dVP_n}{dp} < 0$ for all $p \in (0,1)$ (proposition 4). Combined with the fact that $VP_{rn} = VP_n$ at (p =0), verifiable profit is higher with renegotiation for all level of corruption.

(d) Manager's surplus is given by $MS_{rn} = \frac{2\lambda_{rn}+1}{3}(w_{rn}) + \lambda_{rn}(1-\delta)p\gamma V - C(e_{rn})$. We can derive:

$$\frac{dMS_{rn}}{dp} = \begin{cases} \frac{2}{3}(1+p\gamma)V\lambda'_{rn}\frac{dw_{rn}}{dp} + \lambda_{rn}(1-\delta)\gamma V & \text{if } w_{rn} > 0 \\ \lambda_{rn}(1-\delta)\gamma V & \text{if } w_{rn} = 0 \end{cases}$$

When $w_{rn} > 0$ and δ is small, the term $\frac{2}{3}(1+p\gamma)V\lambda'_{rn}\frac{dw_{rn}}{dp} < 0$ overwhelms $\lambda_{rn}(1-\delta)\gamma V$[27] and MS_{rn} falls much faster than that without renegotiation. Therefore, manager may loose surplus with corruption. However, when $w_{rn} = 0$ the manager start earning additional limited liability rent and the surplus increases with $p\gamma$.

(e) When $w_{rn} > 0$ then the principal profit decreases with δ as shown below:

$\frac{d\Pi_{rn}}{d\delta} = \frac{2}{3}\frac{\partial\lambda_{rn}}{\partial\delta}[V - w_{rn} + \frac{3}{2}(\delta - \frac{1}{3})p\gamma V] + \lambda_{rn}p\gamma V$

$= -p\gamma V\lambda'_{rn}[V - w_{rn} + \frac{3}{2}(\delta - \frac{1}{3})p\gamma V] + \lambda_{rn}p\gamma V \quad (\text{as } \frac{\partial\lambda_{rn}}{\partial\delta} = -\frac{3}{2}p\gamma V\lambda'_{rn})$

$= -p\gamma V(\lambda_{rn} + \frac{1}{2}) + \lambda_{rn}p\gamma V$ (from FOC)

$= -\frac{1}{2}p\gamma V < 0$

Therefore, if $w_{rn} > 0$ then manager's higher bargaining power may not increase the payoff. This is because higher δ implies lower payment to the manager which means the manager needs to be compensated by higher incentive wage and corresponding limited liability (or moral hazard) rent. The optimal δ for the principal is δ at which the equilibrium wage level just reduces to zero with no limited liability rent. If $w_{rn} = 0$ then the principal's profit may increase with δ as it reduces the manager's limited liability rent.

□

1.A.9 Proof of proposition 1.10

a) First consider the case of "no-signal-blocking." Principal's advantage of having manager under "no-signal-blocking" is given by:

$$\underbrace{\frac{2\lambda_n(w_n)+1}{3}(V - w_n) - \frac{V}{2}}_{Difference\ in\ verifiable\ profit} + \underbrace{(\frac{1-\lambda_n(w_n)}{3} - \frac{1}{2})p\gamma V}_{Difference\ in\ private\ benefit}$$

The above expression

- is continuously decreasing in γ, as verifiable profit decreases with γ under "no-signal-blocking" (see figure 1.4), and the second term decreases with γ for all p.

[27] can be shown with some algebra

- is positive if $\gamma = 0$, as verifiable profit without corruption is strictly greater than $\frac{V}{2}$ (see figure 1.4).
- is negative if $\gamma = \frac{2}{p}$ because $w_n = 0$ if $p\gamma = 2$ (proposition 1), which means $\lambda_n = \frac{1}{4}$ and hence the first term becomes zero and the second term is negative.

Therefore, from intermediate value theorem there exist $0 < \gamma_o < \frac{2}{p}$ such that for all $\gamma > \gamma_o$ the principal will prefer to not hire a manager under no-signal-blocking.

b) Under signal-blocking, the private benefit term is similar with or without manager however the signal-blocking has strictly higher verifiable profit than $\frac{V}{2}$ for all $p \in (0, 1)$. Therefore, the principal will always hire a manger under signal-blocking.

c) From b) and a) we can conclude that $\gamma_c < \gamma_o$ because at γ_o the profit in signal-blocking is strictly greater than "no-signal-blocking".

1.A.10 Proof of proposition 1.11

Proof. First order condition:

$$[p\frac{4\lambda_b(w_b) - 1}{6} - \frac{2}{3}(1 - bp)\frac{\partial \lambda_b}{\partial b}](V - w_b) + [\frac{2\lambda_b(w_b) + 1}{6} - \frac{1}{3}(1 - b)\frac{\partial \lambda_b}{\partial b}]p\gamma V = 0 \quad (20)$$

We know that $e_b(b, p, w)$ is decreasing function in b and hence $\frac{\partial \lambda_b}{\partial b} < 0$. Consider the LHS of equation (20). It is continuously increasing in γ. If $\gamma = 0$ the expression is negative and if γ is chosen arbitrarily large the expression is positive. Therefore, from intermediate value theorem there exist γ' such that the expression is negative for $\gamma < \gamma'$ which means optimal $b = 0$. Similarly there exist γ'' such that the expression is positive if $\gamma > \gamma''$ which means optimal $b = 1$.

To show that $\gamma'(p)$ and $\gamma''(p)$ is decreasing in p, we put $b = 0$ and $\gamma = \gamma'$ in equation (20) and apply implicit function theorem to derive $\frac{d\gamma'}{dp} < 0$. Similarly, by putting $b = 1$ and $\gamma = \gamma''$ and using implicit function theorem we get $\frac{d\gamma''}{dp} < 0$. For interior solution $b \in (0, 1)$ we apply implicit function theorem to equation (20), which shows that $\frac{d\gamma}{db} > 0$. The Higher the private benefit, higher the level of blocking. □

Chapter 2

Corruptible Principal: Screening of manager for collusion

2.1 Introduction

The first chapter highlighted that a corruptible principal benefits from collusion with a corruptible manager. We have shown that such collusion enhances both the principal's private benefit and the firm's verifiable profit, as the manager exerts higher effort and the principal opts for a more transparent regime. However, we assumed that the corruptible manager could be identified costlessly during hiring and did not impose any cost to the principal. In this chapter, we relax these assumptions and develop a mechanism for the principal to screen the manager. The manager's type (corruptible or honest) is private information, and a corruptible manager can steal from the firm, imposing a cost. The existence of honest and dishonest agents with different behaviors is widely discussed in the corruption literature, e.g, (Becker and Stigler, 1974; Besley and McLaren, 1993; Kofman and Lawarree, 1996; Tirole, 1996).

In our model, the corruption opportunity for the manager arises because the principal lacks information about the productivity states that determine the verifiable return. An informed manager can use this private information to misappropriate funds by reporting a low productivity state when the true state is high. The greater the difference in return across productivity states, the higher the potential for misappropriation by the manager. This aspect is different from the first chapter where verifiable return V is deterministic.

The ability of a corruptible manager to engage in corruption also provides a screening mechanism. Because only a corruptible manager engages in corruption, the principal can detect such corruption through an imperfect audit that provides an ex-post verifiable signal about the state, revealing the manager's type.[1] Once the principal identifies that the manager has engaged in corruption and has information about the principal's corruption, the collusive side contract between them becomes mutually beneficial and self-enforceable. Since such a side contract is illegal in court, its self-enforceability is crucial. A side contract where both parties have incriminating evidence can be self-enforceable.

We also simplify our model from the first chapter to focus on the manager's screening aspect. We assume that the probability (λ) that the manager receives the principal's corruption signal is exogenous. If $\lambda(e)$ were an endogenous function of effort e, the manager's effort could become costly to the principal, as higher effort would increase the principal's risk. This would create additional agency conflict between the manager and the principal, confounding the screening results. We also assume that the principal has already decided to implement a project involving corruption. Therefore, unlike the first chapter, we do not model the principal's project selection based on the manager's information. Hence, the state variables y and z, as well as the projects A, A^c, B, B^c, are not relevant. The manager's effort in this chapter increases the probability of success of this generic project.

The principal's objective is to offer a wage contract that achieves the following: attracts the desired type of manager, provides incentives for the manager to exert the desired level of effort, identifies whether the manager is corruptible to enter a side contract and reduce her exposure risk, and controls the cost of corruption if the manager is corruptible. Since the manager's type will only be revealed if the principal allows corruption to occur, the "collusion proofness" equivalence of Tirole (1986), which asserts that the optimal contract avoids the manager's corruption, does not hold in our model.[2] Avoiding the manager's corruption also means not revealing the manager's type, making a corruption-free contract potentially suboptimal.

Our results demonstrate three different types of outcome depending on the potential cost of misappropriation by the manager(C):

1. When C is not very high, the principal not only enters a collusive agreement with the corruptible manager but also mitigate the cost of the manager's corruption through a suitable wage contract. However, both types of managers participate, whereas the principal would prefer to hire only a corrupt manager.

[1]The corruptible manager has both a different strategy space and different utility than the honest manager, providing a desired screening condition.

[2]Tirole and most corruption models use a principal-supervisor-agent hierarchy where the agent and supervisor collude for corruption. In our model, the manager's corruption does not require agent-supervisor collusion, but the same principles apply. "Collusion proofness" posits that there is always an optimal contract that does not involve collusion.

2. When C is in the medium range, the principal reduces wage below the reservation wage to shut down the honest manager, hiring only a corruptible manager who colludes with the principal. Besley and McLaren (1993) terms such a wage strategy a "capitulation wage," so that only dishonest takes job.

3. When C is high, the manager's corruption becomes costly to the principal due to limited liability constraints that prevent wage adjustment. While the honest type is preferable, both types of managers participate. The corrupt manager's effort increases with C, but he also retains the surplus from his effort. In this scenario, the principal could invest in increasing audit effectiveness to reduce the cost of corruption.

The principal's optimal contract also ensures that the honest manager's, inefficient type in our case, effort is not distorted due to asymmetric information.[3] This contrasts with the canonical adverse selection models, where the effort of the inefficient agent is distorted downward.

Several studies have explored the beneficial aspects of allowing corruption or collusion between supervisors and agents to improve contracting. Strausz (1996) shows that when a principal contracts for a noisy signal to detect collusion, and if this signal is sufficiently informative and the principal cannot make a full commitment, it is optimal to allow collusion, as the gain from renegotiation outweighs the cost of preventing collusion. Kofman and Lawarree (1996) demonstrate that the principal can reduce the cost of collusion by adopting contracts that induce collusion between supervisors and agents when there are different types of supervisors with varying degrees of honesty. By allowing collusion to take place for the more dishonest supervisors, the principal is able to screen among the types. Olsen and Torsvik (1998) find that corruption may be beneficial if only limited long-term commitment is possible, as it acts as a commitment device by relaxing dynamic information revelation constraints, creating long-term gains that can offset short-term static costs. Che (1995) show that tolerating collusion between firms and regulators enables regulators to work harder, develop expertise, and increase monitoring. Tirole (1992) indicates that collusion may help complete contracts and increase overall efficiency when the principal is unable to use a complete contract. These studies demonstrate that collusion avoidance may not always be ideal, as the process of collusion can sometimes provide valuable information.

This paper applies a similar principle to identify manager types, enabling collusive side-contracts between the principal and the manager. To our knowledge, this is the only paper using such a mechanism for principal-agent collusion. Kofman and Lawarree

[3]In our model, the honest manager is the inefficient type, and the corrupt manager is the efficient type, as corrupt managers can conceal their type.

(1996), which also employs screening, offers differing incentives to honest and dishonest auditors to reduce the cost of supervisor-agent collusion.

2.2 Framework

Consider a framework with the following basic elements: a risk-neutral and corruptible principal (she) who runs the firm with the assistance of a risk-neutral manager (he).

2.2.1 States of the World and Project Payoffs

The principal implements a project that involves corruption and provides a private benefit to the principal. The project requires effort from the manager to succeed. Let e denote the endogenous effort by the manager, which is non-observable and hence non-verifiable and has the cost $C(e)$. If the manager exerts effort e, the probability of success of the project is $\pi(e)$.

The state of the world is determined by the productivity state $x \in \{l, h\}$. Both states are equally likely, with $Pr(x = l) = Pr(x = h) = \frac{1}{2}$. $x = l$ implies a low productivity state, and $x = h$ implies a high productivity state.

Assumption 2.1.

(a) $\pi(e)$ *is an increasing and concave function in* e*. We use the functional form* $\pi(e) = e$*, where* $e \in [0, 1]$.

(b) $C(e)$ *is a twice-differentiable, increasing, and strictly convex function in* e *with* $C(0) = 0$*. We use the functional form* $C(e) = \frac{1}{2}e^2$*, where* $e \in [0, 1]$.

We use a simpler functional form in Assumption 2.1 to make all constraints linear. A more general functional form adds significant complexity without changing the result.

The project yields a positive payoff only if it is successful. The payoff includes a verifiable return to the firm $\hat{v} \in \{0, V_h, V_l\}$ and a private benefit to the principal $\hat{b} \in \{0, B\}$. The principal has full ownership of the firm. Table 2.1 below summarizes the project payoffs depending on the success and the state of the world. When the project does not succeed, both the verifiable return and the private benefit are zero. When it succeeds, the verifiable return is V_h if the productivity state is h (high) and V_l if the productivity state is l (low). The principal also receives the private benefit B upon success.[4]

[4]Note: We do not use γV_h and γV_l as the private benefit of the principal because then the principal's private benefit is linked to manager's corruption.

	Success ($\pi(e)$)		Failure ($1 - \pi(e)$)
State $x =$	h	l	
Payoffs	(V_h, B)	(V_l, B)	$(0, 0)$

Table 2.1: Project payoffs in different states.

The principal offers a wage contract (w_h, w_l) to the manager, where w_h and w_l are the wage payments when the realized state is $x = h$ and $x = l$, respectively. The manager is risk-neutral but protected by limited liability, meaning that managerial wages must be non-negative in every state of the world (i.e., $w_h \geq 0$ and $w_l \geq 0$). The manager has a zero reservation wage unless specified otherwise. The principal pays zero wages when the project is not successful.[5]

2.2.2 Manager's Information and Type

During the project execution, the manager receives a perfect signal regarding state x, but the principal does not receive signal on x. The manager also receives a corruption signal with probability λ (exogenous) that provides verifiable evidence of the principal's corruption. Unless the manager destroys this evidence, it can be leaked to expose the principal's corruption with probability q. If the principal's corruption is exposed, she pays a penalty $P > 0$. The magnitude of P is determined by legislation and is taken to be exogenous. Unlike Chapter 1, we consider λ to be exogenous to simplify the model and focus on the screening problem. Endogenous λ would give rise to another type of agency conflict between the manager and the principal confounding the screening result.[6]

The manager can be of two types $t \in \{h, c\}$:

- **honest** ($t = h$)**:** Does not misreport the state information.
- **corruptible** ($t = c$)**:** Can misreport the state information to misappropriate (or steal) funds and is also willing to collude with the principal to destroy evidence of corruption.

More specifically, the corruptible manager can misappropriate $V_h - V_l$ by reporting $x = l$ when the true state signal he receive is $x = h$.[7] The principal's lack of information about the productivity state enables an informed and corruptible agent to engage in corruption and take advantage of this information asymmetry. Both types of managers

[5] Zero wage on failure is optimal when the manager has limited liability protection.

[6] In the first chapter, λ is endogenous and a function of effort, which gives rise to the agency conflict. Higher effort is beneficial to the principal due to better business signals, but it also provides a more informative corruption signal that poses a risk to the principal.

[7] Such misappropriation can occur through inflating costs, taking kickbacks from suppliers, using company resources for private benefits, etc.

are similar in all other aspects, such as talent and productivity. Both are risk-neutral and have the same reservation wage. The manager's type is private information at the time of hiring. The ex-ante probability of a corrupt type manager in the population is $\theta \in (0, 1)$.

2.2.3 Audit and Side contract

The principal has access to costless[8] but imperfect audit technology that detects incorrect state reporting with probability $\nu \in (0, 1)$. The audit does not falsely report a high state when the state is low (i.e., no Type 1 error).

When the audit finds a discrepancy, there are two possible outcomes based on whether the manager has a corruption signal:

- If the manager has a corruption signal, he can negotiate a side contract with the principal to destroy the evidence and share the surplus.
- If the manager does not have a corruption signal, the principal fires the manager and recovers the misappropriated amount.

Assume the principal and the manager negotiate the side contract using Nash bargaining, with weight $\delta \in [0, 1]$ for the principal. If they successfully negotiate, the total surplus is P, as the principal avoids paying this penalty by destroying the corruption evidence. Therefore, in equilibrium, both parties will negotiate the side contract, with the payoff to the principal being δP and to the manager $(1 - \delta)P$. We also assume that the manager cannot provide a false evidence of corruption signal, as the principal can verify the signal.

In the above description, *we assumed that the manager enters a side contract with the principal only when his type is revealed through an audit discrepancy.* This assumes that corruptible managers will hide his type unless forced to reveal it, possibly due to high external costs such as loss of future earnings in repeated games or increased risk of litigation and investigation. In other words, a manager without a pre-existing tie with the principal may not want to engage in such an illegal contract but would be forced to do so when caught in their own corruption.

An alternative mechanism could include a side contract in a situation where a corruptible manager approaches the principal if he has a corruption signal but is not misreporting the state. In a one-shot game and in the absence of other costs, both the principal and the manager benefit from this side contract. Since this assumption does not change the trade-offs, the results will remain similar, except for vertical shifts in the principal's and

[8] This means the audit cost is part of operating expenses and is netted in the verifiable return.

manager's payoffs. We will first show our results assuming that the side contract occurs only when the audit finds a discrepancy, and then demonstrate how the results change if the side contract also occurs when the manager does not misreport.

2.2.4 The Timeline

Stage 1 : Nature determines the manager's type, and the manager learns his type.

Stage 2 : The principal implements the project and offers a wage contract specifying (w_h, w_l).

Stage 3 : The manager decides whether to accept or refuse the contract. If he accepts, the manager chooses effort $e : (w_h, w_l) \to [0, 1]$ and receives signals about the state x and any corruption signal.

Stage 4 : The manager reports the state information.

Stage 5 : The principal audits the report to verify the true state. If there is a discrepancy, the principal either fires the manager (if there is no corruption signal) or negotiates a side contract to destroy the corruption evidence.

Stage 6 : Payoffs are realized. If the evidence of corruption is not destroyed, the principal is caught with probability q and pays the penalty P.

We solve for the perfect Bayesian equilibrium of this game. Throughout this paper, such equilibrium exists if not mentioned otherwise.

2.2.5 Game Tree

The complete decision tree is presented across three figures for simpler visualization. Figure 2.1 illustrates the decision tree up to the manager's effort decision. Figures 2.2 and 2.3 depict the decision tree for the honest and corrupt managers, respectively, following their effort decisions.

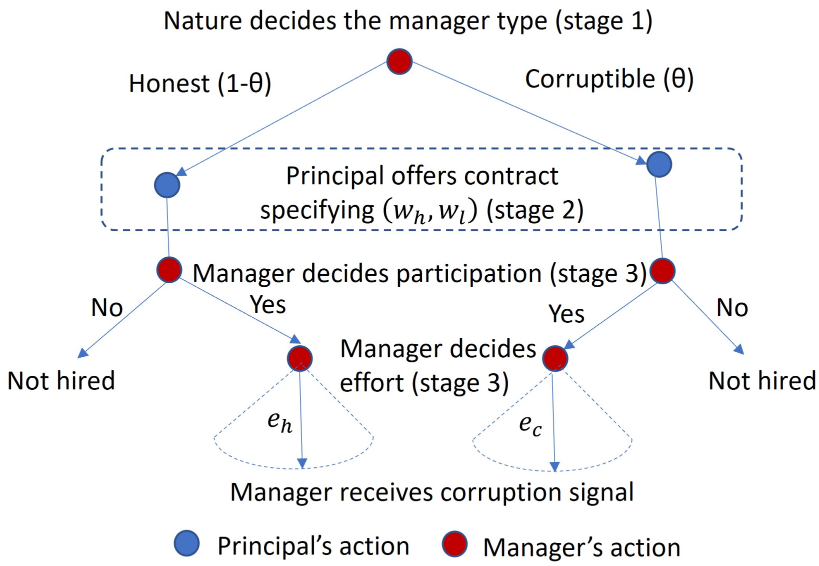


Figure 2.1: Game tree until manager's effort decision

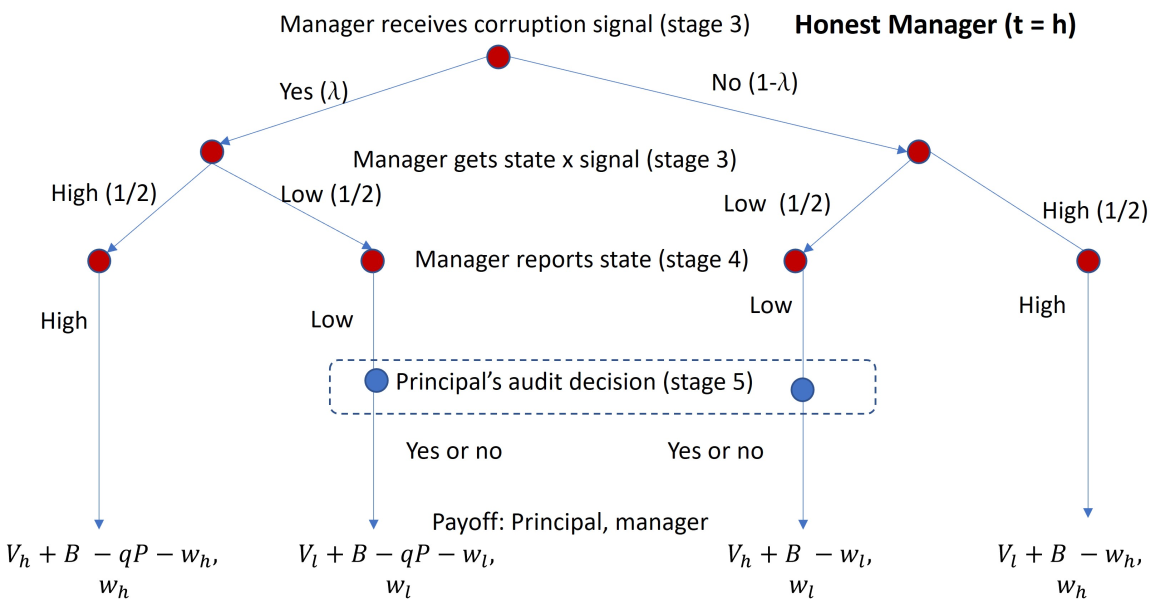


Figure 2.2: Game tree for the honest manager if hired

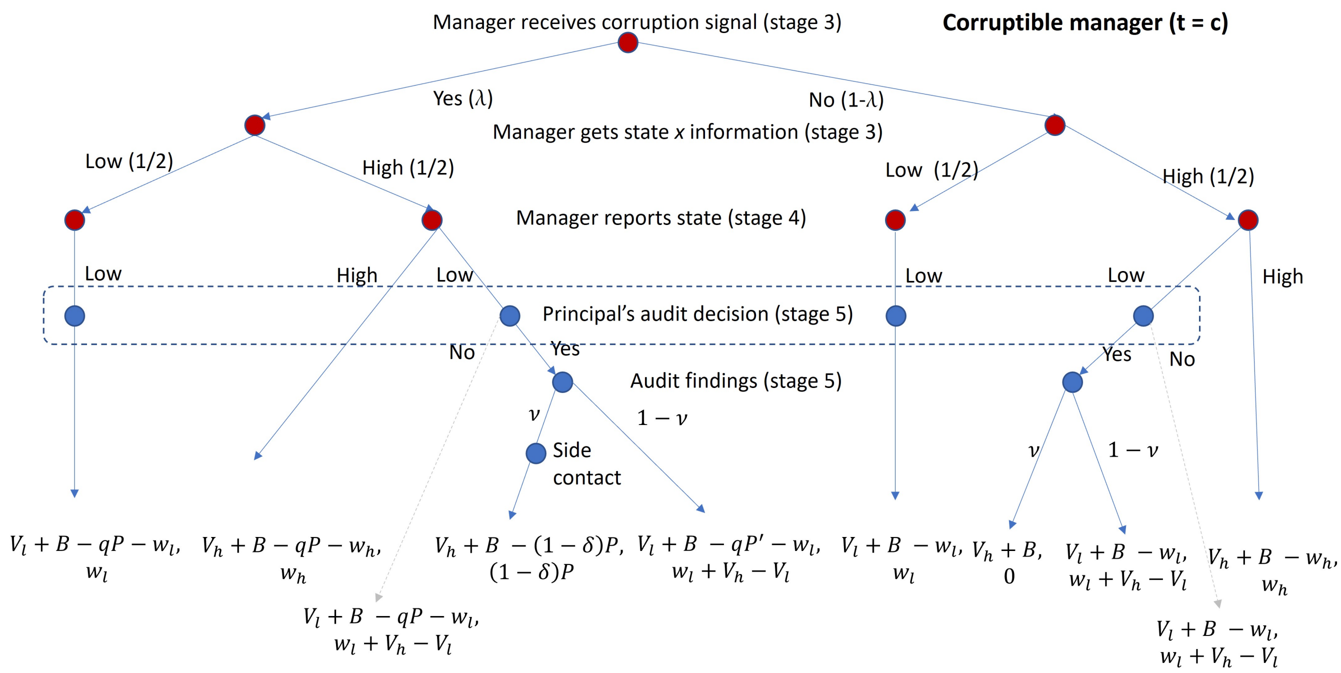


Figure 2.3: Game tree for the corruptible manager if hired

There are a few important points to note in the above figures. First, in the terminal nodes of Figures 2.2 and 2.3, the first element of the payoff represents the principal's payoff, and the second element represents the manager's payoff. Second, when the principal takes action, the corresponding nodes for both the honest and corrupt managers are in the same information set because the principal does not know the manager's type except when a side contract is negotiated in stage 5. Finally, as shown in Figure 2.3, the dotted line indicating the principal's decision not to audit when $\hat{r} = l$ will never be reached, as the principal will always audit when the reported state is $\hat{r} = l$, unless he has perfect information that the manager is honest.

2.2.6 Assumptions and Definitions

In addition to the functional forms of $\pi(e)$ and $C(e)$ as specified in Assumption 2.1, we make the following assumption:

Assumption 2.2. $B - q\,\lambda\,P \geq 0$

Assumption 2.2 ensures that, ex-ante, expected payoff from corruption is non-negative regardless of the manager's type. Therefore, the principal will choose to implement the project that involves corruption. This assumption helps reduce the number of less interesting cases.

Assumption 2.3. *Wage monotonicity: (MC)* $w_h \geq w_l$

Assumption 2.3 is technically not required but is included to avoid scenarios where the principal's type is revealed to external stakeholders, which can be detrimental. This is because only a principal involved in corruption would pay higher wages in the low productivity state compared to the high productivity state to induce the type revelation by a corruptible manager. However, we will demonstrate the impact of relaxing this assumption in section 2.5.

In addition we also assume:

- $q = 1$ to make our model parsimonious. The impact of changes in the exogenous parameter q is equivalent to changes in P.
- The values of V_h, V_l, B, and P are such that $e \in [0, 1]$. This assumption does not change the results but avoids consideration of corner solutions.

We introduce few definitions:

Definition 2.1. *Define* $A \equiv \frac{1}{2}(V_h + V_l) + B - \lambda\, P$ *as the principal's profit if there is no collusion with a corruptible manager.*

Definition 2.2. *Define* $b \equiv \frac{1}{2}\,(1-\nu)\,(V_h - V_l)$ *as the potential amount that a corruptible manager can steal from the firm.*

Note that the corruptible manager misappropriates $V_h - V_l$ when he misreports the high state and is not caught through audit, the probability of which is $\frac{1}{2}(1-\nu)$.

Definition 2.3. *Define* $N \equiv \frac{1}{2}\,\nu\,\lambda\,P$ *as the potential surplus created by the side contract.*

We assume that the manager enters a side contract only when his type is revealed through an audit discrepancy (probability $\frac{1}{2}\nu$), and he has a corruption signal (probability λ). The surplus from the side contract is the penalty amount P.

Definition 2.4. *Define* $s_c \in \{0, 1\}$ *where* $s_c = 1$ *implies that the manager has a corruption signal; otherwise, he does not.*

Definition 2.5. *Define* $r : x \to \{l, h\}$ *is the manager's reporting strategy when the actual state is* x*.*

2.2.7 Actions and Strategies

The principal's role involves three key decisions. First, she specifies the wage contract $(w_h, w_l) \in \mathbb{R}_+^2$ for the manager when the project is successful. Due to the limited liability constraint, the principal will pay zero wages if the project is not successful.

Second, the principal decides whether to audit the state report submitted by the manager. Let's denote the instance of a manager's report as $\hat{r} \in \{l, h\}$. In our model setup, if the corrupt manager participates, the principal will always audit when the manager reports $\hat{r} = l$. This is because the principal benefits from a side contract when the audit reveals a discrepancy in the low state.

Finally, the principal will enter into a side contract with the manager if the audit finds a discrepancy and the manager has corruption evidence, i.e. $s_c = 1$.

The manager takes four types of actions. First, he decide whether to participate based on the wage contract (w_h, w_l) offered by the principal. Second, conditional on participation, the manager chooses the effort $e : (w_h, w_l) \rightarrow [0, 1]$. Suppose p_m is the manager's expected payoff on success, which includes the expected wage payment as specified in the contract, potential benefits from the side contract if the manager type is c (corrupt), and benefits from misappropriation if the manager type is c. The manager's effort decision is based on the solution to the following:

$$e = \arg\max_{e'} \; \pi(e')\, p_m - C(e')$$

and the participation is based on the following individual rationality (IR) constraint:.

$$\pi(e)\, p_m - C(e) \geq 0$$

Third, the manager reports the state information using reporting strategy $r : x \rightarrow \{l, h\}$. For the honest manager, $r(x) = x$, for all $x \in \{l, h\}$. The corrupt manager may misreport the state depending on the offered wage contract. Finally, the manager enters a side contract with the principle if the audit finds a discrepancy and $s_c = 1$.

2.3 Benchmark

Let us consider two benchmark cases where the principal has perfect information about the manager's type at the time of hiring.

2.3.1 Honest manager hired

Consider that the principal can identify the manager's type at the recruitment stage and hires an honest manager.

Truthful reporting will be the dominant strategy for the honest manager if the monotonic wage constraint (MC) is satisfied (see Figure 2.2). The audit report will have no impact, regardless of whether the principal conducts an audit or not.

$$w_h \geq w_l \tag{MC}$$

The principal's profit function is given by:[9]

$$\pi(e)\,[\underbrace{\frac{1}{2}(V_h+V_l)}_{\text{Verifiable return on success}} + \underbrace{B-\lambda\,P}_{\text{Expected private benfit}} - \underbrace{\frac{1}{2}(w_h+w_l)]}_{\text{Wage payment}}$$

The principal's problem (denoted as PH) when she hires an honest manager:

$$\max_{w_h,w_l}\ \pi(e)\,[\frac{1}{2}(V_h+V_l)+B-\lambda\,P-\frac{1}{2}(w_h+w_l)] \tag{PH}$$

subject to:

$$w_h \geq w_l \tag{MC}$$

$$e = \arg\max_{e'}\ \frac{1}{2}\,\pi(e')\,(w_h+w_l)-C(e') \tag{GIC}$$

$$\pi(e)\,(\frac{1}{2}(w_h+w_l))-C(e)\geq 0 \tag{IR}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

where, as is standard, GIC, IR and LL1-LL2 are the manager's (global) incentive compatibility, individual rationality, and limited liability constraints, respectively. Given LLs and GIC, IR holds.[10] If $(w_h+w_l)>0$ then one can replace GIC by the local incentive compatibility condition (LIC) which is the manager's first order condition.[11] Using functional form in Assumption 2.1 we have:

$$e=\frac{1}{2}(w_h+w_l) \tag{LIC}$$

A few important points to note, which we will use across the paper:

a) Given Assumption 2.1, the effort in the GIC condition will equate to the manager's expected payoff on success. Henceforth, we will use this fact for all GIC conditions without further elaboration.

b) Given Assumption 2.1 and point (a), the manager's surplus is given by $\frac{1}{2}e^2$.[12] Therefore, the IR (individual rationality) condition will be satisfied as an inequality if the effort is positive. This is because the manager will earn positive rent due to limited

[9] Since, the honest manager receives the corruption evidence with probability λ but does not destroy it in which case the principal will be exposed (Note: $q=1$) and pays the penalty P, the net payoff from corruption is $B-\lambda P$.

[10] Given LLs the manager can always opt for $e=0$ to get non-negative payoff.

[11] Manager's problem is strictly concave so first order condition will given unique e and fully characterizes GIC

[12] If the manager's expected payoff is p_m, the manager's surplus is $\pi(e)p_m-\frac{1}{2}e^2$. From (a), $p_m=e$, giving a surplus of $\frac{1}{2}e^2$.

liability when the effort is positive. Henceforth, we will use the fact that positive effort results in positive moral hazard rent.

c) Given Assumption 2.1, the principal's objective function is concave in w_h and w_l, and all constraints are linear. Therefore, the Kuhn-Tucker conditions are both necessary and sufficient. This will also apply to all the principal's problems discussed in this paper.

Let's denote expected wage on success as $w_e \equiv \frac{1}{2}(w_h + w_l)$ and use $A \equiv \frac{1}{2}(V_h + V_l) + B - \lambda\, P$ (see Definition 2.1). So the principal's objective function is given by:

$$\pi(w_e)\,(A - w_e)$$

The optimal wage structure using Kuhn Tucker, if MC and LLs are not binding, is given by the Lerner's equation:

$$\frac{\pi^{'}(w_e)}{\pi(w_e)} = \frac{1}{w_e} = \frac{1}{A - w_e} \text{ subject to } w_h \geq w_l \geq 0$$

which implies that optimal wage offered by the principal is:

$$w_e = e = \frac{1}{2}(w_h + w_l) = \frac{A}{2} \tag{2.1}$$

If the principal offers $w_h = w_l = \frac{A}{2}$, then the MC (Monotonicity Condition), LL1 (Limited Liability 1), and LL2 (Limited Liability 2) constraints are satisfied. This is one possible contract. Another solution could be $w_h = A, w_l = 0$. Other linear combinations of w_h and w_l could also be optimal as long as they satisfy (2.1), and the MC and LL constraints. The corresponding principal's profit and the manager's effort are given by:

$$\Pi_{ph} = \frac{A^2}{4} \tag{2.2}$$

$$e = w_e = \frac{A}{2} \tag{2.3}$$

Let's evaluate how this effort compares with the first best. which is given by:

$$e_{fb} = \arg\max_{e^{'}}\ \pi(e^{'})\,A - C(e^{'}) = A \tag{2.4}$$

We can observe that the managerial effort $e = \frac{A}{2}$ is lower than the first-best effort e_{fb} due to moral hazard rent. Proposition 2.1 states our result when the principal knows that the manager is honest before offering the contract.

Proposition 2.1. *If the principal knows the manager is honest before offering the contract, she will offer a wage contract where* $w_e = \frac{1}{2}(w_h + w_l) = \frac{A}{2}$ *and* $w_h \geq w_l \geq 0$*. The princi-*

pal's profit will be $\frac{A^2}{4}$*. Two possible contracts are* $(w_h, w_l) = (\frac{A}{2}, \frac{A}{2})$ *and* $(w_h, w_l) = (A, 0)$*. Other linear combinations of* w_h *and* w_l *that meet these conditions are also optimal.*

2.3.2 Corrupt manager hired with ex-ante contract for collusion

Consider that the principal can identify the manager's type at the recruitment stage and hires a corrupt manager. The principal offers an ex-ante contract that not only provides wage payments conditional on state but also agrees to share the surplus if the manager destroys the corruption evidence that he may receive. To ensure enforcement, the surplus is shared after the principal can verify that the corruption evidence is destroyed. The surplus is shared using the Nash Bargaining as discussed in section 2.2.3.

Since the collusion for corruption has been agreed upon ex-ante, there is no side contract at the time of the audit review, and the principal's problem is to control the manager's misappropriation through a suitable wage contract. The corrupt manager's decision for reporting the state is the same whether he has a corruption signal or not. Thus, we collapse the two sides of the game tree, with or without a corruption signal, into one, as shown in Figure 2.4.

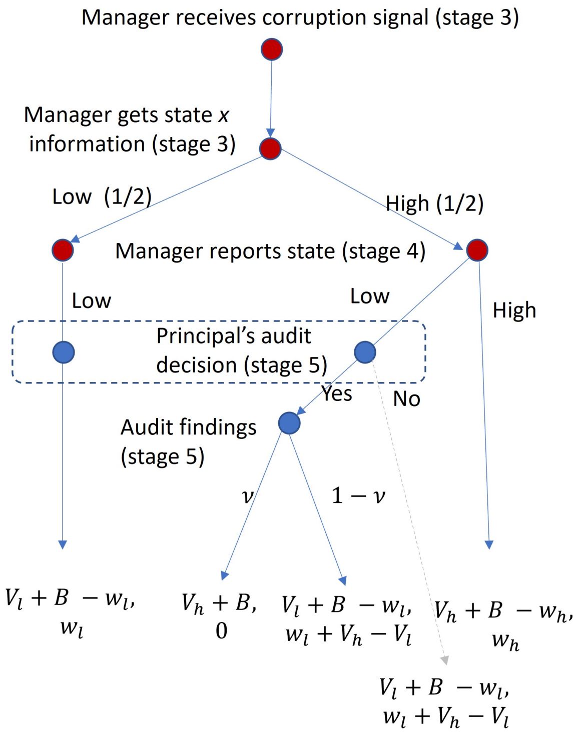


Figure 2.4: Game tree if corrupt manager is hired with an ex-ante contract for collusion

Under full commitment by the principal and no side contract, we can, without loss of generality (revelation principle), restrict our attention to contracts that elicit truthful reporting of the state. To ensure that the manager reports the state truthfully, the incentive compatibility constraints are given by:

$$w_h \geq (1-\nu)\,(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \leq w_l + V_h - V_l \tag{IC2}$$

IC1 ensures that the manager reports the high state correctly and does not misappropriate. IC2 ensures that the manager reports the low state correctly. The manager will report the low state incorrectly when w_h is so high that it is beneficial to report the high state and pay the return difference, $V_h - V_l$, from their own pocket. The principal's objective function under truthful reporting is given by:

$$\pi(e)[\underbrace{\frac{1}{2}(V_h + V_l) + B}_{\text{Payoff on success}} - \underbrace{(1-\delta)\,\lambda\,P}_{\text{The surplus shared with the manager}} - \underbrace{\frac{1}{2}(w_h + w_l)]}_{\text{wage payment}}$$

By using an ex-ante contract, the principal avoids the full risk of exposure, resulting in a total surplus of λP. Since the principal's Nash bargaining weight is δ, she shares $(1-\delta)\lambda P$ with the manager. The principal's objective function can be rewritten as:

$$\max_{w_h, w_l} \pi(e)[\underbrace{\frac{1}{2}(V_h + V_l) + B - \lambda\,P}_{\text{A:Profit without collusion}} + \underbrace{\delta\,\lambda\,P}_{\text{gain from collusion}} - \underbrace{\frac{1}{2}(w_h + w_l)]}_{\text{wage payment}}$$

Therefore, the principal's problem (PC) for the optimal contract can be written as:

$$\max_{w_h, w_l} \pi(e)[A + \delta\,\lambda\,P - \frac{1}{2}(w_h + w_l)] \tag{PC}$$

subject to:

$$w_h \geq (1-\nu)\,(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \leq w_l + V_h = V_l \tag{IC2}$$

$$e = \arg\max_{e'}\ \pi(e')\,[\frac{1}{2}(w_h + w_l) + (1-\delta)\,\lambda\,P] - C(e') \tag{GIC}$$

$$w_h \geq w_l \tag{MC}$$

$$\pi(e)\,[\frac{1}{2}(w_h + w_l) + (1-\delta)\,\lambda\,P] - C(e) \geq 0 \tag{IR}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

As discussed earlier, GIC can be replaced by LIC, and IR is satisfied if $w_h + w_l > 0$.

$$e = \frac{1}{2}(w_h + w_l) + (1-\delta)\,\lambda\,P \tag{LIC}$$

If IC1 and IC2 are not binding then the unconstrained solution of PC is given by:

$$w_e = \frac{1}{2}(w_h + w_l) = \frac{A}{2} + (\delta - \frac{1}{2})\,\lambda\,P \tag{2.5}$$

The principal can choose a combination of w_h and w_l, for instance $(w_h, w_l) = (A + (2\delta - 1)\,\lambda\,P, 0)$, such that MC, and LLs are satisfied. Another possible combination could be $w_h = w_l = \frac{A}{2} + (\delta - \frac{1}{2})\,\lambda\,P$ which also satisfies the MC and LLs, and yields the same profit.[13] Therefore, if IC1 and IC2 are not binding then optimal wage structure is given by (2.5) and corresponding principal profit is given by (2.6):

$$\Pi_{pc} = \frac{(A + \lambda\,P)^2}{4} \tag{2.6}$$

Remark: An important point to note in (2.6) is that the principal's profit is independent of δ. If $\delta > \frac{1}{2}$, then the manager is paid an extra wage (see (2.5)) in addition to the surplus, whereas if $\delta < \frac{1}{2}$, the manager's wage is reduced. The net effect is that the surplus is shared equitably, irrespective of δ. This is because the collusion surplus shared with the manager increases the manager's effort, as the payment is contingent on success, which ultimately benefits the principal.

Next we determine the condition when either IC1 or IC2 is binding, as this will result in a different wage structure. Notice that at most one of IC1 or IC2 will be binding. IC1 is most relaxed when w_h is maximum and w_l is minimum in (2.5). This implies that at the boundary $(w_h, w_l) = (A + (2\delta - 1)\,\lambda\,P, 0)$. IC1 will be non-binding as long as:

$$A + (2\delta - 1)\,\lambda\,P \geq (1 - \nu)\,(V_h - V_l)$$

which implies

$$V_h - V_l > \gamma_c \text{ where } \gamma_c \equiv \frac{A}{1-\nu} + \frac{2\delta - 1}{1 - \nu}\,\lambda\,P \tag{2.7}$$

When IC1 is binding, the optimal wage is given by (2.8), which also satisfies the MC and LL constraints. The corresponding principal's profit is given by (2.9).

$$w_h = (1 - \nu)\,(V_h - V_l),\, w_l = 0 \tag{2.8}$$

$$\Pi_{pc} = (\frac{1}{2}w_h + (1 - \delta)\,\lambda\,P)(A + \delta\,\lambda\,P - \frac{1}{2}w_h) \tag{2.9}$$

From (2.8)-(2.9), wages increase with $V_h - V_l$, and profits decrease with $V_h - V_l$ if IC1 is binding. Next we check the condition when IC2 is binding. IC2 is most relaxed when $w_h - w_l$ is minimum, i.e., $w_h = w_l = \frac{A}{2} + (\delta - \frac{1}{2})\,\lambda\,P$. This implies that IC2 is non-binding if $V_h - V_l \geq 0$. Therefore, IC2 does not impact the optimal contract. We state our result

[13] There could be other feasible linear combination of w_h and w_l that satisfies MC and LLs.

in Proposition 2.2.

