EFFECT OF HOUSEHOLD COMPOSITION ON CONSUMPTION PATTERN : A NOTE

Summary

In this Note it has been shown that analytically it is possible to separate the specific and overall effects of a change in family size on family expenditures. Some empirical results have also been reported to support the theoretical conclusion.

1. *Introduction*

Forsyth [1960] examined the relationship between Family size and consumer expenditure and stressed the analytical impossibility of separating the specific and overall effects of a change in family size on family expenditures. His conclusion essentially emerges from the emphasis he puts on the restrictions that follow when the budget constraint is remembered. Precisely, he discovers that the relevant specific and overall coefficients cannot be identified when the engel functions fulfill the adding-up criterion. i.e., no amount of informations about the engel functions would suffice to determine all the coefficients uniquely.

In the present note it has been shown that Forsyth's conclusion is not valid in general and both sets of scale coefficients can, in principle, be estimated without making unwarranted assumptions. This note also presents an empirical verification of our arguments. The empirical results reported here are based on Forsyth's own estimates of household compositionwise constant elasticity engel curves fitted to the household survey data from the U. K. Ministry of Labour (1953-54). Section 2 presents an outline of Forsyth's problem and shows how this problem can be solved. Section 3 illustrates the method of solution with numerical examples.

2. *Forsyth's problem and solution*

The general form of engel curve considered by Forsyth is

$$\frac{y_{it}}{(c+z_t)^{d_i}} = f_i\left(\frac{x_t}{(c+z_t)^{d_o}}\right) \quad \ldots \quad \ldots \quad (1)$$

where y_{it} is the mean weekly expenditure of families with a couple and z_t children on commodity i,

x_t is the mean weekly total expenditure of these families,

$(c + z_t)$ measures the physical size of the family in units of child, where a couple is assumed to be equivalent to c children, and finally d_i and d_o measure the specific and overall effects of change in physical size of the family as a result of adding successive children to the family.

In fact, here the problem considered in (1) is essentially one of estimation of economies of scale in family expenditure pattern, when allowances have already been made for variations in the number of children in the families concerned. An increase in z_t raises the physical size of the family and results in both specific and overall economies of scale. $(c+z_t)^{d_i}$ and $(c+z_t)^{d_o}$, therefore, measure the effective family size for commodity i and total expenditure respectively. Once d_i and d_o are estimated a set of family equivalences could be estimated as

$$\left(\frac{c+z_t}{c}\right)^{d_i} \text{ and } \left(\frac{c+z_t}{c}\right)^{d_o}$$

for families with different z_t. Forsyth's conclusion rules out such a possibility since it was shown that d_i and d_o could not be estimated separately.

Given (1) it can be shown that

$$\gamma_i = d_i - \lambda_i d_o \quad \ldots \quad (2)$$

where $\gamma_i = \dfrac{\delta \; log \; y_{it}}{\delta \; log \; (c+z_t)}$ and $\lambda_i = \dfrac{\delta \; log \; y_{it}}{\delta \; log \; x_t}$.

The budget constraint $\Sigma_i \, y_{it} = x_t$ implies

$\Sigma_i \dfrac{\delta \, y_{it}}{\delta \, x_t} = 1$ and therefore

$$\Sigma_i \, w_i \, \lambda_i = 1 \quad \ldots \quad (3)$$

The budget constraint also implies

$\Sigma_i \dfrac{\delta \, y_{it}}{\delta \, (c+z_t)} = 0$. This, together with (2) then implies

$$\Sigma \, w_i \, d_i = d_o \quad \ldots \quad (4)$$

where w_i's$\left[= \dfrac{y_i}{x}\right]$ are the engel ratios.

In Forsyth's constant elasticity formulation of (1)

$$\frac{y_{it}}{(c+z_t)^{d_i}} = a_i \left(\frac{x_t}{(c+z_t)^{d_o}} \right)^{b_i}$$

or, alternatively

$$y_{it} = a_i \, (c+z_t)^{\gamma_i} x_t^{\, b_i} \qquad \ldots \quad (5)$$

γ_i and b_i (i.e., λ_i in this case) are estimable from the regression of log y_{it} on log $(c+z_t)$ and log x_t. But, in view of (3) and (4) it is argued that since even for (5) the adding-up property would be approximately satisfied by the estimated values of y_{it}, restrictions (3) and (4) would also be approximately satisfied and hence the system of equations

$$\left. \begin{aligned} \gamma_i &= d_i - b_i d_o \\ \Sigma\, w_i d_i &= d_o \end{aligned} \right\} \qquad \ldots \quad (6)$$

which involves $(n-1)$ independent linear equations in n unknown d_i 's and hence could not be solved uniquely.