Proposition 2.2. *If the principal hires a corrupt manager and signs an ex-ante contract for collusion with the manager to destroy corruption evidence, the principal will offer the following wage contract:*

1. *If* $V_h - V_l \in [0, \gamma_c]$, *then optimal wage contract will satisfy* $\frac{1}{2}(w_h + w_l) = \frac{A}{2} + (\delta - \frac{1}{2})\,\lambda\,P$ *and* $0 \leq w_l \leq w_h$, *and the principal's profit will be* $\frac{(A+\lambda\,P)^2}{4}$. *If* $V_h - V_l = 0$ *then* $w_h = w_l = \frac{A}{2} + (\delta - \frac{1}{2})\,\lambda\,P$. *If* $V_h - V_l = \gamma_c$ *then* $(w_h, w_l) = (A + (2\delta - 1)\,\lambda\,P, 0)$. *In between, multiple solutions of* (w_h, w_l) *are possible.*

2. *If If* $V_h - V_l > \gamma_c$, *then* $w_h = (1-\nu)(V_h - V_l)$ *and* $w_l = 0$, *and the principal's profit is* $\left(\frac{1}{2}w_h + (1-\delta)\,\lambda\,P\right)\left(A + \delta\,\lambda\,P - \frac{1}{2}w_h\right)$, *which is lower than* $\frac{(A+\lambda\,P)^2}{4}$ *and decreases as* $V_h - V_l$ *increases.*

where $\gamma_c = \frac{1}{1-\nu}\,(A + (2\delta - 1)\,P)$

Figure 2.5 depicts the profit and wages graphically, where the x-axis represents $V_h - V_l$, representing the potential leakage through the manager's corruption.

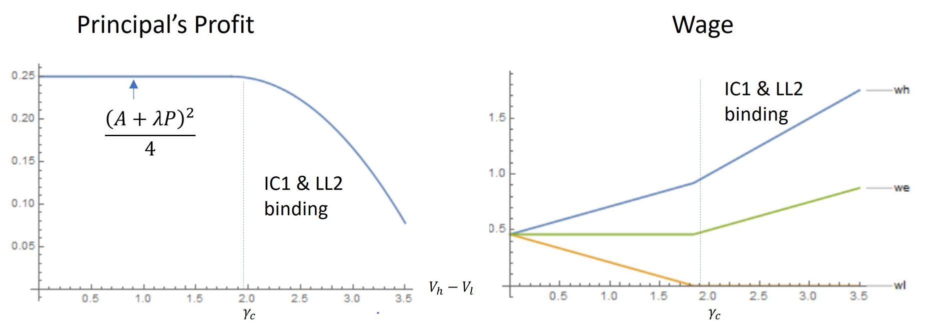


Figure 2.5: Principal's profit and wages when a corruptible manager is hired with an ex-ante collusion contract ($\delta = 0.75$, $\nu = 0.5$, $\lambda = 0.4$, $\frac{P}{B} = 1.6$)

Note that the principal's profit does not reduce with the potential for leakage $V_h - V_l$ until LL2 binds because the principal can control corruption costlessly through a suitable contract structure.

The first-best effort, if the principal can verify and contract on effort, is given by $e_{fb} = A + \lambda P$. The manager's effort relative to the first-best is shown in Figure 2.6.

Figure 2.6: Manager's effort and the first best effort($\delta = 0.75$, $\nu = 0.5$, $\lambda = 0.4$, $\frac{P}{B} = 1.6$)

As we can observe, the principal's profit declines once the IC1 and LL2 binds. However, the manager's effort and surplus rise, due to higher effective wage and higher limited liability rent.

Remark: There are two types of limited liability rent in our model. The first arises because the manager needs to be paid a non-negative wage when the project is unsuccessful. This limited liability rent reduces the effort of both types of managers below the first-best level. The second type arises because the wage in the low productivity state must be non-negative when the project is successful. This constraint restricts the principal's choice of contract structure to control the manager's corruption. This rent increases with the potential leakage from corruption $V_h - V_l$, which in turn increases the manager's effort and surplus. Both types of rent negatively impact the principal's profit.

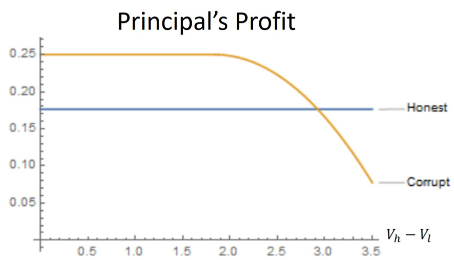


Figure 2.7: Comparison of profit when the principal hires honest vs. corrupt manager

Figure 2.7 illustrates that the principal will prefer to hire a corrupt manager unless the potential cost of the manager's corruption is critically high. We state this result below.[14]

Corollary 2.1. *Suppose the principal can identify the manager's type at the time of hiring. Then, there exists a threshold μ_c such that if $V_h - V_l < \mu_c$, the principal will hire a corrupt manager; otherwise, the principal will hire an honest manager.*

[14]Proof is straightforward as the principal's profit function with the corrupt manager is continuously decreasing and concave when IC1 is binding, whereas the profit function with the honest manager is constant.

2.4 Screening when the manager's type is private information

In this section, we consider our main screening model where the manager's type is private information at the time of hiring. The principal's objective is to minimize the loss of income due to misappropriation by the corrupt manager (moral hazard) while inducing the corrupt manager to reveal his type (asymmetric information) so the principal can benefit from collusion. Since the corrupt manager will not reveal his type while misappropriating funds unless his type is revealed through an audit, the principal tolerates some level of corruption to gain information on the manager's type.

Unlike the case of perfect information about the manager's type, we cannot invoke the revelation principle for two reasons. First, there is a side contract. Second, there is a conflict between inducing the manager to reveal its type and reporting the true state. If the manager reports the true state, his type will not be revealed.

We classify the principal's contracting strategy based on how she specifies w_h in relation to w_l, which determines the manager's reporting strategy. Four possible but exhaustive set of cases are shown below. Figures 2.2 and 2.3 depicts the optimal reporting strategy under various scenarios.

Case 1: $(1-\delta)\,P + (1-\nu)\,(w_l + V_h - V_l) \leq w_h \leq w_l + V_h - V_l$. In this case, the manager's reporting strategy will be: $r(x) = x$ for all t, x. Both type of manager will always report the true state, which means the corrupt manager will not reveal his type so the principal will not benefit from collusion.

Case 2: $(1-\nu)\,(w_l + V_h - V_l) \leq w_h < (1-\delta)\,P + (1-\nu)\,(w_l + V_h - V_l)$. In this case, the manager's reporting strategy will be: If $t = c$ and $s_c = 1$ then $r(h) = l$, otherwise $r(x) = x$. The corrupt manager will misreport the high state when he has a corruption signal (left side in Figure 2.3), in which case the audit can reveal the manager's type to enable collusion through side contract.

Case 3: $0 < w_h < (1-\nu)\,(w_l + V_h - V_l)$. In this case, the manager's reporting strategy will be: If $t = c$ then $r(h) = l$, otherwise $r(x) = x$. The corrupt manager misreport the high state regardless of whether he has a corrupt signal or not. Audit is used to control the manager and reveal its type. The difference between Case 2 and Case 3 is that in Case 3, there are more opportunities for the corrupt manager to steal, but the principal can offer lower wages to both type of managers to reduce rent.

Case 4: With $w_h = 0$, which also implies $w_l = 0$ (due to Assumption 2.3 about wage monotonicity), the manager is paid zero wages in both states. This may lead to the honest manager opting out if his reservation wage is even marginally positive, while the corrupt manager will still join as he receives a positive payoff from corruption. The corrupt manager's reporting strategy will always misreport the high state, i.e. $r(h) = l$ and $r(l) = l$ for all s_c.

For ease of referencing we introduce the following definitions.

Definition 2.6. *Define a Type 1 contract as a wage structure* (w_h, w_l) *such that the manager's optimal reporting strategy is:* $r(x) = x$ *for all* t, x.

Definition 2.7. *Define a Type 2 contract as a wage structure* (w_h, w_l) *such that the manager's optimal reporting strategy is: If* $t = c$ *and* $s_c = 1$ *then* $r(h) = l$*, otherwise* $r(x) = x$.

Definition 2.8. *Define a Type 3 contract as a wage structure* (w_h, w_l) *such that the manager's optimal reporting strategy is: If* $t = c$ *then* $r(h) = l$*, otherwise* $r(x) = x$.

Definition 2.9. *Define a Type 4 contract as a wage structure* (w_h, w_l) *such that the honest manager does not participate, while the corrupt manager does.*

For a given set of exogenous parameters, one or more of the above contract structures may dominate the others. Therefore, we need to identify the local optimum for each type of contract and then determine the globally optimal contracting structure for the principal. We first characterize each of the above contract types and then find the optimal contracting strategy for the principal.

2.4.1 Type 1 contract: Both managers report the true state

Since there is no benefit of collusion in this case and no corruption by the corrupt manager, both managers behave similarly. There is no difference in two manager's wage-effort equation, i.e. single crossing property not satisfied, and hence no screening possible, which means we will get a pooling contract. The principal's profit function is given by:[15]

$$\pi(e)\,[\underbrace{\frac{1}{2}(V_h + V_l) + B - \lambda\,P}_{\text{A: Profit without collusion}} - \frac{1}{2}(w_h + w_l)]$$

[15]Profit is not dependent on θ (exogenous probability of corrupt type) as both managers behave similarly.

The principal's problem (denoted as PP1) in this scenario can be represented as:

$$\max_{w_h, w_l, e} \ \pi(e)\,(A - \frac{1}{2}(w_h + w_l)) \tag{PP1}$$

subject to:

$$w_h \geq \nu\,(1-\delta)\,P + (1-\nu)\,(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \leq w_l + V_h - V_l \tag{IC2}$$

$$w_h \geq w_l \tag{MC}$$

$$e = \arg\max_{e'} \ \pi(e')\,[\frac{1}{2}(w_h + w_l)] - C(e') \tag{GIC}$$

$$\pi(e)\,[\frac{1}{2}(w_h + w_l)] - C(e) \geq 0 \tag{IR}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

We follow the standard procedure to derive the optimal contract structure for PP1. The solution is provided in Appendix 2.A.1. We summarize the result in Lemma 2.1.

Lemma 2.1. *Suppose the principle sets a wage structure so that both types of manager report the true state. Then, there exists* $\gamma_1 > 0$ *such that:*

1. *If* $V_h - V_l \in [0, \gamma_1]$ *then the optimal wage contract will satisfy* $\frac{1}{2}(w_h + w_l) = A$ *and* $0 \leq w_l \leq w_h$*, and the principal's profit will be* $\frac{A^2}{4}$*. If* $V_h - V_l = 0$ *then* $w_h = w_l = \frac{A}{2}$*, and if* $V_h - V_l = \gamma_1$ *then* $w_h = A, w_l = 0$*. If* $V_h - V_l \in (0, \gamma_1)$ *multiple solutions satisfying above conditions are possible.*

2. *If* $V_h - V_l > \gamma_1$ *then the optimal wage contract is* $w_h = \nu\,(1-\delta)\,P + (1-\nu)\,(V_h - V_l)$*,* $w_l = 0$ *and the principal's profit will be* $w_h\,(A - w_h)$ *which is lower than* $\frac{A^2}{4}$ *and is decreasing in* $V_h - V_l$

where $\gamma_1 = \frac{\nu}{1-\nu}\,(\frac{A}{\nu} - (1-\delta)\,P)$

Figure 2.8 shows the principal's profit and the wage structure graphically.

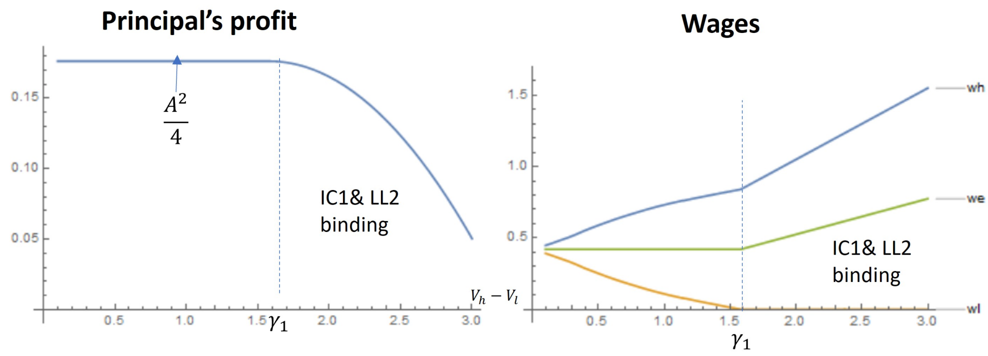


Figure 2.8: Optimal principal's profit and wages for Type 1 contract (PP1).

When $V_h - V_l < \gamma_1$, the principal's earning is similar to that with the honest manager, as there is no collusion. In addition, the principal is able to effectively control the manager by creating a wedge between the high state and low state wages while keeping the effective wage rate constant. When $V_h - V_l > \gamma_1$, IC1 and LL2 bind, and the high state wage increases with $V_h - V_l$ to prevent the corrupt manager from stealing. This results in both managers earning higher limited liability rent, and the principal's profit declines.

Remark: Both type of manager earns similar wages and exerts similar effort, reflecting pooling contract.

2.4.2 Type 2 contract: Corrupt manager misreports high state when $s_c = 1$

For the Type 2 contract structure, the manager's reporting strategy is: If $t = c$ and $s_c = 1$, then $r(h) = l$; otherwise, $r(x) = x$. The corrupt manager will misreport the high state when he has a corruption signal (left side in Figure 2.3). The principal will audit whenever the low state is reported, and this audit will find a discrepancy with probability ν, in which case the manager and the principal will enter into a side contract. On the right side of Figure 2.3, the corrupt manager will report the state correctly. Therefore, the incentive compatibility constraints for this reporting strategy are:

$$w_h \leq \nu\,(1-\delta)\,P + (1-\nu)\,(w_l + V_h - V_l) \qquad \text{(IC1)}$$

$$w_h \geq (1-\nu)\,(w_l + V_h - V_l) \qquad \text{(IC2)}$$

The IC1 constraint ensures that the corrupt manager reports $r(h) = l$ when $s_c = 1$. The IC2 constraint ensures that the corrupt manager reports $r(x) = x$ if $s_c = 0$. Let us denote the corrupt manager's effort as e_c if he decides to participate and the honest manager's effort as e_h. Given the above constraints, the principal's expected payoff if a corrupt manager is hired is:

$$\pi(e_c)[\underbrace{\frac{1}{2}(V_h+V_l)+B-\lambda\,P}_{\text{A: Profit without collusion}}+\underbrace{\delta\,(\frac{1}{2}\,\nu\,\lambda\,P)}_{\delta N\text{: collusion gain}}-\underbrace{\frac{1}{2}\,\lambda\,(1-\nu)\,(V_h-V_l)}_{\lambda b\text{: manager's corruption cost}}$$
$$\underbrace{-\frac{1}{2}\,(1-\lambda)\,w_h-\frac{1}{2}\,(1+\lambda\,(1-\nu))\,w_l]}_{\text{Managerial wage}}$$

and the corrupt manager's expected payoff:

$$\pi(e_c)\,[\underbrace{(1-\delta)\,(\frac{1}{2}\nu\,\lambda\,P)}_{(1-\delta)N\text{: collusion gain}}+\underbrace{\frac{1}{2}\lambda\,(1-\nu)\,(V_h-V_l)}_{\lambda b\text{: corruption benefit}}+\underbrace{\frac{1}{2}\,(1-\lambda)\,w_h+\frac{1}{2}(1+\lambda\,(1-\nu))\,w_l}_{\text{wage}}]-C(e_c)$$

Note: a) We have replaced $\frac{1}{2}\,(1-\nu)\,(V_h-V_l)$ with b (Definition 2.2), which represents the maximum benefit to the manager from stealing. In this case, there is a factor λ since the manager steals only when $s_c=1$; b) Since the side contract is entered when the audit finds a discrepancy and $s_c=1$ and this contract helps principal avoid penalty P, the total surplus from the side contract is $N\equiv\frac{1}{2}\,\lambda\,\nu\,P$ (see Definition 2.3). The principal retains δN of this surplus, and the manager retains $(1-\delta)\,N)$. The manager's wage for this reporting strategy will be: $\frac{1}{2}\,(1-\lambda)\,w_h+\frac{1}{2}(1+\lambda\,(1-\nu))\,w_l$. This expression reflects that the manager does not receive a wage when the audit finds a discrepancy and that he reports a high state only when $s_c=0$.

If the honest manager is hired, then the principal's and the manager's payoffs are similar to those described in Section 2.3.1. By combining the scenarios for both manager types, assuming both are willing to participate, we obtain the expected principal's payoff, which forms the principal's objective function. Hence, the principal's problem (PP2) can be written as:

$$\max_{w_h,w_l}\;(1-\theta)\,\pi(e_h)\,[A-\frac{1}{2}(w_h+w_l)]$$
$$+\,\theta\,\pi(e_c)\,[A-\lambda b+\delta N-\frac{1}{2}(1-\lambda)w_h-\frac{1}{2}\,(1+\lambda(1-\nu))\,w_l] \quad \text{(PP2)}$$

subject to:

$$w_h \leq \nu\,(1-\delta)\,P + (1-\nu)\,(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \geq (1-\nu)\,(w_l + V_h - V_l) \tag{IC2}$$

$$w_h \geq w_l \tag{MC}$$

$$e_h = \arg\max_{e'}\ \frac{1}{2}\,\pi(e')\,(w_h + w_l) - C(e') \tag{GIC1}$$

$$e_c = \arg\max_{e'}\ \pi(e')\,[\lambda b + (1-\delta)\,N + \frac{1}{2}(1-\lambda)\,w_h + \frac{1}{2}\,(1+\lambda(1-\nu))\,w_l] - C(e') \tag{GIC2}$$

$$\frac{1}{2}\,\pi(e_h)\,(w_h + w_l) - C(e_h) \geq 0 \tag{IR1}$$

$$\pi(e_c)\,[\lambda b + (1-\delta)\,N + \frac{1}{2}(1-\lambda)\,w_h + \frac{1}{2}\,(1+\lambda(1-\nu))\,w_l] - C(e_c) \geq 0 \tag{IR2}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

The detailed step-by-step solution to the above problem is provided in Appendix 2.A.2. We summarize the results in Lemma 2.2 and Lemma 2.3.

Lemma 2.2. *Suppose the principal sets a wage structure so that the corrupt manager misreports high signal only when he has a corruption signal. Then there exists* $0 < \beta_2 < \gamma_2$ *such that if* $V_h - V_l \in [\beta_2, \gamma_2]$*, then:*

1. *If* $\delta \leq \frac{1}{2}$ *then the optimal wage structure* (w_h, w_l) *is:*

$$(1-\frac{\nu}{2})\,w_h = (1-\nu)\,\frac{A}{2} + b - \frac{\delta-\frac{1}{2}}{}\lambda N \tag{2.10}$$

$$(1-\frac{\nu}{2})\,w_l = \frac{A}{2} - b + \frac{\delta-\frac{1}{2}}{\lambda}\,N \tag{2.11}$$

and the optimal profit is:

$$(1-\theta)\,\frac{A^2}{4} + \theta\,\frac{(A+N)^2}{4} \tag{2.12}$$

2. *If* $\delta > \frac{1}{2}$ *then the optimal wage structure is:*

$$(1-\frac{\nu}{2})\,w_h = (1-\nu)\,\frac{A}{2} + b + \theta\,(1-\nu)\,(\delta-\frac{1}{2})\,N \tag{2.13}$$

$$(1-\frac{\nu}{2})\,w_l = \frac{A}{2} - b + \theta(\delta-\frac{1}{2})\,N \tag{2.14}$$

and the optimal profit is:

$$(1-\theta)\,\frac{A^2}{4}+\theta\,\frac{(A+N)^2}{4}-\theta\,(1-\theta)\,(\delta-\frac{1}{2})^2\,N^2 \tag{2.15}$$

Intuition: When $V_h - V_l$ is in the interval $[\beta_2, \gamma_2]$ and $\delta \leq \frac{1}{2}$, then no constraints are binding. The total profit is the weighted average of the profit with a corrupt manager, $\frac{(A+N)^2}{4}$, and the profit with an honest manager, $\frac{A^2}{4}$. The principal can simultaneously optimize wages for both types of managers.[16] The principal's profit is also independent of b (manager's corruption) and δ (manager's bargaining weight). This is because any benefit the manager receives from corruption or better bargaining also increases the manager's effort, which cancels out the principal's cost from these effects. When $\delta > \frac{1}{2}$, the IC2 constraint is binding. The principal adjusts by increasing w_h and reducing w_l to relax IC2, which benefits the honest manager by increasing his effort, while it reduces the wages of the corrupt manager, whose effort decreases.[17] This reduces the optimal profit by $\theta\,(1-\theta)\,(\delta-\frac{1}{2})^2\,N^2$.[18]

Next we look at the solution when $V_h - V_l$ is outside of the interval $[\beta_2, \gamma_2]$.

Lemma 2.3. *Suppose the principal sets a wage structure so that the corrupt manager misreports high signal only when he has a corruption signal. Then there exists $\alpha_2 \in (0, \beta_2)$ such that:*

1. *If $V_h - V_l < \alpha_2$, then IC1 and MC bind, and profit is lower than (2.12). Profit decreases when $V_h - V_l$ decreases below α_2.*
2. *If $V_h - V_l \in (\alpha_2, \beta_2)$, then profit is constant and given by (2.12) if $\delta \leq \frac{1}{2}$ or (2.15) if $\delta > \frac{1}{2}$. MC binds, and both w_h and w_l decrease as $V_h - V_l$ increases.*
3. *If $V_h - V_l > \gamma_2$, then LL2 binds, and profit declines as $V_h - V_l$ increases.*

Please refer to Appendix 2.A.2 for the expression of profit and wages. We list the expression for cut-off points:

[16] Notice that $w_h + w_l = A$, which is the optimal wage structure for the honest manager.

[17] The corrupt manager reports the low state when the true state is high, so his compensation is more dependent on w_l and less on w_h.

[18] Reduction is a very small amount as the fraction $\theta\,(1-\theta)\,(\delta-\frac{1}{2})^2$ has the maximum value of $\frac{1}{16}$.

	$\delta \leq \frac{1}{2}$	$\delta > \frac{1}{2}$
β_2	$\frac{\nu}{1-\nu}\left(\frac{A}{2} + (\delta - \frac{1}{2})\, P\right)$	$\frac{\nu}{1-\nu}\left(\frac{A}{2} + (\delta - \frac{1}{2})\,(\frac{1}{2}\,\theta\,\lambda\,\nu)\, P\right)$
γ_2	$\frac{\nu}{1-\nu}\left(\frac{A}{\nu} + (\delta - \frac{1}{2})\, P\right)$	$\frac{\nu}{1-\nu}\left(\frac{A}{\nu} + (\delta - \frac{1}{2})\,(\theta\,\lambda)\, P\right)$
α_2	$\alpha_2 \in (0, \beta_2)$	$\alpha_2 \in (0, \beta_2)$

Table 2.2: Expressions for β_2 and γ_2

The results of Lemma 2.2 and 2.3 is depicted visually in Figure 2.9.

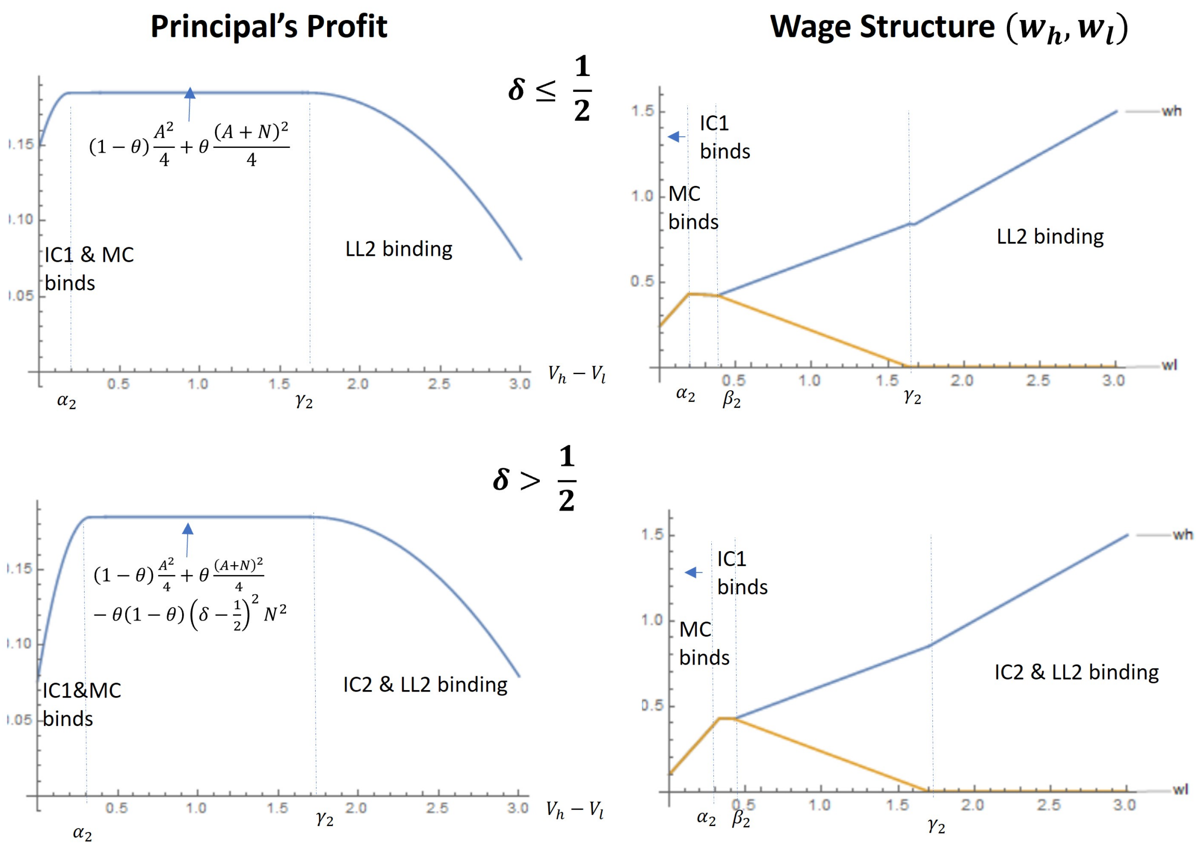


Figure 2.9: Optimal principal's profit and wages for Type 2 contract (PP2).

The profit is lower than (2.12) or (2.15) when $V_h - V_l < \alpha_2$ because IC1 binds, reducing wages below what was optimal without this constraint. This decreases the effort of both managers. This effect is amplified by the MC constraint ($w_h \geq w_l$), which hardens the IC1 constraint. Without the MC constraint, the principal could reduce w_h and increase w_l to relax the IC1 constraint while still maintaining the desired expected wage compensation for managers.

The profit is also lower than (2.12) or (2.15) when $V_h - V_l > \gamma_2$, which means LL2 binds. The profit decreases sharply as $V_h - V_l$ increases above γ_2. When LL2 is not binding, any increase in w_h is accompanied by a reduction in w_l so that all constraints are satisfied without giving extra rent to the managers. However, when LL2 binds, w_h

keeps increasing with $V_h - V_l$ to satisfy the IC2 constraint (see Figure 2.9) without a corresponding reduction in w_l. As a result, both the honest and the corrupt managers receive higher limited liability rent as $V_h - V_l$ increases.

Also the honest and the corruption manager has different effort and payoffs, showing that this is a separating equilibrium.

2.4.3 Type 3 contract: Corrupt manager always misreports high state

As discussed earlier, in this scenario, the manager's reporting strategy will be: If $t = c$, then $r(h) = l$; otherwise, $r(x) = x$. The corrupt manager will misreport the high state regardless of he having corruption signal (both side in Figure 2.3). The principal will audit whenever the low state is reported. If this audit finds a discrepancy when $s_c = 1$ (left side of Figure 2.3) then the manager and the principal will enter into a side contract. If this audit finds a discrepancy when $s_c = 0$ (right side of Figure 2.3) then the principal fire the manager after recovering misappropriated amount. Therefore, the incentive compatibility constraint for this reporting strategy is:

$$w_h \leq (1-\nu)\,(w_l + V_h - V_l) \tag{IC1}$$

The IC1 constraint ensures that the corrupt manager reports $r(h) = l$ for all $s_c \in \{0, 1\}$. Given the above reporting strategy, the principal's expected payoff:

$$\pi(e_c)[\underbrace{\frac{1}{2}(V_h + V_l) + B - \lambda\, P}_{\text{A: Profit without collusion}} + \underbrace{\delta(\frac{1}{2}\,\nu\,\lambda\,P)}_{\delta N\text{: collusion gain}} - \underbrace{\frac{1}{2}(1-\nu)\,(V_h - V_l)}_{b\text{: cost of manager's corruption}} - \underbrace{(1-\frac{\nu}{2})\,w_l}_{\text{Managerial wage}}\ ]$$

and the corrupt manager's expected payoff:

$$\pi(e_c)[\underbrace{(1-\delta)\,(\frac{1}{2}\nu\,\lambda\,P)}_{(1-\delta)N\text{: collusion gain}} + \underbrace{\frac{1}{2}(1-\nu)\,(V_h - V_l)}_{b\text{: corruption benefit}} + \underbrace{(1-\frac{\nu}{2})\,w_l}_{\text{wage}}] - C(e_c)$$

Note the key differences between Type 2 and Type 3: a) The manager's corruption benefit is b instead of $\lambda\, b$, as the corrupt manager misappropriates under both $s_c = 0$ and $s_c = 1$; b) The manager's wage for his reporting strategy will be $(1 - \frac{\nu}{2})\, w_l$. He never reports a high state, so w_h is not part of this wage expression, and he does not receive a wage when the audit finds a discrepancy. If the honest manager is hired, then the principal's and the manager's payoffs are similar to those described in Section 2.3.1. Therefore, the principal's problem (PP3) can be written as:

$$\max_{w_h, w_l} (1-\theta)\,\pi(e_h)\,[A - \frac{1}{2}(w_h + w_l)] + \theta\,\pi(e_c)\,[A - b + \delta N - (1 - \frac{\nu}{2})\,w_l] \tag{PP3}$$

subject to:

$$w_h \leq (1-\nu)(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \geq w_l \tag{MC}$$

$$e_h = \arg\max_{e'} \frac{1}{2}\,\pi(e')\,(w_h + w_l) - C(e') \tag{GIC1}$$

$$e_c = \arg\max_{e'} \pi(e')\,[b + (1-\delta)\,N + (1 - \frac{\nu}{2})\,w_l] - C(e') \tag{GIC2}$$

$$\frac{1}{2}\pi(e_h)(w_h + w_l) - C(e_h) \geq 0 \tag{IR1}$$

$$\pi(e_c)\,[\frac{1}{2}\,(1-\nu)\,(V_h - V_l) + \frac{1}{2}\,\lambda\,\nu\,(1-\delta)\,P + (1 - \frac{\nu}{2})\,w_l] - C(e_c) \geq 0 \tag{IR2}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

The detailed solution of the above problem is provided in Appendix 2.A.3. We summarize the results in Lemma 2.4 and 2.5

Lemma 2.4. *Suppose the principal sets a wage structure so that the corrupt manager always gives a false report when he receives $x = h$. Then there exists $0 < \beta_3 < \gamma_3$ such that If $V_h - V_l \in [\beta_3, \gamma_3]$ then*

1. *If $\delta \geq \frac{1}{2}$ then the optimal wage structure (w_h, w_l):*

$$(1 - \frac{\nu}{2})\,w_h = (1-\nu)\,\frac{A}{2} + b - (\delta - \frac{1}{2})\,N \tag{2.16}$$

$$(1 - \frac{\nu}{2})\,w_l = \frac{A}{2} - b + (\delta - \frac{1}{2})\,N \tag{2.17}$$

   and the optimal profit is:

$$(1-\theta)\,\frac{A^2}{4} + \theta\,\frac{(A+N)^2}{4} \tag{2.12}$$

2. *If $\delta < \frac{1}{2}$ then the optimal wage structure:*

$$(1 - \frac{\nu}{2})\,w_h = (1-\nu)\,\frac{A}{2} + b + \theta\,(1-\nu)\,(\delta - \frac{1}{2})\,N \tag{2.18}$$

$$(1 - \frac{\nu}{2})\,w_l = \frac{A}{2} - b + \theta\,(\delta - \frac{1}{2})\,N \tag{2.19}$$

   and the optimal profit is:

$$(1-\theta)\,\frac{A^2}{4} + \theta\,\frac{(A+N)^2}{4} - \theta\,(1-\theta)\,(\delta - \frac{1}{2})^2\,N^2 \tag{2.15}$$

Lemma 2.4 shows that if $V_h - V_l \in [\alpha_3, \gamma_3]$, the principal's profit remains the same. She adjusts w_h and w_l to achieve the second-best optimal effort from both types of managers with the same contract.[19] For example, w_l in (2.17) decreases by exactly the amount lost to the corrupt manager's misappropriation, thereby recovering the cost. Simultaneously, w_h increases so that $w_h + w_l = A$, which is optimal for the honest manager (see (2.1)).

Also, note that in Type 3 contract, IC1 ($w_h \leq (1-\nu)(w_l + V_h - V_l)$) binds when $\delta < \frac{1}{2}$, whereas in Type 2 contract, IC2 ($w_h \geq (1-\nu)(w_l + V_h - V_l)$) binds when $\delta > \frac{1}{2}$ because the inequality is in the reverse direction. This also results in Type 3 having the same wage structure when $\delta < \frac{1}{2}$ (see (2.18)-(2.19)) as Type 2 when $\delta > \frac{1}{2}$ (see (2.13)-(2.14)).

Lemma 2.5. *Suppose the principal sets a wage structure so that the corrupt manager always gives a false reports when he receives $x = h$. Then there exists $\alpha_3 \in (0, \beta_3)$ such that:*

1. *If $V_h - V_l < \alpha_3$ then MC binds and profit is lower than (2.12) which decreases when $V_h - V_l$ decreases.*
2. *If $V_h - V_l \in (\alpha_3, \beta_3)$ then the profit is constant and is given by (2.12). MC binds and both w_h and w_l decreases when $V_h - V_l$ increases.*
3. *If $V_h - V_l > \gamma_3$ then LL2 binds and profit declines when $V_h - V_l$ increases.*

Please refer to Appendix 2.A.3 for the expression of profit and wages. We list the expression for cut-off points:

	$\delta \geq \frac{1}{2}$	$\delta < \frac{1}{2}$
β_3	$\frac{\nu}{1-\nu}\left(\frac{A}{2} + (\delta - \frac{1}{2})\,\lambda\, P\right)$	$\frac{\nu}{1-\nu}\left(\frac{A}{2} + (\delta - \frac{1}{2})\,(\frac{1}{2}\theta\,\lambda\,\nu)\, P\right)$
γ_3	$\frac{\nu}{1-\nu}\left(\frac{A}{\nu} + (\delta - \frac{1}{2})\,\lambda\, P\right)$	$\frac{\nu}{1-\nu}\left(\frac{A}{\nu} + (\delta - \frac{1}{2})\,(\theta\,\lambda)\, P\right)$
α_3	$\alpha_2 \in (0, \beta_3)$	$\alpha_3 = \beta_3$

Table 2.3: Expressions for β_3 and γ_3

The results of Lemma 2.4 and 2.5 is shown visually in Figure 2.10.

[19]This optimal effort is below the first-best for each manager because the manager's effort is non-verifiable and the manager is protected under limited liability.

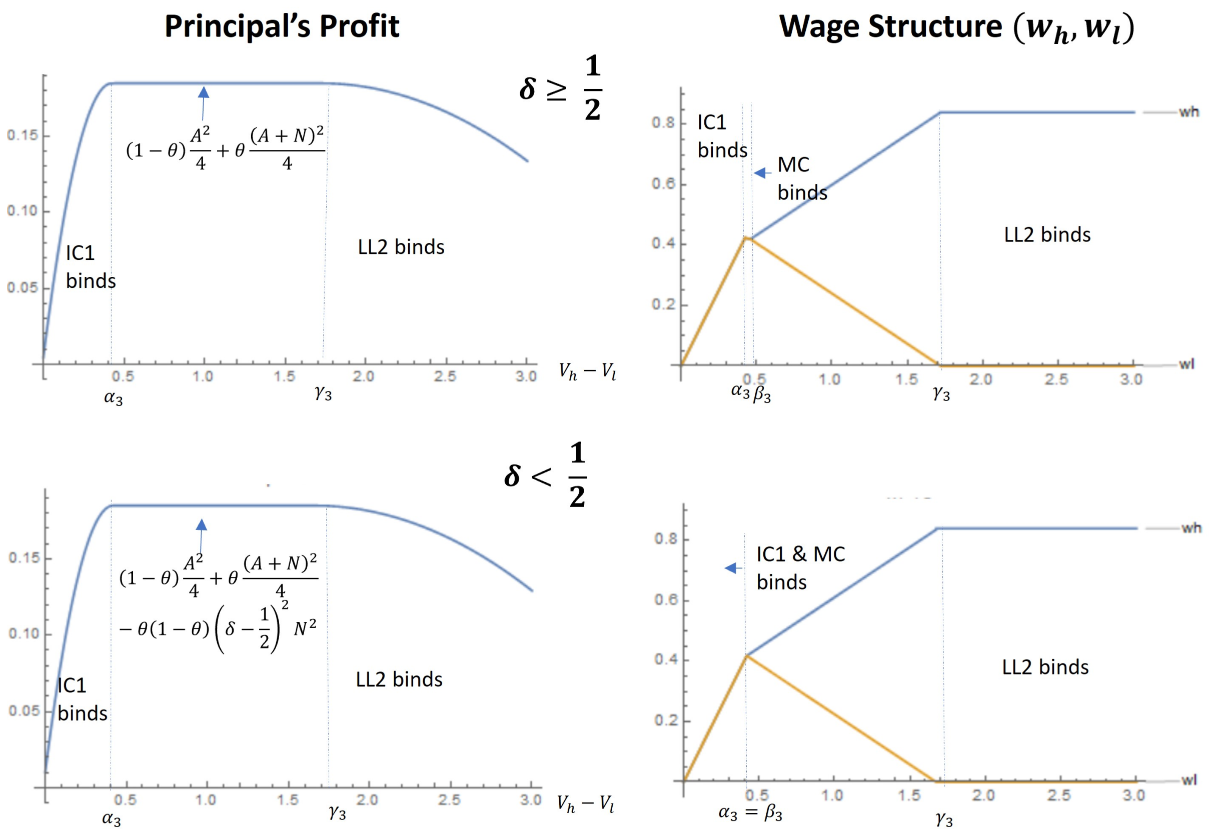


Figure 2.10: Optimal principal's profit and wages for Type 3 contract (PP3).

Notice the two key differences between Type 3 (Figure 2.10) and Type 2 (Figure 2.9). First, when LL2 binds, $w_h(= A)$ remains constant. This ensures that the honest manager does not receive additional rent when LL2 binds. However, the corrupt manager does receive additional limited liability rent since the principal cannot recover the stolen amount by reducing w_l. In contrast, in Type 2, w_h keeps increasing with $V_h - V_l$, providing higher limited liability rent to both managers. Therefore, the profit drops more steeply with $V_h - V_l$ when LL2 binds in Type 2 compared to Type 3.

Second, α_3, the cutoff below which the profit drops if $V_h - V_l$ decreases, is greater than α_2. This is because it is harder to satisfy IC1 in Type 3 ($w_h \leq (1-\nu)(w_l + V_h - V_l)$) than IC1 in Type 2 ($w_h \leq (1-\delta)P + (1-\nu)(w_l + V_h - V_l)$) due to the additional term $(1-\delta)P$.

2.4.4 Type 4 contract: Shutdown of the honest manager

In this scenario, the honest manager does not participate, while the corrupt manager does. This is only possible if the reservation wage is positive, even if it is infinitesimally close to zero.

Assumption 2.4. *The reservation wage of the manager, denoted as r_w, is positive but very* *close to zero, i.e.,* $r_w \to 0^+$*.*

Since the reservation wage is close to zero, Assumption 2.4 does not affect participation considerations in any other cases. In Section 2.5, we will examine the impact if r_w is higher. Under Assumption 2.4, if the principal sets $w_h = w_l = 0$, the honest manager's IR constraint (shown below) is not satisfied.

$$\frac{1}{2}\pi(e_h)\,(w_h + w_l) - C(e_h) \geq r_w \tag{IR}$$

The optimal effort $e_h = 0$ when $w_h + w_l = 0$, resulting in the left-hand side being zero, which is lower than r_w. Therefore, only the corrupt manager participates, as his participation constraint is satisfied because his payoff is positive even at zero wage due to his ability to earn through corruption and collusion.[20] The principal's objective function when only a corrupt manager participates with wages $w_h = w_l = 0$ is given by:

$$\pi(e_c)\,(A - b + \delta N) \text{ where}$$

$$\text{(GIC)}\ e_c = \arg\max_{e'} \pi(e)\,(b + (1-\delta)\,N) - C(e')$$

The manager earns b through his corruption and $(1-\delta)N$ through collusion, and the principal's payoff includes the cost of the manager's corruption (b) and the benefit from collusion (δN). From the first-order condition of GIC, we get the manager's effort $e_c = b + (1-\delta)N$, which gives the principal's profit function as:

$$(b + (1-\delta)N)(A - b + \delta N) \tag{2.20}$$

Equation (2.20) is a concave function and represents an inverted parabola, with the peak at $V_h - V_l = \gamma_4 \equiv \frac{\nu}{1-\nu}\left(\frac{A}{\nu} + \left(\delta - \frac{1}{2}\right)\lambda\,P\right)$ and the maximum profit is $\frac{(A+N)^2}{4}$ which is higher than the maximum profit of Type 2 or Type 3 contracts (see (2.12)) as $\theta < 1$. The principal's profit declines on both side of γ_4.

Lemma 2.6. *Suppose the managers have a reservation wage as specified in Assumption 2.4. Then, there exists an interval of* $V_h - V_l$ *where only the corrupt manager is hired.*

The proof is straight forward as the profit function is continuous and the maximum profit of Type 4 is higher than that of other contracts. We will discuss more precise characterization of this interval in the next section.

Note: γ_4 is the exact value of $V_h - V_l$ where LL2 just binds in the Type 3 contract, i.e. $\gamma_4 = \gamma_3$.

[20] We have shown in Section 2.3.1 that positive payoff results in positive surplus for the manager.

2.4.5 Principal's optimal contracting strategy

In this section, we evaluate which of the contract structures is globally optimal for the principal across different parameters. Our main exogenous variable is $V_h - V_l$, which determines the potential leakage through the manager's corruption. Proposition 2.3 and 2.4 state our main result, with the proof provided in Appendix 2.A.4. For a clearer understanding, this result is visually represented in Figure 2.11.

Proposition 2.3. *There exist* $0 \leq \mu_1 < \mu_2$ *such that:*

1. *If* $V_h - V_l < \mu_1$*, the principal offers a Type 1 contract. There is no corruption by the manager and no collusion between the principal and the manager. The principal's profit is given by* $\frac{A^2}{4}$*. Both managers exert similar effort and receive similar wages (pooling equilibrium).*

2. *If* $V_h - V_l \in (\mu_1, \mu_2)$*, the principal offers a Type 2 contract. The profit is strictly increasing in* $V_h - V_l$ *and lies in the interval* $[\frac{A^2}{4}, (1-\theta)\ \frac{A^2}{4} + \theta\ \frac{(A+N)^2}{4}]$*. The corrupt manager exerts higher effort (separating equilibrium).*

Notes: If δ is lower than a critical value then $\mu_1 = 0$ and Type 1 pooling contract will not exist for any parameter values.

Intuition: When $V_h - V_l < \mu_2$, the principal benefits from collusion and seeks to incentivize the corrupt manager to reveal his type. If $V_h - V_l$ is small, the manager lacks sufficient incentive to steal and expose himself. The principal raises w_l and lowers w_h to provide this incentive without distorting the honest manager's effort. However, once the MC condition binds, she must either raise w_h, increasing rent for the honest manager, or lower both w_h and w_l, reducing efforts. Consequently, Type 2 or Type 3 contracts are inefficient when $V_h - V_l < \mu_1$, leading the principal to offer a pooling contract (Type 1) with no collusion. Without the MC condition, Type 2 or Type 3 contracts would dominate for $V_h - V_l < \mu_2$ (see Section 2.5).

Proposition 2.4. *Suppose Assumption 2.4 holds. There exist* $\mu_2 \leq \mu_3 < \mu_4$ *such that:*

1. *If* $V_h - V_l \in (\mu_2, \mu_3)$*:*
   - *If* $\delta < \frac{1}{2}$*, the principal offers a Type 2 contract with the wage structure as in (2.10)-(2.11).*
   - *If* $\delta \geq \frac{1}{2}$*, the principal offers a Type 3 contract with the wage structure as in (2.16)-(2.17).*

   In both cases, the profit is given by $(1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4}$*, and the corrupt manager exerts higher effort.*

2. *If $V_h - V_l \in (\mu_3, \mu_4)$, the principal offers a contract with zero wage ($w_h = w_l = 0$) such that only the corrupt manager participates, and the profit is given by $\frac{(A+N)^2}{4}$.*

3. *If $V_h - V_l > \mu_4$, the principal offers a Type 3 contract with $w_h = A$ and $w_l = 0$. The profit declines with $V_h - V_l$. The corrupt manager earns significant rent due to limited liability, and their effort increases with $V_h - V_l$, while the honest manager's effort remains constant.*

Figure 2.11 highlights the results of Proposition 2.3 and 2.4 visually:

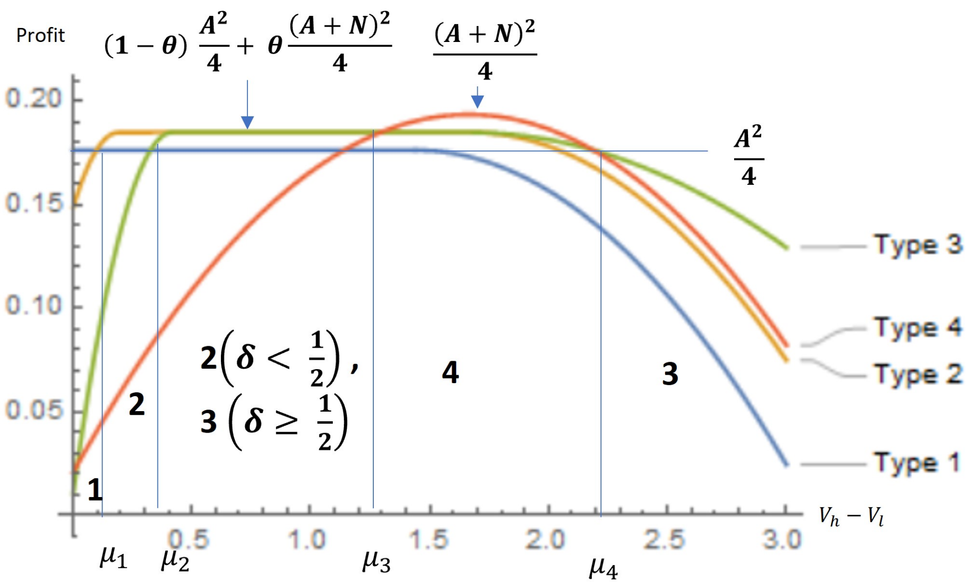


Figure 2.11: Principal's profit for different types of contracts and the dominating contract. Parameter values: $\nu = 0.5$, $\lambda = 0.4$, $\frac{P}{B} = 1.6$

Note: The numbers in the middle in Figure 2.11 shows which type of contract dominates in each interval.

Let's understand the intuition behind the Proposition 2.4.

a) When $V_h - V_l \in (\mu_2, \mu_3)$, the principal can adjust w_h and w_l to solicit second-best effort from both types of managers while controlling corruption. For example, the principal reduces w_l by the amount b the corrupt manager would appropriate (see (2.17)) and raises w_h by the same amount. This ensures that the corrupt manager's payoff and effort, which is only influenced by w_l, does not get distorted by his corruption. At the same time, keeping $w_h + w_l$ constant ensures no distortion of the honest manager's payoff and effort. This keeps the profit constant and independent of $V_h - V_l$. Similarly, the principal adjusts the wages so that the corrupt manager and the principal share the collusion surplus equally, irrespective of δ, because the principal values the manager's effort. When the principal has higher bargaining power, he compensates the corrupt manager through higher w_l, balanced by adjustments in w_h to prevent distortion in the honest manager's effort (see (2.17)). Thus, there is no effort distortion due to asymmetric information. However, due to zero reservation wage, the principal cannot shut down the honest manager, resulting in lower expected profit than $\frac{(A+N)^2}{4}$, achievable with only the corrupt manager.[21]

b) When $V_h - V_l \in (\mu_3, \mu_4)$, only the corrupt manager is hired. Near the point where LL2 just binds ($w_l = 0$) in Type 2 or Type 3 contracts, the principal can offer $w_h = 0$ to shut down the honest manager while maintaining the same level of effort from the corrupt manager, thus increasing her profit. However, the reservation wage must be positive as in Assumption 2.4. The profit from Type 4 is maximized when $V_h - V_l = \gamma_4$, where LL2 just binds, and it declines on both sides of γ_4. When $V_h - V_l < \mu_3$, the profit from the Type 4 contract drops below that of Type 2 or Type 3, as zero wages do not elicit optimal effort from the corrupt manager. When $V_h - V_l > \mu_4$, the limited liability becomes very costly, as explained in d) below.

d) When $V_h - V_l$ is above μ_4, the limited liability constraint becomes very costly, and the profit from the corrupt manager's effort falls below $\frac{A^2}{4}$, making the Type 4 contract suboptimal. In this scenario, the principal would prefer hiring the honest manager but lacks a mechanism to shut down the corrupt manager. However, she ensures that the honest manager participates and exerts optimal effort by setting $w_h = A$, which is a Type 3 contract.

Figure 2.12 shows the effort of each type of manager.

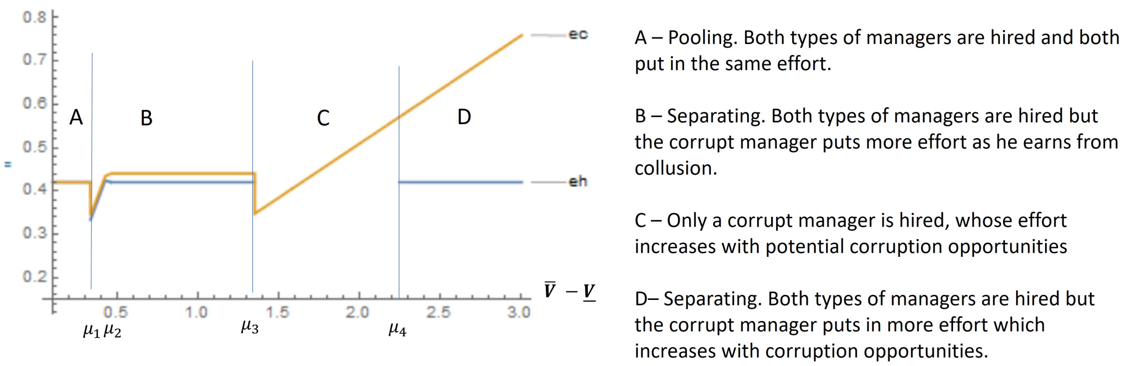


Figure 2.12: Effort Level of manager by their type

Two important points to note in Figure 2.12: a) The honest manager's effort is not distorted due to asymmetric information except when $V_h - V_l \in (\mu_1, \mu_2)$, which is due to the MC constraint. This is because the corrupt manager wage does not depend on w_h,

[21] This differs slightly from Section 2.3.2, where the corrupt manager signs an ex-ante contract to collude whenever $s_c = 1$. Here, collusion occurs when $s_c = 1$, $x = h$, and the audit finds a discrepancy.

which can be used to give optimal incentive to the honest manager.[22] b) The corrupt manager's effort increases when LL is binding, and he captures the benefit of higher effort through higher rent.

Figure 2.13 plots the principal's profit function if she offers the optimal contract for all values of $V_h - V_l$, and compare it with two scenarios: a) The principal hires an honest manager during recruitment (Section 2.3.1); b) The principal hires a corrupt manager without an ex-ante contract, so the corrupt manager engages in side contracts only when the audit finds a discrepancy. Therefore, his potential gain from collusion is $N \equiv \frac{1}{2}\lambda\nu P$ and not λP as it was in Section 2.3.2. We introduce this scenario without a formal model to provide a fair comparison when the benefit from collusion remains the same across models.[23] The comparison with the ex-ante contract is made in Section 2.5.

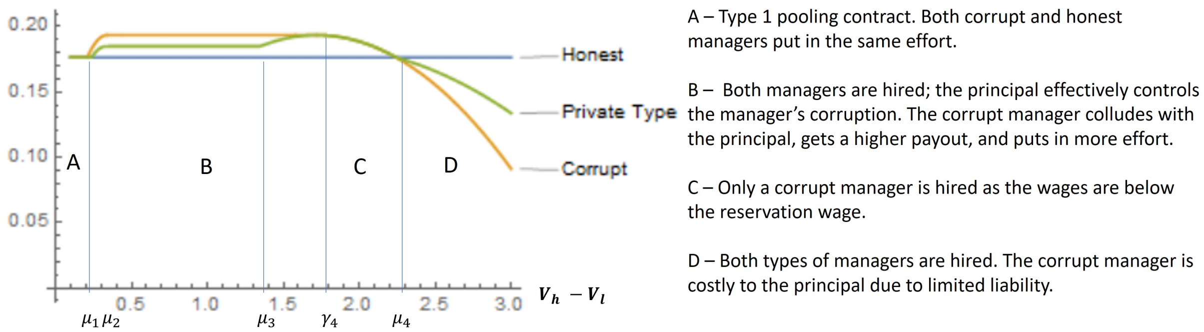


Figure 2.13: Principal's profit across different values of $V_h - V_l$

2.5 Comparative statics

To visualize the impact of changes in assumptions and/or exogenous parameters, we will continue to use the three different types of curves shown in Figure 2.13.

2.5.1 Increase in reservation wage

Assumption 2.4 in section 2.4.4 assumed that the reservation wage is positive but just above zero, limiting the principal to offering a wage structure of $w_h = w_l = 0$ to exclude the honest manager. Now, we relax Assumption 2.4 and treat r_w as an exogenous parameter. Figure 2.14 shows that increasing the reservation wage decreases μ_3, thereby expanding the range of $V_h - V_l$ where only the corrupt manager is hired.

[22]Note the effort distortion due to moral hazard from unobservable effort still persist.

[23]This is mainly for the comparison and intuition.

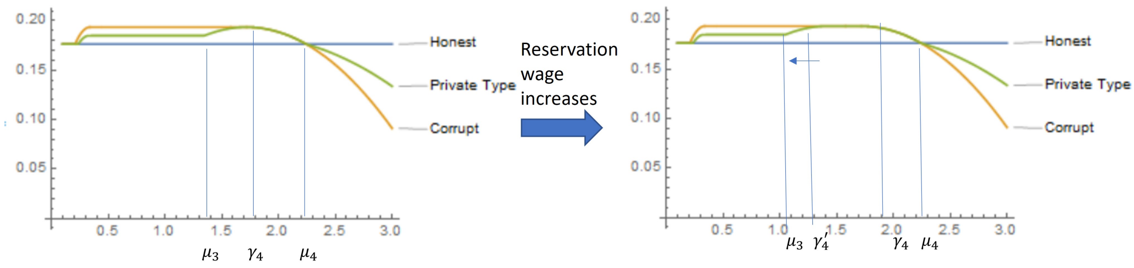


Figure 2.14: Principal's profit change when reservation wage increases

With higher r_w, the principal can raise w_l up to the reservation wage[24] without satisfying the honest manager's participation constraint. As noted earlier, until LL2 binds, the principal can adjust wages to offset corruption's impact while still colluding with the corrupt manager, keeping the profit from the corrupt manager's effort constant at $\frac{(A+N)^2}{4}$. In Figure 2.14, γ_4 marks the point where LL2 just binds if a Type 3 contract was offered. To the left of γ_4, if the principal can raise wages per (2.17), the profit remains constant at the maximum $\frac{(A+N)^2}{4}$. γ_4' indicates the point where w_l cannot be raised further without satisfying the honest manager's participation constraint. This result is summarized in Proposition 2.5.

Proposition 2.5. *If the reservation wage of managers increase then the interval (μ_3, μ_4) in which only corrupt manager is hired increases as μ_3 decreases. Within this interval there exist a sub-interval $(\gamma_4', \gamma_4) \subset (\mu_3, \mu_4)$ where the principal's profit is $\frac{(A+N)^2}{4}$*

2.5.2 Without monotonicity constraint (MC)

Assumption 2.3 enforced the wage monotonicity constraint ($w_h \geq w_l$). This was assumed to prevent the principal from exposing her type to external stakeholders, as a higher wage in the low productivity state than in the high productivity state could reveal the principal as corruptible. Only a corruptible principal would offer such a wage contract.

As discussed earlier, the MC constraint distorts incentives when $V_h - V_l < \mu_2$. Let's relax Assumption 2.3. The left panel of Figure 2.15 shows the principal's contracting strategy without the MC constraint. The pooling contract of Type 1 disappears, as the principal can incentivize the manager to reveal his type without any cost to the firm. Also, observe in the right panel of Figure 2.15 that w_l is greater than w_h when $V_h - V_l < \mu_2$.

[24] Honest manager's compensation $\frac{1}{2}(w_h + w_l) < r_w$ with $w_h \geq w_l$ implies $w_l < r_w$.

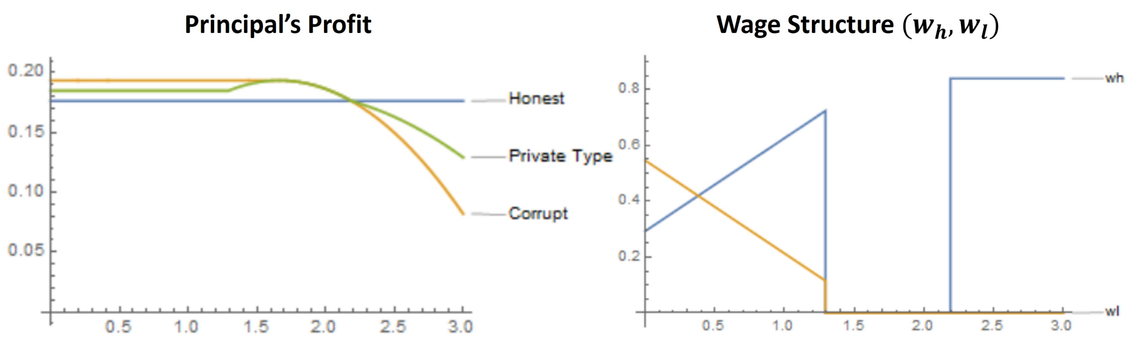


Figure 2.15: Principal's profit and wage structure without MC constraint

2.5.3 Increase in Audit effectiveness

Figure 2.16 shows the impact of increase in audit effectiveness (ν) on the principal's profit. Increase in ν has four main impacts:

a) Increases the benefit from collusion because of higher probability of type revelation by the corrupt manager, as a result higher profit when $V_h - V_l \in (\mu_2, \mu_4)$

b) Limited liability becomes less costly as the leakage from misappropriation $\left(\frac{1}{2}(1-\nu)(V_h - V_l)\right)$ is reduced. Consequently, the profit declines less steeply when $V_h - V_l > \mu_4$.

c) μ_4 increases because limited liability binds at a higher $V_h - V_l$. This extends the interval (μ_3, μ_4) where the principal can shut down the honest manager.[25]

d) μ_1 increases because it becomes harder to incentivize the manager to reveal information when $V_h - V_l$ is small (due to the MC constraint). This expands the interval $(0, \mu_1)$ where the Type 1 pooling contract dominates.

Therefore, higher audit effectiveness benefits the principal except when $V_h - V_l$ is small. If the potential loss from manager's corruption is high , the principal would invest in increasing the audit effectiveness.

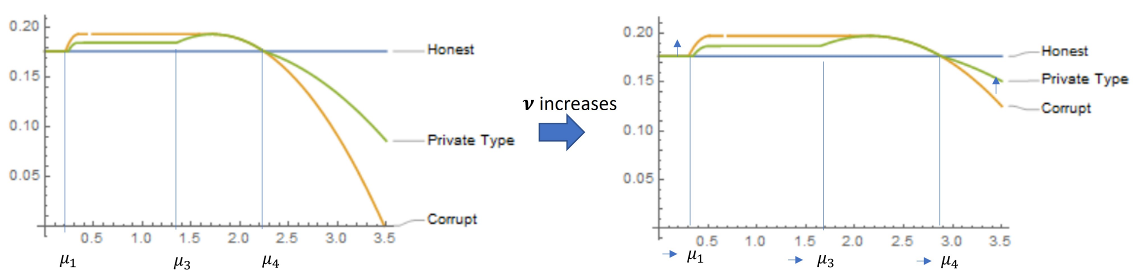


Figure 2.16: Principal's profit change when audit effectiveness ν increases

[25] μ_3 also increases but to a lesser extent because the profit with only a corrupt manager is higher.

2.5.4 Increase in λ

Suppose the manager is more inquisitive and gains access to corruption information with a higher probability (λ). This reduces the principal's overall profit because a larger proportion of the private benefit is at risk. However, the increased access to corruption information also enhances the benefit from collusion, making the corrupt manager more desirable by the principal. Consequently, although the overall profit decreases, the range (μ_3, μ_4), where only corrupt manager join the firm, increases. In addition, the Type 1 pooling contract could vanish as μ_1 decrease (see Figure 2.17).

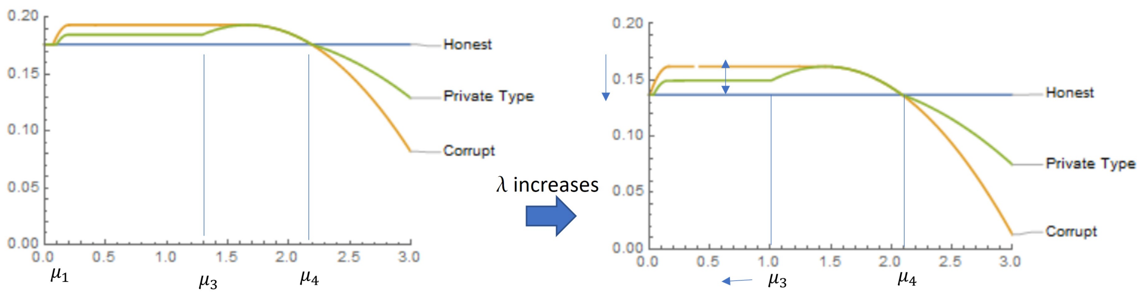


Figure 2.17: Principal's profit change when λ increases

2.5.5 Impact of change in P and δ

The effect of a change in P is exactly similar to that of λ because both increase the cost of exposure λP.

We have already discussed that the principal's profit is independent of δ when $V_h - V_l > \mu_2$. The effect of a change in δ is primarily relevant when IC1 and MC bind, i.e., $V_h - V_l < \mu_2$. IC1 in the Type 2 contract is:

$$\text{(IC1) } w_h \leq \nu(1-\delta)P + (1-\nu)(w_l + V_h - V_l)$$

This constraint is hardest to satisfy when $(1-\delta)P$ is small and MC is binding. This implies that higher P and lower δ relax the constraint, causing μ_2 and μ_1 to shift leftward. Therefore, when δ is below a critical value, $\mu_1 = 0$, and there is no Type 1 pooling contract.

2.5.6 Side-contract when manager does not misreport

In our base model, we assumed that the side contract takes place only when the manager's type is revealed through an audit signal. We assumed this because a manager without a pre-existing ties with the principal would not want to engage in such an illegal contract but would be forced to do so when caught in their own corruption. This provides a more self-enforceable mechanism.

Now, we assume that the manager can also approach the principal for a side contract if he has a corruption signal, and he does not engage in corruption. We refer this as *extended model*. This assumption leads to more instances of collusion, resulting in greater benefits from collusion.

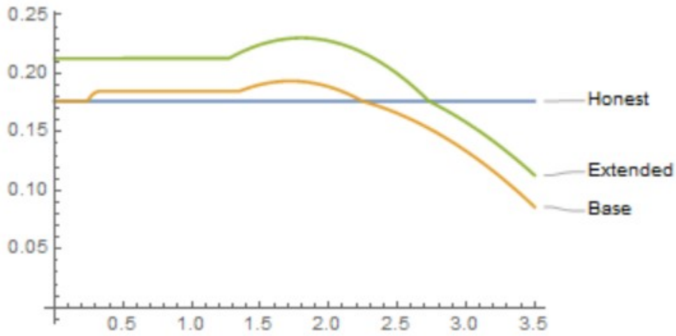


Figure 2.18: Principal's profit when side-contract take place a) After audit (Base) and b) After audit and when the manager has principal's corruption signal but do not engage in corruption (Extended)

As we can see in Figuure 2.18 that the principal's payoff curve shifts vertically with the new assumption. In addition, the distortion at the lower end vanishes as the IC1 condition in Type 2 contract becomes easier to satisfy.

2.5.7 Corruptible manager with ex-ante contract

Suppose the principal hires a corruptible manager with an ex-ante contract for collusion (discussed in section 2.3.2). This means the manager will always destroy corruption evidence upon receiving a corruption signal. As shown in Figure 2.19, the principal's payoff is higher in this scenario compared to even the extended model. This is because the benefit from collusion with an ex-ante contract is λP, whereas in the extended model, it is $\frac{1}{2}(1+\nu)\lambda P$, because the manager will not reveal his type when engaged in corruption and the audit does not find a discrepancy. Therefore, the principal would always prefer an ex-ante contract with a corrupt manager if she can identify such a manager and the contract is self-enforceable. Ex-ante contract also suffers from commitment problem from both the principal and the manager. For example, once the manager receives a corruption signal, his bargaining power is higher and he may demand higher payout from the principal.

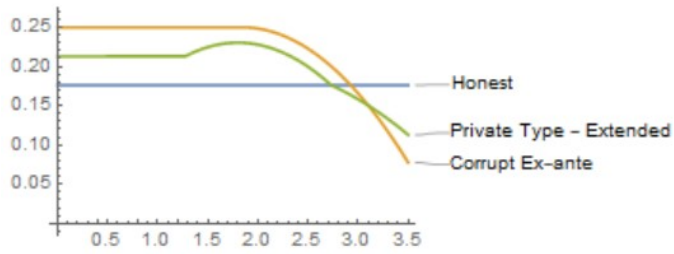


Figure 2.19: Principal's profit when a) the principal hires corrupt manager with ex-ante contract and b) when the manager type is private with the assumption of the extended model as discussed above.

2.6 Summary of Results

We have demonstrated a screening mechanism where a corruptible principal enables a corrupt manager to reveal his type and enter into a collusive side-contract. Our model predicts the following:

1. If $V_h - V_l$ is not too high, i.e., $V_h - V_l \in (\mu_2, \mu_3)$, the principal can control the manager's corruption through a suitable wage contract, ensuring corruption does not impact profit. The principal adjusts the corrupt manager's wage by the same amount as the manager would steal. Similar to the first chapter, the corruptible manager's wage decreases with corruption if limited liability is not binding. Both types of managers are hired, although the principal would strictly prefer a corruptible manager.

2. If $V_h - V_l$ is in the medium range, i.e., $V_h - V_l \in (\mu_3, \mu_4)$, the principal sets the wage below the reservation wage to ensure only the corrupt manager takes the offer, who is compensated through collusion and corruption. This configuration maximizes the principal's profit. Higher reservation wages expand this interval. Besley and McLaren (1993) refers to such a wage structure as capitulation wage, where only dishonest individuals take the job. While this benefits the principal, it may negatively impact the organization. In hierarchical organizations with multiple management levels, this could lead to widespread corruption throughout the organization.

3. If $V_h - V_l$ is high, i.e., $V_h - V_l > \mu_4$, the manager's corruption starts impacting the principal's profit. The principal would prefer to hire an honest manager, but both types of managers participate. One way the principal can address this additional cost of corruption is by investing to increase the audit effectiveness. Manager effort increases with corruption, but the manager retains the surplus from this effort. Che (1995) discussed the beneficial aspect of such corruption induced effort in the

context of bureaucracy. They demonstrated that tax inspectors and regulators work harder, develop expertise, and increase monitoring.

4. The monotonicity condition creates distortion when $V_h - V_l$ is very low, i.e., $V_h - V_l < \mu_2$. It becomes difficult to provide the manager with proper incentives for type revelation. A lower δ (principal's bargaining weight) alleviates this distortion.

5. The principal's profit is independent of her bargaining weight, δ. When $\delta > \frac{1}{2}$, she pays the manager extra wages to incentivize effort. Conversely, when $\delta < \frac{1}{2}$, she reduces the manager's wage. The net effect is an equitable sharing of the surplus from collusion. We highlighted in the first chapter that higher bargaining power isn't always beneficial to the principal, as the principal's profit is non-monotonic in relation to her bargaining weight, a situation referred to by Besley and McLaren (1993) as the "persuasion paradox."

6. Unlike other adverse selection models, the honest manager's effort is not distorted.[26] This is because the corrupt manager's compensation is independent of the high-state wage. Thus, the principal can design a wage structure where the honest manager is paid fully on the high state, while the corrupt manager's compensation is based on the low state wage.

7. The principal would prefer an ex-ante contract with a corrupt manager if the manager's type can be identified during recruitment and the contract can be self-enforced. However, such contracts are illegal and not enforceable in court, leading to commitment problems. The manager might demand more compensation ex-post when he possesses evidence of the principal's corruption or might have more incentive to collude with the investigating agency if he remains corruption-free. Therefore, such contracts become self-enforcing only when the principal and the manager have pre-contractual ties, as highlighted by Kofman and Lawarree (1996). This explains why a corrupt owner often appoints a trusted family member as the manager, even if that individual is not the most competent person (Morck et al., 2000; Bertrand and Schoar, 2006).

As discussed in section 2.5, our results are robust to assumptions about parameters, with some exceptions:

- If the reservation wage of managers r_w is equal to zero, or more precisely equal to the limited liability (i.e., Assumption 2.4 is violated), the principal will not be able to shut down the honest manager for any value of $V_h - V_l$.

[26] Exception: when $V_h - V_l \in (\mu_1, \mu_2)$

- The pooling contract at the lower value of $V_h - V_l$ will not exist if δ is small and/or the benefit from collusion λP is high. It will also fail to exist if there is no wage monotonicity constraint (Assumption 2.3 violated).
- The principal's profit may not remain independent of the bargaining weight if $\pi(e)$ and $C(e)$ (Assumption 2.1) are such that the manager's efforts are non-linear (i.e., strictly concave) in their compensation. However, the "persuasion-paradox" that a higher bargaining weight δ may not always benefit the principal still holds.

2.A Appendices

2.A.1 Solution of PP1 problem

PP1 problem is represented as:

$$\max_{w_h, w_l, e} \quad \pi(e)(A - \frac{1}{2}(w_h + w_l)) \tag{PP1}$$

subject to:

$$w_h \geq \nu\,(1-\delta)\,P + (1-\nu)(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \leq w_l + V_h - V_l \tag{IC2}$$

$$w_h \geq w_l \tag{MC}$$

$$e = \arg\max_{e'} \; \pi(e')\,[\frac{1}{2}(w_h + w_l)] - C(e') \tag{GIC}$$

$$\pi(e)\,[\frac{1}{2}(w_h + w_l)] - C(e) \geq 0 \tag{IR}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

As discussed earlier, IR will be satisfied if $w_h + w_l > 0$ and GIC can be replaced by LIC:

$$e = \frac{1}{2}(w_h + w_l) \tag{LIC}$$

If both IC1 and IC2 are not binding then we can find the optimal wage using unconstrained FOC, which yields $w_e = \frac{1}{2}(w_h + w_l) = \frac{A}{2}$ and the corresponding principal's profit is $\frac{A^2}{4}$. The principal can choose various combinations of (w_h, w_l) so that MC and LLs are satisfied, e.g. $(w_h, w_l) = (\frac{A}{2}, \frac{A}{2})$, $(w_h, w_l) = (A, 0)$.

Let's now consider the situation when IC1 binds and IC2 does not bind. IC1 is most relaxed when there is a maximal difference between w_h and w_l, which implies $w_h = A$ and $w_l = 0$ (LL2 binding) when IC1 just binds. IC1 will bind iff:

$$A < \nu(1-\delta)P + (1-\nu)(V_h - V_l) \text{ which implies}$$

$$\text{IC1 binds iff } V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} - (1-\delta)P) \equiv \gamma_1 \tag{2.21}$$

The wage structure and the profit when IC1 binds is given by $w_h = \nu(1-\delta)P + (1-\nu)(V_h - V_l)$, $w_l = 0$ and $w_h(A - w_h)$, respectively. Note that MC and LLs are satisfied with this wage structure.

Claim: IC2 does not bind for all $V_h - V_l \geq 0$

Proof: Suppose IC1 is binding then $w_h = \nu(1-\delta)P + (1-\nu)(V_h - V_l)$. IC2 will not bind if $V_h - V_l > (1-\delta)P$. If $A > (1-\delta)P$ than using the necessary condition of IC1 binding

(2.21), we get $V_h - V_l > (1-\delta)P$. This means IC2 is not binding. Now suppose IC1 is not binding then IC2 is most relaxed when $w_h - w_l = 0$ which implies that $w_h \leq w_l + V_h - V_l$ because $V_h - V_l \geq 0$. So IC2 is not binding whether IC1 binds or not. Hence Proved.

We summarize the above results which proves Lemma 2.1.

- If $V_h - V_l \in (0, \gamma_1)$ then IC1 is not binding and the principal's wage satisfies $\frac{1}{2}(w_h + w_l) = \frac{A}{2}$ and $0 \leq w_l \leq w_h$. The principal's profit is $\frac{A^2}{4}$. The could be multiple solutions for the above wage conditions.
- If $V_h - V_l = 0$ then $w_h = w_l = \frac{A}{2}$ and profit is $\frac{A^2}{4}$ as IC2 just binds.
- If $V_h - V_l = \gamma_1$ then $w_h = A, w_l = 0$ and profit is $\frac{A^2}{4}$ as IC1 just binds.
- If $V_h - V_l > \gamma_1$, IC1 binds. Wage: $w_h = \nu(1-\delta)P + (1-\nu)(V_h - V_l),\ w_l = 0$ and Profit: $w_h(A - w_h)$

2.A.2 Solution of PP2 problem

PP2 problem is represented as:

$$\max_{w_h, w_l}(1-\theta)\,\pi(e_h)[A - \frac{1}{2}(w_h + w_l)] + \theta\pi(e_c)\,[A - \lambda b + \delta N - \frac{1}{2}(1-\lambda)w_h - \frac{1}{2}(1+\lambda(1-\nu))w_l] \tag{PP2}$$

subject to:

$$w_h \leq \nu\,(1-\delta)\,P + (1-\nu)\,(w_l + V_h - V_l) \tag{IC1}$$

$$w_h \geq (1-\nu)\,(w_l + V_h - V_l) \tag{IC2}$$

$$w_h \geq w_l \tag{MC}$$

$$e_h = \arg\max_{e'}\ \frac{1}{2}\,\pi(e')\,(w_h + w_l) - C(e') \tag{GIC1}$$

$$e_c = \arg\max_{e'}\ \pi(e')\,[\lambda b + (1-\delta)\,N + \frac{1}{2}(1-\lambda)\,w_h + \frac{1}{2}\,(1+\lambda(1-\nu))\,w_l] - C(e') \tag{GIC2}$$

$$\frac{1}{2}\,\pi(e_h)\,(w_h + w_l) - C(e_h) \geq 0 \tag{IR1}$$

$$\pi(e_c)\,[\lambda b + (1-\delta)\,N + \frac{1}{2}(1-\lambda)\,w_h + \frac{1}{2}\,(1+\lambda(1-\nu))\,w_l] - C(e_c) \geq 0 \tag{IR2}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

IR1 will satisfy if $w_h + w_l \geq 0$. Therefore given GIC1 and LLs IR1 will satisfy. Similarly, Given LLs and GIC2 IR2 will satisfy because $\lambda b + (1-\delta)N + \frac{1}{2}(1-\lambda)w_h + \frac{1}{2}(1+\lambda(1-\nu))w_l > 0$. We can replace GIC1 and GIC2 with first order conditions LIC1 and LIC2, as the manager's payoff function is strictly concave.

$$e_h = \frac{1}{2}(w_h + w_l) \tag{LIC1}$$

$$e_c = \lambda b + (1-\delta)N + \frac{1}{2}(1-\lambda)w_h + \frac{1}{2}(1+\lambda(1-\nu))w_l \tag{LIC2}$$

Replacing e_h and e_c from LIC1 and LIC2 in the principal's objective function we get

$(1-\theta)\frac{1}{2}(w_h+w_l)(A-\frac{1}{2}(w_h+w_l)) + \theta(\lambda b + (1-\delta)N + \frac{1}{2}(1-\lambda)w_h + \frac{1}{2}(1+\lambda(1-\nu))w_l)(A - \lambda b + \delta N - \frac{1}{2}(1-\lambda)w_h - \frac{1}{2}(1+\lambda(1-\nu))w_l)$

If IC1, IC2, MC and LLs constraints are not binding then the unconstrained first order condition gives the following two equations:

$w_h + w_l = A$

$\frac{1}{2}(1-\lambda)w_h + \frac{1}{2}(1+\lambda(1-\nu))w_l = \frac{A}{2} - \lambda b + (\delta - \frac{1}{2})N$

Solving the above two equations we get the optimal wage structure if IC1, IC2 and MC are not binding:

$$(1-\frac{\nu}{2})w_h = (1-\nu)\frac{A}{2} + b - \frac{\delta - \frac{1}{2}}{\lambda}N \tag{2.10}$$

$$(1-\frac{\nu}{2})w_l = \frac{A}{2} - b + \frac{\delta - \frac{1}{2}}{\lambda}N \tag{2.11}$$

Principal profit when IC1, IC2 and MC are not binding:

$$(1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4} \tag{2.12}$$

Next we check under what conditions IC1, IC2, MC and LLs are binding. IC1 will satisfy the wage structure in (2.10)-(2.11) if:

$w_h \leq \nu(1-\delta)P + (1-\nu)(w_l + V_h - V_l)$

$\rightarrow (1-\frac{\nu}{2})w_h \leq (1-\frac{\nu}{2})\nu(1-\delta)P + (1-\frac{\nu}{2})(1-\nu)(w_l + V_h - V_l)$

$\rightarrow (1-\nu)\frac{A}{2} + b - \frac{\delta-\frac{1}{2}}{\lambda}N \leq (1-\frac{\nu}{2})\nu(1-\delta)P + (1-\nu)\frac{A}{2} - (1-\nu)(b - \frac{\delta-\frac{1}{2}}{\lambda}N) + (1-\frac{\nu}{2})(1-\nu)(V_h - V_l)$

$\rightarrow (1-\frac{\nu}{2})\nu(1-\delta)P + (2-\nu)\frac{\delta-\frac{1}{2}}{\lambda}N \geq 0$

$\rightarrow (1-\frac{\nu}{2})\nu(1-\delta)P + (1-\frac{\nu}{2})\nu(\delta-\frac{1}{2})P \geq 0$

$\rightarrow \frac{1}{2}(1-\frac{\nu}{2})\nu P \geq 0$

which implies that IC1 will not bind.

MC will satisfy the wage structure in (2.10)-(2.11) if

$\rightarrow (1-\nu)\frac{A}{2} + b - \frac{\delta-\frac{1}{2}}{\lambda}N \geq \frac{A}{2} - b + \frac{\delta-\frac{1}{2}}{\lambda}N$

$\rightarrow 2b \geq \nu\frac{A}{2} + 2\frac{\delta-\frac{1}{2}}{\lambda}N$

$\rightarrow V_h - V_l \geq \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})P)$

This implies MC will bind if $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})P)$

IC2 will be satisfied for the wage structure in (2.10)-(2.11) iff
$w_h \geq (1-\nu)(w_l + V_h - V_l)$
$\rightarrow (1-\frac{\nu}{2})w_h \geq (1-\frac{\nu}{2})(1-\nu)(w_l + V_h - V_l)$
$\rightarrow (1-\nu)\frac{A}{2} + b - \frac{\delta-\frac{1}{2}}{\lambda}N \geq (1-\nu)\frac{A}{2} - (1-\nu)(b - \frac{\delta-\frac{1}{2}}{\lambda}N) + (1-\frac{\nu}{2})(1-\nu)(V_h - V_l)$
$\rightarrow (1-\frac{\nu}{2})\nu(\delta-\frac{1}{2})P \leq 0$
$\rightarrow \delta \leq \frac{1}{2}$ This implies that IC2 will bind if $\delta > \frac{1}{2}$
LL2 is satisfied with the wage structure in (2.10)-(2.11) iff
$\frac{A}{2} - b + \frac{\delta-\frac{1}{2}}{\lambda}N \geq 0$
$\rightarrow b \leq \frac{A}{2} + \frac{\delta-\frac{1}{2}}{\lambda}N \geq 0$
replacing b and N from Definition 2.2 and 2.3 respectively
$V_h - V_l \leq \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta-\frac{1}{2})P$
Therefore LL2 will bind if $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})P)$. LL1 will always satisfy.

First let let us consider the cases with $\delta \leq \frac{1}{2}$ i.e. when IC2 is not binding.

Case 2a. $\delta \leq \frac{1}{2}$ and $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})P)$

As shown above MC is binding but IC1 and IC2 and LL2 not binding. We replace w_h with w_l in the principal's objective function and find the optimal w_h and w_l through first order condition. This optimal w_h, w_l and principal's profit is given by:

$w_h = w_l = min[\frac{1}{(1-\theta)+\theta(1-\frac{\nu\lambda}{2})^2}[(1-\frac{\nu\lambda\theta}{2})\frac{A}{2} - \theta(1-\frac{\nu\lambda}{2})(\lambda b - (\delta-\frac{1}{2})N)], \frac{1-\nu}{\nu}(V_h - V_l) + (1-\delta)P]$

Principal's profit $= (1-\theta)w_l(A - w_l) + \theta(\lambda b + (1-\delta)N + (1-\frac{\nu\lambda}{2})w_l)(A - \lambda b + \delta N - (1-\frac{\nu\lambda}{2})w_l)$

Note when $w_h = w_l$, IC1 constraint hardens and may bind if $V_h - V_l$ is sufficiently low. For this reason the $w_h = w_l$ is capped at $\frac{1-\nu}{\nu}(V_h - V_l) + (1-\delta)P$. Let's call this cutoff point as α_2 such that if $V_h - V_l < \alpha_2$, IC1 binds. If $V_h - V_l > \alpha_2$ then the above profit function simplifies to (2.12).

If $V_h - V_l < \alpha_2$ then $w_h = w_l = \frac{1-\nu}{\nu}(V_h - V_l) + (1-\delta)P$ and the profit will decline if $V_h - V_l$ decreases, and not remain constant as specified in (2.12)

Case 2b. : $\delta \leq \frac{1}{2}$ and $V_h - V_l \in [\frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})P), \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})P)]$

IC1, IC2, MC and LL2 are not binding and we get w_h, w_l and principal's profit using (2.10), (2.11) and (2.12)

$(1-\frac{\nu}{2})w_h = (1-\nu)\frac{A}{2} + b - \frac{\delta-\frac{1}{2}}{\lambda}N$

$(1-\frac{\nu}{2})w_l = \frac{A}{2} - b + \frac{\delta-\frac{1}{2}}{\lambda}N$

Principal profit $= (1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4}$

Case 2c. $\delta \le \frac{1}{2}$ and $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})P)$

IC1, IC2, and MC is not biding but LL2 binding. We solve the optimal w_h by putting $w_l = 0$ in the objective function. We get the optimal w_h and w_l and principal's profit as follows:

$w_l = 0$ and $w_h = \max[\frac{2}{(1-\theta)+\theta(1-\lambda)^2}[(1-\lambda\theta)\frac{A}{2} - \theta(1-\lambda)(\lambda b - (\delta-\frac{1}{2})N)], (1-\nu)(\bar{V} - \underline{V})]$

Principal's profit $= (1-\theta)\frac{w_h}{2}(A - \frac{w_h}{2}) + \theta(\lambda b + (1-\delta)N + \frac{1}{2}(1-\lambda)w_h)(A - \lambda b + \delta N - \frac{1}{2}(1-\lambda)w_h)$

Next we consider the situation when $\delta > \frac{1}{2}$ which means IC2 is binding. Let's consider if MC and LL2 is not binding Then solving for optimal wage after replacing $w_h = (1-\nu)(w_l + V_h - V_l)$ in the profit function and using w_l as independent variable we get.

$$(1-\frac{\nu}{2})w_h = (1-\nu)\frac{A}{2} + b + \theta(1-\nu)(\delta - \frac{1}{2})N \tag{2.13}$$

$$(1-\frac{\nu}{2})w_l = \frac{A}{2} - b + \theta(\delta - \frac{1}{2})N \tag{2.14}$$

Principal's profit when IC2 is binding but not MC and LL2:

$$(1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4} - \theta(1-\theta)(\delta - \frac{1}{2})^2 N^2 \tag{2.15}$$

Note that this is similar to (2.12). However, the cutoff for MC and LL2 binding changes. MC binds if $(1-\nu)\frac{A}{2} + b + \theta(1-\nu)(\delta-\frac{1}{2})N < \frac{A}{2} - b + \theta(\delta - \frac{1}{2})N$

which implies $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\lambda\nu)P)$

Similarly, LL2 binds if

$\frac{A}{2} - b + \theta(\delta - \frac{1}{2})N < 0$
which implies $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\theta\lambda P)$

Case 2d. $\delta > \frac{1}{2}$ and $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\lambda\nu)P)$

As shown above IC2 and MC is binding but LL2 is not binding. We replace w_h with w_l in the principal's objective function and find the optimal w_l through first order condition. This optimal w_h, w_l and principal's profit is given by:

$w_h = w_l = \min[\frac{1}{(1-\theta)+\theta(1-\frac{\nu\lambda}{2})^2}[(1 - \frac{\nu\lambda\theta}{2})\frac{A}{2} - \theta(1 - \frac{\nu\lambda}{2})(\lambda b - (\delta - \frac{1}{2})N)], \frac{1-\nu}{\nu}(V_h - V_l) + (1 - \delta)P]$

Principal's profit $= (1 - \theta)w_l(A - w_l) + \theta(\lambda b + (1 - \delta)N + (1 - \frac{\nu\lambda}{2})w_l)(A - \lambda b + \delta N - (1 - \frac{\nu\lambda}{2})w_l)$

We also need to check as $V_h - V_l$ decreases IC2 satisfies but IC1 binds. Therefore, the check for $w_h = w_l \leq \frac{1-\nu}{\nu}(V_h - V_l) + (1 - \delta)P$ is added.