In principle, however, Forsyth's problem can be solved. In this formulation itself d_i 's and d_o are assumed to be independent of the common income level x_t. But even if the d_i 's do not vary with x_t, restriction (4) might imply that d_o is dependent upon the income level through the engel ratios w_i (that are constant only in the extremely trivial special case where all the engel curves are lines through the origin). Thus to ensure the independence of d_o of x_t we have the additional restriction

$$O = \frac{\delta d_o}{\delta x_t} = \frac{\delta \Sigma w_i d_i}{\delta x_t} \longrightarrow \Sigma w_i b_i d_i = d_o \qquad \ldots \quad (7)$$

Incorporating (7) in (6) we have the following determinate system of equations (in vector-matrix notation)

$$\begin{bmatrix} I & -b \\ (w\hat{b} - w) & o \end{bmatrix} \begin{bmatrix} d \\ d_o \end{bmatrix} = \begin{bmatrix} \gamma \\ o \end{bmatrix} \qquad \ldots \quad (8)$$

where I is an $n \times n$ identity matrix (n being the number of commodities),
b is $n \times 1$ column vector of engel elasticities,
w is $1 \times n$ row vector of engel ratios,
$\hat{b}$ is $n \times n$ diagonal matrix with b_i as the ith diagonal element,

d is $n \times 1$ column vector of unknown specific coefficients (d_i), and γ is the $n \times 1$ column vector of γ_i 's.

(8) describes a system of $(n+1)$ equations involving n unknown d_i's and d_o. Since the inverse of the partitioned matrix on the *l. h. s.* of (8) usually exists, the solution for the system is

$$\begin{bmatrix} d \\ d_o \end{bmatrix} = \begin{bmatrix} I - bD^{-1}(w'\hat{b} - w') & b\,D^{-1} \\ -\,D^{-1}(w'\hat{b} - w') & D^{-1} \end{bmatrix} \begin{bmatrix} \gamma \\ o \end{bmatrix} \quad \ldots \quad (9)$$

where $D = (w'\hat{b} - w')b$.

Thus if D is non-singular, i.e.,

$$\Sigma w_i b_i^2 \neq \Sigma w_i b_i$$

we have unique solutions of d_i's and d_o from (8).

3. *A numerical illustration*

To illustrate the argument presented in Section 2 we have attempted to estimate d_i 's and d_o from actual budget data. Precisely, the value of w_i, b_i and γ_i in (8) for different items of expenditure have been adopted from Forsyth's results, and on the basis of these values we have estimated d_i and d_o and constructed family equivalence scales for different items of expenditure as also for total expenditure. Finally a comparison of these equivalence scales is made with those obtained by setting $d_o = 1$.

Table 1 gives the average expenditures on twelve commodities by family type at the geometric mean level of total expenditure over all sample families of different types estimated from the constant elasticity engel curves.[1] The average w_i 's have been estimated as follows: (*i*) for each item the average level of expenditure over all family types has been calculated by weighting family type-wise average expenditures by corresponding number of families, (ii) the average engel ratios w_i have been taken as the ratio of average item expenditure and the corresponding total expenditure (obtained as the sum of average item wise expenditures).

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[1] Forsyth has considered four family-types, viz., those consisting of (i) a couple (c), (ii) a couple and a child (c + 1), (iii) a couple and two children (c + 2) and (iv) a couple and three children (c + 3).

Table—1

Average expenditure on twelve commodities by family type estimated from the constant elasticity engel curves at the geometric mean level total expenditure over all families.

family type	housing	fuel, light and power	total food*	alcoholic drink	tobacco	clothing and footwear	household durables**
	commodity expenditure (pence per week)						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
c	210.7	135.7	777.7	95.9	167.7	248.9	269.1
c + 1	190.4	138.3	891.0	73.0	182.0	258.8	211.3
c + 2	189.6	150.0	977.0	61.4	164.6	266.6	212.1
c + 3	188.2	156.4	1046.6	66.2	182.9	246.6	166.4
all	199.2	141.2	871.9	80.2	171.9	255.0	233.6

Table 1 (*contd.*)

family type	other goods	trans-port	entertain-ment	essential*** services	other services	total expen-diture +	no. of families
	commodity expenditure (pence per week)						
(1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
c	176.9	166.0	42.0	103.8	87.5	2481.9	3011
c + 1	190.8	166.7	44.2	81.5	81.2	2509.2	1665
c + 2	189.5	124.8	34.2	76.9	67.4	2514.1	1447
c + 3	185.0	114.9	32.7	66.8	70.1	2522.8	571
all	183.8	152.9	40.1	89.3	80.1	2499.2	6694

* includes meals bought away from home.

** includes furniture and furnishings, radio and T.V., gas and electric appliances, china, glass and hardware.

*** includes postage, telephone, telegraph, hairdressing, laundry, cleaning and domestic help etc.

\+ obtained by summing columns (2) — (13).

Forsyth's estimates of family-type-wise constant engel elasticity b_i for different items have been reproduced in Table 2. The average elasticities over all family-types for different items have been obtained as weighted averages of the family-type-wise elasticities where aggregate item expenditure by different types of families have been taken as weights. Table 2 also gives Forsyth's estimates of common constant elasticity for different items for all types of families when the elasticities were forced to be equal for different types of families. These two sets of elasticities compare favourably except for fuel and light for which Forsyth's estimate of common elasticity seems absurd, since it falls outside the range of family type-wise elasticities.