Case 2e. $\delta > \frac{1}{2}$ and $V_h - V_l \in [\frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\lambda\nu)P), \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\theta\lambda P)]$

In this case IC2 is binding but IC1, MC and LL4 are not binding. The optimal wage and profit as already shown in (2.13), (2.14) and (2.15).

$(1 - \frac{\nu}{2})w_h = (1 - \nu)\frac{A}{2} + b + \theta(1 - \nu)(\delta - \frac{1}{2})N$

$(1 - \frac{\nu}{2})w_l = \frac{A}{2} - b + \theta(\delta - \frac{1}{2})N$

Principal's profit $= (1 - \theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4} - \theta(1 - \theta)(\delta - \frac{1}{2})^2N^2$

Due to constraint IC2, the principal's payoff is reduced from the maximum when IC2 is not binding, (2.12). However, this extra loss is small due to factor $\theta(1-\theta)(\delta = \frac{1}{2})^2$ which is a small fraction.

Case 2f. $\delta > \frac{1}{2}$ and $V_h - V_l \geq \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\theta\lambda P)$

In this case IC2 and LL2 is binding but IC1 and MC are not binding. IC2 and LL2 binding provides the value of w_h and w_l.

$w_l = 0$ and $w_h = (1 - \nu)(V_h - V_l)$

Principal's profit $= (1 - \theta)b(A - b) + \theta(b + (1 - \delta)N)(A - b + \delta N)$

We summarize our results for lemma 2.2

If $\delta \leq \frac{1}{2}$ then IC2 is not binding and the wage structure and profit is given as follows:

- There exist α_2 such that if $V_h - V_l < \alpha_2$ then IC1 and MC binds (see Case 2a for expression of wage and profit). Profit is lower than (2.12) and it decreases when $V_h - V_l$ decreases. (See case 2a)
- If $V_h - V_l \in (\alpha_2, \beta_2)$ then the profit is constant and given by (2.12). MC binds and both w_h and w_l decreases when $V_h - V_l$ increases. (See case 2a)
- If $V_h - V_l \in (\beta_2, \gamma_2)$ then the profit is constant and given by (2.12). No constraints bind. w_h increases and w_l decreases when $V_h - V_l$ decreases. (See Case 2b).
- If $V_h - V_l > \gamma_2$ then LL2 binds and profit declines when $V_h - V_l$ increases. w_h increases with $V_h - V_l$ and $w_l = 0$ (See Cace 2c).

where

$$\beta_2 = \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})P)$$

$$\alpha_2 \in (0, \beta_2)$$

$$\gamma_2 = \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})P)$$

If $\delta > \frac{1}{2}$ then IC2 is binding and the wage structure follows very similar to that with $\delta < \frac{1}{2}$, except that the profit reduces when $V_h - V_l \in (\alpha_2, \gamma_2)$ as shown in (2.15) and the cutoff point changes as shown below:

$$\beta_2 = \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\lambda\nu)P)$$

$$\alpha_2 \in (0, \beta_2)$$

$$\gamma_2 = \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})(\theta\lambda)P)$$

The expressions for wages and profits when $\delta > \frac{1}{2}$ are shown in Case 2d, 2e, 2f instead of 2a, 2b, 2c.

2.A.3 Solution of PP3 problem

PP3 problem is represented as:

$$\max_{w_h,w_l} (1-\theta)\,\pi(e_h)\,[A-\frac{1}{2}(w_h+w_l)] + \theta\,\pi(e_c)\,[A-b+\delta N-(1-\frac{\nu}{2})\,w_l] \tag{PP3}$$

subject to:

$$w_h \leq (1-\nu)(w_l+V_h-V_l) \tag{IC1}$$

$$w_h \geq w_l \tag{MC}$$

$$e_h = \arg\max_{e'} \ \frac{1}{2}\,\pi(e')\,(w_h+w_l) - C(e') \tag{GIC1}$$

$$e_c = \arg\max_{e'} \ \pi(e')\,[b+(1-\delta)\,N+(1-\frac{\nu}{2})\,w_l] - C(e') \tag{GIC2}$$

$$\frac{1}{2}\pi(e_h)(w_h+w_l) - C(e_h) \geq 0 \tag{IR1}$$

$$\pi(e_c)\,[\frac{1}{2}\,(1-\nu)\,(V_h-V_l) + \frac{1}{2}\,\lambda\,\nu\,(1-\delta)\,P + (1-\frac{\nu}{2})\,w_l] - C(e_c) \geq 0 \tag{IR2}$$

$$w_h \geq 0 \tag{LL1}$$

$$w_l \geq 0 \tag{LL2}$$

As discussed earlier Given GICs and LLs, IR1 and IR2 will satisfy when $w_h + w_l > 0$. We can replace GIC1 and GIC2 with first order conditions LIC1 and LIC2, as the manager's payoff function is strictly concave.

$$e_h = \frac{1}{2}(w_h+w_l) \tag{LIC1}$$

$$e_c = b+(1-\delta)N+(1-\frac{\nu}{2})w_l \tag{LIC1}$$

Replacing e_h and e_c from LIC1 and LIC2 in the principal's objective function we get

$$(1-\theta)\tfrac{1}{2}(w_h+w_l)(A-\tfrac{1}{2}(w_h+w_l)) + \theta(b+(1-\delta)N+(1-\tfrac{\nu}{2})w_l)(A-b+\delta N(1-\tfrac{\nu}{2})w_l)$$

If IC1, MC and LLs constraints are not binding then the unconstrained first order condition gives the following two equations:

$$w_h + w_l = A$$

$$(1-\tfrac{\nu}{2})w_l = \tfrac{A}{2} - b + (\delta - \tfrac{1}{2})N$$

Solving the above two equations we get the optimal wage structure if IC1, MC and LLs are not binding:

$$(1-\frac{\nu}{2})w_h = (1-\nu)\frac{A}{2} + b - (\delta - \frac{1}{2})N \tag{2.16}$$

$$(1-\frac{\nu}{2})w_l = \frac{A}{2} - b + (\delta - \frac{1}{2})N \tag{2.17}$$

Principal profit when IC1, IC2 and MC are not binding:

$$(1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4} \tag{2.12}$$

Next we check under what conditions IC1, MC and LLs are binding. IC1 will not bind as long as w_h and w_l from (2.16)-(2.17) satisfies:

$w_h \leq (1-\nu)(w_l + V_h - V_l)$

$\rightarrow (1-\frac{\nu}{2})w_h - (1-\nu)(1-\frac{\nu}{2})w_l \leq (1-\frac{\nu}{2})(1-\nu)(V_h - V_l)$

$\rightarrow (1-\frac{\nu}{2})w_h - (1-\nu)(1-\frac{\nu}{2})w_l \leq (1-\frac{\nu}{2})(1-\nu)(V_h - V_l)$

$\rightarrow (1-\nu)\frac{A}{2} + b - (\delta - \frac{1}{2})N - (1-\nu)(\frac{A}{2} - b + (\delta - \frac{1}{2})N) \leq (2-\nu)b$ (replacing value of w_l, w_h, b)

$\rightarrow (2-\nu)b - (2-\nu)(\delta - \frac{1}{2})N \leq (2-\nu)b$

$\rightarrow -(2-\nu)(\delta - \frac{1}{2})N \leq 0$

$\rightarrow \delta \geq \frac{1}{2}$. This implies that IC1 will bind if $\delta < \frac{1}{2}$

MC will satisfy the wage structure in (2.16)-(2.17) if

$(1-\nu)\frac{A}{2} + b - (\delta - \frac{1}{2})N \geq \frac{A}{2} - b + (\delta - \frac{1}{2})N$

$\rightarrow 2b \geq \nu\frac{A}{2} + 2(\delta - \frac{1}{2})N$

$\rightarrow (1-\nu)(V_h - V_l) \geq \nu\frac{A}{2} + 2(\delta - \frac{1}{2})\frac{1}{2}\lambda\nu P$

$\rightarrow V_h - V_l \geq \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})\lambda P)$

This implies MC will bind if $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})\lambda P)$

LL2 will satisfy the wage structure in in (2.16)-(2.17) if

$b \geq \frac{A}{2} + (\delta - \frac{1}{2})N$ or $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\lambda P)$. This implies LL2 will bind if $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\lambda P)$

Now consider that $\delta < \frac{1}{2}$ so that IC1 is binding. Replacing $w_h = (1-\nu)(w_l + V_h - V_l)$ in the principal's objective function and using FOC to identify optimal wage structure we get.

$$(1-\frac{\nu}{2})w_h = (1-\nu)\frac{A}{2} + b + (1-\nu)\theta(\delta - \frac{1}{2})N \tag{2.18}$$

$$(1-\frac{\nu}{2})w_l = \frac{A}{2} - b + \theta(\delta - \frac{1}{2})N \tag{2.19}$$

The principal's profit is given by (2.15).

With the wage structure provided by (2.18) and (2.19), MC will bind if $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\nu\lambda)P$ and LC will bind if $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})(\theta\lambda)P$

Case 3a. $\delta \geq \frac{1}{2}$ and $V_h - V_l \in [\frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})\lambda P), \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\lambda P]$

As shown above IC1, MC and LL2 are not we get wage structure and principal profit as shown in (2.16), (2.17), and (2.12).

$(1-\frac{\nu}{2})w_h = (1-\nu)\frac{A}{2} + b - (\delta - \frac{1}{2})N$

$(1-\frac{\nu}{2})w_l = \frac{A}{2} - b + (\delta - \frac{1}{2})N$

Principal's profit $= (1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4}$

Case 3b. $\delta \geq \frac{1}{2}$ and $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\lambda P)$

IC1 not binding but LL2 is binding and we get w_h, w_l and principal profit as follows:

$w_h = A$, $w_l = 0$

Principal's profit $= (1-\theta)\frac{A^2}{4} + \theta(b + (1-\delta)N)(A - b + \delta N)$

Case 3c. $\delta \geq \frac{1}{2}$ and $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})\lambda P)$

MC is binding i.e. $w_h = w_l$ then the wage structure and profit is given by

$w_h = w_l = min[\frac{1}{(1-\theta)+\theta(1-\frac{\nu}{2})^2}(1 - \frac{\nu\theta}{2})\frac{A}{2} - \theta(1-\frac{\nu}{2})(b - (\delta - \frac{1}{2})N)), \frac{1-\nu}{\nu}(V_h - V_l)]$

Principal's profit $= (1-\theta)w_l(A - w_l) + \theta(b + (1-\delta)N + (1-\frac{\nu}{2})w_l)(A - b + \delta N - (1-\frac{\nu}{2})w_l)$

Note that we need to ensure $w_l \leq \frac{1-\nu}{\nu}(V_h - V_l)$ so that IC1 is not binding with new wage structure. Also note that unconstrained w_l is given by $[(1-\theta)+\theta(1-\frac{\nu}{2})^2]w_l = (1 - \frac{\nu\theta}{2})\frac{A}{2} - \theta(1-\frac{\nu}{2})(b - (\delta - \frac{1}{2})N)$

Case 3d. $\delta < \frac{1}{2}$ and $V_h - V_l \in [\frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\nu\lambda)P), \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\theta\lambda P)]$

As shown above IC1 is binding, but MC and LL2 not binding. The solution of the wage structure is given by (2.18) and (2.19), and the profit by (2.15).

Case 3e. $\delta < \frac{1}{2}$ and $V_h - V_l > \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\theta\lambda P)$

The solution is same as in Case 3b because $w_h = A$ and $w_l = 0$ also satisfies IC1.

Case 3f. $\delta < \frac{1}{2}$ and $V_h - V_l < \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\nu\lambda)P)$

Both IC1 and MC binding. Replacing $w_h = w_l$ in IC1 we get,

$w_h = w_l = \frac{1-\nu}{\nu}(V_h - V_l)$

Principal's profit $= (1-\theta)w_l(A - w_l) + \theta(b + (1-\delta)N + (1-\frac{\nu}{2})w_l)(A - b + \delta N - (1-\frac{\nu}{2})w_l)$

We summarize our results to be used in Lemma 2.4 and 2.5.

If $\delta \geq \frac{1}{2}$ then IC1 is not binding and the wage structure and profit is given as follows:

- There exist α_3 such that if $V_h - V_l < \alpha_3$ then IC1 and MC binds (see Case 3c for expression of wage and profit). Profit is lower than (2.12) and it decreases when $V_h - V_l$ decreases.
- If $V_h - V_l \in (\alpha_3, \beta_3)$ then the profit is constant and given by (2.12). MC binds and both w_h and w_l decreases when $V_h - V_l$ increases. (See case 3c)
- If $V_h - V_l \in (\beta_3, \gamma_3)$ then the profit is constant and given by (2.12). No constrains bind. w_h increases and w_l decreases when $V_h - V_l$ increases. (See Case 3a).
- If $V_h - V_l > \gamma_3$ then LL2 binds and profit declines when $V_h - V_l$ increases. $w_h = A$ and $w_l = 0$ (See Cace 3b).

where

$$\beta_3 = \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})\lambda P)$$

$$\alpha_3 \in (0, \beta_3)$$

$$\gamma_3 = \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})\lambda P)$$

If $\delta < \frac{1}{2}$ then IC1 is binding and the wage structure follows very similar to that with $\delta \geq \frac{1}{2}$, except that cutoff point changes.

$$\beta_3 = \frac{\nu}{1-\nu}(\frac{A}{2} + (\delta - \frac{1}{2})(\frac{1}{2}\theta\lambda\nu)P)$$

$$\alpha_3 = \beta_3$$

$$\gamma_3 = \frac{\nu}{1-\nu}(\frac{A}{\nu} + (\delta - \frac{1}{2})(\theta\lambda)P)$$

2.A.4 Proof of Proposition 2.3 and 2.4

Proof of Proposition 2.3

Step 1: Claim $\alpha_2 < \alpha_3$

If $\delta < \frac{1}{2}$, we have $\alpha_2 < \beta_2 = \frac{\nu}{1-\nu}\left(\frac{A}{2} + \left(\delta - \frac{1}{2}\right) P\right) < \frac{\nu}{1-\nu}\left(\frac{A}{2} + \left(\delta - \frac{1}{2}\right)\left(\frac{1}{2}\theta\lambda\nu\right) P\right) = \beta_3 = \alpha_3$ (Note $\delta - \frac{1}{2} < 0$ and see Table 2.2 and Table 2.3).If $\delta \geq \frac{1}{2}$, we have $\alpha_2 < \beta_2 = \frac{\nu}{1-\nu}\left(\frac{A}{2} + \left(\delta - \frac{1}{2}\right)\left(\frac{1}{2}\theta\lambda\nu\right) P\right) < \frac{\nu}{1-\nu}\left(\frac{A}{2} + \left(\delta - \frac{1}{2}\right)\lambda P\right) = \beta_3$. However, since α_3 could be lower than β_3, we need to consider an additional factor. α_2 is determined when the IC1 constraint binds in Type 2, which is $w_h = w_l = \frac{1-\nu}{\nu}(V_h - V_l) + (1-\delta)P$, whereas α_3 is determined when the IC1 condition binds in Type 3, which is $w_h = w_l = \frac{1-\nu}{\nu}(V_h - V_l)$. Since $(1-\delta)P > 0$, $V_h - V_l$ needs to be lower to satisfy the constraint in Type 2, which implies $\alpha_2 < \alpha_3$.

Step 2: There exists $\mu_2 > 0$ such that if $V_h - V_l \in [0, \mu_2)$, the Type 2 contract dominates the Type 3 contract for all δ.

Since α_2 and α_3 are the points at which the respective profit functions reach their maximum, and from Step 1, $\alpha_2 < \alpha_3$, the profit function of Type 2 reaches its maximum earlier than the profit function of Type 3. If $\delta > \frac{1}{2}$, the maximum profit of Type 2 is lower than that of Type 3 (see Lemma 2.2 and Lemma 2.4). Therefore, the Type 3 profit function will intersect the Type 2 profit function from below. Let's call this point μ_2, which will be greater than α_2. Additionally, the slope of the Type 2 profit function with respect to $V_h - V_l$ is smaller than that of Type 3 if $V_h - V_l < \mu_2$ (0 between α_2 and μ_2, and smaller below α_2 can be easily shown). Therefore, the Type 2 profit function is above that of Type 3 for all $V_h - V_l < \mu_2$.

If $\delta \leq \frac{1}{2}$, the maximum profit of the Type 2 function is greater than that of Type 3. Type 2 profit function reaches its maximum at α_2, and the slope of Type 2 profit function is smaller than that of Type 3 if $V_h - V_l < \alpha_2$. These points imply that if $\mu_2 = \alpha_2$, then the Type 2 contract dominates Type 3 for all $V_h - V_l < \mu_2$

Step 3: There exists $0 \leq \mu_1 < \mu_2$ such that if $V_h - V_l \in (0, \mu_1)$, the Type 1 contract dominates Type 2, and if $V_h - V_l \in (\mu_1, \mu_2)$, Type 2 dominates Type 1. Also, $\mu_1 = 0$ if δ is critically low.

Profit from the Type 1 contract is equal to $\frac{A^2}{4}$ and remains constant for all $V_h - V_l < \alpha_2 < \gamma_1$ (see Lemma 2.1). Note: $\alpha_2 < \beta_2 < \gamma_1$ if $A \geq (1-\delta)P$, which we assume. At $V_h - V_l = \alpha_2$, the Type 2 contract achieves a maximum profit greater than $\frac{A^2}{4}$. For $V_h - V_l < \alpha_2$, Type 2 profit increases continuously. When $V_h - V_l = 0$, the Type 2 profit can be simplified to $(1-\delta)P(A-(1-\delta)P) + \theta(1-\delta)NP$. (Note: set $V_h - V_l = 0$ in $w_h = w_l = \frac{1-\nu}{\nu}(V_h - V_l) + (1-\delta)P$ and calculate profit). If $\delta = 1$, this profit is zero, which implies from the Intermediate Value Theorem (IVT) that such $\mu_1 > 0$ exists. If $\delta = 0$ and $A = \frac{1}{2}P$, then this profit is greater than $\frac{A^2}{4}$, in which case Type 2 dominates for all $V_h - V_l$, and we call this case $\mu_1 = 0$. We can calculate a critical δ_c for a given P such that if $\delta > \delta_c$, then $\mu_1 > 0$; otherwise, $\mu_1 = 0$.

From Step 1, 2 and 3 Proposition 2.3 follows. □

Proof of Proposition 2.4

Step 1: There exists $\mu_3 > \mu_2$ such that if $V_h - V_l \in (\mu_2, \mu_3)$, then the Type 2 contract *dominates if $\delta < \frac{1}{2}$, and the Type 3 contract dominates if $\delta \geq \frac{1}{2}$.*

This follows from two conditions, which we will prove: a) The Type 4 profit function intersects the horizontal profit line $(1-\theta)\frac{A^2}{4}+\theta\frac{(A+N)^2}{4}$ (the maximum profit of the Type 2 or Type 3 contract) from below as it increases towards its peak. We call this intersection point μ_3. b) μ_3 lies in the interior of the interval (μ_2, γ_2) if $\delta < \frac{1}{2}$ and in the interior of the interval (μ_2, γ_3) if $\delta \geq \frac{1}{2}$. This is because the Type 2 or Type 3 profit function is flat at its maximum value in these intervals, and Type 2 has a higher maximum if $\delta < \frac{1}{2}$, while Type 3 has a higher maximum if $\delta > \frac{1}{2}$ (see Lemmas 2.2 and 2.4). Now we will show that these two conditions are satisfied.

The Type 4 profit function is given by $(b+(1-\delta)N)(A-b+\delta N)$. The maximum value of this profit function is $\frac{(A+N)^2}{4}$, which is greater than the maximum value for the Type 2 and Type 3 contracts (see (2.12)). The Type 4 profit function declines on both sides of its peak, which is at $V_h - V_l = \frac{\nu}{1-\nu}(\frac{A}{\nu}+(\delta-\frac{1}{2})\lambda P) \equiv \gamma_4$. Let's determine if and where this curve crosses the line $\frac{A^2}{4}$, the lowest maximum profit under Type 2 or Type 3 when $\theta = 0$. We solve the quadratic equation $(b+(1-\delta)N)(A-b+\delta N) - \frac{A^2}{4} = 0$. Using the solution of this quadratic equation, the left intersection point is given by $V_h - V_l = \gamma_4 - \frac{\sqrt{N(2A+N)}}{1-\nu}$. This intersection point will be closer to γ_4 if the profit line is $(1-\theta)\frac{A^2}{4} + \theta\frac{(A+N)^2}{4}$ and not $\frac{A^2}{4}$. Since on the left side of γ_4, the Type 4 profit function is continuously increasing, this proves condition (a).

If we take $A > P$ and $\nu = \frac{1}{2}$, then $N \leq \frac{A}{4}$. From the above expression, the point of intersection μ_3 is greater than $\gamma_4 - \frac{3}{2}A$. We can easily verify from Table 2.2 and Table 2.3 that this point is in the interior of (μ_2, γ_2) and (μ_2, γ_3), because the distance between γ_4 and μ_2 is greater than $\frac{3}{2}A$ if $\nu = \frac{1}{2}$, and the distance between γ_4 and γ_3 is 0 if $\delta \geq \frac{1}{2}$, and the distance between γ_4 and γ_2 is less than $\frac{A}{2}$ if $\delta < \frac{1}{2}$. This proves condition (b) as well, however under certain assumption that $A \geq P$.

Step 2: If $V_h - V_l < \mu_3$, then the Type 4 contract is dominated by one of the other types of contracts.

As shown in Step 4, at $V_h - V_l = \mu_3$, the Type 4 profit function intersects the Type 2 or Type 3 profit function from below. We also demonstrated in Step 4 that the Type 4 profit function intersects the Type 1 profit function ($\frac{A^2}{4}$) from below. Let's call this point μ_3', which is less than μ_3. We also showed that $\mu_3' > \mu_2$. Therefore, if $V_h - V_l \in (\mu_3', \mu_3)$, Type 2 and Type 3 dominate as they are at their maximum, which is greater than $\frac{A^2}{4}$. If $V_h - V_l \in [0, \mu_3']$, the Type 1 contract will dominate Type 4 because the Type 1 profit is constant while the Type 4 profit is declining.

Step 3: There exists $\mu_4 > \mu_3$ such that if $V_h - V_l \in (\mu_3, \mu_4)$, the Type 4 contract domi-

nates, and if $V_h - V_l > \mu_4$, the Type 3 contract dominates.

The above statement follows if we show: a) If $V_h - V_l > \gamma_4$, then the Type 3 contract is above Type 2. b) The Type 4 profit function intersects the Type 3 profit function from above as it declines when $V_h - V_l > \gamma_4$. We call this point μ_4.

First, consider the case where $\delta \geq \frac{1}{2}$. The Type 3 profit function is above Type 2 at $V_h - V_l = \gamma_3$ because Type 3 has a higher maximum value (see Lemma 2.4), and the Type 3 profit function is at its maximum when $V_h - V_l = \gamma_3$. When $V_h - V_l > \gamma_3$, both Type 2 and Type 3 profits are continuously declining concave functions, but Type 2 declines faster and has higher concavity. (Note: The second derivative of the Type 2 profit with respect to $V_h - V_l$ is $-2b$, while that of Type 3 is $-2\theta b$.) This implies that Type 3 profit is above Type 2 for all $V_h - V_l > \gamma_3 = \gamma_4$. If $\delta < \frac{1}{2}$, then Type 2 is above Type 3 at $V_h - V_l = \gamma_2 < \gamma_3$, but it declines faster than Type 3 if $V_h - V_l > \gamma_2$, crossing Type 3 before $V_h - V_l = \gamma_3$. Therefore, if $V_h - V_l > \gamma_3$, the Type 3 contract dominates Type 2, proving point (a). At $\gamma_4 = \gamma_3$, the profit function of the Type 4 contract is above Type 3, and it declines faster with higher concavity than the Type 3 profit function (second derivative $-2b$ vs. $-2b\theta$). This implies that the Type 4 profit function will intersect the Type 3 profit function from above at μ_4. If $V_h - V_l > \mu_4$, Type 3 dominates. Since we have already shown in Step 4 that when $V_h - V_l = \mu_3$, the Type 4 profit function intersects the maximum profit line of Type 2 or Type 3 from below and increases until γ_4, it implies that if $V_h - V_l \in (\mu_3, \mu_4)$, the Type 4 contract dominates.

Type 1 dominates when $V_h - V_l \in (0, \mu_1)$ because it dominates Type 2 (Step 3), Type 4 (Step 5), and Type 3 (Step 2). Type 2 dominates when $V_h - V_l \in (\mu_1, \mu_2)$ because Type 2 dominates Type 1 (Step 3), Type 3 (Step 2), and Type 4 (Step 5). Step 4 shows that if $V_h - V_l \in (\mu_2, \mu_3)$, Type 2 dominates if $\delta < \frac{1}{2}$, and Type 3 dominates if $\delta \geq \frac{1}{2}$. From Step 6, we see that Type 4 dominates if $V_h - V_l \in (\mu_3, \mu_4)$, and Type 3 dominates if $V_h - V_l > \mu_4$. This proves Proposition 2.4.

Chapter 3

Newspaper Market: Impact of advertisement on quality and market structure

3.1 Introduction

Newspaper markets in the USA and OECD countries are highly concentrated. While at the national level there may be multiple players with relatively equal shares, local and regional markets are often monopolies (Rosse, 1980; Dertouzos and Trautman, 1990). For instance, even though the USA has over 1,000 daily newspapers, more than 95% of cities have only one daily paper. In larger cities with two or more papers, these papers often differ in format (tabloid versus broadsheet) or political alignment (left- or right-leaning editorials). This pattern is also observed in developing countries. For example, metropolitan cities in India typically have one dominant English daily newspaper with more than a 60% market share.[1]

Such market power for a leading firm is unique to print media and is not seen in other types of media. The extensive literature on print media has been driven by the need to explain this concentration, particularly the prevalence of "one-newspaper cities." Most studies attribute this to the network externality effect, which occurs when consumers derive positive utility from advertising.[2] The positive feedback loop between circulation

[1]Hindustan Times in New Delhi, Times of India in Mumbai and Bangalore, The Hindu in Chennai, and Deccan Chronicle in Hyderabad.

[2]see (Furhoff, 1973; Bucklin et al., 1989; Gabszewicz et al., 2007; Häckner and Nyberg, 2008; Chaudhri, 1998; Merrilees, 1983; Blair and Romano, 1993).

and advertising can lead to a monopoly market, unless consumer has a strong preference for variety. In this regard, print media differs from other media such as television and radio, where consumers often view ads as a nuisance. In print media, consumers can easily ignore advertisements, and in some cases, such as classifieds, they may even welcome more ads. Several studies (Rosse, 1970; Dertouzos and Trautman, 1990; Thompson, 1989) provide empirical support for the view that consumers appreciate advertisements.

However, the theory relying purely on the network effect of advertisements fails to explain why such concentration continues to exist despite the significant decline in classified ads with the arrival of online platforms like Craigslist and Monster.com. Moreover, recent studies found that readers' attitudes toward advertisements differ across countries and regions (Sonnac, 2000; Van Cayseele and Vanormelingen, 2009; Filistrucchi et al., 2012). Readers in many European countries do not like commercial advertisements in newspapers (Gabszewicz et al., 2002).

Recent empirical evidence suggests that endogenous investment in quality could be a determinant of concentration. Berry and Waldfogel (2010) using cross-sectional data of metropolitan dailies in the USA, found evidence that firms invest in quality as the market grows and that these costs are fixed in nature. They measured quality by the number of pages (more content), the number of journalist staff (more news produced rather than relying on wire reports), and the quality of staff by counting the number of Pulitzer awards. They found that when the market size increases, the number of newspapers changes relatively little, but the nature and quality of newspapers change dramatically. This corroborates the argument by Shaked and Sutton (1987) that when the burden of quality improvement falls on fixed costs, product proliferation will not occur when market size increases. They argue that as markets grow larger in industries where quality is produced mainly through outlays on fixed costs, at least one firm will have an incentive to invest in quality. A firm producing a higher-quality product can undercut its rivals' prices and attain substantial market share.[3]

Angelucci and Cage (2019) provided further evidence that quality plays a major role in the newspaper market. Using data on French dailies and an exogenous shock to newspaper advertising, they showed that as advertising revenue declines, newspapers produce less journalistic-intensive content (or quality), measured by the size of newsroom staffs.

However, the literature on the newspaper industry has very few papers that model the newspaper market structure based on the interaction of product quality choice and advertisements. Gabszewicz et al. (2012) is one such study that shows the interaction between newspaper quality and advertisement, but their primary focus is to explain the

[3]Berry and Waldfogel (2010) also compared their findings with the restaurant industry, where the burden of quality falls on marginal costs. They found that product variety increases with market size because the high-quality firm cannot easily undercut the low-quality firm with lower marginal costs.

rise of free daily newspapers. Gabszewicz and Wauthy (2014) is another paper that extend the vertical differentiation model for a two-sided platform, but their model relies on the exogenous presence of network externalities across 2-sides rather than on endogenous investment in quality. They show that in the presence of cross-network externalities, if consumers are willing to pay more for a platform with a larger network size, an asymmetric equilibrium can be sustained.

This paper adopts an approach similar to Gabszewicz et al. (2012) and extends the standard vertical differentiation models (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1983) to include interaction with the advertisement side of the market. We demonstrate how different types of market structures and quality choices of players evolve as the advertisement level increases. Like other studies on vertical differentiation (see Gabszewicz and Thisse (1979); Wauthy (1996)), our results show that high-quality firms have an advantage due to their investment in quality, allowing them to attain a significant market share. However, they will not serve the lower end of the market unless consumer preference for quality is homogeneous. Since the high-quality firm does not cater to the lower-end market, it creates an opportunity for the low-quality firm to fill the product gap and serve the niche lower-end market, provided the advertisement level is not too low. Therefore, a natural monopoly occurs when consumer preference is homogeneous and/or the advertisement level is low. When the advertisement level is moderately high, the low-quality firm will serve the lower end of the market as a free product with the lowest quality, while the high-quality firm behaves as a monopolist without competition, similar to what is suggested by Gabszewicz et al. (2012). These results are also consistent with Johnson and Myatt (2006), who highlight that the dispersion in consumers' valuations determines the monopolist's product strategy (mass or niche). When consumers' valuations are homogeneous, the high-quality firm serves the mass market, leaving no room for a new entrant. In contrast, when valuations are heterogeneous, the high-quality firm targets higher-valuation consumers, creating an opportunity for a low-quality firm to serve the low-end niche.

A novel and interesting finding in this paper is that as the advertisement level increases further, the low-quality player can challenge the high-quality firm's market leadership. This forces the high-quality firm to significantly raise its quality—much more than the monopolist level—to protect its customer base. In extreme cases, the high-quality firm might even drive out the competitor and deter further entry. Consequently, the high-quality firm offers a premium product with a lower price-to-quality ratio,[4] and both the quality and the price-to-quality ratio improve with higher advertisements, benefiting consumers. This finding aligns with the empirical evidence provided by Angelucci and

[4] We use the price-to-quality ratio to effectively represent price because standalone price could be driven by changes in quality. A lower price-to-quality ratio more directly conveys higher consumer surplus.

Cage (2019) and Pattabhiramaiah (2014), which shows that as advertisement revenue declines, the quality of the leading newspaper decreases and the price may rise. To our knowledge, this aspect of the impact of advertisements has not been considered in any other papers.

Furthermore, we model the quality choices of duopoly players when there is a entry threat of a third player. The entry threat increases market competition, leading both existing players to raise their quality further and reduce their price-to-quality ratios. In fact, when advertisement levels are high, the profits of the top two players decline with increasing advertisement, which is the opposite of what happens in the duopoly model. This has a testable implication for entry-barriers in the newspaper market. This is similar to Donnenfeld and Weber (1995)'s finding that under vertical differentiation, product competition among duopoly incumbents leads to entry deterrence.

Though our model shares some similarities with Gabszewicz et al. (2012), it is distinct in several key aspects: a) *Quality-dependent fixed costs:* This allows us to model the quality choice of firms and makes our results robust;[5] b) *Advertisers prefer affluent consumers:* to incorporate empirical findings; c) *Sequential entry for firms:* This introduces a new set of equilibria and reflects market dynamics where the leading firm has a significant advantage that can be used for preempting quality space or entry deterrence;[6] d) *Impact of third-player entry:* This has significant implications for the duopoly results. Additionally, we test the robustness of our results under simultaneous entry, different levels of consumer preference heterogeneity, and when consumers receive positive or negative utility from advertisements. Our model also shares some similarities with Lutz (1997), which is one of the few papers that model sequential entry with quality-dependent fixed costs under vertical differentiation. Our model adds advertisement side interaction to that framework and studies broader aspects of market structure, whereas Lutz (1997) focuses solely on entry deterrence. We also complement the Johnson and Myatt (2006), who examines the impact of consumers' valuation preference on product characteristics. We extend it to include the additional impact of advertisement revenue on product characteristics of the 2-sided market. Our results show that high level of advertisement may break the monopolist's niche strategy even when consumers are heterogeneous.

[5]Most models assume the cost of providing quality is L-shaped. Under such assumptions, the high-quality firm always chooses the upper bound quality, which does not capture variations in quality choices with advertisements.

[6]Simultaneous entry models do not have an equilibrium when advertisement levels are high, which is also missing in Gabszewicz et al. (2012).

3.2 Related Literature

An important aspect of the newspaper market is that it is two-sided, catering to two types of customers: readers and advertisers. Advertisers value circulation, so advertisement demand is linked to readers demand. At the same time, readers may like or dislike advertising, leading to interdependencies between the two sides. Initial literature used a "structural" model, deriving interdependent inverse demand equations for circulation and advertising, which are then estimated using empirical data. Rosse (1967) was one of the earlier papers to use the structural model. They estimated that there are economies of scale in production costs. However, they also indicated that returns to scale have remained fairly constant since 1939, which may not explain the rising concentration in the newspaper market. Rosse (1970) estimated positive cross-effects from advertising to consumers. Similarly, Dertouzos and Trautman (1990) used structural equations to show that there are economies of scale in production as well as positive cross-effects from advertisements to consumers. They also showed that product quality positively affects circulation demand and that circulation demands are higher in high-income markets. However, they concluded that chain newspapers do not have any advantage over independent newspapers, suggesting that this scale effect is likely localized to content and distribution. Thompson (1989) followed a similar structural equation model and found that readers appreciate advertising. They also identified that advertisers value affluent consumers, creating a tradeoff between newspaper circulation and the share of high-income readers.

Since these papers identified positive effects of advertisements on circulation, much of the theoretical literature that followed explained the "one-newspaper cities" phenomenon using network effects. Furhoff (1973) was among the first to propose the theory of the circulation spiral, which is based on the positive feedback loop between advertising and circulation. In the limit, these spirals can lead to a monopoly situation. Bucklin et al. (1989) argued that such network effects make the market prone to predatory behavior by firms with a cost advantage, driving other firms out of the market. Merrilees (1983) used a descriptive study of a price war between Sydney-based newspapers to explain a similar effect. Gabszewicz et al. (2007) analyzed the positive effect of advertising on consumers in a duopoly where consumers also have preferences for the political stance of newspapers. He showed that in such scenarios, a weaker newspaper with differentiation may still survive if the advertising intensity is not high. Similarly, Häckner and Nyberg (2008) suggested that either a monopoly equilibrium or an asymmetric market share equilibrium exists if horizontal differentiation is low or consumer preference for quality content is high, in which case advertisements play a smaller role. These two papers demonstrate that a positive valuation for advertising alone is insufficient for monopoly if consumers prefer differentiation and/or the advertisement effect is small. Therefore, smaller cities with homogeneous political preferences would have a monopoly, while larger cities with heterogeneous preferences would have a duopoly.

Chaudhri (1998) analyzed pricing when consumers have a positive valuation for advertising. He examined the two cases of monopoly and perfect competition and showed that the monopoly market has a much lower circulation price, resulting in higher circulation and social welfare. Blair and Romano (1993) focused on the monopoly case and reached a similar conclusion. However, both these papers assumed the market structure as exogenous.

Some recent studies suggest that consumers' attitudes towards advertisements in newspapers can differ across countries and regions (Sonnac, 2000). Gabszewicz et al. (2002) highlighted that newspaper readers in many European countries are ad-avoiders. Filistrucchi et al. (2012) found that Dutch readers appreciate advertising, while Van Cayseele and Vanormelingen (2009) showed that Belgian readers are ad-neutral. Incorporating these new findings, Anderson and Gabszewicz (2006) modeled the newspaper market assuming a mix of consumers, some ad-haters and some ad-lovers. They concluded that under stronger ad-attraction, concentration should be expected, but with weaker ad-attraction, two newspapers with different horizontal characteristics can survive.

As newspapers' ad revenues declined rapidly after the advent of online platforms like Craigslist, market concentration continued to persist, casting doubt on whether the positive effect of advertisements was the primary driver of concentration. Recent empirical studies have identified that product quality plays a major role in determining newspaper market structure. Berry and Waldfogel (2010), using cross-sectional data of metropolitan dailies in the USA, found empirical evidence that firms invest in quality as the market grows and that these costs are fixed in nature. They found that when the market size increases, the number of newspapers changes relatively little (aside from horizontally differentiated suburban dailies), but the nature and quality of newspapers change dramatically across different market sizes. This highlights the vertical differentiation nature of the newspaper market. Under horizontal differentiation, the number of newspapers will increase as the market size increases. This also corroborates the argument of Shaked and Sutton (1987) that when the burden of quality improvement falls on fixed costs, product proliferation will not occur as the market size increases.

Angelucci and Cage (2019) provided further evidence that quality plays a major role in the newspaper market. Using data on French dailies and an exogenous shock to newspaper advertising, they showed that as advertising revenue declines, newspapers produce less journalistic-intensive content (or quality), measured by the size of newsroom staffs. Pattabhiramaiah (2014) showed similar evidence in the US newspaper market, demonstrating that as ad revenues decline, newspapers increase prices and reduce quality.

However, very few theoretical papers model the newspaper market using vertical differentiation. Gabszewicz et al. (2012) is one such paper, but they focused on explaining the entry of free newspapers. Gabszewicz and Wauthy (2014) extended the vertical differentiation model to a two-sided platform, but without investment in quality. They assumed the presence of cross-network externalities across two-side and that consumers are willing to pay more for a platform with a larger network size, which leads to an asymmetric equilibrium.

Most recent theoretical literature in the media market has used the platform market framework developed by Armstrong (2006) and Rochet and Tirole (2003). This framework has been extended for print media where newspapers are horizontally differentiated based on political leaning (Gabszewicz et al., 2001, 2007; Häckner and Nyberg, 2008; Anderson and Gabszewicz, 2006). For example, Anderson and Gabszewicz (2006) used the two-sided market with a Hotelling setup and assumed that viewers dislike advertisements, showing that advertisements result in newspapers locating their political opinions in the center. Johnson and Myatt (2006) models the product characteristics of firms under vertical differentiation, but without the impact of advertisement revenue.

This paper models the newspaper market under vertical differentiation by extending the standard vertical differentiation model to include the advertisement side. Though our model shares some similarities with Gabszewicz et al. (2012), it is distinct in several key aspects, enabling us to draw a new set of insights. We also complement Johnson and Myatt (2006) by analyzing the product characteristics when the firms have advertisement revenue, in additional to dispersion of consumers valuation.

3.3 The model

A newspaper market consists of three types of agents: firms (newspapers), consumers (newspaper readers), and advertisers. The market has one or more firms, with each firm offering one newspaper. Firms are vertically differentiated by the choice of quality of their newspapers. The quality of a newspaper represents its effort in producing information content desired by consumers. Higher effort results in more researched and relevant content, which is perceived to be of higher quality by the readers. All firms in the market have the access to same production technology, with a constant unit printing and circulation cost, c, that is quality-independent and a fixed production cost that is a convex function of quality, $K(\theta) = \alpha\theta^2$. Each firm that enters the market chooses a subscription price (s) per reader, quality (θ), and advertisement price (p) per reader.

There are M consumers in the market. Consumers differ in their disposable income and preferences for reading news. Consumers' disposable incomes follow log-normal distribution with parameters (μ, σ^2). The consumer with higher income has a higher willingness to pay for quality. At the same time, each consumer may have a different willingness to pay due to her outside options or preference for news reading. Consumer i with income Y_i receives utility U_i from reading a newspaper with quality θ and subscription price s. U_i is represented by the utility function:

$$U_i = v_i \, Y_i \, \theta - s$$

where the product $v_i \, Y_i$ represents consumers' willingness to pay for quality. This multiplicative form allows us to consider both factors, income Y_i and consumer preference for quality content v_i, in determining the willingness to pay for quality. For example, some high-income consumers do not prefer subscribing to newspaper (low v_i) as they get news from alternative sources or do not like reading news. Similarly, some low income consumers have a higher willingness to pay due to their strong preference for quality news (high v_i).[7] v_i follows a uniform distribution $\sim U(0, 1)$. For robustness, we also parameterize the level of heterogeneity using the distributional form $\sim U(b-1, b)$ $1 \leq b \leq 2$ (see section 3.8.2). The higher the b, the more homogeneous the preference. Consumers do not get utility from advertisements, which means that the advertisements do not cause nuisance as consumers can ignore advertisements, and consumers do not subscribe to a newspaper to see advertisements. The extension in section 3.8.3 and 3.8.4 discusses the result when consumers receive negative or positive utility from advertisements, respectively. A consumer subscribes to at most one newspaper, which means consumers single-home.[8] Therefore, the consumer choice problem can be represented as:

$$\max_k \; v_i \, Y_i \, \theta_k - s_k \;\; \text{subject to } U_i \geq 0$$

Advertisers are homogeneous within the market and are willing to pay βY_i to target a consumer with income Y_i. We assume β is exogenous.[9] Since advertisers are willing to pay more for high-income consumers, a newspaper that attracts disproportionately high-income consumers will have higher advertisement revenue per customer, consistent with empirical evidence (Thompson, 1989; Dertouzos and Trautman, 1990).[10] We use a representative advertiser to model a set of homogeneous advertisers. Advertisers place ads in multiple newspapers to reach different sets of consumers, meaning advertisers

[7]Alternatively, we could have chosen the additive form $v_i + Y_i$, which changes the form of the demand function but does not change the result.