Table—2

Estimated engel elasticities from constant elasticity engel curves—by commodity and family type

commodity	family type					
	c	c + 1	c + 2	c + 3	all *	all ** (Forsyth)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
housing	0.76	0.91	0.88	0.54	0.81	0.79
fuel, light and power	0.35	0.45	0.52	0.40	0.42	0.33
total food	0.53	0.53	0.47	0.58	0.53	0.52
drink	1.23	1.48	1.09	2.06	1.33	1.30
tobacco	0.86	0.54	0.38	0.48	0.65	0.71
clothing and footwear	1.30	1.43	1.30	1.79	1.38	1.35
household durables	1.61	1.34	1.70	1.43	1.56	1.57
other goods	0.99	0.99	0.91	0.98	0.97	0.98
transport	1.91	1.75	1.71	1.43	1.80	1.83
entertainment	1.65	1.22	1.11	1.24	1.41	1.47
essential services	1.47	1.95	1.92	2.15	1.71	1.66
other services	2.12	2.20	2.68	2.68	2.28	2.24

* Weighted average of family type-wise elasticities with share of aggregate commodity expenditure for each family type as weights.

** The common slope estimates from the constant elasticity engel curve.

Finally, the estimates of γ_i have been obtained from Forsyth's estimates of family equivalences for different items based on total effects γ_i. Precisely Forsyth reports the values of $\left(1+\frac{z_i}{2}\right)^{\gamma_i}$ for different values of z_i in Table 4 of his paper. We have computed the γ_i's from these figures. In Table 3 we present the values of γ_i along with those of w_i and b_i for different items.[8]

Table—3

Estimated w_i, b_i, γ_i and d_i by commodity

commodity	w_i	b_i	γ_i	d_i
(1)	(2)	(3)	(4)	(5)
housing	0.080	0.810	−0.152	0.255
fuel, light and power	0.057	0.420	0.130	0.324
total food	0.349	0.530	0.324	0.591
drink	0.032	1.330	−0.555	0.114
tobacco	0.069	0.650	0.056	0.383
clothing and footwear	0.102	1.380	0.056	0.750
household durables	0.094	1.560	−0.456	0.328
other goods	0.074	0.970	0.096	0.584
transport	0.060	1.800	−0.342	0.563
entertainment	0.015	1.410	−0.227	0.482
essential services	0.036	1.710	−0.478	0.381
other services	0.032	2.280	−0.286	0.860
total expenditure	—	—	—	0.503

[8] It may be mentioned that for the estimated w_i, b_i and γ_i for different items reported in Table 3 $\Sigma w_i b_i = 0.9840$ and $\Sigma w_i \gamma_i = 0.0143$, i.e., the underlying restrictions $\Sigma\, w_i b_i = 1$ and $\Sigma\, w_i \gamma_i = 0$ are approximately satisfied by these estimates.

Table—4

Estimated specific and overall effective family sizes

commodity	specific effective size of family type							
	c		c+1		c+2		c+3	
	d_o unrestricted	$d_o=1$	d_o unrestricted	$d_o=1$	d_o unrestricted	$d_o=1$	d_o unrestricted	$d_o=1$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
housing	1.000	1.000	1.108	1.290	1.193	1.550	1.262	1.790
fuel, light and power	1.000	1.000	1.140	1.210	1.251	1.380	1.345	1.530
total food	1.000	1.000	1.271	1.410	1.506	1.800	1.718	2.170
alcholic drink	1.000	1.000	1.047	1.350	1.081	1.680	1.109	1.980
tobacco	1.000	1.000	1.168	1.370	1.303	1.700	1.420	2.020
clothing and. footwear	1.000	1.000	1.355	1.770	1.683	2.650	1.983	3.630
household durables	1.000	1.000	1.143	1.570	1.256	2.160	1.351	2.760
other goods	1.000	1.000	1.266	1.550	1.500	2.100	1.706	2.670
transport	1.000	1.000	1.256	1.830	1.476	2.810	1.675	3.920
entertainment	1.000	1.000	1.215	1.660	1.396	2.370	1.554	3.130
essential services	1.000	1.000	1.167	1.620	1.302	2.270	1.418	2.960
other services	1.000	1.000	1.417	2.200	1.815	3.860	2.198	5.970
total expenditure	1.000	1.000	1.226	1.500	1.417	2.000	1.585	2.500

The estimates of d_i's for different items and that for d_o from (9) appear to be quite sensible (vide Table 3). As for example, large specific economies of scale are observed for items such as Alcoholic Drink, Housing, Fuel, Light and Power, Durables, Essential Services and Tobacco. On the other, hand, for items such as other services and clothing the magnitude of specific economies are relatively small. The figures for food, transport, entertainment and total expenditure hold intermediate positions.

Table 4 presents the family equivalance scales for different items based on the specific economies d_i and also on d_e estimated from (9). The estimates of $\left(1+\frac{z_i}{2}\right)^{d_i}$ for different types of families (with $z_i = 0$, 1, 2 and 3) and for different items seem to provide sensible scales of item-wise family equivalences. As the results suggest, these scales are clearly more reasonable than those obtained by setting d_e equal to 1 (which yields rather unrealistic estimates of equivalences for most of the luxury items).

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Indian Statistical Institute
Calcutta.

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