[8]This follows from the vertical differentiation. If a consumer subscribes to two newspapers, say $\{1, 2\}$, then their utility is given by $\max(\theta_1, \theta_2) - s_1 - s_2$. So unless the low-quality newspaper is free, the utility maximizing consumer will never buy both products.

[9]This is equivalent to endogenous price when advertiser's utility is linear in the number of users; higher demand market has higher β.

[10]This has been observed in Indian market as well. English dailies that target affluent customers has advertisement rates 3 times that of other dailies.

multi-home. The advertiser's utility u_k in advertising in newspaper k is given by:

$$u_k = N_k\,(\beta\, E_k(Y) - p_k)$$

where N_k is the number of subscribers,$E_k(Y)$ is the expected income of subscribers, and p_k is the advetisement price per unit of circulation of newspaper k.

Each market is characterized by a set of exogenous market factors (M, μ, σ^2), production technology (c, α), and advertisement level β. Each firm in this market chooses endogenous parameters: subscription price (s), newspaper quality (θ), and advertisement price (p) per unit of circulation. We use the price-to-quality ratio to effectively represent firms' pricing strategies, as this ratio is more directly related to consumer surplus. Standalone price changes could be driven by changes in quality.

Firms enter the market sequentially. The timing of their decisions is as follows:

Entry Stage: Firms enter sequentially and each firm chooses the quality before the next firm makes entry decision.

Price Stage: Firms that have entered the market simultaneously set their subscription prices (s) and advertisement prices (p).

We solve for the pure strategy Subgame Perfect Nash Equilibrium. We also assume that firms do not incur fixed entry cost other than the cost to establish quality.

3.3.1 Key assumptions and rationale

We make following assumptions in our model.

1 *Firms are only vertically differentiated.* This paper focuses on the impact of advertising and consumer heterogeneity on the quality of newspapers, which is one of the key factors determining market structure. Therefore, it is natural to focus on vertical differentiation and abstract away from the variety due to horizontal differentiation. In many contexts, horizontal differentiation can be treated as a separate market, in which case our results will still hold. This applies when the consumers of two differentiated products do not overlap or when they do not have to choose one product over the other. For example, in the Indian context, English and Vernacular newspapers can be considered different markets with distinct competitive dynamics and consumer profiles. Similarly, a financial newspaper can be considered a separate market from general dailies when a consumer's choice of a general newspaper does not preclude her from subscribing to a financial newspaper. However, this does not apply in cases of horizontal differentiation due to partisanship, in which case the demand function is a mixture of pure vertical models, and the product variety

will increase. See Gabszewicz and Thisse (1986) and Neven and Thisse (1989) for the models that use both horizontal and vertical differentiation, though not in a newspaper context.

2 *Quality improvement is through fixed cost of production.* The content quality of a newspaper is primarily determined by the number of journalists and the quality of staff (e.g., award-winning journalists), which are part of the fixed costs. While quality could also be related to the quality of printing and/or the number of pages, both of which impact variable costs, empirical evidence clearly suggests that newspaper quality is primarily driven by fixed costs (Reddaway, 1963; Berry and Waldfogel, 2010; Angelucci and Cage, 2019).

3 *Income follows a log-normal distribution.* The income distribution of a population is widely modeled using a log-normal distribution, as it fits many income datasets (see reference). The Pareto distribution is another commonly used model for income, but its moments are restricted for certain parameter ranges, making it unsuitable for some situations, specifically at lower income levels. The log-normal distribution also has the advantageous property that if the pre-tax income is log-normal and the tax schedule is progressive in the form ax^b, then the disposable income also follows a log-normal distribution. The log-normal distribution allows for calculating the Gini-coefficient, a standard measure of income inequality, using the single parameter σ, which is given by $G(\sigma) = 2\Phi(\frac{\sigma}{\sqrt{2}}) - 1$. This enables us to calibrate σ across markets with varying levels of income-inequality.

4 *Consumers are neutral to advertisements.* This has been widely discussed in the literature. Some studies find that newspaper and magazine readers appreciate advertisements, specifically classifieds (Rosse, 1970; Thompson, 1989; Filistrucchi et al., 2012; Dertouzos and Trautman, 1990). However, other studies argue that readers are ad-neutral as they can easily ignore advertisements (Gabszewicz et al., 2001). Fan (2013) and Van Cayseele and Vanormelingen (2009) find empirical evidence supporting ad-neutrality. Sonnac (2000) conducts a cross-country analysis and finds that readers' attitudes vary across countries. Nonetheless, most structural analyses of newspaper and magazine markets model readers as being indifferent to advertising (Fan, 2013; Gabszewicz et al., 2012; Gentzkow et al., 2014). We assume readers are ad-neutral in our base model; however, we discuss the impact of consumers being ad-lovers or ad-haters on our results in the robustness section.

5 *Consumers' willingness to pay depends on both income and preference for quality content.* We consider both factors to generalize the demand function and compare markets with varying income and reader characteristics. Additionally, we parameterize the level of consumer heterogeneity in the market using the distribution

$U(b-1, b)$. The literature typically considers only one factor, which is either income (Gabszewicz and Thisse, 1979) or preference for quality (Wauthy, 1996; Gabszewicz et al., 2012).

6 *Advertisers are homogeneous in the market:* Advertiser heterogeneity is not modeled for simplicity, as it does not impact our results. Our model is equivalent to one with heterogeneous advertisers having linear utility functions: an advertiser of type μ receives utility $\mu N_i - p_i$ by advertising in newspaper (i) with N_i readers and an advertisement price p_i. The newspaper's profit from advertisements in such cases is equivalent to a constant unit advertising price, which in our case is β (see Gabszewicz et al. (2012)). Due to single-homing customers, all firms charge monopoly pricing to advertisers; hence, advertiser heterogeneity does not change competitive dynamics. We also don't analyze the advertisers welfare except calculating the social planner's choice of quality.

7 *Firms enter sequentially.* Since quality improvement is achieved through fixed costs, the vertical differentiation model confers an endogenous advantage to the higher-quality firm. Therefore, the first mover gains a significant advantage by preempting the profitable higher-quality niche. The sequential entry model captures this dynamic. Additionally, sequential entry ensures that a pure strategy equilibrium exists across all parameter values. Shaked and Sutton (1987) pointed out that sequential entry guarantees a pure strategy equilibrium at the product choice stage if the price stage has an equilibrium, which is not necessarily true for simultaneous entry. However, for completeness, we also provide the simultaneous entry results in section 3.8.1 for comparison.

3.4 Benchmark: Social Planner's Problem

Consider a social planner who sets the subscription price s and the quality θ such that $(s, \theta) \in \mathbb{R}^2_+$ to maximize the total surplus, which includes subscription profit, consumer surplus, and advertiser surplus. A consumer i will subscribe to the newspaper if and only if she derives non-negative utility:

$$U_i = v_i\, Y_i\, \theta - s \geq 0 \text{ which implies } v_i \geq \frac{s}{Y_i\, \theta}$$

For uniform distribution of v and log-normal income distribution $ln(Y) \sim N(\mu, \sigma^2)$, the demand function $N(s, \theta)$ is given by:[11]

$$N(s,\theta) = \begin{cases} M \int_0^\infty Pr(v \geq \frac{s}{Y\theta}) dF(Y) = M(1 - \frac{s}{\kappa\,\theta}) & \text{if } 0 \leq s \leq \kappa\,\theta, \\ 0 & \text{if } s \geq \kappa\,\theta \end{cases} \tag{3.1}$$

$$\text{where } \kappa = e^{\mu - \frac{1}{2}\sigma^2}$$

Equation (3.1) highlights that the market will be fully covered only when the subscription price is zero, as some consumers do not value reading a newspaper. When the market is uncovered, the demand is higher in markets with higher median income (μ), ceteris paribus,[12] and lower in markets with higher income inequality (σ), ceteris paribus. A higher σ signifies a higher proportion of consumers in the lower tail of the income distribution, who do not subscribe to the newspaper in an uncovered market.

Given the demand function $N(s, \theta)$, the social planner's objective function $W(s, \theta)$ is given by:

$$W(s,\theta) = \underbrace{N(s,\theta)(s-c)}_{\text{subscription net revenue}} + \underbrace{M \int_0^\infty \Big(\int_{\frac{s}{Y\theta}}^1 (vY\theta - s) dv \Big) dF(Y)}_{\text{consumer surplus}} + \underbrace{M\,\beta \int_0^\infty Y(1 - \frac{s}{Y\theta}) dF(Y)}_{\text{advertiser surplus}} - \underbrace{\alpha\,\theta^2}_{\text{fixed cost}} \tag{3.2}$$

Equation (3.2) simplifies to:[13]

$$W(s,\theta) = N(s,\theta)(s - c + \kappa\beta) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\,\beta(\delta - \kappa) - \alpha\,\theta^2 \tag{3.3}$$

$$\text{where } \kappa = E[\frac{1}{Y}] = e^{\mu - \frac{\sigma^2}{2}} \text{ and } \delta = E[Y] = e^{\mu + \frac{\sigma^2}{2}}$$

Proposition 3.1. *A social planner that optimizes the total welfare would set optimal subscription price s_{SP} and quality θ_{SP} such that:*

$$s_{SP} = \max(0, c - \kappa\beta) \tag{3.4}$$

$$\theta_{SP} = \begin{cases} \frac{M\,\delta}{4\,\alpha} \; \textit{if } s_{SP=0} \\ \theta^* \textit{ that solves } 4\alpha\,\kappa\,\theta^3 - M\,e^{2\mu}\,\theta^2 + M\,(c - \kappa\beta)^2 = 0, \;\; \textit{otherwise} \end{cases} \tag{3.5}$$

Further, if $c - \kappa\beta$ is sufficiently high then the social planner will not serve the market.

[11] $\int_0^\infty Pr(v \geq \frac{s}{\theta Y}) dF(Y) = \int_0^\infty (1 - \frac{s}{\theta Y}) dF(Y) = \int_0^\infty dF(Y) - \frac{s}{\theta} \int_0^\infty \frac{1}{Y} dF(Y) = 1 - \frac{s}{\theta} E(\frac{1}{Y}) = 1 - \frac{s}{\theta} e^{-\mu + \frac{1}{2}\sigma^2}$

[12] Higher μ implies a higher κ, which implies a higher $1 - \frac{s}{\kappa\,\theta}$.

[13] see Appendix 3.A.1

Proof of Proposition 3.1 follows from the first order condition of (3.3) (see Appendix 3.A.1). Proposition 3.1 highlights that the social planner sets the subscription price to cover the variable costs of printing and circulation (c), less offset from advertiser's benefit ($\kappa\beta$). If the advertiser's benefit fully covers the variable costs, then the planner will offer the newspaper for free, achieving full market coverage. Please note that $\kappa\beta$ in true sense is the advertiser's benefit from the marginal consumer and not from every consumer. The total advertiser's benefit is $N(\kappa\beta) + M\,\beta(\delta - \kappa)$

Definition 3.1. *We refer the term $\kappa\beta - c$ where $\kappa = e^{\mu - \frac{\sigma^2}{2}}$ as 'advertisement intensity', and use the invertible function $\phi : \beta \rightarrow \mathbb{R} \equiv \kappa\beta - c$ to compute it.*[14]

The advertisement intensity measures the contribution from advertisement per new subscriber, net of the variable cost of printing and circulation. This term arises because advertises are willing to pay higher for higher-income consumers. It increases with the median market income (μ) and the advertiser's willingness to pay (β), and decreases with the income inequality (σ). It's important to note that high income inequality reduces advertisement intensity because high proportion of customers are in the lower end of the income.

Note: Since ϕ is a strictly increasing invertible function of β both ϕ and β can be used interchangeably to represent the advertisement level. We describe propositions in terms of exogenous β, whereas equations and cutoff values are defined in terms of ϕ as it simplifies the expressions.

3.5 Monopolist

Consider that there is only one firm in the market that sets the non-negative subscription price s, quality θ, and advertisement price p per unit of circulation to maximize its profit. A consumer i will subscribe to the newspaper if and only if she receives non-negative utility, meaning:

$$U_i = v_i\, Y_i\, \theta - s \geq 0 \text{ implies } v_i \geq \frac{s}{Y_i\, \theta}$$

Since the consumer decision in this scenario is similar to that of the social planner case, the monopolist's demand function for the uniform distribution of v and log-normal income distribution will be given by (3.1). The corresponding monopolist's profit function $\Pi(s, \theta, p)$ is:

$$\Pi(s, \theta, p) = \underbrace{N(s, \theta)}_{\text{Unit demand}} \underbrace{(s - c + p)}_{\text{per-unit margin}} - \underbrace{\alpha\, \theta^2}_{\text{fixed cost}} \tag{3.6}$$

[14] $\phi(\beta)$ is an invertible function as it is a a well defined strictly increasing function of β.

Therefore, the monopolist's problem can be written as:

$$\max_{s,\theta,p} \; N(s,\theta)\,(s-c+p)-\alpha\,\theta^2$$

subject to:

$$p\,N(s,\theta)\leq M\int_0^{\infty}\beta\,Y\,(1-\frac{s}{Y\theta})\,dF(Y) \tag{3.7}$$

$$s\geq 0, \theta\geq 0$$

(3.7) is the participation constraint (non-negative surplus)[15] of the advertiser.[16]

Monopolist will set p such that the constraint (3.7) is binding i.e. it will extract full surplus. If not then the monopolist can increase price p by a small amount and increase the profit while still meeting the participation constraint.[17] This implies that

$$p\,N(s,\theta)=M\,\beta(E(Y)-\frac{s}{\theta})=M\,\beta\,(\delta-\frac{s}{\theta}) \tag{3.8}$$

Replacing p from (3.8) in (3.6) and using $\phi\equiv\kappa\,\beta-c$ (Definition 3.1) we get

$$\Pi(s,\theta)=N(s,\theta)\,(s+\phi)+M\,\beta\,(\delta-\kappa)-\alpha\,\theta^2 \tag{3.9}$$

where $\delta=e^{\mu+\frac{\sigma^2}{2}}$, $\kappa=e^{\mu-\frac{\sigma^2}{2}}$, $N(s,\theta)$ given by (3.1)

It is important to note that the term $M\,\beta\,(\delta-\kappa)$ arises due to income inequality and is a fixed rent that the monopolist earns by serving customers in the right tail of income distribution. If there is no income-inequality, i.e. $\sigma=0$, then this term vanishes. The monopolist problem is to choose s and θ that maximizes its profit function (3.9) subjected to non-negative profit:

$$\max_{s,\theta} \; \Pi(s,\theta) \text{ subject to } \Pi(s,\theta)\geq 0,\; s\geq 0,\; \theta\geq 0 \tag{3.10}$$

See Appendix 3.A.2 for the solution of (3.10). The results are summarized below.

Lemma 3.1. *Suppose the exogenous parameters are such that the monopolist cover the market partially (interior solution of* (3.10)*), then the equilibrium quality θ_{MP} is given by the unique solution of* (3.11)

$$8\alpha\,\kappa\,\theta^3-M\,\kappa^2\,\theta^2+M\,\phi^2=0 \;\; \textit{subject to} \;\; \theta>\frac{M\,\kappa}{12\,\alpha} \tag{3.11}$$

[15]The advertiser gets surplus of βY_i from a subscribing consumer with income Y_i.

[16]Note that there is one representative advertiser.

[17]This is a standard result in platform market theory when consumers single-home and advertisers multi-home.

and the subscription price s_{MP} is given by (3.12)

$$s_{MP} = \frac{1}{2}\kappa\,\theta_{MP} - \frac{1}{2}\phi \tag{3.12}$$

The quality set by monopolist is increasing in market size (M) and income-level (μ), decreasing in quality cost(α) and income-inequality (σ), and non-monotonic in β and c with single-peak at $\phi = 0$.

Proof of Lemma 3.1 follows from the first order condition of (3.9) and implicit function theorem on (3.11) (see Appendix 3.A.2). The condition $\theta > \frac{M\,\kappa}{12\,\alpha}$ meets the necessary second order condition, and the condition (3.13) below ensures that the (3.11) has a solution.[18]

$$|\phi| < \frac{M\,\kappa^2}{12\sqrt{3}\,\alpha} \tag{3.13}$$

(3.12) highlights that the higher advertisement intensity, $\phi \equiv \kappa\beta - c$, reduces the subscription price for the consumer. In other words, consumers are subsidized for the externality they exert on advertisers, which is a standard results in the platform market (Armstrong, 2006).

We now identify the critical conditions for corners solutions when the monopolist will not serve the market (zero market coverage) or serve the market with zero subscription price and hence the full market coverage.[19]

Proposition 3.2. *There exist $(\underline{\beta}, \overline{\beta})$ with $0 \leq \underline{\beta} < \overline{\beta}$ such that*

a If $\beta < \underline{\beta}$, the monopolist will not serve the market; the $\underline{\beta}$ is positive only if the marginal cost c is sufficiently high.

b If $\beta > \overline{\beta}$, the monopolist will set the subscription price $s_{MP} = 0$ and cover the full market, but will produce the lowest quality $\theta_{MP} = 0$.

c If $\beta \in [\underline{\beta}, \overline{\beta}]$ then the monopolist will set the subscription price and quality as provided in Lemma 3.1, and the market remains uncovered with coverage increasing in advertisement intensity ϕ.

Further $\overline{\beta}$ is increasing in the market size (M) and marginal cost (c), and decreasing in quality cost (α), and the relationship is reverse for $\underline{\beta}$.[20] Relationship of $\overline{\beta}$ is ambiguous with μ and σ and depends on c,[21] while $\underline{\beta}$ is decreasing in μ and increasing in σ

[18]Appendix 3.A.2 shows that (3.11) has a unique solution whenever there is an interior solution.

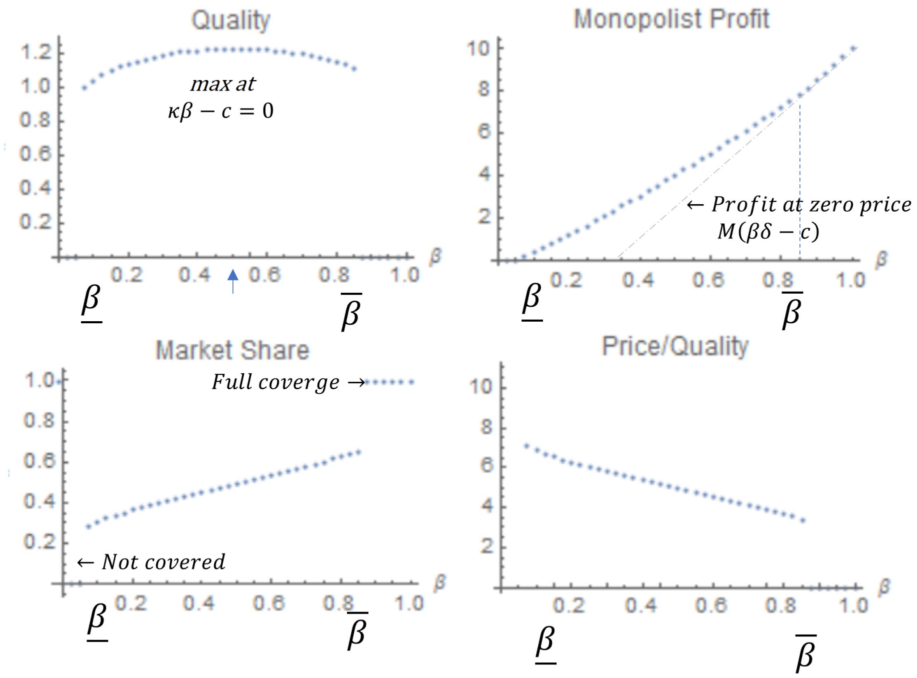
[19]When subscription price is zero all consumers get non-negative utility.

[20]Expressions for $\overline{\beta}$ and $\underline{\beta}$ is given by: $\phi^{-1}(\frac{M\kappa^2}{27\alpha})$ and $max(0, \phi^{-1}(-\frac{M\kappa^2}{27\alpha}))$, respectively.

[21]If μ increases and σ decreases, both advertisement revenue and subscription revenue increases with the opposing effect. If c is small the first effect outweigh and $\overline{\beta}$ increases, reverse otherwise.

Proof of Proposition 3.2 is given in the Appendix 3.A.2. Figure 3.1 depicts the result graphically.

Figure 3.1: Monopoly equilibrium at different value of β



The profit and the market coverage of the monopolist is given by:

$$\Pi(\theta) \quad = \quad \begin{cases} 0 & \text{if } \beta \leq \underline{\beta}, \\ \frac{M}{4\kappa\,\theta}(\kappa\,\theta + \phi)^2 + M\,\beta\,(\delta - \kappa) - \alpha\,\theta^2 & \text{if } \beta \in [\underline{\beta}, \overline{\beta}], \\ M\,(\beta\,\delta - c) & \text{if } \beta > \overline{\beta} \end{cases} \tag{3.14}$$

$$\frac{N}{M} \quad = \quad \begin{cases} 0 & \text{if } \beta < \underline{\beta}, \\ \frac{1}{2}(1 + \frac{\phi}{\kappa\,\theta_{MP}}) & \text{if } \beta \in [\underline{\beta}, \overline{\beta}], \\ 1 & \text{if } \beta > \overline{\beta} \end{cases} \tag{3.15}$$

Intuitively, when $\beta \in [\underline{\beta}, \overline{\beta}]$, the monopolist faces a trade-off between acquiring a marginal customer through price reduction (or higher quality) and incurring revenue loss (or higher quality cost) from existing customers. If β increases, enhancing the value of a marginal customer, the monopolist will adjust the price, quality, or both, depending on the elasticity of demand with respect to price and quality. If $\phi < 0$, the marginal customer is acquired by both reducing the price and improving the quality. If $\phi = 0$, the marginal customer is acquired solely through a price reduction. If $\phi > 0$, the marginal customer is acquired by reducing the price but with an offsetting reduction in quality to lower costs. Therefore, the quality is non-monotonic in β even though the market coverage increases monotonically with β. The monopolist covers one-third the market when $\beta = \underline{\beta}$, one-half the market when $\kappa\beta - c = 0$, and two-third the market when $\beta = \overline{\beta}$.

When $\beta > \overline{\beta}$ the potential loss of advertisement revenue from all non-subscribing customer outweighs the subscription revenue through higher-priced quality product. Consequently, the monopolist opts to forego all subscription revenue in order to capture the full advertisement revenue, which means it covers the market by setting the price to zero and offering the minimum quality.[22]

Remark: There is a tension between advertisement and subscription revenue. Optimizing subscription revenue means maintaining a quality product with a positive subscription price, which leads to many low-end customers not subscribing, resulting in a loss of advertisement revenue. This tension generates a corner solution when advertisement levels are high, as the firm offers a free product and foregoes subscription revenue to capture the full advertisement revenue.

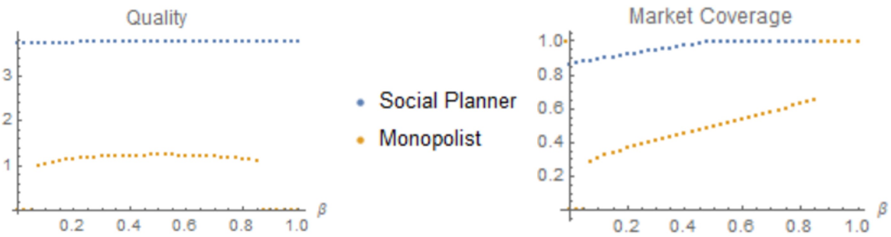
Corollary 3.1. *Monopolist strictly under provisions quality relative to the social planner $\theta_{MP} < \theta_{SP}$ and weakly covers less market.*

Proof: Using implicit function theorem on (3.5) and (3.11), we can show that $\theta_{MP} < \theta_{SP}$.[23] Comparison between (3.4) and (3.12) shows the the $s_{MP} > s_{SP}$ when $\beta < \overline{\beta}$. Higher price and lower quality results in lower market coverage by the monopolist when $\beta < \overline{\beta}$. When $\beta \geq \overline{\beta}$, both a monopolist and the social planner will cover full market but the monopolist will provide a very low quality product.

□

The comparison between the social planner and the monopolist is shown in Figure 3.2.

Figure 3.2: Comparison between social planner and monopolist



Corollary 3.2. *Smaller market (lower M) has lower quality for all values of β. The same is true for market with lower median income (μ) and higher income inequality (σ), provided that c is not critically high.*

[22] The zero quality refers to the product which do not invest in building quality by hiring editorial staff but rather use wire services. Metro or 20 minutes are examples of such newspapers in Europe.

[23] Desired solution of $a\theta^3 - b\theta^2 + c^2 = 0$ is decreasing in a and increasing in b. (3.11) has higher a and lower b relative to (3.5) and hence $\theta_{MP} < \theta_{SP}$, if $s_{SP} > 0$. If $s_{SP} = 0$, the social planner sets $\theta_{SP} = \frac{M\delta}{4\alpha}$, which is strictly greater than the maximum quality set by monopolist across all parameter values.

The proof derives from two observations: a) when the monopolist sets a positive quality level, i.e., $\beta \in [\underline{\beta}, \overline{\beta}]$, the quality level increases with M and μ, and decreases with σ (as stated in Lemma 3.1); b) the interval $[\underline{\beta}, \overline{\beta}]$ expands at both ends if either μ or M increase or σ decreases (as per Proposition 3.2). An exception occurs when c is critically high, causing $\overline{\beta}$ to decrease with μ. In such scenarios, for some values of β, the monopolist may transition from positive to zero quality if μ increases and/or σ decreases.

To summarize, the key aspects of the monopolist market are: a) the monopolist under-provides quality relative to the social planner and sets prices higher than the social planner; b) the market remains uncovered unless the advertisement level is high, i.e., $\beta > \overline{\beta}$; c) when the advertisement level is low to moderate, i.e., $\beta \in [\underline{\beta}, \overline{\beta}]$, higher advertising subsidizes consumers as they pay a lower subscription price. However, when the advertisement level becomes sufficiently high, i.e. $\beta > \overline{\beta}$, consumers are offered very low-quality products and lose all their surplus. Thus, high advertising revenue in a non-competitive market does not necessarily entail high investment in quality and can result in the undesirable outcome of a poor-quality product; d) Corollary 3.2 demonstrates that consumers with similar preferences in a smaller market may receive a lower quality product, consistent with Berry and Waldfogel (2010)'s empirical observation. It also shows that higher income inequality leads to a lower quality product, as a higher proportion of consumers falls into the lower tail of the income distribution, prompting the monopolist to lower both price and quality to capture a sufficient market share.

3.6 Duopoly

Now we consider competition in the market but only two potential entrants, denoted as $k \in \{1, 2\}$. The timing for the sequential entry of firms is as follows:

Stage 1: Firm 1 (or leader) makes the entry decision and choose the quality θ_1

Stage 2: Firms 2 (or follower) makes the entry decision and choose the quality θ_2.

Stage 3: If both firms enter they simultaneously choose the price, otherwise Firm 1 sets price as a monopolist.

We conjecture four distinct types of equilibrium in such a market.

Definition 3.2. *We call the equilibrium Type A (natural monopoly) when only one firm enters the market and sets prices and quality at monopolistic levels.*

Definition 3.3. *We call the equilibrium Type B (uncovered or interior solution) when both firms enter and set strictly positive subscription prices and quality levels, that is,* $(s_k, \theta_k) \in \mathbb{R}^2_{++}$ *for* $k \in \{1, 2\}$*, and the market remains uncovered.*

Definition 3.4. *We call the equilibrium Type C (corner solution) when both firms enter, and the follower firm (Firm 2) sets both price and quality to zero* $(s_2, \theta_2) = (0, 0)$*, while the leader (Firm 1) sets monopolistic price and quality* $(s_1, \theta_1) = (s_{MP}, \theta_{MP})$*. Under this equilibrium, the market is fully covered.*

Let us denote quality infinitesimally greater than θ *as* θ_+ *that is* $\theta_+ \equiv \theta + \epsilon$ *where* $\epsilon \to 0$.

Definition 3.5. *Define* $\theta_c : \phi \to \mathbb{R}_{++}$ *such that if Firm 1 chooses* $\theta_1 = \theta_c$ *then Firm 2 is indifferent between choosing* $\theta_2 = 0$ *and* $\theta_2 = \theta_{c+}$*.*

Definition 3.6. *We call the equilibrium type D (contestable) when both firms enter and the leader (Firm 1) sets the quality* θ_c*, and the follower (Firm 2) sets its quality to zero, that is* $(\theta_1, \theta_2) = (\theta_c, 0)$*. Firm 1 sets the price as a monopolist would for* θ_c *quality, and Firm 2 sets the price to zero.*

First, we characterize the equilibrium types B, C, and D. Subsequently, we will identify the conditions under which each type of equilibrium exists. Type A equilibrium is equivalent to the monopoly equilibrium described in the monopolist section 3.5.

3.6.1 Type B Equilibrium

Let's assume that the entering firm sets lower quality than the leading firm under equilibrium, $0 < \theta_2 < \theta_1$, which we will validate. This implies, $s_2 < s_1$; otherwise, all consumers will switch to the high-quality newspaper (Firm 1) and the low-quality newspaper (Firm 2) will make a negative profit. The utility of a consumer (i) who is indifferent between the two newspapers will be given by:

$$U_i = v_i\, y_i\, \theta_2 - s_2 = v_i\, y_i\, \theta_1 - s_1 \Rightarrow v_i\, y_i = \frac{s_1 - s_2}{\theta_1 - \theta_2}$$

Consumers with a higher preference for quality than that of the indifferent consumer will buy the high-quality newspaper, while those with a lower preference will buy low-quality newspaper provided they receive non-negative utility. Therefore, the demand functions for the two firms are given by:

$$N_1(s_1, s_2, \theta_1, \theta_2) = M \int_0^{\infty} \int_{\frac{s_1 - s_2}{y(\theta_1 - \theta_2)}}^{1} dv dy \quad = \quad M(1 - \frac{s_1 - s_2}{\kappa\, (\theta_1 - \theta_2)}) \tag{3.16}$$

$$N_2(s_1, s_2, \theta_1, \theta_2) = M \int_0^{\infty} \int_{\frac{s_2}{y\theta_2}}^{\frac{s_1 - s_2}{y(\theta_1 - \theta_2)}} dv dy \quad = \quad M(\frac{s_1 - s_2}{\kappa\, (\theta_1 - \theta_2)} - \frac{s_2}{\kappa\, \theta_2}) \tag{3.17}$$

$$\text{where } \kappa = E[\frac{1}{Y}] = e^{\mu - \frac{1}{2}\sigma^2}$$

The firms will set the advertisement prices such that they capture the full surplus from the advertisers, because each firm provides a unique, non-overlapping set of consumers. Therefore, the advertisement prices are determined as follows:

$$\begin{aligned} p_1 N_1 &= M\,\beta \int_0^\infty (1 - \frac{s_1 - s_2}{y\,(\theta_1 - \theta_2)})\, y\, dy \Rightarrow p_1 N_1 = M\,\beta\,(\delta - \frac{s_1 - s_2}{\theta_1 - \theta_2}) \\ p_2 N_2 &= M\,\beta \int_0^\infty (\frac{s_1 - s_2}{y\,(\theta_1 - \theta_2)} - \frac{s_2}{y\,\theta_2})\, y\, dy \Rightarrow p_2 = \kappa\,\beta \end{aligned}$$

$$\text{where } \delta = E[Y] = e^{\mu + \frac{1}{2}\sigma^2}$$

Using above advertisement prices and $\phi \equiv \kappa\beta - c$ (Definition 3.1), we derive the profit functions:

$$\Pi_1(s_1, \theta_1, s_2, \theta_2) = N_1\,(s_1 + \phi) + M\,\beta\,(\delta - \kappa) - \alpha\,\theta_1^2 \tag{3.18}$$

$$\Pi_2(s_1, \theta_1, s_2, \theta_2) = N_2(s_2 + \phi) - \alpha\,\theta_2^2 \tag{3.19}$$

Since this is a two-stage strategic game. We use backward induction to find the sub-game perfect Nash equilibrium. The first-order conditions provide the reaction functions in the price stage (stage 2):

$$s_1(s_2) = \frac{1}{2}s_2 + \frac{1}{2}\kappa\,(\theta_1 - \theta_2) - \frac{1}{2}\phi \tag{3.20}$$

$$s_2(s_1) = \frac{\theta_2}{2\theta_1}\,s_1 - \frac{1}{2}\phi \tag{3.21}$$

Since both profit functions are strictly concave for $\theta_2 < \theta_1$,[24] the first-order conditions are also sufficient. Equilibrium prices, given by the unique solution of the above two linear equations:

$$s_1 = \frac{2\kappa\,\theta_1(\theta_1 - \theta_2)}{4\theta_1 - \theta_2} - \frac{3\theta_1}{4\theta_1 - \theta_2}\,\phi \tag{3.22}$$

$$s_2 = \frac{\kappa\,\theta_2(\theta_1 - \theta_2)}{4\theta_1 - \theta_2} - \frac{2\theta_1 + \theta_2}{4\theta_1 - \theta_2}\,\phi \tag{3.23}$$

The subscription price of both firms decreases with the advertisement intensity ϕ, as firms reduce their subscription prices to acquire marginal customers and increase advertisement revenue.

Substituting (3.22)-(3.23) in (3.18)-(3.19) we can derive the expression of the profit

[24] $\frac{\partial^2 \Pi_1}{\partial s_1^2} = -\frac{2M}{(\theta_1 - \theta_2)}$ and $\frac{\partial^2 \Pi_2}{\partial s_2^2} = -\frac{2M\theta_1}{\theta_2(\theta_1 - \theta_2)}$

function of each player at stage 1 as a function of quality:

$$\Pi_1(\theta_1,\theta_2) = M\frac{\theta_1-\theta_2}{\kappa\,(4\theta_1-\theta_2)^2}(2\kappa\,\theta_1+\phi)^2 + M\,\beta\,(\delta-\kappa) - \alpha\,\theta_1^2$$
$$\Pi_2(\theta_1,\theta_2) = M\frac{\theta_1(\theta_1-\theta_2)}{\kappa\,\theta_2\,(4\theta_1-\theta_2)^2}(\kappa\,\theta_2+2\phi)^2 - \alpha\,\theta_2^2$$

Notice that only the high-quality firm earns a fixed rent $M\,\beta(\delta-\kappa)$, which arises from serving consumers in the right tail of the income distribution. If we define $\gamma \equiv \frac{\theta_2}{\theta_1}$, then the above equation can be rewritten as follows:

$$\Pi_1 = M\frac{1-\gamma}{\kappa\,\theta_1\,(4-\gamma)^2}\,(2\kappa\,\theta_1+\phi)^2 + M\,\beta\,(\delta-\kappa) - \alpha\,\theta_1^2 \tag{3.24}$$
$$\Pi_2 = M\frac{1-\gamma}{\kappa\,\theta_2\,(4-\gamma)^2}\,(\kappa\,\theta_2+2\phi)^2 - \alpha\,\theta_2^2 \tag{3.25}$$

The corresponding market shares and the price-to-quality ratios of firms are given by:

$$\frac{N_1}{M} = \frac{2}{4-\gamma} + \frac{1}{\kappa\,\theta_1\,(4-\gamma)}\,\phi \tag{3.26}$$
$$\frac{N_2}{M} = \frac{1}{4-\gamma} + \frac{2}{\kappa\,\theta_2\,(4-\gamma)}\,\phi \tag{3.27}$$
$$\frac{s_1}{\theta_1} = 2\frac{k\,(1-\gamma)}{4-\gamma} - \frac{1}{\theta_1}\frac{3}{4-\gamma}\,\phi \tag{3.28}$$
$$\frac{s_2}{\theta_2} = \frac{k\,(1-\gamma)}{4-\gamma} - \frac{1}{\theta_2}\frac{2+\gamma}{4-\gamma}\phi \tag{3.29}$$

As advertisement intensity ϕ increases, both firms lower the price-to-quality ratio to acquire marginal customers and increase market share. However, the price-to-quality ratio of the lower-quality firm is more responsive because it has a smaller market share, making the reduction in prices for existing customers less costly.

The first-order conditions below provide the reaction functions for firms.[25]

$$\frac{4M\kappa}{(4-\gamma)^3}\,(1+\frac{\phi}{2\kappa\,\theta_1})\big[(2\gamma^2-3\gamma+4) - \frac{\phi}{2\kappa\,\theta_1}\,(4-7\gamma)\big] = 2\alpha\,\theta_1 \tag{3.30}$$
$$\frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\,\theta_2})\big[4-7\gamma - \frac{2\phi}{\kappa\,\theta_2}\,(2\gamma^2-3\gamma+4)\big] = 2\alpha\,\theta_2 \tag{3.31}$$

Firm 1 will find the most profitable choice of its own quality after considering the reaction function of Firm 2. The characteristics of the Type B equilibrium, if it exists, are described in the following lemmas, and their proofs are provided in Appendix 3.A.3. We first show that the leading firm will take the high quality position so that the follower enters with lower quality.

[25]Necessary second order conditions are verified while finding the solutions.

Lemma 3.2. *Firm 1 will set the quality so that the Firm 2 enters with lower quality i.e.* $\theta_2 < \theta_1$.

Intuitively, the high-quality firm (Firm 1) has an inherent advantage, so as a first mover, Firm 1 will preempt that position. Firm 1 will attract consumers who have higher willingness to pay for the quality and thus has ability to charge higher price (see (3.22)-(3.23)). In addition, it earns extra rent through advertisement by serving the right tail customer in the income distribution, the term $M\,\beta(\delta-\kappa)$. Next we consider the equilibrium solution with the benchmark case $\phi = 0$.

Lemma 3.3. *If* $\phi = 0$ *then there exist a unique solution such that two firms choose quality in the ratio* $\gamma = 0.195064$*, which is a constant for all* (μ, σ^2, α)*. The market share of the high quality firm is twice that of the low quality firm. The high-quality firm sets quality lower than the monopoly level but cover larger market share.*

The corresponding θ_1 and θ_2 can be derived from (3.31).

$$\theta_1 = \rho\frac{M\kappa}{\alpha} \text{ and } \theta_2 = \rho\gamma\frac{M\kappa}{\alpha} \text{ where}$$

$$\gamma = 0.195064, \rho = \frac{4-7\gamma}{2\gamma\,(4-\gamma)^3} = 0.1226$$

The corresponding market shares of the two players using (3.26) and (3.27) yields:

$$\frac{N_1}{M} = \frac{2}{4-\gamma} = 52.56\%, \frac{N_2}{M} = \frac{1}{4-\gamma} = 26.28\%$$

The market remains uncovered, with the total market coverage 78.84%. Compare this to the monopolist case, where the market is only 50% covered when $\phi = 0$. The entry of a low-quality player results in the expansion of market coverage by targeting consumers with lower valuations. Furthermore, the high-quality firm lowers both its price-to-quality ratio and its quality relative to that of a monopolist.[26] Therefore, the competitive entry reduces the price set by a monopolist and expands overall market coverage, resulting in higher consumer surplus.

The constant ratio, $\frac{4}{7}$, is established in the vertical differentiation literature when there are no fixed or variable costs of quality (see Choi and Shin (1992) and Lutz (1997)). In our model, due to the convex quality costs, the ratio is significantly smaller.

Lemma 3.4. *There exist* $\phi(\underline{\beta}) < \phi_{il} < 0$ *and* $0 < \phi_{ir} < \phi(\overline{\beta})$ *such that an interior solution exist iff* $\phi \in (\phi_{il}, \phi_{ir})$ *and this solution is unique for a given* ϕ.

[26] $\theta_1 = \frac{0.1226 M\kappa}{\alpha} < \frac{M\kappa}{8\alpha} = \theta_{MP}$, when $\phi = 0$.

Recall $[\underline{\beta}, \overline{\beta}]$ is the interval in which a monopolist will serve the market with positive quality and price (i.e., interior solution). We can easily observe that the first-order conditions do not have a solution if ϕ is sufficiently negative or sufficiently positive. When $\frac{\phi}{\kappa\,\theta_2} \to -\frac{1}{2}$, the left side of (3.31) approaches zero, indicating that no positive solution for θ_2 is possible. Similarly, when $\phi > 0$, Firm 1 reduces the price-to-quality ratio with higher ϕ until $\frac{s_2}{\theta_2} \to 0$ (see (3.29)), at which point a positive θ_2 is not optimal. Additionally, the interval (ϕ_{il}, ϕ_{ir}) is a subset of the interval $(\phi(\underline{\beta}), \phi(\overline{\beta}))$ in which the monopolist chooses positive quality.[27]

However, the existence of an interior solution is a necessary but not sufficient condition for Type B equilibrium because: a) Firm 2 may have negative profit and will not enter and Type A equilibrium occurs (see lemma 3.5); b) Firm 2 may have profitable deviation to $\theta_2 = 0$ and Type C equilibrium occurs (see lemma 3.11); or c) Firm 2 may have profitable deviation to $\theta_2 = \theta_1+$ (see lemma 3.10).

Lemma 3.5. *There exist a critical $\phi_0 \in (\phi_{il}, 0)$ such that for all $\phi \in (\phi(\underline{\beta}), \phi_0)$ only one firm enters the market (Type A equilibrium).*

Intuitively, if the advertisement revenue per customer is sufficiently small, the low-quality firm may need to significantly raise its prices to achieve a positive contribution margin per customer. However, this firm might not attract enough demand at these higher prices to cover the costs associated with the required quality. Consequently, the low-quality firm may choose not to enter the market, resulting in a natural monopoly (Type A equilibrium). Note that for $\phi < \phi(\underline{\beta})$, the market does not sustain even a single player, as stated in Proposition 3.2. It is also important to note that if the cost parameter c is low such that $\phi(0) > \phi_0$, then both firms would enter for all $\beta \geq 0$.

Lemma 3.6. *γ decreases continuously with ϕ for $\phi \in [0, \phi_{ir}]$, and the high-quality firm sets a lower quality than that of a monopolist for all $\phi \in (\phi_{il}, \phi_{ir})$.*

As the advertisement revenue becomes more valuable, the low-quality firm expands market by attracting customers with lower valuation. It is more optimal for this firm to reduce quality with this expansion to relax the competition. The result that the high-quality firm set lower quality than that of monopolist is due to the sequential entry (or Stackelberg) model. The high-quality firm crowds out the quality space of the low-quality firm by reducing quality. In simultaneous entry model (see section 3.8.1), the high-quality firm sets higher quality higher than the monopolist level in this interval of ϕ.

Lemma 3.7. *For any given ϕ, the equilibrium quality level of both firms increase with M and μ, and decreases with α and σ, ceteris paribus.*

[27] The cutoff value approximates to $\phi_{il} = -\frac{M\kappa^2}{187\alpha}$, $\phi_{ir} = \frac{M\kappa^2}{255\alpha}$. We don't have close form solution of these cutoff values, so we identified the cut-off values using numerical methods with precision $0.005\frac{M\kappa^2}{\alpha}$.

The increase in M or μ or a decrease in σ increases marginal revenue (as indicated in the LHS of (3.30)-(3.31)), while a reduction in α decreases the marginal cost. Consequently, both firms in the Type B equilibrium have higher quality levels, which increases the marginal cost (RHS of (3.30)-(3.31)) to match the marginal revenue.

Lemma 3.8. *Profit and the market coverage of both firms increases continuously with ϕ under Type B equilibrium.*

As ϕ increases, there is a direct effect of increased advertisement revenue for both firms. In addition, there is a positive strategic effect due to relaxed competition when $\phi \geq 0$ because γ decreases.[28] Therefore, profits of both firms increase. Higher ϕ leads to market expansion because marginal customers become more attractive, prompting both firms to lower their price-to-quality ratios to acquire these customers. The market share of the two firms at the right extreme of the interior solution, when $\phi \to \phi_{ir}$:[29]

$$\frac{N_1}{M} = 52.75\% \quad \frac{N_2}{M} = 38.86\%$$

Comparing the above numbers with those at $\phi = 0$, we can infer that the main impact of increase in advertisement is the increase in market coverage of the low quality product.

3.6.2 Type C Equilibrium

In Type C equilibrium, the low-quality firm (Firm 2) chooses zero quality and the high-quality firm (Firm 1) chooses monopolist level quality.

Lemma 3.9. *Suppose low-quality firm (Firm 2) chooses $\theta_2 = 0$ and $\phi \in [0, \frac{M\,\kappa^2}{12\sqrt{3}\,\alpha})$. Then, the best response form Firm 1 is to set $\theta_1 = \theta_{MP}$. The corresponding subscription prices would then be $s_2 = 0$, $s_1 = s_{MP}$.*

Proof is shown in Appendix 3.A.3. The condition $\phi >= 0$ ensures that Firm 2 has positive profit with $(s_2, \theta_2) = (0, 0)$, and $0 \leq \phi \leq \frac{M\,\kappa^2}{12\sqrt{3}\,\alpha}$ satisfies condition (3.13) so that the $\theta_{MP} > 0$.

Given $\theta_2 = 0$ and Lemma 3.9, Firm 1 behaves as a monopolist with the profit function (3.14) and the quality given by (3.11). Firm 2 serves all the customers not served by Firm 1 and earns revenue solely from advertisements. Its profit function Π_{2c} is given by:

$$\Pi_{2c} = \frac{M}{2}\,(1 - \frac{\phi}{\kappa\,\theta_{MP}})\,\phi \tag{3.32}$$

[28]Even when γ increases with ϕ, which occurs near the Type A cutoff (ϕ_0), the direct effect still outweighs the negative strategic effect.

[29]Evaluating θ_1 and θ_2 with $\phi_{ir} \approx \frac{M\kappa^2}{255\alpha}$ and substituting in (3.26)-(3.27)

Under type C equilibrium, the market is fully covered and the market share of Firm 1 increases with ϕ while the market share of Firm 2 decreases with ϕ.

$$\frac{N_1}{M} = \frac{1}{2}(1 + \frac{\phi}{\kappa\,\theta_{MP}}) \text{ and } \frac{N_2}{M} = 1 - \frac{N_1}{M} = \frac{1}{2}(1 - \frac{\phi}{\kappa\,\theta_{MP}}) \tag{3.33}$$

Type C equilibrium does not exist if $\phi < 0$ because Firm 2 will have negative profits. Type C equilibrium will also break if ϕ is high enough that the Firm 2 can earn higher profit by setting a quality infinitesimally greater than θ_{MP}, denoted as θ_{MP+}.

Suppose Firm 1 sets $\theta_1 = \theta_{MP}$ and Firm 2 responds by setting $\theta_2 = \theta_{MP+}$. Since the quality is infinitesimally close (firms are not differentiated), the prices in stage 2 will be zero for both firms, $s_1 = 0$, $s_2 = 0$ (as per Equations (3.22)-(3.23)). Firm 2 with higher quality but same price will capture the full market and the full advertisement revenue, $M(\beta\delta - c)$,[30] but will have no subscription revenue. Its profit function is given by:

$$\Pi_{2d} = M\,(\beta\,\delta - c) - \alpha\,\theta_{MP}^2 = M\,\phi + M\,c\,(e^{\sigma^2} - 1) - \alpha\,\theta_{MP}^2 \tag{3.34}$$

The indifference point for Firm 2 to choose between $\theta_2 = 0$ and $\theta_2 = \theta_{MP+}$ is given by the solution of following equation:

$$F(\phi) = \Pi_{2d} - \Pi_{2c} = M\,\phi\,e^{\sigma^2} + M\,c\,(e^{\sigma^2} - 1) - \alpha\,\theta_{MP}^2 - \frac{M}{2}\,(1 - \frac{\phi}{\ }\kappa\,\theta_{MP})\,\phi \equiv 0 \tag{3.35}$$

We verify that $F(\phi)$ is a continuous and strictly increasing function of ϕ if $\phi \geq 0$,[31] and $F(\frac{M\,\kappa^2}{12\sqrt{3}\,\alpha}) > 0$.[32] Therefore, (3.35) will have a non-negative solution iff

$$F(0) \leq 0 \rightarrow c\,(e^{\sigma^2} - 1) \leq \frac{M\,\kappa^2}{64\alpha}$$

We introduce few additional parameters ϕ_2 and σ_c, as follows:

Definition 3.7. *Define ϕ_2 such that $F(\phi_2) = 0$, which means that if $\phi = \phi_2$ and the Firm 1 chooses $\theta_1 = \theta_{MP}$ then Firm 2 is indifferent between choosing $\theta_2 = 0$ and $\theta_2 = \theta_{MP+}$.*

Lemma 3.10. *The necessary condition for type C Equilibrium is $\phi \in [0, \phi_2]$ and this interval is non-empty iff $c(e^{\sigma^2} - 1) \leq \frac{M\,\kappa^2}{64\,\alpha}$.*

Proof follows from the facts: a) Type C equilibrium does not exist if $\phi < 0$, as Firm 2 will have negative profit and b) if $\phi > \phi_2$, $\theta_2 = 0$ is not the best response of Firm 2 when $\theta_1 = \theta_{MP}$. In addition, $\phi_2 \geq 0$ iff $c\,(e^{\sigma^2} - 1) \leq \frac{M\,\kappa^2}{64\,\alpha}$.

[30] Note that δ is expected income per consumer.

[31] $\frac{dF}{d\phi} = Me^{\sigma^2} - \frac{M}{2}(1 - \frac{2\phi}{\theta_{MP}}) - (2\alpha\theta_{MP} + \frac{M}{2}(\frac{\phi}{\kappa\,\theta_{MP}})^2)\frac{d\theta_{MP}}{d\phi} > 0$ because $\frac{d\theta_{MP}}{d\phi} \leq 0$, when $\phi \geq 0$ (Lemma 3.1).

[32] If $\phi = \frac{M\kappa^2}{12\sqrt{3}\alpha}$ then $\theta_{MP} = \frac{M\kappa}{12\alpha}$ (see Appendix 3.A.3) and we can easily verify that $F > 0$.

The condition outlined in Lemma 3.10 is not sufficient for establishing a Type C equilibrium, as Firm 2's best response might be a strictly positive θ_2 when Firm 1 chooses $\theta_1 = \theta_{MP}$, which is a Type B equilibrium. Conversely, the presence of an interior solution does not necessarily lead to a Type B equilibrium, as Firm 2 may find a profitable deviation to $\theta_2 = 0$, thereby sustaining a Type C equilibrium. This condition is identified in Lemma 3.11 (see Appendix 3.A.3 for proof).

Lemma 3.11. *There exists a critical $\phi_1 \in (0, \phi_{ir})$ such that for $\phi \in (\phi_1, \phi_2]$ there exists a unique Type C equilibrium, and for $\phi \in (\phi_0, \phi_1)$ there exist a unique Type B equilibrium.*

Note that ϕ_0 is cutoff below which only one firm enters (Type A equilibrium). Also ϕ_2 could be lower than ϕ_1, making the interval $\phi \in (\phi_1, \phi_2]$ empty. If $\phi_2 \geq \phi_{ir}$, then there is a non-empty interval $[\phi_{ir}, \phi_2]$ where a Type C equilibrium is guaranteed to occur. From the implicit function theorem on $F(\phi_2) = 0$, we get that ϕ_2 is decreasing in σ. We define a critical σ_c at which $\phi_2 = \phi_{ir}$

Definition 3.8. *Define $\sigma_c : (c, \mu, \alpha) \to \mathbb{R}_{++}$ such that if $\sigma = \sigma_c$, then $\phi_2 = \phi_{ir}$.*

Next we make the following assumption so that the interval $[\phi_{ir}, \phi_2]$ is not empty, which ensures that Type C equilibrium will exist for some ϕ. Later we highlight the implications when this assumption does not hold.

Assumption 3.1. *$\sigma < \sigma_c$ so that $\phi_2 > \phi_{ir}$*

3.6.3 Type D Equilibrium

Now we consider the equilibrium characteristics when the advertisement intensity is sufficiently high, i.e. $\phi > \phi_2$, so that Type C equilibrium is not sustained (Lemma 3.10).

If $\phi > \phi_2$ Firm 2 can contest the leadership of Firm 1. It can get higher profit by marginally exceeding the quality of Firm 1 when Firm 1 sets $\theta_1 = \theta_{MP}$, in which case Firm 1 will make negative profit. Anticipating this, Firm 1 will set the quality level high enough that Firm 2 does not find profitable to adopt such strategy. θ_c as specified in Definition 3.5 is this quality level of Firm 1 that makes Firm 2 indifferent between choosing the strategy of maximal differentiation, $\theta_2 = 0$, and maximal competition $\theta_2 = \theta_{c+}$. θ_c is implicitly defined by (3.36):

$$G(\theta_c(\phi)) = \underbrace{M\,\phi\, e^{\sigma^2} + M\,c\,(e^{\sigma^2} - 1) - \alpha\,\theta_c^2}_{\Pi_2 \text{ if } \theta_2=\theta_{c+} \text{ and } \theta_1=\theta_c} - \underbrace{\frac{M}{2}\,(1 - \frac{\phi}{\kappa\,\theta_c})\,\phi}_{\Pi_2 \text{ if } \theta_2=0 \text{ and } \theta_1=\theta_c} \equiv 0 \tag{3.36}$$

We can easily verify that θ_c is increasing in ϕ and that $\theta_c(\phi_2) = \theta_{MP}$.

Lemma 3.12. *There exist a unique type D equilibrium if $\phi > \phi_2$. In this equilibrium, Firm 1 sets its quality θ_c that increases with ϕ and Firm 2 sets its quality to zero.*

Note: The proof of all lemmas under duopoly is provided in Appendix 3.A.3.

3.6.4 Duopoly Market Configurations

Proposition 3.3 states our main result, which includes all types of possible market configurations. It uses the parameters: $\beta_0 \equiv \phi^{-1}(\phi_0)$, $\beta_1 \equiv \phi^{-1}(\phi_1)$, $\beta_2 \equiv \phi^{-1}(\phi_2)$. The proof follows directly from Lemmas 3.5, 3.6, 3.11, and 3.12.

Proposition 3.3. *There exist $(\beta_0, \beta_1, \beta_2)$ with $\underline{\beta} < \beta_0 < \beta_1 < \beta_2 < \overline{\beta}$ such that*

a *If $\beta \in (\underline{\beta}, \beta_0)$, then a unique Type A equilibrium exists. One firm enters, setting the monopolistic price and quality as given by (3.12) and (3.11), and the market remains uncovered.*

b *If $\beta_0 < \beta < \beta_1$, then a unique Type B equilibrium exists. Both firms set positive and differentiated qualities, and the quality differential increases with β if $\phi(\beta) \geq 0$. The market remains uncovered.*

c *If $\beta_1 < \beta \leq \beta_2$, then a unique Type C equilibrium exists. The high-quality firm sets monopolistic price and quality levels, while the low-quality firm offers a free product of minimum quality. The market is fully covered.*

d *If $\beta > \beta_2$, then a unique Type D equilibrium exists. The high-quality firm chooses a premium product with quality higher than the monopolistic level, which increases with β, while the low-quality firm provides a free product of minimum quality. The market is covered.*

Figure 3.3 depicts endogenous parameters under duopoly graphically.

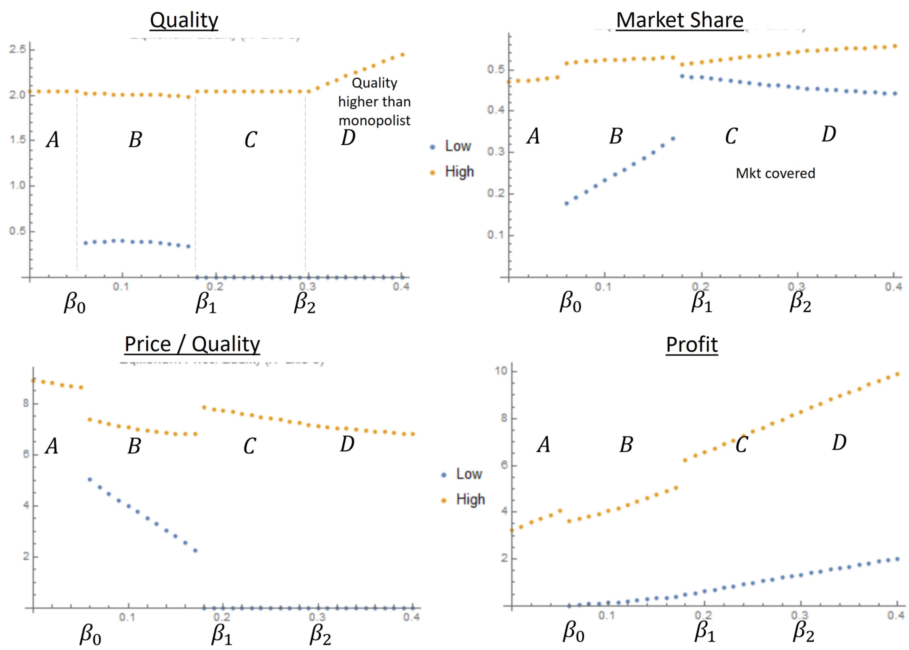


Figure 3.3: Quality, Market Share, Price and Profit of two firms under Duopoly

If the advertisement level is low, $\beta < \beta_0$, the market does not support two players and becomes a natural monopoly with a higher price-to-quality ratio. The market is partially covered, as many consumers with a low value for quality content do not subscribe. Note that β_0 could be zero when c is low enough or μ or M is high enough, in which case the Type A equilibrium does not exist.

As the advertisement level increases, the market transitions from Type A to Type B, where both players enter and compete. This competition lowers the price-to-quality ratio of the high-quality firm, while the low-quality firm attracts new subscribers at the lower end. Both higher market coverage and lower price-to-quality ratios increase consumer surplus. Although market coverage expands significantly, the market still remains partially covered. This expansion is primarily driven by the low-quality firm, which fills a product gap at the lower end. The differentiation between the two firms increases with advertisement as the low-quality firm strives to capture the niche lower-end market. Higher advertisement increases the value of marginal consumers, and the low-quality firm lowers its price-to-quality ratio to acquire these customers. However, it also reduces quality to lessen competition. Thus, while advertisement enables higher market coverage and lower prices, it also leads to a reduction in quality.

With a further increase in the advertisement level, the market transitions from Type B to Type C, where the second firm enters offering a free product of the lowest quality. As advertisement increases, the low-quality firm faces a tension between subscription revenue and advertisement revenue. When β increases above β_1, the potential advertisement revenue from non-subscribers becomes more valuable than the subscription revenue from its current subscribers. As a result, this firm switches to a free product to capture these non-subscribers and foregoes all subscription revenue. Although the market is fully covered — enabling advertisers to reach all consumer — consumer surplus decreases because the low-quality firm provides the lowest quality product, yielding zero surplus to its consumers. Additionally, competition reduces, and the high-quality firm behaves like a monopolist with a higher price-to-quality ratio, which also reduces consumer surplus. As a result consumer surplus becomes non-monotonic in advertisement level β, as illustrated in Figure 3.4.

With very high advertisement levels, i.e., $\beta > \beta_2$, the low-quality firm can contest the high-quality firm's leadership if the high-quality firm continues to set the monopolist level quality. It becomes profitable for the low-quality firm to marginally exceed the monopolist level quality. This is because advertisement revenue is sufficient to cover the cost of high quality, even after losing subscription revenue due to aggressive price competition. This compels the high-quality firm to enhance its quality beyond the monopolistic level to protect its market share. As advertisement levels increase, so does competitive pressure, leading to higher quality and a lower price-to-quality ratio (i.e., better reach) for the premium product. In other words, higher advertisement levels drive the premiumization of the market.

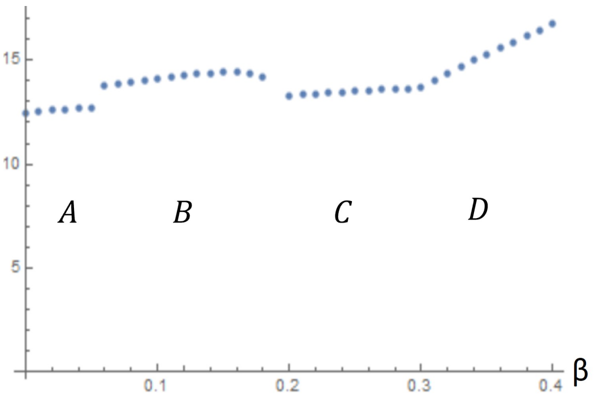


Figure 3.4: Consumer Surplus

Next we observe how equilibrium changes with exogenous parameter:

Proposition 3.4. *The equilibrium cut-off points β_0, β_1, β_2 changes when M, α, μ or σ increases, ceteris paribus, as shown in the table below:*

	β_0	β_1	β_2
$M \Uparrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
$\alpha \Uparrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$
$\mu \Uparrow$	$\Downarrow$	*depends on c*	$\Uparrow$
$\sigma \Uparrow$	$\Uparrow$	*depends on c*	$\Downarrow$
$c \Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$

Intuitively, an exogenous change that either increases subscription demand or reduces quality cost —such as an increase in M or μ, or a decrease in σ or α— increases the quality level of the high-quality firm across all β values, thus relaxing competition. This combined effect of increased demand (or reduced cost) and diminished competition boosts the profitability of both firms. The higher profitability of the low-quality firm reduces β_0, the threshold for non-negative profit for the low-quality firm. Further, as the high-quality firm raises its quality, β_2 also increases, making it more costly for the low-quality firm to contest its leadership. Meanwhile, β_1 marks the point at which the low-quality firm is indifferent between subscription earnings from current subscribers and potential advertising revenue from non-subscribers. Increases in M or decreases in α enhance the former without affecting the latter, thus raising β_1. However, changes in μ or σ impact both earnings, leading to an ambiguous effect on β_1. If the variable cost of circulation c is small, the first effect dominates and β_1 tends to increase with higher μ or lower σ.

Corollary 3.3. *For any given β, the quality produced by the high-quality firm increases when M or μ increases, and decreases when α or σ increases.*

The statement follows from Lemmas 3.1 and 3.7. The factors that raise marginal revenue or reduce marginal cost for the firm increase its quality.

Corollary 3.4. *If the median income (μ) or market size (M) is sufficiently low, a Type C equilibrium does not exist. The same is true if α or σ is sufficiently high.*

The above result highlights the situation when Assumption 3.1 is violated.[33] When μ or M decrease, or α or σ increase, the reduction in β_2 (Proposition 3.4) is much larger than that in β_1, hence the observed result. These changes lead to a reduction in the quality of the high-quality firm, which in turn makes it easier for the low-quality player to contest, thereby lowering β_2. Since the cost is convex in quality, these changes have a higher impact on β_2. Conversely, the impact on β_1 is smaller and arises through an indirect competitive effect on the low-quality firm.

3.6.5 Implications of Duopoly Result

The duopoly results highlight how equilibrium characteristics and market configurations change with the advertisement level (β). The tension between subscription and advertisement revenue leads to many corner solutions, creating different market configurations. Key aspects of the duopoly results include:

1. *Natural monopoly:* The market becomes a natural monopoly when the advertisement level is low relative to the marginal cost of printing and circulation. Smaller or lower-income markets are more likely to be monopolies. In section 3.8.2, we will show that a market can also become a natural monopoly at higher advertisement levels if consumer preferences are homogeneous.

2. *Market expansion:* Advertisements reduce the subscription price.[34] They also increase market coverage by attracting lower-end consumers through price reductions or the entry of a low-quality firm targeting these consumers.

3. *Concentrated market even when consumers are ad-neutral:* In our model, endogenous fixed investment in quality drives market concentration and advantages to the leading firm. This occurs irrespective of whether there is a positive feedback effect of advertisement on the consumer side.

4. *Non-monotonic consumer surplus:* Higher market coverage does not necessarily mean higher consumer surplus. Specifically, consumer surplus decreases when the market configuration changes from Type B to Type C (see Figure 3.4), even though advertisers reach more consumers. Policies that subsidize unit costs (e.g., subsidized

[33] A decrease in μ or M, or an increase in α, reduces σ_c.

[34] In two-sided markets, one side benefits if it has a positive effect on the other side.

postal rates) to increase consumer welfare can have the opposite effect if the market shifts from Type B to Type C. Anderson and Peitz (2020) termed such an effect as the "see-saw effect," where a change in market fundamentals causes one side to lose and the other to gain, highlighting the need for careful consideration in policy-making.

5. *Entry of Free newspapers:* Moderately high advertisement levels (i.e. $\beta \in (\beta_1, \beta_2)$) lead to the entry of second firm as a free newspaper with minimal quality (e.g., relying on wire reports instead of editorial staff). Gabszewicz et al. (2012) first explained this phenomenon. They noted that the rise of free newspapers like Metro or 20 Minutes in Europe and Boston Metro and Philadelphia Metro in the USA accompanied increased ad revenue or reduced printing costs.

6. *Premium products:* Higher advertisement levels (i.e., $\beta > \beta_2$ or Type D equilibrium) challenge the high-quality firm's leadership, prompting it to raise its quality beyond the monopolistic level, potentially reaching the social planner level in the limit. At the same time, the price-to-quality ratio decreases, reducing the market power of the high-quality firm. This aligns with empirical evidence from Angelucci and Cage (2019), which shows that newspapers reduce quality when advertisement revenue declines.

   However, a question could be raised about the feasibility of contesting leadership in the newspaper industry, given the effort required to build a consumer base and quality. In other words, is the threat of a challenge credible? This may require further empirical investigation. However, one piece of information that supports the feasibility of such a challenge is that most newspapers are owned by national-level chains, which often have leadership in one market while being the second player in another market.[35] These chains have the resources and technology to establish quality.

7. *Income inequality effect on firms*: Higher income inequality reduces the quality and profit of both firms, contrary to the effect of higher median income. With higher inequality, more consumers place lower value on quality, forcing firms to lower the price-to-quality ratio to retain marginal consumers, thereby lowering subscription revenue. However, the high-quality firm's advertisement revenue increases due to more affluent consumer base, making it more attractive for the low-quality firm to challenge the high-quality firm's leadership. This leads to a more likely occurrence of Type D equilibrium (i.e., β_2 decreases).

[35] For example, *The Times of India* and *Hindustan Times*, where they have challenged each other in the Delhi and Mumbai markets.

In a vertical differentiation model without advertisement (Gabszewicz and Thisse, 1979; Wauthy, 1996) we would not see Type C or Type D equilibrium. It is the advertisement revenue that makes free product attractive, or makes it profitable to contest the leadership of high-quality firm even if it means loosing all subscription revenue. The vertical differentiation model without advertisement also has a corner configuration with full market coverage (similar to Type C), but that type of equilibrium arises when consumer preference is homogeneous which we discuss in section 3.8.2. The Type D equilibrium also shows that as advertisement increases, the high-quality firm transitions from a niche to a mass market product by lowering its price and increasing its market share. This highlights how ad revenue influences the result found in Johnson and Myatt (2006).

3.7 Third player entry

In this section, we evaluate how the market configuration changes if a third firm is allowed to enter.[36] Suppose there are only three potential entrants, denoted as $k \in \{1, 2, 3\}$. Firms enter sequentially, with each firm making its quality choice, θ_k, before the next firm makes its entry decision. In the price stage, firms that have entered the market simultaneously set their subscription price s_k and p_k.

Since with three players there could be several different combinations of corner solutions, we focus our analysis on specific β intervals. The first interval is near the neighborhood of $\phi = 0$, where the advertisement intensity is not high, and the equilibrium is likely to be an interior solution (see Proposition 3.5). The second interval is when the advertisement intensity is large enough that the third firm can contest the two high-quality players. This includes all $\beta > \beta_1$ (see Proposition 3.6).

Proposition 3.5. *There exists $\epsilon > 0$ such that for all $\phi \in (-\epsilon, \epsilon)$, there exists a unique equilibrium in which Firm 3 enters with the lowest but positive quality. Relative to the duopoly equilibrium:*

- *The quality of Firm 1 and Firm 2 increases.*
- *Firm 1 and Firm 2 are placed closer to each other, i.e., $\frac{\theta_2}{\theta_1}$ increases.*
- *The prices of both Firm 1 and Firm 2 decrease, and their market coverage increases.*

The steps to prove Proposition 3.5 is given in Appendix 3.A.5. The entry of the third firm pushes up the quality of Firm 1 and Firm 2 as they try to reduce the business-stealing effect of Firm 3 by differentiation. However, firms are placed closer (quality ratios are higher) as the quality space is reduced, which leads to reduced prices and profits. Higher

[36] Finding equilibrium qualities and market structures under free entry is intractable in vertical differentiation models, so we draw some inferences from the entry of a third firm.

quality and lower prices increase the market coverage of Firm 1 and Firm 2. Additionally, Firm 3 attracts more lower-end consumers to the market, further increasing total market coverage. Total market coverage increase from 79% to 92% as shown below:

$$\text{Duopoly: } \frac{N_1}{M} = 52.6\% \; \frac{N_2}{M} = 26.3\%$$

$$\text{Three Firms: } \frac{N_1}{M} = 53.3\% \; \frac{N_2}{M} = 27.3\% \; \frac{N_3}{M} = 11.4\%$$

Next, we consider what happens if β (or corresponding ϕ) is such that Firm 3 does not enter with positive quality. Such a point will exist in the interval $(\phi^{-1}(0), \beta_1)$.[37] Let's denote the market share of Firm 1 and Firm 2 when they choose positive prices as N_1 and N_2, and as N_{1c} and N_{2c} when Firm 2 sets a zero price. Let A represent advertisement revenue net of marginal cost when a firm captures all demand. The expressions for these variables are given by (3.26), (3.27), (3.33), and (3.34).

$$\begin{aligned}
N_1 &= \frac{2}{4-\gamma} + \frac{1}{\kappa\,\theta_1\,(4-\gamma)}\,\phi \\
N_2 &= \frac{1}{4-\gamma} + \frac{1}{\kappa\,\theta_2\,(4-\gamma)}\,\phi \\
N_{1c} &= \frac{1}{2}(1 + \frac{1}{\kappa\,(\theta_1-\theta_2)}\,\phi) \\
N_{2c} &= \frac{1}{2}(1 - \frac{1}{\kappa\,(\theta_1-\theta_2)}\,\phi) \\
A &= M\,(\beta\,\delta - c)
\end{aligned}$$

Since Firm 3 can contest the duopoly leadership of both Firm 1 and Firm 2, causing them to lose their demand, Firm 1 and Firm 2 will protect their profits by ensuring the following constraints are met while making quality decisions:[38]

$$\begin{aligned}
&(I) && M\,N_{2c}\,\phi - \alpha\,\theta_2^2 - M\,(1-N_1-N_2)\,\phi \le 0 \\
&(II) && A - \alpha\,\theta_1^2 - M\,(1-N_1-N_2)\,\phi \le 0 \\
&(III) && M\,N_{2c}\,\phi - \alpha\,\theta_2^2 \ge 0
\end{aligned}$$

Constraint (I) implies that Firm 3 should not get higher profit if it contests Firm 2

[37] As β increases beyond $\phi^{-1}(0)$, Firm 2 decreases its price-to-quality ratio, leaving very little of the market uncovered, making it suboptimal for Firm 3 to enter, except with a free newspaper earning advertisement revenue from residual demand. This point is before β_1, when Firm 2, with a larger market share, finds it suboptimal to produce a quality newspaper.

[38] Note that when Firm 3 produces a free newspaper with zero quality, the market is effectively a duopoly.

by setting $\theta_3 = \theta_{2+}$.[39] Constraint (II) implies that Firm 3 should not get higher profit if it contests Firm 1 by setting $\theta_3 = \theta_{1+}$. Constraint (III) implies that Firm 2 gets non-negative profit when its price reaches zero and Firm 3 is driven out.[40] We solve the duopoly problem with these three constraints. Proposition 3.6 states the result.

Proposition 3.6. *Suppose three firms can enter and $\beta > \beta_1$. Then:*

- *Firm 2 will set a positive quality which increases with β*
- *Firm 1 will set higher quality and lower price, and will earn lower profit than in a duopoly, with its profit declining as β increases.*
- *If β is sufficiently high, Firm 2 will either earn zero profit or will not enter the market.*

Figure 3.5 depicts this result graphically. It shows how Firm 1 (high-quality) and Firm 2 (low-quality) product choices, prices, and profits change when Firm 3 can enter. 'D' refers to duopoly and 'T' refers three firms case. The yellow (blue) line represents Firm 1 (Firm 2) under duopoly, and the red (green) line represents Firm 1 (Firm 2) with three firms. The top left box of Figure 3.5 shows that the quality of both Firm 1 and Firm 2 increases. Under duopoly, Firm 2 was producing the lowest quality newspaper, but with Firm 3's entry, it differentiates by increasing quality. This also means that it does not necessarily set a zero price, as shown in the bottom left box. Firm 1 increases quality for two reasons: a) Firm 2 has higher quality, so the competitive response is to differentiate and reduce the business-stealing effect, b) To ensure that the Firm 3 does not contest its product choice. Firm 3 has a lower profit than Firm 2 under duopoly, meaning higher gains from setting $\theta_3 = \theta_{1+}$. Higher competition from Firm 2 also lowers the price of Firm 1, thus reducing its profit. Under duopoly, Firm 1's profit was rising with β, but with Firm 3's entry, it decreases with β.

Finally, when β is sufficiently high, Firm 2 sets a zero price while having positive quality.[41] In this case, there is no demand left for Firm 3, and it will exit the market. Firm 2 covers a large market but earns zero profit. If Firm 2 decides not to enter, there is a loss of consumer surplus as Firm 1 will only partially cover the market.

Note: Figure 3.5 does not show Firm 3's endogenous values to reduce clutter and to focus on the impact of Firm 3's entry on Firm 1 and Firm 2. Firm 3 will be a free newspaper with zero quality if $\beta > \beta_1$, and it will exit the market in the region where Firm 2 has zero price and profit.

[39] Firm 3's profit when it sets zero price and other firms set positive prices is $M(1 - N_1 - N_2)\phi$, which is residual demand times per unit contribution.

[40] If Firm 2 sets zero price and positive quality, then there is no residual demand for Firm 3.

[41] In this case all three constraints are binding

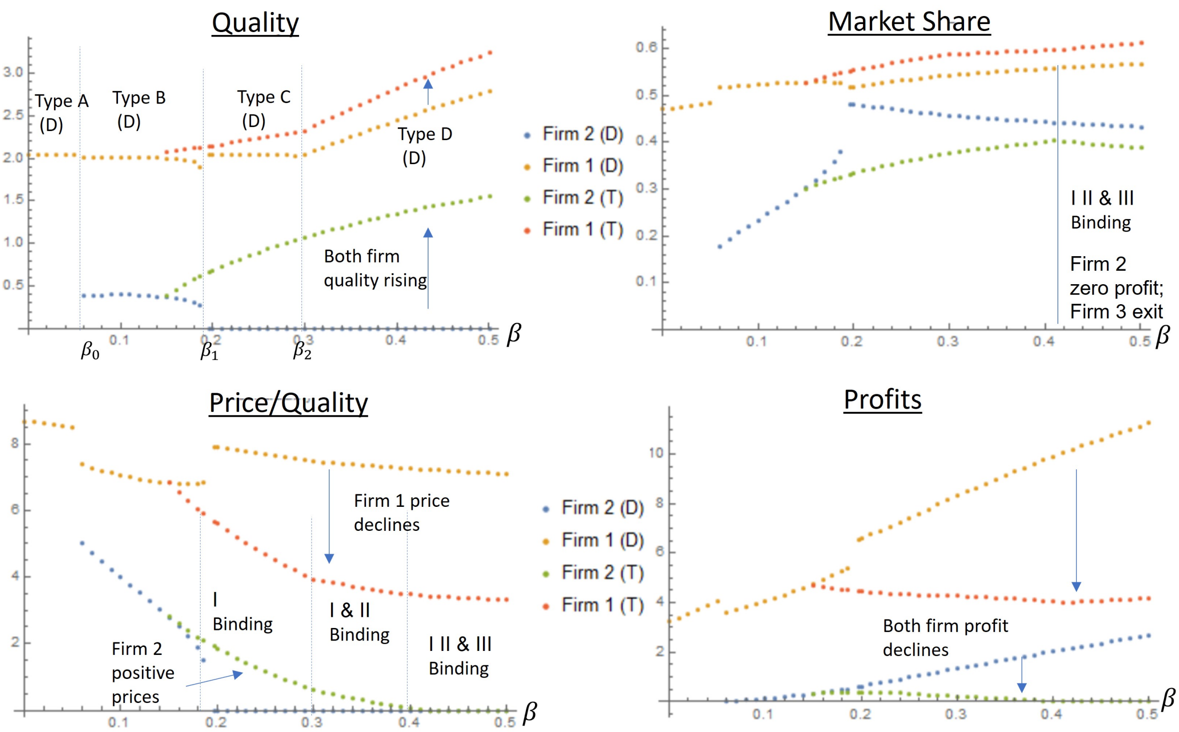


Figure 3.5: Market configurations with Three Firms

To summarize this section, the entry of the third firm has two key aspects. First, it expands the market where it was previously uncovered (Proposition 3.5). Even after the entry of three players, the market is not fully covered, allowing for further entry. When the advertisement does not provide a sufficiently positive per unit contribution (i.e., in the small neighborhood of $\phi = 0$), the tension between advertisement and subscription revenue is diminished, and we don't see corner solutions. The market will support multiple but finite numbers of vertically differentiated firms (Shaked and Sutton, 1983).

More importantly, the entry of the third firm increases competition and raises product qualities when the advertisement level is high (Proposition 3.6). Both Firm 1 and Firm 2 offer much higher quality newspapers at lower prices, thus increasing consumer surplus. Additionally, their profits decrease with the advertisement level. In contrast, their profits were increasing with advertisement under duopoly. In this sense, advertisements make the newspaper industry more efficient even when the market is concentrated with few players. This is consistent with the empirical evidence provided by Angelucci and Cage (2019) and Pattabhiramaiah (2014), which shows that with the decline in advertisement, firms raised their prices and lowered the quality. However, it is of empirical importance to verify if the profits of these firms also increased when the advertisement revenue declined. This is similar to Donnenfeld and Weber (1995)'s finding that under vertical differentiation, product competition among duopoly incumbents leads to entry deterrence.

3.8 Robustness

In this section, we consider the robustness of our duopoly results under different assumptions such as: a) Firms make entry decisions and quality decisions simultaneously; b) The distribution of consumer preferences for reading is more homogeneous; c) Consumers are not ad-neutral. We analyze our results with these new assumptions only with respect to duopoly and, in some cases, monopoly, but not for the three-firm scenario to reduce complexity. However, it can be easily inferred that none of these assumptions change the result for three-firm case except when consumer preferences are homogeneous, in which case Type D equilibrium could vanish.

3.8.1 Duopoly with Simultaneous Entry

We change the timing of the game so that the firms enter and choose quality simultaneously:

> *Stage 1:* Both firms make entry decisions and simultaneously choose the quality (θ) of their own product.
>
> *Stage 2:* Firms simultaneously set subscription price (s) and advertisement price (p).

The solution of the simultaneous entry model is detailed in Appendix 3.A.4. Lemmas 3.13 summarizes how a Type B equilibrium under simultaneous entry compares to that under sequential entry, and Lemma 3.15 shows that no Type D equilibrium exist in simultaneous entry model. There are no changes in Type A and Type C equilibria, as in these cases the high-quality firm behaves like a monopolist. We continue to maintain Assumption 3.1 so that $\phi_2 > \phi_{ir}$, which guarantees the existence of Type C equilibrium.

Lemma 3.13. *Suppose a Type B equilibrium exists for a given ϕ. In this equilibrium:*

1. *The high-quality firm sets a higher quality and price-to-quality ratio but has lower market coverage and profit compared to those under sequential entry. This quality is higher than the monopolist level.*
2. *The low-quality firm sets a higher quality and price-to-quality ratio, but earns higher profits despite lower market shares compared to those under sequential entry.*
3. *The quality ratio $\gamma \equiv \frac{\theta_2}{\theta_1}$ is lower than that in sequential entry/*

The competition reduces under simultaneous entry as firms are located farther (lower γ), and therefore both firms achieve a higher price-to-quality ratio, which results in lower market coverage. Under sequential entry, the high-quality firm lower quality to crowd out the quality space (closer substitute) of the low-quality firm, making its entry less profitable. This is reversed in simultaneous entry, and the high-quality firm sets higher quality, even higher than the monopoly, to distance itself from the entrant. This results in higher profit for the low-quality firm, but the profit of the high-quality firm reduces as it loses first mover advantage. Simultaneous entry also lowers consumer surplus due to both higher price and lower market coverage. This comparison is exactly similar to the comparison between the Cournot equilibrium (simultaneous) and the Stackelberg equilibrium (sequential). Next we evaluate how the cutoff points ϕ_0 and ϕ_1 change relative to that of sequential entry.

Lemma 3.14. *There exist critical cut-off points ϕ_0 and ϕ_1 such that a unique Type B equilibrium exists if $\phi \in (\phi_0, \phi_1)$. Further, ϕ_0 is lower and ϕ_1 is higher than those in the sequential entry model.*

Notice that the length of the interval (ϕ_0, ϕ_1) increases from both sides because the lower competition results in higher profit for the low-quality firm. Conversely, this implies that the leading firm in sequential entry is able to deter the entrant for some advertisement level. The cut-off point ϕ_2 does not change between the two models as this point depends on the θ_{MP}.

Lemma 3.15. *There does not exist any pure strategy equilibrium if $\phi > \phi_2$.*

The proof of Lemma 3.15 is provided in Appendix 3.A.4. Intuitively, when the advertisement intensity exceeds ϕ_2, the vertical differentiation strategy breaks, as both firms compete aggressively to capture the full advertisement revenue, each setting quality levels marginally higher than the other's. This intense competition results in the absence of a pure strategy equilibrium, similar to the one observed in the Hotelling model (D'Aspremont et al., 1979).

Proposition 3.7 outlines the duopoly market configurations under simultaneous entry. The proof is derived directly from Lemmas 3.5, 3.11,3.14 and 3.15. We define: $\beta_0 \equiv \phi^{-1}(\phi_0)$, $\beta_1 \equiv \phi^{-1}(\phi_1)$, $\beta_2 \equiv \phi^{-1}(\phi_2)$.

Proposition 3.7. *There exist $(\beta_0, \beta_1, \beta_2)$ with $\underline{\beta} < \beta_0 < \beta_1 < \beta_2 < \overline{\beta}$ such that*

a *If $\beta \in (\underline{\beta}, \beta_0)$ then unique equilibrium of type A exist. One firm enters and sets the monopolist price and quality as given by (3.12) and (3.11) and the market remains uncovered.*

b *If $\beta_0 < \beta < \beta_1$ then unique equilibrium of type B exist. Both firm set positive and differentiated quality and the quality differential increases with β if $\phi(\beta) \geq 0$. Market remains uncovered.*

c *If $\beta_1 < \beta \leq \beta_2$ then unique equilibrium of type C exist. The high quality firm sets the monopolist level price and quality, and the low quality firm provides a free product with minimum quality. Market is fully covered.*

d *There does not exist any pure strategy equilibrium if $\beta > \beta_2$.*

Note: In the small right-side neighborhood of β_1, there may not exist an equilibrium as firms switch between Type B and Type C configuration (see Appendix 3.A.4 for explanation).

Figure 3.6 graphically depicts the endogenous parameters across all values of β under a duopoly with simultaneous entry.

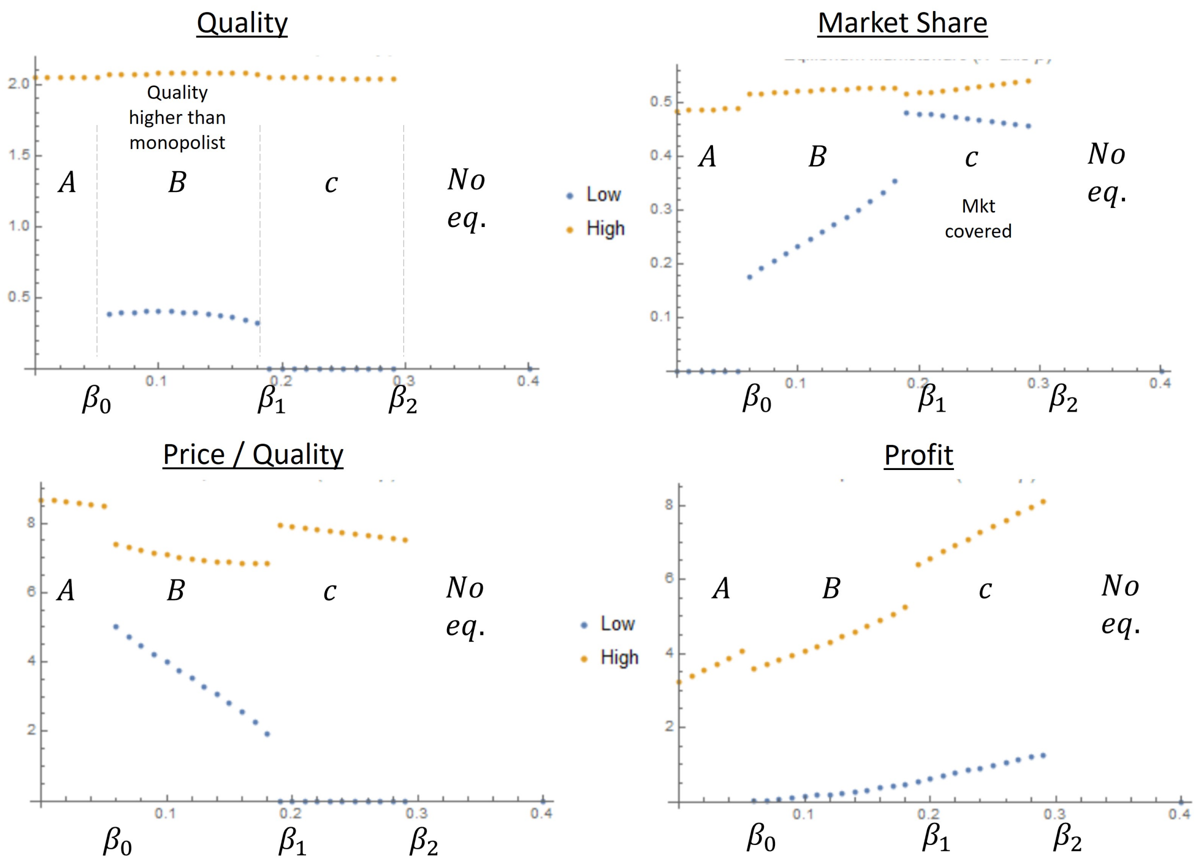


Figure 3.6: Quality, Market Share, Price and Profit of two firms under Simultaneous Entry Duopoly

To summarize, the simultaneous entry model differs from the sequential entry model in three aspects. First, under the Type B equilibrium (i.e., $\beta \in (\beta_0, \beta_1)$), there is less competition in simultaneous entry. As a result, both firms raise their price-to-quality ratio, which reduces consumer surplus and market coverage. This lower competition benefits the low-quality firms, but the high-quality firm, losing its first-mover advantage, earns lower profits. Second, under sequential entry, the first-mover firm deters entry for some parameter values close to β_0. This deterrence is absent in simultaneous entry, allowing some markets to transition from Type A to Type B, which has a higher market coverage and consumer surplus. Finally, the simultaneous entry model lacks a pure strategy equilibrium for $\beta > \beta_2$ and in the vicinity of β_1, which is not an issue under sequential entry. Shaked and Sutton (1987) pointed out that in a vertical differentiation model, if the price equilibrium exist on the second stage then sequential entry guarantees the existence of a pure strategy equilibrium, while such existence problem may arise on the product choice stage under simultaneous entry.

3.8.2 Duopoly and consumer preference heterogeneity

In this section, we parameterize consumer preferences to reflect varying degrees of heterogeneity (or conversely, homogeneity). Suppose the consumer's preference for quality content, v, follows a uniform distribution $v \sim U(b-1, b)$ where $1 \leq b \leq 2$. The ratio $\frac{b}{b-1}$ measures the heterogeneity across consumers, and it decreases when b increases. The lower the b, the more heterogeneous is the preference. In the previous section, we assumed $b = 1$, which implies maximum heterogeneity.

Our findings in this section demonstrate that as consumer preferences become more homogeneous (i.e., b increases), the market becomes more concentrated. Specifically, the high-quality firm becomes more dominant and captures a larger share of the market. As b approaches 2, the market evolves into a natural monopoly, irrespective of the advertisement level. This is a standard result in the vertical differentiation literature (Gabszewicz and Thisse, 1979; Wauthy, 1996). Wauthy (1996) used a duopoly model under vertical differentiation and showed that as consumer preferences become more homogeneous, the market transitions from an uncovered configuration to a covered configuration and finally to a monopoly. Our model validates the same result, even in the presence of advertising. Our results are also consistent with the findings of Johnson and Myatt (2006), which show that a monopolist will offer a mass market product when consumers are homogeneous (higher b) and a niche market product when consumers are heterogeneous (lower b).Higher advertisement levels facilitate the market's transition from uncovered (Type B) to covered configuration (Type C) at a lower level of homogeneity. In addition, when preferences are sufficiently homogeneous, we observe only the Type C market configuration. In contrast, with the heterogeneous preferences of our base model, we observed four distinct types of market configurations, depending on the advertisement level.

We first examine how a monopolist's behavior changes as b increases, since this factor plays a crucial role in determining the structure of the duopoly.

Monopoly

The demand function when $v \sim U(b, b-1)$:

$$N(s,\theta) = M\,(\mathbf{b} - \frac{s}{\kappa\,\theta}) \qquad \text{if } (\boldsymbol{b}-1)\,\kappa\,\theta \leq s \leq \mathbf{b}\,\kappa\,\theta \tag{3.37}$$

(3.37) indicates that the demand curve shifts outwards when b increases. If the monopolist problem has an interior solution, i.e. market is uncovered, the quality, θ_{MP} is given by the unique solution of (3.38).

$$8\alpha\,\kappa\,\theta^3 - M\,\boldsymbol{b}^2\,\kappa^2\,\theta^2 + M\,\phi^2 = 0 \ \text{ subject to } \ \theta > \frac{M\,\kappa\boldsymbol{b}^2}{12\alpha} \tag{3.38}$$

The subscription price and the profit function when there is an interior solution:

$$s_{MP} = \frac{1}{2}\boldsymbol{b}\,\kappa\,\theta_{MP} - \frac{1}{2}\,\phi \tag{3.39}$$

$$\Pi_{MP} = \frac{M}{4\kappa\,\theta_{MP}}\,(\boldsymbol{b}\,\kappa\,\theta_{MP} + \phi)^2 + M\,\boldsymbol{b}\,\beta\,(\delta-\kappa) - \alpha\,\theta_{MP}^2 \tag{3.40}$$

If the monopolist problem has a corner solution then

$$\theta_{MP} = (b-1)\,\frac{M\,\kappa}{2\alpha} \tag{3.41}$$

$$s_{MP} = (b-1)\,\kappa\,\theta_{MP} \tag{3.42}$$

$$\Pi_{MP} = (b-1)^2\,\frac{M^2\,\kappa^2}{4\,\alpha} + M\,b\,\beta\,(\delta-\kappa) + M\,\phi \tag{3.43}$$

The cutoff advertisement level when the monopolist does not serve the market, $\underline{\beta}$, and when it opts for a corner solution, $\overline{\beta}$, are given by:

$$\phi(\underline{\beta}) = -\boldsymbol{b}^3\,\frac{M\,\kappa^2}{27\,\alpha} \qquad \phi(\overline{\beta}) = \rho(b)\,\frac{M\,\kappa^2}{27\,\alpha} \text{ where } \rho \text{ is strictly increasing in } b \text{ and } \rho \in [1, 3\sqrt{6}]$$

We can infer from the above equations that as b increases, quality, price-to-quality, and market coverage also increase. This is because a higher b causes the demand curve to shift outward. Consider the benchmark case where $\phi = 0$: $\theta_{MP} = b^2\,\frac{M\,\kappa}{8\,\alpha}$ increases quadratically with b, and the market coverage $\frac{N}{M} = \frac{1}{2}b$ increases linearly with b. The market is fully covered when $b = 2$. Additionally, the cutoff $\underline{\beta}$ decreases as b increases, enabling more markets with lower advertisement level to be served by a monopolist. Proposition 3.8 states this result.

Proposition 3.8. *As consumer preferences for quality content become more homogeneous (i.e. b increase), the market coverage by the monopolist also increases. The market becomes fully covered when $b = 2$. Furthermore, $\underline{\beta}$ decreases with b, enabling the monopolist to serve more markets that have lower advertisement levels.*

Also note that the corner solution has a positive quality and subscription revenue if $b > 1$. This occurs because a monopolist can cover the full market at a higher price. For $b = 2$, the monopolist chooses the same quality across all values of β. This implies that as consumer preferences become more homogeneous, the monopolist does not significantly reduce quality when $\beta > \overline{\beta}$. Figure 3.7 displays monopolist quality and market coverage for three different levels of b.

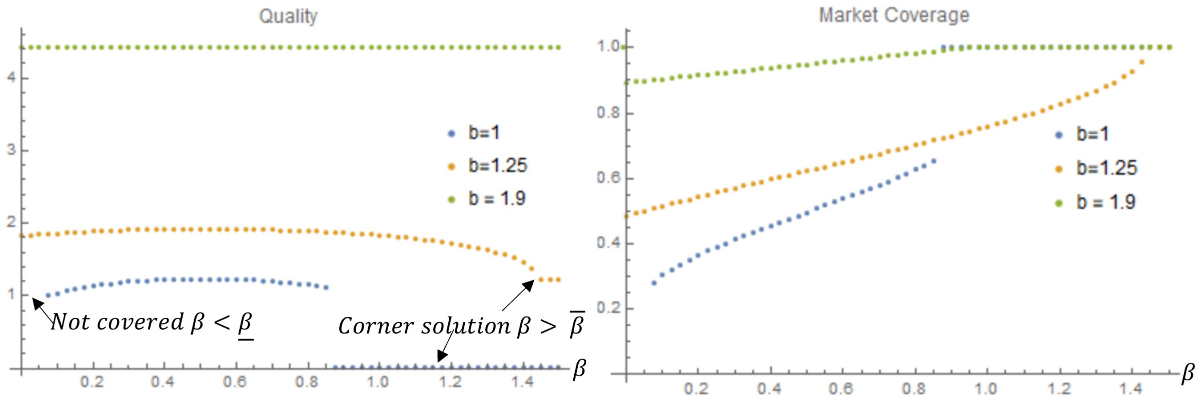


Figure 3.7: Monopolist's quality and market coverage for different levels of consumer preference heterogeneity(b)

As depicted in Figure 3.7, three distortions of the monopolist model from section 3.5 are partially addressed when consumer preferences become more homogeneous: a) market coverage increases; b) markets that are not served earlier are served when b increases; and c) the monopolist does not significantly reduce quality, even when the advertisement level is high.

Duopoly

The duopoly market also demonstrates that the dominance of the high-quality firm increases as consumer preference becomes more homogeneous (b increases), transitioning the market toward a natural monopoly when $b = 2$. All markets shift to Type C when b is sufficiently high,[42] irrespective of the advertisement level.

Type B equilibrium (uncovered market)

Under Type B equilibrium (interior solution), both firm enters and the market remain uncovered. The corresponding equations for demand, profit, market share, and price-to-quality are shown below. The parameter **b** is shown in the bold to highlight difference

[42]In Type C market, the high-quality firm chooses monopolist level quality while the low-quality firm serves the residual demand at lowest quality.

from the model in section 3.6.

$$N_1 = M\,(\mathbf{b} - \frac{s_1 - s_2}{\kappa\,(\theta_1 - \theta_2)}) \tag{3.44}$$

$$N_2 = M\,(\frac{s_1 - s_2}{\kappa\,(\theta_1 - \theta_2)} - \frac{s_2}{\kappa\,\theta_2}) \tag{3.45}$$

$$\Pi_1 = M\frac{1-\gamma}{\kappa\,\theta_1\,(4-\gamma)^2}(2\,\mathbf{b}\,\kappa\,\theta_1 + \phi)^2 + M\,\mathbf{b}\,\beta\,(\delta - \kappa) - \alpha\,\theta_1^2 \tag{3.46}$$

$$\Pi_2 = M\,\frac{1-\gamma}{\kappa\,\theta_2\,(4-\gamma)^2}(\mathbf{b}\,\kappa\,\theta_2 + 2\phi)^2 - \alpha\,\theta_2^2 \tag{3.47}$$

$$\frac{N_1}{M} = \frac{2\,\mathbf{b}}{4-\gamma} + \frac{1}{\kappa\,\theta_1\,(4-\gamma)}\,\phi \tag{3.48}$$

$$\frac{N_2}{M} = \frac{\mathbf{b}}{4-\gamma} + \frac{2}{\kappa\,\theta_2\,(4-\gamma)}\,\phi \tag{3.49}$$

$$\frac{s_1}{\theta_1} = 2\,\mathbf{b}\,\frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_1}\frac{3}{4-\gamma}\,\phi \tag{3.50}$$

$$\frac{s_2}{\theta_2} = \mathbf{b}\,\frac{k\,(1-\gamma)}{4-\gamma} - \frac{1}{\theta_2}\frac{2+\gamma}{4-\gamma}\,\phi \tag{3.51}$$

$$\text{where } \gamma = \frac{\theta_2}{\theta_1}\ , \kappa = e^{\mu - \frac{1}{2}\sigma^2}\ , \delta = e^{\mu + \frac{1}{2}\sigma^2}$$

We characterize the Type B equilibrium by solving the first-order conditions of both firms under sequential entry, using the exact same approach as detailed in Appendix 3.A.3. The result is stated in Lemmas 3.16. The proofs can be easily derived from Equations (3.44)-(3.51); however, we discuss the intuitions behind the results.

Lemma 3.16. *Suppose a Type B equilibrium exists for a given β and b. If b increases marginally, indicating more homogeneous consumer preferences, then the quality, market share, price-to-quality and profit of both firm increases.*

The above results stem directly from the demand curve shifting outward due to an increase in b. These effects benefit both firms as the market is uncovered. However, once the market becomes fully covered, it transitions to a Type C equilibrium, which involves different dynamics as stated in Lemma 3.17.

Type C equilibrium (corner configuration)

Under a Type C equilibrium, the market is covered, so any increase in the market share of the high-quality firm will lower the market share of the low-quality firm. In other words, the low-quality firm serves the residual demand. This implies that higher demand does not necessarily benefit the low-quality firm, as stated in Lemma 3.17. We also modify the definition of Type C, as defined in Definition 3.4, to allow the low-quality firm to set non-zero quality. The equations for price, market share, and profit under Type

C are given by:

$$s_1 = \frac{1}{2}\mathbf{b}\,\kappa\,\theta_1 - \frac{1}{2}\kappa\,\theta_2 - \frac{1}{2}\phi \quad (3.52)$$

$$s_2 = (\mathbf{b}-1)\,\kappa\,\theta_2 \quad (3.53)$$

$$\frac{N_1}{M} = \frac{\mathbf{b}\,\theta_1 - \theta_2}{2(\theta_1-\theta_2)} + \frac{1}{2\kappa\,(\theta_1-\theta_2)}\,\phi \quad (3.54)$$

$$\frac{N_2}{M} = \frac{(2-\mathbf{b})\,\theta_1 - \theta_2}{2(\theta_1-\theta_2)} - \frac{1}{2\kappa\,(\theta_1-\theta_2)}\,\phi \quad (3.55)$$

$$\Pi_1 = \frac{M}{4\kappa\,(\theta_1-\theta_2)}(\kappa\,(\mathbf{b}\theta_1-\theta_2)+\phi)^2 + M\,\mathbf{b}\,\beta\,(\delta-\kappa) - \alpha\,\theta_1^2 \quad (3.56)$$

$$\Pi_2 = M\underbrace{\left(\frac{\kappa\,((2-\mathbf{b})\,\theta_1-\theta_2)-\phi}{2\,\kappa\,(\theta_1-\theta_2)}\right)}_{\text{Market share}}\underbrace{((\mathbf{b}-1)\,\kappa\,\theta_2+\phi)}_{\text{Revenue per subscriber}} - \alpha\,\theta_2^2 \quad (3.57)$$

Lemma 3.17. *Suppose a Type C equilibrium exists for a given β and b. Under this equilibrium:*

a) The market share of the high-quality firm increases with b, while the market share of the low-quality firm decreases with b.

b) The quality and the profit of high-quality firm increase with b.

c) The quality and profit of the low-quality firm are non-monotonic in b, initially increasing and then decreasing.

d) When $b = 2$, the low-quality firm is driven out of the market.

As observed in Type B, the quality, profit, and market share of the high-quality firm increase with b due to the outward shift of the demand curve. However, since the low-quality firm serves the residual demand, its market share decreases as the market share of the high-quality firm increases (see (3.54)-(3.55)). The quality and profit of the low-quality firm are concave and non-monotonic in b due to two opposing effects when b increases:

a) The market share decreases (first term in (3.57)), which reduces revenue and lowers both the equilibrium quality and profit.

b) Revenue per subscriber increases (second term in (3.57)) as the valuation of the lowest-value customer rises. This raises both equilibrium quality and profit.

The latter effect predominates when b is small, while the former effect prevails when b is large, making the quality and profit non-monotonic. When $b = 2$, the high-quality firm fully covers the market, leaving no residual demand for the low-quality firm. Proposition 3.9 states our main result.

Proposition 3.9. *For a given β:*

a) *If the market is Type A at $b = 1$, then there exist two thresholds b_0 and b_1 with $1 < b_0 < b_1 < 2$ such that the market transitions to Type B when $b > b_0$ and to Type C when $b > b_1$.*

b) *If the market is Type B at $b = 1$, then there exists a threshold $b_1 \in (1, 2)$ such that the market transitions to Type C when $b > b_1$.*

c) *If the market is Type D at $b = 1$, then there exists a threshold $b_2 \in (1, 2)$ such that the market transitions to Type C when $b > b_2$.*

d) *If $b = 2$, the market becomes a natural monopoly.*

Further, b_0 and b_1 are decreasing in β, and b_2 is increasing in β.

If b increases, then the demand curve shifts outward, which in turn increases the profit of both firms, including that of a potential entrant, provided that the market is not uncovered (see Lemma 3.16). b_0 represents the threshold b at which the entrant just starts making a positive profit, marking the transition from a Type A to a Type B market. If b continues to increase, market coverage expands until b reaches the threshold b_1, at which point the market is just covered.

An increase in b raises θ_{MP}, the monopolist's quality level. This hardens the constraint for a Type D equilibrium, wherein the low-quality firm must make higher profit by marginally exceeding θ_{MP} when the high-quality firm chooses θ_{MP} (see Definition 3.7). When $b > b_2$, this constraint is violated, and the market moves to Type C.

When β increases and moves closer to the cut-off points β_0 (for Type A) or β_1 (for Type B), smaller increases in b are required for the transition, hence b_0 and b_1 decreases with β. Conversely, the transition from Type D to Type C requires a reduction in β to move closer to the cut-off point β_2, hence b_2 increases with β_2.

Figure 3.8 displays endogenous parameters when market transitions from Type A to Type B to Type C as b increases.

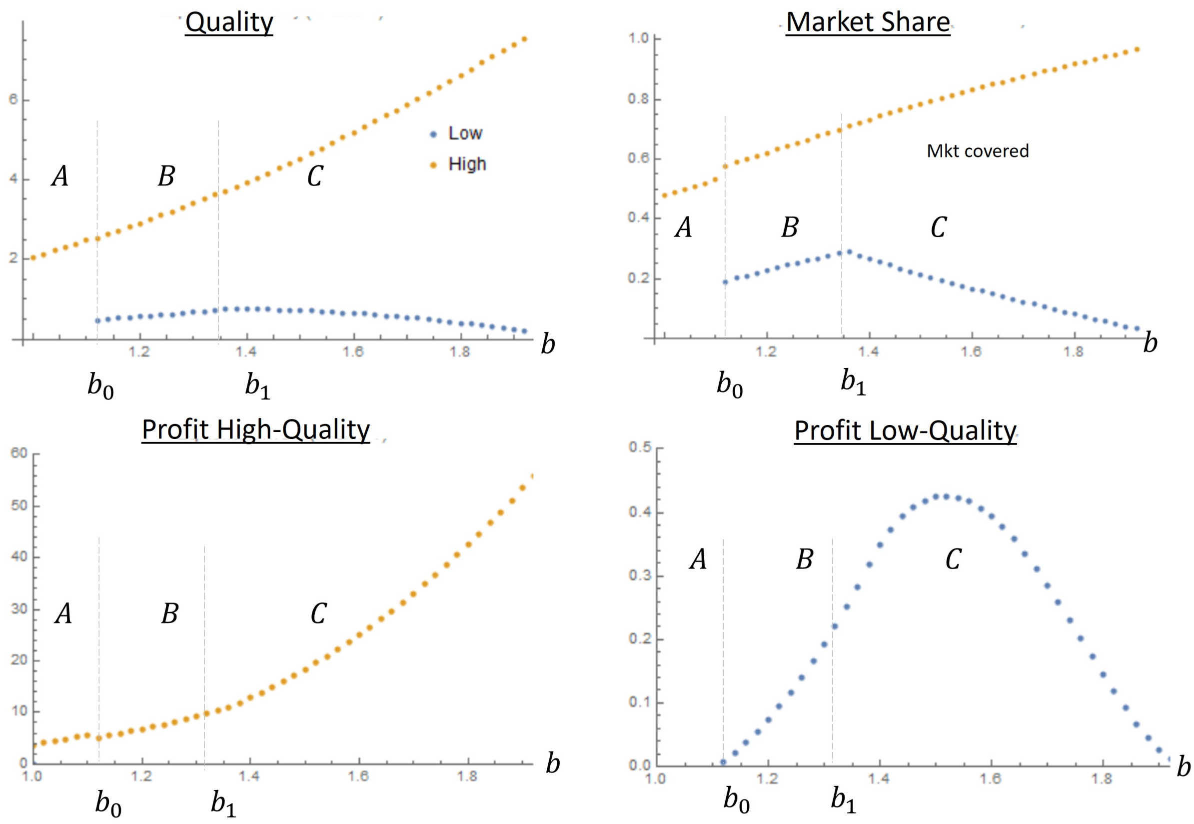


Figure 3.8: Quality, market share and profit of firms when b increases, showing market transitions from Type A to B to C

3.8.3 Consumers see advertisement as nuisance

Let's assume that consumers see advertisements as a nuisance and experience negative utility from them. This utility loss is proportional to the advertisement level, as shown below:

$$U_i = v_i\, Y_i\, \theta - \eta\, \beta - s \text{ where } \eta > 0 \tag{3.58}$$

We refer to the parameter η as the nuisance factor. We first summarize our findings of this section before detailing the model. The key results when η increases are:

a) Total market coverage decreases because consumers with lower utility from reading drop off from subscribing. Even in Type C or Type D markets, where the low-quality product is free, the market is not fully covered.

b) Profits for both firms decrease across all market configurations, with a larger impact on the low-quality firm. Marginal consumers of the low-quality firm are deciding between subscribing and not subscribing, while marginal consumers of the high-quality firm are deciding between the two firms' products, both of which include advertisements. The low-quality firm reduces its price-to-quality ratio to compensate for the utility loss and retain some consumers, prompting a competitive response from the high-quality firm to also lower price-to-quality ratio to retain its customers, leading to reduced profits for both firms. The lower profit of the low-profit firm also results in a lower β_0, as marginal firms exit the market.

c) The quality of both firms is higher under Type C and Type D configurations, which is counter-intuitive given the lower profits. According to the utility function (3.58), setting a zero quality is no longer optimal since demand drops to zero even when subscriptions are free. To maintain its customer base and earn advertisement revenue, the low-quality firm must raise its quality when η increases, triggering a competitive response from the high-quality firm to also increase quality. However, rising quality without subscription revenue (price is zero) disproportionately reduces the low-quality firm's profit under Type C and Type D.

d) The range of advertisement levels, (β_1, β_2), for which Type C configuration exists decreases because β_1 increases and β_2 decreases. Lower profits for the low-quality firm under Type C raise the cut-off point β_1 for switching to a corner solution. The decreased profit in Type C also relaxes the constraint for Type D equilibrium, making it more attractive for the low-quality firm to contest the leadership of high-quality firm, thus lowering β_2.

These results suggest that the high-quality firm becomes even more dominant relative to the low-quality firm.

Next, we detail the model with the modified utility function (3.58). Since all four types of market configurations are possible, we highlight the changes in each configuration.

Type A equilibrium (monopoly)

The demand function of monopolist will be given by:

$$N(s,\theta) = M\left(1 - \frac{s + \boldsymbol{\eta}\,\boldsymbol{\beta}}{\kappa\,\theta}\right) \qquad \text{if } 0 \leq s \leq \kappa\,\theta - \eta\,\beta \tag{3.59}$$

(3.59) indicates that the demand curve shifts inward when η increases. If the monopolist's problem has an interior solution, i.e., the market is uncovered, the quality, θ_{MP}, is given by the unique solution of: (3.38).

$$8\,\alpha\,\kappa\,\theta^3 - M\,\kappa^2\,\theta^2 + M\,(\phi - \eta\,\beta)^2 = 0 \;\text{ subject to }\; \theta > \frac{M\,\kappa}{12\,\alpha} \tag{3.60}$$

The subscription price and the profit function when there is an interior solution:

$$s_{MP} = \frac{1}{2}\kappa\,\theta_{MP} - \frac{1}{2}(\phi + \eta\,\beta) \tag{3.61}$$

$$\Pi_{MP} = \frac{M}{4\kappa\,\theta_{MP}}(\kappa\,\theta_{MP} + \phi - \boldsymbol{\eta}\,\boldsymbol{\beta})^2 + M\,\beta\,(\delta - \kappa) - \alpha\,\theta_{MP}^2 \tag{3.62}$$

If the monopolist's problem has a corner solution then

$$s_{MP} = 0 \tag{3.63}$$

$$\theta_{MP} = \sqrt[3]{\frac{M\,\phi\,\eta\,\beta}{2\,\alpha\,\kappa}} \tag{3.64}$$

$$\Pi_{MP} = M\,(1 - \frac{\eta\,\beta}{\kappa\,\theta_{MP}})\,\phi + M\,\beta(\delta - \kappa) - \alpha\,\theta_{MP}^2 \tag{3.65}$$

The cutoff advertisement level when the monopolist does not serve the market, $\underline{\beta}$, and when it opts for a corner solution, $\overline{\beta}$, are given by:

$$\phi(\underline{\beta}) = \eta\,\beta - \frac{M\,\kappa^2}{27\,\alpha} \qquad \phi(\overline{\beta}) = \eta\,\beta + \frac{M\,\kappa^2}{27\,\alpha}$$

We can make following inferences from the above equations:

a) As the nuisance factor η increases, price-to-quality ratio, profit, and market coverage decrease. This is the direct effect of the inward shift of the demand curve. Consumers need to be compensated for the loss of utility through price reduction and/or quality increase.

b) The market is not fully covered even when the subscription price is zero (see (3.65)) because consumers who derive very little utility from reading will not subscribe even free product.

c) The cutoff point $\underline{\beta}$ for not serving the market increases due to lower profit, as some marginally profitable markets will not be served.

d) The cutoff point for a corner solution $\overline{\beta}$ also increases because of the lower profit under corner solution due to both lower demand and the higher cost of quality (note: quality in a corner solution is not zero).

Note: Quality may increase or decrease with η depending on the advertisement level. It remains single-peaked with respect to β, with the peak at $\phi = \eta\,\beta$ (see (3.60)). With higher η, the peak shifts to the right, but the maximum quality level remains at $\frac{M\kappa}{8\alpha}$.

Type B (uncovered configuration) equilibrium Demand for firms are derived using the utility of indifferent customers and is shown below:

$$N_1 = M(1 - \frac{s_1 - s_2}{\kappa\,(\theta_1 - \theta_2)}) \tag{3.66}$$

$$N_2 = M(\frac{s_1 - s_2}{\kappa\,(\theta_1 - \theta_2)} - \frac{s_2 + \boldsymbol{\eta}\,\boldsymbol{\beta}}{\kappa\,\theta_2}) \tag{3.67}$$

Note that the demand for Firm 1 does not directly depend on η because the marginal consumers are deciding between the products of Firm 1 and Firm 2, and they incur this utility loss with both firms. In contrast, the marginal consumers for Firm 2 are deciding between subscribing and not subscribing. However, the strategic actions of Firm 2 may change the demand for Firm 1 and consequently its profit, price-to-quality ratio, and market coverage, as shown in equations (3.68)-(3.75).

$$s_1 = \frac{2\kappa\,\theta_1(\theta_1-\theta_2)}{4\theta_1-\theta_2} - \frac{3\theta_1}{4\theta_1-\theta_2}\,\phi - \boldsymbol{\frac{\theta_1-\theta_2}{4\theta_1-\theta_2}\,\eta\,\beta} \tag{3.68}$$

$$s_2 = \frac{\kappa\,\theta_2(\theta_1-\theta_2)}{4\theta_1-\theta_2} - \frac{2\theta_1+\theta_2}{4\theta_1-\theta_2}\,\phi - \boldsymbol{\frac{2(\theta_1-\theta_2)}{4\theta_1-\theta_2}\,\eta\,\beta} \tag{3.69}$$

$$\Pi_1 = M\frac{1-\gamma}{\kappa\,\theta_1(4-\gamma)^2}\,(2\kappa\,\theta_1 + (\phi-\boldsymbol{\eta\,\beta}))^2 + M\,\beta\,(\delta-\kappa) - \alpha\,\theta_1^2 \tag{3.70}$$

$$\Pi_2 = M\frac{1-\gamma}{\kappa\,\theta_2\,(4-\gamma)^2}\,(\kappa\,\theta_2 + 2(\phi-\boldsymbol{\eta\,\beta}))^2 - \alpha\,\theta_2^2 \tag{3.71}$$

$$\frac{N_1}{M} = \frac{2}{4-\gamma} + \frac{1}{\kappa\,\theta_1\,(4-\gamma)}\,(\phi-\boldsymbol{\eta\,\beta}) \tag{3.72}$$

$$\frac{N_2}{M} = \frac{1}{4-\gamma} + \frac{2}{\kappa\,\theta_2\,(4-\gamma)}\,(\phi-\boldsymbol{\eta\,\beta}) \tag{3.73}$$

$$\frac{s_1}{\theta_1} = 2\frac{\kappa\,(1-\gamma)}{4-\gamma} - \frac{1}{\theta_1}\frac{3}{4-\gamma}\,\phi - \boldsymbol{\frac{1}{\theta_1}\frac{1-\gamma}{4-\gamma}\,\eta\,\beta} \tag{3.74}$$

$$\frac{s_2}{\theta_2} = \frac{\kappa\,(1-\gamma)}{4-\gamma} - \frac{1}{\theta_2}\frac{2+\gamma}{4-\gamma}\phi - \boldsymbol{\frac{2}{\theta_1}\frac{1-\gamma}{4-\gamma}\,\eta\,\beta} \tag{3.75}$$

$$\text{where } \gamma = \frac{\theta_2}{\theta_1}\;,\; \kappa = e^{\mu-\frac{1}{2}\sigma^2}\;,\; \delta = e^{\mu+\frac{1}{2}\sigma^2}$$

We solve the first-order conditions of both firms under sequential entry and determine the equilibrium θ_1 and θ_2 and thereby prices and profits, using the exact same approach as detailed in Appendix 3.A.3. The result is stated in Lemmas 3.18.

Lemma 3.18. *Suppose the market is in a Type B equilibrium for a given exogenous parameter values. If the nuisance factor η increase, then price-to-quality, market coverage and profit of both firm decreases.*

As η decreases, the marginal customers of low-quality firm (Firm 2), who have a lower value for quality content, need to be compensated for the utility loss from advertisements either by reducing price or increasing quality (i.e., lowering the price-to-quality ratio), or they will not subscribe. Firm 2 lowers the price-to-quality ratio to retain some of these customers, depending on elasticity, but not all. The lower price and market share reduce the profit of Firm 2. As Firm 2 lowers its price, Firm 1 (high-quality firm) loses some of its consumers. To protect its customer base, Firm 1 also lowers its price-to-quality ratio, but to a smaller extent than Firm 2. This reduces Firm 1's profit and market coverage.

Type C (corner) market configuration

$$
\begin{aligned}
s_1 &= \frac{1}{2}\kappa\,(\theta_1-\theta_2)-\frac{1}{2}\phi\ ;\ s_2=0 && (3.76)\\
\frac{N_1}{M} &= \frac{1}{2}+\frac{1}{2\kappa\,(\theta_1-\theta_2)}\,\phi && (3.77)\\
\frac{N_2}{M} &= \frac{1}{2}-\frac{1}{2\kappa\,(\theta_1-\theta_2)}\,\phi-\frac{\boldsymbol{\eta\,\beta}}{\boldsymbol{\kappa\,\theta_2}} && (3.78)\\
\Pi_1 &= \frac{M}{4\kappa\,(\theta_1-\theta_2)}\,(\kappa(\theta_1-\theta_2)+\phi)^2+M\,\beta\,(\delta-\kappa)-\alpha\,\theta_1^2 && (3.79)\\
\Pi_2 &= M\,\underbrace{(\frac{1}{2}-\frac{\phi}{2\kappa\,(\theta_1-\theta_2)}-\frac{\boldsymbol{\eta\,\beta}}{\boldsymbol{\kappa\,\theta_2}})}_{\text{Market share}}\,\phi-\alpha\,\theta_2^2 && (3.80)
\end{aligned}
$$

Two important differences to note in the above equations compared to the Type C configuration in section 3.6 are: a) The market is never fully covered if $\eta > 0$, as the lowest value consumers ($\frac{\eta\,\beta}{\kappa\,\theta_2}$ term) will not subscribe even at a zero price; the higher the η, the greater the loss of subscribers. b) The low-quality firm will set a non-zero quality in Type C, and this quality increases with η.

Lemma 3.19. *Suppose the market is in a Type C equilibrium for given exogenous parameter values. If the nuisance factor η increases, then:*

a) The price-to-quality ratio and profit of both firms decrease.

b) The quality of both firms increases.

c) The market share of the high-quality firm increases, while that of the low-quality firm decreases.

As η increases, more and more consumers with a low value for quality content decide not to subscribe. Since the price cannot go below zero, the low-quality firm (Firm 2) raises its quality to retain some of these consumers. The higher cost of quality and a reduced consumer base decrease its profit. In a competitive response to Firm 2's action, Firm 1 lowers its price-to-quality ratio, partially by lowering the price and partially by raising quality to differentiate from the competition, thereby reducing its own profit.

Type D equilibrium

The quality set by the high-quality firm, θ_c, is implicitly defined by the equation:

$$
\underbrace{M\,(1-\frac{\eta\,\beta}{\kappa\,\theta_c})\,\phi+M\,\beta\,(\delta-\kappa)-\alpha\,\theta_c^2}_{\text{Firm 2 Profit if it marginaly exceeds }\theta_c}-\underbrace{\left[\frac{M}{2}\,(1-\frac{2\eta\,\beta}{\kappa\,\theta_2}-\frac{\phi}{\kappa\,(\theta_c-\theta_2)})\,\phi-\alpha\,\theta_2^2\right]}_{\text{Firm 2 profit it opts for corner solution}}=0
$$

$$
\Rightarrow \frac{M}{2}\,\phi+M\,(\frac{\eta\,\beta}{\theta_2}-\frac{\eta\,\beta}{\theta_c})+\alpha\,(\theta_2^2-\theta_c^2)+M\,\beta\,(\delta-\kappa)+\frac{M}{2}\,\frac{1}{2\kappa\,(\theta_c-\theta_2)}=0 \qquad (3.81)
$$

Lemma 3.20. *Suppose a Type D equilibrium exists for a given set of parameters. If the nuisance factor η increases, the quality set by the high-quality firm θ_c increases.*

When η increases, both θ_2 and $\frac{\eta\beta}{\theta_2}$ increase as per Lemma 3.19. Therefore, we can infer from (3.81) that the left side of the equation increases, which means θ_c has to increase to equate it to zero.

Proposition 3.10. *If the nuisance factor η increases, the profit of both firms and the total market coverage decrease. Further, β_0 and β_1 increase, while β_2 decreases*

β_0 increases as the profit of the low-quality firm increases. β_1 increases because the profit of Firm 2 under the corner solution is more significantly reduced by an increase in η compared to the profit under the uncovered configuration (Lemma 3.19). β_2 decreases because the profit under the corner configuration decreases, which relaxes the Type D constraint that the low-quality firm must achieve higher profit when it deviates by exceeding the quality of the high-quality firm. Two implications of Proposition 3.10 are: a) The range of the interval (β_0, β_1) when Type C equilibrium exists decreases; b) The advertisement nuisance increases the quality of the premium product under Type D.

3.8.4 Consumers get positive utility from advertisements

Let's assume that consumers get positive utility from advertisements as they learn about new products, or search classifieds for new business opportunities. This utility gain is proportional to the advertisement level as shown below:

$$U_i = v_i\, Y_i\, \theta + \lambda\, \beta - s \text{ where } \lambda > 0 \tag{3.82}$$

This model will be similar to what we discussed in the previous section, with η replaced by $-\lambda$, so we will not discuss it in detail. The key difference is that the low-quality firm will set a positive price $s_2 = \lambda\beta$ in the corner solution when $\theta_2 = 0$. The effect will be opposite of what we discussed when ads were a nuisance. More specifically:

- The profit and the market coverage of both firms increase as the demand curve shifts outward.
- The interval (β_1, β_2) increases, i.e., the range on which Type C equilibrium exists increases.
- The high-quality firm sets lower quality under Type D because the constraint for Type D becomes difficulty to satisfy. This implies that the high-quality firm need not raise quality as much under Type D equilibrium to protect its market share.

When advertisements were a nuisance, some customers dropped off, but the competition for the remaining consumers increased, reducing prices and profits. Whereas, when consumers get positive utility from advertisements, demand increases, and competition relaxes.

Another important point to note is that in our model, the market is concentrated irrespective of whether there is a positive effect of advertisements on consumers.

3.9 Concluding Remark

We developed a model to analyze two-sided newspaper markets based on vertical differentiation. Our analysis shows that the tension between advertisement and subscription revenue leads to various market configurations. High advertisement levels can make the market highly competitive, as firms are willing to sacrifice subscription revenue to compete for advertisement revenue. As a result, the leading firm provides a very high-quality product at a lower price to protect its customer base. This market configuration results from our assumptions of a sequential entry model and quality-dependent fixed costs. We also demonstrated that our results are robust to varying consumer attitudes towards advertisements.

3.A Appendices

3.A.1 Social planner objective function and optimal choice

We simplify (3.2) and we get

$$W(s,\theta) = N(s,\theta)(s-c) + M\int_0^{\infty}\left[Y\theta\frac{v^2}{2} - sv\right]_{\frac{s}{Y\theta}}^{1} dF(Y) + M\beta(E[Y] - \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M\int_0^{\infty}(\frac{Y\theta}{2} - s + \frac{s^2}{2Y\theta})dF(Y) + M\beta(E[y] - \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}E[Y] + \frac{s^2}{2\theta}E[\frac{1}{Y}] - s) + M\beta(E[Y] - \frac{s}{\theta}) - \alpha\theta^2$$

Substituting $E[Y] = e^{\mu+\frac{\sigma^2}{2}} = \delta$ and $E[\frac{1}{Y}] = e^{-\mu+\frac{\sigma^2}{2}} = \frac{1}{\kappa}$ for log-normal distribution we get:

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa + \kappa + \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) + \underbrace{M\beta\kappa(1 - \frac{s}{\theta\kappa})}_{\kappa\beta N(s,\theta)} - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c+\kappa\beta) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) - \alpha\theta^2 \tag{3.83}$$

First order condition after putting the value of $N(s,\theta)$:

$$\frac{\partial W}{\partial s} = 0 \rightarrow M(-\frac{1}{\theta\kappa})(s-c+\kappa\beta) + M(1 - \frac{s}{\theta\kappa}) - M(1 - \frac{s}{\theta\kappa}) = 0$$

$$s = c - \kappa\beta \tag{3.84}$$

Notice that s is independent of θ. *Case I* $s = c - \kappa\beta \leq 0$*:* We get corner solution as demand $N(s,\theta) = M$. $W(s,\theta)$ becomes:

$$W(s,\theta) = M(s-c+\kappa\beta) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) - \alpha\theta^2$$

Since $\frac{\partial W}{\partial s} = M\frac{s}{\theta\kappa}$ is increasing in s, therefore the social planner will set the price $s = 0$ and the objective function becomes:

$$W(s,\theta) = \frac{1}{2}M\delta\theta + M\beta(\delta - \kappa) - \alpha\theta^2$$

and therefore the optimal quality θ_{SP} is given by:

$$\frac{\partial W}{\partial \theta} = 0 \Rightarrow \frac{1}{2} M\delta = 2\alpha\theta_{SP}$$

$$\theta_{SP} = \frac{M\delta}{4\alpha} \tag{3.85}$$

Case II $s = c - \kappa\beta > 0$:. We will have interior solution and the optimal quality is given by the first order condition:

$$\frac{\partial W}{\partial \theta} = 0 \to M\frac{s}{\kappa\theta^2}(s - c + \kappa\beta) + \frac{M}{2}(\delta - \frac{s^2}{\kappa\theta^2}) - 2\alpha\theta = 0$$

Using (3.84) we have: $4\kappa\alpha\theta^3 - M\delta\kappa\theta^2 + M(c - \kappa\beta)^2 = 0$

$$4\alpha\kappa\theta^3 - Me^{2\mu}\theta^2 + M(c - \kappa\beta)^2 = 0 \tag{3.86}$$

We need to check second order condition. Hessian is given by:

$$H_w = \begin{vmatrix} \frac{-M}{\kappa\theta} & \frac{M}{\kappa\theta^2}(s - c + \kappa\beta) \\ \frac{M}{\kappa\theta^2}(s - c + \kappa\beta) & \frac{Ms}{\kappa\theta^3}(2(c - \kappa\beta) - s) - 2\alpha \end{vmatrix}$$

The sufficient second order condition for interior solution demands that

$$\theta > \frac{M\delta}{6\alpha}$$

Implicit function theorem on (3.86) implies that the optimal quality decreases with higher price $s = c - \kappa\beta$ when the second order condition is satisfied.

Now, we check the critical quality level that demand approaches zero i.e. $\frac{s}{\theta} = \kappa$. Substituting this value in (3.86) we get

$$\theta = \frac{M\delta}{4\alpha}(1 - e^{-\sigma^2})$$

This implies that if the solution of (3.86) lies in the interval $(\frac{M\delta}{4\alpha}(1 - e^{-\sigma^2}), \frac{M\delta}{4\alpha}]$ then the social planner will serve the market with positive price. Further, if $s = c - \kappa\beta$ is sufficiently high then the social planner will not serve the market.

3.A.2 Solution of Monopolist problem

a) First we characterize the interior solution that is the solution with $N(s, \theta) \in (0, M)$ (uncovered market). Replacing $N(s, \theta)$ from the demand function (3.1) in the profit function (3.9) we have

$$\Pi(s, \theta) = M(1 - \frac{s}{\kappa\theta})(s + \phi) + M\beta(\delta - \kappa) - \alpha\theta^2 \tag{3.87}$$

The first order conditions:

$$\frac{\partial \Pi}{\partial s} = 0 \Rightarrow -\frac{M}{\kappa\theta}(s+\phi) + M(1-\frac{s}{\kappa\theta}) = 0$$

$$s = \frac{1}{2}\kappa\theta - \frac{1}{2}\phi \tag{3.12}$$

$$\frac{\partial \Pi}{\partial \theta} = 0 \Rightarrow -\frac{Ms}{\kappa\theta^2}(s+\phi) - 2\alpha\theta = 0$$

Replacing the value of s from (3.12) and with $\theta > 0$ for interior solution we get:

$$8\alpha\kappa\theta^3 - M\kappa^2\theta^2 + M\phi^2 = 0 \tag{3.11}$$

The sufficient condition for the first order condition to have a local maximum requires that the Hessian H_m is negative definite at the solution of (3.12) and (3.11).

$$H_m = \begin{vmatrix} \frac{-2M}{\kappa\theta} & \frac{M}{\kappa\theta^2}(2s+\phi) \\ \frac{M}{\kappa\theta^2}(2s+\phi) & -\frac{2Ms}{\kappa\theta^3}(s+\phi) - 2\alpha \end{vmatrix}$$

Putting the value of s from (3.12) we get

$$H_m = \begin{vmatrix} \frac{-2M}{\kappa\theta} & \frac{M}{\theta} \\ \frac{M}{\theta} & -\frac{M}{2\kappa\theta^3}(\kappa^2\theta^2 - \phi^2) - 2\alpha \end{vmatrix}$$

If θ_{MP} is the solution of (3.11) then the sufficient condition for local maximum (i.e. negative definite Hessian) requires

$$\theta_{MP} > \frac{M\kappa}{12\alpha}$$

Using implicit function theorem on (3.11) we can identify the relation between θ_{MP} and ϕ for a given M, k, α. We can verify that the θ_{MP} is decreasing in $|\phi|$ with maximum value of $\theta_{MP} = \frac{M\kappa}{8\alpha}$ when $\phi = 0$. The condition $\theta_{MP} > \frac{M\kappa}{12\alpha}$ requires

$$|\phi| < \frac{M\kappa^2}{12\sqrt{3}\alpha} \tag{3.13}$$

We can verify that (3.11) has no positive real solution when condition (3.13) is not satisfied. When condition (3.13) is satisfied, there exist only one real solution of (3.11) that satisfies the second order condition $\theta > \frac{M\kappa}{12\alpha}$ for local maximum, and hence if this is an interior solution then this solution is a unique solution. This proves lemma 3.1. The profit as a function of quality for the interior solution is given by

$$\Pi_{int}(\theta) = \frac{M}{4\kappa\theta}(\kappa\theta+\phi)^2 + M\beta(\delta-\kappa) - \alpha\theta^2 \tag{3.88}$$

Also notice that the condition (3.13) results in the market coverage range between $(\frac{1}{2}-\frac{1}{2\sqrt{3}}, \frac{1}{2}+\frac{1}{2\sqrt{3}})$.

Using implicit function theorem on (3.11) we can show that under interior solution the monopolist's optimal choice of quality is

- increasing in market size (M) and median income (μ)
- decreasing in quality cost (α) and income inequality (σ)
- non-monotonic in advertiser's willingness to pay β and the marginal cost c with single peak at $\phi = 0$

b) Next we consider the possible corner solution when $N = M$(full market coverage) and find the set of parameter values when this solution dominates the interior solution. For full market coverage, the monopolist must set the subscription price $s = 0$ (from (3.1)). The profit function becomes:

$$\Pi_{zp} = M\beta\delta - Mc - \alpha\theta^2$$

Since increasing θ increases the fixed cost without additional revenue as the market is fully covered, the optimal θ for the monopolist with zero subscription price is $\theta = 0$. Therefore,

$$\Pi_{zp} = \underbrace{M\beta\delta}_{advetisement\ revenue} - \underbrace{Mc}_{variable\ cost} \tag{3.89}$$

Intuitively, $M\delta$ is the average income for the whole market and hence $M\beta\delta$ is the total advertisement revenue when the market is fully covered.

When $\beta = 0$, $\Pi_{int} > \Pi_{zp} = 0$. However, using envelope theorem we can show that the optimal profit with zero price increases faster with β than the optimal profit with the interior solution.

$$\frac{d\Pi_{int}}{d\beta} = M\delta - \frac{s}{\theta} < M\delta = \frac{d\Pi_{zp}}{d\beta}$$

Using intermediate value theorem, we can conclude that there exist a critical $\overline{\beta} > 0$ such that $\Pi_{zp} > \Pi_{int}$ whenever $\beta > \overline{\beta}$.

We now show that $\phi(\overline{\beta}) < \frac{M\kappa^2}{12\sqrt{3}\alpha}$ (maximum value for which we have interior solution) so that there is an internal solution at the critical point. To do so, we evaluate Π_{int} at $\phi = \frac{M\kappa^2}{12\sqrt{3}\alpha}$ when $\theta_{MP} = \frac{M\kappa}{12\alpha}$.

$$\Pi_{int}|_{\kappa\beta - c = \frac{M\kappa^2}{12\sqrt{3}\alpha}} = \frac{3\alpha}{\kappa^2}\left(\frac{M\kappa^2}{12\alpha}+\frac{M\kappa^2}{12\sqrt{3}\alpha}\right)^2 + M\beta(\delta-\kappa) - \alpha\left(\frac{M\kappa}{12\alpha}\right)^2 = \underbrace{M\beta\delta - Mc}_{\Pi_{zp}} + \underbrace{(3-2\sqrt{3})\alpha\left(\frac{M\kappa}{12\alpha}\right)^2}_{<0}$$

$$\Rightarrow \Pi_{int} < \Pi_{zp} \text{ when } \phi = \frac{M\kappa^2}{12\sqrt{3}\alpha}$$

$$\Rightarrow \phi(\overline{\beta}) < \frac{M\kappa^2}{12\sqrt{3}\alpha}$$

Although the above result is sufficient for the proof of Proposition 3.2, we can get exact cut-off point by equating $\Pi_{int} = \Pi_{zp}$. Without showing the detail solution steps, we state that $\overline{\beta}$ is given by

$$\phi(\overline{\beta}) = \frac{M\kappa^2}{27\alpha} \Rightarrow \overline{\beta} = \phi^{-1}(\frac{M\kappa^2}{27\alpha}) \Rightarrow \overline{\beta} = \frac{c}{\kappa} + \frac{M\kappa}{27\alpha}$$

Above equation shows that $\overline{\beta}$ increases with M and decreases with α, but ambiguous with μ and σ. If c is small then $\overline{\beta}$ increases with μ and decreases with σ.

c) Now we evaluate the cutoff point for the corner solution when market is not served by the monopolist, which means $N(s, \theta) = 0$ and hence the profit is zero. We evaluate Π_{int} when $\phi = -\frac{M\kappa^2}{12\sqrt{3}\alpha}$ (lowest value for the interior solution) and the corresponding $\theta_{MP} = \frac{M\kappa}{12\alpha}$

$$\Pi_{int}|_{\phi=-\frac{M\kappa^2}{12\sqrt{3}\alpha}} = \frac{3\alpha}{k^2}(\frac{M\kappa^2}{12\alpha} - \frac{M\kappa^2}{12\sqrt{3}\alpha})^2 + M\beta(\delta-\kappa) - \alpha(\frac{M\kappa}{12\alpha})^2 = \underbrace{(3-2\sqrt{3})\alpha(\frac{M\kappa}{12\alpha})^2}_{<0} + M\beta(\delta-\kappa)$$

If we assume $\beta = 0$ and $c = \frac{M\kappa^2}{12\sqrt{3}\alpha}$ then $\Pi_{int} < 0$. Since Π_{int} is monotonically increasing in β and is greater than 0 when $\beta = \overline{\beta}$, there exist $0 < \underline{\beta} < \overline{\beta}$ such that $Pi_{int}|_{\underline{\beta}} = 0$. If $\beta < \underline{\beta}$ then the monopolist will not serve the market. We had assumed $c = \frac{M\kappa^2}{12\sqrt{3}\alpha}$. If we decrease c then $\underline{\beta}$ also decreases, and will continue to be positive only if c is sufficiently high. Specifically, the following equation characterizes the relationship:

$$\underline{\beta} = \max(0, \phi^{-1}(-\frac{M\kappa^2}{27\alpha}))$$

The above equation implies $\underline{\beta}$ decreases with M, mu and increases with c, α and σ.

3.A.3 Solution of Duopoly Market

Let's consider that at-most two firms, $k \in \{1, 2\}$, can enter the market. The firms choose the non-negative subscription price, quality level, and the advertisement price, $(s_k, \theta_k, p_k) \in \mathbb{R}^3_+$.

Equilibrium Type B

Suppose Firm 1 (leader) decides to enter and chooses θ_1 in the stage 1, and the Firm 2 (follower) decides to enter in the second stage with the positive quality $\theta_2 > 0$. If we assume $\theta_1 > \theta_2$ the profit function of two firms are given by the equation (3.24)-(3.25) as it was shown in the section 3.6.

$$\Pi_1 = \underbrace{M\frac{1-\gamma}{\kappa\theta_1(4-\gamma)^2}(2\kappa\theta_1+\phi)^2}_{R_1} + M\beta(\delta-\kappa) - \alpha\theta_1^2 \tag{3.24}$$

$$\Pi_2 = \underbrace{M\frac{1-\gamma}{\kappa\theta_2(4-\gamma)^2}(\kappa\theta_2+2\phi)^2}_{R_2} - \alpha\theta_2^2 \tag{3.25}$$

Define the revenue expression R_1 and R_2 as shown above. We can verify that R_1 is a concave function in θ_1 and R_2 is a concave function in θ_2 if $\gamma \leq \frac{4}{7}$. Given θ_1, the optimal choice of θ_2 by Firm 2 is given by the first order condition:[43]

$$\frac{\partial R_2}{\partial \theta_2} - 2\alpha\theta_2 = 0 \Rightarrow \frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})\left[4-7\gamma-\frac{2\phi}{\kappa\theta_2}(2\gamma^2-3\gamma+4)\right] = 2\alpha\theta_2 \tag{3.31}$$

Lemma 3.2: Firm 1 will set the quality so that the Firm 2 enters with lower quality i.e. $\theta_2 < \theta_1$.
Proof: From equations (3.24) to (3.25), $\Pi_1 > \Pi_2$ for all $\theta_1 > \theta_2$. This implies that the high-quality firm has an advantage, and Firm 1 will choose a quality level that positions it in the high-quality space. Due to the concavity of the profit functions, for any given choice of θ_1, the profit function of Firm 2 will exhibit two strictly concave segments: one for $\theta_2 < \theta_1$ and another for $\theta_2 > \theta_1$. Each segment has a unique local maximum, as shown on the left side of Figure 3.A.3. The left maximum point ($\theta_2 < \theta_1$) continuously increases with θ_1 due to relaxed competition, while the right maximum point ($\theta_2 > \theta_1$) continuously decreases with θ_1 due to increased competition. Therefore, there exist a θ_1^* such that Firm 2 will enter as a high-quality firm if $\theta_1 > \theta_1^*$. The right side of Figure 3.A.3 shows that Firm 1 has higher profit when it preempts as a high quality firm, thus Firm 1 will choose $\theta_1 > \theta_1^*$, and Firm 2 will enter as a low-quality firm.

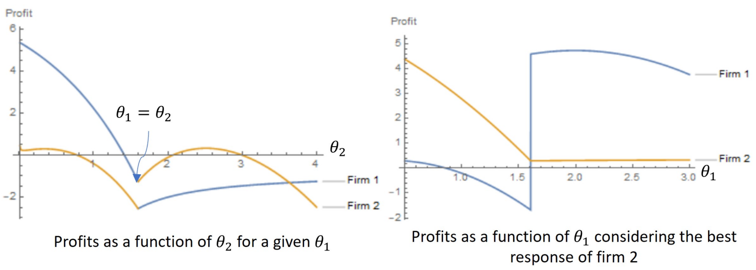


Figure 3.9:

If Firm 1 sets its quality to the monopolistic level, then the right maximum on the left side of Figure 3.A.3 is negative. This effectively blocks new entry as a high quality

[43]We later verify the necessary second order condition and check for uniqueness.

player. □

From the reaction function (3.31) of Firm 2 we calculate $\frac{\partial\theta_2}{\partial\theta_1}$ using implicit function theorem.[44]

$$\frac{\partial\theta_2}{\partial\theta_1} = -\frac{\frac{\partial^2 R_2}{\partial\theta_1\partial\theta_2}}{\frac{\partial^2 R_2}{\partial\theta_2^2} - 2\alpha}$$

Optimal choice for Firm 1 given the response function of Firm 2 is given by the first order condition:

$$\frac{d\Pi_1}{d\theta_1} = \frac{\partial\Pi_1}{\partial\theta_1} + \frac{\partial\Pi_1}{\partial\theta_2}\frac{\partial\theta_2}{\partial\theta_1} = 0 \Rightarrow (\frac{\partial R_1}{\partial\theta_1} - 2\alpha\theta_1)(\frac{\partial^2 R_2}{\partial\theta_2^2} - 2\alpha) - \frac{\partial R_1}{\partial\theta_2}\frac{\partial^2 R_2}{\partial\theta_1\partial\theta_2} = 0$$

Multiplying both side by $\gamma^2\theta_1 \neq 0$ and replacing $2\alpha\theta_2 = \frac{\partial R_2}{\partial\theta_2}$ we get

$$(\gamma\frac{\partial R_1}{\partial\theta_1} - \frac{\partial R_2}{\partial\theta_2})(\frac{\partial^2 R_2}{\partial\theta_2^2} - \frac{\partial R_2}{\partial\theta_2}) - \gamma^2\theta_1\frac{\partial^2 R_2}{\partial\theta_1\partial\theta_2}\frac{\partial R_1}{\partial\theta_2} = 0$$

We evaluate the left side of the above equation and replace $\frac{\phi}{\kappa\theta_1} \equiv x$ to further simplify:

$$\begin{aligned} x^4(-3072 + 5376\gamma - 5312\gamma^2 + 3920\gamma^3 - 3232\gamma^4 + 1968\gamma^5 - 852\gamma^6 + 168\gamma^7 - 8\gamma^8) \\ + x^3\gamma^2(-2048 + 1664\gamma - 1952\gamma^2 + 896\gamma^3 - 480\gamma^4 - 288\gamma^5 + 124\gamma^6 - 4\gamma^7) \\ + x^2\gamma^2(1024 - 5632\gamma + 7488\gamma^2 - 6400\gamma^3 + 3112\gamma^4 - 1908\gamma^5 + 327\gamma^6 + 18\gamma^7 \\ + x\gamma^4(256 - 928\gamma + 1048\gamma^2 - 984\gamma^3 - 184\gamma^4 + 108\gamma^5 \\ + \gamma^4(-64 + 432\gamma - 644\gamma^2 + 675\gamma^3 - 556\gamma^4 + 112\gamma^5) = 0 \end{aligned} \quad (3.90)$$

Even though θ_1 and θ_2 may be non-monotonic in ϕ, we claim that the ratios $\frac{\phi}{\kappa\theta_1(\phi)}$ and $\frac{\phi}{\kappa\theta_2(\phi)}$ are continuously increasing in ϕ, that is elasticity of θ_1 and θ_2 with respect to ϕ is less than 1.

Claim: $\frac{\phi}{\kappa\theta_1(\phi)}$ and $\frac{\phi}{\kappa\theta_2(\phi)}$ is continuously increasing in ϕ when first order condition has a solution.

Proof: Let's assume that the (3.90) has a solution for a given range of ϕ. Continuity is derived from the implicit function theorem and the fact that both θ_1 and θ_2 are positive in the Type B equilibrium. Now, suppose the ratio $\frac{\phi}{\kappa\theta_1}$ is decreasing in ϕ. An increase in ϕ would imply decrease in $\frac{\phi}{\kappa\theta_1}$ and increase in θ_1. As θ_1 increases, θ_2 also increases, but by a proportionately smaller amount because $\frac{d\theta_2}{d\theta_1} < 1$, thus lowering γ. This results in lowering of all components on the left-hand side (LHS) of the equation below (first order condition), while the right-hand side (RHS) increases. Hence, this cannot be the solution

[44]Note that second order condition $\frac{\partial^2 R_2}{\partial\theta_2^2} - 2\alpha < 0$ ensures that implicit function theorem can be applied.

of the first order condition.

$$\frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})\big[4-7\gamma-\frac{2\phi}{\kappa\theta_2}(2\gamma^2-3\gamma+4)\big]+\frac{\partial\Pi_1}{\partial\theta_2}\frac{\partial\theta_2}{\partial\theta_1}=2\alpha\theta_2$$

Therefore, $\frac{\phi}{\kappa\theta_1}$ is increasing in ϕ. We can use the same logic on (3.31) to show that $\frac{\phi}{\kappa\theta_2}$ is increasing in ϕ. □

First, we find the solution of (3.90) at $\phi = 0$ i.e. $x = 0$.

Lemma 3.3: If $\phi = 0$ then there exist a unique solution such that two firms choose quality in the ratio $\gamma = 0.195064$, which is a constant for all (μ, σ^2, α). The market share of the high quality firm is twice that of the low quality firm. The high-quality firm sets its quality lower than the monopoly level but achieves higher market coverage due to a lower price-to-quality ratio.

Proof: Replacing $x = 0$ simplifies (3.90) to $112\gamma^5-556\gamma^4+675\gamma^3-644\gamma^2+432\gamma-64 = 0$ which has a unique solution $\gamma = 0.195064$ that satisfies second order condition and is constant irrespective of μ, σ, α. The corresponding θ_1 and θ_2 can be derived from (3.31).

$$\theta_1 = \rho\frac{M\kappa}{\alpha} \text{ and } \theta_2 = \rho\gamma\frac{M\kappa}{\alpha} \text{ where}$$

$$\gamma = 0.195064, \rho = \frac{4-7\gamma}{2\gamma(4-\gamma)^3} = 0.1226$$

$\theta_2 = 0.1225\frac{M\kappa}{\alpha}$ is lower than $\theta_{MP} = \frac{M\kappa}{8\alpha}$. The corresponding market shares of the two players using (3.26) and (3.27) yields

$$\frac{N_1}{M} = \frac{2}{4-\gamma} = 52.56\%, \frac{N_2}{M} = \frac{1}{4-\gamma} = 26.28\%$$

□

Now we characterize the solution of (3.90) for $\phi < 0$ and $\phi > 0$.

Lemma 3.4: There exist $\phi(\underline{\beta}) < \phi_{il} < 0$ and $0 < \phi_{ir} < \phi(\overline{\beta})$ such that an interior solution exist iff $\phi \in (\phi_{il}, \phi_{ir})$ and this solution is unique for a given ϕ.

Steps to prove lemma 3.4: It is easy to verify that γ is a continuous function of x using implicit function theorem on (3.90). We have already established that $\frac{\phi}{\kappa\theta_2}$ is increasing in ϕ. So when ϕ decreases below 0 so that $\frac{\phi}{\kappa\theta_2}$ approaches $-\frac{1}{2}$, the lhs of (3.31) becomes zero which implies that (3.31) does not have a solution. Therefore, there exist $\phi_{il} < 0$ such that if $\phi < \phi_{il}$ no interior solution exist. Similarly on the right side, $\frac{s_2}{\theta_2}$ decreases as ϕ increases (see (3.29)). When $\frac{s_2}{\theta_2} \to 0$, $\frac{\phi}{\kappa\theta_2} \to \frac{1-\gamma}{2+\gamma}$, and the profit function (3.25) of the low quality firm becomes:

$$\Pi_2 = \frac{M}{2+\gamma}\phi - \alpha\theta_2^2 \Rightarrow \frac{d\Pi_2}{d\theta_2} < 0$$

Hence the optimal quality is zero, which means that there is no interior solution. Therefore, there exist $\phi_{ir} > 0$ such that no interior solution exist.

For uniqueness and the cutoff points, we need to find the solution of equation (3.92) which is a complex equation and does not have a close form solutions. However, we already have found a solution $\phi = 0$ (lemma 3.3). We now change x numerically with small precision on both directions, and find roots of the polynomial of γ that satisfy the necessary second order condition. Then we we use (3.31) to find θ_1 and θ_2. We find that no solution exist if $\phi \geq \frac{M\kappa^2}{255\alpha}$ and $\phi \leq -\frac{M\kappa^2}{187\alpha}$, and there is a unique solution when the solution exists. This proves that $= \frac{-M\kappa^2}{27\alpha} = \phi(\underline{\beta}) < \phi_{il} = -\frac{M\kappa^2}{187\alpha}$ and $\phi_{ir} = \frac{M\kappa^2}{255\alpha} < \phi\overline{\beta} = \frac{M\kappa^2}{27\alpha}$. Please note that the cutoffs are not precise due to numerical method but within the reasonable precision limit of $0.005\frac{M\kappa^2}{\alpha}$. □

Lemma 3.6: γ decreases continuously with ϕ when $\phi > 0$ and the high-quality firm sets lower quality than the monopolist for all ϕ.

Proof: Using implicit function theorem on (3.92) we evaluate that γ is a continuously decreasing function of x when $x > 0$. We have already established that x is a continuously increasing function of ϕ, which implies that γ is decreasing in ϕ. $\theta_2(\phi) < \theta_{MP}(\phi)$ for all ϕ is verified through the solution.

□

Lemma 3.5: There exist a critical $\phi_0 \in (\phi_{il}, 0)$ such that for all $\phi \in (\phi(\underline{\beta}), \phi_0)$ only one firm enters the market (type A equilibrium).

Proof: Proof follows from the IVT and the following statements:

a) Low quality firm makes a negative profit when $\phi = \phi_{il} + \epsilon = -\frac{M\kappa^2}{188\alpha}$. [45]

b) Profit function of the low quality firm, Π_2, is continuously increasing in ϕ when $\phi \in (\phi_{il}, \phi_{ir})$(see Lemma 3.8).

c) Low quality firm makes positive profit when $\phi = 0$ (shown above).

d) A monopolist will enter the market if $\phi > -\frac{M\kappa^2}{27\alpha}$.

□

Using (3.31) and (3.92) we can show that this cutoff point is $\phi_1 \approx -\frac{M\kappa^2}{242\alpha}$. Therefore, when $\frac{-M\kappa^2}{27\alpha} \leq \phi \leq -\frac{M\kappa^2}{242\alpha}$ only one firm (monopolist) exist and further entry is blockaded.

Lemma 3.8: Profit of both firms increases continuously with ϕ under Type B equilibrium.

[45] Corresponding $\gamma = 0.158679, \theta_2 = 0.01896\frac{M\kappa}{\alpha}$; Plugging these values in profit function (3.34) we get the negative profit value.

Proof: We evaluate

$$\frac{\partial \Pi_1}{\partial \theta_2} = \frac{\partial \Pi_1}{\partial \gamma}\frac{\partial \gamma}{\partial \theta_2} = -\frac{M(2+\gamma)}{\kappa\theta_1(4-\gamma)^3}(2\kappa\theta_1+\phi)^2(\frac{1}{\theta_1}) = -\frac{4Mk(2+\gamma)}{(4-\gamma)^3}(1+\frac{\phi}{2\kappa\theta_1})^2$$

$$\frac{\partial \Pi_2}{\partial \theta_1} = \frac{\partial \Pi_2}{\partial \gamma}\frac{\partial \gamma}{\partial \theta_1} = -\frac{M(2+\gamma)}{\kappa\theta_2(4-\gamma)^3}(\kappa\theta_2+2\phi)^2(-\frac{\theta_2}{\theta_1^2}) = \frac{Mk\gamma^2(2+\gamma)}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})^2$$

$$\frac{\partial \Pi_1}{\partial \phi} = \frac{4M(1-\gamma)}{(4-\gamma)^2}(1+\frac{\phi}{2\kappa\theta_1}) > 0$$

$$\frac{\partial \Pi_2}{\partial \phi} = \frac{4M(1-\gamma)}{(4-\gamma)^2}(1+\frac{2\phi}{\kappa\theta_2}) > 0$$

Using envelope theorem,

$$\frac{d\Pi_1(\theta_1,\theta_2,\phi)}{d\phi} = \underbrace{\frac{\partial \Pi_1}{\partial \theta_2}\frac{\partial \theta_2}{\partial \phi}}_{\text{strategic effect}} + \underbrace{\frac{\partial \Pi_1}{\partial \phi}}_{\text{direct effect}} > 0$$

$$\frac{d\Pi_2(\theta_1,\theta_2,\phi)}{d\phi} = \underbrace{\frac{\partial \Pi_2}{\partial \theta_1}\frac{\partial \theta_1}{\partial \phi}}_{\text{strategic effect}} + \underbrace{\frac{\partial \Pi_1}{\partial \phi}}_{\text{direct effect}} > 0$$

Direct effect is always positive. It can be easily shown that for all values of γ strategic effect either reinforces the direct effect or is much smaller than the direct effect because $|\kappa\frac{\partial\theta_1}{\partial\phi}| < 1$ and $|\kappa\frac{\partial\theta_2}{\partial\phi}| < 1$. We have already shown that when $\phi \geq 0$, there is a positive strategic effect because γ decreases with ϕ (Lemma 3.6). γ increases with ϕ at small value of ϕ when $\frac{\phi}{\kappa\theta_2} = -\frac{1}{2}$ when the strategic effect is small.

Type C Equilibriurm

Lemma 3.9: Suppose Firm 2 chooses $\theta_2 = 0$ and $\phi \in [0, \frac{M\kappa^2}{12\sqrt{3}\alpha})$. Then, the best response of Firm 1 is $\theta_1 = \theta_{MP}$, and corresponding subscription prices are $s_1 = s_{MP}$, $s_2 =$ 0.

Proof: If Firm 2 chooses $\theta_2 = 0$, then to maintain positive demand, it must also set $s_2 = 0$ in the second stage for all $\theta_1 \in \mathbb{R}+$. $\phi \geq 0$ ensures that Firm 2 will have non-negative profit even without subscription revenue. With $s_2 = 0$ and $\theta_2 = 0$, the consumers of Firm 2 receive zero utility. Hence, the demand for Firm 1 is comprised of all consumers who gain positive utility from subscribing to its product, i.e., $v_i Y_i \theta_1 - s_1 > 0$, which yields the same demand curve as that of a monopolist, as described in Equation (3.1). Therefore, the optimal response for Firm 1 is (s_{MP}, θ_{MP}), as detailed in Lemma 3.1, provided the necessary condition $|\phi| < \frac{M\kappa^2}{12\sqrt{3}\alpha}$ for the monopolist's interior solution is satisfied (see (3.13)).

Note that $\theta_1 = 0$ is never the best response for Firm 1, because if both firms set their subscription prices to zero, they would have to share the advertising revenue. However, Firm 1 can achieve higher profits by capturing the full advertising revenue if it increases its quality by an infinitesimally small amount. □

Lemma 3.11: There exists a critical $\phi_1 \in (0, \phi_{ir})$ such that for $\phi \in (\phi_1, \phi_2]$ there is a unique equilibrium of type C.

Proof: Let us use the subscript 'b', 'c','o' to denote endogenous parameter under type B, type C, and when Firm 2 unilaterally deviates from type B to zero quality, respectively. Profit of Firm 2 in type B is given by (3.25)

$$\Pi_{2b} = M\frac{1-\gamma}{\kappa\theta_{2b}(4-\gamma)^2}(\kappa\theta_{2b}+2\phi)^2 - \alpha\theta_{2b}^2 \tag{3.25}$$

If Firm 2 unilaterally deviates from type B to zero quality its profit is given by (3.91)

$$\Pi_{2o} = \frac{M}{2}(1-\frac{\phi}{\kappa\theta_{1b}})\phi \tag{3.91}$$

We evaluate Π_{2b} at $\phi = 0$ and at $\phi = \phi_{ir} = \frac{M\kappa^2}{255\alpha}$ using (3.90)

$$\Pi_{2b}|_{\phi=0} = 0.00076\frac{M^2\kappa^2}{\alpha} \text{ and } \Pi_{2b}|_{\phi=\phi_{ir}} = 0.0019\frac{M^2\kappa^2}{\alpha}$$

Similarly, Π_{2o} when $\phi = 0$ and $\phi = \phi_{ir} = \frac{M\kappa^2}{255\alpha}$ using (3.91)

$$\Pi_{2o}|_{\phi=0} = 0 \text{ and } \Pi_{2o}|_{\phi=\phi_{ir}} = 0.002\frac{M^2\kappa^2}{\alpha}$$

Therefore

- If $\phi = 0$ then $\Pi_{2b} > \Pi_{2o}$ and $\frac{d\Pi_{2o}}{d\phi} = \frac{M}{2} > \frac{d\Pi_{2b}}{d\phi}$. Therefore, Π_{2o} is lower than Π_{2b} but increasing faster.
- If $\phi = \phi_{ir} = \frac{M\kappa^2}{255\alpha}$ then $\Pi_{2b} < \Pi_{2o}$.
- Π_{2b} is a continuously increasing and convex function of ϕ when $\phi \in [0, \phi_{ir}]$ (from (3.25)), and Π_{2o} is a continuously increasing and concave function of ϕ (from (3.91)).

Therefore, Π_{2o} must cross Π_{2b} from below, and there is exactly one cutoff point $\phi_1 \in (0, \phi_{ir})$ such that for $\phi < \phi_1$, we have $\Pi_{2o} < \Pi_{2b}$. This implies that if $< \phi_0 < \phi < \phi_1$ there is no Type C equilibrium, and we get a unique type B equilibrium.

If $\phi > \phi_1$, then Firm 2 has a profitable deviation to zero quality. Should it choose zero quality, Firm 1 will set the monopolistic level of price and quality, θ_{MP} (Lemma 3.9), which is greater than θ_{1b} (Lemma 3.6). This relaxes competition and reduces the market coverage of Firm 1, thereby increasing the profit of Firm 2 in both scenarios: a) when Firm 2 acts according to the reaction function of Type B (3.31), and b) if it maintains zero quality under Type C (note: $\Pi_{2c} > \Pi_{2o}$ because $\theta_{MP} > \theta_{1b}$). Except for a small interval when ϕ is closer to ϕ_1, the latter profit (b) is greater, and thus a Type C equilibrium is sustained because both Firm 1 and Firm 2 are acting optimally after deviation. Therefore, we have a unique Type C equilibrium when $\phi \in (\phi_1, \phi_2)$, with the exception noted below. □

Exception: In the small interval where ϕ is very close to ϕ_1, the profit under scenario a) is greater, and thus the Type C equilibrium with $\theta_1 = \theta_{MP}$ is not sustained. In this case, Firm 2 will increase its quality, making θ_{MP} suboptimal for Firm 1. However due to lower competition, Firm 1 profits more when Firm 2 opts for zero quality. Consequently, Firm 1 preempts by setting a quality level lower than θ_{MP} but greater than θ_{1d} to make Firm 2 indifferent between zero and positive quality levels under Type B. For simplicity, we categorize this scenario also as a Type C equilibrium, despite Firm 1's quality being lower than θ_{MP}.

Type D Equilibriurm

Lemma 3.12: There exist a unique type D equilibrium if $\phi > \phi_2$

Proof: If $\phi > \phi_2$, Firm 2 will respond with $\theta_2 = \theta_{MP+}$ when Firm 1 sets $\theta_1 = \theta_{MP}$, which will result in Firm 1 losing all demand and will have negative profit. This is because the price of both firms will become zero, but Firm 2 has better quality and hence is preferred by consumers. Anticipating this, Firm 1, who is a first mover, will increase quality and set $\theta_1 = \theta_c$ (see definition 3.5) when Firm 2 is indifferent between choosing zero quality and θ_{c+}. If Firm 1 chooses $\theta_1 = \theta_c$, the best response for Firm 2 is $\theta_2 = 0$.

It is not profitable for Firm 2 to set $\theta \in (0, \theta_c)$ because $\phi_2 > \phi_{ir}$, which means no interior solution exists (Lemma 3.4). This is evident from the reaction function of Firm 2 (Equation (3.31)). As ϕ increases and approaches ϕ_{ir}, the ratio $\frac{\phi}{\kappa\theta_2}$, which increases with ϕ, approaches $\frac{1}{2}$, and the last factor on the LHS becomes negative for all γ. Therefore, for $\phi > \phi_{ir}$, $\frac{d\Pi_2}{d\theta_2} < 0$, and the optimal quality for Firm 2 is zero.

$$\frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})\big[4-7\gamma-\frac{2\phi}{\kappa\theta_2}(2\gamma^2-3\gamma+4)\big] = 2\alpha\theta_2 \tag{3.31}$$

It is also not profitable for Firm 2 to set $\theta_2 > \theta_c$ because we have shown in Lemma 3.2 that if Firm 1 sets quality greater than or equal to monopoly, then entry of Firm 2 as a high quality player is not profitable under Type B equilibrium. Since there are no profitable deviations for Firm 2 from $\theta_2 = 0$, a unique Type D equilibrium exist. □

3.A.4 Solution of Simultaneous Duopoly Market

Reaction functions of Firm 1 and Firm 2 at stage 1:

$$\frac{4M\kappa}{(4-\gamma)^3}(1+\frac{\phi}{2\kappa\theta_1})\big[(2\gamma^2-3\gamma+4)-\frac{\phi}{2\kappa\theta_1}(4-7\gamma)\big]=2\alpha\theta_1 \quad (3.30)$$

$$\frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})\big[4-7\gamma-\frac{2\phi}{\kappa\theta_2}(2\gamma^2-3\gamma+4)\big]=2\alpha\theta_2 \quad (3.31)$$

Necessary condition for the Type B equilibrium is that the above two equation has a solution. Assuming that the above reaction function has a solution, we can infer (using implicit function theorem on (3.30)-(3.31)) that θ_1 and θ_2 are non-monotonic in ϕ. However, the ratios $\frac{\phi}{\kappa\theta_1(\phi)}$ and $\frac{\phi}{\kappa\theta_2(\phi)}$ are continuously increasing in ϕ. In other words, elasticity of θ_1 and θ_2 with respect to ϕ is less than 1.[46]

To solve the above equations, we further simplify it by defining $x \equiv \frac{\phi}{\kappa\theta_1}$ and eliminating M and α from the above two equations we get:

$$x^2(16-12\gamma+8\gamma^2-4\gamma^3+7\gamma^4)+x\gamma^2(2+\gamma+2\gamma^2+\gamma^3)+\gamma^2(-4+23\gamma-12\gamma^2+8\gamma^3)=0 \quad (3.92)$$

In place of (3.90) for sequential entry we have (3.92) as FOC for the simultaneous entry. We solve this in a similar manner starting with $x=0$ and then changing x on both positive and negative side by a small value, identifying γ that satisfies the second order condition, and then using (3.31) to calculate value of θ_1 and θ_2. We do this until (3.92) has no solution, which will provide the cutoff points ϕ_{il} and ϕ_{ir}.

First, we find the solution of (3.92) at the $\phi=0$ i.e. $x=0$. Replacing $x=0$ simplifies (3.92) to $8\gamma^3-12\gamma^2+23\gamma-4=0$ which has a unique solution $\gamma=0.194031$ and is constant irrespective of μ,σ,α. We verify that the necessary second order condition is also satisfied for this solution. The corresponding θ_1 and θ_2 can be derived from (3.30) and (3.31).

$$\theta_1=\rho\frac{M\kappa}{\alpha} \text{ and } \theta_2=\rho\gamma\frac{M\kappa}{\alpha} \text{ where}$$

$$\gamma=0.1904,\ \rho=\frac{4-7\gamma}{2\gamma(4-\gamma)^3}=\frac{2(2\gamma^2-3\gamma+4)}{(4-\gamma)^3}=0.1267$$

The corresponding market shares of the two players using (3.26) and (3.27) yields

$$\frac{N_1}{M}=\frac{2}{4-\gamma}=52.4\%,\ \frac{N_2}{M}=\frac{1}{4-\gamma}=26.2\%$$

[46] Suppose that when ϕ increases θ_1 (resp. θ_2) increases proportionately by larger amount so that $\frac{\phi}{\theta_1}$ (resp. $\frac{\phi}{\theta_2}$) decreases then we can easily see that the LHS (marginal revenue) of (3.30) (resp. (3.31)) decreases as long as $\gamma<\frac{4}{7}$ while the RHS (marginal cost) increases, which is a contradiction. In equilibrium, $\gamma<\frac{4}{7}$ is satisfied in our model.

The following table compares the solution for simultaneous and sequential entry model for $\phi = 0$.

	Sequential Entry	Simultaneous Entry
$\gamma \equiv \frac{\theta_2}{\theta_1}$	0.1951	0.1904
θ_1	$0.1226\frac{M\kappa}{\alpha} < \theta_{MP}$	$0.1265\frac{M\kappa}{\alpha} > \theta_{MP}$
$(\frac{N_1}{M}, \frac{N_2}{M})$	$(52.6\%, 26.3\%)$	$(52.4\%, 26.2\%)$
$(\frac{s_1}{\theta_1}, \frac{s_2}{\theta_2})$	$(0.4231\kappa, 0.2116\kappa)$	$(0.4254\kappa, 0.2127\kappa)$
(Π_1, Π_2)	$(12.235\frac{M^2\kappa^2}{1000\alpha}, 0.758\frac{M^2\kappa^2}{1000\alpha})$	$(12.22\frac{M^2\kappa^2}{1000\alpha}, 0.764\frac{M^2\kappa^2}{1000\alpha})$

Table 3.1: Difference between simultaneous entry and sequential entry when $\phi = 0$

Table 3.1 clearly establishes the properties of Lemma 3.13. This can be verified for all ϕ in (ϕ_{il}, ϕ_{ir}).

Next we evaluate the cut-off points ϕ_0 and ϕ_1 for the simultaneous entry model by identifying the lowest ϕ at which the profit of Firm 2 is positive, and the lowest ϕ at which Firm 2 has profitable deviation to $\theta_2 = 0$. Table 3.2 compares the cut-off points of sequential and simultaneous entry model, establishing the properties of Lemma 3.14.

	Simultaneous Entry	Sequential Entry
Interior solution interval (ϕ_{il}, ϕ_{ir})	$-(\frac{M\kappa^2}{185\alpha}, \frac{M\kappa^2}{240\alpha})$	$-(\frac{M\kappa^2}{187\alpha}, \frac{M\kappa^2}{255\alpha})$
ϕ_0	$-\frac{M\kappa^2}{243\alpha}$	$-\frac{M\kappa^2}{242\alpha}$
ϕ_1	$\frac{M\kappa^2}{297\alpha}$	$\frac{M\kappa^2}{302\alpha}$

Table 3.2: Cut-off points comparison between simultaneous entry and sequential entry

Note: There exist ϵ, however small, such that there does not exist any equilibrium when $\phi = [\phi_1, \phi_1 + \epsilon)$.

$\phi = \phi_1$ is the lowest ϕ for which Firm 2 has a profitable deviation to $\theta_2 = 0$. If it deviates at $\phi = \phi_1$, then Firm 1 will set the monopolistic quality (see Lemma 3.9), which is lower than the one before the deviation (see Lemma 3.13). This reduces the profit of Firm 2, prompting it to switch back to positive quality. Thus, in the small right-side neighborhood of ϕ_1, the two firms cycle between Type B and Type C configurations without reaching any equilibrium.

Lemma 3.15: There does not exist any pure strategy equilibrium if $\phi > \phi_2$.

Proof: We have already discussed that if two firms are infinitesimally close in quality, the firm with the lower quality will lose all demand and earn negative profit. Suppose $\phi > \phi_2$. If Firm 1 sets $\theta_1 = \theta_{MP}$, Firm 2 will respond with $\theta_2 = \theta_{MP+}$ (by definition of ϕ_2, see Definition 3.7), which will result in negative profits for Firm 1. Subsequently, each firm will attempt to outdo the other by marginally increasing quality until one of the firms, say Firm 1, sets $\theta_1 > \theta_c$. At this point, Firm 2 will deviate to $\theta_2 = 0$ (by definition of θ_c, see Definition 3.5). However, when Firm 2 sets $\theta_2 = 0$, the best response for Firm 1 is to revert to θ_{MP}. Thus, this cycle will continue without reaching an equilibrium.

Furthermore, according to Assumption 3.1 and Lemma 3.4, a Type B equilibrium does not exist when $\phi > \phi_2$. Additionally, both firms cannot have the same quality in equilibrium because this would imply that both firms have zero subscription price and merely share the advertisement revenue. Therefore, any firm can deviate by slightly increasing its quality to capture full advertisement revenue. Since $\phi > 0$, a Type A equilibrium does not exist either.

Therefore, there does not exist any pure strategy equilibrium. □

3.A.5 Solution for Three Firms Model

Steps to prove Proposition 3.5

Suppose there exist an equilibrium where all three firms set positive prices. Since the higher quality firm has higher profit (see lemma 3.2), early entrants will choose higher quality, which means $0 < \theta_3 < \theta_2 < \theta_1$. The demand function based on indifferent consumers:

$$\begin{aligned}
N_1(s_1, s_2, s_3, \theta_1, \theta_2, \theta_3) &= M\left(1 - \frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)}\right) \\
N_2(s_1, s_2, s_3, \theta_1, \theta_2, \theta_3) &= M\left(\frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)} - \frac{s_2 - s_3}{\kappa(\theta_2 - \theta_3)}\right) \\
N_3(s_1, s_2, s_3, \theta_1, \theta_2, \theta_3) &= M\left(\frac{s_2 - s_3}{\kappa(\theta_2 - \theta_3)} - \frac{s_3}{\kappa\theta_3}\right)
\end{aligned}$$

The price stage solution for subscription prices when firms simultaneously choose prices:

$$s_1 = \frac{\kappa(\theta_1 - \theta_2)(4\theta_1\theta_2 - \theta_3(3\theta_2 + \theta_1))}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))} - \frac{\theta_2(7\theta_1 - \theta_2) - \theta_3(5\theta_2 + \theta_1)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))}\phi \quad (3.93)$$

$$s_2 = \frac{\kappa\theta_2(\theta_1 - \theta_2)(\theta_2 - \theta_3)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))} - \frac{3\theta_2(\theta_1 - \theta_3)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))}\phi \quad (3.94)$$

$$s_3 = \frac{\kappa\theta_3(\theta_1 - \theta_2)(\theta_2 - \theta_3)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))} - \frac{\theta_2(4\theta_1 - \theta_2) + 2\theta_3(\theta_1 - \theta_2) - 3\theta_3^2}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))}\phi \quad (3.95)$$

Using above equations, the corresponding profit functions on the product choice stage are given by:

$$\Pi_1 = \frac{M(\theta_1-\theta_2)}{4\kappa}\Big(\frac{\kappa(4\theta_1\theta_2-\theta_3(\theta_1+3\theta_2))+c(\theta_2-\theta_3)}{\theta_2(4\theta_1-\theta_2)-\theta_3(2\theta_2+\theta_1)}\Big)^2 + M\beta(\delta-\kappa)-\alpha\theta_1^2 \quad (3.96)$$

$$\Pi_2 = \frac{M(\theta_1-\theta_2)(\theta_2-\theta_3)(\theta_3-\theta_1)}{\kappa}\Big(\frac{\kappa\theta_2+c}{\theta_2(4\theta_1-\theta_2)-\theta_3(2\theta_2+\theta_1)}\Big)^2-\alpha\theta_2^2 \quad (3.97)$$

$$\Pi_3 = \frac{M\theta_2(\theta_2-\theta_3)}{4\kappa\theta_3}\Big(\frac{\kappa\theta_3(\theta_1-\theta_2)+c(4\theta_1-\theta_2-3\theta_3)}{\theta_2(4\theta_1-\theta_2)-\theta_3(2\theta_2+\theta_1)}\Big)^2-\alpha\theta_3^2 \quad (3.98)$$

To solve this problem, we begin by calculating the reaction function of Firm 3. Firm 2 will then choose its optimal quality based on the reaction function of Firm 3, given θ_1, thereby forming Firm 2's reaction function. Subsequently, Firm 1 will choose its optimal quality based on the reaction function of Firm 2. Given the complexity of the resulting set of equations, we utilize Wolfram Mathematica for the solution.[47] Additionally, we simplify the problem by solving it for $\phi = 0$.

At $\phi = 0$, the above equations yield a unique solution where all three firms make positive profits. Similar to the duopoly case described in Lemma 3.3, the ratio $\frac{\theta_3}{\theta_2} = 0.198$ and $\frac{\theta_2}{\theta_1} = 0.196$ remains constant for all μ, σ, and α. These ratios are greater than the corresponding duopoly ratio, which is $\gamma = \frac{\theta_2}{\theta_1} = 0.195$.

Also, no firm will deviate to a corner solution, as setting a zero price will result in zero profit. Similarly, none of the two firms will choose the same quality, as in that case, the price will be zero in the price stage, and the firm will make zero profit. Hence, the above interior solution is a unique pure strategy equilibrium. Since the profit function is a continuous function of ϕ (see Lemma 3.8) and firms make positive profit at $\phi = 0$, there exists a neighborhood, however small, where the above equilibrium configuration will persist. This proves Proposition 3.5.

[47]Equations and codes can be provided on request.

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